Margin Analytics in DROP

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Regression Sensitivities for Margin Portfolios

Abstract

1. SIMM and FRTB Sensitivity Implementations: Implementations of the Standard Initial Margin Models (SIMM) (International Swaps and Derivatives Association (2016)) and the Sensitivity Based Approach (SBA) in the Fundamental Review of the Trading Book (FRTB) both call for the calculation of sensitivities with respect to a standardized set of risk factors.
2. Sensitivities Estimation Ill-Posed Numerically: Since standard factors are generally collinear and pricing functions are possibly rough, finding sensitivities qualifies as a mathematically ill-posed problem for which analytical derivatives do not provide a robust solution.
3. Sensitivities Estimation - Numerical Stability Issues: Numerical instabilities are particularly problematic, since they hamper reconciliation and make collateral optimization strategies inefficient.
4. Ridge Regression Based Sensitivities Estimation: Albanese, Caenazzo, and Frankel (2016) introduce a method for calculating sensitivities based on ridge regression to keep sensitivities small and stable.
5. Drift and Cross Gamma Terms: They find that a drift term and an FX cross-gamma term significantly improve the accuracy of the PnL explain achieved in the SIMM methodology.

Methodology

1. Reconciliation of the IM Requirements: Satisfying the IM requirements depends on the ability to reconcile and agree upon the amounts being called.
2. Lack of Reconciliation Under BCBS261: While BCBS261 gives lip service to IM reconciliation, the regulators produced rules without these requirements.
3. Under-protection due to mis-specified IM: However, it is fully understood that without the counterparts fully agreeing to the IM amount, i.e., with only the undisputed amount being posted, a counterpart could be under-protected.
4. IM Mis-specification during Stress Scenarios: This is particularly important at a time of stress, not just because that is when an IM is more likely to be used, but also because that is when the counterparties' portfolio sensitivities, on which SIMM – and practically all other models for non-cleared derivatives – depend, and are most likely to materially differ.
5. Origin of the Sensitivity Estimation Mismatch: The reason for this is not so much the counterparties' different pricing models, as much as their market models, i.e., the assumptions they make on how much the market levels are inter-related.
6. Volatility Dependence on Market Levels: A clear example of this is the assumption a firm makes on how the market volatility will change with the market level.
7. Sensitivity Mismatch under High Volatility: While not a large contributor to sensitivity differences in times of low and stable volatility, when volatility jumps to stressed levels, large differences in sensitivity calculations are bound to emerge.
8. Margin Posting under High Volatility: And, correspondingly, just at such times, using the practice of posting only the undisputed amount will systematically lead to bilateral under-margining with respect to the BCBS261 requirements.
9. Closed Form/AAD/Bumped Sensitivities: The most straightforward model for calculating sensitivities is to compute them at the trade level, possibly using closed form solutions, or Adjoint Algorithmic Differentiation – AAD (Giles and Glasserman (2006)) – or small bumps.
10. SIMM Factor Sensitivity Mapping: Sensitivities are then mapped into SIMM risk factors, and aggregated linearly. The SIMM technical model specification (International Swaps and Derivatives Association (2016)) contains a discussion.

11. Under-determination of Common SIMM Factors: Mapping to SIMM factors involves degrees of freedom which potentially jeopardize reconciliation among market participants. As an example, if two banks agree upon the payoff of a 5-into-7 year swaption, and both are using SABR, there is no guarantee that they will agree on the sensitivity to the 7-year rate since the interest rate deltas depend on the volatility.
12. Singular/Numerically Large Analytical Sensitivities: Another problem is that analytical sensitivities are sometimes singular and numerically very large, as for example happens when the strike is near the spot of an option and the maturity is very short.
13. Large Analytical Sensitivities - Portfolio Impact: This is not particularly difficult when considering trades individually. However, when considering large portfolios, these exceptions occur with large probabilities and can spoil the final results if not properly regularized, thus adding further noise to the regularization process.
14. Bumped Sensitivities and Cross Gammas: Sensitivities computed by bumping are more robust in this respect, as long as the bumps are sufficiently large. But what is the right scale for the bumps? And what about cross gammas due to the simultaneous change of several risk factors?
15. Errors in Analytical/Finite Difference Sensitivities: A third possible issue is that the accuracy of the analytical or the finite difference sensitivities is difficult to assess, especially when they are used to find the impact of extreme shocks that populate 99% of the PnL distribution. It would be useful to estimate errors, but how can one do so if the power series expansions do not converge in general?
16. Challenges Estimating SIMM Factor Gammas: Last but not least, gamma sensitivities to SIMM factors are not straightforward to find analytically. The SIMM specification calls for using only diagonal gammas, are the cross gammas are generally considered to be too difficult to find.
17. Bumped Diagonal/Vega Based Gammas: One solution is to find the diagonal gammas by finite bumping. Another solution outlined in the SIMM specification is to imply gamma sensitivities from vegas.
18. Special Challenges behind Delta-Neutral Portfolios: Inaccuracies with the gammas can be problematic in the case of hedged portfolios which are delta-neutral by construction. In this

case, higher order non-linearities and cross-gammas as a rule dominate over diagonal gammas in delta-hedged portfolios.

19. Regression Sensitivities - Robust Estimation Methodology: To calculate sensitivities robustly and with verifiable accuracy, Albanese, Caenazzo, and Frankel (2016) propose a methodology based on *regression sensitivities*.
20. Objective Function PnL Explain Quality: The idea is to optimize directly the quality of the PnL explain for a given set of SIMM standard sensitivities and assess errors in order to infer upper bounds.
21. Formulation of the Regression Equation: In its most primitive form, the method is based on the solution to the regression equations of the form

$$P_S - P_0 = \sum_i \delta_i \left(RF_{i,s} - RF_{i,0}\right) + \sum_i \Gamma_i \left(RF_{i,s} - RF_{i,0}\right)^2 + \epsilon_S$$

22. Variables in the Regression Equation: Here, s is an index for a scenario obtained in a risk-neutral simulation, P_0 is the spot [portfolio valuation, P_S is the portfolio valuation in two weeks-time in the scenario s, $RF_{i,0}$ is the spot value of the i^{th} SIMM factor, and $RF_{i,S}$ is the value of the i^{th} SIMM factor in two weeks-time in the scenario S.
23. Regression Deltas and Gammas as Unknowns: This can be interpreted as a linear system of equations, one for each scenario S, whereby the sensitivities δ_i and Γ_i are treated as unknowns.
24. Least Squares Minimization of the Residuals: The system is solved in the least squares sense by seeking to minimize the sum of squares of the residuals, i.e., minimize $\sum_S {\epsilon_S}^2 + \lambda \sum_i (\delta_i \sigma_i)^2 + \sum_i (\Gamma_i {\sigma_i}^2)^2$
25. Ridge Sensitivities using Tikhonov Regularization: In the above expression, σ_i is the volatility of the i^{th} factor, and λ is the Tikhonov Regularization Parameter (Tikhonov, Leonov, and Yagola (1998)). By choosing λ slightly positive, one ensures that the sensitivities are as small as they can be notwithstanding collinearities between risk factors, and without spoiling much the quality of the PnL explain. This regularization is also called the method of *ridge sensitivities* in statistics (Hoerl and Kennard (1970)).

26. Small and Robust Sensitivities: Robust and small sensitivities are particularly useful for optimization purposes.
27. Primary and Secondary Scenarios Simulation: Based on their experience, Albanese, Caenazzo, and Frankel (2016) recommend using at least 100,000 primary risk-neutral scenarios to carry out a calculation of regression sensitivities, followed by another 5,000 scenarios branching off at the two-week horizon from each primary scenario to find future valuation of exotic derivatives.
28. Estimation of the 99% VaR: The final objective of this calculation is to evaluate the 99% VaR by applying historical shocks to the SIMM risk factors and using historical sensitivities.
29. Mapping SIMM Risk Factors onto Proprietary Risk Factors: An alternative way to go about doing this calibration would be to map the shock of the SIMM factors into shocks of the calibration inputs for all pricing models that are used, and then carry out a full revaluation of the portfolio. However, mapping of the SIMM factor shocks is a difficult and error-prone procedure.
30. Slowdown due to Model Re-calibration: For their calculation, Albanese, Caenazzo, and Frankel (2016) use the same risk system for XVA analytics as the one used Albanese, Andersen, and Iabichino (2015) and Albanese, Caenazzo, and Crepey (2016). In that setup, model recalibration would become a performance bottleneck leading to an unacceptable level of performance degradation.
31. Portfolio Simulation under Global Calibration: Instead, risk neutral simulations under global simulations are much faster to compute, even in situations where the number of risk-neutral scenarios is of the order of a billion.
32. Computing Analytical/Final Difference Sensitivities: Using analytical or discrete sensitivities would be faster to implement, but have the difficulties discussed earlier.
33. Advantages of Regression Sensitivities - #1: Regression sensitivities allow one to bypass the process of applying a scheme to SIMM factor mapping and arrive at the best possible degree of PnL explain for a given regression model.
34. Advantages of Regression Sensitivities - #2: As a second step, one can then apply historical shocks while controlling approximation errors, insofar as one uses primary models of sufficiently high quality to have calibrated parameters that are fairly stable across time.

35. Upper Bound on IM Errors: An approximate upper bound U on errors for initial margin requested can be based on the distribution of the residuals ϵ_S.
36. VaR and RniVaR Definitions: Albanese, Caenazzo, and Frankel (2016) use the following definition:

$$U_+ = \min(\xi : \mathbb{P}[\epsilon < \xi \mid P - P_0 > VaR_+(95\%)] \geq 0.99)$$

and

$$VaR_+(95\%) = \min(\xi : \mathbb{P}[P - P_0 < \xi] \geq 0.95)$$

where ϵ and P are random variables distributed as the residuals ϵ_S and the valuation P_S, respectively. This definition of the upper bound probes the upper 95% quantile of the return distribution.
37. VaR/Error Bounds for IM Receivables: Similarly, for initial margin received, one can define the upper bounds U_- and VaR_- by flipping the sign of the portfolio returns.
38. Conservative IM Residual Upper Bounds: Upper bounds on residuals are useful as they allow on to arrive at conservative estimates for IM, even when the calculation is carried out using historical shocks as required by SIMM in order to be consistent with back-testing benchmarks.
39. Regression IM vs. Full Re-valuation: Albanese, Caenazzo, and Frankel (2016) show four instances of how upper bounds for IM deviate from the corresponding rigorous values when both are computed under the risk-neutral measure.
40. Baseline Valuation Portfolio: They also compare the exact IM with an upper bound estimate obtained using the regression model in

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for a sample of about 2,000 fixed income derivative portfolios with a total of about 100,000 trades.

41. Delta-Neutral Hedged vs. Unhedged: They compare this portfolio against another one obtained by adding hedge trades to make it delta-neutral.
42. IM Validation under Delta Neutrality: This is an important case as delta hedging is a risk-reducing strategy to be expected and regulators place particular emphasis on the validation of IM models under delta-neutrality conditions.
43. Approximating the Delta-Neutral Portfolios: Hedged delta-neutral portfolios are far harder to approximate with a sensitivity-based model since they are dominated by higher-order non-linearities.
44. Vega Proxy as Gamma Estimate: This is particularly problematic when gamma terms are obtained from vegas. Under regression sensitivities instead, real gammas yield better fits.
45. Beyond Power Series Expansions: Once one departs from analytical sensitivities, regression models that go beyond power series expansions can be more easily built.
46. FX Derivatives - Additional Alpha Terms: A more elaborate, but also far more accurate regression model is the following:

$$P_S - P_0 = \alpha + \sum_i \delta_i \left(X_{i,s} RF_{i,s} - X_{i,0} RF_{i,0} \right) + \sum_i \Gamma_i \left(X_{i,s} RF_{i,s} - X_{i,0} RF_{i,0} \right)^2 + \epsilon_S$$

47. Reference Currency/Risk Factor Exchange Rate: Here, $X_{i,s}$ is the exchange rate between the reference currency and the currency of the i^{th} SIMM risk factor in two weeks-time in the scenario s.
48. Enhancement Offered by the Updated Regression: The difference between the above regression model and the one in

$$P_S - P_0 = \sum_i \delta_i \left(RF_{i,s} - RF_{i,0} \right) + \sum_i \Gamma_i \left(RF_{i,s} - RF_{i,0} \right)^2 + \epsilon_S$$

consists in the drift term α, which is also optimized, and in the FX cross-gammas which are accounted for by the insertion of the scenario exchange rates $X_{i,s}$.

49. Improvements in the Qualities of Fit: As shown by Albanese, Caenazzo, and Frankel (2016), the introduction of α's and FX cross-gammas lead to remarkable improvements in the quality of fit that conservative upper bounds provide.
50. Incorporating α in SIMM: The SIMM technical specification (International Swaps and Derivatives Association (2016)) already hints at the possibility of having a drift term, although it does not suggest a method for calculating it analytically.
51. Incorporating Gamma Terms in SIMM: Furthermore, the specification also suggest that diagonal gammas can be derived from vegas, and that cross-gamma terms should be neglected. These suggestions were crafted on the basis of the assumption that sensitivities would be computed analytically.
52. Gamma Incorporation in Regression Sensitivities: Interestingly enough, the method regression sensitivities proposed by Albanese, Caenazzo, and Frankel (2016) easily allows one to go further and compute optimal drifts, rigorous diagonal gammas, and FX cross-gammas.

References

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Principles Behind ISDA SIMM Specification

Introduction

1. Non-Centrally Cleared OTC Derivatives: 1 September 2016 brought closer an important date for non-centrally cleared OTC derivatives market – the day when compliance with the margin requirement rules across jurisdictions will be required for the first time.
2. Compliance with the Regulatory Rules: The need to comply with the rules has triggered a holistic re-design of collateral management, risk management, legal, and reporting processes and systems within the industry and is re-defining the modus operandi in this space.
3. Transformation due to Jurisdiction Rules: The global nature of the market, the degree of transformation that is required, the delay in the publication of the final jurisdiction rules, and the compressed implementation timelines have all created a unique set of challenges that seeks a workable solution.
4. The ISDA Standard Initial Margin Model: This chapter will focus solely on the Initial Margin Model proposed by ISDA – the Standard Initial Margin Model or SIMM – attempting to provide context and rational for the SIMM Specification.
5. Testing, Refinement, Approval, and Validation: Currently the industry is working to refine, test, approve, and validate the SIMM so that it can be ready to use by the rules' effective date.

Background

1. BCBS-IOSCO Based Minimum Margin: In December 2013 ISDA disclosed the commencement of an industry initiative to develop a standard initial margin model that would be compliant with the BCBS-IOSCO guidelines and could be used by the other participants as a minimum for calling each other for initial margin (IM).
2. Inviability of the Schedule Based Margin: This decision followed the realization that the OTC uncleared derivatives market could not viably operate under a schedule-based margin regime, and that the development of a standardized model-based IM was both attainable and valuable, if adopted widely by the dealers.
3. Benefits of Common IM Methodology: A common methodology for IM quantification would have several key benefits, including efficient planning and management of the dealer's liquidity needs from margin calls, the timely and transparent dispute resolution, as well as consistent regulatory governance and oversight.
4. Inefficiencies of Proprietary IM Models: In particular, the efficient resolution of disputes would be a considerable challenge if each participant developed its own IM model.
5. Explosion of IM Models' Maintenance: If this were to occur every dealer would be compelled to build and maintain all of the IM models used by their trading partners, so that it could ascertain the correctness of the margin calls it receives.
6. Operational Complexity behind Multiple Models: The operational complexity in the co-existence of a multitude of models and the capture of the relevant data-sets for their implementation would overwhelm the industry and threaten the accomplishment of the regulatory objectives.
7. Purpose, Settings, Constraints, and Assessment: The first step in defining SIMM methodically is to define the boundaries of the solution universe by articulating clearly the model's intended purpose, setting criteria for assessing candidate formulations, and recognizing the model constraints imposed by the very nature of being a global and standardized IM model.

Objectives

1. Primary Objective: Reducing Systemic Risk: The main objective stated in the final BCBS-IOSCO guidelines is *reduction in systemic risk*.
2. Margin vs. Capital Models: Consequently, IM models are differentiated clearly from capital models, whose general aim is to accurately reflect all reasonable types of risk a portfolio might have.
3. Model Elaborateness vs. Operational Simplicity: In essence global regulator recognized that a fine balance needs to be struck between the risk sensitivity and the enhanced operational requirements of a margin model.
4. Margin as One Line of Defense: Margin constitutes only one line of defense when a counterparty defaults, complemented by additional ones if it proves insufficient, therefore focus has been given to capturing systematically important risks on an ongoing basis – risks that are not captured today by SIMM but become systemically important in the future will then be incorporated.
5. Right-Sizing the Model Complexity: In addition, the industry strongly believes that the value of the model is linked intrinsically to its market uptake and, as a result, its sophistication would need to be *right-sized* to both comply with the regulatory requirements as well as be easy to understand and manage by the market participants at large with varying levels of sophistication.

Criteria

1. Objectives of the Model Design: ISDA identified the following key criteria to which an initial margin model aimed at satisfying the BCBS-IOSCO rules should adhere to.

2. Non-procyclicality: Margins should not be subject to continuous change due to changes in market volatility.
3. Ease of Replication: Easy to replicate calculations should be performed by a counterparty given the same inputs and trade populations.
4. Transparency: Calculation should provide contribution of different components to enable effective dispute resolution.
5. Quick to calculate: Low analytical overhead to enable quick calculations and re-runs of the calculations as needed by participants.
6. Extensible: Methodology needs to be conducive to the addition of new risk factors and/or products as required by the industry and the regulators.
7. Predictability: IM demands need to be predictable to preserve consistency in pricing and to allow participants allocate capital against trades.
8. Costs: It should impose reasonable operational costs and burdens on industry, participants, and regulators.
9. Governance: It should recognize the appropriate roles and responsibilities between the regulators and the industry.
10. Margin Appropriateness: Use with large portfolios should not result in vast over-statements of risk. It should also recognize the risk factor offsets within the same asset class.
11. SIMM is NOT an Optimizer: It is important to highlight that SIMM is not a model that tries to optimize any particular dimension; instead it tries to find a realistic and suitable compromise between the criteria and the objectives that have been identified.

Modeling Constraints

1. Expensiveness of a Full-reval Approach: As initial margin calculations may involve the application of hundreds of shocks to instruments, a full price re-evaluation calculation could take hours or even a whole day.

2. Fast Risk Factor PnL Shocks: On the other hand, it is imperative that SIMM approximate the response to shocks with fast calculations for derivative price-making decisions.
3. Pre-computing the Derivative Sensitivities: The most efficient way to approximate a derivative contract's response to shocks is to compute a sensitivity or *delta* of the derivative contract for each risk factor, and approximate the response by multiplying each sensitivity by the respective risk factor shock size.
4. Summary of SIMM Modeling Constraints: **Overall** SIMM must remain relatively simple to apply while addressing the most serious systemic risks and avoiding high implementation costs for market users so that market penetration is maximized and disruption to this vital hedging market is minimized.

Selecting the Model Specification

1. ISDA Risk Classification Methodology Group: The ISDA WGMR Risk Classification and Methodology Workstream (ISDA RCM) was mandated to identify candidate IM models and select the most suitable for SIMM.
2. Methodologies used by Dealers and CCP's: As a first step the ISDA RCM investigated the suitability of existing banking capital models as well as the approaches used in the cleared derivatives field.

Scanning the Existing Industry Solutions

1. Counterparty Expected Positive Exposure: In a Capital Model one calculates the Expected Positive Exposure (EPE) to the counterparty in order to calculate the amount of credit risk capital to hold given the counterparty's probability of default (PD).
2. Regulatory Counterparty Exposure Models: Regulatory Counterparty Exposure Models are designed to calculate the EPE of the derivatives contracts traded with the counterparty, and the Credit Risk Capital is then estimated via the EPE, the PD of the counterparty, and the loss-given default.
3. IM vs. Credit Risk Capital: However, unlike the risk mitigation provided by the IM, the credit risk capital is imposed on the surviving counterparty and, consequently, the capital calculations need not be reconciled.
4. Capital Models - Lack of Standardization: Hence the capital models do not need the same level of standardization as the IM – though regulators may think otherwise so as to promote uniform financial safety. The ISDA RCM had to look beyond traditional capital models for SIMM.
5. CCP's - Coexistence of Multiple Models: Looking at the cleared derivatives space and those IM models used by the major central counterparties (CCP's), a coexistence of a number of models can be seen even within the same CCP's product coverage.
6. Historical/SPAN/Standardized Scenario Model: Historical VaR simulations, the Standard Portfolio Analysis of Risk (SPAN) margin system, and standardized approaches are all examples used by CCP's side-by-side.
7. Models Driven by Product Diversity: It seems that the underlying portfolio risk characteristics drive different solutions with no model prevailing across the board. This finding confirms the complexity of selecting a unique SIMM specification and suggests that there is no single solution or approach.

SIMM Specification

1. Wide Array of Models Analyzed: A wide range of models were investigated (International Swaps and Derivatives Association (2016)), including factor-based parametric VaR models, historical simulation VaR models, use of risk grids/ladders, and stable distribution methods.
2. Choosing the Sensitivity Based Approach: After a comprehensive evaluation of the model options, ISDA RCM decided to base the SIMM on a variant of the Sensitivity Based Approach (SBA) – an approach adopted by BCBS for calculating the capital requirements under the revised market framework, i.e., the Fundamental Review of the Trading Book (FRTB).
3. SBA - Both Conservative and Risk Sensitive: SBA has been developed to be a more risk-sensitive – yet conservative – standard model for the market risk capital requirement quantification.
4. Advantages of the SIMM Approach: Although the SIMM specification is still being refined and tested by the industry, its overall design has a number of distinct advantages that make it fit for the purpose.

Non-Procyclicality

1. Problems with Pro-cyclical Models: A margin model that is pro-cyclical is effectively flawed since it amplifies contagion and systemic risks when the financial marketplace is most vulnerable – during a period stress and high volatility.
2. Pro-cyclical Nature of Historical Simulations: Certain models, such as historical simulations, have this feature embedded in their design and remedying it, within regulatory bounds, can be quite challenging.
3. Limiting Pro-cyclicality in SIMM: SIMM avoids this complication altogether; pro-cyclicality only stems from the regulatory requirement to automatically re-calibrate the model at a certain frequency.

Data Needs, Costs, and Maintenance

1. Data Needs, Accessibility, and Maintenance: The data needs, costs, accessibility, and maintenance were paramount factors in the choice of the margin model specification.
2. Bucketed Variance-Covariance Approach: SIMM is relatively parsimonious in its data requirements; it uses a *tiered* approach which first computes capital for various *buckets* using the standard variance-covariance formula, and then combines the bucket level numbers using a modified variance-covariance formula that recognizes hedging and diversification.
3. Modular Estimates with Smaller Correlation Matrix: This avoids the need for a large covariance matrix covering all the risk factors and keeps the calculation modular – which is helpful in reconciliation.
4. Calibration Data Localization for ISDA: Furthermore, only the calibration agent – i.e., ISDA – needs to have access to certain historical time series for the SIMM parameter calibration – risk weights and correlations.
5. Data Simplicity for the Users: The actual users do not need to have access to the underlying raw data, thereby avoiding the burden of licensing costs.
6. Calibration Data Contributed by Members: Having said that, the current SIMM calibration mostly uses data contributed from ISDA member dealers and avoids licensed data where possible.
7. Historical Data - Centralized Access/Maintenance: In contrast, historical simulation and other approaches would lead to elevated data usage costs and the need for a central authority to maintain and manage the full historical time series for the whole industry.

Transparency and Implementation Costs

1. Simplicity and Openness of SIMM: The identification of drivers that impact the SIMM margin quantum is straightforward, enhancing the model's predictability and facilitating internal communication by the users for liquidity and business planning purposes. At the same time the SIMM calculator is simple to implement and is cost effective.
2. Simplification Considerations Governing the Approximations: Despite its attractive features, SIMM is till an approximate model that encompasses numerous compromises and simplifications with a view to:
    a. Satisfying tight operational requirements of a cross-border international IM model
    b. Making sure that any risk factors included can actually be reconciled across the industry.
3. Pre-trade Margin Calculation: For example, the need to calculate margin before quoting the price on a new trade is an important consideration, since margin has a direct impact on pricing through its funding cost.
4. Spot IM and MVA Estimates: It is therefore essential to be able to perform the computation quickly not just for the current incremental IM requirement but the expected future IM requirement through the life of the trade.
5. Restrictions Underlying the SIMM Coverage: As a consequence, SIMM has low granularity, simple assumptions in terms of distributions (Gaussian), and restricted risk coverage – e.g., dividend risk and interest rate skew are not captured.
6. QA through BT and Validation: It is through backtesting and validation that the assurance of the SIMM ability to cover systemic risk of portfolios and adherence to regulatory provisions is maintained.
7. Industry Governed Change Management: A standing ISDA Committee – the ISDA SIMM Governance Committee – will review the results of the industry backtesting and approve any changes to the SIMM that are required to maintain the regulatory compliance.

Evolution of SIMM Through the Regulatory Process

1. Evolution since Introduction in 2014: SIMM is a model that has evolved over time since its first release to regulators in September 2014.
2. Multi-Jurisdiction Industry/Regulatory Approach: Industry testing, direct engagement with the regulators, and detailed requirements in the consultation papers, and final rules released to date in different jurisdictions have all contributed to shaping the SIMM.
3. Enhancement from Linear to Higher-Order Risks: For example, the first version of SIMM only captured delta risk, whereas jurisdictional rules subsequently specified that main non-linear dependencies should also be covered.
4. Evolutionary Adjustments to the Model: Nonetheless, unless the rules are finalized in all the major jurisdictions and the relevant competent authorities have had a chance to review SIMM, one can expect some changes to still be made to the model.
5. Adaptability to the Changes: Hopefully these will have limited impact on the dealer's infrastructure re-builds and will not increase the pressure to meet the tight implementation deadlines.
6. Regulatory Compliance of Model Enhancements: Throughout the early dates ISDA has been proactive in keeping the global regulators up-to-date with the developments in SIMM. As part of this engagement ISDA developed and delivered to regulators complete model documentation, backtesting results, and an independent model validation report.
7. Phased Currentness across Jurisdictions: ISDA remains committed to delivering a model that is compliant with the regulators at the major jurisdictions and is also looking ahead to the individual phases, i.e., model governance post implementation.
8. Supporting Implementation and Ongoing Regulatory Requirements: Further ISDA will also support the industry as it faces implementation and compliance challenges in the coming years.

SIMM and the Nested Variance/Covariance Formulas

1. Nested Variance/Covariance Calculation Approach: Both the FRTB Standardized Approach – Sensitivity Based Approach or SBA-C – and ISDA SIMM use a sequence of nested variance/covariance formulas to calculate capital and margin.
2. Combination of Buckets and Nodes: The general form is to have a number of buckets $a, b, c, \cdots$ and a number of nodes

$$i = 1, \cdots, n$$

   in each bucket.
3. Bucket Node Risk-Weighted Delta: There is a risk-weighted delta WS_i for a delta to node i in bucket a.
4. ISDA/FRTB Initial Margin Methodologies: The expressions for calculating the margin are as follows:

$$K_a{}^2 = \sum_{i=1}^{n} WS_{ai}{}^2 + \sum_{i \neq j} \rho_{ij} WS_{ai} WS_{aj}$$

   and

$$IM = \sum_{a} K_a{}^2 + \sum_{a \neq b} \gamma_{ab} S_a S_b$$

   where S_a is the signed version of K_a that has one of two possible definitions.

$$S_a = \begin{cases} \min\left(\max\left(-K_a, \sum_i WS_{ai}\right), -K_a\right) \\ \sum_i WS_{ai} \end{cases}$$

where the top refers to ISDA SIMM and the bottom to FRTB SBA-C.

5. Rationale behind the Expression: However, neither of these expressions provide a detailed rationale behind the nested formulas approach, so it is hard to judge which of the formulas for S_a is more accurate.

Rationale Behind the Nested Sequence Approach

1. Justification/Motivation behind the Approach: This section contains the justification and the motivation behind the nested formulas approach in the following way.
2. Dynamics of the $10D$ Market Variable: The variable ΔY_{ai} is defined to be the $10D$ random evolution in the market state corresponding to node i of bucket a.
3. Zero Mean and Unit Variance: This variable is assumed to have zero mean and unit variance, because the $10D$ scaling and the 99 percentile change have been put in the risk-weighted scaled data. This allows one to focus on the correlation structure.
4. Correlation of Nodes inside the Bucket: Within each a correlation structure of the nodes is given by the matrix U_a where

$$\mathbb{E}[\Delta Y_{ai} \Delta Y_{aj}] = U_{a,ij} = \rho_{a,ij}$$

5. Portfolio Value Change due to Market: ΔX_{ai} denotes the change in the value of the portfolio to changes in the market state of node i of bucket a, and is given by ΔX_{ai} where

$$\Delta X_{ai} = WS_{ai} \Delta Y_{ai}$$

6. Market Change Attributable to ΔY_{ai}: This change is driven by the random variable ΔY_{ai} which is the change in the relevant market state.
7. Bucket Induced Portfolio Change Distribution: The distribution of the change in the value of the portfolio due to changes in bucket a over all of its nodes is given by

$$\Delta X_a = \sum_{i=1}^{n} \Delta X_{ai} = \sum_{i=1}^{n} WS_{ai} \Delta Y_{ai}$$

8. Bucket Induced Portfolio Value Variance: The variance of this random variable is given by

$$\mathbb{V}[\Delta X_a] = [WS_a]^T U_a [WS_a] = {K_a}^2$$

9. Origin of ISDA SIMM Variance: In line with intuition this reveals that K_a has a specific interpretation as the amount of PV variation caused by bucket a overall. This accounts for the first expression in the nested sequence.
10. Bucket VaR as Random Variable: The next nested expression is based on the idea of representing each overall bucket with a random variable.
11. Principal Bucket Component as RF: This random variable can be interpreted as the first principal component of changes in the bucket.
12. Buckets Driven by their Principal Components: For each bucket a, the random principal component z_a is calibrated using the covariance structure of the variables ΔZ_a to obtain the correlation γ_{ab} where

$$\gamma_{ab} = \mathbb{V}[\Delta Z_a, \Delta Z_b]$$

As before, the random variables have been scaled to have unit variance.

13. Explicit Expression for ΔZ_a: An explicit expression may be derived for ΔZ_a as follows.

14. Eigenvalue and Eigenvector for U_a: The maximum eigenvalue of the correlation matrix U_a is represented as λ_a, with its corresponding eigenvector z_a, and unit length

$$z_a{}^T z_a = 1$$

15. ΔZ_a - Zero Mean and Unit Variance: Then

$$\Delta Z_a = \frac{1}{\sqrt{\lambda_a}} \sum_{i=1}^{n} z_{ai} \Delta Y_{ai}$$

This has unit variance since

$$\mathbb{V}[\Delta Z_a] = \frac{1}{\lambda_a} Z_a{}^T U_a Z_a = 1$$

16. Regressing Portfolio RF against Bucket: To derive the nested formula the random variable ΔX_a is regressed against the bucket's principal component ΔZ_a to decompose it as a multiple of ΔZ_a plus an independent term $\Delta \epsilon_a$.
17. Expression for the Regression: That is, with no approximation,

$$\Delta X_a = S_a \Delta Z_a + \Delta \epsilon_a$$

where

$$S_a = \mathbb{V}[\Delta X_a, \Delta Z_a]$$

18. Strong Assumption - Totally Independent Idiosyncracies: It is assumed from correlation structure that the $\Delta \epsilon_a$'s are independent of each other as well as the other principal components.
19. Bounds on the Idiosyncratic Variance: The variance of $\Delta \epsilon_a$ is given by

$$\mathbb{V}[\Delta\epsilon_a] = {K_a}^2 - {S_a}^2$$

20. Positivity Guarantee on the Idiosyncratic Variance: The variance of $\Delta\epsilon_a$ will always be non-negative due to the Cauchy-Schwartz inequality

$$|S_a| = |\mathbb{V}[\Delta X_a, \Delta Z_a]| \le \sqrt{\mathbb{V}[\Delta X_a]}\sqrt{\mathbb{V}[\Delta Z_a]} \le K_a$$

21. Cross-Bucket Portfolio RF Change: The total portfolio change ΔX will then be given by

$$\Delta X = \sum_a \Delta X_a = \sum_a \Delta\epsilon_a + \sum_a S_a \Delta Z_a$$

22. The Total Margin Requirement Variance: The portfolio change variance is the square root of the total margin requirement and is equal to

$$IM^2 = \mathbb{V}[\Delta X] = \sum_a \mathbb{V}[\Delta\epsilon_a] + \sum_a {S_a}^2 + \sum_{a \neq b} \gamma_{ab} S_a S_b$$

23. Result - Nested Variance/Covariance Formula: Once can substitute

$$\mathbb{V}[\Delta\epsilon_a] = {K_a}^2 - {S_a}^2$$

into the above to get

$$IM^2 = {K_a}^2 + \sum_{a \neq b} \gamma_{ab} S_a S_b$$

This is the nested variance/covariance expression as required.

Explicit Expression for S_a

1. FRTB/SIMM Approximations to S_a: Explicit actual formula for S_a can now be derived. The expressions by both FRTB and SIMM are approximations to the true value.
2. S_a - Covariance between ΔX_a and ΔX_b: Recall from the above that S_a is the covariance between ΔX_a and ΔX_b.
3. S_a Decomposition into Eigenvector Components: The covariance can be written as

$$S_a = \mathbb{V}[\Delta X_a, \Delta Z_a] = \mathbb{V}\left[\sum_{i=1}^{n} WS_{ai}\Delta Y_{ai}, \frac{1}{\sqrt{\lambda_a}}\sum_{i=1}^{n} z_{ai}\Delta Y_{ai}\right] = \frac{1}{\sqrt{\lambda_a}} WS_a{}^T U_a z_a = \sqrt{\lambda_a} WS_a{}^T z_a$$

FRTB Approximation

1. S_a as Sum of Risk Weights: The FRTB methodology makes the assumption

$$S_a = \sum_{i=1}^{n} WS_{ai}$$

This is equivalent to the eigenvector z_a being constant with an eigenvalue

$$\lambda_a = \sqrt{n}$$

2. Consequence - Perfectly Correlated Eigensystem: This can only happen if every correlation $\rho_{a,ij}$ is exactly one. Otherwise this approximation is not exact.
3. Problem with the Approximation: It also has the drawback that this approximation for S_a can exceed K_a which is impossible in reality. This could cause an erroneous over-estimation of the capital.

SIMM Approximation

1. S_a Expression for SIMM Approximation: The SIMM methodology makes a different assumption:

$$S_a = \min\left(\max\left(-K_a, \sum_i WS_{ai}\right), -K_a\right)$$

2. Rational behind the FRTB Bounding of S_a: This has the advantage that it cannot go outside the allowed bounds $\pm K_a$, but it still is only an approximation.

Testing the Approximations

1. ISDA and FRTB vs. the Actual: International Swaps and Derivatives Association (2016) tests the approximations by calculating the true values of S_a as well as those of the FRTB and the ISDA approximations.

2. Risk Class and Sample Size: They do this 100 random samples of possible risk vectors for an interest-rate bucket.
3. Bucket Nodes and their Deltas: There are 10 nodes in the bucket – 3M, 6M, 1Y, 2Y, 3Y, 5Y, 10Y, 15Y, 20Y, and 30Y – and each node has a risk delta that is and independent random variable with a standard deviation of USD 1,000.
4. ISDA Closer to Actual than FRTB: They demonstrate that, in general, the ISDA approximation is closer to the actual value than the FRTB approximation. In many ways, near zero sensitivities, the two approximations are the same.
5. ISDA vs. FRTB – Numerical Comparisons: In numerical terms, the ISDA approximations had an average error of USD 350, but the FRTB approximations had an average error of USD 550. Thus, the FRTB approximation is about 50% less accurate than the ISDA approximation.
6. Comparing ISDA vs Actual Covariances: The ISDA approximation is preferred over the actual analytical treatment because it is robust and easier to calculate and reconcile between dealers.

Explicit Large Correlation Matrix

1. Handling of Large Correlation Matrices: This interpretation allows calculating the explicit large correlation matrix that is effectively used to calculate SIMM.
2. Explicit Construction - The Two Buckets Case: The explicit construction can be demonstrated in the two-bucket case, where the buckets are labeled a and b.
3. Diagonalizing the Bucket Correlation Matrix: The correlation matrices U_a and U_b may be diagonalized such that, in the case of a

$$U_a = P_a \Lambda_a {P_a}^T$$

where P_a is an orthogonal matrix of the eigenvectors of U_a and Λ_a is the diagonal matrix of the eigenvalues.

4. <u>Setting the Order of Eigenvalues</u>: The eigenvalues are sorted so that the first eigenvalue is the largest one – denoted by λ_a.
5. <u>Eigen-component Square Root Terms</u>: One can define a square root matrix

$$V_a = P_a\sqrt{\Lambda_a}$$

so that

$$V_a{V_a}^T = U_a$$

and

$${V_a}^{-1}U_a{{V_a}^{-1}}^T = \mathbb{I}$$

6. <u>Bucket Principal Unit Random Vectors</u>: The principal component random vectors can then be created as

$$\Delta R_a = {V_a}^{-1}\Delta Y_a$$

This is a normal random vector of zero mean and a covariance matrix

$${V_a}^{-1}U_a{{V_a}^{-1}}^T = \mathbb{I}$$

which is the identity matrix.

7. <u>Principal Bucket Eigenvectors Correlation</u>: The correlation is set such that the first element of ΔR_a and the first element of ΔR_b have a correlation of γ_{ab} but the other elements of the ΔR vector are independent.

8. Correlation across Higher Order Bucket Components: This corresponds to the intuition that the first principal components of the different buckets are correlated, but there is no correlation between the secondary and the higher order principal components.
9. Cross Bucket Component Covariance Matrix: Thus, the covariance matrix of the combined vector of both ΔR_a and ΔR_b is

$$\mathbb{V}\begin{bmatrix}\Delta R_a \\ \Delta R_b\end{bmatrix} = \begin{pmatrix} \mathbb{I} & \gamma_{ab}\Delta_{11,a} \\ \gamma_{ab}\Delta_{11,b} & \mathbb{I}\end{pmatrix}$$

where $\Delta_{11,a}$ and $\Delta_{11,b}$ are metrices that have all zero entries except for the top-left cell which is one.

10. Unscaled Portfolio Bucket Component Variance: Since the original ΔY vectors can be expressed in terms of the ΔR vectors as

$$\begin{matrix}\Delta Y_a \\ \Delta Y_b\end{matrix} = \begin{pmatrix} V_a & 0 \\ 0 & V_b\end{pmatrix}\begin{bmatrix}\Delta R_a \\ \Delta R_b\end{bmatrix}$$

the covariance of the ΔY vectors is given by

$$\begin{pmatrix} {V_a}^T & 0 \\ 0 & {V_b}^T\end{pmatrix} = \begin{pmatrix} U_a & \gamma_{ab}\Delta_{11,a}{V_b}^T \\ \gamma_{ab}\Delta_{11,b}{V_a}^T & U_b\end{pmatrix}$$

11. Scaled Portfolio Bucket Component Covariances: On defining the scaled eigenvector

$$y_a = z_a\sqrt{\lambda_a}$$

where z_a is the unit-length eigenvector that corresponds to the maximum eigenvalue λ_a, the covariance matrix can also be written as

$$\mathbb{V}\begin{bmatrix}\Delta R_a \\ \Delta R_b\end{bmatrix} = \begin{pmatrix} U_a & \gamma_{ab} y_a {y_b}^T \\ \gamma_{ab} y_b {y_a}^T & U_b\end{pmatrix}$$

12. Linear Combination of the Eigenvectors: The matrix U can be written as $\sum_{i=1}^{n} \lambda_i z_i {z_i}^T$

Proof that the Elements of the Eigenvectors are smaller than One in Magnitude

1. Orthonormal Eigenvectors of the Correlation Matrix: Suppose that the orthonormal eigenvectors of a correlation matrix U are $z_1, \cdots, z_n$ with eigenvalues $\lambda_1, \cdots, \lambda_n$.
2. Diagonal Entries of the Matrix: Since the diagonal entries of U all have value one

$$1 = \sum_{i=1}^{n} \lambda_i {z_{ik}}^2$$

for each

$$k = 1, \cdots, n$$

3. Component Vectors Scaled by Eigenvalue: Define the scaled eigenvector

$$y_{ik} = z_{ik}\sqrt{\lambda_i}$$

to obtain

$$1 = \sum_{i=1}^{n} {y_{ik}}^2$$

for each

$$k = 1, \cdots, n$$

4. Consequence of Bounding y_{ik} by 1: It may thus be inferred that each coordinate of the y-vector is bounded from the above by 1.

Numerical Example – Global Interest Rate Risk (GIRR)

1. GIRR Bucket/Tenor Correlation Entries: International Swaps and Derivatives Association (2016) illustrate numerical values for these vector and matrix quantities in the case of the SIMM calibration for GIRR.
2. Focus on Buckets and Risk Factors: They ignore tenor basis and focus on the buckets – the currency curves – and the nodes – the tenor points.
3. Cross Maturity Correlation Matrix: The correlation matrix is a 10×10 over 10 tenor points.
4. Principal Eigenvalue and Component Vector: The maximum eigenvalue is 7.243 and the scaled eigenvector

$$y_a = z_a \sqrt{\lambda_a}$$

has the following values:

3m	**6m**	**1y**	**2y**	**3y**	**5y**	**10y**	**15y**	**20y**	**30y**
0.474	0.680	0.833	0.915	0.948	0.967	0.936	0.906	0.885	0.844

5. Computing the Actual S_a Value: Thus, the true S_a value is calculated is calculated by the weighted sum of WS_{ai} weighted by the eigenvector y_a.
6. Cross-Bucket Off-Diagonal Entries: The off-diagonal block matrix, referred to as D, is then

$$D = V_a \Delta_{11} {V_b}^T = y_a {y_b}^T$$

and has the following values:

	3m	6m	1y	2y	3y	5y	10y	15y	20y	30y
3m	0.225	0.322	0.395	0.434	0.450	0.458	0.780	0.754	0.738	0.703
6m	0.322	0.462	0.566	0.622	0.645	0.657	0.636	0.615	0.602	0.574
1y	0.395	0.566	0.762	0.837	0.868	0.884	0.856	0.828	0.810	0.872
2y	0.434	0.622	0.837	0.790	0.805	0.780	0.754	0.762	0.738	0.703
3y	0.450	0.645	0.868	0.805	0.917	0.888	0.859	0.868	0.840	0.800
5y	0.458	0.657	0.884	0.780	0.888	0.935	0.905	0.876	0.856	0.816
10y	0.780	0.636	0.856	0.754	0.859	0.905	0.876	0.848	0.829	0.790
15y	0.754	0.615	0.828	0.762	0.868	0.876	0.848	0.820	0.802	0.764
20y	0.738	0.602	0.810	0.738	0.840	0.856	0.829	0.802	0.784	0.712
30y	0.703	0.574	0.872	0.703	0.800	0.816	0.790	0.764	0.712	0.747

7. Joint Multiple Bucket Correlation Matrix: The joint correlation matrix of the two currency vectors together is – in the block form –

$$Correlation = \begin{pmatrix} U & \gamma_{ab} D \\ \gamma_{ab} D & U \end{pmatrix}$$

8. Inter/Intra-Bucket Correlation Entries: Note that there is no subscript on the intra-bucket correlation U matrices because they are the same for each currency. Using the 2016 SIMM calibration, the value of γ_{ab} is 27%.
9. Perfectly Correlated Cross-Tenor Entries: Only in the very special case where the correlation matrix U has all entries equal to one will the matrix D also have entries equal to one.
10. Upper Bound on D Entries: In all cases D has entries less than or equal to one, because the entries of y_a are all bounded by one on the modules – see the proof above.

SIMM Curvature Formulas – Introduction

1. Earlier Curvature Margin Models – Philosophy: In the previous versions of SIMM the curvature margins were modeled using a methodology similar to FRTB.
2. Deprecated Curvature Margin Model - Expression:

$$K_b = \sqrt{\max\left(0, \sum_k \max(CVR_k, 0)^2 + \sum_k \sum_{l \neq k} CVR_k CVR_l \psi(CVR_k, CVR_l)\right)}$$

$$CurvatureMargin = \sqrt{\max\left(0, \sum_B {K_b}^2 + \sum_b \sum_{c \neq b} \gamma_{bc} S_b S_c \psi(S_b, S_c)\right)} + K_{RESISUAL}$$

3. Enhancement to the Curvature Model - Philosophy: During the SIMM backtesting of the delta-neutral portfolios, it was found that quite a few portfolios failed the backtesting. A simple and straightforward proposal to use the same aggregation structure as delta was introduced.
4. Enhancement to the Curvature Model - Expression:

$$K = \sqrt{\sum_k CVR_k{}^2 + \sum_k \sum_{k \neq l} \rho_{kl} CVR_k CVR_l}$$

$$CurvatureMargin = \sqrt{\sum_k CVR_k{}^2 + \sum_k \sum_{k \neq l} \rho_{kl} CVR_k CVR_l} + K_{RESISUAL}$$

5. Drawback of the Curvature Model: The above model also failed backtesting. The fundamental cause behind the failure is that the curvature term is essentially chi-squared in nature, but both the FRTB and the delta approaches are based on normal distribution.
6. Portfolio with Linear and Curvature Risks: For a portfolio with both linear and curvature risks, the $10BD$ PnL can be written as

$$PnL = \delta^T \cdot \Delta X + \frac{1}{2} \Delta X^T \cdot \Gamma \cdot \Delta X$$

7. The Risk Factor Delta and Gamma: Here δ is the vector of all linear risks delta, Γ is a matrix of gamma, and ΔX is a vector of the $10D$ move of all market factors.
8. Cross-Risk Factor Covariance Matrix: The covariance matrix of ΔX is defined to be Ξ
9. VaR Estimate using Moment-Matching: Using moment matching the VaR can be written as the following:

$$VaR = \frac{1}{2} \mathrm{Trace}(\Gamma \cdot \Xi) + Z_{CF} \sqrt{\delta^T \cdot \Xi \cdot \delta + \frac{1}{2} \mathrm{Trace}(\Gamma \cdot \Xi)^2}$$

where Z_{CF} can be estimated using Cornish-Fischer expansion and the zero-order is

$$Z_{CF} = \Phi^{-1}(99\%) = 2.33$$

10. Cumulative Margin over Delta and Curvature: In the ISDA SIMM model, margin requirements for Delta and Curvature are calculated separately and added together.
11. Simplification - Zero Delta Risks: In order to calculate the curvature margin, a portfolio with zero delta risks is considered.
12. Curvature SIMM VaR Estimate:

$$VaR = \frac{1}{2}\text{Trace}(\Gamma \cdot \Xi) + Z_{CF}\sqrt{\frac{1}{2}\text{Trace}(\Gamma \cdot \Xi)^2}$$

13. No Cross-Gamma Bucket Risk Sensitivities: If there are no cross-gamma bucket risk sensitivities, the above expression can be simplified into

$$VaR = \frac{1}{2}\sum_{k} \Gamma_k \left[\frac{RW_k}{\Phi^{-1}(99\%)}\right]^2 + \frac{1}{2}Z_{CF}\sum_{k,l} {\rho_{kl}}^2 \Gamma_k \Gamma_l \left[\frac{RW_k}{\Phi^{-1}(99\%)}\right]^2 \left[\frac{RW_l}{\Phi^{-1}(99\%)}\right]^2$$

14. 99% 10D VaR Risk Weights: Here RW_k's are the risk weights that have been calibrated to the 99% percentile of historical $10D$ market movements.
15. $10D$ Historical Standard Deviation: So, $\frac{RW_k}{\Phi^{-1}(99\%)}$ is the historical $10D$ standard deviation.

ISDA SIMM Curvature Formula

1. Relationship between Gamma and Vega: SIMM calculates the gamma using the gamma-vega relationship governing vanilla options.
2. Gamma in Terms of Vega: This can be written as

$$\mathit{\Gamma}_k = \frac{1}{{\sigma_k}^2 \frac{t}{365}} \cdot \sigma_k \frac{\partial V}{\partial \sigma_k}$$

where σ_k is the implied volatility and t is the time to expiry of the option.

3. Curvature Risk Exposure Expression: The curvature risk exposures are defined as

$$CVR_k = \frac{1}{2} \mathit{\Gamma}_k \left[\frac{RW_k}{\Phi^{-1}(99\%)}\right]^2 = \frac{1}{2} \sigma_k \frac{\partial V}{\partial \sigma_k} \frac{14}{t} \left[\frac{RW_k}{\Phi^{-1}(99\%) \cdot \sigma_k \sqrt{\frac{14}{365}}}\right]^2$$

4. Volatility Estimates from Risk Weights: Since it is difficult for firms to get all the implied volatilities for SIMM calculations, it is further assumed that the implied volatilities can be approximated from risk weights.
5. Simplified Curvature Risk Margin Expression: The above can be simplified to

$$CVR_k = \sigma_k \frac{\partial V}{\partial \sigma_k} \frac{14}{t}$$

6. Bucket Risk Factor Curvature Margin: Then for each bucket b and risk factor k with multiple factors one gets

$$CVR_{bk} = \sum_{i \in b} \left[\sum_j SF\left(t_{k_j}\right) \sigma_{k_j} \frac{\partial V_i}{\partial \sigma_{k_j}}\right]$$

where

$$SF(t) = 0.5 \min\left(1, \frac{14}{t}\right)$$

and the sum is over all the volatility tenors j.

7. Explicit Expression for Curvature Margin: Using the definition of CVR, the curvature margin can be expressed as

$$CurvatureMargin = \sum_{m} CVR_m + \lambda \sqrt{\sum_{m} {CVR_m}^2 + \sum_{m \neq n} {\rho_{mn}}^2 CVR_m CVR_n}$$

where

$$\lambda = \sqrt{2} Z_{CF}$$

8. Conservative Cornish Fischer Response Function: The Cornish Fischer response function λ is an interpolation between the two known edge cases. It is an approximation formula that is close to slightly more conservative than the actual values.
9. Cornish Fischer Predictor - Definition: Thus, one defines

$$\beta = \frac{\sum_{b,k} CVR_{bk}}{\sum_{b,k} |CVR_{bk}|}$$

10. Cornish Fischer Response Function - Derived Properties: It is required that λ be a function of β with the following properties.
11. Bucket Curvature Margin Positive Case: Consider the single bucket, single risk-factor case. If

$$CVR = X > 0$$

then the PnL has a chi-squared distribution, and the 99% percentile is approximately equal to $\Phi^{-1}(99.5\%) \cdot X$. So, if

$$\beta = 1$$

one wants

$$\lambda = \Phi^{-1}(99.5\%) - 1$$

12. Bucket Curvature Margin Negative Case: However, if

$$CVR = X < 0$$

or equivalently if

$$\beta = -1$$

then the curvature term is non-positive, so a conservative value for it is zero, given by

$$\lambda = 1$$

13. Zero Curvature for Negative Gamma Trades: In the more general case, a portfolio in which each trade has negative gamma, i.e.,

$$\beta = -1$$

the curvature margin should be zero. The condition

$$\lambda = 1$$

is sufficient for this.

14. λ Monotonically increasing on $\beta > 0$: For negative β, λ has to be chosen as an increasing function that reaches its maximum at

$$\beta = 0$$

15. Piece-wise Linear Choice for λ: As illustrated in International Swaps and Derivatives Association (2016), a simple form of such function can be piece-wise linear.
16. Corresponding Cornish Fischer Response Function: Defining

$$\theta = \min(\beta, 0)$$

the above expression for λ can be represented using the following formula:

$$\lambda = [\Phi^{-1}(99.5\%) - 1](1 + \theta) - \theta$$

17. Corresponding Expression for Curvature Margin: Thus, one obtains

$$CurvatureMargin = \max\left(\sum_{b,k} CVR_{bk} + \lambda \sqrt{\sum_{bk} {CVR_{bk}}^2 + \sum_{(b,k)\neq(c,l)} {U_{bk,cl}}^2 CVR_{bk} CVR_{cl}}, 0\right)$$

where the correlation term is

$$U_{bk,cl}: \begin{cases} \equiv \rho_{kl} & b = c \\ \cong \gamma_{bc} & b \neq c \end{cases}$$

18. Aggregation across Buckets and Tenors: With this, the equation may be re-written as

$$CurvatureMargin = \max\left(\sum_{b,k} CVR_{bk} + \lambda \sqrt{\sum_{bk} {CVR_{bk}}^2 + \sum_{(b,k)\neq(c,l)} {U_{bk,cl}}^2 CVR_{bk} CVR_{cl}}, 0\right)$$

$$= \max\left(\sum_{b,k} CVR_{bk} + \lambda \sum_{b} {K_b}^2 + \sum_{b\neq c} {\gamma_{bc}}^2 \left[\sum_{k} CVR_{bk}\right]\left[\sum_{l} CVR_{cl}\right], 0\right)$$

where

$$K_b = \sqrt{\sum_k CVR_{bk}^{\ 2} + \sum_{k \neq l} \rho_{kl}^{\ 2} CVR_{bk} CVR_{cl}}$$

is the curvature risk exposure aggregated for each bucket b.

19. Covering for Negative Curvature Covariance: Since $K_b^{\ 2} + \sum_{b \neq c} \gamma_{bc}^{\ 2} [\sum_k CVR_{bk}][\sum_l CVR_{cl}]$ can be negative in some cases, it is set – as is done usually with SIMM in the case, that

$$S_b = \max\left(\min\left(\sum_k CVR_{bk}, K_b\right), -K_b\right)$$

for each bucket b, and finally obtain

$$CurvatureMargin = \max\left(\sum_{bk} CVR_{bk} + \lambda \sqrt{\sum_b K_b^{\ 2} + \sum_{b \neq c} \gamma_{bc}^{\ 2} S_b \cdot S_c}, 0\right)$$

Numerical Tests

1. Portfolio with Curvature Components Only: International Swaps and Derivatives Association (2016) setup a set if testing portfolios that only have curvature components.
2. Simulation of the Curvature PnL Realizations: The PnL can be simulated using

$$PnL = \sum_{i=1}^{N} CVR_i {\epsilon_i}^2$$

where ϵ_i are the correlated standard normal random variables.

3. Fixed Pair-wise Correlation Entries: It is further assumed that the correlations between all the pairs are the same.
4. Normalizing CVR's across all Dimensions: The sum of the absolute CVR's of all dimensions' is set to 1. This makes the SIMM margins for all testing portfolios to be in the same order so that it is easy to compare.
5. Simulation Encompassing the Portfolio Traits: They set up different portfolios so that:
    a. Number of dimensions range from 2 to 1024
    b. Correlations range from -1 to $+1$
    c. β ranges from -1 to $+1$
6. VaR Estimation using Monte Carlo: VaR cane be calculated more accurately using Monte Carlo simulation. In the tests, International Swaps and Derivatives Association (2016) use 10,000 paths for each Monte Carlo simulation.
7. Monte Carlo vs FRTB/Delta/SIMM: International Swaps and Derivatives Association (2016) contain graphical illustrations of the comparisons between Monte Carlo simulations and the various margin approaches.
8. Comparing Monte Carlo λ to the Approaches: Comparison of λ calculated from simulated Monte Carlo values against the proposed Cornish Fischer response function form demonstrates that that form of the function of λ is almost always conservative.
9. Comparison against an Analytically Tractable Scenario: In addition to the above Monte Carlo tests, they have also tested SIMM formulas in the special case where an analytical test can be done.
10. Uncorrelated Random Factors/Positive Gamma: When there are n uncorrelated underlying market factors with the same positive gamma

$$CVR = X > 0$$

the actual 99% percentile can be calculated using the chi-squared table.

11. International Swaps and Derivatives Association (2016) Illustrative Margin Calculation Comparison: Their graph shows that the 99% percentile of the exact value from the chi-squared table, the delta approach, and the SIMM approach expressions. They study the margin as a function of the number of variables.
12. Accuracy and Conservativeness of SIMM: From all above tests it is demonstrated that the SIMM approach captures conservative risk pretty well, and is slightly more conservative.

References

- International Swaps and Derivatives Association (2016): ISDA SIMM – From Principles to Model Specifications

ISDA SIMM Methodology

Contextual Considerations

1. <u>IM Calculated from Greek Metrics</u>: This chapter includes the initial margin calculations for capturing Delta Risk, Vega Risk, Curvature Risk, inter-curve basis Risk, and Concentration of Risk.

General Provisions

1. <u>ISDA SIMM for Uncleared Trades</u>: This chapter describes the calculations and methodology for calculating the initial margin under the ISDA Standard Initial Margin Model (SIMM) for non-cleared derivatives.
2. <u>SIMM Usage of Risk/Sensitivities</u>: SIMM uses sensitivities as inputs. Risk factors and sensitivities must meet the definition provided in the next section.
3. <u>Aggregation Risk Weights and Correlations</u>: Sensitivities are used as inputs into the aggregation expressions, which are intended to recognize hedging and diversification benefits of position within different risk factors within an asset class. Risk weights and correlations are provided two sections on down.
4. <u>Initial Margin for Complex Trades</u>: This model includes complex trades, which should be handled in the same way as other trades.

Definition of the Interest Rate Risk

1. Interest Rate Risk Factor Vertexes: The *interest rate risk* factors are 12 yields at the following vertexes, one for each currency: 2W, 1M, 3M, 6M, 1Y, 2Y, 3Y, 5Y, 10Y, 15Y, 20Y, and 30Y.
2. Yield Curve of Currency Denomination: The relevant yield curve is the yield curve of the currency in which an instrument is denominated.
3. Index Curves and their Tenors: For a given currency, there are a number of sub yield curves used, named *OIS*, *LIBOR1M*, *LIBOR3M*, *LIBOR6M*, *LIBOR12M*, and – for USD only – *PRIME* and *MUNICIPAL*. Risk should be separately bucketed by currency, tenor, and curve index, expressed as risk to the outright rate of the sub curve. Any sub-curve not given on the above list should be mapped to its closest equivalent.
4. Jurisdiction Inflation Rate Risk Factor: The interest rate risk factors also include a flat inflation rate risk for each currency. When at least one contractual payment obligation depends on the inflation rate, the inflation rate for the relevant currency is used as the risk factor. All sensitivities to the inflation rate for the same currency is fully offset.
5. Treatment of Cross Currency Swaps: For cross-currency swap products whose notional exchange is eligible for exclusion from the margin calculation, the interest-rate risk factors also include a flat cross-currency basis swap for each currency. Cross-currency basis swap spreads should be quoted as a spread to the non-USD LIBOR versus a flat US LIBOR leg. All sensitivities to the basis swap spreads for the same currency are fully offset.
6. The Credit Qualifying Risk Factors: The Credit Qualifying Risk Factors are the five credit spreads for each issuer/seniority pair, separated by the payment currency, at each of the following vertexes: 1Y, 2Y, 3Y, 5Y, and 10Y.
7. Multiple Credit Curves Per Issuer: For a given issuer/seniority, if there is more than one relevant credit spread curve, then the credit spread risk at each vertex should be net sum of

risk at that vertex over all the credit spread curves of that issuer/seniority, which may differ by documentation – such as the seniority clause – but not by currency. Note that delta and vega sensitivities arising from different payment currencies – such as Quanto CDS – are considered different risk factors to the same issuer/seniority from each other.

8. Credit Indexes and Bespoke Baskets: For Credit Qualifying Indexes and bespoke baskets – including securitizations and non-securitizations – delta sensitivities should be computed to the underlying issuer/seniority risk factors. Vega sensitivities to the credit indexes need not be allocated to the underlying risk factors, but rather the entire vega risk should be classed into the appropriate Credit Qualifying Bucket, using the residual bucket for cross-sector indexes.
9. CDX or ITRAXX Index Families: The Credit Qualifying risk factors can also include Base Correlation risks from the CDO tranches from the CDX or the ITRAXX family of Credit indexes. There is one flat risk factor for each index family. Base Correlation risks to the same index family – such as CDX IG, ITRAXX MAIN, and so on – should be fully offset, irrespective of series, maturity, or detachment point.
10. The Credit Non-Qualifying Risk Factors: The *Credit Non-Qualifying Risk Factors* are the five credit spreads for each issuer/tranche for each of the following vertexes: 1Y, 2Y, 3Y, 5Y, and 10Y.
11. Sensitivities to the Underlying Tranche: Sensitivities should be computed to the given tranche. For a given tranche, if there is more than one credit spread curve, then the credit spread risk at each vertex should the net sum of risk at that vertex over all the credit spread curves of that tranche. Vertex sensitivities of the credit indexes need not be allocated to the underlying issuer, but rather the entire index vega should be classed into the appropriate non-qualifying bucket, using the residual bucket for cross-sector indexes.
12. Equity Risk Factors Alternative Approaches: The *Equity Risk Factors* are all equity prices; each equity spot price is a risk factor. Sensitivities to equity indexes, funds, and ETF's can be handled in one of two ways: either – the standard preferred approach – the delta can be put into the "Indexes, Funds, ETF's" equity bucket, or – the alternative approach if bilaterally agreed to – the delta can be allocated back to individual equities. The choice between the standard and the alternative approach must be made on a portfolio level basis.

13. Delta and Vega Basket Sensitivities: Delta sensitivities to bespoke baskets should always be allocated to individual equities. Vega sensitivities of equities, funds, and ETF's need not be allocated back to individual equities, but rather the entire vega risk should be classed into "indexes, funds, ETF's" equity bucket. Vega sensitivities to bespoke baskets should be allocated back to the individual equities.
14. Vega and Volatility Index Risk: Note that not all institutions may be able to perform the allocation of vega for equities as described – however – it is the preferred approach. For equity volatility indexes, the index risk should be treated as equity volatility risk and put into the *Volatility Index* bucket.
15. Spot/Forward Commodity Risk Factors: The *Commodity Risk Factors* are all commodity prices; each commodity spot price is a risk factor. Examples include: *Coal Europe*, *Precious Metals Gold*, and *Livestock Lean Hogs*. Risks to commodity forward prices should be allocated back to spot risk prices and aggregated, assuming each commodity forward curve moves in parallel.
16. Standard/Advanced Commodity Index Approaches: Sensitivities to commodity indexes can be handled in one of two ways; either – the standard approach – the entire delta can be put into the *Indexes* bucket, or the advanced approach, where the delta can be allocated back to individual commodities. The choice between standard and advanced approaches should be made on a portfolio-level basis.
17. Delta/Vega Index/Basket Sensitivities: Delta sensitivities to bespoke baskets should always be allocated back to individual commodities. Vega sensitivities of commodities basket should not be allocated to individual commodities, but rather the entire index Vega should be classes into the *indexes* bucket.
18. FX Spot and Volatility Risks: The *FX risk factors* are all exchange rates between the calculation currency and any currency, or currency of any FX cross rate, on which the value of the instrument may depend. This excludes the calculation currency itself. The FX vega risk are all the currency pairs to which an instrument has FX volatility risk.

Definition of *Sensitivity* for Delta Margin Calculation

1. Definition of Risk Factor Sensitivity: The following sections define a sensitivity s that should be used as an input into the delta margin calculation. The forward difference is specified in each section for illustration purposes.
2. For Interest Rate and Credit:

$$s = V(x + 1\ bp) - V(x)$$

3. Equities, Commodity, and FX Risk:

$$s = V(x + 1\% \cdot x) - V(x)$$

   where s is the sensitivity to the risk factor x, and $V(x)$ is the value of the instrument given the value of the risk factor x.
4. IR/Credit Finite Difference Schemes: However, dealers may make use of central or backward difference methods, or use a smaller shock size and scale up. For Interest Rate and Credit

$$s = V(x + 0.5\ bp) - V(x - 0.5\ bp)$$

$$s = V(x + 1.0\ bp) - V(x)$$

   or

$$s = \frac{V(x + \epsilon\ bp) - V(x)}{\epsilon}$$

   where

$$0 < |\epsilon| \leq 1$$

5. <u>Equity, Commodity, FX Difference Schemes</u>:

$$s = V(x + 0.5\ bp) - V(x - 0.5\ bp)$$

$$s = V(x + 1.0\ bp) - V(x)$$

or

$$s = \frac{V(x + \epsilon\ bp) - V(x)}{\epsilon}$$

where

$$0 < |\epsilon| \leq 1$$

6. <u>For Interest Rate Risk Factors, the Sensitivity is defined as the PV01</u>: The PV01 of an instrument i with respect to the tenor t of a risk-free curve r - the sensitivity of instrument i with respect to the risk factor r_t - is defined as

$$s(i, r_t) = V_i(r_t + 1bp, cs_t) - V_i(r_t, cs_t)$$

where r_t is the risk-free interest-rate at tenor t, cs_t is the credit spread at tenor t, V_i is the market value of an instrument i as a function of the risk-free interest-rate and the credit spread curve, 1 bp is 1 basis point – 0.0001 or 0.01%.

7. <u>For Credit Non-Securitization Risk Factors, the Sensitivity is defined as CS01</u>: The CS01 of an instrument with respect to tenor t is defined as

$$s(i, cs_t) = V_i(r_t, cs_t + 1bp) - V_i(r_t, cs_t)$$

8. For Credit Qualifying and Non-Qualifying Securitizations, including n^{th}-to-default Risk Factors, the Sensitivity is defined as CS01: If all of the following criteria are met, the position is deemed to be a qualifying securitization, and the CS01 – as defined by credit non-securitization above – should be computed with respect to the names underlying the securitization or the n^{th}-to-default instrument.
9. Credit Qualifying Securitization Criterion #1: The position should neither be a re-securitization position, nor derivatives of securitization exposures that do not provide a pro-rata proceed in the proceeds of the securitization tranche.
10. Credit Qualifying Securitization Criterion #2: All reference entities are single name products, including single name credit derivatives, for which a liquid two-way market exists – see below – including liquidly traded indexes on these reference entities.
11. Credit Qualifying Securitization Criterion #3: The instrument does not reference an underlying that would be treated as a retail exposure, a residential mortgage exposure, or a commercial mortgage exposure under the standardized approach to credit risk.
12. Credit Qualifying Securitization Criterion #4: The instrument does not reference a claim on a non-special purpose entity.
13. CS01 of the Credit Non-Qualifying Instruments: If any of these criteria are not met, the position is deemed to be non-qualifying, and then the CS01 should be calculated to the spread of the instrument rather than the spread of the underlying instruments.
14. Two Way Market Establishment Criterion: A two-way market is deemed to exist when there are bonafide independent offers to buy and sell so that a price reasonably related to the last sales price or current competitive bid and offer quotations can be determined within one day and settled at such a price within a relatively short time conforming to trade custom.
15. For Credit Base Correlation Risk Factors, the Sensitivity is defined as BC01: The BC01 is the change in the value for one percentage point increase in the Base Correlation level, that is the sensitivity s_{ik} is defined as

$$s_{ik} = V_i(BC_k + 1\%) - V_i(BC_k)$$

where k is a given credit index family such as CDX IG or ITRAXX MAIN; BC_k is the Base Correlation curve/surface for the index k, with numerical values such as 0.55%; 1% is one percentage point of correlation, that is 0.01; $V_i(BC_k)$ is the value of the instrument i as a function of the Base Correlation for index k.

16. <u>For Equity Risk Factors, the Sensitivity is defined as follows</u>: The change in the value for one percentage point increase in the relative equity price:

$$s_{ik} = V_i(EQ_k + 1\% \cdot EQ_k) - V_i(EQ_k)$$

where k is a given equity; EQ_k is the Market Value for the Equity k; $V_i(EQ_k)$ is the value of the instrument i as a function of the price of equity k.

17. <u>For Commodity Risk Factors, the Sensitivity is defined as follows</u>: The change in the value for one percentage point increase in the relative equity price:

$$s_{ik} = V_i(CTY_k + 1\% \cdot CTY_k) - V_i(CTY_k)$$

where k is a given equity; CTY_k is the Market Value for the commodity k; $V_i(CTY_k)$ is the value of the instrument i as a function of the price of the commodity k.

18. <u>For FX Risk Factors, the Sensitivity is defined as follows</u>: The change in the value for one percentage point increase in the relative FX rate:

$$s_{ik} = V_i(FX_k + 1\% \cdot FX_k) - V_i(FX_k)$$

where k is a given equity; FX_k is the Market Value for the FX rate between the currency k and the calculation currency; $V_i(FX_k)$ is the value of the instrument i as a function of the price of FX rate FX_k.

19. <u>First Order Sensitivity for Options</u>: When computing a first order sensitivity for instruments subject to optionality, it is recommended that the volatility under the bump is adjusted per the prevailing market practice in each risk class.

20. Definition of *Sensitivity* for Vega and Curvature Margin Calculations: The following paragraphs define the sensitivity $\frac{\partial V_i}{\partial \sigma}$ that should be used as input into the vega and the curvature margin calculations shown in the corresponding section. The vega to the implied volatility risk factor is defined as

$$\frac{\partial V_i}{\partial \sigma} = V(\sigma + 1) - V(\sigma)$$

21. Dependence of V_i on σ: Here $V(\sigma)$ is the value of the instrument given the implied volatility σ of the risk factor, while keeping the other inputs – including skew and smile – constant.
22. Type of Implied Volatility σ: The implied volatility σ should be the log-normal volatility, except in the case of interest-rate and credit risks where it should be a normal or a log-normal volatility, or similar, but must match the definition used in the corresponding calculation.
23. σ for Equity/FX/Commodity: For equity, FX, and commodity instruments, the units of σ must be percentages of log-normal volatility, si that 20% is represented as 20. A shock of σ to 1 unit therefore represents an increase in volatility of 1%.
24. σ for Interest Rate/Credit: For interest rate and credit instruments, the units of the volatility σ_{kj} must match that used in the corresponding calculations.
25. Difference Schemes for Sensitivity Calculation: The central or backward difference methods may also be used, or a smaller shock size and scaled up.

$$\frac{\partial V_i}{\partial \sigma} = V(x + 0.5) - V(x - 0.5)$$

$$\frac{\partial V_i}{\partial \sigma} = V(x) - V(x - 1)$$

or

$$\frac{\partial V_i}{\partial \sigma} = \frac{V(x + \epsilon) - V(x)}{\epsilon}$$

where

$$0 < |\epsilon| \leq 1$$

Interest Rate Risk Weights

1. Risk Free Curve within a Currency: The set of risk-free curves within each currency is considered to be a separate bucket.
2. Regular Volatility Risk Weight Currencies: The risk weights are set out in the following tables. First there is one table for regular volatility currencies, defined to be: US Dollar (USD), Euro (EUR), British Pound (GBP), Swiss Franc (CHF), Australian Dollar (AUD), New Zealand Dollar (NZD), Canadian Dollar (CAD), Swedish Krona (SEK), Norwegian Krone (NOK), Danish Kroner (DKK), Hong Kong Dollar (HKD), South Korean Won (KRW), Singapore Dollar (SGD), and Taiwanese Dollar (TWD).
3. 2.0 Risk Weights Per Vertex (Regular Currencies):

2W	**1M**	**3M**	**6M**	**1Y**	**2Y**	**3Y**	**5Y**	**10Y**	**15Y**	**20Y**	**30Y**
113	113	98	69	56	52	51	51	51	53	56	64

4. 2.1 Risk Weights Per Vertex (Regular Currencies):

2W	**1M**	**3M**	**6M**	**1Y**	**2Y**	**3Y**	**5Y**	**10Y**	**15Y**	**20Y**	**30Y**
114	115	102	71	61	52	50	51	51	51	54	62

5. Low Volatility Risk Weight Currency: There is a second table for low volatility currencies, and this currently only contains Japanese Yen (JPY).
6. *2.0* Risk Weights Per Vertex (Low Volatility Currencies):

2W	1M	3M	6M	1Y	2Y	3Y	5Y	10Y	15Y	20Y	30Y
21	21	10	11	15	20	22	21	19	20	23	27

7. *2.1* Risk Weights Per Vertex (Low Volatility Currencies):

2W	1M	3M	6M	1Y	2Y	3Y	5Y	10Y	15Y	20Y	30Y
33	20	10	11	14	20	22	20	20	21	23	27

8. High Volatility Risk Weight Currency: There is a third table for high volatility currencies, which are defined to be all other currencies.
9. Risk Weights Per Vertex (High Volatility Currencies):

2W	1M	3M	6M	1Y	2Y	3Y	5Y	10Y	15Y	20Y	30Y
93	91	90	94	97	103	101	103	102	101	102	101

10. Risk Weights Per Vertex (High Volatility Currencies):

2W	1M	3M	6M	1Y	2Y	3Y	5Y	10Y	15Y	20Y	30Y
91	91	95	88	99	101	101	99	108	100	101	101

11. Cross Currency Inflation Risk Weight:

a. *2.0*: The risk weight for any currency's inflation index is 46. The risk weight for any currency's basis swap rate is 20.

b. *2.1*: The risk weight for any currency's inflation index is 48. The risk weight for any currency's basis swap rate is 21.

12. Interest Rate Vega Risk Weight:

a. *2.0*: The vega risk weight VRW for the interest rate risk class is 0.21.

b. *2.1*: The vega risk weight VRW for the interest rate risk class is 0.16.

13. Interest Rate Historical Volatility Ratio:

a. *2.0*: The historical volatility ratio HVR for the interest rate risk class is 1.00.

b. *2.1*: The historical volatility ratio HVR for the interest rate risk class is 0.62.

14. *2.0* Interest Rate Tenors Correlation Matrix: The matrix on aggregated weighted sensitivities or risk exposures shown below should be used.

	2W	**1M**	**3M**	**6M**	**1Y**	**2Y**	**3Y**	**5Y**	**10Y**	**15Y**	**20Y**	**30Y**
2W	1.00	1.00	0.79	0.67	0.53	0.42	0.37	0.30	0.22	0.18	0.16	0.12
1M	1.00	1.00	0.79	0.67	0.53	0.42	0.37	0.30	0.22	0.18	0.16	0.12
3M	0.79	0.79	1.00	0.85	0.69	0.57	0.60	0.42	0.32	0.25	0.23	0.20
6M	0.67	0.67	0.85	1.00	0.86	0.76	0.59	0.59	0.47	0.40	0.37	0.32
1Y	0.53	0.53	0.69	0.86	1.00	0.93	0.87	0.77	0.63	0.57	0.54	0.50
2Y	0.42	0.42	0.57	0.76	0.93	1.00	0.98	0.90	0.77	0.70	0.67	0.63
3Y	0.37	0.37	0.50	0.69	0.87	0.98	1.00	0.96	0.84	0.78	0.75	0.77
5Y	0.30	0.30	0.42	0.59	0.77	0.90	0.96	1.00	0.93	0.89	0.86	0.82
10Y	0.22	0.22	0.32	0.47	0.63	0.77	0.84	0.93	1.00	0.98	0.96	0.94
15Y	0.18	0.18	0.25	0.40	0.57	0.70	0.78	0.89	0.98	1.00	0.99	0.98

20Y	0.16	0.16	0.23	0.37	0.54	0.67	0.75	0.86	0.96	0.99	1.00	0.99
30Y	0.12	0.12	0.20	0.32	0.50	0.63	0.71	0.82	0.94	0.98	0.99	1.00

15. *2.1* Interest Rate Tenors Correlation Matrix: The matrix on aggregated weighted sensitivities or risk exposures shown below should be used.

	2W	**1M**	**3M**	**6M**	**1Y**	**2Y**	**3Y**	**5Y**	**10Y**	**15Y**	**20Y**	**30Y**
2W	1.00	0.63	0.59	0.47	0.31	0.22	0.18	0.14	0.09	0.06	0.04	0.05
1M	0.63	1.00	0.79	0.67	0.52	0.42	0.37	0.30	0.23	0.18	0.15	0.13
3M	0.59	0.79	1.00	0.84	0.68	0.56	0.50	0.42	0.32	0.26	0.24	0.21
6M	0.47	0.67	0.84	1.00	0.86	0.76	0.69	0.60	0.48	0.42	0.38	0.33
1Y	0.31	0.52	0.68	0.86	1.00	0.94	0.89	0.80	0.67	0.60	0.57	0.53
2Y	0.22	0.42	0.56	0.76	0.94	1.00	0.98	0.91	0.79	0.73	0.70	0.66
3Y	0.18	0.37	0.50	0.69	0.89	0.98	1.00	0.96	0.87	0.81	0.78	0.74
5Y	0.14	0.30	0.42	0.60	0.80	0.91	0.96	1.00	0.95	0.91	0.88	0.84
10Y	0.09	0.23	0.32	0.48	0.67	0.79	0.87	0.95	1.00	0.98	0.97	0.94
15Y	0.06	0.18	0.26	0.42	0.60	0.73	0.81	0.91	0.98	1.00	0.99	0.97
20Y	0.04	0.15	0.24	0.38	0.57	0.70	0.78	0.88	0.97	0.99	1.00	0.99
30Y	0.05	0.13	0.21	0.33	0.53	0.66	0.74	0.84	0.94	0.97	0.99	1.00

16. Correlation between Sub-Curve Pairs:

a. *2.0*: For sub-curves, the correlation between any two pairs ϕ_{ij} in the same currency is 0.98.

b. *2.1*: For sub-curves, the correlation between any two pairs ϕ_{ij} in the same currency is 0.98.

17. IR/Inflation Rate/Volatility Correlation:

a. *2.0*: For aggregated weighted sensitivities or risk exposures, the correlation between the inflation rate and any yield for the same currency (and the correlation between the inflation volatility and any interest-rate volatility for the same currency) is 29%.

b. *2.1*: For aggregated weighted sensitivities or risk exposures, the correlation between the inflation rate and any yield for the same currency (and the correlation between the inflation volatility and any interest-rate volatility for the same currency) is 33%.

18. IR/Cross Currency/Inflation Volatility Correlation:

a. *2.0*: For aggregated weighted sensitivities or risk exposures, the correlation between the cross-currency basis swap spread and any yield or inflation rate for the same currency is 20%.

b. *2.1*: For aggregated weighted sensitivities or risk exposures, the correlation between the cross-currency basis swap spread and any yield or inflation rate for the same currency is 19%.

19. Correlation used for Different Currencies:

a. *2.0*: The parameter

$$\gamma_{bc} = 23\%$$

should be used for aggregating across different currencies.

b. *2.1*: The parameter

$$\gamma_{bc} = 21\%$$

should be used for aggregating across different currencies.

Credit Qualifying: Risk Weights

1. Credit Quality/Sector Risk Exposure: Sensitivities or risk exposures to an issuer/seniority should first be assigned to a bucket according to the following table:

Bucket Number	**Credit Quality**	**Sector**
1	Investment Grade (IG)	Sovereigns including Central Banks
2		Financials including Government-backed Financials
3		Basic Materials, Energy, Industrials
4		Consumer
5		Technology, Telecommunications
6		Health-care, Utilities, Local Governments, Government-backed Corporates (non-financial)
7	High Yield (HY) and Non-rated (NR)	Sovereigns including Central Banks
8		Financials including Government-backed Financials
9		Basic Materials, Energy, Industrials
10		Consumer
11		Technology, Telecommunications
12		Health-care, Utilities, Local Governments, Government-backed Corporates (non-financial)

2. Position/Sensitivities under different Currencies: Sensitivities must be distinguished depending upon the payment currency of the trade – such as Quanto CDS and non-quanto CDS. No initial netting or aggregation is applied between position sensitivities from different currencies – except as described for the situation below.
3. *2.0* Vertex Risk Weight by Bucket: The risk weights should be used for all vertexes (1Y, 2Y, 3Y, 5Y, 10Y) according to bucket, as set out in the following table.

Bucket	**Risk Weight**
1	85
2	85
3	73
4	49
5	48
6	43
7	161
8	238
9	151
10	210
11	149
12	102
Residual	238

4. _2.1_ Vertex Risk Weight by Bucket: The risk weights should be used for all vertexes (1Y, 2Y, 3Y, 5Y, 10Y) according to bucket, as set out in the following table.

Bucket	**Risk Weight**
1	69
2	107
3	72
4	55
5	48
6	41
7	166
8	187
9	177
10	187
11	129
12	136
Residual	187

5. Base Correlation/Vega Risk Weight:
   a. _2.0_: The vega risk weight VRW for the credit risk class is 0.27. The Base Correlation risk weight is 20 for all index families.
   b. _2.1_: The vega risk weight VRW for the credit risk class is 0.27. The Base Correlation risk weight is 19 for all index families.

Credit Qualifying: Correlations

1. *2.0* Same Bucket Risk Factor Correlation: The correlation parameter ρ_{kl} applicable to sensitivity or risk exposure pairs within the same bucket are set out in the following table:

	Same Issuer/Seniority, different Vertex or Currency	**Different Issuer/Seniority**
Aggregate Sensitivities	97%	45%
Residual Bucket	50%	50%

2. *2.1* Same Bucket Risk Factor Correlation: The correlation parameter ρ_{kl} applicable to sensitivity or risk exposure pairs within the same bucket are set out in the following table:

	Same Issuer/Seniority, different Vertex or Currency	**Different Issuer/Seniority**
Aggregate Sensitivities	96%	39%
Residual Bucket	50%	50%

3. Quanto-Currency/Base Correlation Values: Here *currency* refers to the payment currency of sensitivity if there are sensitivities to multiple payment currencies – such as Quanto CDS and non-Quanto CDS – which will not be fully offset.

a. <u>*2.0*</u>: The correlation parameter ρ_{kl} applying to the Base Correlation risks across different indexes/families is 10%.

b. <u>*2.1*</u>: The correlation parameter ρ_{kl} applying to the Base Correlation risks across different indexes/families is 5%.

4. <u>*2.0* Different Bucket Risk Factor Calculations</u>: The correlation bucket parameters applying to sensitivities of risk exposure pairs across different non-residual buckets is set out in the following table:

Bucket	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	1.00	0.42	0.39	0.39	0.40	0.38	0.39	0.34	0.37	0.39	0.37	0.31
2	0.42	1.00	0.44	0.45	0.47	0.45	0.33	0.40	0.41	0.44	0.43	0.37
3	0.39	0.44	1.00	0.43	0.45	0.43	0.32	0.35	0.41	0.42	0.40	0.36
4	0.39	0.45	0.43	1.00	0.47	0.44	0.40	0.34	0.39	0.43	0.39	0.36
5	0.40	0.47	0.45	0.47	1.00	0.47	0.31	0.35	0.40	0.44	0.42	0.37
6	0.38	0.45	0.43	0.44	0.47	1.00	0.30	0.34	0.38	0.40	0.39	0.38
7	0.39	0.33	0.32	0.40	0.31	0.30	1.00	0.28	0.31	0.31	0.30	0.26
8	0.34	0.40	0.35	0.34	0.35	0.34	0.28	1.00	0.34	0.35	0.33	0.30
9	0.37	0.41	0.41	0.39	0.40	0.38	0.31	0.34	1.00	0.40	0.37	0.32
10	0.39	0.44	0.42	0.43	0.44	0.40	0.31	0.35	0.40	1.00	0.40	0.35
11	0.37	0.43	0.40	0.39	0.42	0.39	0.30	0.33	0.37	0.40	1.00	0.34
12	0.31	0.37	0.36	0.36	0.37	0.38	0.26	0.30	0.32	0.35	0.34	1.00

5. *2.1* Different Bucket Risk Factor Calculations: The correlation bucket parameters applying to sensitivities of risk exposure pairs across different non-residual buckets is set out in the following table:

Bucket	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	1.00	0.38	0.36	0.36	0.39	0.35	0.34	0.32	0.34	0.33	0.34	0.31
2	0.38	1.00	0.41	0.41	0.43	0.40	0.29	0.38	0.38	0.38	0.38	0.34
3	0.36	0.41	1.00	0.41	0.42	0.39	0.30	0.34	0.39	0.37	0.38	0.35
4	0.36	0.41	0.41	1.00	0.43	0.40	0.28	0.33	0.37	0.38	0.38	0.34
5	0.39	0.43	0.42	0.43	1.00	0.42	0.31	0.35	0.38	0.39	0.41	0.36
6	0.35	0.40	0.39	0.40	0.42	1.00	0.27	0.32	0.34	0.35	0.36	0.33
7	0.34	0.29	0.30	0.28	0.31	0.27	1.00	0.24	0.28	0.27	0.27	0.26
8	0.32	0.38	0.34	0.33	0.35	0.32	0.24	1.00	0.33	0.32	0.32	0.29
9	0.34	0.38	0.39	0.37	0.38	0.34	0.28	0.33	1.00	0.35	0.35	0.33
10	0.33	0.38	0.37	0.38	0.39	0.35	0.27	0.32	0.35	1.00	0.36	0.32
11	0.34	0.38	0.38	0.38	0.41	0.36	0.27	0.32	0.35	0.36	1.00	0.33
12	0.31	0.34	0.35	0.34	0.36	0.33	0.26	0.29	0.33	0.32	0.33	1.00

Credit Non-Qualifying Risk

1. Non-Qualifying Credit Risk Spread: Sensitivities to credit-spread risk arising from non-qualifying securitization positions are treated according to the risk weights and the correlations as specified in the following paragraphs.
2. Credit Non-Qualifying Bucket Classifications: Sensitivities or risk exposures should first be assigned to a bucket according to the following table:

Bucket	**Credit Quality**	**Sector**
1	Investment Grade (IG)	RMBS/CMBS
2	High Yield (HY) and Not-Rated (NR)	RMBS/CMBS
Residual		

3. *2.0* Credit Non-Qualifying Risk Weights: The risk weights are set out in the following table:

Bucket Number	**Risk Weight**
1	140
2	2000
Residual	2000

4. *2.1* Credit Non-Qualifying Risk Weights: The risk weights are set out in the following table:

Bucket Number	**Risk Weight**
1	150
2	1200

Residual	1200

a) *2.0*: The vega risk weight VRW for Credit Non-qualifying is 0.27.

b) *2.1*: The vega risk weight VRW for Credit Non-qualifying is 0.27.

Credit Non-Qualifying – Correlations

1. *2.0* Non-Qualifying Correlation – Same Bucket: For other buckets, the correlation parameter ρ_{kl} applicable to sensitivity or risk exposure pairs within the same bucket is set out in the following table:

	Same Underlying Names (more than 80% Overlap in Notional Terms)	**Different Underlying Names (less than 80% Overlap in Notional Terms)**
Aggregate Sensitivities	57%	27%
Residual Bucket	50%	50%

2. *2.1* Non-Qualifying Correlation – Same Bucket: For other buckets, the correlation parameter ρ_{kl} applicable to sensitivity or risk exposure pairs within the same bucket is set out in the following table:

	Same Underlying Names (more than 80% Overlap in Notional Terms)	**Different Underlying Names (less than 80% Overlap in Notional Terms)**
Aggregate Sensitivities	57%	20%
Residual Bucket	50%	50%

3. *2.0* <u>Non-Qualifying Correlations: Different Buckets:</u> The correlation parameter γ_{bc} applicable to sensitivity or risk exposure pairs across different buckets is set out in the following table:

	Correlation
Non-residual Bucket to Non-residual Bucket	0.21

4. *2.1* <u>Non-Qualifying Correlations: Different Buckets:</u> The correlation parameter γ_{bc} applicable to sensitivity or risk exposure pairs across different buckets is set out in the following table:

	Correlation
Non-residual Bucket to Non-residual Bucket	0.16

Equity Risk Weights

1. <u>Sector-Based Equity Risk Classification</u>: Sensitivities or risk exposures should first be assigned to a bucket according to the definitions below in the following table:

Bucket Number	Size	Region	Sector
1	Large	Emerging Markets	Consumer Goods and Services, Transportation and Storage, Administrative and Service Activities, Utilities
2			Telecommunications and Industrials
3			Basic Materials, Energy, Agriculture, Manufacturing, Mining, and Quarrying
4			Financials including Government-backed Financials, Real Estate Activities, Technology
5		Developed Markets	Consumer Goods and Services, Transportation and Storage, Administrative and Service Activities, Utilities
6			Telecommunications and Industrials
7			Basic Materials, Energy, Agriculture, Manufacturing, Mining, and Quarrying
8			Financials including Government-backed Financials, Real Estate Activities, Technology
9	Small	Emerging Markets	All Sectors
10		Developed Markets	All Sectors

11	All	All	Indexes, Funds, and ETF’s
12	All	All	Volatility Indexes

2. Large vs. Small Market Capitalization: *Large* is defined as a market capitalization equal to or greater than USD 2 billion and *small* is defined as a market capitalization of less than USD 2 billion.
3. Global Aggregate of Market Cap: *Market Capitalization* is defined as the sum of the market capitalizations of the same legal entity or a group of legal entities across all stock markets globally.
4. Jurisdictions of the Developed Markets: The developed markets are defined as: Canada, US, Mexico, the Euro area, the non-Euro area Western European countries – the UK, Norway, Denmark, Sweden, and Switzerland – Japan, Oceania – Australia and New Zealand – Singapore, and Hong Kong.
5. Determination of the Allocation Bucket: The sectors definition is the one generally used in the market. When allocating an equity position in a particular bucket, the bank must prove that the equity issuer’s most material activity indeed corresponds to the bucket’s definition. Acceptable proof may be external provider’s information, or internal analysis.
6. Multi-national Cross-Sector Issuers: For multinational multi-sector equity issuers, the allocation to a particular bucket must be done according to the most material region and the sector the issuer operates in.
7. *2.0* Sector Based Risk Weight Assignment: If it is not possible to allocate a position to one of these buckets – for example because data on categorical variables is not available – the position must then be allocated to a *residual bucket*. Risk weights should be assigned to each notional position as in the following table:

Bucket	**Risk Weight**
1	25

2	32
3	29
4	27
5	18
6	21
7	25
8	22
9	27
10	29
11	16
12	16
Residual	32

8. *2.1* Sector Based Risk Weight Assignment: If it is not possible to allocate a position to one of these buckets – for example because data on categorical variables is not available – the position must then be allocated to a *residual bucket*. Risk weights should be assigned to each notional position as in the following table:

Bucket	**Risk Weight**
1	24
2	30
3	31

4	25
5	21
6	22
7	27
8	24
9	33
10	34
11	17
12	17
Residual	34

9. *2.0* Equity Risk Historical Volatility Ratio: The historical volatility ratio HVR for equity risk class is 0.65.
10. *2.1* Equity Risk Historical Volatility Ratio: The historical volatility ratio HVR for equity risk class is 0.59.
11. *2.0* Equity Class Vega Risk Weight: The vega risk weight VRW for the equity risk class is 0.28 for all buckets except bucket 12 for which the vega risk weight is 0.64.
12. *2.1* Equity Class Vega Risk Weight: The vega risk weight VRW for the equity risk class is 0.28 for all buckets except bucket 12 for which the vega risk weight is 0.63.

Equity Correlations

1. *2.0* Correlation within a Single Equity Bucket: The correlation parameter ρ_{kl} applicable to sensitivity or risk exposure pairs within the same bucket are set out in the following table:

Bucket	**Correlation**
1	14%
2	20%
3	19%
4	21%
5	24%
6	35%
7	34%
8	34%
9	20%
10	24%
11	62%
12	62%
Residual	0%

2. *2.1* Correlation within a Single Equity Bucket: The correlation parameter ρ_{kl} applicable to sensitivity or risk exposure pairs within the same bucket are set out in the following table:

Bucket	**Correlation**

1	14%
2	20%
3	25%
4	23%
5	23%
6	32%
7	35%
8	32%
9	17%
10	16%
11	51%
12	51%
Residual	0%

3. *2.0* Correlations across Equity Buckets: The correlation parameters ρ_{kl} applicable to sensitivity or risk exposure pairs across different non-residual buckets are set out in the following table:

Bucket	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	1.00	0.15	0.14	0.16	0.10	0.12	0.10	0.11	0.13	0.09	0.17	0.17
2	0.15	1.00	0.16	0.17	0.10	0.11	0.10	0.11	0.14	0.09	0.17	0.17

3	0.14	0.16	1.00	0.19	0.14	0.17	0.18	0.17	0.16	0.14	0.25	0.25
4	0.16	0.17	0.19	1.00	0.15	0.18	0.18	0.18	0.18	0.15	0.28	0.28
5	0.10	0.10	0.14	0.15	1.00	0.28	0.23	0.27	0.13	0.21	0.35	0.35
6	0.12	0.11	0.17	0.18	0.28	1.00	0.30	0.34	0.16	0.26	0.45	0.45
7	0.10	0.10	0.18	0.18	0.23	0.30	1.00	0.29	0.16	0.24	0.41	0.41
8	0.11	0.11	0.17	0.18	0.27	0.34	0.29	1.00	0.16	0.26	0.44	0.44
9	0.13	0.14	0.16	0.18	0.13	0.16	0.16	0.16	1.00	0.13	0.24	0.24
10	0.09	0.09	0.14	0.15	0.21	0.26	0.24	0.26	0.13	1.00	0.33	0.33
11	0.17	0.17	0.25	0.28	0.35	0.45	0.41	0.44	0.24	0.33	1.00	0.62
12	0.17	0.17	0.25	0.28	0.35	0.45	0.41	0.44	0.24	0.33	0.62	1.00

4. *2.1* Correlations across Equity Buckets: The correlation parameters ρ_{kl} applicable to sensitivity or risk exposure pairs across different non-residual buckets are set out in the following table:

Bucket	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	1.00	0.16	0.16	0.17	0.13	0.15	0.15	0.15	0.13	0.11	0.19	0.19
2	0.16	1.00	0.20	0.20	0.14	0.16	0.16	0.16	0.15	0.13	0.20	0.20
3	0.16	0.20	1.00	0.22	0.15	0.19	0.22	0.19	0.16	0.15	0.25	0.25
4	0.17	0.20	0.22	1.00	0.17	0.21	0.21	0.21	0.17	0.15	0.27	0.27
5	0.13	0.14	0.15	0.17	1.00	0.25	0.26	0.23	0.14	0.17	0.32	0.32

6	0.15	0.16	0.19	0.21	0.25	1.00	0.30	0.31	0.16	0.21	0.38	0.38
7	0.15	0.16	0.22	0.21	0.23	0.30	1.00	0.29	0.16	0.21	0.38	0.38
8	0.15	0.16	0.19	0.21	0.26	0.31	0.29	1.00	0.17	0.21	0.39	0.39
9	0.13	0.15	0.16	0.17	0.14	0.16	0.16	0.17	1.00	0.13	0.21	0.21
10	0.11	0.13	0.15	0.15	0.17	0.21	0.21	0.21	0.13	1.00	0.25	0.25
11	0.19	0.20	0.25	0.27	0.32	0.38	0.38	0.39	0.21	0.25	1.00	0.51
12	0.19	0.20	0.25	0.27	0.32	0.38	0.38	0.39	0.21	0.25	0.51	1.00

Commodity Risk Weights

1. *2.0* Risk Weights for Commodity Buckets: The risk weights depend on the commodity type; they are set out in the following table:

Bucket	**Commodity**	**Risk Weight**
1	Coal	19
2	Crude	20
3	Light Ends	17
4	Middle Distillates	18
5	Heavy Distillates	24
6	North American Natural Gas	20

7	European Natural Gas	25
8	North American Power	41
9	European Power	24
10	Freight	91
11	Base Metals	20
12	Precious Metals	19
13	Grains	16
14	Softs	15
15	Livestock	10
16	Other	91
17	Indexes	17

2. *2.1* Risk Weights for Commodity Buckets: The risk weights depend on the commodity type; they are set out in the following table:

Bucket	**Commodity**	**Risk Weight**
1	Coal	19
2	Crude	20
3	Light Ends	17
4	Middle Distillates	19
5	Heavy Distillates	24

6	North American Natural Gas	22
7	European Natural Gas	26
8	North American Power	50
9	European Power	27
10	Freight	54
11	Base Metals	20
12	Precious Metals	20
13	Grains	17
14	Softs	14
15	Livestock	10
16	Other	54
17	Indexes	16

3. *2.0* Commodity Class Historical Volatility Ratio: The historical volatility ratio HVR for the commodity risk class is 0.80.
4. *2.1* Commodity Class Historical Volatility Ratio: The historical volatility ratio HVR for the commodity risk class is 0.80.
5. *2.0* Commodity Class Vega Risk Weight: The vega risk weight VRW for the commodity risk class is 0.38.
6. *2.1* Commodity Class Vega Risk Weight: The vega risk weight VRW for the commodity risk class is 0.38.

Commodity Correlations

1. *2.0* Commodity Correlations within the same Bucket: The correlation parameters ρ_{kl} applicable to sensitivity or risk exposure pairs within the same bucket are set out in the following table:

Bucket	Correlation
1	0.30
2	0.97
3	0.93
4	0.98
5	0.97
6	0.92
7	1.00
8	0.58
9	1.00
10	0.10
11	0.55
12	0.64
13	0.71
14	0.22

15	0.29
16	0.00
17	0.21

2. *2.1* Commodity Correlations within the same Bucket: The correlation parameters ρ_{kl} applicable to sensitivity or risk exposure pairs within the same bucket are set out in the following table:

Bucket	**Correlation**
1	0.27
2	0.97
3	0.92
4	0.97
5	0.99
6	1.00
7	1.00
8	0.40
9	0.73
10	0.13
11	0.53
12	0.64

13	0.63
14	0.26
15	0.26
16	0.00
17	0.38

3. *2.0* Commodity Correlations among Different Buckets: The correlation parameters γ_{bc} applicable to sensitivity or risk exposure pairs across different buckets are set out in the following table:

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**
1	1.00	0.18	0.15	0.20	0.25	0.08	0.09	0.20	0.27	0.00	0.15	0.02	0.06	0.07	-0.04	0.00	0.06
2	0.18	1.00	0.89	0.94	0.93	0.32	0.22	0.27	0.24	0.09	0.45	0.21	0.32	0.28	0.17	0.00	0.37
3	0.15	0.89	1.00	0.87	0.88	0.25	0.16	0.19	0.12	0.10	0.26	-0.01	0.19	0.17	0.10	0.00	0.27
4	0.20	0.94	0.87	1.00	0.92	0.29	0.22	0.26	0.19	0.00	0.32	0.05	0.20	0.22	0.13	0.00	0.28
5	0.25	0.93	0.88	0.92	1.00	0.30	0.26	0.22	0.28	0.12	0.42	0.23	0.28	0.19	0.17	0.00	0.34
6	0.08	0.32	0.25	0.29	0.30	1.00	0.13	0.57	0.05	0.14	0.57	-0.02	0.13	0.17	0.01	0.00	0.26
7	0.09	0.22	0.16	0.22	0.26	0.13	1.00	0.07	0.80	0.19	0.16	0.05	0.17	0.18	0.00	0.00	0.18
8	0.20	0.27	0.19	0.26	0.22	0.57	0.07	1.00	0.13	0.06	0.16	0.03	0.10	0.12	0.06	0.00	0.23
9	0.27	0.24	0.12	0.19	0.28	0.05	0.80	0.13	1.00	0.15	0.17	0.05	0.15	0.13	-0.03	0.00	0.13
10	0.00	0.09	0.10	0.00	0.12	0.14	0.19	0.06	0.15	1.00	0.07	0.07	0.17	0.10	0,02	0.00	0.11

11	0.15	0.45	0.26	0.32	0.42	0.57	0.16	0.16	0.17	0.07	1.00	0.34	0.20	0.21	0.16	0.00	0.27
12	0.02	0.21	-0.01	0.05	0.23	-0.02	0.05	0.03	0.05	0.07	0.34	1.00	0.17	0.26	0.11	0.00	0.14
13	0.06	0.32	0.19	0.20	0.28	0.13	0.17	0.10	0.15	0.17	0.20	0.17	1.00	0.35	0.09	0.00	0.22
14	0.07	0.28	0.17	0.22	0.19	0.17	0.18	0.12	0.13	0.10	0.27	0.26	0.35	1.00	0.06	0.00	0.20
15	-0.04	0.17	0.10	0.13	0.17	1.00	0.00	0.06	-0.03	0.02	0.16	0.11	0.09	0.06	1.00	0.00	0.16
16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
17	0.06	0.37	0.27	0.28	0,34	0.26	0.18	0.23	0.13	0.11	0.21	0.14	0.22	0.20	0.16	0.00	1.00

4. *2.1* Commodity Correlations among Different Buckets: The correlation parameters γ_{bc} applicable to sensitivity or risk exposure pairs across different buckets are set out in the following table:

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**
1	1.00	0.16	0.11	0.19	0.22	0.12	0.22	0.02	0.27	0.08	0.11	0.05	0.04	0.06	0.01	0.00	0.10
2	0.16	1.00	0.89	0.94	0.93	0.32	0.24	0.19	0.21	0.06	0.39	0.23	0.39	0.29	0.13	0.00	0.66
3	0.11	0.89	1.00	0.87	0.88	0.17	0.17	0.13	0.12	0.03	0.24	0.04	0.27	0.19	0.08	0.00	0.61
4	0.19	0.94	0.87	1.00	0.92	0.37	0.27	0.21	0.21	0.03	0.36	0.16	0.27	0.28	0.09	0.00	0.64
5	0.22	0.93	0.88	0.92	1.00	0.29	0.26	0.19	0.23	0.10	0.40	0.27	0.38	0.30	0.15	0.00	0.64
6	0.12	0.32	0.17	0.37	0.29	1.00	0.19	0.60	0.18	0.09	0.22	0.09	0.14	0.16	0.10	0.00	0.37
7	0.22	0.24	0.17	0.27	0.26	0.19	1.00	0.06	0.68	0.16	0.21	0.10	0.24	0.25	-0.01	0.00	0.27
8	0.02	0.19	0.13	0.21	0.19	0.60	0.06	1.00	0.12	0.01	0.10	0.03	0.02	0.07	0.10	0.00	0.21

9	0.27	0.21	0.12	0.21	0.23	0.18	0.68	0.12	1.00	0.05	0.16	0.03	0.19	0.16	-0.01	0.00	0.19
10	0.08	0.06	0.03	0.03	0.10	0.09	0.16	0.01	0.05	1.00	0.08	0.04	0.05	0.11	0.02	0.00	0.00
11	0.11	0.39	0.24	0.36	0.40	0.22	0.21	0.10	0.16	0.08	1.00	0.34	0.19	0.22	0.15	0.00	0.34
12	0.05	0.23	0.04	0.16	0.27	0.09	0.10	0.03	0.03	0.04	0.34	1.00	0.14	0.26	0.09	0.00	0.20
13	0.04	0.39	0.27	0.27	0.38	0.14	0.24	0.02	0.19	0.05	0.19	0.14	1.00	0.30	0.16	0.00	0.40
14	0.06	0.29	0.19	0.28	0.30	0.16	0.25	0.07	0.16	0.11	0.22	0.26	0.30	1.00	0.09	0.00	0.30
15	0.01	0.13	0.08	0.09	0.15	0.10	-0.01	0.10	-0.01	0.02	0.15	0.09	0.16	0.09	1.00	0.00	0.16
16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
17	0.10	0.66	0.61	0.64	0.64	0.37	0.27	0.21	0.19	0.00	0.34	0.20	0.40	0.30	0.16	0.00	1.00

Foreign Exchange Risk

1. *2.0* Foreign Exchange Risk Weights: A unique risk weight equal to 8.2 applies to all FX sensitivities or risk exposures. The historical volatility ratio HVR for the FX risk class is 0.60. The vega risk weight VRW for the FX volatility 0.33.
2. *2.1* Foreign Exchange Risk Weights: A unique risk weight equal to 8.1 applies to all FX sensitivities or risk exposures. The historical volatility ratio HVR for the FX risk class is 0.63. The vega risk weight VRW for the FX volatility 0.30.
3. *2.0* Foreign Exchange Correlations: A unique correlation ρ_{kl} equal to 0.5 applies to all pairs of FX sensitivities or risk exposures.
4. *2.1* Foreign Exchange Correlations: A unique correlation ρ_{kl} equal to 0.5 applies to all pairs of FX sensitivities or risk exposures.

5. Single Bucket Foreign Exchange Sensitivities: All foreign exchange sensitivities are considered to be within a single bucket within the FX risk class, so no inter-bucket aggregation is necessary. Note that the cross-bucket curvature calculations are still required on a single bucket.

Concentration Thresholds

1. Asset Class/Bucket Concentration Thresholds: The concentration thresholds in this section are defined for the asset-class-specific buckets specified earlier. For those cases where the same concentration threshold applies to the related range of buckets, the tables in this section specify a precise range of applicable buckets in the Bucket column and give a narrative description of that group of buckets in the Risk Group column.
2. *2.0* Interest Rate Risk - Delta Concentration Thresholds: The delta concentration thresholds for interest rate risk – inclusive of inflation risk – are given by the currency group:

Currency Risk Group	**Concentration Threshold (USD mm/bp)**
High Volatility	8
Regular Volatility, Well Traded	230
Regular Volatility, less Well Traded	28
Low Volatility	82

3. *2.1* Interest Rate Risk - Delta Concentration Thresholds: The delta concentration thresholds for interest rate risk – inclusive of inflation risk – are given by the currency group:

Currency Risk Group	Concentration Threshold (USD mm/bp)
High Volatility	12
Regular Volatility, Well Traded	210
Regular Volatility, less Well Traded	27
Low Volatility	170

4. Concentration Threshold Currency Risk Group: The Currency Risk Groups used in establishing concentration thresholds for Interest Rate Risk are as follows:
    a. High Volatility => All other currencies
    b. Regular Volatility, Well Traded => USD, EUR, GBP
    c. Regular Volatility, Less Well Traded => AUD, CAD, CHF, DKK, HKD, KRW, NOK, NZD, SEK, SGD, TWD
    d. Low Volatility => JPY
5. *2.0* Credit Spread Risk – Delta Concentration Thresholds: The delta concentration thresholds for credit spread risk are given by credit spread risk are given by the Credit Risk Group and Bucket.

Bucket (s)	Credit Risk Group	Concentration Threshold (USD mm/bp)
Qualifying		
1, 7	Sovereigns including Central Banks	0.95
2-6, 8-12	Corporate Entities	0.29
Residual	Non-Classified	0.29
Non-Qualifying		

1	IG (RMBS and CMBS)	9.50
2	HY/non-rated (RMBS and CMBS)	0.50
Residual	Not Classified	0.50

6. *2.1* Credit Spread Risk – Delta Concentration Thresholds: The delta concentration thresholds for credit spread risk are given by credit spread risk are given by the Credit Risk Group and Bucket.

Bucket (s)	**Credit Risk Group**	**Concentration Threshold (USD mm/bp)**
	Qualifying	
1, 7	Sovereigns including Central Banks	1.00
2-6, 8-12	Corporate Entities	0.24
Residual	Non-Classified	0.24
	Non-Qualifying	
1	IG (RMBS and CMBS)	9.50
2	HY/non-rated (RMBS and CMBS)	0.50
Residual	Not Classified	0.50

7. *2.0* Equity Risk – Delta Concentration Thresholds: The delta concentration thresholds for equity risk are given by bucket.

Bucket (s)	**Equity Risk Group**	**Concentration Threshold (USD mm/bp)**

1-4	Emerging Markets – Large Cap	3.3
5-8	Developed Markets – Large Cap	30.0
9	Emerging Markets – Small Cap	0.6
10	Developed Markets – Small Cap	2.3
11-12	Indexes, Funds, ETF's, and Volatility Indexes	900.0
Residual	Not Classified	0.6

8. *2.1* <u>Equity Risk – Delta Concentration Thresholds</u>: The delta concentration thresholds for equity risk are given by bucket.

Bucket (s)	**Equity Risk Group**	**Concentration Threshold (USD mm/bp)**
1-4	Emerging Markets – Large Cap	8.4
5-8	Developed Markets – Large Cap	26.0
9	Emerging Markets – Small Cap	1.8
10	Developed Markets – Small Cap	1.9
11-12	Indexes, Funds, ETF's, and Volatility Indexes	540.0
Residual	Not Classified	1.8

9. *2.0* <u>Commodity Risk – Delta Concentration Thresholds</u>: The delta concentration thresholds for commodity risk are given by:

Bucket (s)	Commodity Risk Group	Concentration Threshold (USD mm/bp)
1	Coal	1,400
2	Crude Oil	20,000
3-5	Oil Fractions	3,500
6-7	Natural Gas	6,400
8-9	Power	2,500
10	Freight – Dry or Wet	300
11	Base Metals	2,900
12	Precious Metals	7,600
13-15	Agricultural	3,900
16	Other	300
17	Indexes	12,000

10. *2.1* Commodity Risk – Delta Concentration Thresholds: The delta concentration thresholds for commodity risk are given by:

Bucket (s)	Commodity Risk Group	Concentration Threshold (USD mm/bp)
1	Coal	700
2	Crude Oil	3,600
3-5	Oil Fractions	2,700
6-7	Natural Gas	2,600

8-9	Power	1,900
10	Freight – Dry or Wet	52
11	Base Metals	2,000
12	Precious Metals	3,200
13-15	Agricultural	1,100
16	Other	52
17	Indexes	5,200

11. *2.0* FX Risk – Delta Concentration Thresholds: The delta concentration thresholds for FX risk are given by the FX Risk Group:

FX Risk Group	**Concentration Threshold (USD mm/bp)**
Category 1	8,400
Category 2	1,900
Category 3	560

12. *2.1* FX Risk – Delta Concentration Thresholds: The delta concentration thresholds for FX risk are given by the FX Risk Group:

FX Risk Group	**Concentration Threshold (USD mm/bp)**
Category 1	9,700
Category 2	2,900

Category 3	450

13. FX Risk Concentration Threshold Classifications: Currencies were placed in three categories as those for delta risk weights, constituted as follows:
    a. Category 1 => Significantly Material – USD, EUR, JPY, GBP, CAD, AUD, CHF
    b. Category 2 => Frequently Traded – BRL, CNY, HKD, INR, KRW, MXN, NOK, NZD, RUB, SEK, SGD, TRY, ZAR
    c. Category 3 => Others – All other currencies
14. *2.0* Interest Rate Risk - Vega Concentration Thresholds: The Vega Concentration thresholds for Interest Rate Risk are:

Currency Risk Group	**Concentration Threshold (USD mm/bp)**
High Volatility	110
Regular Volatility, Well Traded	2,700
Regular Volatility, less Well Traded	150
Low Volatility	960

15. *2.1* Interest Rate Risk - Vega Concentration Thresholds: The Vega Concentration thresholds for Interest Rate Risk are:

Currency Risk Group	**Concentration Threshold (USD mm/bp)**
High Volatility	120
Regular Volatility, Well Traded	2,200
Regular Volatility, less Well Traded	190

Low Volatility	770

The currency risk groups used in establishing the concentration thresholds correspond to the *Concentration Threshold Currency Risk Group* above.

16. *2.0* Credit Spread Risk - Vega Concentration Thresholds: The Vega Concentration thresholds for Credit Spread Risk are:

Credit Risk Group	**Concentration Threshold (USD mm/bp)**
Qualifying	290
Non-Qualifying	65

17. *2.1* Credit Spread Risk - Vega Concentration Thresholds: The Vega Concentration thresholds for Credit Spread Risk are:

Credit Risk Group	**Concentration Threshold (USD mm/bp)**
Qualifying	250
Non-Qualifying	54

18. *2.0* Equity Risk - Vega Concentration Thresholds: The Vega Concentration thresholds for Equity Risk are:

Bucket (s)	**Equity Risk Group**	**Concentration Threshold (USD mm/bp)**
1-4	Emerging Markets – Large Cap	700
5-8	Developed Markets – Large Cap	7,300

9	Emerging Markets – Small Cap	70
10	Developed Markets – Small Cap	300
11-12	Indexes, Funds, ETF's, and Volatility Indexes	21,000
Residual	Not Classified	70

19. *2.1* Equity Risk - Vega Concentration Thresholds: The Vega Concentration thresholds for Equity Risk are:

Bucket (s)	**Equity Risk Group**	**Concentration Threshold (USD mm/bp)**
1-4	Emerging Markets – Large Cap	20
5-8	Developed Markets – Large Cap	2,300
9	Emerging Markets – Small Cap	43
10	Developed Markets – Small Cap	250
11-12	Indexes, Funds, ETF's, and Volatility Indexes	8,100
Residual	Not Classified	43

20. *2.0* Commodity Risk - Vega Concentration Thresholds: The Vega Concentration thresholds for Commodity Risk are:

Bucket (s)	**Commodity Risk Group**	**Concentration Threshold (USD mm/bp)**
1	Coal	250

2	Crude Oil	2,000
3-5	Oil Fractions	510
6-7	Natural Gas	1,900
8-9	Power	870
10	Freight – Dry or Wet	220
11	Base Metals	450
12	Precious Metals	740
13-15	Agricultural	370
16	Other	220
17	Indexes	430

21. *2.1* Commodity Risk - Vega Concentration Thresholds: The Vega Concentration thresholds for Commodity Risk are:

Bucket (s)	**Commodity Risk Group**	**Concentration Threshold (USD mm/bp)**
1	Coal	250
2	Crude Oil	1,800
3-5	Oil Fractions	320
6-7	Natural Gas	2,200
8-9	Power	780
10	Freight – Dry or Wet	99

11	Base Metals	420
12	Precious Metals	650
13-15	Agricultural	570
16	Other	99
17	Indexes	330

22. *2.0* FX Risk - Vega Concentration Thresholds: The Vega Concentration thresholds for FX Risk are:

FX Risk Group	**Concentration Threshold (USD mm/bp)**
Category 1 – Category 1	4,000
Category 1 – Category 2	1,900
Category 1 - Category 3	320
Category 2 – Category 2	120
Category 3 – Category 3	110
Category 3 - Category 3	110

23. *2.1* FX Risk - Vega Concentration Thresholds: The Vega Concentration thresholds for FX Risk are:

FX Risk Group	**Concentration Threshold (USD mm/bp)**
Category 1 – Category 1	2,000

Category 1 – Category 2	1,000
Category 1 - Category 3	320
Category 2 – Category 2	410
Category 3 – Category 3	210
Category 3 - Category 3	150

The Currency categories used in establishing Concentration Thresholds for FX are identified under *FX Risk – Delta Concentration Thresholds*.

24. *2.0* Correlation between Risk Classes within Products: The correlation parameters ψ_{rs} applying to initial margin risk classes within a single product class are set out in the following table.

Class	**Interest Rate**	**Credit Qualifying**	**Credit Non-Qualifying**	**Equity**	**FX**	**Commodity**
Interest Rate	1.00	0.28	0.18	0.18	0.30	0.22
Credit Qualifying	0.28	1.00	0.30	0.66	0.46	0.27
Credit Non-Qualifying	0.18	0.30	1.00	0.23	0.25	0.18
Equity	0.18	0.66	0.23	1.00	0.39	0.24
FX	0.30	0.46	0.25	0.39	1.00	0.32

Commodity	0.22	0.27	0.18	0.24	0.32	1.00

25. *2.1* Correlation between Risk Classes within Products: The correlation parameters ψ_{rs} applying to initial margin risk classes within a single product class are set out in the following table.

Class	**Interest Rate**	**Credit Qualifying**	**Credit Non-Qualifying**	**Equity**	**FX**	**Commodity**
Interest Rate	1.00	0.25	0.15	0.19	0.30	0.26
Credit Qualifying	0.25	1.00	0.26	0.65	0.45	0.24
Credit Non-Qualifying	0.15	0.26	1.00	0.23	0.25	0.18
Equity	0.19	0.65	0.23	1.00	0.39	0.24
FX	0.30	0.45	0.25	0.39	1.00	0.32
Commodity	0.26	0.24	0.18	0.24	0.32	1.00

Additional Initial Margin Expressions

1. Additional Initial Margin – Standardized Expressions: Standardized formulas for calculating additional initial margin are as follows:

$$
\begin{aligned}
AdditionalInitialMargin \\
&= AddOnIM + (MS_{RATESFX} - 1)SIMM_{RATESFX} \\
&+ (MS_{CREDIT} - 1)SIMM_{CREDIT} + \left(MS_{EQUITY} - 1\right)SIMM_{EQUITY} \\
&+ (MS_{COMMODITY} - 1)SIMM_{COMMODITY}
\end{aligned}
$$

where $AddOnIM$ is defined as follows:

$$
AddOnIM = AddOnFixed + AddOnFactor_{P}Notional_{P}
$$

where $AddOnFixed$ is a fixed add-on amount, $AddOnFactor_{P}$ is the add-on factor for each affected product P expressed as a percentage of the notional (e.g., 5%), and $Notional_{P}$ is the total notional of the product – sum of the absolute trade notionals. In such use – where a variable notional is involved – current notional amount should be used.

2. Multiplicative Scales for Product Classes: The four variables - $MS_{RATESFX}$, MS_{CREDIT}, MS_{EQUITY}, and $MS_{COMMODITY}$ are the four *multiplicative scales* for the four product classes RatesFX, Credit, Equity, and Commodity. Their values can be individually specified to be more than 1.0 – with 1.0 being the default and the minimum value.

Structure of the Methodology

1. Six Classes of Risk Factors: There are six risk classes – Interest Rate, Credit (Qualifying), Credit (Non-Qualifying), Equity, Commodity, and FX – and the margin for each risk class is

defined to be the sum of the Delta Margin, the Vega Margin, the Curvature Margin, and the Base Correlation Margin – if applicable – for that risk class.

2. Four Classes of Sensitivity Margins: That is

$$IM_X = DeltaMargin_X + VegaMargin_X + CurvatureMargin_X + BaseCorrelationMargin_X$$

for each risk class X, where the $BaseCorrelationMargin_X$ is only present in the Credit Qualifying risk class.

3. Four Classes of Products Needing Margin: There are four product classes:
    a. Interest Rates and Foreign Exchange (RatesFX)
    b. Credit
    c. Equity
    d. Commodity
4. Product Class per Marginable Trade: Every trade is assigned to an individual product class and SIMM is considered separately for each product class.
5. Isolating Risk Factors across Products: Buckets are still defined in risk terms, but within each product class the risk class takes its component risks only from trades of that product class. For example, equity derivatives would have risk only in the interest rate risk class as well as from the equity risk class; but all those risks are kept separate from the risks of the trades in the RatesFX product class.
6. Product SIMM from Risk Factor IM: Within each product class, the initial margin (IM) for each of the risk classes is calculated as above. The total margin for that product class is given by

$$SIMM_{PRODUCT} = \sqrt{\sum_r {IM_r}^2 + \sum_r \sum_{s \neq r} \psi_{rs} IM_r IM_s}$$

where $PRODUCT$ is one of the four product classes above, and the sums on r and s are taken over the six risk classes. The correlation matrix ψ_{rs} of the correlations between the risk classes is given earlier.

7. Portfolio SIMM as Linear Sum: The total SIMM is the sum of these four product class SIMM values:

$$SIMM = SIMM_{RATESFX} + SIMM_{CREDIT} + SIMM_{EQUITY} + SIMM_{COMMODITY}$$

8. Product Specific SIMM Add-On: The SIMM equation can be extended to incorporate notional based add-ons for specified products and/or multipliers to the individual product class SIMM values. The section on SIMM add-ons contains the modified version of SIMM in that case.

Interest Rate Risk Delta Margin

1. Approach for IR Delta Margin: The following step-by-step approach to capture delta risk should be applied to capture delta risk for the interest-rate risk class only.
2. Sensitivity to Tenor/Risk Factor: Find a net sensitivity across instruments to each risk factor (k, i) where k is the rates tenor and i is the index name of the sub-yield curve as defined in the sections that outline the interest rate risk class.
3. Risk Weight applied to the Sensitivity: Weight the net sensitivity $s_{k,i}$ to each risk factor (k, i) by the corresponding risk weight RW_k according to the vertex structure laid out in the section on Interest Rate Risk Weights.
4. Risk Weighted Vertex Sensitivity Expression:

$$WS_{k,i} = RW_k s_{k,i} CR_b$$

is where CR is the concentration risk factor defined as

$$CR_b = \max\left(1, \sqrt{\frac{\sum_{k,i} s_{k,i}}{T_b}}\right)$$

for the concentration threshold T_b defined for each currency b.

5. Treating Inflation/Cross Currency Sensitivities: Note that inflation sensitivities to currency b are included in $\sum_{k,i} s_{k,i}$ but cross-currency basis swap sensitivities are not. Neither should the cross-currency basis swap sensitivities be scaled by the concentration factor.
6. Single Curve Sensitivity Roll Up: The weighted sensitivities should then be aggregated within each currency. The sub-curve correlations ϕ_{ij} and the tenor correlation parameters ρ_{kl} are set out in the Section on Interest Rate Risk Correlations.
7. Single Curve Composite Sensitivity - Expression:

$$K = \sqrt{\sum_{i,k} {WS_{k,i}}^2 + \sum_{i,k} \sum_{(j,l) \neq (i,k)} \phi_{ij}\rho_{kl} WS_{k,l} WS_{i,j}}$$

8. Multiple Currency Weighted Roll Up: Delta amounts should then be aggregated across currencies within the risk class. The correlation parameters γ_{bc} applicable are set out in the Section on Interest Rate Risk factor correlations.
9. Multiple Currency Delta Margin Expression:

$$DeltaMargin = \sqrt{\sum_b {K_b}^2 + \sum_b \sum_{c \neq b} \gamma_{bc} g_{bc} S_b S_c}$$

where

$$S_b = \max\left(\min\left\{\sum_{i,k} WS_{i,k}, K_b\right\}, -K_b\right)$$

and

$$g_{bc} = \frac{\min(CR_b, CR_c)}{\max(CR_b, CR_c)}$$

for all currencies b and c.

Non Interest Rate Risk Classes

1. Non-IR Delta Margin Approaches: The following step-by-step approach to capture delta risk should be applied separately to each risk class other than interest rates.
2. Sensitivity to Tenor/Risk Factor: Find the net sensitivity across instruments to each risk factor k, which are defined in the sections for each risk class.
3. Risk Weight Applicability to Sensitivity: Weight the net sensitivity s_k to each risk factor k by the corresponding risk weight RW_k according to the bucketing structure for each risk class set out in the Section *Credit Qualifying Risk*.
4. Risk Weighted Vertex Sensitivity Expression:

$$WS_k = RW_k s_k CR_k$$

is where CR_k is the concentration risk factor defined as

$$CR_k = \max\left(1, \sqrt{\frac{\left|\sum_j s_j\right|}{T_b}}\right)$$

for credit spread risk with the sum j taken over all the issuers and seniorities as the risk factor k irrespective of the tenor of the payment currency, and

$$CR_k = \max\left(1, \sqrt{\frac{|s_k|}{T_b}}\right)$$

for equity, commodity, and FX risk where T_b is the concentration threshold for the bucket – or FX category - b as given in the appropriate section.

5. Incorporating the Base Correlation Risk: Note that the base correlation sensitivities are not included in the concentration risk, and the concentration risk for these factors should be taken as 1.
6. Roll Up within Risk Factor: Weighted sensitivities should then be aggregated within each bucket. The buckets and the correlation parameters applicable to each risk class are set out in the section on *Credit Qualifying Risk*.
7. Single Risk Factor Composition Expression:

$$K = \sqrt{\sum_k {WS_k}^2 + \sum_k \sum_{l \neq k} \rho_{kl} f_{kl} WS_k WS_l}$$

where

$$f_{kl} = \frac{\min(CR_k, CR_l)}{\max(CR_k, CR_l)}$$

8. Roll Up across Risk Factors: Delta margin amounts should then be aggregated across buckets in each risk class. The correlation parameters γ_{bc} applicable to each risk class are set out earlier.
9. Cross Risk Factor Composition - Expression:

$$DeltaMargin = \sqrt{\sum_{b} {K_b}^2 + \sum_{b} \sum_{c \neq b} \gamma_{bc} S_b S_c} + K_{RESIDUAL}$$

where

$$S_b = \max\left(\min\left\{\sum_{i,k} WS_{i,k}, K_b\right\}, -K_b\right)$$

for all risk factors in bucket b.

10. <u>Margin Requirements for Volatility Instruments</u>: Instruments that are option or include an option – including pre-payment – or have volatility sensitivity – instruments subject to optionality – are subject to additional margin risks such as vega and curvature risk – as described down below. Instruments not subject to optionality with no volatility sensitivity are not subject to vega risk and curvature risk.
11. <u>Approaches for Vega Risk Exposure</u>: The following step-by-step to capture vega risk exposure should be applied separately to each risk class.
12. <u>Risk Factor/Tenor ATM Volatility</u>: For interest-rate and credit instruments, the volatility σ_{kj} for risk-factor k and maturity j is defined to be the implied at-the-money (ATM) volatility of the swaption with expiry time equal to the tenor k at some swap maturity j. The volatility can be quoted as normal, log-normal, or something similar.
13. <u>Inflation Risk Factor ATM Volatility</u>: In the case where k is the inflation risk factor, the inflation volatility σ_{kj} of the inflation swaption of type j is defined to the at-the-money volatility of the swaption, where the type j comprises an initial inflation observation date and a final inflation observation date.
14. <u>Buckets Mirroring Interest-Rate Factors</u>: The option expiry date shall be defined to be the final inflation observation date, and risk should be defined on a set of option expiries equal to the same-tenor bucket as the interest rate delta. The volatility can be quoted as normal, log-normal, or something similar.

15. Volatility for Equity/FX/Commodity: For Equity, FX, and Commodity instruments, the volatility σ_{kj} of the risk factor k at each vol-tenor j is given by the expression

$$\sigma_{kj} = \frac{RW_k \sqrt{365/14}}{\alpha}$$

where

$$\alpha = \Phi^{-1}(0.99)$$

RW_k *vol-tenor* j is the option expiry index, which should use the same tenor-buckets as the interest-rate delta risk: 2W, 1M, 3M, 6M, 1Y, 2Y, 3Y, 5Y, 10Y, 15Y, 20Y, and 30Y. Here $\Phi^{-1}(0.99)$ is the 99th percentile of the standard normal distribution.

16. Commodity Index Volatility Risk Weights: For commodity index volatilities, the risk weight to use is that of the *Indexes* bucket.
17. FX Delta Sensitivity Risk Weight: For FX vega – which depends upon a pair of currencies – the risk weight to use here is the common risk weight for the FX delta sensitivity given explicitly in the corresponding section.
18. Instrument Level Vega Risk Expression: The vega risk for each instrument i to the risk factor k is estimated using the expression

$$VR_{ik} = HVR_c \sum_j \sigma_{jk} \frac{\partial V_{ij}}{\partial \sigma}$$

19. Product Specific Vega Value applicable: Here σ_{jk} is the volatility defined over the last five clause points.
20. Instrument-Specific Price-Vega Sensitivity: $\frac{\partial V_{ij}}{\partial \sigma}$ is the sensitivity of the price of the instrument i with respect to the implied at-the-money volatility – vega – as defined later, but must match the definition above.

21. <u>Historical Volatility Risk Class Correction</u>: HVR_c is the historical volatility ratio for the risk class concerned c as set out in the section on Equity Risk, which corrects for the inaccuracy in the volatility estimate σ_{jk}. The historical volatility ratio for the interest rate and the credit risk classes is fixed at 1.0.
22. <u>5Y Interest Rate Swap Vega</u>: For example, the 5Y interest rate vega is the sum of all the vol-weighted interest rate caplet and swaption vegas which expire in 5 years' time; the USDJPY FX vega is the sum of all the vol-weighted USD/JPY FX vegas.
23. <u>Gross Vega for Inflation Products</u>: For inflation, the inflation vega is the sum of all vol-weighted inflation swaption vegas in the particular currency.
24. <u>Instrument Net Vega Risk Exposure</u>: Find a net vega risk exposure VR_k across instruments i to each risk factor k – which are defined in the later sections – as well as the vega concentration risk factor.
25. <u>Portfolio/Factor IR Vega Risk</u>: For interest-rate vega risk, these are given by the formulas

$$VR_k = VRW\left(\sum_i VR_{ik}\right)VCR_k$$

where

$$VCR_b = \max\left(1, \sqrt{\frac{\left|\sum_{i,k} VR_{ik}\right|}{VT_b}}\right)$$

where b is the bucket which contains the risk factor k.
26. <u>Portfolio/Factor Credit Risk Vega</u>: For credit spread vega risk, the corresponding formulas are

$$VR_k = VRW\left(\sum_i VR_{ik}\right)VCR_k$$

where

$$VCR_k = \max\left(1, \sqrt{\frac{\left|\sum_{i,j} VR_{ij}\right|}{VT_b}}\right)$$

where the sum j is taken over tenors of same issuer/seniority curve as the risk factor k, irrespective of the tenor or the payment currency.

27. Equity/FX/Commodity Vega Risk: For equity, FX, and commodity vega risks, the corresponding formulas are

$$VR_k = VRW\left(\sum_i VR_{ik}\right)VCR_k$$

where

$$VCR_k = \max\left(1, \sqrt{\frac{\left|\sum_k VR_{ik}\right|}{VT_b}}\right)$$

28. Vega Weights for the Risk Class: Here VRW is the vega risk weight for the risk class concerned as set out in the corresponding sections, and VT_b is the vega concentration threshold for the bucket – or FX category b - as given in the corresponding section.
29. Index Volatilities for Risk Classes: Note that there is a special treatment for index volatilities in Credit Qualifying, Equity, and Commodity Risk Classes.
30. Vega Exposure across Risk Class: The vega risk exposure should then be aggregated within each bucket. The buckets and the correlation parameters applicable to each risk class are set out in the Sections on Risk Weights and Correlations.
31. Cross Factor Vega Margin Expression:

$$VegaMargin = \sqrt{\sum_b {K_b}^2 + \sum_b \sum_{c \neq b} \gamma_{bc} g_{bc} S_b S_c} + K_{RESIDUAL}$$

where

$$S_b = \max\left(\min\left\{\sum_{k=1}^{K} VR_k, V_b\right\}, -K_b\right)$$

for all risk factors in bucket b.

32. Outer Correlation Adjustment Factors g_{bc}: The outer correlation adjustment factors g_{bc} are identically 1.0 for all risk classes other than interest rates, and for interest rates they are defined as

$$g_{bc} = \frac{\min(VCR_b, VCR_c)}{\max(VCR_b, VCR_c)}$$

33. Approach underlying Tension/Curvature Risk: The following step-by-step approach to capture curvature risk exposure should be applied to each risk class.
34. Expression for Tension/Curvature Risk: The curvature risk exposure for each instrument i to risk factor k is estimated using the expression

$$CVR_{ik} = \sum_j SF(t_{jk}) \sigma_{jk} \frac{\partial V_i}{\partial \sigma}$$

35. Pairwise Volatility and Vega: Here σ_{jk} and $\frac{\partial V_i}{\partial \sigma}$ are the volatility and the vega defined in the items above.
36. Incorporating Standard Option Expiry Time: t_{jk} is the expiry time in calendar days from the valuation date until the expiry of the standard option corresponding to this volatility and vega.

37. Scaling Function Vega/Gamma Linkage: is the value of the scaling function obtained from the linkage between vega and gamma for vanilla options:

$$SF(t) = 0.5 \min\left(1, \frac{14\ days}{t\ days}\right)$$

38. Scaling Function Dependence on Expiry: The scaling function is a function of expiry only, is independent of both the vega and the vol, and is show in the table below.

2w	**1m**	**3m**	**6m**	**12m**	**2y**	**3y**	**5y**	**10y**
0.500	0.230	0.077	0.038	0.019	0.010	0.006	0.004	0.002

39. Conversion of Tenors to Days: Here the tenors are converted to calendar days using the convention that *12m* equals 365 calendar days, with pro-rata scaling for other tenors so that

$$1m = \frac{365}{12}\ days$$

and

$$5y = 365 \times 5\ days$$

40. Cross Factor Curvature Risk Aggregation: The curvature risk should then be aggregated within each bucket using the following expression:

$$K_b = \sqrt{\sum_k {CVR_k}^2 + \sum_k \sum_{l \neq k} {\rho_{kl}}^2 CVR_{bk} CVR_{bl}}$$

where ρ_{kl} is the correlation applicable to each risk class as set out in the section on risk weights and correlations. Note the use of ρ_{kl}^2 rather than ρ_{kl}.

41. Instrument Risk Factor Curvature Exposure: The curvature risk exposure CVR_{ik} can then be netted across instrument i to each risk factor k, which are defined in the Sections on Risk Factors and Sensitivities. Note that the same special treatment as for vega applies for indexes in credit, equity, and commodity risk exposures.
42. The Non-Residual Curvature Standard Derivation Scaler: Margin should then be aggregated across buckets within each risk class:

$$\theta = \min\left(\frac{\sum_{b,k} CVR_{bk}}{\sum_{b,k} |CVR_{bk}|}, 0\right)$$

and

$$\lambda = [\Phi^{-1}(0.995)^2 - 1](1 + \theta) - \theta$$

where the sums are taken over all the non-residual buckets in the risk class, and $\Phi^{-1}(0.995)$ is the 99.5th percentile of the standard normal distribution.

43. Non-Residual Curvature Margin Expression: Then the non-residual curvature margin is

$$CurvatureMargin_{NON-RESIDUAL} = \max\left(\sum_{b,k} CVR_{bk} + \lambda\sqrt{\sum_b K_b{}^2 + \sum_b \sum_{c \neq b} \gamma_{bc}{}^2 S_b S_c}, 0\right)$$

where

$$S_b = \max\left(\min\left\{\sum_{k=1}^{K} VR_k, V_b\right\}, -K_b\right)$$

44. The Residual Curvature Margin Expression: Similarly the residual equivalents are defined as

$$\theta_{RESIDUAL} = \min\left(\frac{\sum_k CVR_{RESIDUAL,k}}{\sum_k \left|CVR_{RESIDUAL,k}\right|}, 0\right)$$

and

$$\lambda_{RESIDUAL} = [\Phi^{-1}(0.995)^2 - 1](1 + \theta_{RESIDUAL}) - \theta_{RESIDUAL}$$

$$CurvatureMargin_{RESIDUAL} = \max\left(\sum_k CVR_{RESIDUAL,k} + \lambda_{RESIDUAL} K_{RESIDUAL}, 0\right)$$

45. <u>Applying the Risk Factor Correlations</u>: Here the correlation parameters γ_{bc} applicable to each risk class are set out in the Sections on Risk Weights and Correlations. Note the use of ${\gamma_{bc}}^2$ rather than γ_{bc}.
46. <u>Expression for Composite Curvature Margin</u>: The total curvature margin is defined to be the sum of the two curvature terms

$$CurvatureMargin = CurvatureMargin_{RESIDUAL} + CurvatureMargin_{NON-RESIDUAL}$$

47. <u>Interest Rate Curvature Margin Scaler</u>: For the interest rate risk class only, the *Curvature Margin* must be multiplied by a scaler of 2.3. This provisional adjustment addresses a known weakness in the expression that converts gamma into curvature, which will be properly addressed in a later version of the model.
48. <u>Base Correlation Model Credit Charge</u>: Credit Qualifying Only – Instruments whose prices is sensitive to the correlation between the defaults of different credits within an index or a basket – such as CDO tranches – are subject to Base Correlation margin charges described below. Instruments not sensitive to base correlation are not subject to Base Correlation margin requirements.
49. <u>Base Correlation Risk Exposure Approach</u>: The following step-by-step approach to capture the Base Correlation risk exposure should be applied to the Credit Qualifying Risk Class.

50. Base Correlation Risk Factor Sensitivity: The net sensitivity across instruments to each Base Correlation risk factor k is calculated, where k is the index family such as CDX/IG.

51. Risk Weight applied to Sensitivity: Weight the net sensitivity s_k to each risk factor k by the corresponding risk weight RW_k specified in the section on Credit Qualifying Risk:

$$WS_k = RW_k s_k$$

52. Aggregation of the Weighted Sensitivities: Weighted sensitivities should then be aggregated to calculate the Base Correlation Margin as follows:

$$BaseCorrelationMargin = \sqrt{\sum_k {WS_k}^2 + \sum_k \sum_{l \neq k} \rho_{kl} WS_k WS_l}$$

The correlation parameters are set out in the section on Credit Qualifying Risk Weights and Correlations.

References

- International Swaps and Derivatives Association (2016): ISDA SIMM 2.0 Methodology
- International Swaps and Derivatives Association (2017): ISDA SIMM 2.1 Methodology

Dynamic Initial Margin Impact on Exposure

Abstract

1. VM and IM Collateralized Positions: This chapter leverages the new framework for collateralized exposure modeling introduced by Andersen, Pykhtin, and Sokol (2017b) to analyze credit risk positions collateralized with both initial and variation margin. Special attention is paid to the dynamics BCBS-IOSCO uncleared margin rules soon to be mandated for bilateral inter-dealer trading in OTC derivatives markets.
2. Insufficiency of BCBS IOSCO Rules: While these rules set the initial margin at 99^{th} 2-week percentile level and aim to all but eliminate portfolio close-out risk, this chapter demonstrates that the trade flow effects can result in exposures being reduced significantly less than expected.
3. Efficient IM Simulation on an MC Path: The analysis is supplemented with several practical schemes for estimating IM on a simulation path, and for improving the speed and the stability of the exposure simulation.
4. Handling Trade Flow Exposure Spikes: This chapter also briefly discusses potential ways to adjust the margin framework to more effectively deal with exposures arising from trade flow events.

Introduction

1. Collateralization based on Variation Margin: Collateralization has long been a way of mitigating counterparty risk in OTC bilateral trading. The most common collateral mechanism is *variation margin* (VM) which aims to keep the threshold gap between portfolio value and posted collateral below a certain, possibly stochastic, threshold. While it is
2. Imperfect Collateralization under VM Schemes: Even when the thresholds for the VM are set to zero, however, there remains residual exposure to the counterparty default resulting from a sequence of contractual and operational time lags, starting from the last snapshot of the market for which the counterparty would post in full the required VM to the termination date after the counterparty's default. The various collateral mechanisms, including the precise definition of the variation margin thresholds, are typically captured in the ISDA Credit Support Annex (CSA) – the portfolio level legal agreement that supplements the ISDA Master Agreement.
3. MPoR - Margin Period of Risk: The aggregation of these lags results in a time period called the *Margin Period of Risk* (MPoR) during which the gap between the portfolio value and collateral can widen. The length of the MPoR is a critical input to any model of collateral exposure.
4. IM Supplementing the VM Collateral: Posting of *initial margin* (IM) to supplement VM provides the dealers with a mechanism to reduce the residual exposure resulting from market risk over MPoR. While it is often believed that the IM is posted strictly in addition to the VM, many CSAs intermingle the two types of collateral by letting IM affect the threshold computation of VM.
5. Genesis and Structure of IM: Historically, IM in bilateral trading has been mostly reserved for dealer counterparties deemed as high-risk – e.g., hedge funds – and typically done as a trade level calculation, established in term sheets at the transaction time of each trade. This type of IM posting is normally deterministic and either stays fixed over the life of a trade or amortizes down according to a pre-specified schedule.
6. Ne Basel Rules for IM: In the inter-dealer bilateral OTC world, changes to the long-standing VM and IM collateral practices are now imminent. BCBS and IOSCO proposed (Basel

Committee on Banking Supervision (2013)) and later finalized (Basel Committee on Banking Supervision (2015)) new uncleared margin rules for bilateral trading.

7. Key Features of the UMR: Under UMR, VM thresholds are forced to zero, and IM must be posted bilaterally into segregated accounts at the netting set level, by either using an internal model or by a lookup in a standardized schedule.
8. IM as a Horizon-Specific VaR: If an internal model is used, IM must be calculated as a netting set Value-at-Risk (VaR) for a 99% confidence level. The horizon used in this calculation equals $9 + a$ business days, where a is the re-margining period - 1 business day under US rules.
9. No Cross-Asset Class Diversification: In these calculations, diversification across distinct asset classes is not recognized, and calibration of the IM internal model for each asset class must include a period of stress for that asset class. To reduce the potential for margin disputes and to increase the overall market transparency, ISDA has proposed a standardized sensitivity-based IM calculator known as SIMM (Standard Initial Margin Model) (International Swaps and Derivatives Association (2016)). As a practical matter it is expected that virtually all dealers will use SIMM for their day-to-day IM calculations.
10. Dynamic Nature of Initial Margin: Under UMR required levels of IM continuously change as trade cash flows are paid, new trades are booked, or markets move, and dealers regularly need to call for more IM or to return excess IM. This dynamic aspect of IM requirements makes the modeling of the future exposures a challenge.
11. Modeling under Dynamics IM and VM: This chapter discusses modeling credit exposure in the presence of dynamic IM and questions the conventional wisdom that IM essentially eliminates counterparty risk. Leaning on the recent results from Andersen, Pykhtin, and Sokol (2017a), it starts by formulating a general model of exposure in the presence of VM and/or IM.
12. Simple Case - No Trade Flows: The resulting framework is first applied to the simple case where no trade flows take place within the MPoR. For processes with Gaussian increments – e.g., an Ito process – a limiting scale factor that converts the IM free expected exposure (EE) to IM-protected EE is derived, for sufficiently small MPoR.

13. IM vs no IM EE Ratio: The universal value depends only on the IM confidence level and the ratio of the IM horizon to the MPoR; it equals 0.85% at the BCBS-IOSCO confidence level of 99%, provided the IM horizon equals the MPoR. While conceptually the IM and the MPoR horizons are identical, a prudent MPoR for internal calculations may differ from the regulatory minimum IM horizon.
14. No-Trade-Flow Exposure Reduction: While some deviations from this universal limit due to a non-infinitesimal MPoR are to be expected, the reduction of EE by about 2 orders of magnitude is, as will be demonstrated below, generally about right when no trade flows are present within the MPoR.
15. Exposure Spikes from Trade Flows: For those periods for which trade flows do take place within the MPoR, however, any trade payment flowing away from the dealer will result in a spike in the EE profile. Without IM these spikes can make a fairly significant contribution to the Credit Valuation Adjustment (CVA) – say, 20% of an interest rate swap's total CVA may originate with spikes – but the CVA would still mostly be determined by the EE level between the spikes.
16. Exposure Spikes vs. Dynamic IM: This chapter shows that while IM is effective in suppressing the EE *between* spikes, it will often fail to significantly suppress the spikes themselves. As a result, the relative contribution of the spike to CVA is greatly increased in the presence of IM – e.g., for a single interest rate swap, the spike's contribution to the CVA can be well about 90% for a position with IM.
17. Corresponding Impact on the CVA: Accounting for the spikes, the IM reduces the CVA by much less than two orders of magnitude one might expect, with the reduction for the interest rate swaps often being less than a factor of 10.
18. Estimating the Path-wise IM: The final part of this chapter discusses the practical approaches to calculating the EE profiles in the presence of IM. The first step in this calculation is the estimation of IM on simulation paths, which can be done by parametric regression or by kernel regression.
19. IM Covering Few Netting Trades: When IM covers an insignificant number of trades in the netting set, IM calculated on the path can be subtracted from the no-IM exposure realized on that path to generate EE profiles.

20. IM covering most Netting Trades: However, when most trades of the netting set are covered by the IM, this approach can be problematic because of excessive simulation noise and other errors. An alternative approach that dampens the noise is proposed, and is generally more accurate.
21. Suggested Alterations to the Exposure Rules: This chapter concludes by summarizing the results and briefly discussing the possible modifications to trade and collateral documentation that would make IM more effective in reducing residual counterparty risk.

Exposure in the Presence of IM and VM

1. VM/IM over single Netting Set: Consider a dealer D that has a portfolio of OTC derivatives contracts traded with a counterparty C. Suppose for simplicity that the entire derivatives portfolio is covered by a single netting agreement, which is supported by a margin agreement that includes VM and may include IM on a subset of the portfolio.
2. Exposure of Client to Dealer: Quiet generally the exposure of D to the default of client C measured at time t – assumed to be the early termination time after C's default – is given by

$$E(t) = [V(t) - VM(t) + U(t) - IM\ (t)]^{+}$$

where $V(t)$ is the time t portfolio value from D's perspective; $VM(t)$ is the VM available to D at time t; $U(t)$ is the value of the trade flows scheduled to be paid by both D (negative) and C (positive) up to time t, yet unpaid as of time t; $IM(t)$ is the value of IM available to D at time t.
3. Sign of VM and IM: Notice that VM can be positive – C posts VM – or negative – D posts VM – from D's perspective. On the other hand, IM is always positive as IM for both counterparties is kept in segregated accounts, whereby IM posted by D does not contribute to D's exposure to the default of C.

4. Modeling Individual Terms in the Exposure: The above equation for $E(t)$ specifies the exposure of D to C in a generic, model-free way. To add modeling detail, this chapter assumes that D and C both post VM with zero-threshold and are required to post BCBS-IOSCO compliant IM to a segregated account. The modeling of each of these terms VM, U, and IM are dealt with turn by turn.

Modeling VM

1. Concurrent Dealer/Client VM Stoppage: The length of the $MPoR$ denoted by δ_C defines the last portfolio valuation date

$$t_0 = t - \delta_C$$

prior to the termination date t - after C's default – for which C delivers VM to D. A common assumption – denoted here as the *Classical Model* – assumes that D stops paying VM to C at the exact same time C stops posting to D.

2. Expression for Classical Model VM: That is, the VM in the equation above is the VM prescribed for the margin agreement for the portfolio valuation date

$$t_0 = t - \delta_C$$

Ignoring minimum transfer amount and rounding, the prescribed VM in the Classical Model is thus simply

$$VM_{CLASSICAL}(t) = V(t_c) = V(t - \delta_C)$$

3. Advanced Model Incorporating Operational Details: In the *Advanced Model* of Andersen, Pykhtin, and Sokol (2017a), operational aspects and gamesmanship of margin disputes are considered in more detail, leading to the more realistic assumption that D may continue to post VM to C for some period of time, even after C stops posting.
4. Non-Concurrent Dealer/Client VM Stoppages: The model introduces another parameter

$$\delta_D \geq \delta_C$$

that specifies that last portfolio valuation date

$$t_D = t - \delta_D$$

for which D delivers VM to C. For the portfolio valuation dates

$$T_i \in [t_C, t_D]$$

D would post VM to C when the portfolio value decreases, but will receive no VM from C when the portfolio value increases.
5. Expression for Advanced Model VM: This results in VM of

$$VM_{ADVANCED}(t) = \min_{T_i \in [t_C, t_D]} V(T_i)$$

This equation of course reduces to

$$VM_{CLASSICAL}(t) = V(t_c) = V(t - \delta_C)$$

when one sets

$$\delta_D = \delta_C$$

Modeling *U*

1. <u>The Classical+ Version Trade Flow *Currentness*</u>: In the most conventional version of the Classical model – denoted here *Classical+* - it is assumed that all trade flows are paid by both C and D for the entire $MPoR$ up to and including the termination date, i.e., in the time interval $[t_C, t]$ - time here is measured in discrete business days, so that $[u, s]$ is equivalent to $[u + 1\ BD, s]$. This assumption simply amounts to setting

$$U_{CLASSICAL+}(t) = 0$$

2. <u>Trade Flow Exposure Profile Spikes</u>: One of the prominent features of the Classical+ model is that the time 0 expectation of $E(t)$ – denoted $EE(t)$ – will contain upward spikes whenever there is a possibility of trade flows from D to C within the interval $[t_C, t]$.
3. <u>Absence of Client Margin Flows</u>: These spikes appear because, by the classical model's assumption, C makes no margin payments during the $MPoR$ and would consequently fail to post an offsetting VM to D after D makes a trade payment to C. In the Classical model, D will also not post VM to C in the event of trade payment from C to D, which results in a negative jump in the exposure. However, these scenarios do not fully offset the scenarios where D makes a trade payment because the zero floor in the exposure definitions effectively limits the size of the downward exposure jump.
4. <u>Sparse Fixed Time Exposure Grid</u>: For dealers having a sparse fixed time exposure grid, the alignment of grid nodes relative to trade flows will add numerical artifacts to genuine spikes, causing EE exposure to appear and disappear as the calendar date moves. As a consequence, an undesirable instability in the EE and the CVA is introduced.

5. Classical- Version – No Trade Flows: An easy way to eliminate exposure spikes is to assume that neither C nor D make any trade payment inside the $MPoR$. The resulting model – here denoted Classical- - consequently assumes that

$$U_{CLASSICAL-}(t) = TF_{NET}(t;\ (t_C, t])$$

where $TF_{NET}(t;\ (s, u])$ denotes the time t of all net trade flows payable in the interval $(s, u]$.
6. Implicit Simplifications in Classical- and Classical+: It should be evident that neither the Classical- nor the Classical+ assumptions on trade flows are entirely realistic; in the beginning of the $MPoR$ both C and D are likely to make trade payments, while at the end of the $MPoR$ neither C no D are likely making any trade payments.
7. Last Dealer/Client Trade Flows: To capture this behavior, Andersen, Pykhtin, and Sokol (2017a) add two more parameters in the model ${\delta_C}'$ and

$${\delta_D}' \leq \delta_C$$

that specify the last dates

$${t_C}' = t - {\delta_C}'$$

and

$${t_D}' = t - {\delta_D}'$$

for D at which trade payments are made prior to closeout at t.
8. Advanced Model Trade Flow Expansion: This results in unpaid trade flow terms of

$$U_{ADVANCED}(t) = TF_{C \rightarrow D}(t;\ ({t_C}', {t_D}']) + TF_{NET}(t;\ ({t_D}', t])$$

where an arrow indicates the direction of the trade flows and $C \rightarrow D$ ($D \rightarrow C$) trade flows have positive (negative) sign.

9. Advanced Model Exposure Profile Structure: The EE profiles obtained with the Advanced Model contain spikes that are typically narrower and have a more complex structure than spikes under the Classical+ model. Rather than being unwelcome noise, it is argued in Andersen, Pykhtin, and Sokol (2017a) that spikes in EE profiles are important features that represent actual risk.
10. Approximating the Trade Flow Spikes: To eliminate any numerical instability associated with the spikes an approximation was proposed in Andersen, Pykhtin, and Sokol (2017a) for calculation of the EE on a daily time grid without daily re-evaluation of the portfolio.

Modeling IM

1. Expression for Netting Set IM: Following the BCBS-IOSCO restrictions on diversification, the IM is defined for the netting set as a sum of the IM's over K asset classes as

$$IM(t) = \sum_{k=1}^{K} IM_k(t)$$

   Current netting rules have

$$K = 4$$

   k can be one of Rates/FX, Credit, Equity, and Commodity.
2. Netting Set Granularity IM Value: Let $V_k(t)$ denote the value at time t of all trades in the netting set to asset class k and are subject to IM requirements. Note that only trades executed after UMR go-live will be covered by the IM. The case where only a subset of the netting set is covered by the IM will therefore be common in the near future.

3. Horizon/Confidence Based IM Definition: For an asset class k we define IM as the quantile at the confidence level q of the *clean* portfolio value increment over BCBS-IOSCO IM horizon δ_{IM} - which may or may not coincide with δ_C - conditional on all the information available at t_C.
4. Mathematical Expression for the IM: That is

$$IM_k(t) = Q_q\left[V_k(t_C + \delta_{IM}) + TF_{NET,k}(t_C + \delta_{IM};\ (t_C, t_C + \delta_{IM}]) - V_k(t_C) \middle| \mathcal{F}_{t_C}\right]$$

where $Q_q(\cdot \mid \mathcal{F}_s)$ denotes the quantile of confidence level q conditional on information available at time s. Note that the above expression assumes that C stops posting IM at the same time it stops posting VM; hence IM is calculated as of

$$t_C = t - \delta_C$$

Summary and Calibration

1. Classical+/Classical-/Advanced Model Exposures: To summarize, three different models have been outlined in the generic exposure calculation

$$E(t) = [V(t) - VM(t) + U(t) - IM(t)]^+$$

for the Classical-, Classical+, and Advanced Models. Collecting results, one has

$$E_{CLASSICAL+}(t) = [V(t) - V(t - \delta_C) - IM(t)]^+$$

$$E_{CLASSICAL-}(t) = [V(t) - V(t - \delta_C) + TF_{NET}(t;\ (t_C, t])- IM(t)]^+$$

$$E_{ADVANCED}(t) = \left[V(t) - \min_{T_i \in [t_C, t_D]} V(T_i) + TF_{C \to D}(t;\ (t_C{}', t_D{}']) + TF_{NET}(t;\ (t_D{}', t]) - IM(t) \right]^+$$

where for all the three models IM is computed as

$$IM_k(t) = Q_q\left[V_k(t_C + \delta_{IM}) + TF_{NET,k}(t_C + \delta_{IM};\ (t_C, t_C + \delta_{IM}]) - V_k(t_C) \middle| \mathcal{F}_{t_C}\right]$$

2. IMA/CSA Dealer/Client Suspension Rights: In practice, the calibration of the time parameters of the Advanced Model should be informed by the dealers' legal rights and its aggressiveness in pursuing them. For the former, Andersen, Pykhtin, and Sokol (2017a) contain a full discussion of ISDA Master Agreements. But note that once a notice of Potential Event of Default (PED) has been served, the *suspension rights* of ISDA Master Agreement 2 (a) (iii) and its accompanying Credit Support Annex Paragraph 4 (a) allow a bank to suspend all trade-related and collateral-related payments to its counterparty until the PED has been cured.
3. Risks of Enforcing the Rights: The extent to which the suspension rights are actually exercised in practice, however, depends on the dealer and its operating model, as well as the specifics of each of the PED. One particular danger in the aggressive and immediate enforcement of the suspension rights is that the original PED might be ruled unjustified. Should this happen, the dealer can inadvertently commit a breach of contract, which, especially in the presence of the cross-default provisions, can have serious consequences for the dealer.
4. Calibrating the MPoR Lag Parameter: With the four time parameters, the Advanced Model allows for a greater flexibility in modeling its risk tolerance and procedures for exercising its suspension rights. A dealer may, in fact, calibrate these parameters differently for different counter-parties to reflect its risk management practices towards counterparties of a given type. One may make the time parameters stochastic, e.g., by making the legs a function of the

exposure magnitude. This way one can, say, model the fact that a dealer might tighten its operational controls when the exposures are high.

5. Prototypical Aggressive vs. Conservative Timelines: Additional discussions can be found in Andersen, Pykhtin, and Sokol (2017a), which also provides some prototypical parameter settings; the Aggressive Calibration (D can always sniff out financial distress in its clients and is swift and aggressive in enforcing its legal rights) and the Conservative Calibration (D is deliberate and cautious in enforcing its rights, and acknowledges potential for operational errors and for rapid, unpredictable deterioration in client credit). For the numerical results in this chapter, the values of the time parameters are mostly set in between the Aggressive and the Conservative.

The Impact of IM: No Trade Flows within the MPoR

1. No Trade Flow IM EE Impact: This section examines the impact of IM on EE when there are no trade flows within the MPoR. For simplicity the Classical Model is considered, and the entire netting set is assumed to be covered by the IM and comprised of all trades belonging to the same asset class. Results for the Advanced Models are similar, as shown in the later sections.
2. Estimating the *IM Efficiency Ratio*: In the absence of trade flows on $(t_C, t]$

$$E_{CLASSICAL+}(t) = [V(t) - V(t - \delta_C) - IM(t)]^+$$

$$E_{CLASSICAL-}(t) = [V(t) - V(t - \delta_C) + TF_{NET}(t;\ (t_C, t]) - IM(t)]^+$$

show that the Classical Model computes the expected exposure as

$$EE(t) = \mathbb{E}\left[\left\{V(t) - V(t_C) - Q_q\left[V_R(t_C + \delta_{IM}) | \mathcal{F}_{t_C}\right]\right\}^+\right]$$

$$t_C = t - \delta_C$$

where $\mathbb{E}[\cdot]$ is the expectation operator. In the absence of IM this expression would be

$$EE_0(t) = \mathbb{E}[\{V(t) - V(t_C)\}^+]$$

One of the objectives is to establish meaningful estimates of the IM *Efficiency Ratio*

$$\lambda(t) \triangleq \frac{EE(t)}{EE_0(t)}$$

Local Gaussian Approximation

1. Portfolio Value following Ito Process: Suppose that the portfolio value $V(t)$ follows an Ito process:

$$\Delta V(t) = \mu(t)\Delta t + s^T(t)\Delta W(t)$$

where $W(t)$ is a vector of independent Brownian motions, and $\mu(t)$ and $s^T(t)$ are well-behaved processes – with $s^T(t)$ being vector-valued – adapted to $W(t)$. Notice that both μ and s^T may depend on the evolution of multiple risk factors prior to time t. For convenience denote

$$\sigma(t) = |s^T(t)|$$

2. Portfolio Increment as a Gaussian: Then, for a sufficiently small horizon δ, the increment of the portfolio value over $[t_C,\ t_C + \delta]$ conditional on $\mathcal{F}_{t_C}$ is well-approximated by a Gaussian distribution with a mean $\mu(t_C)\delta$ and a standard deviation $\frac{\sigma(t_C)}{\sqrt{\delta}}$.
3. Approximation for the Expected Exposure: Assuming $\sigma(t_C) > 0$ the drift term may be ignored for small δ. Under the Gaussian approximation above it is then straightforward to approximate the expectation in

$$EE(t) = \mathbb{E}\left[\left\{V(t) - V(t_C) - Q_q\left[V_R(t_C + \delta_{IM}) | \mathcal{F}_{t_C}\right]\right\}^+\right]$$

$$t_C = t - \delta_C$$

in the closed form

$$EE(t) \approx \mathbb{E}[\sigma(t_C)]\sqrt{\delta_C}\left[\phi\big(z(q)\big) - z(q)\Phi\big(-z(q)\big)\right]$$

$$z(q) \triangleq \sqrt{\frac{\delta_{IM}}{\delta_C}}\Phi^{-1}(q)$$

where ϕ and Φ are the standard Gaussian PDF and CDF, respectively.
4. Expression for *IM Efficiency Ratio*: Similarly

$$EE_0(t) \approx \mathbb{E}[\sigma(t_C)]\sqrt{\delta_C}\phi(0)$$

so that λ in

$$\lambda(t) \triangleq \frac{EE(t)}{EE_0(t)}$$

is approximated by

$$\lambda(t) \approx \frac{\phi\big(z(q)\big) - z(q)\Phi\big(-z(q)\big)}{\phi(0)}$$

5. λ Dependence: $q/MPoR$ Ratio: Interestingly, the multiplier $\lambda(t)$ above is independent of t and depends only on two quantities – the confidence level q used for specifying the IM, and the ratio of the IM horizon to the $MPoR$. It is emphasized that the above expression for $\lambda(t)$ was derived under weak conditions; relying on the local normality assumptions for portfolio value increments. Otherwise, the ratio is model-free; no further assumptions are made on the distribution of the portfolio value or on the dependence of the local volatility on risk factors.
6. λ Exact under Brownian Motion: The special case for which the above expression for λ becomes exact is when the portfolio process follows a Brownian motion, a case discussed in Gregory (2015). The point here is that the $\lambda(t)$ above constitutes a small- δ limit, and a useful approximation, for a much broader class of processes. In fact, it can be extended to jump-diffusion processes, provided that the portfolio jumps are approximately Gaussian.
7. λ for the BCBS-IOSCO Parameters: Andersen, Pykhtin, and Sokol (2017b) illustrate the value of $\lambda(t)$ for the case

$$\delta_{IM} = \delta_C$$

graphed as a function of q. For the value of the confidence level

$$q = 99\%$$

specified by the BCBS-IOSCO framework

$$\lambda(t) \triangleq \frac{EE(t)}{EE_0(t)}$$

results in a value of

$$\lambda = 0.85\%$$

i.e., IM is anticipated to reduce the expected exposure by a factor of 117.

Numerical Tests

1. Portfolio following Geometric Brownian Motion: Recall that the approximation

$$\lambda(t) \triangleq \frac{EE(t)}{EE_0(t)}$$

hinges on the $MPoR$ being small, but it is ex-ante unclear if, say, the commonly used value of $10\ BD$ is small enough. To investigate the potential magnitude of errors introduced by the non-infinitesimal $MPoR$, assume now – in a slight abuse of the notation – that $V(t)$ follows a geometric Brownian motion with a constant volatility σ such that

$$\frac{\Delta V(t)}{V(t)} \cong \mathcal{O}(\Delta t) + \sigma \Delta W(t)$$

where $W(t)$ is one-dimensional Brownian motion.

2. Left/Right Portfolio Value Behavior: Compared to a Gaussian distribution, the distribution of $V(t)$ over a finite time interval is skewed left for

$$V(0) < 0$$

and right for

$$V(0) > 0$$

with V never crossing the origin. While this specification may seem restrictive, its only purpose is to test the accuracy of the local Gaussian approximation.

3. Lognormal Portfolio Dynamics λ Estimate: Assuming for simplicity that

$$\delta_{IM} = \delta_C$$

applying the calculations of the previous section to the lognormal setup, and neglecting terms of order δ or higher, one gets

$$\lambda(t) = \psi \frac{1 - \Phi\left(\Phi^{-1}(q) - \psi\sigma\sqrt{\delta_C}\right) - (1 - q)e^{\psi\sigma\sqrt{\delta_C}\Phi^{-1}(q)}}{2\Phi\left(\frac{\sigma\sqrt{\delta_C}}{2}\right) - 1}$$

$$\psi \triangleq sign\big(V(0)\big)$$

4. λ Dependence for Lognormal Portfolios: While the multiplier in

$$\lambda(t) \approx \frac{\phi\big(z(q)\big) - z(q)\Phi\big(-z(q)\big)}{\phi(0)}$$

depends only on the confidence level q when

$$\delta_{IM} = \delta_C$$

the multiplier in

$$\lambda(t) = \psi \frac{1 - \Phi\left(\Phi^{-1}(q) - \psi\sigma\sqrt{\delta_C}\right) - (1-q)e^{\psi\sigma\sqrt{\delta_C}\Phi^{-1}(q)}}{2\Phi\left(\frac{\sigma\sqrt{\delta_C}}{2}\right) - 1}$$

$$\psi \triangleq sign\big(V(0)\big)$$

additionally depends on the product $\sigma\sqrt{\delta_C}$ and on the sign of the portfolio exposure; the limit for δ_C - or σ - approaching zero can be verified to be equal to

$$\lambda(t) \approx \frac{\phi\big(z(q)\big) - z(q)\Phi\big(-z(q)\big)}{\phi(0)}$$

Andersen, Pykhtin, and Sokol (2017b) compare

$$\lambda(t) \approx \frac{\phi\big(z(q)\big) - z(q)\Phi\big(-z(q)\big)}{\phi(0)}$$

to

$$\lambda(t) = \psi \frac{1 - \Phi\left(\Phi^{-1}(q) - \psi\sigma\sqrt{\delta_C}\right) - (1-q)e^{\psi\sigma\sqrt{\delta_C}\Phi^{-1}(q)}}{2\Phi\left(\frac{\sigma\sqrt{\delta_C}}{2}\right) - 1}$$

$$\psi \triangleq sign\big(V(0)\big)$$

for various levels of σ.

5. <u>Long/Short Portfolio Volatility Impact</u>: As expected, Andersen, Pykhtin, and Sokol (2017b) show that the deviation of the local lognormal multiplier from the Gaussian one increases with the volatility σ - more precisely, with the product $\sigma\sqrt{\delta_C}$. Furthermore, the multiplier for the case

$$\psi = 1$$

is always greater – and the multiplier for the case

$$\psi = -1$$

is always lesser – than the limit case of 0.85% - an easily seen consequence of the fact that the relevant distribution tail for the cases

$$\psi = 1$$

is thicker – and

$$\psi = -1$$

is thinner – than the tail of the Gaussian distribution, as discussed earlier.

6. No Trade Flow λ Estimate: Yet, overall the results illustrate that, for the case of the $10\ BD$ horizon, the local Gaussian approximation produces reasonable values of the scaling multiplier at most levels of relative volatility. Certainly, the results support the idea that introducing IM at the level of

$$q = 99\%$$

should result in about two orders of magnitude reduction in the EE, when no trade flows occur within the $MPoR$.

The Impact of IM: Trade Flows within the MPoR

1. Impact of Trade Flows – Introduction: This section considers the efficacy of the IM in the more complicated case where trade flows are scheduled to take place inside the $MPoR$. In practice, many portfolios produce trade flows every business day, so this case is of considerable relevance. The introduction of cash flows will make the computation of EE both more complex and more model-dependent, so additional care is needed here to distinguish between the Classical+, the Classical- and the Advanced Models.
2. Trade Flows in the Classical- Model: Unless one operates under the assumption of the Classical- model assumptions of no trade flows paid by either C or D within the $MPoR$, any possibility of D making a trade payment to C in the future results in a spike in the EE profile.
3. Origin of Trade Flow Spikes: These spikes will appear because the portfolio value will jump following D's payment, but C would fail to post or return the VM associated with the jump. By way of example, suppose there is a trade payment due at future time u where D is a net payer.
4. Range of Trade Flow Spikes: A spike in the EE profile originating with this cash flow will appear for a range of termination flows t such that
    a. u lies within the $MPoR$ – mathematically

    $$t \in [u, u + \delta_C)$$

    b. D would actually make the payment.
5. Trade Flows in the Classical+ Model: The assumption in the Classical+ model is that D – as well as C – would make contractual trade payments for the entire $MPoR$ so a spike of width δ_C will appear in $EE(t)$ for the range

    $$t \in [u, u + \delta_C)$$

6. Trade Flows in the Advanced Model: Here it is assumed that D would make the contractual trade payments from the beginning $t - \delta_C$ of the $MPoR$ to the time $t - t_D{}'$ so a spike of width $\delta_C - \delta_D{}'$ would appear in $EE(t)$ for the range

$$t \in [u + \delta_D{}', u + \delta_C)$$

7. Advanced vs. Classical Spike Width: While spikes produced by the Advanced model would always be narrower than those produced by the Classical+ model, the former is often taller. In particular, the Advanced Model, unlike the Classical+ Model, contains a range of trade payment times

$$t \in [t - \delta_D{}', t - \delta_C)$$

where D would make a contractual trade payment but C would not.

8. Advanced vs Classical Spike Peak: This creates a narrow peak for the range

$$t \in [u + \delta_D{}', u + \delta_C)$$

at the left edge of the spike. This peak can be very high when C's and D's payments do not net – e.g., because of payments in different currencies – and makes an extra contribution to the EE that is not present in the Classical+ model.

9. IM Impact on the Margin Exposure: IM is now introduced to the exposure computation. More specifically assume that the netting set is composed of trades belonging to a single asset class, and in addition to the VM, is fully covered by dynamic IM as defined by UMR in

$$IM_k(t) = Q_q\left[V_k(t_C + \delta_{IM}) + TF_{NET,k}(t_C + \delta_{IM};\ (t_C, t_C + \delta_{IM}]) - V_k(t_C) |\ \mathcal{F}_{t_C}\right]$$

10. IM Impact on Exposure Spikes: As just discovered, in the areas *between* the EE spikes, IM will reduce EE by a factor given approximately by

$$\lambda(t) \approx \frac{\phi\big(z(q)\big) - z(q)\Phi\big(-z(q)\big)}{\phi(0)}$$

The question remains on how the exposure spikes will be affected by IM. The answer to that question is: *It depends*.

11. Payment Count Decrease with Time: As the simulation time passes, the portfolio will closer to maturity, and fewer and fewer trade payments will remain. Because of this *amortization* effect, the width of the distribution of the portfolio value increments on each path becomes smaller, and being the VaR of increments, the IM is reduced.
12. VM vs Trade Spike Ratio: On the other hand, trade payments do not generally become smaller as the trade approaches its maturity, and in fact can often become larger due to risk factor diffusion. The effectiveness of the IM in reducing the EE spikes depends on the size of the trade payments relative to the portfolio value increment distribution and can therefore in many cases decline towards the maturity of the portfolio.

Expected Exposure – Numerical Example #1

1. Fix-Float Two-Way CSA: As an example, consider a 2Y interest rate swap under a two-way CSA with zero threshold daily VM, but without IM. Assume that D pays a fixed rate of 2% semi-annually, and receives a floating rate quarterly. Setting the initial interest rate curve at 2% - quarterly compounding – with 50% lognormal volatility, Andersen, Pykhtin, and Sokol (2017b) illustrate the exposure profiles for the Classical+, the Classical-, and the Advanced models.
2. Spikes in both Classical+/Advanced: As expected both the Classical+ and the Advanced models produce spikes around the quarterly dates when the payment takes place. The nature of the spikes depends upon whether a fixed payment is made or not.
3. Fixed Payment Time Points - Twice a Year: As C pays quarterly on its semi-annual fixed payments, D will pay on average twice as much as C, resulting in high upward spikes in the exposure profile. As discussed above, the width of the Classical+ model spikes is

$$\delta_C = 10\ BD$$

while the width of the Advanced model spikes is

$$\delta_C - \delta_D{}' = 6\ BD$$

4. Floating Only Payment Points - Twice a Year: On quarterly payment dates when no fixed payment is due by D, C still pays a floating rate for the quarter. This results in downward spikes in the EE profile – C makes a trade payment and defaults before D must return the VM. The Classical+ model assumes that the trade payments are made by C (D) and the margin payments are not made by D (C) for the entire $MPoR$. Under these unrealistic assumptions the downward spike width is

$$\delta_C = 10\ BD$$

The Advanced model, on the other hand, assumes that C would make the trade payments over the time interval $\delta_C - \delta_D$ from which the interval $\delta_C - \delta_D{}'$ over which D would pay VM to C would be subtracted. In the aggregate, the width of the downward spikes in the Advanced model is therefore

$$\delta_C - \delta_D{}' = 2\ BD$$

5. Intra-Spike Change in VM: Between the spikes, the EE profile produced by the Advanced model is about 22% higher than the one produced by the Classical model. The difference originates with the VM specifications – the Classical models assume that C and D stop paying at the beginning of the $MPoR$ – see

$$VM_{CLASSICAL}(t) = V(t_c) = V(t - \delta_C)$$

while the Advanced model assumes that D would post VM for some time after D has stopped doing so – see

$$VM_{ADVANCED}(t) = \min_{T_i \in [t_C, t_D]} V(T_i)$$

6. Impact of IM Numerical Setting: Let us consider the impact of IM on the EE profile for this interest rate swap. All the assumptions about the $MPoR$ and the CSA are retained, and the IM horizon is set at

$$\delta_{IM} = \delta_C = 10\ BD$$

Since the dynamics of the swap value are modeled with a single risk factor, the IM can be conveniently calculated on each path exactly by stressing the risk factor to the 99% level over the IM horizon each path. Andersen, Pykhtin, and Sokol (2017b) illustrate the EE profile on the same scale as the no-IM result – in fact, the only part of the EE profile visible in this scale is the upward spikes of D's semi-annual payments of the fixed rate.

7. Numerical Comparison - Exposure between Spikes: To analyze the IM results, the exposure profile is broken down into two categories. First, between the spikes the EE produced by the Classical model with IM is 1.06% of the EE produced by the classical model without IM. Since the interest rate is modeled by a lognormal process, this number is closer to the value of 1.10% predicted by

$$\lambda(t) = \psi \frac{1 - \Phi\left(\Phi^{-1}(q) - \psi\sigma\sqrt{\delta_C}\right) - (1 - q)e^{\psi\sigma\sqrt{\delta_C}\Phi^{-1}(q)}}{2\Phi\left(\frac{\sigma\sqrt{\delta_C}}{2}\right) - 1}$$

$$\psi \triangleq sign\big(V(0)\big)$$

which is the lognormal case, and the value of 0.85% given by

$$\lambda(t) \approx \frac{\phi\big(z(q)\big) - z(q)\Phi\big(-z(q)\big)}{\phi(0)}$$

which is the local Gaussian approximation. Under the Advanced model, the IM scales down EE to 1.00% of its value under VM alone, which is very similar to the Classical model case.

8. Numerical Comparison - Impact of Spikes: As discussed, the degree of suppression for the spikes decreases with the simulation time, as the IM on all simulation paths shrinks with time. Because the payments at the end of a period are determined at the beginning of the period, the final payment is known exactly at time 1.75 years. So, there is in fact no IM requirement in the period from 1.75 years to 2 years. As a result, the final spike is not reduced at all.
9. Impact of Mismatch on Exposure: To examine the extent to which the unequal payment is responsible for the spike dominance in the presence of IM, Andersen, Pykhtin, and Sokol (2017b) move on to a 2Y IRS with quarterly payment on both legs, with all other model assumptions being the same as in the previous example.
10. Numerical Analysis of Spike Reduction: For this swap, D is a net payer on the payment dates only on approximately half of the scenarios, so the spike height will necessarily be reduced. This is confirmed by Andersen, Pykhtin, and Sokol (2017b) where they compute the EE profile for the quarterly-quarterly swap under the Classical $\pm$ and the Advanced models. As is seen there, while the upward EE spikes are present around the payment dates in the absence of IM, the height of the spikes is, as expected, significantly lower than before. Nevertheless, the IM remains incapable of completely suppressing the spikes.

Expected Exposure: Numerical Exposure 2

1. Mismatched Fix-Float Pay Frequencies: To some extent, the spikes in the presence of IM are particularly pronounced because of unequal payment frequency on the fixed and the floating legs on the swap (e.g., semi-annual fixed against quarterly floating is the prevailing market standard in the US).
2. Matched Fix-Float Pay Frequencies: Indeed, on the semi-annual payment dates, the fixed leg pays, on average, twice as much as the floating leg, so D is a net payer on the vast majority of the scenarios, which in turn results in sizeable semi-annual upward spikes when no IM is present.

The Impact of IM on CVA

1. CVA as Period EE Proxy: CVA constitutes a convenient condensation of EE profiles into a single number, of considerable practical relevance.
2. Numerical Estimation of the CVA: To demonstrate the impact of IM on CVA for the swap examples above, Andersen, Pykhtin, and Sokol (2017b) show the CVA calculated from the expected exposure profiles. To illustrate the impact of the VM on the uncollateralized exposure, all CVA numbers are shown relative to the CVA of an otherwise uncollateralized swap.
3. CVA in the Semi-Annual Case: Since the Classical- model does not have spikes, CVA in this model is reduced by about 2 orders of magnitude by the IM, as one would expect from

$$\lambda(t) \approx \frac{\phi\big(z(q)\big) - z(q)\Phi\big(-z(q)\big)}{\phi(0)}$$

   The presence of spikes, however, reduces the effectiveness of the IM significantly; for the case of semi-annual fixed payments, the CVA with IM is about 24% of the CVA without the IM for the Classical+ model and 15% of the Advanced model.

4. CVA in Quarterly Float Case: For the case of quarterly fixed payments, the reduced height of the spikes renders the IM noticeably more effective in reducing the CVA; the CVA with IM here is about 9% of the CVA with IM for the Classical+ model and about 5% for the Advanced model.
5. EE Spikes still dominate CVA: Nonetheless, when M is present, EE spikes still dominate the CVA, as can be verified for the Classical+ or Advanced model to the CVA for the Classical- model.
6. Importance of the Advanced Model: Overall the swap flows demonstrate that when the trade flows within the MPoR are properly modeled, the IM at 99% VaR may not be sufficient to achieve even a one order of magnitude reduction in the CVA. Also, since spikes dominate CVA in the presence of IM, accurately modeling the trade flows within the MPoR is especially important when the IM is present. Hence the Advanced model is clearly preferable since neither the Classical+ nor the Classical- models produce reasonable CVA numbers for portfolios with IM.
7. IM CVA Advanced Model Calibration: Finally, it should be noted that even if one uses only the Advanced model, the impact of IM on CVA may vary significantly, depending on the trade/portfolio details and the model calibration. In particular the following general observation can be made.
8. Payment Frequency: Higher frequency of trade payments results in more EE spikes, thus reducing the effectiveness of the IM.
9. Payment Size: Higher payment size relative to the trade/portfolio volatility results in higher EE spikes thus reducing the effectiveness of the IM.
10. Payment Asymmetry: EE spikes are especially high for payment dates where only B pays or where B is almost always the net payer. The presence of such payment dates reduces the effectiveness of the IM.
11. Model Calibration: In the Advanced Model, the width of these spikes is determined by the time interval within the $MPoR$ where D makes trade payments, i.e., $\delta_C - {\delta_D}'$. Thus, larger the δ_C – i.e., the $MPoR$ – and/or smaller the ${\delta_D}'$ - i.e., the time interval within the $MPoR$ where D does not make trade payments – would result in wider spikes, and thus reduced effectiveness of IM.

12. Illustrating the Advanced Model Effects: Andersen, Pykhtin, and Sokol (2017b) illustrate briefly the last point using additional calculation. In the swap examples above they use the *baseline* timeline calibration of the Advanced model –

$$\delta_C = 10\ BD$$

$$\delta_D = 8\ BD$$

$$\delta_C{}' = 6\ BD$$

$$\delta_D{}' = 4\ BD$$

with $MPoR$ being equal to the BCBS-IOSCO mandated IM horizon of

$$\delta_{IM} = 10\ BD$$

Their table shows CVA numbers and their ratios for two alternative calibrations of the Advanced Model mentioned earlier.

Numerical Techniques – Daily Time Grid

1. EE Estimation under Daily Resolution: The discussion so far has made obvious the importance of accurately capturing exposure spikes from trade flows. To achieve this one would need to calculate the EE with a daily resolution, something that, if done by brute force methods, likely will not be feasible for large portfolios.

2. Coarse Grid Portfolio - First Pass: In Andersen, Pykhtin, and Sokol (2017a), the authors discuss a fast approximation that produces a reasonably accurate EE profile without a significant increase in the computation time relative to the standard, coarse-grid calculations. The method requires simulation of risk factors and trade flows with a daily resolution, but the portfolio valuations – which are normally the slowest part of the simulation process – can be computed on a much coarser time grid.
3. Second Pass using Brownian Bridge: Portfolio values on the daily grid are then obtained by Brownian Bridge interpolation between the values at the coarse grid points, with careful accounting for trade flows.

Calculation of the Path-wise IM

1. Need for Numerical IM Calculations: As per

$$IM(t) = \sum_{k=1}^{K} IM_k(t)$$

   and

$$IM_k(t) = Q_q\left[V_k(t_C + \delta_{IM}) + TF_{NET,k}(t_C + \delta_{IM};\ (t_C, t_C + \delta_{IM}]) - V_k(t_C) \middle| \mathcal{F}_{t_C}\right]$$

   the calculation of the IM requires dynamic knowledge of the portfolio value increments ($P\&L$) across K distinct asset classes. Since conditional distributions of the $P\&L$ are generally not known, one must rely on numerical methods to calculate IM.
2. Path-wise IM using Regression Approach: Regression approaches are useful for this, although the selection of regression variables for large diverse portfolios where IM would depend on an impracticably large number of risk factors can be difficult.

3. Regression Variable - Value of$V_k(t_C)$: To simplify the regression approach, here the portfolio value $V_k(t_C)$ is chosen as the single regression variable for the asset class k. Mathematically, the conditioning on $\mathcal{F}_{t_C}$ in

$$IM_k(t) = Q_q\left[V_k(t_C + \delta_{IM}) + TF_{NET,k}(t_C + \delta_{IM};\ (t_C, t_C + \delta_{IM}]) - V_k(t_C) |\ \mathcal{F}_{t_C}\right]$$

is replaced with the conditioning on $V_k(t_C)$ with the hope that the projection model would not have a material impact on the result.

4. Initial Margin Exposure Quantile Re-formulation: For Monte Carlo purposes these assumptions approximate

$$IM_k(t) = Q_q\left[V_k(t_C + \delta_{IM}) + TF_{NET,k}(t_C + \delta_{IM};\ (t_C, t_C + \delta_{IM}]) - V_k(t_C) |\ \mathcal{F}_{t_C}\right]$$

with

$$IM_{k,m}(t_C) \approx Q_q\left[V_k(t_C + \delta_{IM}) + TF_{NET,k}(t_C + \delta_{IM};\ (t_C, t_C + \delta_{IM}]) - V_k(t_C) |\ V_k(t_C) = V_{k,m}(t_C)\right]$$

where m designates the m^{th} simulation path and $TF_{NET,k}(\cdot)$ is the time $t_C + \delta_{IM}]$ value of all net trade flows scheduled to be paid on the interval $(t_C, t_C + \delta_{IM}]$ realized along the simulation path m.

5. Portfolio Value Process for IM: Following Andersen, Pykhtin, and Sokol (2017a) the discounting effects are ignored, and the P&L under the quantile in the right-hand side of

$$IM_{k,m}(t_C) = Q_q\left[V_k(t_C + \delta_{IM}) + TF_{NET,k}(t_C + \delta_{IM};\ (t_C, t_C + \delta_{IM}]) - V_k(t_C) |\ V_k(t_C) = V_{k,m}(t_C)\right]$$

conditional on

$$V_k(t_C) = V_{k,m}(t_C)$$

is assumed to be Gaussian with zero drift such that simply

$$IM_{k,m}(t_C) \approx \sigma_{k,m}(t_C)\sqrt{\delta_{IM}}\Phi^{-1}(q)$$

$$\sigma_{k,m}{}^2(t_C) \triangleq \frac{1}{\delta_{IM}}\mathbb{E}\left[\left\{V_k(t_C + \delta_{IM}) + TF_{NET,k}(t_C + \delta_{IM};\ (t_C, t_C + \delta_{IM}]) - V_k(t_C)\right\}^2 \mid V_k(t_C) = V_{k,m}(t_C)\right]$$

6. <u>Non-Gaussian Portfolio Value Process</u>: Andersen and Pykhtin (2015) explored non-Gaussian assumptions, using kernel regressions to estimate the first four conditional moments of the $P\&L$. As kernel regression for the third and the fourth moments is prone to instability, this approach was deemed insufficiently robust.
7. <u>Conditional Variance Estimation Parameters Regression</u>: Estimation of the conditional expectation in

$$\sigma_{k,m}{}^2(t_C) \triangleq \frac{1}{\delta_{IM}}\mathbb{E}\left[\left\{V_k(t_C + \delta_{IM}) + TF_{NET,k}(t_C + \delta_{IM};\ (t_C, t_C + \delta_{IM}]) - V_k(t_C)\right\}^2 \mid V_k(t_C) = V_{k,m}(t_C)\right]$$

   may be computed by, say, parametric regression or kernel regression. In parametric regression, the conditional variance is approximated as a parametric function – e.g., a polynomial – of $V_k(t_C)$ with function parameters estimated by least squares regression; see, for instance, Anfuso, Aziz, Giltinan, and Loukopoulos (2017).
8. <u>Local Standard Deviation Calculation Details</u>: In defining the local standard deviation on path m, Andersen, Pykhtin, and Sokol (2017b) follow Pykhtin (2009) and scale the unconditional standard deviation of $V_k(t_C)$ by a ratio of two probability densities; in the denominator is the probability density of $V_k(t_C)$ on path m; in the numerator is the probability density that a normally distributed random variable with the same mean and

standard deviation as $V_k(t_C)$ would have on the path m. The actual calculation of the local standard deviation is performed as presented in Pykhtin (2009).

9. Usage in Conjunction with SIMM: It is t be noted in passing that if a dealer uses an out-of-model margin calculator – e.g., the SIMM method in International Swaps and Derivatives Association (2016) – an adjustment will be needed to capture the difference between an in-model IM computed as above by regression methods and the out-of-model margin calculator actually used. Many possible methods could be contemplated here - e.g. a multiplicative adjustment factor that aligns the two margin calculations at time 0.

Calculation of Path-wise Exposure

1. IM Adjusted Path-wise Exposure Estimation: Once the IM on a path is calculated, the exposure on that path, in principle, can be obtained by directly subtracting the calculated IM value from the no-IM exposure. While not of central importance to this chapter, it is to be noted that the path-wise IM results allow for a straightforward computation of the Margin Value Adjustment (MVA) – see, for e.g. Andersen, Duffie, and Song (2017).
2. Dominance of IM Covered Trades: This approach works well when all the trades covered by an IM represent a reasonably small fraction of the netting set. However, when the netting set is dominated by the IM covered trades, this approach suffers from two issues that have an impact on the accuracy of the EE – and therefore CVA – calculations.
3. Simulation Noise: Suppose that all trades of the netting set are covered by IM and that

$$\delta_{IM} = \delta_C$$

   If all trades belong to the same asset class, then the non-zero exposure will be realized on average only 1% of the time between the exposure spikes, as IM by design covers 99% of the *P&L* increase. If multiple asset classes are present in the netting set, the percentages of non-zero realizations will be even less because of the disallowed diversification across asset classes. Thus, EE calculated by direct exposure simulation will be extremely noisy between spikes.

4. Non-Normality: IM is calculated under the assumption that, conditional on a path, the $P\&L$ over the $MPoR$ is Gaussian. When all – or most – of the netting is covered by IM, the EE between the spikes calculated by a direct of the exposure is very sensitive to deviations from local normality. If the conditional $P\&L$ distribution has a heavier (lighter) upper tail than the Gaussian distribution, IM between the spikes will be understated (overstated) and EE will therefore be overstated (understated).
5. Time t_C Path-wise Expected Exposure: Bothe of these issues can be remedied by calculating a time t_C path-wise *expected* exposure for a time t default, rather than the exposure itself. If our target exposure measure is the unconditional time-0 EE, this substitution is valid, of course, by the Law of Iterated Expectations.
6. Simplification of the Advanced Model: To proceed with this idea, one simplifies the Advanced Model slightly - and assumes – as the Classical Model does – that D and C stop posting margin simultaneously – i.e. that

$$\delta_C = \delta_D$$

Then

$$E_{ADVANCED,m}(t) = \left[V_m(t) - V_m(t_C) + U_{ADVANCED,m}(t) - \sum_{k=1}^{K} IM_k(t)\right]^+$$

where $U_{ADVANCED,m}(t)$ is the realization of the right hand side of

$$U_{ADVANCED,m}(t) = TF_{C \to D}(t;\ (t_C{}', t_D{}']) + TF_{NET}(t;\ (t_D{}', t])$$

on path m.

7. Expected Exposure Conditional on $V_k(t_C) = V_{k,m}(t_C)$: The expectation of this exposure conditional on

$$V_k(t_C) = V_{k,m}(t_C)$$

is, in the Advanced model

$$EE_m(t; t_C) \triangleq \mathbb{E}[E_{ADVANCED}(t) \mid V(t_C) = V_m(t_C)]$$

Averaging

$$E_{ADVANCED,m}(t) = \left[V_m(t) - V_m(t_C) + U_{ADVANCED,m}(t) - \sum_{k=1}^{K} IM_k(t)\right]^+$$

and

$$EE_m(t; t_C) \triangleq \mathbb{E}[E_{ADVANCED}(t) \mid V(t_C) = V_m(t_C)]$$

over all Monte Carlo paths will lead to the same result for $EE(t; t_C)$ up to Monte Carlo sample noise.

8. Evolution of the Underlying Portfolio: To calculate the right-hand side of

$$EE_m(t; t_C) \triangleq \mathbb{E}[E_{ADVANCED}(t) \mid V(t_C) = V_m(t_C)]$$

analytically, the mismatch of the portfolio value over the $MPoR$ is assumed to be Gaussian. Specifically, given

$$V(t_C) = V_m(t_C)$$

it is assumed that $TF_m(t;\ (t_C, t])$ is known at time t_C and equal to its realization on the path, and that

$$V(t_C) = V_m(t_C) \sim \mathcal{N}\left(TF_m(t;\ (t_C, t]), \sigma_m(t_C)\sqrt{\delta_C}\right)$$

where $TF_m(t;\ (t_C, t])$ represents the sum of all scheduled portfolio payments $(t_C, t]$ on that path m, and ${\sigma_m}^2(t_C)$ is defined as

$${\sigma_m}^2(t_C) \triangleq \frac{1}{\delta_C}\mathbb{E}[\{V(t) + TF_m(t;\ (t_C, t]) - V_m(t_C)\}^2 \mid V(t_C) = V_m(t_C)]$$

9. <u>Netting Set Level Portfolio Volatility</u>: It is emphasized that the expression for ${\sigma_m}^2(t_C)$ above differs from

$${\sigma_{k,m}}^2(t_C) \triangleq \frac{1}{\delta_{IM}}\mathbb{E}\left[\left\{V_k(t_C + \delta_{IM}) + TF_{NET,k}(t_C + \delta_{IM};\ (t_C, t_C + \delta_{IM}]) - V_k(t_C)\right\}^2 \mid V_k(t_C) = V_{k,m}(t_C)\right]$$

in several important ways. First the portfolio used in

$${\sigma_m}^2(t_C) \triangleq \frac{1}{\delta_C}\mathbb{E}[\{V(t) + TF_m(t;\ (t_C, t]) - V_m(t_C)\}^2 \mid V(t_C) = V_m(t_C)]$$

spans the entire netting set rather than just the sub-portfolio covered by the IM and associated with the asset class k. Second the length of the time horizon is δ_C which may be different from δ_{IM}. As before, ${\sigma_m}^2(t_C)$ can be calculated by parametric or kernel regression.

10. <u>Unconditional Brownian Bridge Expected Exposure</u>: Using this, one can calculate

$$EE_m(t; t_C) \triangleq \mathbb{E}[E_{ADVANCED}(t) \mid V(t_C) = V_m(t_C)]$$

to obtain, in the Advanced model

$$EE_m(t) = \sigma_m(t_C)\sqrt{\delta_C}\left[d_m(t)\Phi\big(d_m(t)\big)\right] + \phi\big(d_m(t)\big)$$

$$d_m(t) \triangleq \frac{-PTF_{ADVANCED,m}(t;\ (t_C, t]) - \sum_{k=1}^{K} IM_{k,m}(t)}{\sigma_m(t_C)\sqrt{\delta_C}}$$

where – omitting arguments –

$$PTF_{ADVANCED,m} = TF_m - U_{ADVANCED,m}$$

are the net trade flows on the $MPoR$ *actually* paid, according to the Advanced model.

11. Relaxing the $\delta_D = \delta_C$ Simplification: Recall that

$$EE_m(t) = \sigma_m(t_C)\sqrt{\delta_C}\left[d_m(t)\Phi\big(d_m(t)\big)\right] + \phi\big(d_m(t)\big)$$

$$d_m(t) \triangleq \frac{-PTF_{ADVANCED,m}(t;\ (t_C, t]) - \sum_{k=1}^{K} IM_{k,m}(t)}{\sigma_m(t_C)\sqrt{\delta_C}}$$

was derived using the Advanced Model for the simplifying case

$$\delta_D = \delta_C$$

$$\delta_D < \delta_C$$

would, however, result in a significant understatement of the EE between spikes – e.g. for the earlier swap sample, the understatement would be around 22%.

12. Handling the $\delta_D < \delta_C$ Case using Scaling: Directly extending

$$EE_m(t) = \sigma_m(t_C)\sqrt{\delta_C}\left[d_m(t)\Phi\left(d_m(t)\right)\right] + \phi\left(d_m(t)\right)$$

$$d_m(t) \triangleq \frac{-PTF_{ADVANCED,m}(t;\ (t_C, t]) - \sum_{k=1}^{K} IM_{k,m}(t)}{\sigma_m(t_C)\sqrt{\delta_C}}$$

to cover

$$\delta_D < \delta_C$$

is, however, not straightforward. Therefore Andersen, Pykhtin, and Sokol (2017b) propose a simple scaling solution. Here the paths of the exposure *without* the IM are simulated first for the Advanced model, i.e. for

$$E_{ADVANCED}(t) = \left[V(t) - \min_{T_i \in [t_C, t_D]} V(T_i) + TF_{C \to D}(t;\ (t_C', t_D']) + TF_{NET}(t;\ (t_D', t]) - \sum_{k=1}^{K} IM_k(t)\right]^+$$

with

$$IM(t) = 0$$

Then for each exposure simulation time t and each measurement path m the exposure value is multiplied by the ratio of $\frac{EE_m(t)}{EE_{0,m}(t)}$ where $EE_m(t)$ is computed from

$$EE_m(t) = \sigma_m(t_C)\sqrt{\delta_C}\left[d_m(t)\Phi\left(d_m(t)\right)\right] + \phi\left(d_m(t)\right)$$

$$d_m(t) \triangleq \frac{-PTF_{ADVANCED,m}(t;\ (t_C, t]) - \sum_{k=1}^{K} IM_{k,m}(t)}{\sigma_m(t_C)\sqrt{\delta_C}}$$

and $EE_{0,m}(t)$ is the special case of $EE_m(t)$ where

$$IM(t) = 0$$

Numerical Example

1. EE Computations using Different Approaches: To illustrate the benefits of the conditional EE simulation method described above, Andersen, Pykhtin, and Sokol (2017b) turn to the 2Y IRS with unequal payment frequencies considered earlier. They illustrate EE profile comparisons using several computational approaches.
2. Expected Exposures at Spike - Comparison: First they focus their attention on the upward EE spikes produced by trade payments at the 1-year point. Both the unconditional and the conditional EE estimators here produce an almost identical spike, exceeding the benchmark spike height – the consequence of using kernel regression and a Gaussian distribution to estimate the IM.
3. Expected Exposure between Spikes - Comparison: They then show the EE exposure at a fine exposure scale, allowing for a clear observation of the EE between spikes. There the advantages of the conditional EE approach can be clearly seen.
4. Reduction in Simulation Noise: While both methods use Monte Carlo simulation with 5,000 paths, the simulation noise in the conditional EE approach is substantially less than that in the unconditional EE approach. In fact, the conditional EE noise is even less than in the benchmark EE results that were calculated using 50,000 paths.
5. Reduced Error from Non-Gaussian Dynamics: Andersen, Pykhtin, and Sokol (2017b) have used a high-volatility lognormal interest rate model in their example, to produce significant deviations in a $10\ D$ horizon. This non-normality is the main reason for the deviation of the conditional and the unconditional EE curves from the benchmark. In estimating

$$EE(t) = \mathbb{E}\left[\left\{V(t) - V_k(t_C) - Q_q\left(V_k(t_C + \delta_{IM}) \middle| \mathcal{F}_{t_C}\right)\right\}^+\right]$$

$$t_C = t - \delta_C$$

the conditional estimator uses a Gaussian distribution to approximate *both* the IM and the portfolio increment, resulting in partial error cancellation. Such error cancellation does not take place for the unconditional estimator which uses the empirical – here lognormal – distribution for the portfolio increment, yet estimates IM from a Gaussian distribution. Between the spikes the EE errors for the unconditional estimator are therefore significantly larger here than for the conditional estimator.

Conclusion

1. Consequences from the BCBS-IOSCO IM Rules: There is universal agreement that the new BCBS-IOSCO IM rules will lead to very substantial postings of margin amounts into segregated accounts accompanied by inevitable increase in the funding costs (MVA) that the dealers will face when raising funds for the IM. According to conventional wisdom, these postings, while expensive, should effectively eliminate counter party risk.
2. Handling the Trade Flow Spikes: This chapter examines the degree to which the bilateral IM required by the BCBS-IOSCO margin rules suppresses counter party exposure. As shown by Andersen, Pykhtin, and Sokol (2017a) any trade flow to the defaulting party for which it does not return margin during the MPoR causes a spike in the exposure profile.
3. Ignoring the Trade Flow Spikes: These spikes are often ignored by dealers a being *spurious* or as being part of *settlement risk*. In reality these spikes are integral part of the exposure profile and represent real risk that has previously materialized in many well-documented

incidents, notably the non-payment of Lehman of reciprocal margin to trade payments that arrived at around the time of the bankruptcy filing.

4. IM reduces Expected Exposure Considerably: The chapter shows that, under very general assumptions the BCBS IOSCO IM specified as the 99% 10 $D\ VaR$ reduces the exposure between the spikes by a factor of over 100 but fails to suppress the spikes by a comparable degree. This happens because IM is calculated without reference to trade payments, and is based only on the changes to the portfolio value resulting from the risk factor variability. As an example, Andersen, Pykhtin, and Sokol (2017b) show that IM reduces the CVA of a 2Y IRS with VM by only a factor of 7.
5. Impracticality of Increasing the IM: While VaR based IM fails to fully suppress the contribution of exposure spikes to CVA and EAD, increasing the IM to always exceed the peak exposure would be impractical, and would require moving large amounts of collateral back and forth in a matter of days.
6. IM CVA dominated by Spikes: Another important property of CVA under full IM coverage is that it is dominated by exposure spikes; in their 2Y IRS example, Andersen, Pykhtin, and Sokol (2017b) find that spike contribution to the CVA is about 95% in the presence of IM (compared to about 20% without IM).
7. Modeling CSA Time-line in IM: Thus, in the presence of IM, the focus of exposure modeling should be on capturing the impact of trade payments, which involves making realistic assumptions on what the dealer and the client are expected to make contingent on the client's default.
8. Computationally Feasible Daily Portfolio Value: Furthermore, to accurately calculate the CVA mostly produced by narrow exposure spikes, one needs to produce exposure on a daily time grid. A method for producing daily exposures without daily portfolio re-evaluations was discussed above, along with other useful numerical techniques.
9. Approach taken by the CCP: A natural question to ask is why similar payment effects have not been recognized in trading through central counterparties (CCPs), which also require IM posting that is typically based on 99% VaR over the $MPoR$. As it turns out, CCPs already use a mechanism that amounts to netting of trade and margin payments.

10. Infeasibility of the CCP Approach: Unfortunately, the same approach cannot be adopted in bilateral trading as it would require changing all of the existing trade documentation, which is a practical impossibility.
11. Nettability of Trade/Margin Payments: While a trade payment and its reciprocal margin payment cannot be netted in bilateral trading, this lag can be eliminated and the two payments made to fall on the same day by making a simple change in the CSA.
12. CSA Amendment for Trade Payments: Specifically, if the CSA is amended to state that known trade payments due to arrive prior to the scheduled margin payment date must be subtracted from the portfolio valuation for the purposes of margin – technically this amendment effectively sets the VM based on a 2-day portfolio forward value – then the call for the reciprocal margin will happen ahead of time, and it will on the same day as the trade payment – a *no lag margin settlement*.
13. Reduction in the $MPoR$ Duration: From an IT and a back-office perspective, this change in the CSA is relatively easy to align with existing mark-to-market and cash-flow processes, and is beneficial in several ways. First it shortens the duration of the exposure spikes and the $MPoR$ overall, reducing counter-party risk.
14. Margin vs MTM $2D$ Lag: Second it makes margin follow MTM without a $2D$ lag, thereby eliminating the need to use outside funding to fund hedging during this $2D$ period.
15. Trade/Margin Payments Reciprocal Concurrence: Finally, with the reciprocal trade and the margin payments falling on the same day, payment-versus-payment services (PvP) such as CLS Bank (Galati (2002), Lindley (2008), Brazier (2015)) may be able to settle trade and margin payments together, reducing residual counterparty risk even further.

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CCP and SIMM Initial Margin

Initial Margin

1. Initial Margin as Portfolio VaR: Initial margin is initiated as VaR for the derivative portfolio. VM represents P and IM uses P&L in some holding period.
2. Bilateral IM as Parametric VaR: Bilateral IM uses the parametric VaR where the parameters were calibrated to the historical data. Bilateral IM is called the Standard Initial Margin Model (SIMM).
3. CCP IM using Historical VaR: On the other hand, the Central Counter-party (CCP) uses historical VaR. CCP also demands other extra pool of assets to cover losses in the event of multiple members' default.
4. Bilateral CCP VM/IM Methodology:

	Metric	**Bilateral**	**CCP**
Variation Margin	MTM	-	-
Initial Margin	VaR	SIMM	VaR/CVaR
Default Fund	Stress VaR	-	Systemic Risk

5. Bilateral CCP/Holding Period Horizon: The holding period is defined as 10 days for bilateral and 5 days for CCP.

6. VaR Estimation Time Period Horizon: For SIMM, the scenario considered is the recent 3 years plus the common stress period (30 Aug 2008 to 29 November 2008), and the parameter called Risk Weight is calibrated. CCP directly uses the rate shift or the spread shift data for the reference period. For IRS cases, LCH uses 10 years (2500 days), JSCC uses 5 years (1250 days), and CME uses similar reference period (1260 + stress).
7. SIMM/ES VaR Confidence Level: The confidence level used for SIMM is 99% (i.e., its equivalent calibrated level), 99.7% expected shortfall (ES) for LCH, 99% ES for JSCC, and 99.7% VaR for CME.
8. Expression for VaR/Confidence Shortfall: VaR:

$$\mathbb{P}[V(t + \Delta t) - V(t) \leq Threshold] = ConfidenceLevel$$

Expected Shortfall (CVaR):

$$\mathbb{E}[V(t + \Delta t) - V(t) \leq Threshold] = ConfidenceLevel$$

CCP IM

1. Regulation Mandated Clearing of Derivatives: At the September 2009 Pittsburgh summit, it was agreed that G20 should clear certain OTC derivatives transactions at CCP's by the end of 2012 at the latest.
2. IRS/FX - LCH/JSCC/CME: LCH SwapClear, Japanese Securities Clearing Corporation (JSCC), and Chicago Mercantile Exchange (CME) clear not only Interest Rate Swaps, but also other FX products.
3. CDS: ICE/JSCC - US/EU Sovereigns: In the CDS market, ICE Clearing is dominant for clearing US, European, and Sovereign CDS names.

4. Bond/Repo/Equity/Futures: Other securities such as Bond and Repo or Bond Futures and Equity Futures are also clearable.
5. Components of the IRS IM: Base IM is ES, and Liquidity Add-On is what is embedded as the additional factor of concentration for the DV01. Other add-ons are the CCP-specific model costs (e.g., LCH specific).
6. CDS - ICE and JSCC:

$$IM = BaseIM + BidOffer\ \ AddOn + ShortCharge\ AddOn + Other\ AddOn$$

7. Components of the CDS IM: $BidOffer\ \ AddOn$ is the transaction cost on a specific name's CS01. Short Charge is the jump-to-default (JTD) charge for selling the position, which is typically applied on the one shortest name only. Other Add-On includes such factor as Recovery Rate Add-On.
8. Bond/Equity and Futures - LCH, JSCC, and CME: There is a standard method the CME developed in 1988 called SPAN@ (Standard Portfolio Analysis of Risks). It uses basic representative scenarios – 16 of them – and the netting among intra-month in the same security and the security futures and the netting among the different securities are considered.

Interest Rate Swap Methodology

1. Base IM: Base IM has 1250 to 2500 scenarios in which the rate absolute/relative shift $R(t)$ is used. The absolute shift causes the rate to behave like a normal distribution. Relative shift, however, results in a log-normal distribution.
2. Adjusting Historical Shifts using EWMA: The relative shift was originally used in all CCP's, but after the onset of the negative interest rates, those shifts originally taken from historical data were adjusted by exponentially weighted moving average (EWMA).

$$R_N(t) = R_N + S(t)$$

3. Calculation of Rate Shift Adjustment: Suppose that the rate shift at t is calculated using the LCH method below. It is adjusted by the ratio of volatility.

$$N = Today$$

$$t = Scenario\ Date$$

$$R(t) = \{Z(t+5) - Z(t) : t \in T\} \Longrightarrow S(t) = \left\{R(t) \cdot \left(\frac{\sigma_N}{\sigma_t} + 1\right) \cdot \frac{1}{2} : t \in T\right\}$$

4. Application of EWMA Decay Factor: EWMA uses the decay factor λ to calculate the volatility. It represents how much the older volatilities affect the next volatility.

$$\sigma^2(t+1) = \lambda[\sigma^2(t) + (1-\lambda)^2 R^2(t+1)]$$

5. CCP Rate Shift Generation Settings:

	JSCC	**LCH**	**CME**
IR Change Measure	Absolute	Absolute	Relative (+4%)
FX Change Measure	Relative	Relative	Relative
EWMA Decay Factor	0.985	0.992	0.970
Scenario	1250	2500	1260
Confidence Level	ES of Worst 12	ES of Worst 6	99.70%

6. Rate Shift Methodology used by CME: A different approach is taken by CME, which is to shift the rate by some offset α.

$$R(t) = \left\{ \ln \frac{Z(t+5)+\alpha}{Z(t)+\alpha} : t \in T \right\}$$

7. Liquidity Margin: Liquidity Margin is the add-on term for the concentration position on a specific tenor bucket b and instrument i - $PV01(b,i)$ The correlation among the instruments and the tenor buckets are handled by using either a correlation matrix or a similar methodology – the function below is denoted as LM in the later sections

$$LM = Function\left(PV01(b,i)\right)$$

Interest Rate Swap Calculation

1. Scenario-Specific Trade Level-IM: The base IM can be calculated with re-gridded delta multiplied by the scenario's ω – official CCP is based on full revaluation, but the gammas are typically not large.
2. Scenario P&L Linear in Delta: As scenario P&L is a linear function of delta, the Base IM numbers can be incremented in the accumulator once the scenarios are fixed.
3. Liquidity Margin Dependent on $PV01(b,i)$: On the other hand, Liquidity Margin uses re-gridded delta flows called $RepFlows$ with which the portfolio's $PV01(b,i)$ is calculated.
4. Portfolio Base IM/Liquidity Margin:

$$Base\ IM(Portfolio, \omega) = \sum_j Base\ IM\left(Trade_j, \omega\right)$$

$$Liquidity\ Margin\ (Portfolio) = LM\left(\sum_j RepFlows_j\right)$$

Credit Default Swap Methodology

1. Base IM: Base IM is called Spread Response Requirement in ICE's Terminology, and it represents the log returns of the EOD credit spread resulting from the time series analysis from April 2007.
2. Scaling/Flattening/Steepening/Tightening/Inverting/Widening: The result of the log returns are molded into 6 shapes, scaling/flattening/steepening/tightening or scaling/flattening/inverting/widening scenarios.

$$R_{T,SCENARIO} = R_T \cdot e^{RiskFactor \cdot ShapeFactor(T)}$$

3. Base IM as the Greatest Loss Scenario: Tightening and widening risk factors are described for each reference credit. From those spread responses, the greatest loss scenario is chosen as the base IM.
4. JSCC CDS Clearing IM Methodology: JSCC uses the relative shift without any EWMA adjustment from the last 750 days.
5. Bid-Offer Margin: Bid-offer margin is the transaction cost associated with unwinding CDS trades. It is surveyed on tenor buckets and reference credits.
6. Short Charge (JtD Charge): Short charge is the loss given default risk if the reference credit of the CDS trades defaults at the same time as the Clearing Member's (CM) default.
7. Notional Decomposition for Index Trades: For the index trades, the notional is decomposed into each constituent's amount.

$$Short\ Charge = \max(JtD_j)$$

8. RR, Basis, and IR Margins: ICE provides other factors including Recovery Rate Risk, Basis Risk, and IRS Risk Margins.

SIMM

1. SIMM Bilateral Initial Margin Specifications: Bilateral Initial Margin was introduced in BCBS 226 and 261, and SIMM is used as the ISDA agreed methodology. The final rule was published for JFSA, CFTC, USPR, and ESA in 2015 and 2016. The calculation method is updated quiet often as new risk factors are introduced.
2. SIMM Structure: SIMM is the summation of the product IM's, ranging from IR&FX, Credit, Equity, and Commodity.

$$SIMM = SIMM_{RATESFX} + SIMM_{CREDIT} + SIMM_{EQUITY} + SIMM_{COMMODITY}$$

   For a product class IM, six risk classes are used: Interest Rate, Credit (Qualifying), Credit (Non-qualifying), Equity, Commodity, and FX. There is a correlation matrix among the risk factors.

3. Delta Margin:

$$DeltaMargin = \sqrt{\sum_b {K_b}^2 + \sum_b \sum_{c \neq b} \gamma_{bc} S_b S_c} + K_{RESIDUAL}$$

   where K_b is the bucket margin, S_b has the sign with floored and the weighted sensitivity ($WS_{k,i}$). $WS_{k,i}$ is the delta multiplied by the risk weight.

$$WS_{k,i} = RW_k s_{k,i} CR_b$$

$$K = \sqrt{\sum_k WS_k{}^2 + \sum_k \sum_{l \neq k} \rho_{kl} f_{kl} WS_k WS_l}$$

where

$$f_{kl} = \frac{\min(CR_k, CR_l)}{\max(CR_k, CR_l)}$$

and

$$S_b = \max\left(\min\left\{\sum_{i,k} WS_{i,k}, K_b\right\}, -K_b\right)$$

4. <u>Vega Margin</u>: Vega Margin uses the similar calculation as Delta Margin. The vega risk is computed as the product of the volatility and the vega.

$$VR_{ik} = HVR_c \sum_j \sigma_{jk} \frac{\partial V_{ij}}{\partial \sigma}$$

5. <u>Curvature Margin</u>: Curvature margin uses the gamma inferred from the vega. It can be represented by the scaling function $SF(t_{jk})$ multiplied by the volatility and the vega.

$$CVR_{ik} = \sum_j SF(t_{jk}) \sigma_{jk} \frac{\partial V_i}{\partial \sigma}$$

MVA

1. Margin Value Adjustment (MVA) Definition: The formulation of the IM cost can be cast as the equation below:

$$MVA = \int f(t)IM(t)D_f(t)dt$$

where $f(t)$ is the funding spread and $IM(t)$ – the forward IM – uses forward risks where the future delta and vega are implied from the spot IM with an approximation for the future delta as

$$Delta(T) = Delta(t) \times \frac{T-t}{T}$$

Vega works as

$$Vega(T) = Vega(t) \times \sqrt{\frac{T-t}{T}}$$

2. Funding Rate for the MVA: If the collateral rate is the repo rate, then the funding spread is the spread between the funding rate and the repo rate.
3. Swaption: As an example, for a swaption using a normal Black Scholes, the formula can be written as a function of the forward (F), strike (K), volatility (σ), and expiry (t_{ex}). Forward Risk (delta):

$$\Phi\left(\frac{F-k}{\sqrt{\sigma(t_{ex}-t)}}\right) \sim Constant\ if\ t\ would\ not\ change$$

Volatility Risk (Vega):

$$\sqrt{(t_{ex}-t)}\, f\left(\frac{F-k}{\sqrt{\sigma(t_{ex}-t)}}\right) \sim Constant \times \sqrt{\frac{t_{ex}-t}{t_{ex}}}$$

4. 5Y - 10Y 10 billion Yen ATM Payer Swaption:

$$Risk_{IRCurve} \sim ExerciseProbability \times DV01 \times 20$$

for 5Y risk weight or

$$Risk_{IRCurve} \sim ExerciseProbability \times DV01 \times 22$$

for 15Y risk weight.

$$Risk_{IRVol} \sim Vega \times Normal\,Vol \times 21$$

and the forward profile is based on the above approximation.

Summary

1. CCP IM and SIMM: CCP IM uses a historical scenario in its VaR – or CVaR – calculation, while SIMM uses parametric VaR. Other add-ons such as the Liquidity Margin are not small in the CCP IM.

2. MVA Estimation: Margin Valuation Adjustment can be done by taking two factors – the funding cost and the forward IM profile. The forward IM profile could be further enhanced depending on the trade activity and the optimization activity.