VaR Optimisation and Regression Sensitivities

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Abstract

Infinitesimal sensitivities, computed as derivatives of pricing functions, are useful to find high-frequency hedge ratios. However, they are less useful for the purpose of optimising 2-week VaR, especially if one includes shocks from stressed periods, as is required for applications to margin requirements for bilateral portfolios.

We compute regression sensitivities by using Krylov regularisation and find that they have a better quality P&L explain than infinitesimal sensitivities. RniVaR (Risk-not-in-VaR) is defined as the upper bound on errors in the sensitivities expansion. We suggest that RniVaR should be an add-on for SBA VaR (Sensitivities-Based-Approach VaR). We find that RniVaR is about 20% for unoptimised portfolios but can be as large as VaR itself for delta-neutral, optimised portfolios where the SBA approach breaks down.

We conclude that a full revaluation VaR is preferable for optimisation purposes over SBA VaR and that regression sensitivities are useful to find optimal hedge ratios.

1 Introduction

VaR [1] is an important metric to determine market risk capital and initial margin requirements. Hedging based on VaR optimisation is a very topical subject for both risk capital and collateral management. The challenge is that in some contexts, e.g. the calculation of initial margin for bi-lateral markets, VaR is defined for fairly sizeable shocks typical of a period of 2-weeks and calibrated to include periods of stress. Traditional greeks based on infinitesimal derivatives are fairly inaccurate for this purpose.

Infinitesimal sensitivities are typically computed by differentiating closed form pricing formulas analytically. Chain rule calculations are often implemented by using Adjoint Algorithmic Differentiation (AAD) [3]. Others use finite differentiation based on small bumps.

Infinitesimal sensitivities are sometimes singular and numerically inaccurate. An instance where this happens is whenever the spot is near the strike of an option and the maturity is short. In typical large portfolios, these exceptions occur rather frequently and may spoil accuracy if not properly regularised. Furthermore, AAD methods do not rank

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[1] All considerations in this article extend also to the case of expected shortfall which has been put forward as a VaR replacement in the Fundamental Review of the Trading Book (FRTB).

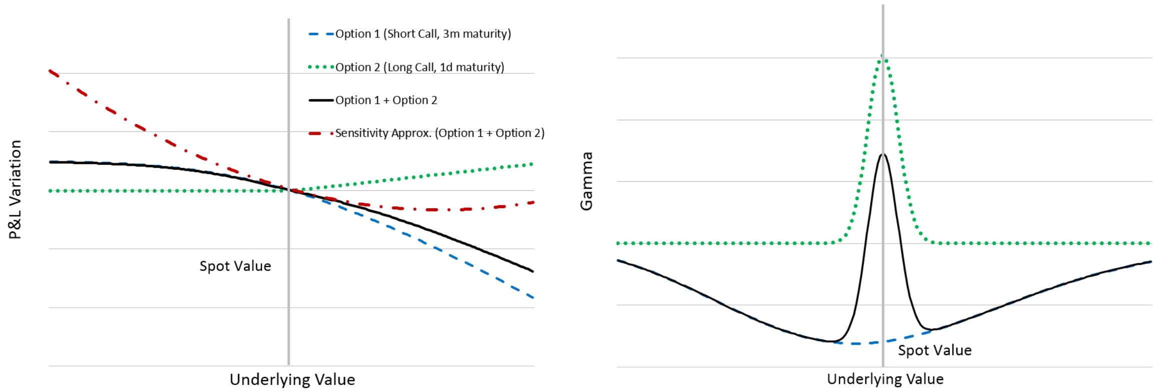

Figure 1: P&L and gammas of a portfolio of two call options. Although the analytical gamma is positive and the analytical approximation is convex (red dash-point curve), the exposure (black solid curve) seems concave by visual inspection over the relevant range for 10-day shocks.

the signal-to-noise ratio of sensitivities, a useful metric for optimisation purposes, see also [1].

In this article, we propose a method to find sensitivities based on a full Monte Carlo simulation and a regression. We generate instantaneous shocks typical of a 2-week period and compute regression sensitivities by means of a partial least-square (PLS) method with Krylov regularisation, see [5], to achieve an accurate fit to simulated P&L changes.

Since regression sensitivities are simulation-based, we find that they also allow us to estimate the P&L mismatch and thus quantify Risk-not-in-VaR (RniVaR, colloquially pronounced Arnie-VaR). By adding RniVaR to the SBA VaR number obtained using sensitivities, we ensure that backtesting benchmarks are satisfied also in difficult situations whereby portfolios are delta neutral and are dominated by higher order non linearities. Numerical experiments indicate that the RniVaR add-on is typically as large as 20% for delta-dominated portfolios and 80% for optimised portfolios.

The paper is organised as follows. In Section 2, we review infinitesimal sensitivities and discuss their use and limitations. In Section 3, we describe our simulation framework and handling of model risk. Regression sensitivities are introduced in Section 4 while in Section 5 we quantify RniVaR precisely. Section 6 discusses an optimisation case study and concludes that full-revaluation VaR is preferable for optimisation purposes since the RniVaR add-on to SBA VaR is very large.

2 Infinitesimal sensitivities

The Taylor series expansion for a function $f(x)$ has the form

$$f(x_0 + \Delta x) - f(x_0) = \sum_k \frac{(\Delta x)^k}{k!} \frac{\partial^k f(x)}{\partial x^k}\bigg|_{x_0}. \tag{1}$$

If the function $f(x)$ is sufficiently well behaved, then a few sensitivities are sufficient to capture the essential features of the profile of the function $f(x)$ around the point x_0. However, if Δx is not sufficiently small or the function $f(x)$ is highly non-linear, the error could be significant, as the simple examples in this section demonstrate.

In Figure 1, the left-hand graph shows the pricing functions of a portfolio consisting of two at-the-money call options, a short position of maturity 3m and nominal \$5 (blue) and a long position of maturity 1d and nominal \$1 (green). The black graph corresponds to the portfolio pricing function computed using a Black-Scholes formula. The right-hand graph shows the gammas. The plotted range is 2.3 times 10-days volatility, i.e. covering up to the 99th percentile of the distribution of the underlying assuming normality. Although the infinitesimal gamma is positive at the spot, the P&L appears to be negative gamma by visual inspection. The red line on the left-hand-side graph shows the Delta-Gamma approximation to the P&L, a convex parabola with opposite curvature. Hence, the short dated at-the-money option totally obfuscates negative gamma exposures and misrepresents risk.

As this example shows, the accuracy of infinitesimal sensitivities is difficult to assess, especially when they are used to find the impact of fairly extreme scenarios contributing to the 99% quantile of the P&L distribution for 2-week shocks calibrated including periods of stress. To estimate errors, nothing short of a full revaluation exercise is sufficiently accurate.

For the case whereby VaR is used to determine initial margin requirements, heritage bilateral portfolios only rarely include short-dated at-the-money options. However, bad actors could in principle trade these positions to manipulate margin amounts once a method based on infinitesimal sensitivities is accepted.

The calculation of infinitesimal gamma sensitivities is fragile and numerically unstable. A possible work-around is to account only for diagonal gammas and neglect cross-gammas. Another is to imply gamma sensitivities from vega sensitivities, which are more readily available, by using a relationship valid in the Black-Scholes model, see [4].

There are classes of trades with no material P&L impact but with a large impact on sensitivities. For instance, positions in binary options and butterfly spreads of immaterial nominal of say one dollar, see Figure 2, can be combined to offset any given delta or gamma. By adjusting the maturity to be short enough, a binary option struck at the spot can have an arbitrarily large delta and zero gamma. Similarly, a butterfly spread option centred at the spot of sufficiently short maturity and nominal of one dollar can have any pre-assigned gamma with zero delta. If a bad actor was to combine two such positions on purpose, he could offset any delta and any gamma and thus reduce SBA VaR to zero.

We conclude that the SBA VaR should be subject to a RniVaR add-on to compensate for the intrinsic inaccuracy. Since the radius of convergence of Taylor expansions can potentially be much smaller than the typical size of 2-weeks' shocks, the RniVaR cannot be found analytically even under Black-Scholes. A full-fledged Monte Carlo simulation is unavoidable to quantify a proper RniVaR metric including:

(i) Errors of the SBA approximation to P&L variation,

(ii) Intrinsic risk factors which enter into pricing models but are not perfectly spanned by the standard risk factors,

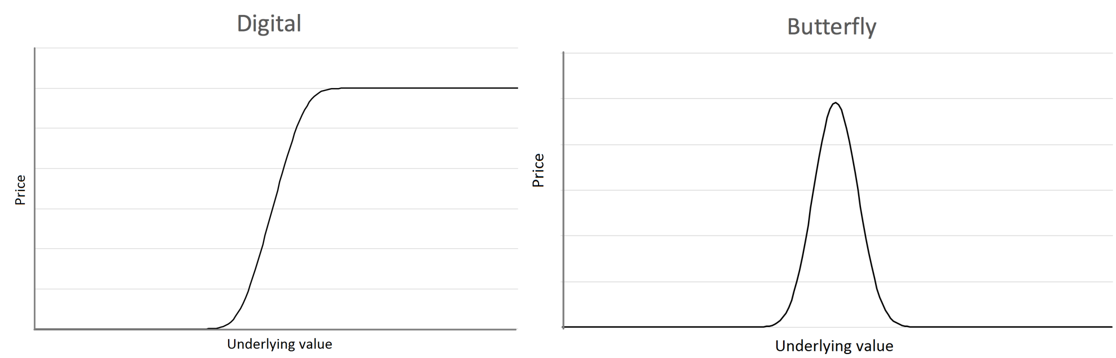

Figure 2: Pricing functions for a short-dated digital payoff and a short-dated butterfly payoff. By combining two such payoffs one can offset any delta-gamma combination even with a fixed nominal of just one dollar.

(iii) Extrinsic risk factors which do not enter into pricing models and are reflected in time-varying calibrations of model parameters.

This leads us to ask what is the most useful notion of sensitivities and optimal hedge ratios. For delta-neutral portfolios, the SBA VaR number itself computed with infinitesimal sensitivities is particularly fragile and inaccurate, giving rise to poor backtesting. We are thus confronted with three questions:

- How inaccurate is SBA VaR?
- Is it worth optimising SBA VaR or should we optimise the more accurate full-revaluation VaR?
- How to compute optimal hedge ratios for risk reduction?

3 The simulation framework

We find that the quantification of RniVaR requires far more scenarios than a few years of by-weekly historical shocks. We thus propose to carry out a hybrid historical/risk-neutral simulation in which recalibration risk is accounted for.

Key to handling the vast variety of OTC contracts traded on a bilateral basis is the ability to have a scalable and consistent cross-asset valuation model.

Pricing models involve one or more risk factors such as the interest rate for the various currencies, foreign exchange rates, stock prices, commodity prices or credit default swap spreads. Our single risk factor models are used consistently throughout the entire portfolio. They may not be the most accurate for trading purposes on any given market segment, but they perform well at the portfolio level.

We model each risk factor f in terms of a lattice model with Markov generator $\mathcal{L}_f(x, y; t, \overrightarrow{c}_f)$. Here, $x, y \in \Lambda$ are state variables on a lattice Λ with typically about 2-3 thousand states, t is calendar time and the risk factor values are given by a map $\Phi(x, t)$. Model parameters $\overrightarrow{c}_f$ are calibrated to market data and updated at regular time intervals.

The simulation includes a first instantaneous step over a time-interval of zero duration since in typical VaR calculations portfolios are not aged over the simulation time step. The transition probabilities for the risk factor f is given by

$$U_f(x,y) = e^{\tau_f \mathcal{L}_f(\overrightarrow{c}_f)}(x,y). \tag{2}$$

The parameter τ_f is named *time dilation factor* and is calibrated in such a way to be consistent with a historical volatility based also on periods of stress, as explained in section 6. The transition matrices U_f for the various risk factors f are then used jointly with a copula correlation construct to generate scenarios.

In order to properly capture model risk, we adopt a Bayesian simulation setup in a hybrid historical/risk-neutral framework. More precisely, let $t_i, i = 0, ..., N-1$, be a sequence of N time-points spaced by bi-weekly periods in the past measured in days, whereby $t_0 = 0$ is the valuation date and $t_i = t_0 - i \cdot 2w$. This bi-weekly time series can be selected from the previous year and can optionally include also periods of stress. We set

$$\delta \overrightarrow{c}_{fi} = \overrightarrow{c}_f(t_i) - \overrightarrow{c}_f(t_i - 2w) \tag{3}$$

to be the change of the calibration parameters at time t_i. We then consider a Bayesian prior distribution supported by models whose generators have shocked parameters at the 2-weeks' time horizon, i.e.:

$$\mathcal{L}_{fi}(x,y;\overrightarrow{c}_{f0} + \delta\overrightarrow{c}_{fi}) \tag{4}$$

for $i = 0, ..., N-1$ with uniform weights equal to $\frac{1}{N}$.

Notice that some risk factors share the same Markov generator and transition probability kernel but differ only by the mapping function. For instance, if one describes interest rate risk by assigning 12 vertices on an interest rate curve, the Markov generators $\mathcal{L}_f(x,y;\overrightarrow{c}_{fi})$ may all be in common and describe a short rate model for the rate. The state variable x is then associated with a number of different functions $\Phi_{fi}(x)$ corresponding to the yield at a particular vertex on the discount curve. Typically, the map $\Phi_{fi}(x)$ depends not only on the currency but also on the vertex. As another example, to model volatility risk $\Phi_{fi}(x)$ would be defined as either the log-normal or the normal implied volatility of a certain reference option.

To define historical model parameter shocks, one could, as an example, consider about N = 52 bi-weekly periods spanning two calendar years and including periods of stress.

Based on our experience, we recommend using a grand total of at least 100,000 Monte-Carlo risk neutral scenarios for accurate quantile estimations. The 100,000 primary scenarios are grouped in 52 batches of about 2,000 scenarios, where each batch is characterised by a specific 2-week shock to model parameters. Future valuations of exotic derivatives are computed by means of nested Monte Carlo simulations, that start at the 2-weeks time horizon and require further 5,000 scenarios.

4 Regression sensitivities

To calculate sensitivities robustly and with optimal accuracy, we tune them specifically to yield the best fit to $P\&L$ variation. Benefits of this approach are to allow one to filter out the most relevant sensitivities and estimate errors.

"Regression sensitivities" optimise the quality of P&L explain for any assigned set of standard risk factor sensitivities. In its most primitive form, the method is based on the solution of a regression equation of the form:

$$(P_s - P_0) = \alpha + \sum_i \Delta_i \cdot \rho(\mathrm{RF}_0^i, \mathrm{RF}_s^i) + \frac{1}{2}\Gamma_i \cdot \rho(\mathrm{RF}_0^i, \mathrm{RF}_s^i)^2 + \epsilon_s \tag{5}$$

Here, s is a scenario index, P_0 is the portfolio spot valuation, P_s is the portfolio valuation in two-weeks time under scenario s, RF_0^i is the spot valuation of the i-th risk factor, RF_s^i is the valuation of risk factors in two-weeks time under scenario s and ϵ_s is the regression residual under scenario s. The function ρ gives a return, which can be either of the arithmetic type as in

$$\rho(\mathrm{RF}_0^i, \mathrm{RF}_s^i) = \mathrm{RF}_s^i - \mathrm{RF}_0^i \tag{6}$$

or a log-return as in

$$\rho(\mathrm{RF}_0^i, \mathrm{RF}_s^i) = \log\left(\frac{\mathrm{RF}_s^i}{\mathrm{RF}_0^i}\right) \tag{7}$$

In the linear system of equations in (5), equations are indexed by the primary scenarios number s and the sensitivities Δ_i and Γ_i are the unknowns. The unknown α is called drift and is a catch-all constant term for higher order non-linearities. Notice that curvature terms for the vegas are often also neglected. Given this constraint, the system is then solved in the least square sense by applying a combined Krylov regularisation, see [5].

4.1 Drift term

It is preferable to rewrite the regression equation (5) including only risk factor sensitivities as the unknowns. An equivalent linear system in the least-square sense can be obtained by removing the drift term α thanks to the following result:

Theorem 4.1. *If α, Δ_i and Γ_i are chosen optimally so that $\sum_s \epsilon_s^2$ is minimum, then we have that*

$$\overline{\epsilon} \equiv \frac{1}{N_s}\sum_s \epsilon_s = 0 \tag{8}$$

and

$$\alpha = \overline{P} - P_0 - \sum_i \Delta_i \overline{\rho(\mathrm{RF}_0^i, \mathrm{RF}^i)} - \sum_i \Gamma_i \overline{\rho(\mathrm{RF}_0^i, \mathrm{RF}^i)^2} \tag{9}$$

where

$$\overline{P} = \frac{1}{N_s}\sum_s P_s, \quad \overline{\rho}(\mathrm{RF}_0^i, \mathrm{RF}^i) = \frac{1}{N_s}\sum_s \rho(\mathrm{RF}_0^i, \mathrm{RF}_s^i), \quad \overline{\rho(\mathrm{RF}_0^i, \mathrm{RF}^i)^2} = \frac{1}{N_s}\sum_s \rho(\mathrm{RF}_0^i, \mathrm{RF}_s^i)^2. \tag{10}$$

Proof. Suppose that $\overline{\epsilon} \neq 0$. The solution with the same Δ_i and Γ_i but with α replaced by $\alpha' = \alpha + \overline{\epsilon}$ has residuals $\epsilon_s' = \epsilon_s - \overline{\epsilon}$. We have that

$$\sum_s \epsilon_s'^2 = \sum_s \epsilon_s^2 - \overline{\epsilon}^2. \tag{11}$$

Hence, the new choice of α leads to a deeper minimum, contradicting the assumed optimality of α.

Equation (9) follows by taking sample averages of all terms. □

4.2 Compression by Krylov Regularisation

To formulate the regularisation criterion, let us recast as follows equation (5):

$$Ax = b + \epsilon \tag{12}$$

whereby b is the vector of components $(P_s - \bar{P})$ and $\bar{P}$ denotes the sample mean of portfolio valuations. The components of the vector x include

$$\rho(\mathrm{RF}_0^i, \mathrm{RF}_s^i) - \overline{\rho(\mathrm{RF}_0^i, \mathrm{RF}_s^i)}. \tag{13}$$

and the curvature terms

$$\rho(\mathrm{RF}_0^i, \mathrm{RF}_s^i)^2 - \overline{\rho(\mathrm{RF}_0^i, \mathrm{RF}_s^i)^2}, \tag{14}$$

limited to the indices i for which a gamma term Γ_i is included in the regression.

The least square problem for equation (12) is equivalent to the following linear system:

$$Cx = A^T b. \tag{15}$$

where $C = A^T A$ and T denotes matrix transposition.

Since the system in Equation (15) is often ill-conditioned, we rebalance it via a change of coordinates. Let D_{jk} be the diagonal matrix of elements

$$D_{jk} = C_{jj}\delta_{jk}. \tag{16}$$

Where δ_{jk} is the Kronecker delta. Let $X = D^{-1/2} C D^{-1/2}$ be the balanced matrix and consider the following balanced version of the system in Equation (15)

$$Xw = b'. \tag{17}$$

where $w = D^{-1/2} x$ and $b' = D^{-1/2} A^T b$. Notice that the matrix X has only ones on the diagonal and the vector w is dimensionless. This transformation typically solves the ill-conditioning due to the possibly vastly different scale of dimensional risk factors, placing them all on an equal footing.

The matrix X is positive semidefinite. We carry out an eigenvalue decomposition and find the projector P_λ on the linear space spanned by the eigenvectors of eigenvalue $\geq \lambda$. We select a regularisation parameter λ and find the following regularised solution:

$$y = X^{-1} P_\lambda b'. \tag{18}$$

or, otherwise stated,

$$x = D^{1/2} X^{-1} P_\lambda D^{-1/2} A^T b. \tag{19}$$

This method leads to compressed sensitivities which are robust with respect to numerical noise. This is particularly useful in cases where there is a high degree of collinearity between risk factors. λ is chosen in such a way to ensure that sensitivities are as small as they can be notwithstanding collinearities between risk factors and without spoiling much the quality of P&L explain.

4.3 FX Cross-gammas

Once we depart from analytical sensitivities and are not limited to applying the chain rule of differentiation, we can easily build regression models that go beyond power series expansions. A more elaborate but more accurate regression model is the following:

$$(P_s - P_0) = \sum_i \Delta_i \cdot \rho(X_0^i \mathrm{RF}_0^i, X_s^i \mathrm{RF}_s^i) + \frac{1}{2}\Gamma_i \cdot \rho(X_0^i \mathrm{RF}_0^i, X_s^i \mathrm{RF}_s^i)^2 + \epsilon_s \tag{20}$$

Here, X_s^i is the exchange rate between the reference currency of the portfolio and the currency of the i-th risk factor in two weeks time under scenario s. The difference between the regression model in Equations (5) and (20) consists in adding the FX-cross gammas which are accounted for by sake of inserting the exchange rate scenarios X_s^i and removing the drift term α by virtue of Theorem 4.1.

5 Quantifying Risk-Not-In-VaR (RniVaR)

A metric for RniVaR can be defined as the upper bound on errors for initial margin estimated based on the distribution of the residuals within a certain confidence level. To prevent overfitting, we compute the sensitivities with one simulation and then apply those sensitivities to the calculation of residuals $\bar{\epsilon}_s$ of a control simulation with the same number of scenarios but obtained with a different random seed.

RniVaR for margin received is defined as follows:

$$\mathrm{RniVaR}_+ = \min\left\{\xi : \mathrm{Prob}\big(\bar{\epsilon} < \xi \ \mid \ \big[\mathrm{VaR}_+(95\%) \le \bar{P} - P_0\big] \ge 0.99\big)\right\} \tag{21}$$

and

$$\mathrm{VaR}_+(\alpha) = \min\{\xi : \mathrm{Prob}(\bar{P} - P_0 < \xi) \ge \alpha\} \tag{22}$$

where probabilities are estimated in the control simulation and where $\bar{\epsilon}$ and $\bar{P}$ are random variables distributed as the residuals of the control simulation $\bar{\epsilon}_s$ and the corresponding valuations $\bar{P}_s$, respectively. This definition of upper bound probes the upper 5% quantile of largest gains in the return distribution and applies to initial margin received. Similarly, for initial margin posted one can define the upper bounds RniVaR_- and $\mathrm{VaR}_-(\alpha)$ by flipping the sign of portfolio returns.

The $\mathrm{RniVaR}_\pm$ metrics are useful as a conservative adjustment for IM when the calculation is carried out using historical shocks as required by VaR in order to compensate for errors in P&L explain.

The adjusted amount for posted initial margin is thus:

$$\mathrm{IM}_+ = \mathrm{VaR}_+ + \mathrm{RniVaR}_+, \tag{23}$$

where VaR_+ is a parametric approximation to $\mathrm{VaR}_+(99\%)$. The terms contributing to margin amounts received VaR_- and RniVaR_- are defined in a similar way.

	Un-optimised	Optimised 1	Optimised 2
SBA VaR with infinitesimal sensitivities	570M	180M	253M
SBA VaR with regression sensitivities	494M	269M	165M
Full revaluation VaR	566M	182M	148M
RniVaR	92M	122M	140M

Table 1: The column "Un-optimised" refers to the swaption portfolio where all receiver swaptions have been replaced by payer swaptions. The column "Optimised 1" refers to the delta-neutral portfolio with both payer and receiver swaptions. The column "Optimised 2" refers to a portfolio optimised by adding swap positions to offset the regression delta sensitivities.

6 VaR Optimisation Case Study

In this section, we discuss case studies referring to numerical experiments with a test portfolio of interest rate swaptions in the USD entailing a grid with both payer and receiver swaptions of maturities: 3m, 1y, 5y, 10y; tenors: 1y, 5y, 10y, 30y; strikes: ATM, +50bp, +100bp, -50bp, -100bp. We also consider a non-symmetric counterpart of this portfolio with no receiver swaptions and double nominal for payer swaptions. While the first portfolio is delta-neutral with respect to infinitesimal sensitivities, the second is not. We consider then a third portfolio which is optimised by adding swaps with hedge ratios such to offset regression deltas.

We estimate the time-dilation factors in Equation 2, we make use of the ISDA SIMM specification for the covariance matrix, see [4]. This standard is very close to the FRTB SBA method, see [2] and is based on Cornish-Fisher formulas for VaR amounts. Total VaR is defined as the sum of an amount in which only delta risk is considered, an amount in which only vega sensitivites are included and a third whereby only the gamma (or curvature) enters. The covariance matrix is estimated historically using time periods which also include periods of stress. To estimate the dilation factor, we find sensitivities and then compare the VaR amount for individual trades with the VaR amount obtained using a full-revaluation approach. We then estimate the dilation factor by requiring that the best fit regression line for this data has unit slope. See the first graph on the left in Figure 3. Dilation factors correspond to approximately 40 days for IR swaptions. This means that, for the selected valuation date, the simulation volatilities are about 70% above risk neutral volatilities. [2] To avoid pro-cyclicality, the covariance matrix in [4] is estimated including also periods of stress and volatilities are thus biased upward in periods characterised by normal volatility levels.

The total VaR numbers produced using the SBA method starting from regression and analytical sensitivities are in good agreement with one another. Regression sensitivities give rise to VaR amounts which are on average 9% lower. See the central graph in Figure 3.

Figure 4 compares the delta, vega and curvature components of the total VaR amount.

[2]We may reasonably assume short-term normality for the stochastic processes involved. Under this assumption, then we may argue that the simulation volatilities are proportional to the square root of the dilated time interval. Since the conventional margin period of risk is 2 weeks, we have that the ratio between simulation and risk neutral volatilities is $\sqrt{40 \text{ days}/14 \text{ days}} \simeq 1.69$.

We notice that the delta is well aligned between regression and analytical methods. Regression sensitivities generally give a slightly lower VaR as should be expected since they are regularised. We find less agreement for curvature, which is also expected since we are not using the vega to approximate gamma as suggested in [4].

RniVaR is substantial and its inclusion triggers a correction worth about the 20% of un-optimised margin, see Table 1 and the rightmost graph in Figure 3. This substantial adjustment is essential to prevent the SBA VaR number to fall below the full-revaluation VaR.

Residuals for the symmetric portfolio with both payer and receiver swaptions are illustrated in Figure 5. The x-axis is the P&L return with a non-linear rescaling to emphasise extreme quantiles. Error bars correspond to the 99%-quantiles which enter in the definition of RniVaR. Figure 6 is similar to Figure 5 except that historical bumps to model parameters have been omitted from the Monte Carlo simulation. This graph showcases a greater quality of fit, although there is still a substantial deviation in the tails.

Table 1 includes three portfolios. The first column refers to the swaption portfolio where all receiver swaptions have been replaced by payer swaptions. The second refers to the delta-neutral portfolio with both payer and receiver swaptions. The third refers to a portfolio optimised by adding swap positions to offset the regression delta sensitivities.

Notice that RniVaR is not decreased by optimisation. This is because RniVaR captures the effect of higher order discrepancies between the sensitivity expansion and the actual portfolio returns: by definition, delta optimisation has no effect on higher order terms of the expansion and in turn it has no effect on RniVaR. Also, RniVaR is a needed add-on over the SBA VaR to ensure that this number is above the more accurate full-revaluation VaR.

Since our procedure effectively calibrates well the full-revaluation VaR number to the given covariance matrix, we propose that the full-revaluation VaR should be used as the most robust, stable and accurate number for optimal portfolios. The full-revaluation VaR computed with shocked models, by definition, does not necessitate a RniVaR add-on. As the results in Table 1 indicate, regression sensitivities are useful to find hedge ratios to optimise the full-revaluation VaR amount.

7 Conclusion

We describe a method for finding robust sensitivities to derivative portfolios which is suitable for applications to VaR type calculations.

The method is based on regression sensitivities computed with the method of Partial Least Squares with Krylov regularisation. We quantify P&L errors by means of a Risk-Not-In-VaR (RniVaR) metric. When used as an add-on to VaR, this allows one to satisfy backtesting benchmarks even for highly convex portfolios dominated by non-linearities. We find that RniVaR is typically about the 20% of VaR for unhedged, delta-dominated portfolios.

We find that a full-revaluation method can be calibrated to reproduce the result of a SBA method for delta dominated portfolios on a trade-by-trade basis. The full revaluation VaR is more accurate and robust and best suited for optimisation purposes. Optimal portfolios are typically delta-neutral and SBA methods break down. We find that regression based sensitivities are useful to find optimal hedge ratios leading to an optimal reduction for full revaluation VaR.

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Figure 3: Case of a test portfolio of USD swaptions. The dilation factor is adjusted in such a way that the regression line in the first scatterplot has unit slope. The second scatterplot compares VaR amount computed using regression sensitivities to the amounts computed using analytical sensitivities but excluding RniVaR adjustments. The latter are included in the third figure.

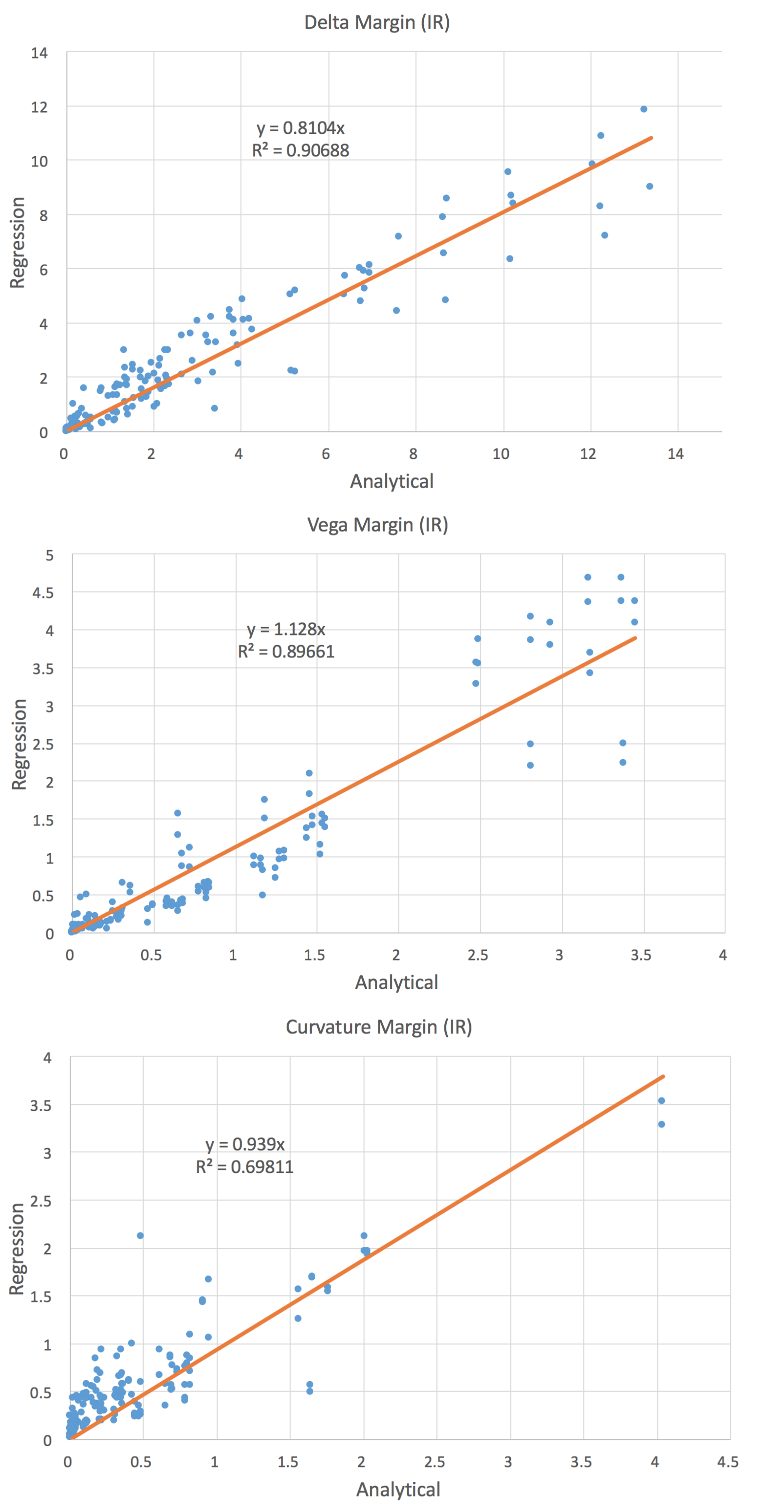

Figure 4: Case of a test portfolio of USD swaptions. The three scatterplots compare the delta, vega and curvature portions of a SBA VaR calculation based on regression and infinitesimal sensitivities.

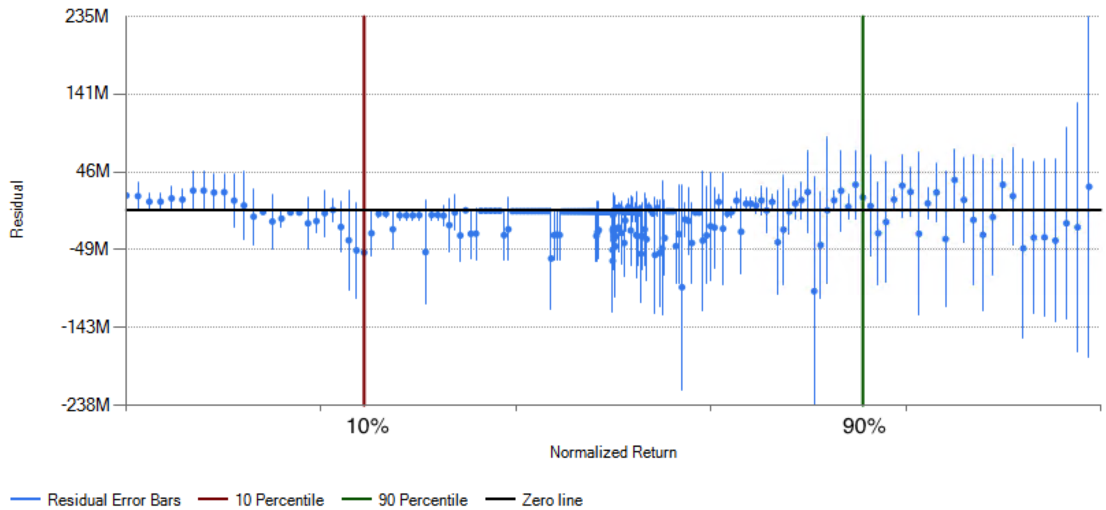

Figure 5: Residuals for the full test portfolio of USD swaptions including historical bumps as described in the paper. The x-axis is the P&L return with a non-linear rescaling to emphasise extreme quantiles. Error bars correspond to the 99%-quantiles which enter in the definition of RniVaR.

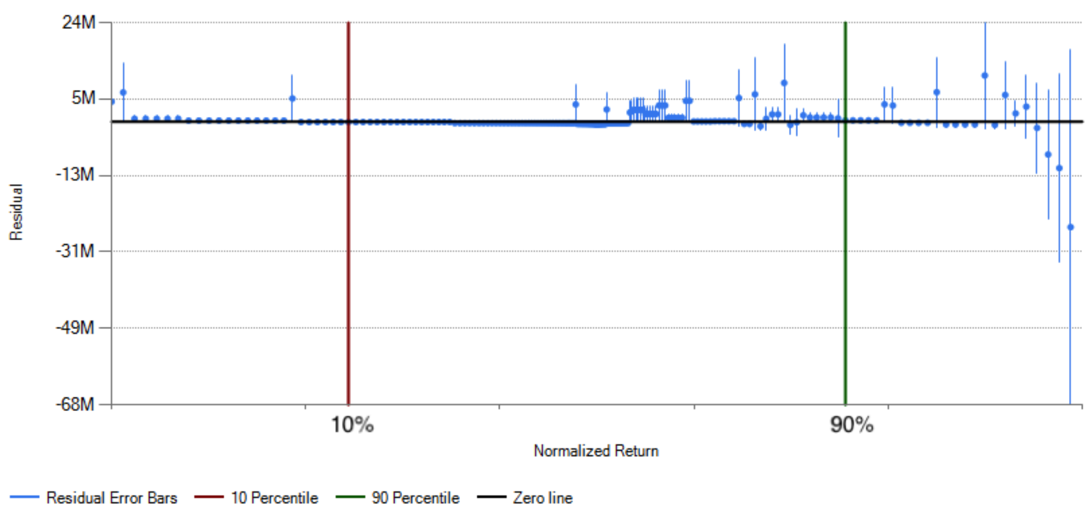

Figure 6: Same as Figure 5 except that historical bumps to model parameters have been omitted from the Monte Carlo simulation. This graph showcases a greater quality fit, although there is still a substantial deviation in the tails.