Universidad Nacional del Altiplano
Facultad de Ingeniería Estadística e Informática
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Trabajo Encargado - N° 004

Ejercicios Gauss Jordan

Código

Repositorio github

Ejercicio 1: Modelo de regresión lineal

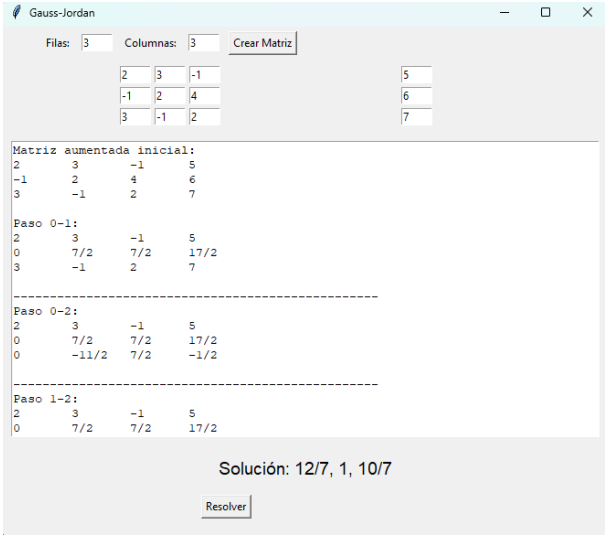
Resolver el siguiente sistema:

$$\begin{aligned} 2w_1 + 3w_2 - w_3 &= 5, \\ -w_1 + 2w_2 + 4w_3 &= 6, \\ 3w_1 - w_2 + 2w_3 &= 7. \end{aligned}$$

Matriz Aumentada:

$$\left[\begin{array}{ccc|c} 2 & 3 & -1 & 5 \\ -1 & 2 & 4 & 6 \\ 3 & -1 & 2 & 7 \end{array}\right].$$

Resolucion Codigo:



Paso a paso:

$$
\begin{aligned}
\text{Paso 0-1:} &\quad \left[\begin{array}{ccc|c} 2 & 3 & -1 & 5 \\ 0 & \frac{7}{2} & \frac{7}{2} & \frac{17}{2} \\ 3 & -1 & 2 & 7 \end{array}\right] \\
\text{Paso 0-2:} &\quad \left[\begin{array}{ccc|c} 2 & 3 & -1 & 5 \\ 0 & \frac{7}{2} & \frac{7}{2} & \frac{17}{2} \\ 0 & -\frac{11}{2} & \frac{7}{2} & -\frac{1}{2} \end{array}\right] \\
\text{Paso 1-2:} &\quad \left[\begin{array}{ccc|c} 2 & 3 & -1 & 5 \\ 0 & \frac{7}{2} & \frac{7}{2} & \frac{17}{2} \\ 0 & 0 & 9 & \frac{90}{7} \end{array}\right] \\
\text{Paso 2-1:} &\quad \left[\begin{array}{ccc|c} 2 & 3 & -1 & 5 \\ 0 & \frac{7}{2} & 0 & \frac{7}{2} \\ 0 & 0 & 9 & \frac{90}{7} \end{array}\right] \\
\text{Paso 2-0:} &\quad \left[\begin{array}{ccc|c} 2 & 3 & 0 & \frac{45}{7} \\ 0 & \frac{7}{2} & 0 & \frac{7}{2} \\ 0 & 0 & 9 & \frac{90}{7} \end{array}\right] \\
\text{Paso 1-0:} &\quad \left[\begin{array}{ccc|c} 2 & 0 & 0 & \frac{24}{7} \\ 0 & \frac{7}{2} & 0 & \frac{7}{2} \\ 0 & 0 & 9 & \frac{90}{7} \end{array}\right] \\
\text{Normalizar fila 0:} &\quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{12}{7} \\ 0 & \frac{7}{2} & 0 & \frac{7}{2} \\ 0 & 0 & 9 & \frac{90}{7} \end{array}\right] \\
\text{Normalizar fila 1:} &\quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{12}{7} \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 9 & \frac{90}{7} \end{array}\right] \\
\text{Normalizar fila 2:} &\quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{12}{7} \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \frac{10}{7} \end{array}\right].
\end{aligned}
$$

Solución: $w_1 = \frac{12}{7}, w_2 = 1, w_3 = \frac{10}{7}$

Ejercicio 2: Calibración de hiperparámetros

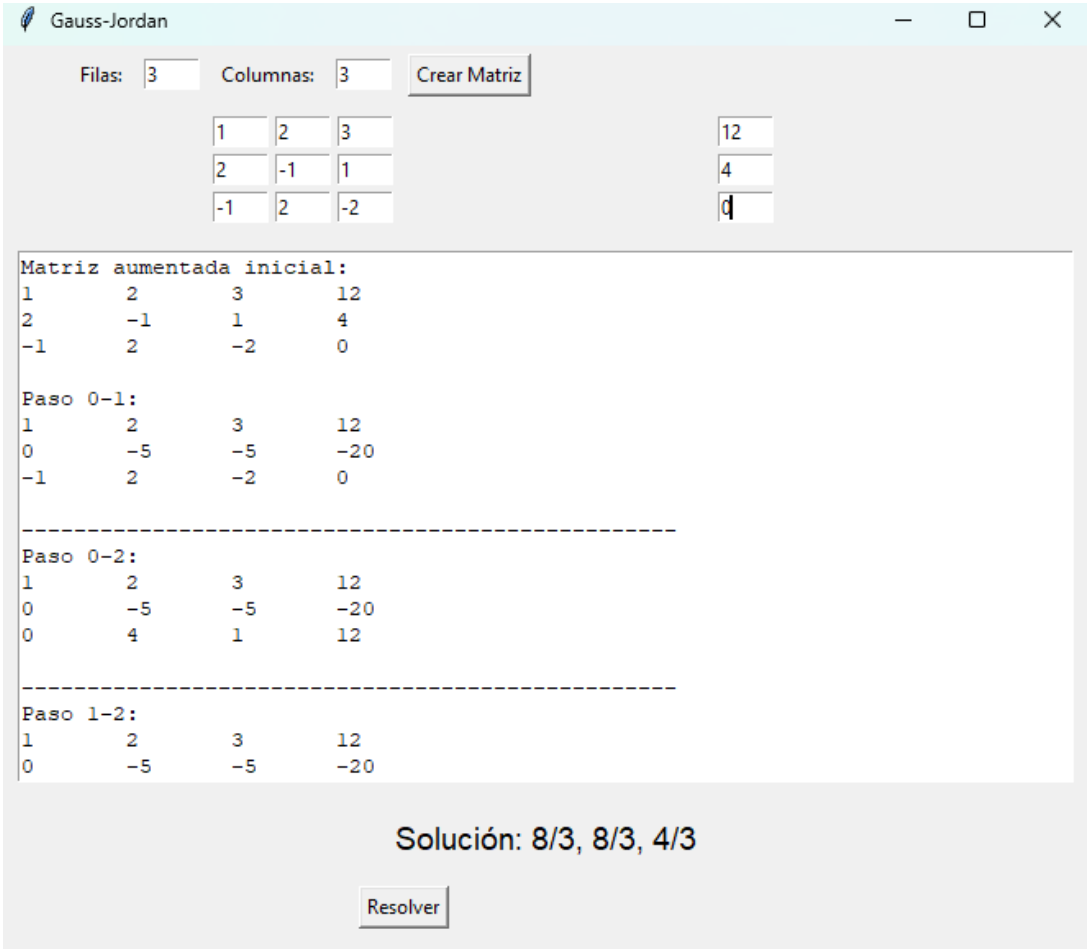
Resolver el sistema:

$$\begin{aligned} x + 2y + 3z &= 12, \\ 2x - y + z &= 4, \\ -x + 2y - 2z &= 0. \end{aligned}$$

Matriz Aumentada:

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 12 \\ 2 & -1 & 1 & 4 \\ -1 & 2 & -2 & 0 \end{array}\right].$$

Resolucion Codigo:



Paso a paso:

$$\text{Paso 0-1:} \quad \left[\begin{array}{ccc|c} 1 & 2 & 3 & 12 \\ 0 & -5 & -5 & -20 \\ -1 & 2 & -2 & 0 \end{array}\right]$$

$$\text{Paso 0-2:} \quad \left[\begin{array}{ccc|c} 1 & 2 & 3 & 12 \\ 0 & -5 & -5 & -20 \\ 0 & 4 & -3 & 12 \end{array}\right]$$

$$\text{Paso 1-2:} \quad \left[\begin{array}{ccc|c} 1 & 2 & 3 & 12 \\ 0 & -5 & -5 & -20 \\ 0 & 0 & -3 & -4 \end{array}\right]$$

$$\text{Paso 2-1:} \quad \left[\begin{array}{ccc|c} 1 & 2 & 3 & 12 \\ 0 & -5 & 0 & -\frac{40}{3} \\ 0 & 0 & -3 & -4 \end{array}\right]$$

$$\text{Paso 2-0:} \quad \left[\begin{array}{ccc|c} 1 & 2 & 0 & 8 \\ 0 & -5 & 0 & -\frac{40}{3} \\ 0 & 0 & -3 & -4 \end{array}\right]$$

$$\text{Paso 1-0:} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{8}{3} \\ 0 & -5 & 0 & -\frac{40}{3} \\ 0 & 0 & -3 & -4 \end{array}\right]$$

$$\text{Normalizar fila 0:} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{8}{3} \\ 0 & -5 & 0 & -\frac{40}{3} \\ 0 & 0 & -3 & -4 \end{array}\right]$$

$$\text{Normalizar fila 1:} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{8}{3} \\ 0 & 1 & 0 & 8 \\ 0 & 0 & -3 & -4 \end{array}\right]$$

$$\text{Normalizar fila 2:} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{8}{3} \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & \frac{4}{3} \end{array}\right].$$

Solución: $x = \frac{8}{3}, y = 8, z = \frac{4}{3}$

Ejercicio 3: Asignación óptima de recursos

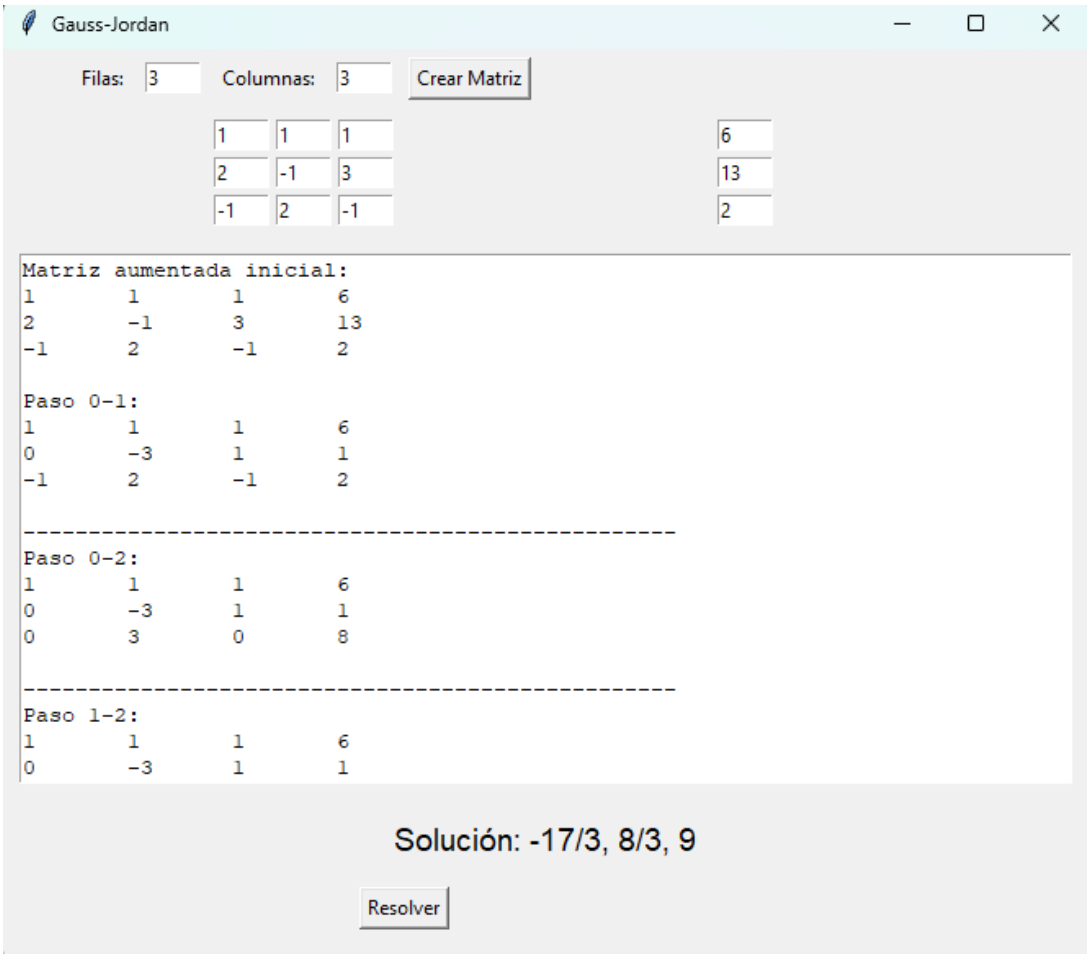
Resolver el sistema:

$$\begin{aligned} a + b + c &= 6, \\ 2a - b + 3c &= 13, \\ -a + 2b - c &= 2. \end{aligned}$$

Matriz Aumentada:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & -1 & 3 & 13 \\ -1 & 2 & -1 & 2 \end{array}\right].$$

Resolucion Codigo:



Paso a paso:

$$\text{Paso 0-1:} \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & 1 \\ -1 & 2 & -1 & 2 \end{array}\right]$$

$$\text{Paso 0-2:} \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & 1 \\ 0 & 3 & 0 & 8 \end{array}\right]$$

$$\text{Paso 1-2:} \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & 1 \\ 0 & 0 & 1 & 9 \end{array}\right]$$

$$\text{Paso 2-1:} \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & 0 & -8 \\ 0 & 0 & 1 & 9 \end{array}\right]$$

$$\text{Paso 2-0:} \quad \left[\begin{array}{ccc|c} 1 & 1 & 0 & -3 \\ 0 & -3 & 0 & -8 \\ 0 & 0 & 1 & 9 \end{array}\right]$$

$$\text{Paso 1-0:} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{17}{3} \\ 0 & -3 & 0 & -8 \\ 0 & 0 & 1 & 9 \end{array}\right]$$

$$\text{Normalizar fila 0:} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{17}{3} \\ 0 & -3 & 0 & -8 \\ 0 & 0 & 1 & 9 \end{array}\right]$$

$$\text{Normalizar fila 1:} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{17}{3} \\ 0 & 1 & 0 & \frac{8}{3} \\ 0 & 0 & 1 & 9 \end{array}\right]$$

$$\text{Normalizar fila 2:} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{17}{3} \\ 0 & 1 & 0 & \frac{8}{3} \\ 0 & 0 & 1 & 9 \end{array}\right].$$

Solución: $a = -\frac{17}{3}, b = \frac{8}{3}, c = 9$

Ejercicio 4: Optimización de parámetros de un Bosque Aleatorio

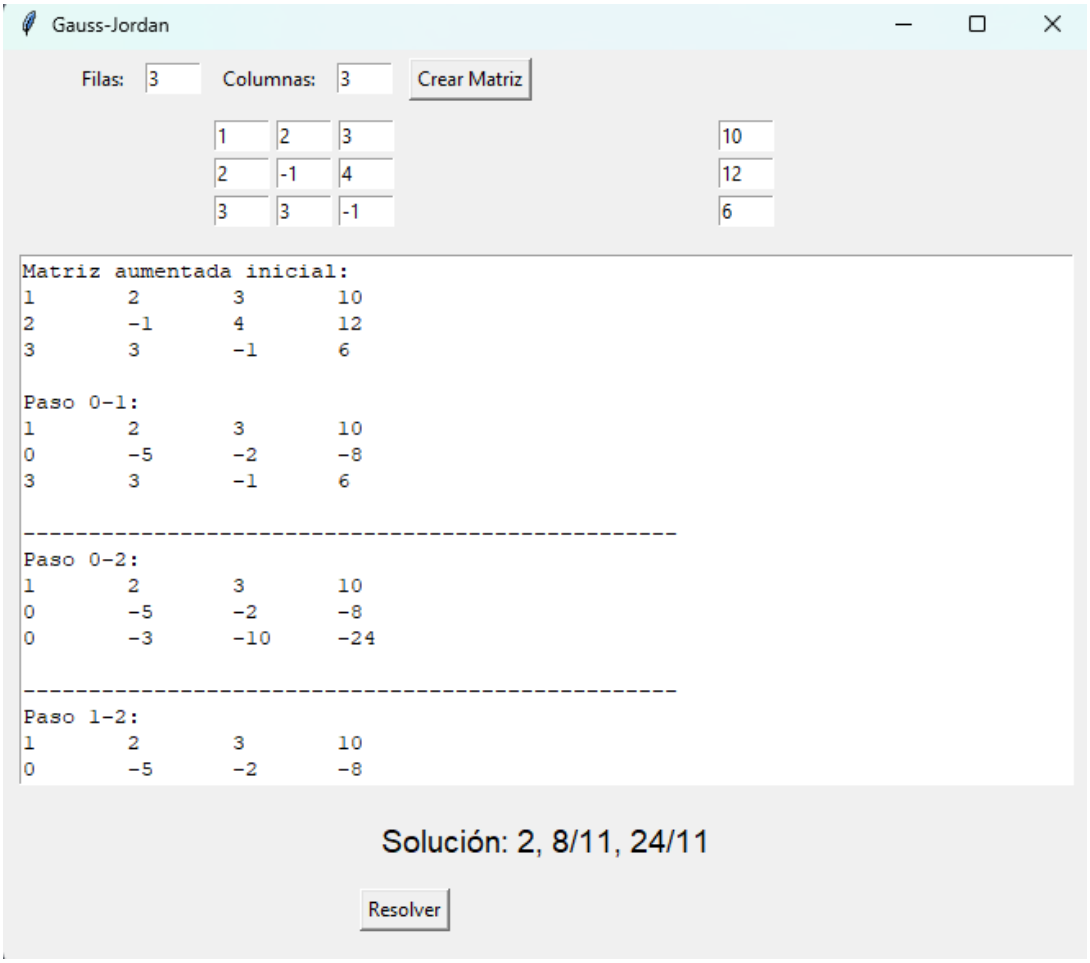
Resolver el sistema:

$$\begin{aligned} p + 2q + 3r &= 10, \\ 2p - q + 4r &= 12, \\ 3p + 3q - r &= 6. \end{aligned}$$

Matriz Aumentada:

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 2 & -1 & 4 & 12 \\ 3 & 3 & -1 & 6 \end{array}\right].$$

Resolucion Codigo:



Paso a paso:

$$\text{Paso 0-1:} \quad \left[\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 0 & -5 & -2 & -8 \\ 3 & 3 & -1 & 6 \end{array}\right]$$

$$\text{Paso 0-2:} \quad \left[\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 0 & -5 & -2 & -8 \\ 0 & -3 & -10 & -24 \end{array}\right]$$

$$\text{Paso 1-2:} \quad \left[\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 0 & -5 & -2 & -8 \\ 0 & 0 & -\frac{44}{5} & -\frac{96}{5} \end{array}\right]$$

$$\text{Paso 2-1:} \quad \left[\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 0 & -5 & 0 & -\frac{40}{11} \\ 0 & 0 & -\frac{44}{5} & -\frac{96}{5} \end{array}\right]$$

$$\text{Paso 2-0:} \quad \left[\begin{array}{ccc|c} 1 & 2 & 0 & \frac{38}{11} \\ 0 & -5 & 0 & -\frac{40}{11} \\ 0 & 0 & -\frac{44}{5} & -\frac{96}{5} \end{array}\right]$$

$$\text{Paso 1-0:} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & -5 & 0 & -\frac{40}{11} \\ 0 & 0 & -\frac{44}{5} & -\frac{96}{5} \end{array}\right]$$

$$\text{Normalizar fila 0:} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & -5 & 0 & -\frac{40}{11} \\ 0 & 0 & -\frac{44}{5} & -\frac{96}{5} \end{array}\right]$$

$$\text{Normalizar fila 1:} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & \frac{8}{11} \\ 0 & 0 & -\frac{44}{5} & -\frac{96}{5} \end{array}\right]$$

$$\text{Normalizar fila 2:} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & \frac{8}{11} \\ 0 & 0 & 1 & \frac{24}{11} \end{array}\right].$$

Solución: $p = 2, q = \frac{8}{11}, r = \frac{24}{11}$

Ejercicio 5: Estimación de demanda de inventario

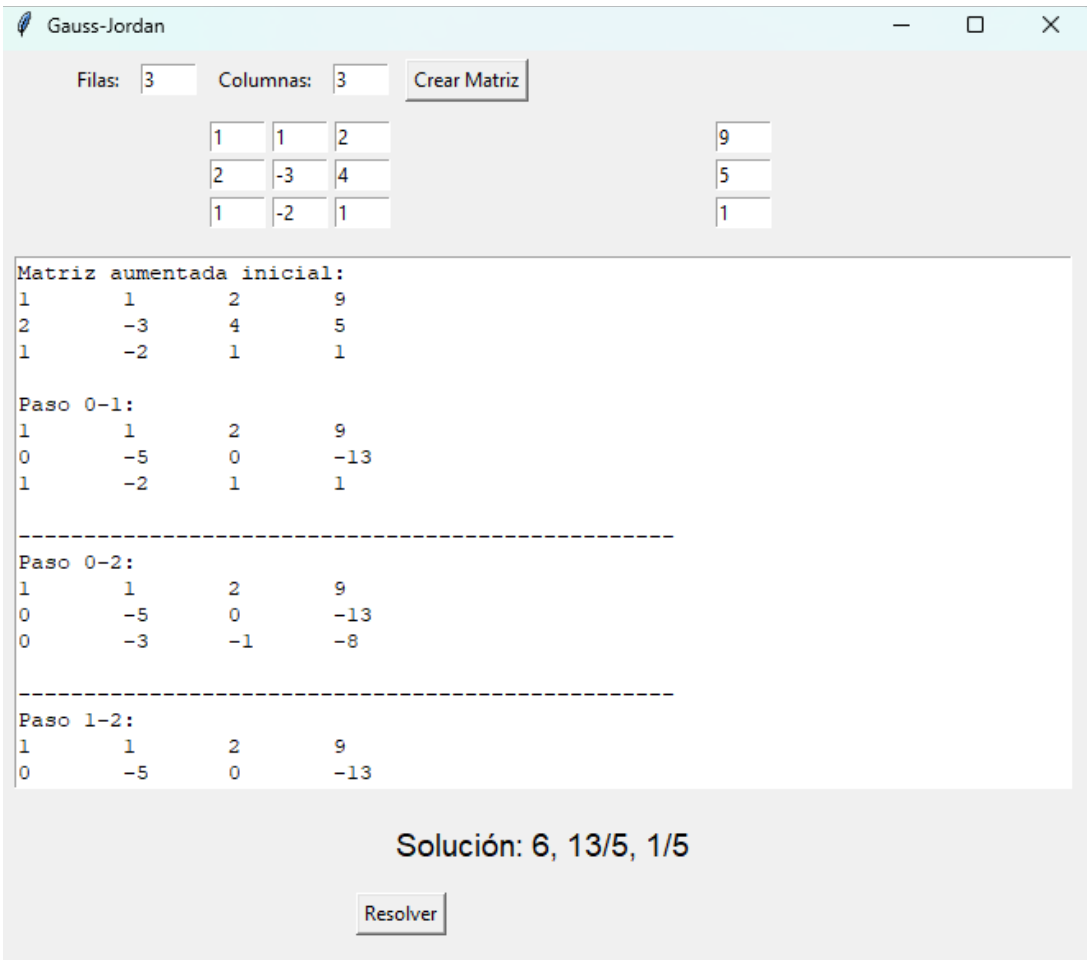
Resolver el sistema:

$$\begin{aligned} u + v + 2w &= 9, \\ 2u - 3v + 4w &= 5, \\ u - 2v + w &= 1. \end{aligned}$$

Matriz Aumentada:

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 2 & -3 & 4 & 5 \\ 1 & -2 & 1 & 1 \end{array}\right].$$

Resolucion Codigo:



Paso a paso:

$$\text{Paso 0-1:} \quad \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & -5 & 0 & -13 \\ 1 & -2 & 1 & 1 \end{array}\right]$$

$$\text{Paso 0-2:} \quad \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & -5 & 0 & -13 \\ 0 & -3 & -1 & -8 \end{array}\right]$$

$$\text{Paso 1-2:} \quad \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & -5 & 0 & -13 \\ 0 & 0 & -1 & -\frac{1}{5} \end{array}\right]$$

$$\text{Paso 2-1:} \quad \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & -5 & 0 & -\frac{13}{5} \\ 0 & 0 & -1 & -\frac{1}{5} \end{array}\right]$$

$$\text{Paso 2-0:} \quad \left[\begin{array}{ccc|c} 1 & 1 & 0 & 6 \\ 0 & -5 & 0 & -\frac{13}{5} \\ 0 & 0 & -1 & -\frac{1}{5} \end{array}\right]$$

$$\text{Paso 1-0:} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & -5 & 0 & -\frac{13}{5} \\ 0 & 0 & -1 & -\frac{1}{5} \end{array}\right]$$

$$\text{Normalizar fila 0:} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & -5 & 0 & -\frac{13}{5} \\ 0 & 0 & -1 & -\frac{1}{5} \end{array}\right]$$

$$\text{Normalizar fila 1:} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & \frac{13}{5} \\ 0 & 0 & -1 & -\frac{1}{5} \end{array}\right]$$

$$\text{Normalizar fila 2:} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & \frac{13}{5} \\ 0 & 0 & 1 & \frac{1}{5} \end{array}\right].$$

Solución: $u = 6, v = \frac{13}{5}, w = \frac{1}{5}$