Exposure, Capital, and Margin Analytics in DROP

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Modeling Counterparty Credit Exposure in the Presence of Margin Agreements

Abstract

1. Margin Based Credit Exposure Reduction: Margin agreements as a means of reducing counterparty credit exposure.
2. Calculating MPoR Collateralized Exposures:
    a. Collateralized Exposure and Margin Period of Risk
    b. Semi-analytical Method for calculating collateralized EE
3. Analysis of Basel Exposure Methods: Analysis of Basel *Shortcut* method for Effective EPE.

Margin Agreements as a Means of Reducing Counterparty Credit Exposure

1. Definition of Counterparty Credit Risk: *Counterparty Credit Risk* is the risk that a counterparty in an *OTC* derivative transaction will default prior to the expiration of the contract and will be unable to make all contractual payments. *Exchange-traded* derivatives bear no counterparty risk.
2. Counterparty vs. Lending Risk Difference: The primary feature that distinguishes counterpart risk from lending risk is the uncertainty of exposure at any future time. For a loan, the exposure at any future date is the outstanding balance, which is certain – not taking into

account pre-payments. For a derivative, the exposure at any future date is the replacement cost, which is determined by the market value at that date, and is, therefore, uncertain.

3. Bilateral Nature of the Counterparty Risk: Since derivative portfolio value can be both positive and negative, counterparty risk is *bilateral*.
4. Spot/Forward Contract Market Value: Market value for counter i with counterparty is known only at the current date

$$t = 0$$

   For any future date t this value $V_i(t)$ is uncertain and should be assumed random.
5. Replacement Cost at Counterparty Default: If a counterparty defaults at a time τ prior to the contract maturity, the economic loss is equal to the replacement loss of the contract. If

$$V_i(\tau) > 0$$

   the dealer does not receive anything from the defaulted counterparty, but has to pay $V_i(\tau)$ to another counterparty to replace the contract. If

$$V_i(\tau) < 0$$

   the dealer receives $V_i(\tau)$ from another counterparty, but has to forward this amount to the defaulted counterparty.
6. Forward Exposure at Contract Level: Combining these two scenarios the contract-level exposure $E_i(t)$ at time t is specified according to

$$E_i(t) = \max(V_i(t), 0)$$

7. Definition of Counterparty Level Exposure: *Counterparty level exposure* at a future time t can be defined as the loss experienced by the dealer if the counterparty defaults at time t under the assumption of no recovery.

8. Unmitigated Counterpart Level Positive Exposure: It the counterparty risk is not mitigated in any way, the *counterparty level* exposure equals the sum of *contract-level* exposure:

$$E(t) = \sum_{i} E_i(t) = \max(V_i(t), 0)$$

9. Counterparty Level Exposure under Netting: If there are *netting agreements*, derivatives with positive values at the time of the default offset the ones with negative values within each netting set NS_k so that the *counterparty-level exposure* is

$$E(t) = \sum_{k} E_{NS_k}(t) = \sum_{k} \max\left(\sum_{i \in NS_k} V_i(t), 0\right)$$

   Each non-nettable trade represents a netting set.

10. Purpose of a Margin Agreement: Margin agreements allow for further reductions to counterparty level exposure.
11. Bilateral Posting under Margin Agreements: Margin agreement is a legally binding contract between two counterparties that requires one or both counterparties to post collateral under certain conditions. A margin threshold is defined for one (unilateral agreement) or both (bilateral agreement) counterparties. If the difference between the net portfolio value and the already posted collateral exceeds the threshold, the counterparty must provide sufficient collateral to cover this excess (subject to the minimum transfer amount).
12. Threshold Value in Margin Agreements: The threshold value depends primarily on the margin value of the counterparty.
13. Netting Set Level Margin Agreement: Assuming that every margin agreement requires a netting agreement, the exposure to the counterparty is

$$E_C(t) = \sum_{k} \max\left(\sum_{i \in NS_k} V_i(t) - C_k(t), 0\right)$$

where $C_k(t)$ is the market value of the collateral for the netting set NS_k at time t. If the netting set is not covered by a margin agreement, then

$$C_k(t) = 0$$

14. Netting Set Collateralized Portfolio Value: To simplify the notations, consider a single netting set

$$E_C(t) = \max(V_C(t), 0)$$

where $V_C(t)$ is the collateralized portfolio value at time t given by

$$V_C(t) = V(t) - C(t) = \sum_i V_i(t) - C(t)$$

Collateralized Exposure and Margin Period of Risk

1. Collateral as Excess Portfolio Value: Collateral covers the excess portfolio value $V(t)$ over the threshold H:

$$V_C(t) = \max(V(t) - H, 0) = -\min(H - V(t), 0)$$

2. Expression for the Collateralized Portfolio Value: Therefore the collateralized portfolio value is

$$V_C(t) = V(t) - C(t) = \min(V(t), H)$$

3. Floor/Ceiling of the Collateralized Exposure:

$$E_C(t) = \max(V_C(t), 0) = \begin{cases} 0 & V(t) < 0 \\ V(t) & 0 < V(t) < H \\ H & V(t) > H \end{cases}$$

   is limited from above and by zero from below.
4. Margin Period of Risk (MPoR): Even with a daily margin call frequency, there is a significant period delay known as the *margin period of risk (MPoR)* between a margin call that the counterparty does not respond to and the start of the default procedures. Margin calls can be disputed, and it may take several days to realize that the counterparty is defaulting rather than disputing the call. Further there is a grace period after the counterparty issues a notice of default. During this grace period the dealer and/or the counterparty may still post collateral.
5. Counterparty Variation Margin Posting Delay: Thus the collateral available at time t is determined by the portfolio value at time $t - \Delta t$.
6. Delay Dependence - Call Frequency and Product Liquidity: While Δt is not known with certainty, it is usually assumed to be a fixed number. The assume value of Δt depends on the margin call frequency and the trade liquidity.
7. MPoR Start - Collateral/Portfolio Values: Suppose that at time $t - \Delta t$ the unilateral collateral value is $C(t - \Delta t)$ and the portfolio value is $V(t - \Delta t)$.
8. Posted Counterparty Collateral at t: Then the amount $\Delta C(t)$ that should be posted at time t is

$$\Delta C(t) = \max\big(V(t - \Delta t) - C(t - \Delta t) - H, -C(t - \Delta t)\big)$$

   negative $\Delta C(t)$ means that the collateral will be returned to the counterparty.
9. Unilateral Counterparty Collateral at t: The unilateral counterparty collateral $C(t)$ available at time t is

$$C(t) = C(t - \Delta t) + \Delta C(t) = \max(V(t - \Delta t) - H, 0)$$

10. <u>The Total Collateralized Portfolio Value</u>: The collateralized portfolio value is

$$V_C(t) = V(t) - C(t) = \min\big(V(t), H + \Delta V(t)\big)$$

where

$$\Delta V(t) = V(t) - V(t - \Delta t)$$

11. <u>Monte Carlo Primary Simulation Points</u>: Suppose one has a set of *primary* simulation points $\{t_k\}$ for modeling non-collateralized exposure.
12. <u>Monte-Carlo Look-back Points</u>: For each

$$t_k > \Delta t$$

define a lookback point at time $t_k - \Delta t$
13. <u>Monte Carlo Primary Plus Lookback</u>: The task is to simulate the non-collateralized portfolio value along the path that includes both the *primary* and the *lookback* simulation times.
14. <u>Collateral PLUS Collateralized/Uncollateralized Portfolio Values</u>: Given $V(t_{k-1})$ and $C(t_{k-1})$ one calculates:
    a. Uncollateralized Portfolio Value $V(t_k - \Delta t)$ at the next lookback time $t_k - \Delta t$
    b. Uncollateralized Portfolio Value $V(t_k)$ at the next primary time t_k
    c. Collateral at t_k:

$$C(t_k) = \max(V(t_k - \Delta t) - H, 0)$$

    d. Collateralized Portfolio Value at t_k:

$$C(t_k) = V(t_k) - C(t_k)$$

e. Collateralized Exposure at t_k:

$$E_C(t_k) = \max(V_C(t_k), 0)$$

15. Simulating the Collateralized Portfolio Value: Collateralized threshold can go above the threshold due to MPR and MTA.

Semi-Analytical Method for Collateralized EE

1. Portfolio Value at Primary Points: Assume that the simulation is only run for the primary time points t and the portfolio distribution has been obtained in the form of M quantities $V_j(t)$, where j (from 1 to M) designates different scenarios.
2. Evaluating the Unconditional Portfolio Distribution: From the set $\{V_j(t)\}$ once can estimate the unconditional expectation $\mu(t)$ and standard deviation $\sigma(t)$ of the portfolio value, as well as any other distributional parameter.
3. Collateralized EE at Lookback Points: Can the collateralized EE profile be estimated without simulating the portfolio value at the lookback time points $\{V_j(t - \Delta t)\}$?
4. Collateralized EE Conditional on Path: Collateralized EE can be represented as

$$EE_C(t) = \mathbb{E}\left[EE_{C,j}(t)\right]$$

where $EE_{C,j}(t)$ is the collateralized EE conditional on $V_{C,j}(t)$:

$$EE_{C,j}(t) = \mathbb{E}\left[\max\left(V_{C,j}(t), 0\right) | V_j(t)\right]$$

5. <u>The Conditional Collateralized Portfolio Value</u>: The collateralized portfolio value $V_{C,j}(t)$ is

$$V_{C,j}(t) = \min\left(V_j(t), H + V_j(t) - V_j(t - \Delta t)\right)$$

6. <u>Goal - Computing the Collateralized EE Analytically</u>: If $EE_{C,j}(t)$ can be computed analytically, the *unconditionally collateralized EE* can be obtained as a simple average of $EE_{C,j}(t)$ across all scenarios j
7. <u>Assumption of Normal Portfolio Value</u>: Assume that the portfolio value $V(t)$ at time t is normally distributed with mean $\mu(t)$ and standard deviation $\sigma(t)$.
8. <u>Brownian Bridge for Secondary Nodes</u>: One can construct a *Brownian Bridge* from $V(0)$ to $V_j(t)$.
9. <u>$V_j(t - \Delta t)$ Mean and Standard Deviation</u>: Conditional on $V_j(t)$, $V_j(t - \Delta t)$ has a *normal distribution* with *expectation*

$$\alpha_j(t) = \frac{\Delta t}{t} V(0) + \frac{t - \Delta t}{t} V_j(t)$$

and *standard deviation*

$$\beta_j(t) = \sigma(t) \sqrt{\frac{\Delta t(t - \Delta t)}{t^2}}$$

10. <u>Closed Form Conditional Collateralized EE</u>: *Conditional Collateralized EE* can be obtained in a closed form.
11. <u>Piece-wise Constant Local Volatility</u>: It is assumed that, conditional on $V_j(t)$, the distribution of $V_j(t - \Delta t)$ is normal, but $\sigma(t)$ will be replaced by the local quantity $\sigma_{LOC}(t)$.
12. <u>Portfolio Value Monotonically Increasing with Z</u>: The portfolio value $V(t)$ at time t is described using

$$V(t) = \vartheta(t, Z)$$

where $\vartheta(t, Z)$ is a monotonically increasing function of the standard normal random variable Z.

13. The *Equivalent Normal* Portfolio Process: A *normal equivalent* portfolio process is defined as

$$W(t) = \omega(t, Z) = \mu(t) + \sigma(t)Z$$

14. Density Scaling to determine $\sigma_{LOC}(t)$: To obtain $\sigma_{LOC}(t)$, $\sigma(t)$ will be scaled by the probability densities of $W(t)$ and $V(t)$.
15. Standard Deviation Scaled Probability Density: The probability density of the quantity X is denoted via $f_X(\cdot)$ and the standard deviation is scaled according to

$$\sigma_{LOC}(t) = \frac{f_{W(t)}\big(\omega(t, Z)\big)}{f_{V(t)}\big(\vartheta(t, Z)\big)}\sigma(t)$$

16. Changing Variables from W/V to Z: Changing the variables from $V(t)$ and $W(t)$ to Z, one gets

$$f_{V(t)}\big(\vartheta(t, Z)\big) = \frac{\phi(Z)}{\partial\vartheta(t, Z)/\partial Z}$$

$$f_{W(t)}\big(\omega(t, Z)\big) = \frac{\phi(Z)}{\sigma(t)}$$

17. Substitution to the Definition of $\sigma_{LOC}(t)$: Substituting to the definition of $\sigma_{LOC}(t)$ above gives

$$\sigma_{LOC}(t) = \frac{\partial\vartheta(t, Z)}{\partial Z}$$

18. Estimating CDF - The Base Methodology: The values of Z_j corresponding to $V_j(t)$ can be obtained from

$$Z_j = \Phi^{-1}\left(F_{V(t)}\left(V_j(t)\right)\right)$$

19. Estimating the CDF - Sorting the Realizations: One sorts the array $V_j(t)$ in increasing order so that

$$V_{[j(k)]}(t) = V_{k,SORTED}(t)$$

where $j(k)$ is the sorting index.

20. Estimating CDF - Piecewise Constant Jump: From the sorted array, one can build a piecewise constant CDF that jumps by $\frac{1}{M}$ as $V(t)$ crosses any of the simulated values.

$$F_{V(t)}\left(V_j(t)\right) \approx \frac{1}{2}\frac{k-1}{M} + \frac{1}{2}\frac{k}{M} = \frac{2k-1}{2M} \rightarrow \frac{k-0.5}{M}$$

where 0.5 is the de-facto bias reducer.

21. Estimation of the Weiner Wanderer: Now one can obtain Z_j corresponding to $V_j(t)$ as

$$Z_{[j(k)]} = \Phi^{-1}\left(\frac{2k-1}{2M}\right)$$

22. Estimating the Local Standard Deviation: Local standard deviation $\sigma_{LOC,j}(t)$ can be estimated as

$$\sigma_{LOC,[j(k)]}(t) \equiv \sigma_{LOC}\left(t, Z_{[j(k)]}\right) \approx \frac{V_{[j(k+\Delta k)]}(t) - V_{[j(k-\Delta k)]}(t)}{Z_{[j(k+\Delta k)]} - Z_{[j(k-\Delta k)]}}$$

23. Choice of the Different Amount Δk: The offset Δk should not be too small (too much noise) or too large (loss of *locality*). This range works apparently well (Pykhtin (2009)):

$$20 \leq \Delta k \leq 0.05M$$

24. The Brownian Bridge Mean and σ: Similar to the above it is assumed that, conditional on $V_j(t)$, $V_j(t - \Delta t)$ has a *normal distribution* with *expectation*

$$\alpha_j(t) = \frac{\Delta t}{t} V(0) + \frac{t - \Delta t}{t} V_j(t)$$

and *standard deviation*

$$\beta_j(t) = \sigma(t) \sqrt{\frac{\Delta t(t - \Delta t)}{t^2}}$$

25. The Collateralized Exposure Mean and σ: The *collateralized exposure* depends on $\Delta V_j(t)$, which is also normal conditional on $V_j(t)$ with the same standard deviation $\beta_j(t)$ and expectation $\alpha_{C,j}(t)$ given by

$$\alpha_{C,j}(t) = V_j(t) - \alpha_j(t) = \frac{\Delta t}{t} \left[V_j(t) - V(0)\right]$$

26. Collateralized EE Conditional on j: Collateralized EE conditional on scenario j at time t is

$$EE_{C,j}(t) = \mathbb{E}\left[\max\left(\min\left(V_j(t), H + \Delta V_j(t)\right), 0\right) | V_j(t)\right]$$

27. Collateralized EE on Negative Exposure: $EE_{C,j}(t)$ equals *zero* whenever

$$V_j(t) > 0$$

so that

$$EE_{C,j}(t) = \mathbb{I}_{V_j(t)>0}\mathbb{E}\left[\min\left(V_j(t), H + \Delta V_j(t)\right)|V_j(t)\right]$$

28. <u>Integral Form for Collateralized EE</u>: Since $\Delta V_j(t)$ has a normal distribution, one can write

$$EE_{C,j}(t) = \mathbb{I}_{V_j(t)>0}\int_{-\infty}^{+\infty}\min\left(V_j(t), H + \alpha_{C,j}(t) + \beta_j(t)Z\right)\phi(Z)dZ$$

$$= \mathbb{I}_{V_j(t_k)>0}\left\{\int_{-d_2}^{-d_1}\left[H + \alpha_{C,j}(t) + \beta_j(t)Z\right]\phi(Z)dZ + V_j(t)\int_{-d_1}^{+\infty}\phi(Z)dZ\right\}$$

29. <u>Conditional Collateralized EE Closed Form</u>: Evaluating the integrals, one obtains

$$EE_{C,j}(t) = \mathbb{I}_{V_j(t_k)>0}\left\{\left[H + \alpha_{C,j}(t)\right][\Phi(d_2) - \Phi(d_1)] + \beta_j(t)[\phi(d_2) - \phi(d_1)] + V_j(t)\Phi(d_1)\right\}$$

where

$$d_1 = \frac{H + \alpha_{C,j}(t) - V_j(t)}{\beta_j(t)}$$

$$d_2 = \frac{H + \alpha_{C,j}(t)}{\beta_j(t)}$$

Analysis of Basel "Shortcut" Method for Collateralized Effective EPE

1. Basel 2 Exposure Capital Requirements: Basel 2 minimal capital requirements for the counterparty risk are determined by wholesale exposure rules with exposure at default obtained from expected exposure profile as follows.
2. Exposure at Default - Basel Variants:
    a. Expected Exposure (EE) – Expected Exposure Profile (EE)
    b. Expected Positive Exposure (EPE) –

$$EPE = \int_{0}^{1\ Year} EE(t)dt$$

    c. Effective EE -

$$Effective\ EE(t_k) = \max\left(EE(t_k), Effective\ EE(t_{k-1})\right)$$

    d. Effective EPE –

$$Effective\ EPE = \int_{0}^{1\ Year} Effective\ EE(t)dt$$

    e. Exposure at Default (EAD) –

$$EAD = \alpha \times Effective\ EPE$$

3. Incorporating the Margin Agreement: For collateralized counterparties, the netting set level Effective EPE must incorporate the effect of margin agreement.
4. Effective EPE using Internal Model of Collateral: Collateralized Effective EPE can be calculated using an *internal model of collateral*.

5. Basel 2 Simple and Conservative Shortcut: Alternatively dealers can use a *simple and conservative approximation* to the effective EPE, and sets the effective EPE for a margined counterparty equal to the lesser of:
    a. The *Threshold*, if positive, under the margin agreement *plus* an *add-on* that reflects the potential increase in exposure over the margin period of risk. The *add-on* is computed as the *expected increase in the netting set's exposure* beginning from the current exposure of zero over the margin period of risk.
    b. *Effective EPE without a margin agreement*.
6. Derivation of the "Shortcut" Method: The Basel "Shortcut" method can be obtained as follows:

$$
\begin{aligned}
EE_C(t) = \mathbb{E}\big[\max\big(\min\big(V(t), H + \Delta V(t)\big), 0\big)\big] &= \mathbb{E}[\min(E(t), H + \max(\Delta V(t), -H))] \\
&\le \mathbb{E}[\min(E(t), H + \max(\Delta V(t), 0))] \le \min(EE(t), H + \mathbb{E}[\max(\Delta V(t), 0)]) \\
&\approx \min(EE(t), H + \mathbb{E}[\max(\Delta V(\Delta t), 0)]) \equiv EE_{C,BSM}(t)
\end{aligned}
$$

7. Enhancing the Exposure Conservativeness: Time averaging adds more conservativeness:

$$
\frac{1}{T}\int_0^T EE_{C,BSM}(t)dt \le \min(EPE, H + \mathbb{E}[\max(\Delta V(\Delta t), 0)])
$$

Conclusion

1. Margin Agreements for Risk Mitigation: Margin agreements are important risk mitigation tools that need to be modeled accurately.
2. Complete MC Doubles Simulation Time: Full Monte Carlo is the most flexible approach, but requires simulating trade values at secondary time points, thus doubling the simulation time.

3. Semi-Analytical Approach Avoids That: Pykhtin (2009) has presented an accurate semi-analytical approach of calculating the EE that avoids doubling of the simulation time.
4. Basel 2 Shortcuts are too Conservative: Basel 2 "Shortcut" method for Effective EPE has sound theoretical grounds, but is too conservative.

References

- Pykhtin, M. (2009): Modeling Counterparty Credit Exposure in the Presence of Margin Agreements

Estimation of Margin Period of Risk

Abstract

1. Enhanced CSA Collateral Exposure Model: Andersen, Pykhtin (2017) describe a new framework for collateral exposure modeling under an ISDA Master Agreement with Credit Support Annex. The proposed model captures the legal and the operational aspects of default in considerably greater detail than models currently used by most practitioners, while remaining fully tractable and computationally feasible.
2. Legal Rights Exercise/Deferral Choices: Specifically, it considers the remedies and the suspension rights available within these legal agreements; the firm's policies of availing itself of these rights; and the typical time it takes to exercise them in practice.
3. Significantly Higher Credit Exposure Revealed: The inclusion of these effects is shown to produce a significantly higher credit exposure for representative portfolios compared to the currently used models. The increase is especially pronounced when dynamic initial margin is also present.

Introduction

1. Margin Period of Risk Overview: In modeling the exposure of collateralized positions, it is well recognized that credit default cannot be treated as a one-time event. Rather the entire

sequence of events following up to the default and beyond need to be considered, from the last successful margin call in advance of the eventual default to the time when the amount of loss becomes known – in the industry parlance, *crystallized*. These events unfold over a period of time called the *margin period of risk* (MPoR).

2. Range of Model Applicability: To properly identify the exposure during the MPoR, a detailed understanding of the contractual obligations is essential. In their paper, Andersen, Pykhtin, and Sokol (2017) focus on collateralized exposures under bilateral trading relationships governed by the *ISDA Master Agreement* (IMA) and its *Credit Support Annex*. The IMA, by far, is the most common legal contract for bilateral over-the-counter (OTC) derivatives trading, although other agreements are sometimes used (such as national forms of agreements used in some jurisdictions for domestic trading). The analysis by Andersen, Pykhtin, and Sokol (2017) is expected to apply to a broad class of contracts, although the model assumptions should be re-examined to confirm that the key legal provisions remain substantially the same as IMA.
3. Refinement for Legal/Operational Impact: It should be noted that the modeling of default exposure and close-out risk arising from a non-zero MPoR has received a fair amount of attention in the past (see, e.g., Gibson (2005), Pykhtin (2009, 2010), and Brigo, Capponi, Pallavicini, and Papatheodorou (2011)), although most past analysis has been conducted under very strong simplifying assumptions about the trade and the margin flows during the MPoR. One exception is Bocker and Schroder (2011), which contains elements of a more descriptive framework, including recognition of the role played by the cash flows close to the default event. Andersen, Pykhtin, and Sokol (2017) use a more detailed framework for legal and operational behavior to refine the classical models for collateralized exposure modeling.
4. Variation Margin Operational Timelines: This chapter is organized as follows. The fundamentals of variation margin posting are first outlined, and the classical collateralized exposure model is then presented. The full timelines of events likely to transpire during a credit default are then discussed from both legal and operational perspectives. This sets the stage for the proposal of a condensed representation of the timeline suitable for analytical and numerical work. The resulting setup results in a more significantly nuanced and flexible definition of the collateralized trading exposure. As fixing the actual model parameters (i.e.,

calibrating the MPoR model) requires taking a stance on operational procedures and corporate behavior, the next section discusses how such parametrizations may be done in practice, for various levels of overall model prudence and counterparty types.

5. Numerical Computation of Collateralized Exposure: Subsequently, the model is fleshed out in more detail, especially as it pertains to numerical implementations and quantitative comparisons with the classical model. As a starting point, exposure models are formulated in mathematical terms, and the key differences to the classical models are highlighted by means of brute-force Monte-Carlo simulations. Computational techniques permitting efficient model implementation are introduced subsequently, along with several test results. Applications to portfolios with risk-based initial margins are briefly discussed, and conclusions are finally drawn.

The Fundamentals of Variation Margin: Basic Definitions

1. Types of Margin: Initial/Variation: In bilateral OTC derivatives trading, it is common for parties to require posting of collateral to mitigate excessive exposures. Although the initial margin is discussed briefly in a later section, this section focuses primarily on the variation margin (VM) as a form of collateral that is regularly re-adjusted based on the changing value of the bilateral portfolio. The VM is calculated and settled in time according to a set of CSA rules discussed a few sections down.
2. Dealer and Client VM Timelines: For concreteness, throughout this chapter, the exposure of dealer D to a client C with whom D engages in bilateral OTC trading under the IMA/CSA legal framework is considered. C is referred to as the *defaulting party*, and D is *the dealer* or the *non-defaulting* party. All present value and exposure amounts throughout this chapter will be calculated from the viewpoint of D. Let the default-free market value to D of the securities portfolio at time t be $V(t)$ and let $A_D(t)$ and $A_C(t)$ be the collateral support amounts stipulated by the CSA to be posted to D and C respectively. In the absence of initial

margin it is virtually always the case that only one of A_D or A_C is positive, i.e., only one party will be required to post margin at a given point in time.

3. Net Collateral and its Posting: Assuming that collateral is netted (rather than posted by both parties in full and held in segregated accounts or by a third party), the total collateral amount in D's possession may be calculated as of time t as

$$c(t) = A_C(t) - A_D(t)$$

Assuming also that the collateral may be treated as *pari passu* with the derivatives portfolio itself for the purposes of bankruptcy claim, it is common to denote the positive part of the difference $V(t) - c(t)$ as the *exposure* $E(t)$:

$$E(t) = [V(t) - c(t)]^+$$

$$c(t) = A_C(t) - A_D(t)$$

where the notation

$$x^+ = \max(x, 0)$$

is used. Normally both the collateral and the portfolio would be treated together as a senior unsecured claim of D against the bankruptcy estate of C. There are several time lags and practical complications that render the above exposure and collateral expressions an imprecise measure, and they shall be substantially refined later on. In particular it is emphasized that the collateral computed at time t is generally not transferred to D until several days after t.

4. VM Designated to Track Portfolio Value: The type of VM encountered in the CSA is typically designed to broadly track the value of the portfolio between the parties, thereby ensuring that $E(t)$ in

$$E(t) = [V(t) - c(t)]^+$$

$$c(t) = A_C(t) - A_D(t)$$

does not grow to be excessive. However, to avoid unnecessary operational expenses, it is common to introduce language in the CSA to relax margin transfer requirements if the amounts are sufficiently small.

5. Reducing the Collateral Posting Events: To that end the typical CSA language for collateral calculations will stipulate:
    a. Collateral posting requirements by each party, h_D and h_C, representing the minimum amount of exposure before D or C, respectively, is required to post collateral
    b. A minimum transfer amount (MTA) establishing a minimum valid amount of a margin call
    c. Rounding, which rounds collateral movements to some reasonable unit (say $1,000).
6. Incorporating Thresholds into Collateral Expressions: Formally the effects of thresholds on the stipulated collateral may be written as

$$A_D(t) = [-V(t) - h_D]^+$$

$$A_C(t) = [V(t) - h_C]^+$$

with the net stipulated credit support amount assigned to D being

$$c(t) = A_C(t) - A_D(t)$$

as before. The actual availability of this amount is then subject to the path dependent effects on collateral by MTA and rounding, of which the former has significant effect only for zero or very small thresholds, and the latter is usually negligible. Both have been omitted in the equation above.

7. Unilateral and Asymmetric Collateral Requirements: Most CSAs are bilateral in nature, but unilateral CSAs do exist in which only one of the two parties is required to post collateral. A CSA may be formally bilateral, but highly asymmetric, requiring both parties to post collateral but with vastly different thresholds, e.g.

$$h_D = \$20\ mm$$

vs.

$$h_C = \$2\ mm$$

Typically, even for asymmetric CSAs, the MTAs and the rounding are the same for both parties.

Margin Calls and Cash Flows

1. Margining Frequency of the Collateral Process: From an exposure perspective, the frequency with which the amount of collateral is adjusted – the *re-margining frequency* – is a critical component of the CSA. Following the financial crisis, most new IMA/CSAs, especially between major financial institutions, have been using daily re-margining frequency in order to reduce the amount by which the exposure can change relative to the collateral between the margin calls. However many small financial institutions or buy-side clients may not be able to cope with the operational burden of frequent margin calls and will often negotiate longer re-margining frequencies, e.g., weekly, monthly, or even longer.
2. Events Constituting the Margining Process: The amount of collateral held by the parties is adjusted to their stipulated values A_D and A_C via the mechanism of a margin call. Many models for exposure treat the margin call as an instantaneous event, taking place on the re-

margining date and completed instantaneously. In practice the margin call is a chain of events that takes several days to complete. With daily re-margining, several such chains run concurrently in an *interlaced* manner; even as one margin call is yet to be settled, another one already may be initiated. The time lag of this settlement process, long with the inherent lag of the re-margining schedule, means that the changes in the VM are always running behind the changes in the portfolio value. This, in turn, implies that the idealized expressions such as

$$E(t) = [V(t) - c(t)]^{+}$$

$$c(t) = A_C(t) - A_D(t)$$

are inaccurate. The detailed events involved in an initiation and the eventual settlement of a margin call will be discussed in a later section.

3. Underlying Trade Cash Flow: With both default processes and margin settlements being non-instantaneous events, it becomes relevant to track what payment flows take place – or not – during the periods close to a default. Two types of payments are needed here. The first type – called using the term *trade flows*, covers the contractual cash flows, physical settlements, and other forms of asset transfers related to the trade themselves. These terms are spelt out in trade documents and *term sheets* for each trade. The term *trade flows* rather than *cash flows* is used to emphasize that term sheets may involve flows other than cash – such as transfers of other non-cash assets, e.g. commodities, physical settlements resulting from the creation of new trades from old ones, e.g. exercise of a physically settled swaption into a swap. A missed trade flow is a serious event under the IMA, and a failure to pay can rapidly result in a default and trade termination unless cured properly. Any missed trade flow is, of course, part of the non-defaulting party's claim.
4. CSA Specified Margin Cash Flow: The second type of flows is that that arises from the exchange of collateral between the parties – *margin flows*. The legal treatment of the margin flows is determined by the IMA/CSA, rather than by the trade documentation between the parties. For purposes of this treatment, the most important aspect of the IMA/CSA is the relatively mild treatment it affords to a party that misses a margin flow. Indeed, partially

missing a margin payment is a common occurrence, as disputes about margin amounts happen regularly, and sometimes persist for years.

5. Delays causing the Default Termination: During a collateral dispute, the CSA protocol calls for the payment of the undisputed components of the collateral, but there is of course the possibility that there will be no undisputed component at all, if one party's counter-proposals are sufficiently frivolous. Should suspicious about *gaming* arise, the CSA does contain a methodology to stop disputes through market quotations, but the resulting leakage of the position information is often a good deterrent to its use. As such, there is potential for abuse by firms experiencing financial difficulties, and a good possibility that such abuse can go on for some time before the dealer takes further efforts to end it. This, in turn, may result in a fairly long period of time between the last fully settled margin call and the eventual termination of a portfolio due to default.

Revised Exposure Definition

1. Stipulated vs. Realized Collateral Amount: In light of the discussion above, this section makes a first effort at improving

$$E(t) = [V(t) - c(t)]^+$$

$$c(t) = A_C(t) - A_D(t)$$

For this consider a default of C at time τ following an early termination of the trade portfolio at time

$$t \geq \tau$$

At time t, let $K(t)$ be the collateral B can *actually* rely on for the portfolio termination; this amount will very likely differ from the CSA stipulated amount $C(t)$ – and from $C(\tau)$ for that matter – due to the margin transfer time lags and some degree of non-performance by C.

2. Exposure Enhanced by Trade Flow: In addition, it is possible that some trade flows are missed; denote their value at time t, including accrued interest, as $UTF(t)$. The exposure generated by a default at time

$$\tau \leq t$$

may be re-defined as

$$E(t) = [V(t) + UTF(t) - K(t)]^{+}$$

Notice that the expression anchors the exposure at the termination date rather than at the default date τ - this will be treated at a later section. For later use, the time-0 expectation of the future time-t exposure is defined as

$$EE(t) = \mathbb{E}_0[E(t)]$$

where $\mathbb{E}$ is the expectation operator in a relevant probability measure.

3. Impact of the Margin Timelines: Determining how $K(t)$ can differ from $c(t)$, and how large can realistically $UTF(t)$ be, will require a more detailed understanding of the settlement and the margining processes, a topic that will be treated in detail in a later section. The next goes about determining how classical approaches go about modeling $K(t)$ and $UTF(t)$ in

$$E(t) = [V(t) + UTF(t) - K(t)]^{+}$$

Classical Model for Collateralized Exposure – Assumptions about Margin Flows

1. The Naïve Collateralized Exposure Model: A naïve, and now outdated, model for collateralized exposure follows the definition

$$E(t) = [V(t) - c(t)]^{+}$$

$$c(t) = A_C(t) - A_D(t)$$

literally, and assumes that the collateral available is exactly equal to its prescribed value at time t. That is, in the language of

$$E(t) = [V(t) + UTF(t) - K(t)]^{+}$$

it is assumed that

$$K(t) = c(t)$$

In addition, the parties are assumed to pay off all of the trade flows as described

$$UTF(t) = 0$$

and it is assumed that the termination date in

$$E(t) = [V(t) + UTF(t) - K(t)]^{+}$$

is the default date τ, i.e., there is no lag between the default date and the termination date. In this model, the assumption of loss crystallized at a time t is the function of the portfolio value at a single point $V(t)$ and does not depend on the earlier history $V(\cdot)$ In the limit of *perfect CSA* where

$$c(t) = V(t)$$

the collateralized exposure in such a model is exactly zero.

2. Lag Induced Classical Exposure Model: Assuming

$$K(t) = c(t)$$

is an idealization that ignores the non-instantaneous nature of collateral settlement protocols and does not capture the fact that firms under stress may stop fully honoring margin calls, resulting in a divergence between the portfolio value and the collateral value at some time lag δ before the termination of the portfolio. In what is denoted here as the *Classical Model* (see, for example, Pykhtin (2010)), this particular lag effect is captured by modifying

$$E(t) = [V(t) - c(t)]^+$$

$$c(t) = A_C(t) - A_D(t)$$

to

$$E(t) = [V(t) - K(t)]^+$$

$$K(t) = c(t - \delta)$$

So, for instance, for a CSA with thresholds h_D and h_C and from

$$A_D(t) = [-V(t) - h_D]^+$$

$$A_C(t) = [V(t) - h_C]^+$$

one gets

$$K(t) = [V(t-\delta) - h_C(t-\delta)]^+ - [-V(t-\delta) - h_D(t-\delta)]^+$$

3. Drawbacks of the Classical Exposure Model: Having a mechanism for capturing the divergence between the collateral and the portfolio value is an important improvement over the older method described above, and the classical model has gained widespread acceptance for both the CVA (Credit Valuation Adjustment) and the regulatory calculations. Nevertheless, it hinges on a number of assumptions that are unrealistic. For instance,

$$E(t) = [V(t) - K(t)]^+$$

$$K(t) = c(t-\delta)$$

   assumes that both D and C will simultaneously stop paying margin at time $t-\delta$ freezing the margin level over the entire MPoR. In reality, if the party due to post collateral at $t-\delta$ happens to be the non-defaulting party D, it will often continue doing so for some time even in the presence of the news about the possible impending default of C. And should C miss a few margin payments (maybe under the guise of a dispute), D would often continue to post collateral while it evaluates its options. This creates an asymmetry between posting and receiving collateral that the classical model fails to recognize.
4. Impact of Lag on Exposure: In

$$E(t) = [V(t) - K(t)]^+$$

$$K(t) = c(t-\delta)$$

the lag parameter δ is clearly critical; the larger δ is, the more $V(t)$ may pull from the frozen margin value at time $t - \delta$ and bigger the expected exposure will become. In practice, the determination of δ will often be done in a simplistic manner, e.g. by using a fixed lag ($10\ BD$ is common), or, more realistically, by adding a universal time delay to the re-margining frequency of the CSA in question. This practice is echoed in regulatory guidelines, e.g., in Basel 3 accord where MPoR is set to the re-margining frequency minus $1\ BD$ plus an MPoR floor that defaults to $10\ BD$. The MPoR floor must be increased in certain cases, e.g., for large netting sets, illiquid trades, illiquid collateral, and recent collateral disputes – however, the increase is specified as a multiplier relative to the same default. With a high proportion of individually negotiated and amended features in real life IMA/CSAs, using a *one size fits all* assumption may, however, lead to significant inaccuracies.

Assumptions about Trade Flows

1. The Classical+ Collateral Exposure Model: Because large trade flows after the start of MPoR may no longer be followed by collateral adjustment, they have the potential to either extinguish or exacerbate the exposure. For this reason, the model assumptions with respect to the date when either party suspends the trade flows are likely to have a significant impact on the counterparty credit loss. In one common interpretation of the classical model, it is simply assumed that both D and C will continue to pay all the trade flows during the entire MPoR. As a consequence, the unpaid trade flow term $UTF(t)$ in

$$E(t) = [V(t) + UTF(t) - K(t)]^{+}$$

will be zero, consistent with

$$E(t) = [V(t) - K(t)]^{+}$$

$$K(t) = c(t - \delta)$$

For ease of reference this version of classical model is denoted as Classical+.

2. The Classical- Collateral Exposure Model: In another, less common, version of the Classical Model, the assumption is that both C and D will stop paying trade flows at the moment the MPoR commences, i.e., at time $t - \delta$. In this case, the unpaid trade flows are set equal to

$$UTF(t) = TF_{NET}(t;\ (t - \delta, t])$$

where $TF_{NET}(t;\ (t', t"])$ is the time t value of all net trade flows scheduled to be paid in the interval $(t', t"]$. Note that the time is measured in discrete units of business days, such that the notation $(u, s]$ is equivalent to $[u + 1BD, s]$. Further, if t is after the margin flow date, the trade flow value accrues from the payment date to t at a contractually specified rate. This version of the classical model is denoted Classical-; it is associated with an exposure definition of

$$E(t) = [V(t) + TF_{NET}(t;\ (t - \delta, t]) - c(t - \delta)]^{+}$$

3. Inadequacies of the Classical Exposure Models: In practice neither the Classical+ exposure equation

$$E(t) = [V(t) - K(t)]^{+}$$

$$K(t) = c(t - \delta)$$

nor the Classical- exposure equation

$$E(t) = [V(t) + TF_{NET}(t;\ (t - \delta, t]) - c(t - \delta)]^{+}$$

are accurate representations of reality. Trade flows are likely to be paid at least by D at the beginning of the MPoR, and are likely not to be paid by C at least at its end. For instance, due to the CSA protocol for collateral calculations (next section), there is typically a $3\ BD$ lag between the start of an MPoR – the market observation date for the last full margin payment – and the date when D definitely observes that C has missed paying a margin flow; during this period D would always make all trade payments unless C commits any additional contract violations. Even after D has determined that C has missed a margin payment, D's nominal rights to suspend payments following a breach would, as mentioned earlier, not always be exercised aggressively. Legal reviews, operational delays, and grace periods can further delay the time when D stops paying trade flows to C.

4. Accrual of Missed Trade Flows: Another trade flow effect arises during the last $2 - 3$ days of the MPoR (just prior to termination) where C has already defaulted and neither party is likely making trade payments. Here, the IMA stipulates that the missed trade flows within this period accrue forward at a contractually specified rate and become part of the bankruptcy claim. This gives rise to a termination period in addition to the $UTF(t)$ term, in turn leading to an adjustment of the exposure.

Full Timeline of IMA/CSA Events

1. Details of the IMA/CSA Processes: Loosely speaking, the IMA concerns itself with the events of default, termination, and close out, and the CSA governs the collateral exchanges, including the concrete rules for collateral amount calculations and posting frequencies. While this chapter so far has touched on the workings of the IMA/CSA in the previous sections, model construction hereafter will require more detailed knowledge of certain provisions regarding the normal exchange of collateral, the legal options available in the case of missed payments, and the common dealer policies with respect to availing itself of these options. A

detailed exposition of the IMA and the CSA legal complexities can be found in multiple sources, including at http://www.isda.org; here only a brief summary to the extent necessary to develop the model is provided. The focus is on the development of a plausible timeline of events taking place around the default, and the subsequent portfolio termination.

Events Prior to Default

1. Calculated vs. Actual Collateral Amounts: It is assumed that the dealer D is the Calculation Agent for the computation of the collateral amounts. As before, A_C and A_D denote the *prescribed* collateral amounts for C and D; as discussed they may differ from the *actually* available collateral amounts M_C and M_D if one of the parties fails to make the margin flow or changes the prescribed amount.
2. CSA Specified Margin Process Timelines: The following list describes the complete sequence of events taking place at times $T_0, T_1, \cdots$ The next section simplifies and condenses these into a tractable model.
3. Collateral/Portfolio Valuation Date T_0: The timeline begins at T_0, the as-of-date at which the value of the portfolio and its collateral are measured, for usage in the T_1 evaluation of the formulas for the Credit Support Amount – plainly, the amount of collateral. Typically, T_0 is the close of business on the business day before T_1.
4. Honored Collateral Invocation Date T_1: For the purposes of this treatment, T_1 is used to refer to the last *undisputed and respected* Valuation Date prior to default. At time T_1, besides officially determining $A_c(T_0)$ and $A_C(T_0)$, the dealer D calculates the incremental payment amounts to itself and to C as

$$m_D = A_D(T_0) - M_D(T_0)$$

and

$$m_C = A_C(T_0) - M_C(T_0)$$

respectively. Taking into account any minimum transfer amounts, the transfer amounts m_D and m_C should normally be communicated by D to C prior to a Notification Time (e.g., 1 PM local time).

5. Collateral Transfer Initiation Date T_2: After receiving the notice of the calculated collateral amount, C must initiate the transfers of the sufficient amount of eligible collateral on the payment date T_2. Assuming that D managed to have the collateral amount notification sent to C prior to the Notification Time, T_2 defaults to 1 BD after T_1. If D is late in its notification, T_2 would be 2 BD after T_1. It is assumed here that the required amounts – recalling that they were calculated at T_1 using market data at time T_0 – are all settled without incident at T_2. However, T_2 will be the *last* time that margin flows settle normally before the default takes place.
6. Non honored Collateral Calculation Date T_3: Let T_3 denote the next scheduled valuation date after T_1. If α is the average scheduled time between collateral calculations, one approximately has – ignoring business calendar effects –

$$T_3 \approx T_1 + \alpha$$

   At T_3 – hopefully before the Notification Time - D will be able to send payment notice to C, but C has fallen into financial distress and will not be able – or willing – to pay further margin flows. Should C simply fail to pay collateral at the next payment date, a *Credit Support Default* could be triggered shortly thereafter (non-payment of collateral is associated with a 2 BD grace period). To prevent this from happening, it is, as discussed earlier, likely that C would attempt to stall by disputing the result of the T_3 collateral calculation by D.
7. Potential Event of Default τ: Exactly how long the margin dispute is allowed to proceed is a largely a behavioral question that requires some knowledge of D's credit policies and its willingness to risk legal disputes with C. Additionally one needs to consider to what extend C

is able to conceal its position of financial stress by using dispute tactics, or, say, blaming operational issues on its inability to pay collateral. Ultimately, however, D will conclude that C is in default of its margin flows (a Credit Support Default), or C will commit a serious contract violation such as failing to make a trade-related payment. At that point D will conclude that a *Potential Event of Default* (PFD) has occurred. The time of this event is identified as the true *default time* τ.

8. Client PED Communication Date T_4: Once the PED has taken place, D needs to formally communicate it to C, in writing. Taking into account mail/courier delays, legal reviews, and other operational lags, it is likely that the communication time, denoted T_4, takes place at a slight delay to the PED.
9. Event of Default Date T_5: After the receipt of the PED notice, C will be granted a brief period of time to cure the PED. The length of this *cure period* is specified in the IMA and depends on both the type of the PED and the specific IMA. For instance, of the PED in question is Failure to Pay, the default cure period is 3 BD in the 1992 IMA and 1 BD in the 2002 IMA – this may very well be overridden in the actual documents. At the end of the cure period – here denoted T_5 – and *Event of Default* (ED) formally crystallizes. It is emphasized here at the T_5 (the *official* default time) is *not* associated with the true default time τ; instead τ is equated to the time of the actual default (the PED) that, after contractual formalities, will lead to the default of C.
10. ED Communication (T_6) and ETD Designation Dates (T_7): After the ED has taken place, D will inform C of the ED at time

$$T_6 \geq T_5$$

and may, at time

$$T_7 \geq T_6$$

elect to designate an *Early Termination Date* (ETD).

11. Early Termination Date T_8: The ETD is denoted T_8; per the IMA it is required that $T_8 \in [T_7, T_7 + 20D]$ The ETD constitutes the as-of-date for the termination of C's portfolio and collateral positions. Many dealers will aim for speedy resolution in order to minimize market risk, and will therefore aim to set the ETD as early as possible. There are, however, cases where this may not be optimal, as described in the Section below.
12. Post Client ETD Establishment Events: Once the portfolio claim has been established as of the ETD, the value of any collateral and unpaid trade flows held by C is added to the amount owed to D. Paragraph 8 of the CSA then allows D to liquidate any securities collateral in its possession and to apply the proceeds against the amount it is owed. Should the collateral be insufficient to cover what is owed to D, the residual amount will be submitted as a claim in C's insolvency. The claim is usually challenged by the insolvency representative, and where parties cannot agree, may be referred to court. It can sometimes take a long time before the claim is resolved by the bankruptcy courts and the realized recovery becomes known. The interest on the recovery amount for this time is added to the awarded amount. Note that this chapter focusses exclusively on modeling the magnitude of exposure and bankruptcy claim, and does not challenge the established way of modeling the amount and the timing of the eventual recovery using a loss-given-default (LGD) fraction.

Some Behavioral and Legal Aspects

1. Margin Exposure Modeling Parametrization Components: With the timeline just having been established, it remains for it to be tied with a proper model for exposure. In order to do so, as already mentioned, the timeline needs to be combined with coherent assumptions about the dealer and the client behavior in each sub-period. The assumptions should be determined not only by the rights available under the IMA/CSA, but also by the degree of operational efficiencies in serving notices and getting legal opinions, and also by the level of prudence

injected into the assumptions about the dealer ability and willingness to strictly uphold contractual terms within each client group as it pertains to margin flows and disputes.

2. Issues with Exercising Suspension Rights: From a legal rights perspective, the most important observation is that once notice of a PED has been served (time T_4) the so called *suspension rights* of IMA (Section 2(a)(iii)) and the CSA (Paragraph 4(a)) will allow D to suspend all trade- and collateral- related payments to C until the PED has been cured. The extent to which the suspension rights are actually exercised, however, is quiet situational. A particular danger is that D exercises its suspension rights due to a Potential Event of Default (PED), but that subsequently the PED is ruled to be not valid. Should this happen, the dealer can inadvertently commit a breach of contract which, especially in the presence of cross-default provisions, can have serious consequences for the dealer.
3. Choice of Designating an ETD: Another, somewhat counter-intuitive, reason for D not to enforce its suspension rights is tied to IMA Section 2(a)(iii)which can sometimes make it favorable for D to *never* designate an ETD. Indeed, if D owes C money, it would seem a reasonable course of action for D to simply:
    a. Never designate and ETD, and
    b. Suspend all the payments in the portfolio until the default gets *cured* – which most likely will never happen.

    This tactic basically allows D to walk away from its obligations on the portfolio when C defaults, effectively making D a windfall gain.
4. Jurisdiction Legality of the ETD Delays: The strategy of delaying the ETD is perpetuity has been tested by UK courts and found legal – although contract language has been proposed by ISDA to prevent the issue. In the US, however, local *safe haven* laws have been ruled to prevent ETD's for more than about one year. Still a one-year delay may prove tempting if D has a big negative exposure to C and is unwilling to immediately fund the large cash flow needed to settle. As most large dealers are presumably unlikely to play legal games with the ETD, this topic shall not be considered further here, but note that there is room to make more aggressive model assumptions around the ETD's than is done here.

Simplified Timeline of IMA/CSA Events

1. Motivation for the Timeline Simplification: It should be evident from the preceding section that the full timeline of IMA/CSA reviewed earlier is in many ways different, and more complex, than what is assumed in the Classical- and the Classical+ versions of the classical model. However, it is equally evident that the timeline is too complex to be modeled in every detail. This section offers a simplification of the timeline designed to extract the events most important for exposure modeling. The resulting model offers several important improvements over the classical model, while remaining practical and computationally feasible.

Identification of Key Time Periods

1. Classical MPOR Start/End Dates: To recap, first the classical model only considers two dates in the timeline of default; the start and the end of the MPoR. The start of the MPoR, denoted by $t - \delta$, is defined as the last observation date for which the margin was settled in full (a few days after the observation date). The end of the MPoR, denoted by t, is the observation date on which D's claim is established. Note that t coincides with the IMA's *Early Termination Date* (ETD) discussed earlier.
2. Classical Model Lag Length Error: In the classical model there is no clear distinction between the observation and the payment dates, making it difficult to cleanly capture the trade flow effects. For instance, in the classical version of the model, $t - \delta$ denotes both the last observation date as well as the dates on which all trade flows cease. In reality, the last margin observation date is unlikely to be contentious and trigger stoppage of trade flows, as

the margin payment to which the observation corresponds to will only be missed by C several business days later. Specifically, if the market data is observed on day 0, the valuation is performed in day 1, then only on day 2 (or 3 if the notification was late) is the initiation of the actual payment expected to take place. The length of this lag is of the same order of magnitude as typical assumptions for the length of the MPoR, and can be a source of considerable model error if not handled properly.

3. Delineating Observations and Payment Dates: In the simplified timeline proposed here, care is kept to take care of the distinction between the observation and the payment dates, and also to consider the possibility that D may take the action of stopping a particular type of flow at a different time than C does. Accordingly, the model includes two potentially different observation dates for which D and C later settle their margin flows in full for the last time; and two potentially different dates when they pay their trade flows respectively for the last time. The end of the MPoR is defined as in the same way as in the classical model, to coincide with the ETD. The table below summarizes the notation for the five dates in the simplified timeline.
4. Notation for the Dates in the Simplified Timeline:

Event	**Date Type**	**Notation**
Observation Date for the Last Margin Flow from C	Observation	$t_C = t - \delta_C$
Observation Date for the Last Margin Flow from D	Observation	$t_D = t - \delta_D$
Observation Date for the Last Trade Flow Payment from C	Settlement	$t_C' = t - \delta_C'$
Observation Date for the Last Trade Flow Payment from D	Settlement	$t_D' = t - \delta_D'$
ETD	Observation	t

5. Current Scheme MPoR Start Date: The start of the MPoR in the current model is $t - \delta$, which in the notation of table above may be defined symmetrically as

$$\delta = \max(\delta_C, \delta_D)$$

C is always expected to stop posting margin no later than the non-defaulting party D, and therefore on would very likely have

$$\delta_C \geq \delta_D$$

and

$$\delta = \delta_C$$

6. Exposure Model Timeline Lag Choices: The second column in the table above specifies which of the dates is the observation date, and which is the settlement or the payment date. According to the notation established in the table, δ_C and δ_D are the lengths of time preceding the ETD during which changes in the portfolio values no longer result in collateral payments by C and D, respectively. Similarly, $\delta_C{}'$ and $\delta_D{}'$ are the lengths of time preceding the ETD during which the respective parties do not pay trade flows. In, say, a Classical 10 day MPoR model

$$\delta_C = \delta_D = 10\ BD$$

with

$$\delta_C' = \delta_D' = 0$$

for Classical+ and

$$\delta_C' = \delta_D' = 10\ BD$$

for Classical-.

Establishing the Sequence of Events

1. Order of the MPoR Events: *A priori*, the four events in the Table between the start event and the end event of the MPoR can occur in any order. However, this section will now explain why the table very likely shows the proper sequence of events.
2. Time Lag between Margin/Trade Flows: As discussed earlier, missing trade flows are recognized as a more serious breach of contractual obligations than missing margin flows, especially since the latter may take the form of a margin valuation dispute. Therefore, it is reasonable to assume that neither party will stop paying the trade flows before stopping the payment of margin flows. Accounting for the margin settlement lag between the observation date and the payment date, this yields

$$\delta_C' \leq \delta_C - Margin\ Settlement\ Lag$$

$$\delta_D' \leq \delta_D - Margin\ Settlement\ Lag$$

3. Lag between Dealer/Client Events: It is also reasonable to assume that either of the two types of flows is first missed by the defaulting party C, and then only by the non-defaulting party D. This leads to the following additional constraints on the sequence of events within the timeline:

$$\delta_C \geq \delta_D$$

$$\delta_C' \geq \delta_D'$$

4. Client Settlement vs. Dealer Observation: Except in rare and unique situations such as outright operational failures, D would not continue to pay margin flows once C commits a more serious violation by missing a trade flow, resulting in

$$\delta_C' \leq \delta_D - Margin\ Settlement\ Lag$$

Combining these inequalities results in the chronological order of events shown in the table above.

Evaluation of the Client Survival Probability

1. Client Survival at MPoR Start: As was the case for the classical model, the setup anchors the exposure date t at the termination date ETD, at the very end of the MPoR. The ETD is the same for both parties, and constitutes a convenient reference point for aligning the actions of one party against those of the other. It needs to be emphasized that the ETD for which the exposure is evaluated does not coincide with the date at which the survival probability is evaluated, e.g. for the computation of the CVA. In the simplified timeline, the counterparty survival probability should be evaluated for $t - \delta_C{}'$, the last date when C stops paying trade flows – effectively assuming that the default is due to failure-to-pay. Hence, if $EE(t)$ is the expected exposure anchored at the ETD t, then the incremental contribution to the unilateral CVA from time t is, under suitable assumptions, $EE(t) \cdot \Delta\mathbb{P}_{t-\delta_C}[\cdot]$ where $\mathbb{P}$ is the survival probability under the model's measure – later sections contain concrete examples.
2. Client Survival at MPoR End: Evaluating the default probability at the anchor date t rather than at $t - \delta_C{}'$ will introduce the slight error in computing the survival probability. While this error is relatively small and is often ignored by practitioners, it takes virtually no effort, and has no impact on model efficiency, to evaluate the survival probability at the right date.

Timeline Calibration

1. <u>Client Customization of IMA/CSA Specifications</u>: As mentioned earlier, the specific IMA/CSA terms for a given counterparty should always be ideally examined in detail, so that any non-standard provisions may be analyzed by their impact on the timeline. For those cases where such bespoke timeline construction is not practical (typically for operational reasons), two standard (*reference*) timelines are proposed here. This will allow the demonstration of the thought process behind the timeline calibration, and will provide some useful base cases for later numerical tests.
2. <u>Parametrizing the Client and the Dealer Timelines</u>: While factors such as portfolio size and dispute history with the counterparty should, of course, be considered in establishing the MPoR, an equally important consideration in calibrating the model is the nature of expected response by B to a missed margin or trade flow by C. Even under plain vanilla IMA/CSA terms, experience shows that the reactions to contract breaches are subject to both human and institutional idiosyncracies, rendering the MPoR quiet variable.
3. <u>Aggressive vs. Conservative Timeline Parametrization</u>: Recognizing that "one size does not fit all", two different calibrations shall therefore be considered; one *aggressive* which assumes the best case scenario for rapidly recognizing the impending default, and taking swift action; and one *conservative*, which takes into consideration not only a likely delay in recognizing that the counterparty default is imminent, but also the possibility that the bank may not aggressively enforce its legal rights afforded under IMA and CSA in order to avoid damaging its reputation. In both scenarios daily re-margining is assumed – if a CSA calls for less frequent margin calls than this, the MPoR must be lengthened accordingly.

Aggressive Calibration

1. Applicability of the Aggressive Calibration: The aggressive calibration applies to trading relationship between two counterparties that both have string operational competence, and where there is little reputational risk associated with swift and aggressive enforcement of the non-defaulting party's legal rights against the defaulting party.
2. Inter-dealer Monitoring and Call-outs: A good example would be trading between two large dealers, both willing to aggressively defend against a possible credit loss. The credit officers here are assumed to be diligent in the monitoring of their counterparties, and generally be able to see a default developing, rather than be caught by surprise.
3. Full Application of Operational Sophistication: Under aggressive calibration, the event of C missing or disputing a margin call by any non-trivial amount will, given C's sophistication, immediately alert D that an impending default is likely. D will not be misled by claims of valuation disputes or other excuses, and will send a Notice of Credit Support Default under the IMA/CSA the next business day after the breach of the margin agreement. At the same time, to protect itself further, D will stop both the margin and the trade flows. The counterparty is assumed to simultaneously stop paying margin and trade flows as well, so that no further payments of any kind are exchanged by the parties.
4. Elimination of Settlement/Herstatt Risk: The simultaneous action by both parties in the Aggressive scenario to stop paying the trade flows at the earliest possible moment results in the elimination of all *settlement* risk – the possibility that the dealer may continue paying on its trade flow obligations while not receiving promised payments in return. In the context of cross-currency trades, this type of settlement risk is frequently referred to as the *Herstatt* risk, after the bank that caused large counterparty losses in this manner (https://en.wikipedia.org/wiki/Settlement_risk). Such risk shall be captured in the Conservative Calibration case below, and shall be discussed in more detail in a later section.
5. Timeline of the corresponding MPoR: Despite D's immediate and aggressive response, the MPoR will still be fairly long due to the way the IMA/CSA operates in practice. In particular,

notice that the first period in the simplified timeline is between the last observation date for which the margin was fully settled, and the first date for which C misses a margin flow.

6. Breakdown of the CSA Steps: As it takes at least 2 business days to settle a margin payment, plus 1 business day between the last margin that was successfully settled and the first margin payment that was not, a minimum of 3 business days will accrue from the start of an MPoR and a margin-related PED. Further, once the margin flow is missed, D must send at least 2 notices and permit a grace period – usually 2 business days – to cure the violation before an event of default (ED) has officially taken place and an ETD has been designated.
7. Comparison with Classical MPoR Timeline: Since an ETD cannot be designated prior to the event of default, it is unlikely that an MPoR can ever be less than 7 business days. It is remarkable that even under the most aggressive set of assumptions, the MPoR is still only 3 business days shorter than the classical 2-week MPoR.
8. Detailed Breakdown of the Aggressive Timeline: The detailed taxonomy of the aggressive timeline is listed in the table down below, and essentially splits the MPoR into two sections; a margin delay period of 3 business days, and a default resolution period of 4 business days. During the latter period, C and D cease paying on the first day, leaving a period of 3 business days where neither party makes any payments. Notice that it is assumed that the ETD is declared to coincide with the ED, i.e., the dealer will terminate as quickly as legally possible.

Conservative Calibration

1. Non-aggressive Enforcement of CSA Rights: The conservative calibration is intended to cover the situation where the dealer's enforcement of its rights under the IMA/CSA is deliberate and cautious, rather than swift. There may be several reasons for such a situation, sometime acing in tandem.
2. Applicable Clients - Less Sophisticated Participants: First, a dealer, if overly trigger-happy, can gain a market-wide reputation as being rigid and litigious, potentially causing clients to

seek other trading partners. In fact, should aggressive legal maneuvers be applied to counterparties that may be considered “unsophisticated”, there is even a potential for the dealer to be perceived as predatory by the larger public.

3. <u>“Leakage” of Dealer Positions</u>: Second, there are situations where exercising the legal rights would cause an unattractive leakage of information into the broader market. As indicated earlier, this may happen for instance if the formal collateral dispute methodology of Paragraph 5 of the IDSA CSA is activated; the market poll inherent in the methodology would inevitably reveal the positions held with the counterparty to competing dealers.
4. <u>Ramification of Aggressive Legal Exercise</u>: Third, sometimes an aggressive interpretation of the legal rights can backfire in the form of lawsuits and counter-measures by the counterparty. For example, even when the dealer may have the rights to withhold payments (e.g., under Section 2(a)(iii)), it would often elect to not exercise this right immediately out of concern that a counter-ED would be raised against it or that withholding payments would exacerbate the liquidity situation of the counterparty potentially exposing the dealer to liabilities and lawsuits.
5. <u>Damage from “Improper PED” Rulings</u>: As mentioned, a particular danger is that the dealer exercises its suspension rights due to a Potential Event of Default (PED), but that subsequently the PED is ruled to not be valid. Should this happen, the dealer can inadvertently commit a breach of contract.
6. <u>Limitations with the Dealers’ Operational Capacity</u>: Of course, even if a dealer may potentially be willing to aggressively exercise its rights, it may not have the operational capacity to do so quickly. For example, the dealer may not be able to perform the required legal review on a short notice, or may not always have the efficiency to get the notices mailed out at the earliest possible date. On top of this there is always potential for technology related and human errors and oversights.
7. <u>Timeline Incurred by Conservative Calibration</u>: While it is harder to get concrete data to estimate a reasonable timeline for the Conservative case (this case being dependent not only on the IMA/CSA details, but also on the specifics of the dealer’s reputational considerations), under a perfectly reasonable set of assumptions the MPoR ends up being more than twice as long as for the Aggressive case above. Under this calibration choice, the Conservative

scenario assumes that the totality of the margin dispute negotiations, operational delays, human errors, legal reviews etc., adds up to 8 business days, yielding an MPoR of a total of 15 business days.

8. Typical Conservative CSA Event Timeline: One plausible scenario with daily re-margining could be:
    a. $t - 15 \implies D$ observes the portfolio value as needed for the margin transfer amount #1 as of $t - 15$.
    b. $t - 14 \implies D$ sends margin call #1 to C; D observes a margin transfer amount #2.
    c. $t - 13 \implies D$ sends margin call #2 to C; C honors margin call #1; D observes a margin transfer amount #3.
    d. $t - 12 \implies C$ fails to honor margin call #2 and initiates dispute; D tries to resolve the dispute while still paying and calculating the margin.
    e. $t - 7 \implies C$ fails to make a trade payment.
    f. $t - 6 \implies D$ stops paying margin and sends a PED notice.
    g. $t - 5 \implies C$ receives PED; D keeps making trade payments.
    h. $t - 3 \implies$ The PED is not cured.
    i. $t - 2 \implies D$ stops making trade payments and sends an ED notice to C, designating t as the ETD.
    j. $t \implies$ ETD.
9. Current/Interlacing Outstanding Margin Process: Notice that a number of different margin processes are simultaneously active (denoted #1, #2, and #3), reflecting the interlacing nature of the daily margin calls. Also, unlike the earlier Aggressive Calibration, the above scenario explicitly involves settlement risk, as a time period exists only where D pays trade flows (from $t - 7$ to $t - 3$, both dates inclusive).
10. Dealer/Client Payment/Settlement Lags: To translate the scenario above into the notation of the earlier sections, first notice that

$$\delta_C = 15$$

since the observation date of the last margin call (#1) honored by C is $t - 15$. Second, as D makes its last possible margin call at $t - 7$ based on an observation at time $t - 9$

$$\delta_D = 9$$

Third, as C fails to make a trade payment at $t - 7$, C's last payment date is $t - 8$, and therefore

$$\delta'_C = 8$$

And finally since D stops its trade payments at $t - 2$

$$\delta'_D = 3$$

Summary and Comparison of Timelines

1. Classical+/Classical-/Aggressive/Conservative Parametrizations: Using the notation above, the Aggressive and the Conservative scenarios are presented in the table below. For reference, the Classical+ and the Classical- versions of the classical model are presented in the table as well. Note that the 10 BD assumption of the classical MPoR lies between the two calibration choices proposed, and is closer to the Aggressive scenario.
2. MPoR Periods for CSA's with Daily Re-margining:

Parameter	Conservative	Aggressive	Classical+	Classical-
δ_C	15 BD	7 BD	10 BD	10 BD

δ_D	$9\ BD$	$6\ BD$	$10\ BD$	$10\ BD$
$\delta_C{}'$	$8\ BD$	$4\ BD$	$0\ BD$	$10\ BD$
$\delta_D{}'$	$3\ BD$	$4\ BD$	$0\ BD$	$10\ BD$

3. Caveats over Aggressive/Conservative Parameters: The Aggressive and the Conservative parameter choices represent two opposite types of dealer-client relationships, and may also be used as two limit scenarios for materiality and model risk analysis. Of course, the best approach would always be to set the model parameters based on prudent analysis of the firm's historical default resolution timelines, to the extent that it is practically feasible. The model could also conceivably treat the various time lags as random variables to be simulated as part of the exposure computations; yet it is debatable whether increasing the number of model parameters this way is warranted in practice.

Unpaid Margin Flows and Margin Flow Gap

1. Margin and Trade Flow Gaps: To formulate the model in more precise mathematical terms, this section returns to

$$E(t) = [V(t) + UTF(t) - K(t)]^+$$

and considers how to draw the analysis of the previous sections to reasonably specify both the collateral amount $K(t)$ as well as the value $UTF(t)$ of net unpaid cash flows.
2. Client Last Margin Posting Date: As with the classical model, it is assumed that the MPoR starts at time

$$t_C = t - \delta_C$$

the portfolio observation date associated with the last regular collateral posting by C. Recall that the classical model further assumes that D will stop posting collateral simultaneously with C so that

$$K(t) = c(t_C)$$

where $c(t_C)$ denotes the CSA prescribed collateral support amount calculated from the market data observed at time t_C. This is to be compared with

$$E(t) = [V(t) - K(t)]^+$$

$$K(t) = c(t - \delta)$$

3. <u>Definition of the Margin Flow Gap</u>: In contrast to

$$K(t) = c(t_C)$$

this model assumes that D will continue posting and returning collateral to C for all contractual margin observations dates t_i whenever required by the CSA, after

$$t_C = t - \delta_C$$

and up to and including the observation date

$$t_D = t - \delta_D$$

The presence of an observation period of non-zero length for which D is posting and returning collateral but C is not is referred to as *margin flow gap*.

4. <u>Choosing the Collateral Computation Date</u>: Here, it is always expected that

$$t_D \geq t_C$$

which therefore in effect assumes the possibility of a time interval $(t_D, t_C]$ where only D honors its margin requirements. In this interval, D can match its contractually stipulated amounts $c(t_i)$ only when they involve transfers from D to C. This asymmetry results in D holding at time t the *smallest* collateral computed in the observation interval $[t_C, t_D]$ i.e.

$$K(t) = \min_{t_i \in [t_C, t_D]} c(t_i)$$

5. <u>Implications of the Collateral Date Choice</u>: The *worst case* form of the above provides a less optimistic view on available collateral than on classical modeling, resulting in larger exposure whenever there are multiple collateral observation dates in $[t_C, t_D]$ – it is assumed that one of the observation dates t_i always coincides with the start of the MPoR t_C. All other things being equal, the difference in exposure relative to the classical model will increase with $\delta_C - \delta_D$. If $\delta_C - \delta_D$ is kept constant, the difference will increase with more frequent re-margining. Note that

$$K(t) = \min_{t_i \in [t_C, t_D]} c(t_i)$$

matches

$$K(t) = c(t_C)$$

when

$$\delta_C = \delta_D$$

Unpaid Trade Flows and Trade Flow Gap

1. Origin of the Trade Flow Gap: According to the assumptions in the earlier sections, the last date when C is still paying the trade flows is

$$t_C{}' = t - \delta_C{}'$$

and the last date when D is still trade flows is

$$t_D{}' = t - \delta_D{}' \geq t_C{}'$$

The period when D is still paying trade flows while C does not is referred to as the *trade flow gap*.

2. Projecting the Unpaid Trade Flow Value: The value of the net trade flows unpaid by the termination date t can be expressed using the notation established so far as

$$\begin{aligned} UTF(t) &= TF_{C\rightarrow D}(t;\ (t_C{}', t]) + TF_{D\rightarrow C}(t;\ (t_D{}', t]) \\ &= TF_{C\rightarrow D}(t;\ (t_C{}', t_D{}']) + TF_{NET}(t;\ (t_D{}', t]) \end{aligned}$$

where an arrow indicates the direction of the trade flows and $C \rightarrow D$ ($D \rightarrow C$) trade flows have positive (negative) sign.

3. Same Date Dealer/Client Flows: In calculating the above equation, care needs to be taken on how the trade flows are aggregated and accrued to the termination date t. Cash flows of opposite direction scheduled to be paid in the same currency on the same date (for instance, the two legs of an ordinary single-currency interest-rate swap) in the period $(t_D{}', t]$ are aggregated (netted) at the cash flow date, therefore only the aggregated amount – their

difference – enters in to the above equation. The aggregated amount of the missed cash flows should be accrued to time t at the interest rate of the currency in question, and then converted to D's domestic currency.

4. Different Currency Dealer/Client Flows: Cash flows in opposite direction scheduled to be paid in *different* currencies on the same date (for instance, the two legs of a cross-currency interest rate swap) are *not* netted at the cash flow date. The missed cash flow amounts in each currency should be accrued to time t at the relevant interest rates, and then converted to D's domestic currency.
5. Physically Settled Dealer/Client Flows: The value of each asset flow (for instance, a swap that would result from exercising a physically settled swaption) should be obtained through pricing at time t of the undelivered asset in D's domestic currency. Generally, asset flows are not aggregated.
6. Collateral available at Termination Date: To analyze the impact of these assumptions on the expression for $UTF(t)$ above, consider for simplicity a zero-threshold margin agreement with no MTA/rounding. Then, from

$$K(t) = \min_{t_i \in [t_C, t_D]} c(t_i)$$

the collateral available to D at the termination date can be written as

$$K(t) = V(t_{COL})$$

$$t_{COL} = \min_{t_i \in [t_C, t_D]} V(t_i)$$

7. Corresponding CSA Implied Client Exposure: Substituting

$$UTF(t) = TF_{C \to D}(t;\ (t_C{}', t_D{}']) + TF_{NET}(t;\ (t_D{}', t])$$

and

$$K(t) = V(t_{COL})$$

$$t_{COL} = \min_{t_i \in [t_C, t_D]} V(t_i)$$

into

$$E(t) = [V(t) + UTF(t) - K(t)]^+$$

yields, for the simple CSA considered

$$E(t) = [V(t) - V(t_{COL}) + TF_{C\to D}(t;\ (t_C{}', t_D{}']) + TF_{NET}(t;\ (t_D{}', t])]^+$$

8. <u>CSA Implied Client Exposure Components</u>: $E(t)$ above implies that trading flows from D to C can occur within the MPoR. Thus trade flows have the potential to generate large spikes in the exposure profiles – especially in the presence of a trade flow gap where only D pays trade flows. To see this, the exposure components of $E(t)$ can be further drilled down as follows. First, ignoring the minor discounting effects inside the MPoR, the portfolio value at time t_{COL} can be represented as a sum of the portfolio's forward value V_F to time t and the value of all the trade flows taking place <u>after t_{COL}</u> and <u>up to and including t</u>:

$$V(t_{COL}) = V_F(t_{COL}; t) + TF_{NET}(t_{COL};\ (t_{COL}, t])$$

9. <u>Trade Flow Post Collateral Date</u>: This may be further re-written as

$$\begin{aligned} TF_{NET}(t_{COL};\ (t_{COL}, t]) \\ &= TF_{NET}(t_{COL};\ (t_{COL}, t_C{}']) + TF_{C\to D}(t_{COL};\ (t_C{}', t_D{}']) \\ &+ TF_{D\to C}(t_{COL};\ (t_C{}', t_D{}']) + TF_{NET}(t_{COL};\ (t_D{}', t]) \end{aligned}$$

which, together with

$$V(t_{COL}) = V_F(t_{COL}; t) + TF_{NET}(t_{COL};\ (t_{COL}, t])$$

allows re-stating

$$E(t) = [V(t) - V(t_{COL}) + TF_{C\to D}(t;\ (t_C', t_D']) + TF_{NET}(t;\ (t_D', t])]^+$$

in the following form:

$$\begin{aligned} E(t) = \{V(t) - V_F(t_{COL}; t) + TF_{C\to D}(t;\ (t_C', t_D']) - TF_{C\to D}(t_{COL};\ (t_C', t_D']) \\ + TF_{NET}(t;\ (t_D', t]) - TF_{NET}(t_{COL};\ (t_D', t]) - TF_{NET}(t_{COL};\ (t_{COL}, t_C']) \\ - TF_{D\to C}(t_{COL};\ (t_C', t_D'])\}^+ \end{aligned}$$

10. <u>The Market Driven Portfolio Change $t_{COL} \to t$</u>: The terms in

$$E(t) = [V(t) - V(t_{COL}) + TF_{C\to D}(t;\ (t_C', t_D']) + TF_{NET}(t;\ (t_D', t])]^+$$

have been re-arranged into five separate components, corresponding to the five different contributions to the exposures. The first is

$$V(t) - V_F(t_{COL}; t)$$

- the change of the portfolio forward value to time t driven by the change in the market factors between t_{COL} and t. This term is driven by the volatility of the market factors between t_{COL} and t; it produces no spikes in the expected exposure profile.

11. <u>The Market Driven Trade Flow $t_C' \to t_D'$</u>:

$$TF_{C\to D}(t;\ (t_C', t_D']) - TF_{C\to D}(t_{COL};\ (t_C', t_D'])$$

represents the change of the value of the trade flows scheduled to be paid – but actually unpaid – by C in the interval $(t_C', t_D']$ resulting from the change of the market factors between t_{COL} and t. This term is driven by the volatility of the market factors between t_{COL} and t; it produces no spikes in the expected exposure profiles.

12. <u>The Market Driven Trade Flow $t_D' \rightarrow t$</u>:

$$TF_{NET}(t;\ (t_D', t]) - TF_{NET}(t_{COL};\ (t_D', t])$$

is the change of the value of the net trade flows between C and D scheduled to be paid – but actually unpaid – in the interval $(t_D', t]$ resulting from the change in the market factors between t_{COL} and t. This term likewise produces no spikes in the expected exposure profile.

13. <u>Net Trade Flow between $t_{COL} \rightarrow t_C'$</u>:

$$-TF_{NET}(t_{COL};\ (t_{COL}, t_C'])$$

is the negative value of the net trade flow between C and D scheduled to be paid – and actually paid – in the interval $(t_{COL}, t_C']$. Paths where D is the net payer – so that TF is negative – contribute to the upward spikes in the EE profile.

14. <u>Dealer Trade Flow across $t_C' \rightarrow t_D'$</u>: The negative value of the trade flows scheduled to be paid – and actually paid – by C to D in the interval $(t_C', t_D']$. Whenever such trade flows are present D is always the payer, leading to upward spikes in the EE profile. Furthermore, in some cases, the spikes arising from this term can be of extreme magnitude, e.g., the scheduled notional exchange in a cross-currency swap where D pays the full notional, but receives nothing.

Numerical Examples

1. Illustrative Exposures and CVA Magnitudes: To gain intuition for the model, this section presents exposure profiles and CVA metrics for several trade and portfolio examples, using both the Aggressive and the Conservative calibrations. The focus is on ordinary and cross-currency swaps, as these instruments are the primary sources of exposures in most dealers. For all the numerical examples, the stochastic yield curves are driven by one-factor Hull-White model; for cross-currency swap examples, the FX rate is assumed to follow a Black Scholes model.
2. Single Swap Classical/Conservative Exposures: Andersen, Pykhtin, and Sokol (2017) first examine how the model differs from the classical exposure approach. They use Monte Carlo simulation on a USD 10 MM 1-year par-valued vanilla interest rate swap to compare exposures of the Conservative calibration with those computed against the Classical+ and the Classical- models – see the Table above. To make comparisons more meaningful, they override the default setting of 10 business days for the Classical model, and instead set it equal to 15 business days – the length of the MPoR for the Conservative calibration.
3. Single Swap - Classical Exposure Estimations: As they note in their figures, the Classical- calibration is, of course, the least conservative setting, as it ignores both the effect of trade flows and that of the margin asymmetry. The Classical+ calibration tracks the Classical- calibration at most times, but contains noticeable spikes around the last 3 cash flow dates. No spikes occur in their figure on the first quarterly cash flow date, as they assume that the floating rate is fixed at the fixed rate, making the net cash flow zero in all scenarios.
4. Single Swap Conservative Model - MPoR End: The conservative calibration results also contain spikes around the cash flow dates, although these differ from the Classical+ calibration in several ways. First, the Conservative calibration always recognizes that there will be a part towards the end of the MPoR (after time $t_D{}'$) where C and D will both have stopped paying margin and coupons; as a result, the spikes of the Conservative calibration start later (here: 3 business days) than those of the Classical+ calibration.
5. Single Swap Conservative Model - Trade Flows: Second, the initial part of the spike – in the period from $t_C{}'$ to $t_D{}'$ - is substantially higher for the Conservative calibration due to the assumption of only D paying cash flows in this sub-period. The remainder of the spike is comparable in height to the Classical+ spike.

6. Single Swap - Classical vs. Conservative: *Between* spikes the Conservative calibration produces higher exposures than both the Classical- and the Classical+ methods – by around 40%. This is, of course, a consequence of the *worst case* margin asymmetry mechanism in

$$K(t) = \min_{t_i \in [t_C, t_D]} c(t_i)$$

the effect of which will grow with the diffusion volatility of the rate process. Of course the last coupon period again has no exposure between spikes, since the volatility of swap prices vanishes after the last coupon rate fixing.
7. Single Swap: Aggressive Calibration Exposures: Comparison the Aggressive calibration to the Classical+ and the Classical- calibrations are qualitatively similar. A detailed comparison is therefore skipped here, but Andersen, Pykhtin, and Sokol note that the pick-up in exposure from margin asymmetry falls to about 15%, rather than the 40% observed for the Conservative calibration – a result of the fact that, for Aggressive calibration, the *worst case* margin result is established over much fewer days. Andersen, Pykhtin, and Sokol (2017) demonstrate the comparison of the exposure profiles for the Aggressive and the Conservative calibrations in their graphs; as expected, the Conservative calibration leads to both bigger and wider exposure spikes, as well as to higher exposure levels between spikes.
8. Single Swap - Maturity/Coupon Effect: While instructive, the 1-year vanilla swap example is quiet benign exposure-wise; not only is the instrument very short-dated, it also allows for netting of coupons on trade-flow dates, thereby reducing the effects of trade flow spikes. Andersen, Pykhtin, and Sokol (2017) relax both effects by increasing the maturity of the swap, and by making the fixed and the floating legs pay on different schedules – and illustrate the exposure results in a separate figure.
9. Single Swap - Coupon Payment Mismatch: The upward exposure spikes occur twice per year, whenever the dealer must make a semi-annual fixed payment. On the dates when the counterparty makes a quarterly floating payment that is not accompanied by a fixed payment by the dealer, a narrow *downward* spike emerges, due to the delay in transferring the coupon back to the counterparty through the margin mechanism.

10. Single Swap - Impact of Maturity: The exposure between spikes is also much larger, a consequence of the higher volatility of the 10-year swap compared to the 1-year swap. Of course, as the swap nears its maturity, its duration and volatility die out, so the non-spike exposure profile predictably gets pulled to zero at the 10-year date. Also, as predicted, the Aggressive calibration produces much lower exposures than the Conservative calibration, by nearly a factor of 2.
11. Single Cross-Currency Swap - Herstatt Risk: A more extreme form of trade flow spikes will occur for cross-currency swaps, where neither the coupon nor the final payment can be netted. The notional payment, in particular, can induce a very significant payment exposure spike (the Herstatt Risk), whenever the exposure model allows for a trade flow gap. To recall, the Conservative calibration has a trade flow gap, but the Aggressive calibration does not.
12. Single Cross-Currency Swap - Coupon Mismatch: As confirmed by Andersen, Pykhtin, and Sokol (2017), the exposure for a conservative calibration has a very large spike that is not present in the Aggressive calibration. Like Conservative calibration, Aggressive calibration will, of course, still produce spikes at the cash flow dates, due to margin effects.
13. Single Cross-Currency Swap - Principal Mismatch: As a consequence, the principal exchange is likely fa away from break-even, resulting in a large exposure spike at maturity. Although smaller than for the Conservative calibration, the spike at maturity is also present for the Aggressive calibration; while both C and D pay the principal exchange, C does not make the margin transfer for the balance of the principal payments.

Portfolio Results

1. Single Swap Portfolio: Setup Overview: For individual trades, the presence of localized spikes in the exposure profiles may ultimately have a relatively modest impact on the credit risk metrics, such as the CVA – after all, the likelihood of the counterparty default in a

narrow time interval around quarterly or semi-annual cash flow event is typically low. For a *portfolio* of swaps, however, the spikes will add up and affect the net exposure profile nearly everywhere.

2. Single Swap Portfolio - Draw Algorithm: To illustrate this, Andersen, Pykhtin, and Sokol (2017) picked 50 interest rate swaps with quarterly floating rate payments and semi-annual fixed rate payments of 2%. The terms of the swap were randomized as follows:
    a. Notionals of the swap are sampled uniformly on the interval from 0 to USD 1 MM.
    b. Duration of the fixed leg payments – payer or receiver – is random.
    c. Start date of each swap is subject to a random offset to avoid complete MPoR overlaps.
    d. Swap maturities are scaled uniformly on the interval from 1 to 10 years.
3. Single Swap Portfolio: Aggressive vs Conservative: They also illustrate the resulting exposure profile in a separate figure. Both the Conservative profile, and to a lesser extent, the Aggressive profile include frequent spikes around the trade-flow times above the *baseline* exposure level. As seen in the next section, these spikes make a significant contribution to the CVA metrics. As before, the exposure under the Conservative calibration is twice as large as that under the Aggressive calibration.
4. XCCY Swaps Portfolios Generation Algorithm: To repeat the portfolio results with a cross-currency swap, Andersen, Pykhtin, and Sokol (2017) constructed a 50 deal portfolio by randomization, using the following rules.
    a. EUR notionals are sampled uniformly in the interval from 0 to USD 10 MM
    b. USD notionals are 1.5 times the EUR notionals
    c. EUR leg has a fixed semi-annual coupon of 3%, and the USD leg floating quarterly coupon
    d. Direction of the fixed leg payments (payer or receiver) is random
    e. Start date of each swap is subject to a random offset to avoid complete MPoR overlaps
    f. Swap maturities are sampled uniformly in the interval from 1 to 10 years
5. XCCY Swap Portfolio: Conservative vs. Aggressive: As shown in their figure for an expected exposure for a 10Y cross-currency swap, Andersen, Pykhtin, and Sokol (2017)

generated the swaps within the portfolio such that the principal exchanged and the fixed coupon are not at-the-money, to mimic a typical situation corresponding to a portfolio of seasoned trades. As demonstrated in another figure for the expected exposure of the cross-currency swap portfolio, the exposure for the conservative calibration is, as expected, dominated by a series of Herstatt risk spikes, one per swap in the portfolio.

CVA Results

1. CVA Computation from Expected Exposure: As mentioned earlier, a common use of the expected exposure results is the computation of the CVA. Under suitable assumptions, D's unilateral CVA may be computed from the expected exposure (EE) profile as

$$CVA = (1 - R) \int_0^\infty P(u + \delta_C{}')EE(u + \delta_C{}')dX(u)$$

where R is the recovery rate, $P(t)$ is the time-0 discount factor to time t, and $X(t)$ is the time-0 survival probability of C to time t. As discussed before, the exposure profile here is offset by $\delta_C{}'$ to properly align it with the default events.

2. CVA Metrics Dealer/Client Settings: The CVA metric serves as a convenient condensation of the exposure profiles of the previous two sections into single numbers, and Andersen, Pykhtin, and Sokol (2017) tabulate the CVA numbers for the corresponding instruments/portfolios. The CVA integral was discretized using a daily grid, assumed at

$$R = 40\%$$

and the forward default intensity is left constant at 2.5% such that

$$X(t) = e^{-0.025t}$$

For reference, the table also includes the results of the Classical method, with the MPoR length equal to both that of the Aggressive calibration (7 BD) and the Conservative Calibration (15 BD).

3. CVA Comparison - Classical/Conservative/Aggressive: Their results confirm what was seen earlier. For instance, the CVA for the Aggressive calibration is 50% to 70% smaller than that for the Conservative calibration. In Addition, the CVA of the Conservative calibration is between 50% and 100% larger than that of the Classical+ calibration – at similar MPoR – which in turn is larger than the CVA for the Classical- calibration by around 5% to 25%. Not surprisingly the CVA results for the XCCY portfolio are particularly high in the Conservative calibration due to the Herstatt risk.

Improvement of the Computation Times

1. Computation Speed-Up using Coarse Grids: In exposure calculations for realistic portfolios, horizons can be very long, often exceeding 30 years. For such lengthy horizons, brute-force Monte-Carlo exposures on a daily, or even weekly, time grid will often be prohibitively slow. It is therefore common to use daily simulation steps only for the earliest parts of the exposure profile (e.g., the first month), and then gradually increase the step-length over time to monthly or quarterly, in order to keep the total number of simulation dates manageable. Unfortunately, such a coarsening of the time-grid will inevitably fail to capture both the *worst case* margin effect and the trade spikes that are key to the exposure model.
2. Coarse Grid Lookback Analysis: The next two sections look at ways to capture exposure without having to resort to brute-force daily simulation. A common speed-up technique for the Classical model – the Coarse Grid Look-back Model – is first reviewed, and its

shortcomings and pitfalls are highlighted. An improved practical technique based on Brownian Bridge is the proposed.

The Coarse Grid Lookback Method and its Shortcomings

1. Layout of the Coarse Grid: Assume that the portfolio is not computed daily, but instead on a coarse grid $\{s_j\}$ where j runs from 1 to J. This section uses s rather than t to distinguish the model grid from the daily margin calculation grid.

   Points on the Coarse Grid: In the classical model, the collateral depends only on the portfolio value at the start and at the end of the MPoR, i.e., $s_j - \delta$ and s_j, as is seen from

$$E(t) = [V(t) - K(t)]^+$$

$$K(t) = c(t - \delta)$$

$$K(t) = [V(t - \delta) - h_C(t - \delta)]^+ - [-V(t - \delta) - h_D(t - \delta)]^+$$

   where MPoR is usually around

$$\delta = 10\ BD$$

for CSA's with daily margining. To achieve acceptable computational performance, the time step of the coarse model grid $s_j - s_{j-1}$ must be significantly greater than the length of the MPoR. This, however, would preclude one from establishing a portfolio value at $s_j - \delta$.

2. Introducing a Lookback Node: The coarse grid lookback method deals with this issue by simply adding a second *lookback* time point $s_j - \delta$ to all *primary* measurement times s_j, in effect replacing each node on the coarse model grid by a pair of closely spaced nodes. For each simulated portfolio path, the portfolio value at the lookback point is then used to determine the collateral available at the corresponding primary time point.
3. Slowdown due to the Lookback: The Coarse Grid Lookback Scheme causes, at worst, a factor of $\times$ 2 slowdown relative to valuing the portfolio once per node of the Coarse model grid. If even a $\times$ 2 performance loss is not acceptable, a Brownian Bridge constructed between the primary coarse grid nodes can be used to interpolate the value of the portfolio at each lookback point, see, for example, Pykhtin (2009). Notice that the use of the Brownian Bridge for this purpose should not be confused with its use in the next section.
4. Shortcoming of the Model: The Coarse Grid Lookback method is a common way of addressing the mismatch between the long time-step of the coarse model grid and the much shorter MPoR. Similar to the commonly used models of uncollateralized exposure, the method produces accurate – with respect to the underlying assumptions of the Classical model – exposure numbers at the coarse grid time points, but provides no information on the exposure between the grid points.
5. Collateralized vs. Uncollateralized Grid Exposures: For uncollateralized positions, the exposure profiles are reasonably smooth, so one can safely interpolate between the grid points for calculating integral quantities, such as the CVA. In collateralized case, however, one cannot rely on such interpolations because the true exposures, as has been seen above, is likely to have spikes and jumps between the grid points. The Coarse Grid Lookback method has no means to determining the position or the magnitude of the irregularities between the grid points, and thus, is not suitable for CVA or capital calculations.
6. Classical+ Model - Coarse Grid Impact: To briefly expand on this, consider the Classical+ version of the classical model. Here it is assumed that all trade flows are paid within the

MPoR, where, as shown before, trade flows often result in exposure spikes. Exposure profiles computed from daily time steps would consequently show spikes from all trade flows until the maturity of the portfolio.

7. Trade Flows outside the Simulated MPoR's: In contrast, in a typical implementation with sparsely spaced MPoR's, only trade flows that happen to be within a sparsely simulated MPoR's may result in spikes; the exposure profile would then miss all other flows.
8. Simulation Calendar Impact on Exposure: Furthermore, as the location of the simulation point will likely change with the advancement in the calendar time, trade flows would move in and out of the simulated MPoR's, and the exposure profile one report on any given day may very well differ significantly from those that were reported the day before. This in turn causes CVA or risk capital to exhibit significant, and entirely spurious, oscillations.
9. Classical- Model - Coarse Grid Impact: While the Classical- exposure model does not exhibit outright spikes, its exposure profiles still exhibit jumps around significant trade flows. The classical coarse grained implementation would not be able to resolve the position of these jumps, instead only showing the conservative jumps between two exposure measurement points often separated by many months. This creates another source of instability, present in both the Classical- and the Classical+ versions of the classical model.
10. Illustration using the Forward CVA: To illustrate the effects described above, Andersen, Pykhtin, and Sokol (2017) define the concept of time t forward CVA, denoted CVA_t, obtained by
    a. Changing the lower integration limit in

$$CVA = (1 - R) \int_0^\infty P(u + \delta_C{}')EE(u + \delta_C{}')dX(u)$$

    from 0 to t, and
    b. Dividing the result by $P(t)X(t)$.

Using the same portfolio of 50 EUR-USD cross-currency swaps, they show the t-dependence of CVA_t on a daily grid to portfolio maturity.

11. Spurious Oscillations from Moving Windows: As CVA is an integral of exposures, spikes in exposure profile profiles should result in jumps rather than oscillations in CVA_t. However, when one of the Coarse Grid Lookback method's sparsely located *MPoR window* moves past a large trade flow, the contribution to the CVA temporarily increases only to drop back when the window moves past the large trade flow. As illustrated by Andersen, Pykhtin, and Sokol (2017), such oscillations are spurious and their presence is highly unattractive when CVA is computed and reported as part of daily P&L.

Brownian Bridge Method

1. Brute Force Portfolio Value Simulation: Overcoming the deficiencies outlined in the previous section is, unfortunately, prohibitively expensive for large portfolios, mostly due to the expense of repricing the entire portfolio at each simulation path and each observation date.
2. Daily Simulation of Risk Factors: On the other hand, merely simulating the risk factors at a daily resolution is generally feasible, as the number of the simulated risk factors is typically relatively small (i.e., several hundred) and the equations driving the risk factor dynamics are usually simple.
3. Generation of Daily Trade Flows: Furthermore, having produced risk factors on a daily grid, one can normally also produce all realized trade flows along each path because trade flows, unlike trade prices, are usually simple functions of the realized risk factors.
4. Risk Factors under Daily Resolution: Based on these observations, Andersen, Pykhtin, and Sokol (2017) propose the following algorithm for generating paths of portfolio values and trade flows on a daily time grid. First, simulate paths of market risk factors with daily resolution.
5. Trade Flow under Daily Resolution: For each path m, use the simulated market risk factors to calculate trade flows on the path with daily resolution.

6. Coarse Grid Path Portfolio Valuation: For each path m and each coarse portfolio valuation time point s_j $(j = 1, \cdots, J)$ use the simulated risk factors to calculate portfolio value on the path $V_m(s_j)$
7. Trade Flow Adjusted Forward Value: For each path m and each time point s_j use the trade flows realized on the path between times s_{j-1} and s_j to calculate the *forward* to s_j portfolio value $V_m{}'(s_{j-1};\ s_j)$:

$$V_m{}'(s_{j-1};\ s_j) = V_m(s_{j-1}) - TF_{m,NET}(s_j;\ (s_{j-1}, s_j])$$

   Note that $V_m{}'(s_{j-1};\ s_j)$ is not a true forward value because the realized trade flows are subtracted from the s_{j-1} portfolio value rather than the true forward value being calculated at time s_{j-1}.
8. Portfolio Value Local Variance Estimation: For each path m and each portfolio measurement time point s_j compute the local variance $\sigma_m{}^2(t_{j-1})$ for the portfolio value *diffusion* $V_m(s_j) - V_m{}'(s_{j-1};\ s_j)$ via a kernel regression estimator – e.g., the Nadaraya-Watson Gaussian kernel estimator (Nadaraya (1964), Watson (1964)) conditional on the realized value of $V_m{}'(s_{j-1};\ s_j)$. The selection of bandwidth for the kernels is covered in, e.g., Jones, Marron, and Sheather (1996). In their numerical results, Andersen, Pykhtin, and Sokol (2017) use the *Silverman's Rule of Thumb* (Silverman (1986)).The term *diffusion* is used to indicate that the portfolio value change has been defined to avoid any discontinuities resulting from trade flows.
9. Brownian Bridge Local Interpolation Scheme: For each path m and each exposure measurement time point s_j, simulate an independent, daily sampled, Brownian Bridge process (see, for instance, Glasserman (2004)) that starts from the value $V_m{}'(s_{j-1};\ s_j)$ at time s_{j-1} and ends at the value $V_m(s_j)$ at time s_j. The volatility of the underlying Brownian motion should be set to $\sigma_m(s_{j-1})$.
10. Brownian Bridge Portfolio Value Approximation: For each path m and each exposure measurement time point s_j, the portfolio values for each time u of the daily grid in the

interval (s_{j-1}, s_j) are approximated from the simulated Brownian bridge $BB_m(u)$ by adding the trade flows realized along the path m between the times u and s_j:

$$V_{m,APPROX}(u) = BB_m(u) + TF_{m,NET}(s_j;\ (u, s_j])$$

11. Rational behind Brownian Bridge Methodology: In a nutshell, the algorithm above uses a Brownian bridge process to interpolate portfolio values from a coarse grid in a manner that ensures that intermediate trade flow events are handled accurately. The algorithm produces paths of portfolio values and trade flows in a daily time grid, wherefore exposure can be calculated as described earlier with daily resolution and overlapping MPoR's. Furthermore, daily sampling allows for further refinements of the proposed model by consistently incorporating thresholds, minimum transfer amount, and rounding.
12. Brownian Bridge Portfolio Weiner Increment: A key assumption made by the Brownian Bridge algorithm is that the portfolio value process within the interpolation interval is a combination of an approximately normal *diffusion* overlaid by trade flows. For Weiner process models without risk factor jumps, this approximation is accurate in the limit of infinitesimal interpolation interval, and is often a satisfactory approximation for monthly or even quarterly interpolation steps.
13. Brownian Bridge Approximation Error #1: Nevertheless, the presence of trade flows that depend on the values of the risk factors between the end points introduces two types of errors. Suppose that there is a trade flow at an end point that depends on the risk factor value at the date when it is paid. The independence of the Brownian Bridge process from the risk factor processes that drive that trade flow would result in an error in the expected exposure profile around the trade flow date. This error is largest for trade flows in the middle of the interpolation interval and disappears for trade flows near the ends of the interval.
14. Brownian Bridge Approximation Error #2: Suppose that there is a trade flow that occurs at the end point of an interpolation interval, but whose values depend entirely on the realization of the risk factor within the interpolation interval. A typical example would be a vanilla interest rate swap where the floating leg payment being paid at the end of the interpolation interval depends on the interest rate on a date within the interval. Even in the absence of a

trade flow within the interpolation interval, the volatility of the swap value drops at the floating rate fixing date as some of the uncertainty is resolved. Thus the *true* swap value process has two volatility values; a higher value before the rate fixing date and a lower value after the rate fixing date. In contrast the approximation algorithm assumes a single value of volatility obtained via kernel regression between the end points. Similar to the de-correlation error discussed above, the error resulting from this volatility mismatch is largest for fixing dates in the middle of the interpolation interval and disappears for fixing dates near the end points.

15. Trade Flow at Mid-Interval: To illustrate the two errors above, Andersen, Pykhtin, and Sokol (2017) compute the expected exposure profile for a one year interest rate swap when a monthly grid for full valuation is situated so that the payments/fixing dates sit roughly in the middle of the interpolation interval, thus maximizing the error of the Brownian Bridge algorithm.
16. Unbiased Nature of the Error: While there are, as expected, some error around the trade flow dates, they are acceptable in magnitude and overall unbiased, in the sense that the over-estimation of the exposure is about as frequent as the under-estimation of the exposure. For, say, CVA purposes, the Brownian Bridge results would therefore be quite accurate.
17. Trade Flows at Interval End: Andersen, Pykhtin, and Sokol (2017) also compute the expected exposure profiles when the monthly valuation points are aligned with the rate fixing/payment dates. In this case, Brownian Bridge approximation is nearly exact.
18. Choice of Valuation Grid Location: Of course, in practice such alignment is only possible for a single trade or a small netting set, and not for large portfolios where trade flows will occur daily. Yet, even for large netting sets the calculation accuracy will improve if the interpolation pillars are aligned with the largest trade flows (e.g., principal exchange dates for the largest notional amounts). In practice, errors can be typically expected to be somewhere between the two extremes discussed above.
19. Performance Gains from Brownian Bridge: While the exact speed up provided by the Brownian Bridge method depends on the implementation, for most portfolios the overhead of building the Brownian Bridge at a daily resolution is negligible compared to computing the exposure on the model's coarse grid.

20. Comparison with Coarse Grid Lookback: In this case, the computational effort of the daily Brownian Bridge method is about half the computational effort of the Coarse Grid Lookback method, as the former does not require adding a *lookback* point to each of the primary coarse grids. Thus the Brownian Bridge is both faster and significantly more accurate than the Standard Coarse Grid Lookback method.

Initial Margin

1. Role of IM: Extra Protection: The posting of initial margin (IM), in addition to the regular variation margin collateral (VM), provides dealers with a mechanism to gain additional default protection. The practice of posting IM has been around for many years, typically with IM being computed on trade inception on a trade level basis.
2. Modeling Static Initial Margin Exposure: This type of IM is entirely deterministic and normally either stays fixed over the lifetime of a trade or amortizes down according to a pre-specified schedule. As a consequence, modeling the impact on the exposure is trivial; for the exposure points of interest all trade level IM amounts are summed across the netting set and the total – which is the same for all paths – is subtracted on the portfolio value from each path.
3. Dynamically Refreshed Initial Margin (DIM): A more interesting type of IM is dynamically refreshed to cover portfolio-level close-out risk at some high percentile, often 99%. This type of margin is routinely applied by Clearinghouses (CCPs) and by margin lenders, and will also soon be required by regulators for inter-dealer OTC transactions.
4. BCBS IOSCO Initial Margin Rules: In particular, in 2015 BCBS and IOSCO issued a final framework on margin requirements (BCBS and IOSCO (2015)) under which two covered entities that are counterparties in non-centrally cleared derivatives are required to:
    a. Exchange VM under a zero threshold margin agreement, and
    b. Post IM to each other without netting the amounts.

Covered entities include all financial firms and systematically important non-financial firms. Central banks and sovereigns are not covered entities.

5. Third Party Management of IM: IM must be held in a default remote way, e.g., by a custodian, so that IM posted by the counter-party should be immediately available to it should the other counter-party default.
6. Internal Model/Standardized Schedule IM: Under the BCBS and IOSCO rules, regulatory VM can be calculated by an internal model or by lookup in a standardized schedule.
7. Internal Models Based IM Calculation: If an internal model is used, the calculation must be made at the netting set level as the value-at-risk at the 99% confidence level. The horizon used in this calculation equals 10 business days for daily exchange of VM or 9 business days plus a re-margining period for less frequent exchange of VM.
8. Denial of Cross-Asset Netting: Diversification across distinct asset classes is not recognized, and the IM internal model must be calibrated to a period of stress of each of the asset classes.
9. Handling Adjustments to the IM: The required levels of IM are changed as the cash flows are paid, new trades are booked, or markets move. To accommodate this, dealers would call for more IM or return the excess IM.
10. Complexities Associated with the IM Estimation: For trades done with CCPs or under the new BCBS-IOSCO rules, one must find a way to estimate the future IM requirements for each simulated path. No matter how simple the IM VaR model is, it will likely be difficult to perform such calculations in practice if one wants to incorporate all the restrictions and twists of the IM rules; stress calibration, limited diversification allowance, and, for CCP's, add-ons for credit downgrades and concentration risk.
11. Estimating Simplified Version of IM: However, it is possible to utilize the model in this chapter to calculate the counter-party exposures if one ignores these complications. Note that ignoring such complications is conservative, as it will always lead to a *lower* of IM, and therefore, to a *higher* level of exposure.
12. t_C as IM Delivery Date: To calculate the exposure at time t the assumption here is that the last observation date for which C would deliver VM to D is

$$t_C = t - \delta_C$$

It is reasonable to assume that this date is also the last date at which C would deliver IM to a custodian.

13. Simplified IM Mechanics Timeline: To simplify modeling, it is assumed that the custodian would not return any amount to C for observation dates after $t - \delta_C$. Thus, to calculate exposure at time t, IM on a path has to be estimated from the dynamics of the exposure model as of time $t - \delta_C$
14. t_C IM Estimate using Gaussian Portfolio Evaluation: Assuming, as is common in practice, that the portfolio values are locally Gaussian, it suffices to know the local volatility for the portfolio value for the period $[t - \delta_C, t]$ estimated at $t - \delta_C$. Denoting the IM horizon by δ_{IM} and the local volatility of the portfolio value at time u on path m via $\sigma_m(u)$, the IM available to D at the ETD date t on path m is given by

$$IM_m(t - \delta) = \sigma_m(t - \delta)\sqrt{\delta_{IM}}\Phi^{-1}(q)$$

where q is a confidence level – often 99% - and $\Phi^{-1}(\cdot)$ is the inverse of the standard normal cumulative distribution function.

15. Kernel Regression Based Local Volatility: Estimating the local volatility can be done via kernel regression, as in the previous section. If the portfolio value is simulated at both $t - \delta_C$ and t, the kernel regression for could be run on the $P\&L$ $V(t) - V(t - \delta_C) + TF_{NET}(t;\ (t - \delta_C, t])$ conditional on the realization of the portfolio value on path m at the beginning of the $MPoR$ $V_m(t - \delta_C)$. If one does not calculate the portfolio value at the beginning of the $MPoR$ but uses the fast approximation outlined earlier instead, $\sigma_m(t - \delta_C)$ can be set equal to the local volatility estimated for the time interval that encloses the given $MPoR$ $[t - \delta_C, t]$.
16. Brownian Bridge IM Plus VM: Thus, the Brownian Bridge framework can now produce not only the collateralized exposure under VM alone, but also a reasonable estimate of the collateralized exposure under a combination of VM and IM.
17. IM Timing and Transfer Mechanics: In calculating the IM, an important consideration is the timing and the mechanics of the adjustment to the IM when C misses a margin flow or a trade flow.

18. Assumption - IM Return to the Client: For instance, when a large trade reaches maturity, the portfolio VaR may be reduced, in which case, some of the IM posted by C must be refunded. The issue of whether this refund can be delayed due to an ongoing margin dispute is not yet fully resolved. To simplify the calculations, it is assumed that no part of the IM is returned to C during the $MPoR$.
19. 10Y OTC Swap VM + IM EE: To show some numerical results, Andersen, Pykhtin, and Sokol (2017) consider the individual trades and portfolios of the earlier section. They use the case of a 10Y vanilla swap for which they calculate the impact of IM on exposure.
20. Time Horizon IM Mechanism Impact: As is evident from their calculations, the IM mechanism strongly reduces exposures away from trade flows, but near the trade flow dates the protection gets progressively weaker and disappears almost completely for the last couple of trade flows. The reason for this uneven benefit of IM on this trade is that the 10 day VaR of this trade bears no direct relationship to the size of the trade flows that determines the exposure spikes in the model.
21. Inter/Intra-Spike IM Exposures: The variance of the $P\&L$ reduces as the swap approaches maturity so that the amount of IM on a given path is also reduced. However, the size of the trade flows is not reduced, but can actually grow with simulation time as larger and larger realizations of the floating rates are possible. Thus, when the swap approaches maturity the amount of IM is greatly reduced relative to the trade flows, so exposure spikes grow larger, while the *diffusion* component of the exposure becomes smaller.
22. Cross-Currency Swap VM + IM EE: Andersen, Pykhtin, and Sokol (2017) compute the impact of IM on the vanilla swap and the cross currency swap portfolios described earlier. As can be seen there the IM strongly suppresses the diffusion component of the portfolio value changes, but proves inadequate in reducing the spikes of exposure for both single currency, and especially, cross-currency portfolios.

Conclusion

1. Fully Collateralized Counterparty Exposure: Industry standard models for collateralized credit risk are well-known to produce non-negligible counterparty credit risk exposure, even under full collateralization of the variation margin. This exposure essentially arises due to the inevitable operational an legal delays (margin period of risk, or $MPoR$) that are *baked into* the workings of ISDA contracts that govern OTC trading.
2. Classical Implementations of the $MPoR$: In the most common industry implementation, the length of the $MPoR$, and precisely what transpires inside it, is, however, often treated in a highly stylized fashion. Often the $MPoR$ is set equal to 10 business days for little reason other than tradition, and often counter-parties are assumed to have oddly synchronized behavior inside the $MPoR$.
3. The Classical+ and Classical- Implementations: For instance, one common approach – denoted Classical- - assumes that the $MPoR$ and the trade flows by both counter-parties terminate at the beginning of the $MPoR$, but the trade flows terminate simultaneously at the end of the $MPoR$. Surprisingly, the Classical+ and the Classical- approaches continue to co-exists in the market, and neither has become the sole market practice.
4. Reasons for the Popularity of the Classical Models: One reason for this state of affairs is that the two models correspond to different choices for the trade-off between implementation complexity and the model stability; Classical+ is easier to implement but is prone to spurious spikes in the daily CVA P&L – as demonstrated earlier – whereas Classical- is more difficult to implement, but is free from such spikes.
5. Objectives of *Well* Designed Models: Ultimately, of course, a model should be selected not on the basis of the implementation ease or on the properties of a specific numerical technique, but on the basis of how well the model captures the legal and the behavioral aspects of the events around a counter-party default. The term *well* means different things in different applications of the exposure model. For regulatory capital purposes, prudence and conservatism may, for instance, be as important as outright precision.
6. Inadequacies of the Classical Approach: To this end, even a cursory analysis suggests that the perfect synchronicity of the Classical $\pm$ models cannot be supported in reality. For instance, due to the way the CSA works in practice, the non-defaulting party will need at

least 3 days after a portfolio valuation date to determine for sure that the corresponding margin payment by its counterparty will not be honored.

7. Detailed Analysis of $MPoR$ Timeline: This chapter carefully dissects the $MPoR$ into a full timeline around the default event, starting with the missed margin call and culminating at the post-default valuation date at which the termination value of the portfolio is established.
8. Model Parameters of the Timeline: For modeling purposes, the timeline of the model has been condensed into 4 model parameters, each specified as the number of days prior to the termination for the events below – in contrast the classical model has only one parameter – the full length of the $MPoR$.
9. Dealer/Client Trade/Margin Dates:
    a. The last market data measurement for which the margin flow is received (δ_C) and paid (δ_B) as prescribed.
    b. The last date when the defaulting party ($\delta_C{}'$) and the dealer ($\delta_D{}'$) make the trade payments as prescribed.
10. Legal Operational Basis behind the Parameters: As shown, each of these parameters has a legal and/or operational interpretation, enabling calibration from the CSA and from the operational setup of the dealer. Note that the proposed model parameterization includes the Classical+ and the Classical- models as the limit cases.
11. Aggressive CSA Timeline for the $MPoR$: For indicative purposes, two particular models are described – Aggressive and Conservative. For former assumes that the non-defaulting dealer always operates at an optimal operational level, and will enforce the legal provisions of the ISDA legal contracts as strictly as possible.
12. Conservative CSA for $MPoR$ Timeline: The latter will allow for some slack in the operations of the dealer, to allow for manual checks of calculations, legal reviews, *gaming* behavior of the counterparty, and so forth.
13. Aggressive/Conservative Timeline Exposure Comparison: The Conservative model setting obviously produces higher exposures than the Aggressive setting, for the following reasons.
    a. The Conservative setting has a longer overall length of $MPoR$
    b. The Conservative setting has a margin flow period where the dealer pays, but does not receive, margin flows

c. The Conservative setting, unlike the Aggressive setting, contains a trade flow gap period where the dealer pays, but does not receive, trade flows

14. Comparison of Margin Flow Exposures: In their numerical tests, Andersen, Pykhtin, and Sokol (2017) found that the first two factors of the Conservative setting to have approximately twice the exposure of both the Aggressive and the Classical $\pm$ settings away from the dates of large trade flows.
15. Comparison of Trade Flow Exposures: The last factor, i.e., the presence of a large trade flow gap may cause exposure spikes of extremely large magnitudes under the Conservative calibration. Despite the fairly short duration of these spikes, they may easily add up to very significant CVA contributions, especially for cross-currency trades with principal exchange – the Herstatt risk.
16. Past Realizations of Trade Flow Default: Credit losses due to trade flow gaps materialized in practice during the financial crisis – especially due to the Lehmann Brothers' default – so their incorporation into the model is both prudent and realistic.
17. Impracticality of Daily Simulation Schemes: Detailed tracking of the margin and the trade flow payments requires the stochastic modeling of the trade portfolio on a daily grid. As brute-force simulations at such a resolution are often impractically slow, it is important that numerical techniques be devised to speed up the calculations.
18. Kernel Regression on *Stripped* Cash Flows: While the focus of this chapter was mainly on establishing the fundamental principles for margin exposure, it also proposed an acceleration method based on kernel regression and applied Brownian Bridge to portfolio values *stripped* of cash flows.
19. Impact on Different Product Types: For ordinary and cross-currency swaps this chapter demonstrates that this method is both accurate and much faster than either brute-force simulation or standard acceleration techniques of the desired model. Further improvements in the acceleration techniques, and expansion of applicability into more exotic products, is an area of future research.
20. Initial Margin – Classical- Settings Impact: Under suitable assumptions, kernel regression may also be used to embed risk-based initial margin into exposure simulations. As

demonstrated in the final section of the chapter, initial margin at 99% exposure greatly succeeds in reducing bilateral exposure for the Classical- calibration.

21. Initial Margin Trade Flow Impact: For all other calibration choices, and especially for the Conservative setting, the reduction in counterparty exposure afforded by initial margin fails around the time of large trade flows, when a sudden change of exposure following an initial trade flow exceeds the initial margin level.
22. Initial Margin Maturity Decay Impact: Note that the already inadequate level of IM protection deteriorates around the maturity of the portfolio, where the local volatility of the trade flow value decreases, but the trade flows themselves do not. Overall accurate modeling of the events within the $MPoR$ becomes critically important for portfolios covered by dynamic IM.

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Dynamic Initial Margin Impact on Exposure

Abstract

1. VM and IM Collateralized Positions: This chapter leverages the new framework for collateralized exposure modeling introduced by Andersen, Pykhtin, and Sokol (2017b) to analyze credit risk positions collateralized with both initial and variation margin. Special attention is paid to the dynamics BCBS-IOSCO uncleared margin rules soon to be mandated for bilateral inter-dealer trading in OTC derivatives markets.
2. Insufficiency of BCBS IOSCO Rules: While these rules set the initial margin at 99^{th} 2-week percentile level and aim to all but eliminate portfolio close-out risk, this chapter demonstrates that the trade flow effects can result in exposures being reduced significantly less than expected.
3. Efficient IM Simulation on an MC Path: The analysis is supplemented with several practical schemes for estimating IM on a simulation path, and for improving the speed and the stability of the exposure simulation.
4. Handling Trade Flow Exposure Spikes: This chapter also briefly discusses potential ways to adjust the margin framework to more effectively deal with exposures arising from trade flow events.

Introduction

1. Collateralization based on Variation Margin: Collateralization has long been a way of mitigating counterparty risk in OTC bilateral trading. The most common collateral mechanism is *variation margin* (VM) which aims to keep the threshold gap between portfolio value and posted collateral below a certain, possibly stochastic, threshold.. While it is
2. Imperfect Collateralization under VM Schemes: Even when the thresholds for the VM are set to zero, however, there remains residual exposure to the counterparty default resulting from a sequence of contractual and operational time lags, starting from the last snapshot of the market for which the counterparty would post in full the required VM to the termination date after the counterparty's default. The various collateral mechanisms, including the precise definition of the variation margin thresholds, are typically captured in the ISDA Credit Support Annex (CSA) – the portfolio level legal agreement that supplements the ISDA Master Agreement.
3. MPoR - Margin Period of Risk: The aggregation of these lags results in a time period called the *Margin Period of Risk* (MPoR) during which the gap between the portfolio value and collateral can widen. The length of the MPoR is a critical input to any model of collateral exposure.
4. IM Supplementing the VM Collateral: Posting of *initial margin* (IM) to supplement VM provides the dealers with a mechanism to reduce the residual exposure resulting from market risk over MPoR. While it is often believed that the IM is posted strictly in addition to the VM, many CSAs intermingle the two types of collateral by letting IM affect the threshold computation of VM.
5. Genesis and Structure of IM: Historically, IM in bilateral trading has been mostly reserved for dealer counterparties deemed as high-risk – e.g., hedge funds – and typically done as a trade level calculation, established in term sheets at the transaction time of each trade. This type of IM posting is normally deterministic and either stays fixed over the life of a trade or amortizes down according to a pre-specified schedule.
6. Ne Basel Rules for IM: In the inter-dealer bilateral OTC world, changes to the long-standing VM and IM collateral practices are now imminent. BCBS and IOSCO proposed (Basel

Committee on Banking Supervision (2013)) and later finalized (Basel Committee on Banking Supervision (2015)) new uncleared margin rules for bilateral trading.

7. Key Features of the UMR: Under UMR, VM thresholds are forced to zero, and IM must be posted bilaterally into segregated accounts at the netting set level, by either using an internal model or by a lookup in a standardized schedule.
8. IM as a Horizon-Specific VaR: If an internal model is used, IM must be calculated as a netting set Value-at-Risk (VaR) for a 99% confidence level. The horizon used in this calculation equals $9 + a$ business days, where a is the re-margining period - 1 business day under US rules.
9. No Cross Asset Class Diversification: In these calculations, diversification across distinct asset classes is not recognized, and calibration of the IM internal model for each asset class must include a period of stress for that asset class. To reduce the potential for margin disputes and to increase the overall market transparency, ISDA has proposed a standardized sensitivity-based IM calculator known as SIMM (Standard Initial Margin Model) (International Swaps and Derivatives Association (2016)). As a practical matter it is expected that virtually all dealers will use SIMM for their day-to-day IM calculations.
10. Dynamic Nature of Initial Margin: Under UMR required levels of IM continuously change as trade cash flows are paid, new trades are booked, or markets move, and dealers regularly need to call for more IM or to return excess IM. This dynamic aspect of IM requirements makes the modeling of the future exposures a challenge.
11. Modeling under Dynamics IM and VM: This chapter discusses modeling credit exposure in the presence of dynamic IM and questions the conventional wisdom that IM essentially eliminates counterparty risk. Leaning on the recent results from Andersen, Pykhtin, and Sokol (2017a), it starts by formulating a general model of exposure in the presence of VM and/or IM.
12. Simple Case - No Trade Flows: The resulting framework is first applied to the simple case where no trade flows take place within the MPoR. For processes with Gaussian increments – e.g., an Ito process – a limiting scale factor that converts the IM free expected exposure (EE) to IM-protected EE is derived, for sufficiently small MPoR.

13. IM vs no IM EE Ratio: The universal value depends only on the IM confidence level and the ratio of the IM horizon to the MPoR; it equals 0.85% at the BCBS-IOSCO confidence level of 99%, provided the IM horizon equals the MPoR. While conceptually the IM and the MPoR horizons are identical, a prudent MPoR for internal calculations may differ from the regulatory minimum IM horizon.
14. No-Trade-Flow Exposure Reduction: While some deviations from this universal limit due to a non-infinitesimal MPoR are to be expected, the reduction of EE by about 2 orders of magnitude is, as will be demonstrated below, generally about right when no trade flows are present within the MPoR.
15. Exposure Spikes from Trade Flows: For those periods for which trade flows do take place within the MPoR, however, any trade payment flowing away from the dealer will result in a spike in the EE profile. Without IM these spikes can make a fairly significant contribution to the Credit Valuation Adjustment (CVA) – say, 20% of an interest rate swap's total CVA may originate with spikes – but the CVA would still mostly be determined by the EE level between the spikes.
16. Exposure Spikes vs. Dynamic IM: This chapter shows that while IM is effective in suppressing the EE *between* spikes, it will often fails to significantly suppress the spikes themselves. As a result the relative contribution of the spike to CVA is greatly increased in the presence of IM – e.g., for a single interest rate swap, the spike's contribution to the CVA can be well about 90% for a position with IM.
17. Corresponding Impact on the CVA: Accounting for the spikes, the IM reduces the CVA by much less than two orders of magnitude one might expect, with the reduction for the interest rate swaps often being less than a factor of 10.
18. Estimating the Path-wise IM: The final part of this chapter discusses the practical approaches to calculating the EE profiles in the presence of IM. The first step in this calculation is the estimation of IM on simulation paths, which can be done by parametric regression or by kernel regression.
19. IM Covering Few Netting Trades: When IM covers an insignificant number of trades in the netting set, IM calculated on the path can be subtracted from the no-IM exposure realized on that path to generate EE profiles.

20. IM covering most Netting Trades: However, when most trades of the netting set are covered by the IM, this approach can be problematic because of excessive simulation noise and other errors. An alternative approach that dampens the noise is proposed, and is generally more accurate.
21. Suggested Alterations to the Exposure Rules: This chapter concludes by summarizing the results and briefly discussing the possible modifications to trade and collateral documentation that would make IM more effective in reducing residual counterparty risk.

Exposure in the Presence of IM and VM

1. VM/IM over single Netting Set: Consider a dealer D that has a portfolio of OTC derivatives contracts traded with a counterparty C. Suppose for simplicity that the entire derivatives portfolio is covered by a single netting agreement, which is supported by a margin agreement that includes VM and may include IM on a subset of the portfolio.
2. Exposure of Client to Dealer: Quiet generally the exposure of D to the default of client C measured at time t – assumed to be the early termination time after C's default – is given by

$$E(t) = [V(t) - VM(t) + U(t) - IM\ (t)]^{+}$$

   where $V(t)$ is the time t portfolio value from D's perspective; $VM(t)$ is the VM available to D at time t; $U(t)$ is the value of the trade flows scheduled to be paid by both D (negative) and C (positive) up to time t, yet unpaid as of time t; $IM(t)$ is the value of IM available to D at time t.
3. Sign of VM and IM: Notice that VM can be positive – C posts VM – or negative – D posts VM – from D's perspective. On the other hand IM is always positive as IM for both counterparties is kept in segregated accounts, whereby IM posted by D does not contribute to D's exposure to the default of C.

4. Modeling Individual Terms in the Exposure: The above equation for $E(t)$ specifies the exposure of D to C in a generic, model-free way. To add modeling detail, this chapter assumes that D and C both post VM with zero-threshold and are required to post BCBS-IOSCO compliant IM to a segregated account. The modeling of each of these terms VM, U, and IM are dealt with turn by turn.

References

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Initial Margin Backtesting Framework

Abstract

1. Mandatory Margins for OTC Transactions: The introduction of mandatory margining for bilateral OTC transactions is significantly affecting the derivatives market, particularly in light of the additional funding costs that financial institutions could face.
2. Initial Margin Forecast Models Backtest: This chapter details the approach by Anfuso, Aziz, Loukopoulos, and Giltinan (2017) for a consistent framework, applicable equally to cleared and non-cleared portfolios, to develop and backtest forecasting models for initial margin.

Introduction

1. BCBS-IOSCO Mandatory Margining Guidelines: Since the publication of the new Basel Committee on Banking Supervision and the International Organizations of Securities Commissions (BCBS-IOSCO) guidance for mandatory margining for non-cleared OTC derivatives (Basel Committee on Banking Supervision (2015)) there has been a growing interest in the industry regarding the development of dynamic initial margin models (DIM) – see, for example, Green Kenyon (2015), Andersen, Pykhtin, and Sokol (2017b). By *DIM model* this chapter refers to any model that can be used to forecast portfolio initial margin requirements.

2. Protection Afforded by BCBS-IOSCO: The business case for such a development is at least two fold. First, the BCBS-IOSCO IMR (B-IMR) rules are expected to protect against potential future exposure at a high-level of confidence (99%) and will substantially affect funding costs, XVA, and capital.
3. IM and VM Based Margining: Second, B-IMR has set a clear incentive for clearing; extensive margining in the form of variation margin (VM) and initial margin (IM) is the main element of the central counter-party (CCP) risk management as well.
4. IMR Impact on Bilateral + Cleared: Therefore, for both bilateral and cleared derivatives, current and future IMR significantly affects the probability and the risk profile of a given trade.
5. B-IMR Case Study - Performance Evaluation: This chapter considers B-IMR as a case study, and shows how to include a suitably parsimonious DIM model on the exposure calculation. It also proposes an end-to-end framework and also defines a methodology to backtest model performance.
6. Organization of this Chapter: This chapter is organized as follows. First, the DIM model for forecasting future IMR is presented. Then methodologies for two distinct levels of back-testing analysis are presented. Finally, conclusions are drawn.

How to Construct a DIM Model

1. Applications of the DIM Model: A DIM model can be used for various purposes. In the computation of the counter-party credit risk (CCR), capital exposure, or credit valuation adjustment (CVA), the DIM model should forecast, in a path-by-path basis, the amount of posted and received IM at any revaluation point.
2. Path Specific IMR Estimation: For this specific application, the key ability of the model is to associate a realistic IMR to any simulated market scenario based on a mapping that makes use of a set of characteristics of the path.

3. RFE Dependence on the DIM: The DIM model is *a priori* agnostic to the underlying risk factor evolution (RFE) models to generate the exposure paths (as shall be seen, dependencies may arise, if for example, the DIM is computed on the same paths that are generated for the exposure).
4. Cross-Probability Measure IMR Distribution: It is a different story if the goal is to predict the IMR distribution (IMRD) at future horizons, either in the real-world P or the market-implied Q measures.
5. IMRD Dependence on the RFE: In this context, the key feature of the model is to associate the right probability weight with a given IMR scenario; hence the forecast IMRD also becomes a measure of the accuracy if the IMRD models (which ultimately determine the likelihood of different market scenarios).
6. P vs. Q Measure IMRD: The distinction between the two cases will become clear later on, in the discussion of how to assess model performance.
7. ISDA SIMM BCBS IOSCO IM: The remainder of this chapter considers the BCBS-IOSCO IM as a case study. For the B-IMR, the current industry proposal is the International Swaps and Derivatives Association Standard Initial Margin Model (SIMM) – a static aggregation methodology to compute the IMR based on first-order delta-vega trade sensitivities (International Swaps and Derivatives Association (2016)).
8. Challenges with SIMM Monte Carlo: The exact replication of SIMM in a capital exposure or an XVA Monte Carlo framework requires in-simulation portfolio sensitivities to a large set of underlying risk factors, which is very challenging in most production implementations.
9. Andersen-Pykhtin-Sokol IM Proposal: Since the exposure simulation provides the portfolio mark-to-market (MTM) on the default (time t) and closeout (time $t + MPoR$, where $MPoR$ is the *margin period of risk*) grids, Andersen, Pykhtin, and Sokol (2017b) have proposed using this information to infer path-wise the size of any percentile of the local $\Delta MTM(t, t + MPoR, Path_i)$ distribution, based on a regression that uses the simulated portfolio $MTM(t)$ as a regression variable.
10. Andersen-Pykhtin-Sokol Proposal Assumptions: The

$$\Delta MTM(t, t + MPoR) = MTM(t + MPoR) - MTM(t)$$

distributed is constructed assuming that no cash flow takes place between the default and the closeout. For a critical review of this assumption, see Andersen, Pykhtin, and Sokol (2017a).

11. Enhancing the Andersen-Pykhtin-Sokol Model: This model can be further improved by adding more descriptive variables to the regression, e.g., values at the default time of the selected risk factors of the portfolio.
12. Optimization: Re-using Exposure Paths: For the DIM model, the following features are desirable. First the DIM should consume the same number of paths as the exposure simulation, to minimize the computational burden.
13. DIM Optimization – B-IMR SIMM Reconciliation: Second, the output of the DIM model should reconcile with the known IMR value for

$$t = 0$$

i.e.

$$IM(Path_i, 0) = IMR_{SIMM}(0)$$

for all i.

14. Key Aspects of IOSCO/SIMM: Before proceeding, this section notes some of the key aspects of the BCBS-IOSCO margining guidelines, and, consequently, of the ISDA SIMM Model (International Swaps and Derivatives Association (2016)).
15. Andersen-Pykhtin-Sokol Proposal Assumptions: First, the $MPoR$ for the IM calculation of a daily margined counter-party is 10 BD. This may differ from the capital exposure calculation, in which, for example

$$MPoR = 20\ BD$$

if the number of trades in the portfolio exceeds 5,000.

16. No Netting across the Asset Classes: Second, the B-IMR in the Basel Committee on Banking Supervision (2015) prescribes calculating the IM by segregating trades from different asset classes. Tis feature is reflected in the SIMM model design.
17. SIMM Methodology Market Volatility Independence: Finally, the SIMM methodology consumes trade sensitivities as its only inputs and has a static calibration that is not sensitive to market volatility.
18. Regression on the ΔMTM Distribution: For the IM calculation, the starting point is similar to that of Andersen, Pykhtin, and Sokol (2017a), i.e.
    a. A regression methodology based on path's $MTM(t)$ is used to compute the moments of the local $\Delta MTM(t, t + MPoR, Path_i)$ distribution, and
    b. $\Delta MTM(t, t + MPoR, Path_i)$ is assumed to be a given probability distribution that can be fully characterized by its first two moments – the drift and the volatility. Additionally, since the drift is immaterial over the $MPoR$ horizon, it is not computed and set to 0.
19. Quadratic Regressor for Local Volatility: There are multiple regression schemes that can be used to determine the local volatility $\sigma(i, t)$ The present analysis follows the standard American Monte Carlo literature (Longstaff and Schwartz (2001)) and uses a least squares method (LSM) with a polynomial basis:

$$\sigma^2(i, t) = \mathbb{E}[\Delta MTM^2(i, t) \mid MTM(i, t)] = \sum_{k=0}^{n} a_{\sigma k} MTM^k(i, t)$$

$$IM_{R/P,U}(i, t) = \Phi^{-1}\big(0.99/0.01, \mu = 0, \sigma = \sigma(i, t)\big)$$

where R/P indicates received and poste, respectively. In this implementation, the n in

$$\sigma^2(i, t) = \mathbb{E}[\Delta MTM^2(i, t) \mid MTM(i, t)] = \sum_{k=0}^{n} a_{\sigma k} MTM^k(i, t)$$

is set equal to 2, i.e., a polynomial regression of order 2 is used.

20. Calculating the Unnormalized IM Value: The unnormalized posted and received $IM_{R/P,U}(i,t)$ and calculated analytically in

$$\sigma^2(i,t) = \mathbb{E}[\Delta MTM^2(i,t) \mid MTM(i,t)] = \sum_{k=0}^{n} a_{\sigma k} MTM^k(i,t)$$

$$IM_{R/P,U}(i,t) = \Phi^{-1}\big(0.99/0.01, \mu = 0, \sigma = \sigma(i,t)\big)$$

by applying the inverse of the cumulative distribution $\Phi^{-1}(x,\mu,\sigma)$ to the appropriate quantiles; $\Phi(x,\mu,\sigma)$ being the probability distribution that models the local $\Delta MTM(t, t + MPoR, Path_i)$.

21. Note on the Distributional Assumptions: The precise choice of Φ does not play a crucial role, since the difference in the quantiles among the distribution can be compensated in calibration be applying the appropriate scaling factors (see the $\alpha_{R/P}(t)$ functions below). For simplicity, in the below Φ is assumed to be normal.

22. Comparative Performance of the LSM: It is observed that the LSM method performs well compared to the more sophisticated kernel methods such as Nadaraya-Watson, which is used in Andersen, Pykhtin, and Sokol (2017a), and it has the advantage of being parameter free and cheaper from a computational stand-point.

23. Applying $t = 0, MPoR$ and SIMM Reconcilers: The next step accounts for the

$$t = 0$$

reconciliation as well as the mismatch between SIMM and the exposure model calibrations – see the corresponding items above.

24. De-normalizing using IM Scaling Parameters: These issues can be tackled by scaling $IM_{R/P,U}(i,t)$ with suitable normalization functions $\alpha_{R/P}(t)$:

$$IM_{R/P}(i,t) = \alpha_{R/P}(t) \times IM_{R/P,U}(i,t)$$

$$\alpha_{R/P}(t) = [1 - h_{R/P}(t)] \times \sqrt{\frac{10\ BD}{MPoR}} \times \left[\alpha_{R/P,\infty} + (\alpha_{R/P,0} - \alpha_{R/P,\infty})e^{-\beta_{R/P}(t)t}\right]$$

$$\alpha_{R/P,0} = \sqrt{\frac{MPoR}{10\ BD}} \times \frac{IMR_{R/P,SIMM}(t=0)}{q(0.99/0.01\,, \Delta MTM(0, MPoR))}$$

25. <u>Differential Calibration for Posted/Received IM</u>: In

$$\alpha_{R/P}(t) = [1 - h_{R/P}(t)] \times \sqrt{\frac{10\ BD}{MPoR}} \times \left[\alpha_{R/P,\infty} + (\alpha_{R/P,0} - \alpha_{R/P,\infty})e^{-\beta_{R/P}(t)t}\right]$$

$$\beta_{R/P}(t) > 0$$

and

$$h_{R/P}(t) < 1$$

with

$$h_{R/P}(t=0) = 0$$

are four functions to be calibrated – two for received and two for posted IM's. As will become clearer later in this chapter, the model calibration generally differs for received and posted DIM models.

26. <u>Scaling IM using RFE MPoR</u>: In

$$IM_{R/P}(i,t) = \alpha_{R/P}(t) \times IM_{R/P,U}(i,t)$$

$$\alpha_{R/P}(t) = [1 - h_{R/P}(t)] \times \sqrt{\frac{10\ BD}{MPoR}} \times [\alpha_{R/P,\infty} + (\alpha_{R/P,0} - \alpha_{R/P,\infty})e^{-\beta_{R/P}(t)t}]$$

$MPoR$ indicates the $MPoR$ relevant for the Basel III exposure. The ratio of $MPoR$ to 10 BD accounts for the VM vs. IM margin period, and it is taken as a square root because the underlying models are typically Brownian, at least for short horizons.

27. Components of the $\alpha_{R/P}(t)$ Term: In

$$\alpha_{R/P,0} = \sqrt{\frac{MPoR}{10\ BD}} \times \frac{IMR_{R/P,SIMM}(t = 0)}{q(0.99/0.01\,, \Delta MTM(0, MPoR))}$$

$IMR_{R/P,SIMM}(t = 0)$ are the $IMR_{R/P}$ computed at

$$t = 0$$

using SIMM; $\Delta MTM(0, MPoR)$ is the distribution of the MTM variations over the first $MPoR$; and $q(x, y)$ is a function that gives quantile x for the distribution y.

28. $t = 0$ chosen to match SIMM: The values of the normalization functions $\alpha_{R/P}(t)$ at

$$t = 0$$

are chosen in order to reconcile $IM_{R/P}(i, t)$ with the starting SIMM IMR.

29. Mean-reverting Nature of the Volatility: The functional form of $\alpha_{R/P}(t)$ at

$$t > 0$$

is dictated by what is empirically observed, as is illustrated by Anfuso, Aziz, Loukopoulos, and Giltinan (2017); accurate RFE models, in both P and Q measures, have either a volatility term structure or an underlying stochastic volatility process that accounts for the mean-reverting behavior to the normal market conditions generally observed from extremely low or high volatility.

30. Reconciliation with Static SIMM Methodology: Since the SIMM calibration is static (independence of market volatility for SIMM), the

$$t = 0$$

reconciliation factor is not independent of the market volatility, and thus not necessarily adequate for the long-term mean level.

31. Volatility Reducing Mean-reversion Speed: Hence, $\alpha_{R/P}(t)$ is an interpolant between the

$$t = 0$$

scaling driven by $\alpha_{R/P,0}$ and the long-erm scaling driven by $\alpha_{R/P,\infty}$, where the functions $\beta_{R/P}(t)$ are the mean-reverting speeds.

32. Estimating from the Long-End: The values of $\alpha_{R/P,\infty}$ can be inferred by a historical analysis of a group of portfolios, or it can be *ad hoc* calibrated, e.g., by computing a different $\Delta MTM(0, MPoR)$ distribution in

$$\alpha_{R/P,0} = \sqrt{\frac{MPoR}{10\ BD}} \times \frac{IMR_{R/P,SIMM}(t = 0)}{q\left(0.99/0.01\,, \Delta MTM(0, MPoR)\right)}$$

using the long-end of the risk-factor implied volatility curves and solving the equivalent scaling equations for $\alpha_{R/P,\infty}$.

33. Interpreting the Haircut $h_{R/P}(t)$ Term: As will be seen below, the interpretation of $h_{R/P}(t)$ can vary depending on the intended application of the model.

34. <u>$h_{R/P}(t)$ for Capital/Risk Models</u>: For capital and risk models, $h_{R/P}(t)$ are two capital and risk functions that can be used to reduce the number of back-testing exceptions (see below) and ensure that the DIM model is conservatively calibrated.
35. <u>$h_{R/P}(t)$ for the XVA Models</u>: For XVA pricing, $h_{R/P}(t)$ can be fine-tuned – together with $\beta_{R/P}(t)$ - to maximize the accuracy of the forecast based on historical performance.
36. <u>Lack of Asset Class Netting</u>: Note that owing to the *No netting across Asset Classes* clause, the $IM_{R/P,x}(i,t)$ can be computed on a stand-alone basis for every asset class x defined by SIMM (IR/FX, equity, qualified and non-qualified credit, commodity) without any additional exposure runs. The total $IM_{R/P}(i,t)$ is then given by the sum of the $IM_{R/P,x}(i,t)$ values.
37. <u>Historical vs. Computed IM Calibrations</u>: A comparison between the forecasts of the DIM model defined in

$$\sigma^2(i,t) = \mathbb{E}[\Delta MTM^2(i,t) \mid MTM(i,t)] = \sum_{k=0}^{n} a_{\sigma k} MTM^k(i,t)$$

$$IM_{R/P,U}(i,t) = \Phi^{-1}\big(0.99/0.01, \mu = 0, \sigma = \sigma(i,t)\big)$$

$$IM_{R/P}(i,t) = \alpha_{R/P}(t) \times IM_{R/P,U}(i,t)$$

$$\alpha_{R/P}(t) = \big[1 - h_{R/P}(t)\big] \times \sqrt{\frac{10\ BD}{MPoR}} \times \big[\alpha_{R/P,\infty} + \big(\alpha_{R/P,0} - \alpha_{R/P,\infty}\big)e^{-\beta_{R/P}(t)t}\big]$$

$$\alpha_{R/P,0} = \sqrt{\frac{MPoR}{10\ BD}} \times \frac{IMR_{R/P,SIMM}(t=0)}{q\big(0.99/0.01\,, \Delta MTM(0, MPoR)\big)}$$

and the historical IMR realizations computed with the SIMM methodology is shown in Anfuso, Aziz, Loukopoulos, and Giltinan (2017) where alternative scaling approaches are considered.

38. Criteria Utilized in the Comparison: A comparison is performed at different forecasting horizons using 7 years of historical data, monthly sampling, and averaging among a wide representation of single-trade portfolios for the posted and the received IM cases.
39. $\mathcal{L}_1$ Error Metric Choice: For a given portfolio/horizon, the chosen error metric is given by $\mathbb{E}_{t_k}\left[\frac{|F_{R/P}(t_k+h)-G_{R/P}(t_k+h)|}{G_{R/P}(t_k+h)}\right]$ where $\mathbb{E}_{t_k}[\cdot]$ indicates an average across historical sampling dates – the definitions of $F_{R/P}$ and $G_{R/P}$ are contained below. Here and throughout this chapter, t_k is used in place of t whenever the same quantity is computed at multiple sampling dates.
40. Comparison of the Tested Universe: The tested universe is made up of 102 single-trade portfolios. The products considered, always at-the-money and of different maturities, include cross-currency swaps, IR swaps, FX options, and FX forwards – approximately 75% of the population is made up of

$$\Delta = 1$$

trades.
41. Calibrated Estimates of the Parameters: As is made evident by Anfuso, Aziz, Loukopoulos, and Giltinan (2017), the proposed term structure of $\alpha_{R/P}(t)$ improves the accuracy of the forecast by a significant amount – they also provide the actual calibration used for their analysis.
42. Conservative Calibration of the Haircut Function: Below contains further discussions on the range of values that the haircut functions $h_{R/P}(t)$ are expected to take for a conservative calibration of DIM to be used for regulatory exposure.
43. Comparison with CCP IMR: Finally, as an outlook, Anfuso, Aziz, Loukopoulos, and Giltinan (2017) show the error metrics for the case of CCP IMR where the Dim forecasts are compared against the Portfolio Approaches to Interest Rate Scenarios (Pairs: LCH.ClearNet) and historical value-at-risk (HVaR; Chicago Mercantile Exchange) realizations.
44. Prototype Replications of CCP Methodologies: The realizations are based on prototype replications of the market risk components of the CCP IM methodologies.

45. Universe Used for the CCP Tests: The forecasting capability of the model is tested separately for Pairs and HVaR IMR as well as for 22 single-trade portfolios (IRS trades of different maturities and currencies). The error at any given horizon is obtained by averaging among 22×2 cases.
46. Accuracy of the Proposed Scaling: Without fine tuning the calibration any further, the time-dependent scaling $\alpha_{R/P}(t)$ drives a major improvement in the accuracy of the forecasts with respect to the alternative approaches.

How to Backtest a DIM Model

1. Assessing Model for Different Applications: The discussion so far has focused on a DIM model for B-IMR without being too specific about how to assess the model performance for different applications, such as CVA and margin valuation adjustment (MVA) pricing, liquidity coverage ratio/net stable funding ratio (LCR/NSFR) monitoring (Basel Committee on Capital Supervision (2013)), and capital exposure.
2. Estimating the IMR Distribution Accurately: As mentioned above, depending upon which application one considers, it may or may not be important to have an accurate assessment of the distribution of *the simulated IM requirements* value (IMRD).
3. Backtesting to measure DIM Performance: This chapter introduces two distinct levels of backtesting that can measure the DIM model performance in two topical cases:
    a. DIM applications that do not depend directly on the IMRD (such as capital exposure and the CVA), and
    b. DIM applications that directly depend on the IMRD (such as MVA calculation and LCR/NSFR monitoring).

    The methodologies are presented below, with a focus on the P-measure applications.

Backtesting DIM Mapping Functions (for Capital Exposure and CVA)

1. Review of the Monte-Carlo Framework: In a Monte-Carlo simulation framework, the exposure is computed by determining the MTM values of a given portfolio on a large number of forward looking risk-factor scenarios.
2. Adequacy of Forecasts across Scenarios: To ensure that a DIM model is sound, one should verify that the IM forecasts associated with the future simulation scenarios are adequate for a sensible variety of forecasting horizons as well as initial and terminal market conditions.
3. Setting up a Suitable Backtesting Framework: A suitable historical backtesting framework so as to statistically assess the performance of the model by comparing the DIM forecast with the realistic exact IMR, e.g., in the case of B-IMR calculated according to the SIMM methodology – for a representative sample of historical dates as well as market conditions and portfolios.
4. Generic IMR of a Portfolio: Let us first define generic IMR of a portfolio p as

$$IMR = g_{R/P}\left(t = t_{\alpha}, \Pi = \Pi\big(p(t_{\alpha})\big), \vec{M}_g = \vec{M}_g(t_{\alpha})\right)$$

   The terms are as follows.
5. Posted/Received IMR Computation Algorithm: The functions g_R and g_P represent the exact algorithm used to compute the IMR for the posted and the received IM's, respectively (e.g., such as SIMM for B-IMR, or in the case of the CCP's, IM methodologies such as Standard Portfolio Analysis of Risk (SPAN), Pairs, or HVaR).
6. Date of the IMR Valuation:

$$t = t_{\alpha}$$

   is the time at which the IMR portfolio p is determined.

7. Portfolio Trade Population at t_α: $\Pi\big(p(t_\alpha)\big)$ is the trade population of portfolio p at time t_α.
8. Market State Information at t_α: $\vec{M}_g(t_\alpha)$ is a generic state variable that characterizes all of the $T \leq t_\alpha$ market information required for the computation of the IMR.
9. DIM Forecast of the Portfolio: Similarly, the DIM forecast for the future IMR of a portfolio p can be defined as

$$DIM = f_{R/P}\left(t_0 = t_k, t = t_k + h, \vec{r}, \qquad \Pi = \Pi\big(p(t_k)\big), \vec{M}_{DIM} = \vec{M}_{DIM}(t_k)\right)$$

The terms are as follows.

10. Posted/Received DIM Computation Algorithm: The functions f_R and f_P represent the DIM forecast for the posted and the received IM's, respectively.
11. Date of the DIM Forecast:

$$t_0 = t_k$$

is the date time at which the DIM forecast is computed.

12. Horizon of the DIM Forecast:

$$t = t_k + h$$

is the time for which the IMR is forecast – over a forecasting horizon

$$h = t - t_0$$

13. Predictor Set of Market Variables: $\vec{r}$ - the *predictor* – is a set of market variables whose forecasted values on a given scenario are consumed by the DIM models as input to infer the IMR.
14. $\vec{r}$ as Simulated Portfolio MTM: The exact choice of $\vec{r}$ depends on the DIM model. For the one considered previously, $\vec{r}$ is simply given by the simulated MTM of the portfolio.

15. Market State Information at t_k: $\vec{M}_{DIM}(t_k)$ is the generic state variable characterizing all the

$$T \leq t_k$$

market information required for the computation of the DIM forecast.
16. Portfolio Trade Population at t_k: $\Pi(\cdot)$ is defined as before.
17. Caveats around f_R and f_P: Despite being computed using the stochastic RFE models, f_R and f_P are not probability distributions, as they do not carry any information regarding the probability weight of a given received/posted IM value. $f_{R/P}$ are instead mapping functions between the set $\vec{r}$ chosen as predictor and the forecast value for IM.
18. Confidence Level Based DIM Calibration: In terms of $g_{R/P}$ and $f_{R/P}$ one can define exception counting tests. The underlying assumption is that the DIM model is calibrated at a given confidence level (CL); therefore it can be tested as a $VaR(CL)$ model.
19. Model Conservatism Linked to CL: This comes naturally in the context of real-world P applications, such as capital exposure or liquidity monitoring, where a notion of model conservatism, and hence of exception, is applicable, since the model will be conservative whenever it understates (overstates) posted (received) IM.
20. The Portfolio Backtesting Algorithm Steps: For a portfolio p, a single forecasting day t_k, and a forecasting horizon h, one can proceed as follows.
21. t_k Estimate of the Forecast Functions: The forecast functions $f_{R/P}$ computed at time t_k are $f_{R/P}\left(t_0 = t_k, t = t_k + h, \vec{r},\ \Pi = \Pi\big(p(t_k)\big), \vec{M}_{DIM} = \vec{M}_{DIM}(t_k)\right)$ Note that $f_{R/P}$ depends exclusively on the predictor $\vec{r}$ –

$$\vec{r} = MTM$$

for the case considered above.
22. Impact of the Horizon on Predictor/Portfolio: The realized value of the predictor

$$\vec{r} = \vec{R}$$

is determined. For the model considered above, $\vec{R}$ is given by the portfolio value $p(t_k + h)$ where the trade population $\Pi\big(p(t_k + h)\big)$ at $t_k + h$ differs from t_k only because of portfolio aging. Aside from aging, no other portfolio adjustments are made.

23. Forecast Received/Posted IMR Estimate: The forecast values for the received and the posted IM's are computed as

$$F_{R/P}(t_k + h) = f_{R/P}\left(t_0 = t_k, t = t_k + h, \vec{r}, \qquad \Pi = \Pi\big(p(t_k)\big), \vec{M}_{DIM} = \vec{M}_{DIM}(t_k)\right)$$

24. Forecast of the Received/Posted IM Estimate: The realized values for the received and the posted IM's are computed as

$$G_{R/P}(t_k + h) = g_{R/P}\left(t = t_k + h, \Pi = \Pi\big(p(t_k + h)\big), \vec{M}_g = \vec{M}_g(t_k + h)\right)$$

25. Exception Case: F/G Mismatch Conservatism: The forecast and the realized values are then compared. The received and the posted DIM models are considered independently, and a backtesting exception occurs whenever $F_R(F_P)$ is larger (smaller) than $G_R(G_P)$. As discussed above, this definition of exception follows from the applicability of a notion of model conservatism.
26. Detecting the Backtesting Exception History: Applying the above steps to multiple sampling points t_k one can detect back-testing exceptions for the considered history.
27. Dimensionality Reduction for the Comparison: The key step is the estimate of the posted/received IMR forecast, where the dimensionality of the forecast is reduced – from a function to a value – making use of the realized value of the predictor, and, hence, allowing for a comparison with the realized IMR.
28. Determining the Test p-value using TVS: The determination of the test p-value requires additional knowledge of the Test Value Statistics (TVS), which can be derived numerically if the forecasting horizons are overlapping (Anfuso, Karyampas, and Nawroth (2017)).

29. Caveats behind Blind TVS Usage: In the latter situation, it can happen that a single change from one volatility regime to another may trigger multiple correlated exceptions; hence the TVS should adjust the beck-testing assessments for the presence of false positives.
30. Accuracy of the $\alpha_{R/P}(t)$ Scaling: The single trade portfolios seen earlier have been tested by Anfuso, Aziz, Loukopoulos, and Giltinan (2017) using the SIMM DIM models with the three choices of scaling discussed earlier. The results confirm the greater accuracy of the term structure scaling of $\alpha_{R/P}(t)$.
31. Accuracy in the Presence of Haircut: In fact, for the same level of the haircut function

$$h_{R/P}(t > 0) = \pm 0.25$$

positive/negative for posted/received – a much lower number of exceptions is detected.
32. Realistic Values for the Haircut: Anfuso, Aziz, Loukopoulos, and Giltinan (2017) also observe that, in this regard, for realistic diversified portfolios and calibration targets of

$$CL = 95\%$$

the functions $h_{R/P}(t)$ take values typically in the range of $10 - 40\%$.
33. Assumptions Underlying the Haircut Assumption: The range of values for $h_{R/P}(t)$ has been calibrated using

$$\beta_{R/P}(t) = 1$$

and

$$\alpha_{R/P,\infty}(t) = 1$$

Both assumptions are broadly consistent with historical data.

34. IOSCO results in Over-collateralization: Note also that the goal of the BCBS-IOSCO regulations is to ensure that the netting sets are largely over-collateralized as a consequence of:
    a. The high confidence level at which the IM is computed, and
    b. The separate requirements for IM and VM.
35. Impact of Over-collateralization: Hence, the exposure generating scenarios are tail events, and the effect on capital exposure of a conservative haircut applied to the received IM is rather limited in absolute terms.
36. Over-collateralization Impact on Exposure: This issue is demonstrated by Anfuso, Aziz, Loukopoulos, and Giltinan (2017) where the expected exposure (EE) at a given horizon t is shown as a function of $h_R(t)$ – the haircut to be applied to the received IM collateral – for different distributional assumptions on $\Delta MTM(t, t + MPoR)$.
37. Distribution Dependence on Haircut Functions: In particular, they compute the expected exposure for

$$h_R(t) = 0$$

and

$$h_R(t) = 1$$

indicating full IM collateral benefit or no benefit at all – and take the unscaled IM as the 99^{th} percentile of the corresponding distribution. For different classes of the ΔMTM distribution, the exposure reduction is practically unaffected up to haircuts of $\approx 50\%$.

Backtesting the IMRD for MVA and LCR/NSFR

1. MC Based DIM IMR Distributions: The same Monte Carlo framework can be used in combination with a DIM model to forecast the IMD at any future horizon – implicit here are the models in which the DIM is not always constant across the scenarios. The applications of the IMRD are multiple.
2. Some Applications using the IMRD: The following are two examples that apply equally to the cases of B-IMR and CCP IMR:
    a. Future IM funding costs in the P measure, i.e., the MVA
    b. Future IM funding costs in the Q measure, e.g., in relation to LCR and NSFR regulations (Basel Committee on Banking Supervisions (2013))
3. Numerically Forecasting the IMR Distributions: The focus here is on the forecasts on the P-measure – tackling the case of the Q-measure may require a suitable generalization of Jackson (2013). The main difference with the backtesting approach discussed above is that the new model forecasts are the numerical distributions of the simulated IMR values.
4. Scenario-specific IM Forecasting: These can be obtained for a given horizon by associating every simulated scenario with its corresponding IMR forecast, computed according to the given DIM model.
5. Posted/Received IMR Density CDF: Using the notation introduced previously, the numerical representations of the received/posted IMRD cumulative density functions (CDF's) of a portfolio p for a forecasting day t_k and a horizon h are given by

$$CDF_{R/P}(x, t_k, h) = \frac{\#\{v \in \mathbb{V} \mid v \leq x\}}{N_{\mathbb{V}}} \; \forall \, \vec{r}_\omega \in \Omega$$

$$\mathbb{V} = \left\{ f_{R/P} \left(t_0 = t_k, t = t_k + h, \vec{r}, \qquad \Pi = \Pi\big(p(t_k)\big), \vec{M}_{DIM} = \vec{M}_{DIM}(t_k) \right) \right\}$$

6. Terms of the CDF Expression: In

$$CDF_{R/P}(x, t_k, h) = \frac{\#\{v \in \mathbb{V} \mid v \leq x\}}{N_{\mathbb{V}}}$$

$N_{\mathbb{V}}$ is the total number of scenarios. In

$$\mathbb{V} = \left\{f_{R/P}\left(t_0 = t_k, t = t_k + h, \vec{r}, \qquad \Pi = \Pi\big(p(t_k)\big), \vec{M}_{DIM} = \vec{M}_{DIM}(t_k)\right) \forall\, \vec{r}_\omega \in \Omega\right\}$$

$f_{R/P}$ are the functions computed using the DIM model, $\vec{r}_\omega$ are the scenarios for the predictor – the portfolio MTM values in the case originally discussed, and Ω is the ensemble of $\vec{r}_\omega$ spanned by the Monte Carlo simulation.

7. Suitability of IMRD for Backtesting: The IMRD in this form is directly suited for historical backtesting using the Probability Integral Transformation (PIT) framework (Diebold, Gunther, and Tay (1998)).
8. Forecasting Horizon PIT Time Series: Referring to the formalism described in one can derive the PIT time series $\tau_{R/P}$ for a portfolio p for a given forecasting horizon h and backtesting history $\mathcal{H}_{BT}$ as:

$$\tau_{R/P} = CDF\left(g_{R/P}\left(t = t_k + h, \Pi = \Pi\big(p(t_k + h)\big), \vec{M}_g = \vec{M}_g(t_k + h)\right), t_k, h\right) \forall\, t_k \in \mathcal{H}_{BT}$$

9. Samples from the Actual IMR Algorithm: In the expression for $\tau_{R/P}$ above, $g_{R/P}$ is the exact IMR algorithm for the IMR methodology that is to be forecast – defined as

$$IMR = g_{R/P}\left(t = t_\alpha, \Pi = \Pi\big(p(t_\alpha)\big), \vec{M}_g = \vec{M}_g(t_\alpha)\right)$$

and t_α are the sampling points in $\mathcal{H}_{BT}$.

10. Probability of t_k-realized IMR: Every element of the PIT time series $\tau_{R/P}$ corresponds to the probability of the realized IMR at time $t_k + h$ according to the DIM forecast built at t_k.
11. Backtesting of the Portfolio Models - Variations: As discussed extensively in Anfuso, Karyampas, and Nawroth (2017) one can backtest $\tau_{R/P}$ using uniformity tests. In particular, analogous to what was shown in Anfuso, Karyampas, and Nawroth (2017) for portfolio backtesting in the context of capital exposure models, one can use test metrics that do not

penalize conservative modeling – i.e., models overstating/understating posted/received IM. In all cases the appropriate TVS can be derived using numerical Monte Carlo simulations.

12. Factors affecting the Backtesting: In this setup the performance of a DIM is not done in isolation. The backtesting results will be mostly affected by the following.
13. Impact of $\vec{r}$ on Backtesting: As discussed earlier, $\vec{r}$ is the predictor used to associate an IMR with a given scenario/valuation time point. If $\vec{r}$ is a poor indicator for the IMR, the DIM forecast will consequently be poor.
14. Mapping of $\vec{r}$ to IMR: If the mapping model is not accurate, then the IMR associated with a given scenario will be inaccurate. For example, the models defined in

$$\sigma^2(i,t) = \mathbb{E}[\Delta MTM^2(i,t) \mid MTM(i,t)] = \sum_{k=0}^{n} a_{\sigma k} MTM^k(i,t)$$

$$IM_{R/P,U}(i,t) = \Phi^{-1}\big(0.99/0.01, \mu = 0, \sigma = \sigma(i,t)\big)$$

$$IM_{R/P}(i,t) = \alpha_{R/P}(t) \times IM_{R/P,U}(i,t)$$

$$\alpha_{R/P}(t) = \left[1 - h_{R/P}(t)\right] \times \sqrt{\frac{10\ BD}{MPoR}} \times \left[\alpha_{R/P,\infty} + \left(\alpha_{R/P,0} - \alpha_{R/P,\infty}\right) e^{-\beta_{R/P}(t)t}\right]$$

$$\alpha_{R/P,0} = \sqrt{\frac{MPoR}{10\ BD}} \times \frac{IMR_{R/P,SIMM}(t=0)}{q\big(0.99/0.01\,, \Delta MTM(0, MPoR)\big)}$$

include scaling functions to calibrate the calculated DIM to the observed

$$t = 0$$

IMR. The performance of the model is therefore dependent on the robustness of this calibration at future points in time.

15. RFE Models used for $\vec{r}$: The models ultimately determine the probability of a given IMR scenario. It may so happen that the mapping functions $f_{R/P}$ are accurate but the probabilities for the underlying scenarios for $\vec{r}$ are misstated, and, hence, cause backtesting failures.
16. Differential Impact of Backtesting Criterion: Note that
    a. The choice of $\vec{r}$, and
    b. The mapping

$$\vec{r} \to IMR$$

are also relevant to the backtesting methodology discussed earlier in this chapter. RFE models used for $\vec{r}$, however, are particular to this backtesting variance, since it concerns the probability weights of the IMRD.

Conclusion

1. Framework to Develop/Backtest DIM: This chapter has presented a complete framework to backtest and develop DIM models. The focus has been on B-IMR and SIMM, and the chapter has shown how to obtain forward-looking IM's from the simulated exposure paths using simple aggregation methods.
2. Applicability of the Proposed Model: The proposed model is suitable for both XVA pricing and capital exposure calculations; the haircut functions in

$$\alpha_{R/P}(t) = [1 - h_{R/P}(t)] \times \sqrt{\frac{10\ BD}{MPoR}} \times [\alpha_{R/P,\infty} + (\alpha_{R/P,0} - \alpha_{R/P,\infty})e^{-\beta_{R/P}(t)t}]$$

can be used to either improve the accuracy (pricing) or to ensure the conservatism of the forecast (capital).

3. CCR Capital using DIM Models: If a financial institution were to compute CCR exposure using internal model methods (IMM), the employment of a DIM could reduce the CCR capital significantly, even after the application of a conservative haircut.
4. Over-collateralization inherent in Basel SA-CCR: This should be compared with the regulatory alternative SA-CCR, where the benefits from over-collateralization are largely curbed (Anfuso and Karyampas (2015)).
5. Backtesting Methodology to Estimate Performance: As part of the proposed framework, this chapter introduced a backtesting methodology that is able to measure model performance for different applications of DIM.
6. Agnosticity of DIM to the Underlying IMR: The DIM model and the backtesting methodology presented are agnostic to the underlying IMR algorithm, and they can be applied in other contexts such as CCP IM methodologies.

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