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Trabajo Encargado - N° 004

Ejercicios Gauss Jordan

Código

Repositorio github

Ejercicio 1: Modelo de regresión lineal

Resolver el siguiente sistema:

$$\begin{aligned}2w_1 + 3w_2 - w_3 &= 5, \\ -w_1 + 2w_2 + 4w_3 &= 6, \\ 3w_1 - w_2 + 2w_3 &= 7.\end{aligned}$$

Matriz Aumentada:

$$\left[\begin{array}{ccc|c} 2 & 3 & -1 & 5 \\ -1 & 2 & 4 & 6 \\ 3 & -1 & 2 & 7 \end{array} \right].$$

ResolucionCodigo:

The screenshot shows a web-based Gauss-Jordan elimination calculator. At the top, it has input fields for 'Filas' (3) and 'Columnas' (3), and a 'Crear Matriz' button. Below this is a grid for the augmented matrix. The initial matrix is displayed as:

2	3	-1	5
-1	2	4	6
3	-1	2	7

The calculator shows the following steps:

Matriz aumentada inicial:

2	3	-1	5
-1	2	4	6
3	-1	2	7

Paso 0-1:

2	3	-1	5
0	7/2	7/2	17/2
3	-1	2	7

Paso 0-2:

2	3	-1	5
0	7/2	7/2	17/2
0	-11/2	7/2	-1/2

Paso 1-2:

2	3	-1	5
0	7/2	7/2	17/2

The final solution is displayed as: **Solución: 12/7, 1, 10/7**. There is a 'Resolver' button at the bottom.

Paso a paso:

$$\text{Paso 0-1: } \left[\begin{array}{ccc|c} 2 & 3 & -1 & 5 \\ 0 & \frac{7}{2} & \frac{7}{2} & \frac{17}{2} \\ 3 & -1 & 2 & 7 \end{array} \right]$$

$$\text{Paso 0-2: } \left[\begin{array}{ccc|c} 2 & 3 & -1 & 5 \\ 0 & \frac{7}{2} & \frac{7}{2} & \frac{17}{2} \\ 0 & -\frac{11}{2} & \frac{7}{2} & -\frac{1}{2} \end{array} \right]$$

$$\text{Paso 1-2: } \left[\begin{array}{ccc|c} 2 & 3 & -1 & 5 \\ 0 & \frac{7}{2} & \frac{7}{2} & \frac{17}{2} \\ 0 & 0 & 9 & \frac{90}{7} \end{array} \right]$$

$$\text{Paso 2-1: } \left[\begin{array}{ccc|c} 2 & 3 & -1 & 5 \\ 0 & \frac{7}{2} & 0 & \frac{7}{2} \\ 0 & 0 & 9 & \frac{90}{7} \end{array} \right]$$

$$\text{Paso 2-0: } \left[\begin{array}{ccc|c} 2 & 3 & 0 & \frac{45}{7} \\ 0 & \frac{7}{2} & 0 & \frac{7}{2} \\ 0 & 0 & 9 & \frac{90}{7} \end{array} \right]$$

$$\text{Paso 1-0: } \left[\begin{array}{ccc|c} 2 & 0 & 0 & \frac{24}{7} \\ 0 & \frac{7}{2} & 0 & \frac{7}{2} \\ 0 & 0 & 9 & \frac{90}{7} \end{array} \right]$$

$$\text{Normalizar fila 0: } \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{12}{7} \\ 0 & \frac{7}{2} & 0 & \frac{7}{2} \\ 0 & 0 & 9 & \frac{90}{7} \end{array} \right]$$

$$\text{Normalizar fila 1: } \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{12}{7} \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 9 & \frac{90}{7} \end{array} \right]$$

$$\text{Normalizar fila 2: } \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{12}{7} \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \frac{10}{7} \end{array} \right] .$$

Solución: $w_1 = \frac{12}{7}, w_2 = 1, w_3 = \frac{10}{7}$

Ejercicio 2: Calibración de hiperparámetros

Resolver el sistema:

$$\begin{aligned}x + 2y + 3z &= 12, \\ 2x - y + z &= 4, \\ -x + 2y - 2z &= 0.\end{aligned}$$

Matriz Aumentada:

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 12 \\ 2 & -1 & 1 & 4 \\ -1 & 2 & -2 & 0 \end{array} \right].$$

ResolucionCodigo:

Gauss-Jordan

Filas: 3Columnas: 3

Crear Matriz

1	2	3	12
2	-1	1	4
-1	2	-2	0

Matriz aumentada inicial:

1	2	3	12
2	-1	1	4
-1	2	-2	0

Paso 0-1:

1	2	3	12
0	-5	-5	-20
-1	2	-2	0

Paso 0-2:

1	2	3	12
0	-5	-5	-20
0	4	1	12

Paso 1-2:

1	2	3	12
0	-5	-5	-20

Solución: 8/3, 8/3, 4/3

Resolver

Paso a paso:

$$\text{Paso 0-1: } \left[\begin{array}{ccc|c} 1 & 2 & 3 & 12 \\ 0 & -5 & -5 & -20 \\ -1 & 2 & -2 & 0 \end{array} \right]$$

$$\text{Paso 0-2: } \left[\begin{array}{ccc|c} 1 & 2 & 3 & 12 \\ 0 & -5 & -5 & -20 \\ 0 & 4 & -3 & 12 \end{array} \right]$$

$$\text{Paso 1-2: } \left[\begin{array}{ccc|c} 1 & 2 & 3 & 12 \\ 0 & -5 & -5 & -20 \\ 0 & 0 & -3 & -4 \end{array} \right]$$

$$\text{Paso 2-1: } \left[\begin{array}{ccc|c} 1 & 2 & 3 & 12 \\ 0 & -5 & 0 & -\frac{40}{3} \\ 0 & 0 & -3 & -4 \end{array} \right]$$

$$\text{Paso 2-0: } \left[\begin{array}{ccc|c} 1 & 2 & 0 & 8 \\ 0 & -5 & 0 & -\frac{40}{3} \\ 0 & 0 & -3 & -4 \end{array} \right]$$

$$\text{Paso 1-0: } \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{8}{3} \\ 0 & -5 & 0 & -\frac{40}{3} \\ 0 & 0 & -3 & -4 \end{array} \right]$$

$$\text{Normalizar fila 0: } \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{8}{3} \\ 0 & -5 & 0 & -\frac{40}{3} \\ 0 & 0 & -3 & -4 \end{array} \right]$$

$$\text{Normalizar fila 1: } \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{8}{3} \\ 0 & 1 & 0 & 8 \\ 0 & 0 & -3 & -4 \end{array} \right]$$

$$\text{Normalizar fila 2: } \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{8}{3} \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & \frac{4}{3} \end{array} \right].$$

Solución: $x = \frac{8}{3}, y = 8, z = \frac{4}{3}$

Ejercicio 3: Asignación óptima de recursos

Resolver el sistema:

$$\begin{aligned}a + b + c &= 6, \\ 2a - b + 3c &= 13, \\ -a + 2b - c &= 2.\end{aligned}$$

Matriz Aumentada:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & -1 & 3 & 13 \\ -1 & 2 & -1 & 2 \end{array} \right].$$

ResolucionCodigo:

Gauss-Jordan

Filas: Columnas:

<input type="text" value="1"/>	<input type="text" value="1"/>	<input type="text" value="1"/>	<input type="text" value="6"/>
<input type="text" value="2"/>	<input type="text" value="-1"/>	<input type="text" value="3"/>	<input type="text" value="13"/>
<input type="text" value="-1"/>	<input type="text" value="2"/>	<input type="text" value="-1"/>	<input type="text" value="2"/>

Matriz aumentada inicial:

```
1      1      1      6
2      -1     3     13
-1     2     -1     2
```

Paso 0-1:

```
1      1      1      6
0      -3     1      1
-1     2     -1     2
```

Paso 0-2:

```
1      1      1      6
0      -3     1      1
0      3      0      8
```

Paso 1-2:

```
1      1      1      6
0      -3     1      1
```

Solución: -17/3, 8/3, 9

Paso a paso:

$$\text{Paso 0-1:} \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & 1 \\ -1 & 2 & -1 & 2 \end{array} \right]$$

$$\text{Paso 0-2:} \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & 1 \\ 0 & 3 & 0 & 8 \end{array} \right]$$

$$\text{Paso 1-2:} \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & 1 \\ 0 & 0 & 1 & 9 \end{array} \right]$$

$$\text{Paso 2-1:} \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & 0 & -8 \\ 0 & 0 & 1 & 9 \end{array} \right]$$

$$\text{Paso 2-0:} \quad \left[\begin{array}{ccc|c} 1 & 1 & 0 & -3 \\ 0 & -3 & 0 & -8 \\ 0 & 0 & 1 & 9 \end{array} \right]$$

$$\text{Paso 1-0:} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{17}{3} \\ 0 & -3 & 0 & -8 \\ 0 & 0 & 1 & 9 \end{array} \right]$$

$$\text{Normalizar fila 0:} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{17}{3} \\ 0 & -3 & 0 & -8 \\ 0 & 0 & 1 & 9 \end{array} \right]$$

$$\text{Normalizar fila 1:} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{17}{3} \\ 0 & 1 & 0 & \frac{8}{3} \\ 0 & 0 & 1 & 9 \end{array} \right]$$

$$\text{Normalizar fila 2:} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{17}{3} \\ 0 & 1 & 0 & \frac{8}{3} \\ 0 & 0 & 1 & 9 \end{array} \right].$$

Solución: $a = -\frac{17}{3}, b = \frac{8}{3}, c = 9$

Ejercicio 4: Optimización de parámetros de un Bosque Aleatorio

Resolver el sistema:

$$p + 2q + 3r = 10,$$

$$2p - q + 4r = 12,$$

$$3p + 3q - r = 6.$$

Matriz Aumentada:

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 2 & -1 & 4 & 12 \\ 3 & 3 & -1 & 6 \end{array} \right].$$

ResolucionCodigo:

Gauss-Jordan

Filas: 3Columnas: 3

Crear Matriz

1	2	3	10
2	-1	4	12
3	3	-1	6

Matriz aumentada inicial:
1 2 3 10
2 -1 4 12
3 3 -1 6

Paso 0-1:
1 2 3 10
0 -5 -2 -8
3 3 -1 6

Paso 0-2:
1 2 3 10
0 -5 -2 -8
0 -3 -10 -24

Paso 1-2:
1 2 3 10
0 -5 -2 -8

Solución: 2, 8/11, 24/11

Resolver

Paso a paso:

$$\begin{array}{ll}
\text{Paso 0-1:} & \begin{bmatrix} 1 & 2 & 3 & | & 10 \\ 0 & -5 & -2 & | & -8 \\ 3 & 3 & -1 & | & 6 \end{bmatrix} \\
\text{Paso 0-2:} & \begin{bmatrix} 1 & 2 & 3 & | & 10 \\ 0 & -5 & -2 & | & -8 \\ 0 & -3 & -10 & | & -24 \end{bmatrix} \\
\text{Paso 1-2:} & \begin{bmatrix} 1 & 2 & 3 & | & 10 \\ 0 & -5 & -2 & | & -8 \\ 0 & 0 & -\frac{44}{5} & | & -\frac{96}{5} \end{bmatrix} \\
\text{Paso 2-1:} & \begin{bmatrix} 1 & 2 & 3 & | & 10 \\ 0 & -5 & 0 & | & -\frac{40}{11} \\ 0 & 0 & -\frac{44}{5} & | & -\frac{96}{5} \end{bmatrix} \\
\text{Paso 2-0:} & \begin{bmatrix} 1 & 2 & 0 & | & \frac{38}{11} \\ 0 & -5 & 0 & | & -\frac{40}{11} \\ 0 & 0 & -\frac{44}{5} & | & -\frac{96}{5} \end{bmatrix} \\
\text{Paso 1-0:} & \begin{bmatrix} 1 & 0 & 0 & | & 2 \\ 0 & -5 & 0 & | & -\frac{40}{11} \\ 0 & 0 & -\frac{44}{5} & | & -\frac{96}{5} \end{bmatrix} \\
\text{Normalizar fila 0:} & \begin{bmatrix} 1 & 0 & 0 & | & 2 \\ 0 & -5 & 0 & | & -\frac{40}{11} \\ 0 & 0 & -\frac{44}{5} & | & -\frac{96}{5} \end{bmatrix} \\
\text{Normalizar fila 1:} & \begin{bmatrix} 1 & 0 & 0 & | & 2 \\ 0 & 1 & 0 & | & \frac{8}{11} \\ 0 & 0 & -\frac{44}{5} & | & -\frac{96}{5} \end{bmatrix} \\
\text{Normalizar fila 2:} & \begin{bmatrix} 1 & 0 & 0 & | & 2 \\ 0 & 1 & 0 & | & \frac{8}{11} \\ 0 & 0 & 1 & | & \frac{24}{11} \end{bmatrix}.
\end{array}$$

Solución: $p = 2, q = \frac{8}{11}, r = \frac{24}{11}$

Ejercicio 5: Estimación de demanda de inventario

Resolver el sistema:

$$\begin{aligned}u + v + 2w &= 9, \\ 2u - 3v + 4w &= 5, \\ u - 2v + w &= 1.\end{aligned}$$

Matriz Aumentada:

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 2 & -3 & 4 & 5 \\ 1 & -2 & 1 & 1 \end{array} \right].$$

ResolucionCodigo:

Gauss-Jordan

Filas:

Columnas:

Crear Matriz

<input type="text" value="1"/>	<input type="text" value="1"/>	<input type="text" value="2"/>	<input type="text" value="9"/>
<input type="text" value="2"/>	<input type="text" value="-3"/>	<input type="text" value="4"/>	<input type="text" value="5"/>
<input type="text" value="1"/>	<input type="text" value="-2"/>	<input type="text" value="1"/>	<input type="text" value="1"/>

Matriz aumentada inicial:

```

1      1      2      9
2     -3      4      5
1     -2      1      1

```

Paso 0-1:

```

1      1      2      9
0     -5      0    -13
1     -2      1      1

```

Paso 0-2:

```

1      1      2      9
0     -5      0    -13
0     -3     -1     -8

```

Paso 1-2:

```

1      1      2      9
0     -5      0    -13

```

Solución: 6, 13/5, 1/5

Resolver

Paso a paso:

$$\text{Paso 0-1:} \quad \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & -5 & 0 & -13 \\ 1 & -2 & 1 & 1 \end{array} \right]$$

$$\text{Paso 0-2:} \quad \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & -5 & 0 & -13 \\ 0 & -3 & -1 & -8 \end{array} \right]$$

$$\text{Paso 1-2:} \quad \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & -5 & 0 & -13 \\ 0 & 0 & -1 & -\frac{1}{5} \end{array} \right]$$

$$\text{Paso 2-1:} \quad \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & -5 & 0 & -\frac{13}{5} \\ 0 & 0 & -1 & -\frac{1}{5} \end{array} \right]$$

$$\text{Paso 2-0:} \quad \left[\begin{array}{ccc|c} 1 & 1 & 0 & 6 \\ 0 & -5 & 0 & -\frac{13}{5} \\ 0 & 0 & -1 & -\frac{1}{5} \end{array} \right]$$

$$\text{Paso 1-0:} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & -5 & 0 & -\frac{13}{5} \\ 0 & 0 & -1 & -\frac{1}{5} \end{array} \right]$$

$$\text{Normalizar fila 0:} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & -5 & 0 & -\frac{13}{5} \\ 0 & 0 & -1 & -\frac{1}{5} \end{array} \right]$$

$$\text{Normalizar fila 1:} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & \frac{13}{5} \\ 0 & 0 & -1 & -\frac{1}{5} \end{array} \right]$$

$$\text{Normalizar fila 2:} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & \frac{13}{5} \\ 0 & 0 & 1 & \frac{1}{5} \end{array} \right].$$

Solución: $u = 6, v = \frac{13}{5}, w = \frac{1}{5}$