Exposure, Capital, and Margin Analytics in DROP

v3.76 28 September 2018

Modeling Counterparty Credit Exposure in the Presence of Margin Agreements

Abstract

1. Margin Based Credit Exposure Reduction: Margin agreements as a means of reducing counterparty credit exposure.
2. Calculating MPoR Collateralized Exposures:
    a. Collateralized Exposure and Margin Period of Risk
    b. Semi-analytical Method for calculating collateralized EE
3. Analysis of Basel Exposure Methods: Analysis of Basel *Shortcut* method for Effective EPE.

Margin Agreements as a Means of Reducing Counterparty Credit Exposure

1. Definition of Counterparty Credit Risk: *Counterparty Credit Risk* is the risk that a counterparty in an *OTC* derivative transaction will default prior to the expiration of the contract and will be unable to make all contractual payments. *Exchange-traded* derivatives bear no counterparty risk.
2. Counterparty vs. Lending Risk Difference: The primary feature that distinguishes counterpart risk from lending risk is the uncertainty of exposure at any future time. For a loan, the exposure at any future date is the outstanding balance, which is certain – not taking into

account pre-payments. For a derivative, the exposure at any future date is the replacement cost, which is determined by the market value at that date, and is, therefore, uncertain.

3. Bilateral Nature of the Counterparty Risk: Since derivative portfolio value can be both positive and negative, counterparty risk is *bilateral*.
4. Spot/Forward Contract Market Value: Market value for counter i with counterparty is known only at the current date

$$t = 0$$

   For any future date t this value $V_i(t)$ is uncertain and should be assumed random.
5. Replacement Cost at Counterparty Default: If a counterparty defaults at a time τ prior to the contract maturity, the economic loss is equal to the replacement loss of the contract. If

$$V_i(\tau) > 0$$

   the dealer does not receive anything from the defaulted counterparty, but has to pay $V_i(\tau)$ to another counterparty to replace the contract. If

$$V_i(\tau) < 0$$

   the dealer receives $V_i(\tau)$ from another counterparty, but has to forward this amount to the defaulted counterparty.
6. Forward Exposure at Contract Level: Combining these two scenarios the contract-level exposure $E_i(t)$ at time t is specified according to

$$E_i(t) = \max(V_i(t), 0)$$

7. Definition of Counterparty Level Exposure: *Counterparty level exposure* at a future time t can be defined as the loss experienced by the dealer if the counterparty defaults at time t under the assumption of no recovery.

8. Unmitigated Counterpart Level Positive Exposure: It the counterparty risk is not mitigated in any way, the *counterparty level* exposure equals the sum of *contract-level* exposure:

$$E(t) = \sum_{i} E_i(t) = \max(V_i(t), 0)$$

9. Counterparty Level Exposure under Netting: If there are *netting agreements*, derivatives with positive values at the time of the default offset the ones with negative values within each netting set NS_k so that the *counterparty-level exposure* is

$$E(t) = \sum_{k} E_{NS_k}(t) = \sum_{k} \max\left(\sum_{i \in NS_k} V_i(t), 0\right)$$

   Each non-nettable trade represents a netting set.

10. Purpose of a Margin Agreement: Margin agreements allow for further reductions to counterparty level exposure.
11. Bilateral Posting under Margin Agreements: Margin agreement is a legally binding contract between two counterparties that requires one or both counterparties to post collateral under certain conditions. A margin threshold is defined for one (unilateral agreement) or both (bilateral agreement) counterparties. If the difference between the net portfolio value and the already posted collateral exceeds the threshold, the counterparty must provide sufficient collateral to cover this excess (subject to the minimum transfer amount).
12. Threshold Value in Margin Agreements: The threshold value depends primarily on the margin value of the counterparty.
13. Netting Set Level Margin Agreement: Assuming that every margin agreement requires a netting agreement, the exposure to the counterparty is

$$E_C(t) = \sum_{k} \max\left(\sum_{i \in NS_k} V_i(t) - C_k(t), 0\right)$$

where $C_k(t)$ is the market value of the collateral for the netting set NS_k at time t. If the netting set is not covered by a margin agreement, then

$$C_k(t) = 0$$

14. Netting Set Collateralized Portfolio Value: To simplify the notations, consider a single netting set

$$E_C(t) = \max(V_C(t), 0)$$

where $V_C(t)$ is the collateralized portfolio value at time t given by

$$V_C(t) = V(t) - C(t) = \sum_i V_i(t) - C(t)$$

Collateralized Exposure and Margin Period of Risk

1. Collateral as Excess Portfolio Value: Collateral covers the excess portfolio value $V(t)$ over the threshold H:

$$V_C(t) = \max(V(t) - H, 0) = -\min(H - V(t), 0)$$

2. Expression for the Collateralized Portfolio Value: Therefore the collateralized portfolio value is

$$V_C(t) = V(t) - C(t) = \min(V(t), H)$$

3. Floor/Ceiling of the Collateralized Exposure:

$$E_C(t) = \max(V_C(t), 0) = \begin{cases} 0 & V(t) < 0 \\ V(t) & 0 < V(t) < H \\ H & V(t) > H \end{cases}$$

   is limited from above and by zero from below.
4. Margin Period of Risk (MPoR): Even with a daily margin call frequency, there is a significant period delay known as the *margin period of risk (MPoR)* between a margin call that the counterparty does not respond to and the start of the default procedures. Margin calls can be disputed, and it may take several days to realize that the counterparty is defaulting rather than disputing the call. Further there is a grace period after the counterparty issues a notice of default. During this grace period the dealer and/or the counterparty may still post collateral.
5. Counterparty Variation Margin Posting Delay: Thus the collateral available at time t is determined by the portfolio value at time $t - \Delta t$.
6. Delay Dependence - Call Frequency and Product Liquidity: While Δt is not known with certainty, it is usually assumed to be a fixed number. The assume value of Δt depends on the margin call frequency and the trade liquidity.
7. MPoR Start - Collateral/Portfolio Values: Suppose that at time $t - \Delta t$ the unilateral collateral value is $C(t - \Delta t)$ and the portfolio value is $V(t - \Delta t)$.
8. Posted Counterparty Collateral at t: Then the amount $\Delta C(t)$ that should be posted at time t is

$$\Delta C(t) = \max\big(V(t - \Delta t) - C(t - \Delta t) - H, -C(t - \Delta t)\big)$$

   negative $\Delta C(t)$ means that the collateral will be returned to the counterparty.
9. Unilateral Counterparty Collateral at t: The unilateral counterparty collateral $C(t)$ available at time t is

$$C(t) = C(t - \Delta t) + \Delta C(t) = \max(V(t - \Delta t) - H, 0)$$

10. <u>The Total Collateralized Portfolio Value</u>: The collateralized portfolio value is

$$V_C(t) = V(t) - C(t) = \min\big(V(t), H + \Delta V(t)\big)$$

where

$$\Delta V(t) = V(t) - V(t - \Delta t)$$

11. <u>Monte Carlo Primary Simulation Points</u>: Suppose one has a set of *primary* simulation points $\{t_k\}$ for modeling non-collateralized exposure.
12. <u>Monte-Carlo Look-back Points</u>: For each

$$t_k > \Delta t$$

define a lookback point at time $t_k - \Delta t$
13. <u>Monte Carlo Primary Plus Lookback</u>: The task is to simulate the non-collateralized portfolio value along the path that includes both the *primary* and the *lookback* simulation times.
14. <u>Collateral PLUS Collateralized/Uncollateralized Portfolio Values</u>: Given $V(t_{k-1})$ and $C(t_{k-1})$ one calculates:
    a. Uncollateralized Portfolio Value $V(t_k - \Delta t)$ at the next lookback time $t_k - \Delta t$
    b. Uncollateralized Portfolio Value $V(t_k)$ at the next primary time t_k
    c. Collateral at t_k:

$$C(t_k) = \max(V(t_k - \Delta t) - H, 0)$$

    d. Collateralized Portfolio Value at t_k:

$$C(t_k) = V(t_k) - C(t_k)$$

e. Collateralized Exposure at t_k:

$$E_C(t_k) = \max(V_C(t_k), 0)$$

15. Simulating the Collateralized Portfolio Value: Collateralized threshold can go above the threshold due to MPR and MTA.

Semi-Analytical Method for Collateralized EE

1. Portfolio Value at Primary Points: Assume that the simulation is only run for the primary time points t and the portfolio distribution has been obtained in the form of M quantities $V_j(t)$, where j (from 1 to M) designates different scenarios.
2. Evaluating the Unconditional Portfolio Distribution: From the set $\{V_j(t)\}$ once can estimate the unconditional expectation $\mu(t)$ and standard deviation $\sigma(t)$ of the portfolio value, as well as any other distributional parameter.
3. Collateralized EE at Lookback Points: Can the collateralized EE profile be estimated without simulating the portfolio value at the lookback time points $\{V_j(t - \Delta t)\}$?
4. Collateralized EE Conditional on Path: Collateralized EE can be represented as

$$EE_C(t) = \mathbb{E}\left[EE_{C,j}(t)\right]$$

where $EE_{C,j}(t)$ is the collateralized EE conditional on $V_{C,j}(t)$:

$$EE_{C,j}(t) = \mathbb{E}\left[\max\left(V_{C,j}(t), 0\right) | V_j(t)\right]$$

5. <u>The Conditional Collateralized Portfolio Value</u>: The collateralized portfolio value $V_{C,j}(t)$ is

$$V_{C,j}(t) = \min\left(V_j(t), H + V_j(t) - V_j(t - \Delta t)\right)$$

6. <u>Goal - Computing the Collateralized EE Analytically</u>: If $EE_{C,j}(t)$ can be computed analytically, the *unconditionally collateralized EE* can be obtained as a simple average of $EE_{C,j}(t)$ across all scenarios j
7. <u>Assumption of Normal Portfolio Value</u>: Assume that the portfolio value $V(t)$ at time t is normally distributed with mean $\mu(t)$ and standard deviation $\sigma(t)$.
8. <u>Brownian Bridge for Secondary Nodes</u>: One can construct a *Brownian Bridge* from $V(0)$ to $V_j(t)$.
9. <u>$V_j(t - \Delta t)$ Mean and Standard Deviation</u>: Conditional on $V_j(t)$, $V_j(t - \Delta t)$ has a *normal distribution* with *expectation*

$$\alpha_j(t) = \frac{\Delta t}{t} V(0) + \frac{t - \Delta t}{t} V_j(t)$$

and *standard deviation*

$$\beta_j(t) = \sigma(t) \sqrt{\frac{\Delta t (t - \Delta t)}{t^2}}$$

10. <u>Closed Form Conditional Collateralized EE</u>: *Conditional Collateralized EE* can be obtained in a closed form.
11. <u>Piece-wise Constant Local Volatility</u>: It is assumed that, conditional on $V_j(t)$, the distribution of $V_j(t - \Delta t)$ is normal, but $\sigma(t)$ will be replaced by the local quantity $\sigma_{LOC}(t)$.
12. <u>Portfolio Value Monotonically Increasing with Z</u>: The portfolio value $V(t)$ at time t is described using

$$V(t) = \vartheta(t, Z)$$

where $\vartheta(t, Z)$ is a monotonically increasing function of the standard normal random variable Z.

13. The *Equivalent Normal* Portfolio Process: A *normal equivalent* portfolio process is defined as

$$W(t) = \omega(t, Z) = \mu(t) + \sigma(t)Z$$

14. Density Scaling to determine $\sigma_{LOC}(t)$: To obtain $\sigma_{LOC}(t)$, $\sigma(t)$ will be scaled by the probability densities of $W(t)$ and $V(t)$.
15. Standard Deviation Scaled Probability Density: The probability density of the quantity X is denoted via $f_X(\cdot)$ and the standard deviation is scaled according to

$$\sigma_{LOC}(t) = \frac{f_{W(t)}\big(\omega(t, Z)\big)}{f_{V(t)}\big(\vartheta(t, Z)\big)}\sigma(t)$$

16. Changing Variables from W/V to Z: Changing the variables from $V(t)$ and $W(t)$ to Z, one gets

$$f_{V(t)}\big(\vartheta(t, Z)\big) = \frac{\phi(Z)}{\partial\vartheta(t, Z)/\partial Z}$$

$$f_{W(t)}\big(\omega(t, Z)\big) = \frac{\phi(Z)}{\sigma(t)}$$

17. Substitution to the Definition of $\sigma_{LOC}(t)$: Substituting to the definition of $\sigma_{LOC}(t)$ above gives

$$\sigma_{LOC}(t) = \frac{\partial\vartheta(t, Z)}{\partial Z}$$

18. Estimating CDF - The Base Methodology: The values of Z_j corresponding to $V_j(t)$ can be obtained from

$$Z_j = \Phi^{-1}\left(F_{V(t)}\left(V_j(t)\right)\right)$$

19. Estimating the CDF - Sorting the Realizations: One sorts the array $V_j(t)$ in increasing order so that

$$V_{[j(k)]}(t) = V_{k,SORTED}(t)$$

where $j(k)$ is the sorting index.

20. Estimating CDF - Piecewise Constant Jump: From the sorted array, one can build a piecewise constant CDF that jumps by $\frac{1}{M}$ as $V(t)$ crosses any of the simulated values.

$$F_{V(t)}\left(V_j(t)\right) \approx \frac{1}{2}\frac{k-1}{M} + \frac{1}{2}\frac{k}{M} = \frac{2k-1}{2M} \rightarrow \frac{k-0.5}{M}$$

where 0.5 is the de-facto bias reducer.

21. Estimation of the Weiner Wanderer: Now one can obtain Z_j corresponding to $V_j(t)$ as

$$Z_{[j(k)]} = \Phi^{-1}\left(\frac{2k-1}{2M}\right)$$

22. Estimating the Local Standard Deviation: Local standard deviation $\sigma_{LOC,j}(t)$ can be estimated as

$$\sigma_{LOC,[j(k)]}(t) \equiv \sigma_{LOC}\left(t, Z_{[j(k)]}\right) \approx \frac{V_{[j(k+\Delta k)]}(t) - V_{[j(k-\Delta k)]}(t)}{Z_{[j(k+\Delta k)]} - Z_{[j(k-\Delta k)]}}$$

23. Choice of the Different Amount Δk: The offset Δk should not be too small (too much noise) or too large (loss of *locality*). This range works apparently well (Pykhtin (2009)):

$$20 \leq \Delta k \leq 0.05M$$

24. The Brownian Bridge Mean and σ: Similar to the above it is assumed that, conditional on $V_j(t)$, $V_j(t - \Delta t)$ has a *normal distribution* with *expectation*

$$\alpha_j(t) = \frac{\Delta t}{t} V(0) + \frac{t - \Delta t}{t} V_j(t)$$

and *standard deviation*

$$\beta_j(t) = \sigma(t) \sqrt{\frac{\Delta t(t - \Delta t)}{t^2}}$$

25. The Collateralized Exposure Mean and σ: The *collateralized exposure* depends on $\Delta V_j(t)$, which is also normal conditional on $V_j(t)$ with the same standard deviation $\beta_j(t)$ and expectation $\alpha_{C,j}(t)$ given by

$$\alpha_{C,j}(t) = V_j(t) - \alpha_j(t) = \frac{\Delta t}{t}\left[V_j(t) - V(0)\right]$$

26. Collateralized EE Conditional on j: Collateralized EE conditional on scenario j at time t is

$$EE_{C,j}(t) = \mathbb{E}\left[\max\left(\min\left(V_j(t), H + \Delta V_j(t)\right), 0\right) | V_j(t)\right]$$

27. Collateralized EE on Negative Exposure: $EE_{C,j}(t)$ equals *zero* whenever

$$V_j(t) > 0$$

so that

$$EE_{C,j}(t) = \mathbb{I}_{V_j(t)>0}\mathbb{E}\left[\min\left(V_j(t), H + \Delta V_j(t)\right)|V_j(t)\right]$$

28. Integral Form for Collateralized EE: Since $\Delta V_j(t)$ has a normal distribution, one can write

$$EE_{C,j}(t) = \mathbb{I}_{V_j(t)>0}\int_{-\infty}^{+\infty}\min\left(V_j(t), H + \alpha_{C,j}(t) + \beta_j(t)Z\right)\phi(Z)dZ$$

$$= \mathbb{I}_{V_j(t_k)>0}\left\{\int_{-d_2}^{-d_1}\left[H + \alpha_{C,j}(t) + \beta_j(t)Z\right]\phi(Z)dZ + V_j(t)\int_{-d_1}^{+\infty}\phi(Z)dZ\right\}$$

29. Conditional Collateralized EE Closed Form: Evaluating the integrals, one obtains

$$EE_{C,j}(t) = \mathbb{I}_{V_j(t_k)>0}\left\{\left[H + \alpha_{C,j}(t)\right][\Phi(d_2) - \Phi(d_1)] + \beta_j(t)[\phi(d_2) - \phi(d_1)] + V_j(t)\Phi(d_1)\right\}$$

where

$$d_1 = \frac{H + \alpha_{C,j}(t) - V_j(t)}{\beta_j(t)}$$

$$d_2 = \frac{H + \alpha_{C,j}(t)}{\beta_j(t)}$$

Analysis of Basel "Shortcut" Method for Collateralized Effective EPE

1. Basel 2 Exposure Capital Requirements: Basel 2 minimal capital requirements for the counterparty risk are determined by wholesale exposure rules with exposure at default obtained from expected exposure profile as follows.
2. Exposure at Default - Basel Variants:
    a. Expected Exposure (EE) – Expected Exposure Profile (EE)
    b. Expected Positive Exposure (EPE) –

$$EPE = \int_0^{1\,Year} EE(t)dt$$

    c. Effective EE -

$$Effective\ EE(t_k) = \max\left(EE(t_k), Effective\ EE(t_{k-1})\right)$$

    d. Effective EPE –

$$Effective\ EPE = \int_0^{1\,Year} Effective\ EE(t)dt$$

    e. Exposure at Default (EAD) –

$$EAD = \alpha \times Effective\ EPE$$

3. Incorporating the Margin Agreement: For collateralized counterparties, the netting set level Effective EPE must incorporate the effect of margin agreement.
4. Effective EPE using Internal Model of Collateral: Collateralized Effective EPE can be calculated using an *internal model of collateral*.

5. Basel 2 Simple and Conservative Shortcut: Alternatively dealers can use a *simple and conservative approximation* to the effective EPE, and sets the effective EPE for a margined counterparty equal to the lesser of:
    a. The *Threshold*, if positive, under the margin agreement *plus* an *add-on* that reflects the potential increase in exposure over the margin period of risk. The *add-on* is computed as the *expected increase in the netting set's exposure* beginning from the current exposure of zero over the margin period of risk.
    b. *Effective EPE without a margin agreement*.
6. Derivation of the "Shortcut" Method: The Basel "Shortcut" method can be obtained as follows:

$$
\begin{aligned}
EE_C(t) = \mathbb{E}\left[\max\left(\min\left(V(t), H + \Delta V(t)\right), 0\right)\right] &= \mathbb{E}[\min(E(t), H + \max(\Delta V(t), -H))] \\
&\le \mathbb{E}[\min(E(t), H + \max(\Delta V(t), 0))] \le \min(EE(t), H + \mathbb{E}[\max(\Delta V(t), 0)]) \\
&\approx \min(EE(t), H + \mathbb{E}[\max(\Delta V(\Delta t), 0)]) \equiv EE_{C,BSM}(t)
\end{aligned}
$$

7. Enhancing the Exposure Conservativeness: Time averaging adds more conservativeness:

$$
\frac{1}{T}\int_0^T EE_{C,BSM}(t)dt \le \min(EPE, H + \mathbb{E}[\max(\Delta V(\Delta t), 0)])
$$

Conclusion

1. Margin Agreements for Risk Mitigation: Margin agreements are important risk mitigation tools that need to be modeled accurately.
2. Complete MC Doubles Simulation Time: Full Monte Carlo is the most flexible approach, but requires simulating trade values at secondary time points, thus doubling the simulation time.

3. Semi-Analytical Approach Avoids That: Pykhtin (2009) has presented an accurate semi-analytical approach of calculating the EE that avoids doubling of the simulation time.
4. Basel 2 Shortcuts are too Conservative: Basel 2 “Shortcut” method for Effective EPE has sound theoretical grounds, but is too conservative.

References

- Pykhtin, M. (2009): Modeling Counterparty Credit Exposure in the Presence of Margin Agreements

Estimation of Margin Period of Risk

Abstract

1. Enhanced CSA Collateral Exposure Model: Andersen, Pykhtin (2017) describe a new framework for collateral exposure modeling under an ISDA Master Agreement with Credit Support Annex. The proposed model captures the legal and the operational aspects of default in considerably greater detail than models currently used by most practitioners, while remaining fully tractable and computationally feasible.
2. Legal Rights Exercise/Deferral Choices: Specifically, it considers the remedies and the suspension rights available within these legal agreements; the firm's policies of availing itself of these rights; and the typical time it takes to exercise them in practice.
3. Significantly Higher Credit Exposure Revealed: The inclusion of these effects is shown to produce a significantly higher credit exposure for representative portfolios compared to the currently used models. The increase is especially pronounced when dynamic initial margin is also present.

Introduction

1. Margin Period of Risk Overview: In modeling the exposure of collateralized positions, it is well recognized that credit default cannot be treated as a one-time event. Rather the entire

sequence of events following up to the default and beyond need to be considered, from the last successful margin call in advance of the eventual default to the time when the amount of loss becomes known – in the industry parlance, *crystallized*. These events unfold over a period of time called the *margin period of risk* (MPoR).

2. Range of Model Applicability: To properly identify the exposure during the MPoR, a detailed understanding of the contractual obligations is essential. In their paper, Andersen, Pykhtin, and Sokol (2017) focus on collateralized exposures under bilateral trading relationships governed by the *ISDA Master Agreement* (IMA) and its *Credit Support Annex*. The IMA, by far, is the most common legal contract for bilateral over-the-counter (OTC) derivatives trading, although other agreements are sometimes used (such as national forms of agreements used in some jurisdictions for domestic trading). The analysis by Andersen, Pykhtin, and Sokol (2017) is expected to apply to a broad class of contracts, although the model assumptions should be re-examined to confirm that the key legal provisions remain substantially the same as IMA.
3. Refinement for Legal/Operational Impact: It should be noted that the modeling of default exposure and close-out risk arising from a non-zero MPoR has received a fair amount of attention in the past (see, e.g., Gibson (2005), Pykhtin (2009, 2010), and Brigo, Capponi, Pallavicini, and Papatheodorou (2011)), although most past analysis has been conducted under very strong simplifying assumptions about the trade and the margin flows during the MPoR. One exception is Bocker and Schroder (2011), which contains elements of a more descriptive framework, including recognition of the role played by the cash flows close to the default event. Andersen, Pykhtin, and Sokol (2017) use a more detailed framework for legal and operational behavior to refine the classical models for collateralized exposure modeling.
4. Variation Margin Operational Timelines: This chapter is organized as follows. The fundamentals of variation margin posting are first outlined, and the classical collateralized exposure model is then presented. The full timelines of events likely to transpire during a credit default are then discussed from both legal and operational perspectives. This sets the stage for the proposal of a condensed representation of the timeline suitable for analytical and numerical work. The resulting setup results in a more significantly nuanced and flexible definition of the collateralized trading exposure. As fixing the actual model parameters (i.e.,

calibrating the MPoR model) requires taking a stance on operational procedures and corporate behavior, the next section discusses how such parametrizations may be done in practice, for various levels of overall model prudence and counterparty types.

5. Numerical Computation of Collateralized Exposure: Subsequently, the model is fleshed out in more detail, especially as it pertains to numerical implementations and quantitative comparisons with the classical model. As a starting point, exposure models are formulated in mathematical terms, and the key differences to the classical models are highlighted by means of brute-force Monte-Carlo simulations. Computational techniques permitting efficient model implementation are introduced subsequently, along with several test results. Applications to portfolios with risk-based initial margins are briefly discussed, and conclusions are finally drawn.

The Fundamentals of Variation Margin: Basic Definitions

1. Types of Margin: Initial/Variation: In bilateral OTC derivatives trading, it is common for parties to require posting of collateral to mitigate excessive exposures. Although the initial margin is discussed briefly in a later section, this section focuses primarily on the variation margin (VM) as a form of collateral that is regularly re-adjusted based on the changing value of the bilateral portfolio. The VM is calculated and settled in time according to a set of CSA rules discussed a few sections down.
2. Dealer and Client VM Timelines: For concreteness, throughout this chapter, the exposure of dealer D to a client C with whom D engages in bilateral OTC trading under the IMA/CSA legal framework is considered. C is referred to as the *defaulting party*, and D is *the dealer* or the *non-defaulting* party. All present value and exposure amounts throughout this chapter will be calculated from the viewpoint of D. Let the default-free market value to D of the securities portfolio at time t be $V(t)$ and let $A_D(t)$ and $A_C(t)$ be the collateral support amounts stipulated by the CSA to be posted to D and C respectively. In the absence of initial

margin it is virtually always the case that only one of A_D or A_C is positive, i.e., only one party will be required to post margin at a given point in time.

3. <u>Net Collateral and its Posting</u>: Assuming that collateral is netted (rather than posted by both parties in full and held in segregated accounts or by a third party), the total collateral amount in D's possession may be calculated as of time t as

$$c(t) = A_C(t) - A_D(t)$$

Assuming also that the collateral may be treated as *pari passu* with the derivatives portfolio itself for the purposes of bankruptcy claim, it is common to denote the positive part of the difference $V(t) - c(t)$ as the *exposure* $E(t)$:

$$E(t) = [V(t) - c(t)]^+$$

$$c(t) = A_C(t) - A_D(t)$$

where the notation

$$x^+ = \max(x, 0)$$

is used. Normally both the collateral and the portfolio would be treated together as a senior unsecured claim of D against the bankruptcy estate of C. There are several time lags and practical complications that render the above exposure and collateral expressions an imprecise measure, and they shall be substantially refined later on. In particular it is emphasized that the collateral computed at time t is generally not transferred to D until several days after t.

4. <u>VM Designated to Track Portfolio Value</u>: The type of VM encountered in the CSA is typically designed to broadly track the value of the portfolio between the parties, thereby ensuring that $E(t)$ in

$$E(t) = [V(t) - c(t)]^+$$

$$c(t) = A_C(t) - A_D(t)$$

does not grow to be excessive. However, to avoid unnecessary operational expenses, it is common to introduce language in the CSA to relax margin transfer requirements if the amounts are sufficiently small.

5. Reducing the Collateral Posting Events: To that end the typical CSA language for collateral calculations will stipulate:
   a. Collateral posting requirements by each party, h_D and h_C, representing the minimum amount of exposure before D or C, respectively, is required to post collateral
   b. A minimum transfer amount (MTA) establishing a minimum valid amount of a margin call
   c. Rounding, which rounds collateral movements to some reasonable unit (say $1,000).
6. Incorporating Thresholds into Collateral Expressions: Formally the effects of thresholds on the stipulated collateral may be written as

$$A_D(t) = [-V(t) - h_D]^+$$

$$A_C(t) = [V(t) - h_C]^+$$

with the net stipulated credit support amount assigned to D being

$$c(t) = A_C(t) - A_D(t)$$

as before. The actual availability of this amount is then subject to the path dependent effects on collateral by MTA and rounding, of which the former has significant effect only for zero or very small thresholds, and the latter is usually negligible. Both have been omitted in the equation above.

7. Unilateral and Asymmetric Collateral Requirements: Most CSAs are bilateral in nature, but unilateral CSAs do exist in which only one of the two parties is required to post collateral. A CSA may be formally bilateral, but highly asymmetric, requiring both parties to post collateral but with vastly different thresholds, e.g.

$$h_D = \$20\, mm$$

vs.

$$h_C = \$2\, mm$$

Typically, even for asymmetric CSAs, the MTAs and the rounding are the same for both parties.

Margin Calls and Cash Flows

1. Margining Frequency of the Collateral Process: From an exposure perspective, the frequency with which the amount of collateral is adjusted – the *re-margining frequency* – is a critical component of the CSA. Following the financial crisis, most new IMA/CSAs, especially between major financial institutions, have been using daily re-margining frequency in order to reduce the amount by which the exposure can change relative to the collateral between the margin calls. However many small financial institutions or buy-side clients may not be able to cope with the operational burden of frequent margin calls and will often negotiate longer re-margining frequencies, e.g., weekly, monthly, or even longer.
2. Events Constituting the Margining Process: The amount of collateral held by the parties is adjusted to their stipulated values A_D and A_C via the mechanism of a margin call. Many models for exposure treat the margin call as an instantaneous event, taking place on the re-

margining date and completed instantaneously. In practice the margin call is a chain of events that takes several days to complete. With daily re-margining, several such chains run concurrently in an *interlaced* manner; even as one margin call is yet to be settled, another one already may be initiated. The time lag of this settlement process, long with the inherent lag of the re-margining schedule, means that the changes in the VM are always running behind the changes in the portfolio value. This, in turn, implies that the idealized expressions such as

$$E(t) = [V(t) - c(t)]^{+}$$

$$c(t) = A_C(t) - A_D(t)$$

are inaccurate. The detailed events involved in an initiation and the eventual settlement of a margin call will be discussed in a later section.

3. Underlying Trade Cash Flow: With both default processes and margin settlements being non-instantaneous events, it becomes relevant to track what payment flows take place – or not – during the periods close to a default. Two types of payments are needed here. The first type – called using the term *trade flows*, covers the contractual cash flows, physical settlements, and other forms of asset transfers related to the trade themselves. These terms are spelt out in trade documents and *term sheets* for each trade. The term *trade flows* rather than *cash flows* is used to emphasize that term sheets may involve flows other than cash – such as transfers of other non-cash assets, e.g. commodities, physical settlements resulting from the creation of new trades from old ones, e.g. exercise of a physically settled swaption into a swap. A missed trade flow is a serious event under the IMA, and a failure to pay can rapidly result in a default and trade termination unless cured properly. Any missed trade flow is, of course, part of the non-defaulting party's claim.
4. CSA Specified Margin Cash Flow: The second type of flows is that that arises from the exchange of collateral between the parties – *margin flows*. The legal treatment of the margin flows is determined by the IMA/CSA, rather than by the trade documentation between the parties. For purposes of this treatment, the most important aspect of the IMA/CSA is the relatively mild treatment it affords to a party that misses a margin flow. Indeed, partially

missing a margin payment is a common occurrence, as disputes about margin amounts happen regularly, and sometimes persist for years.

5. Delays causing the Default Termination: During a collateral dispute, the CSA protocol calls for the payment of the undisputed components of the collateral, but there is of course the possibility that there will be no undisputed component at all, if one party's counter-proposals are sufficiently frivolous. Should suspicious about *gaming* arise, the CSA does contain a methodology to stop disputes through market quotations, but the resulting leakage of the position information is often a good deterrent to its use. As such, there is potential for abuse by firms experiencing financial difficulties, and a good possibility that such abuse can go on for some time before the dealer takes further efforts to end it. This, in turn, may result in a fairly long period of time between the last fully settled margin call and the eventual termination of a portfolio due to default.

Revised Exposure Definition

1. Stipulated vs. Realized Collateral Amount: In light of the discussion above, this section makes a first effort at improving

$$E(t) = [V(t) - c(t)]^{+}$$

$$c(t) = A_C(t) - A_D(t)$$

For this consider a default of C at time τ following an early termination of the trade portfolio at time

$$t \geq \tau$$

At time t, let $K(t)$ be the collateral B can *actually* rely on for the portfolio termination; this amount will very likely differ from the CSA stipulated amount $C(t)$ – and from $C(\tau)$ for that matter – due to the margin transfer time lags and some degree of non-performance by C.

2. Exposure Enhanced by Trade Flow: In addition, it is possible that some trade flows are missed; denote their value at time t, including accrued interest, as $UTF(t)$. The exposure generated by a default at time

$$\tau \leq t$$

may be re-defined as

$$E(t) = [V(t) + UTF(t) - K(t)]^{+}$$

Notice that the expression anchors the exposure at the termination date rather than at the default date τ - this will be treated at a later section. For later use, the time-0 expectation of the future time-t exposure is defined as

$$EE(t) = \mathbb{E}_0[E(t)]$$

where $\mathbb{E}$ is the expectation operator in a relevant probability measure.

3. Impact of the Margin Timelines: Determining how $K(t)$ can differ from $c(t)$, and how large can realistically $UTF(t)$ be, will require a more detailed understanding of the settlement and the margining processes, a topic that will be treated in detail in a later section. The next goes about determining how classical approaches go about modeling $K(t)$ and $UTF(t)$ in

$$E(t) = [V(t) + UTF(t) - K(t)]^{+}$$

Classical Model for Collateralized Exposure – Assumptions about Margin Flows

1. The Naïve Collateralized Exposure Model: A naïve, and now outdated, model for collateralized exposure follows the definition

$$E(t) = [V(t) - c(t)]^{+}$$

$$c(t) = A_C(t) - A_D(t)$$

literally, and assumes that the collateral available is exactly equal to its prescribed value at time t. That is, in the language of

$$E(t) = [V(t) + UTF(t) - K(t)]^{+}$$

it is assumed that

$$K(t) = c(t)$$

In addition, the parties are assumed to pay off all of the trade flows as described

$$UTF(t) = 0$$

and it is assumed that the termination date in

$$E(t) = [V(t) + UTF(t) - K(t)]^{+}$$

is the default date τ, i.e., there is no lag between the default date and the termination date. In this model, the assumption of loss crystallized at a time t is the function of the portfolio value at a single point $V(t)$ and does not depend on the earlier history $V(\cdot)$ In the limit of *perfect CSA* where

$$c(t) = V(t)$$

the collateralized exposure in such a model is exactly zero.

2. Lag Induced Classical Exposure Model: Assuming

$$K(t) = c(t)$$

is an idealization that ignores the non-instantaneous nature of collateral settlement protocols and does not capture the fact that firms under stress may stop fully honoring margin calls, resulting in a divergence between the portfolio value and the collateral value at some time lag δ before the termination of the portfolio. In what is denoted here as the *Classical Model* (see, for example, Pykhtin (2010)), this particular lag effect is captured by modifying

$$E(t) = [V(t) - c(t)]^{+}$$

$$c(t) = A_C(t) - A_D(t)$$

to

$$E(t) = [V(t) - K(t)]^{+}$$

$$K(t) = c(t - \delta)$$

So, for instance, for a CSA with thresholds h_D and h_C and from

$$A_D(t) = [-V(t) - h_D]^{+}$$

$$A_C(t) = [V(t) - h_C]^+$$

one gets

$$K(t) = [V(t-\delta) - h_C(t-\delta)]^+ - [-V(t-\delta) - h_D(t-\delta)]^+$$

3. Drawbacks of the Classical Exposure Model: Having a mechanism for capturing the divergence between the collateral and the portfolio value is an important improvement over the older method described above, and the classical model has gained widespread acceptance for both the CVA (Credit Valuation Adjustment) and the regulatory calculations. Nevertheless, it hinges on a number of assumptions that are unrealistic. For instance,

$$E(t) = [V(t) - K(t)]^+$$

$$K(t) = c(t-\delta)$$

   assumes that both D and C will simultaneously stop paying margin at time $t-\delta$ freezing the margin level over the entire MPoR. In reality, if the party due to post collateral at $t-\delta$ happens to be the non-defaulting party D, it will often continue doing so for some time even in the presence of the news about the possible impending default of C. And should C miss a few margin payments (maybe under the guise of a dispute), D would often continue to post collateral while it evaluates its options. This creates an asymmetry between posting and receiving collateral that the classical model fails to recognize.
4. Impact of Lag on Exposure: In

$$E(t) = [V(t) - K(t)]^+$$

$$K(t) = c(t-\delta)$$

the lag parameter δ is clearly critical; the larger δ is, the more $V(t)$ may pull from the frozen margin value at time $t-\delta$ and bigger the expected exposure will become. In practice, the determination of δ will often be done in a simplistic manner, e.g. by using a fixed lag (10 BD is common), or, more realistically, by adding a universal time delay to the re-margining frequency of the CSA in question. This practice is echoed in regulatory guidelines, e.g., in Basel 3 accord where MPoR is set to the re-margining frequency minus 1 BD plus an MPoR floor that defaults to 10 BD. The MPoR floor must be increased in certain cases, e.g., for large netting sets, illiquid trades, illiquid collateral, and recent collateral disputes – however, the increase is specified as a multiplier relative to the same default. With a high proportion of individually negotiated and amended features in real life IMA/CSAs, using a *one size fits all* assumption may, however, lead to significant inaccuracies.

Assumptions about Trade Flows

1. <u>The Classical+ Collateral Exposure Model</u>: Because large trade flows after the start of MPoR may no longer be followed by collateral adjustment, they have the potential to either extinguish or exacerbate the exposure. For this reason, the model assumptions with respect to the date when either party suspends the trade flows are likely to have a significant impact on the counterparty credit loss. In one common interpretation of the classical model, it is simply assumed that both D and C will continue to pay all the trade flows during the entire MPoR. As a consequence, the unpaid trade flow term $UTF(t)$ in

$$E(t) = [V(t) + UTF(t) - K(t)]^{+}$$

will be zero, consistent with

$$E(t) = [V(t) - K(t)]^{+}$$

$$K(t) = c(t - \delta)$$

For ease of reference this version of classical model is denoted as Classical+.

2. The Classical- Collateral Exposure Model: In another, less common, version of the Classical Model, the assumption is that both C and D will stop paying trade flows at the moment the MPoR commences, i.e., at time $t - \delta$. In this case, the unpaid trade flows are set equal to

$$UTF(t) = TF_{NET}(t;\ (t - \delta, t])$$

where $TF_{NET}(t;\ (t', t"])$ is the time t value of all net trade flows scheduled to be paid in the interval $(t', t"]$. Note that the time is measured in discrete units of business days, such that the notation $(u, s]$ is equivalent to $[u + 1BD, s]$. Further, if t is after the margin flow date, the trade flow value accrues from the payment date to t at a contractually specified rate. This version of the classical model is denoted Classical-; it is associated with an exposure definition of

$$E(t) = [V(t) + TF_{NET}(t;\ (t - \delta, t]) - c(t - \delta)]^{+}$$

3. Inadequacies of the Classical Exposure Models: In practice neither the Classical+ exposure equation

$$E(t) = [V(t) - K(t)]^{+}$$

$$K(t) = c(t - \delta)$$

nor the Classical- exposure equation

$$E(t) = [V(t) + TF_{NET}(t;\ (t - \delta, t]) - c(t - \delta)]^{+}$$

are accurate representations of reality. Trade flows are likely to be paid at least by D at the beginning of the MPoR, and are likely not to be paid by C at least at its end. For instance, due to the CSA protocol for collateral calculations (next section), there is typically a $3\ BD$ lag between the start of an MPoR – the market observation date for the last full margin payment – and the date when D definitely observes that C has missed paying a margin flow; during this period D would always make all trade payments unless C commits any additional contract violations. Even after D has determined that C has missed a margin payment, D's nominal rights to suspend payments following a breach would, as mentioned earlier, not always be exercised aggressively. Legal reviews, operational delays, and grace periods can further delay the time when D stops paying trade flows to C.

4. <u>Accrual of Missed Trade Flows</u>: Another trade flow effect arises during the last $2 - 3$ days of the MPoR (just prior to termination) where C has already defaulted and neither party is likely making trade payments. Here, the IMA stipulates that the missed trade flows within this period accrue forward at a contractually specified rate and become part of the bankruptcy claim. This gives rise to a termination period in addition to the $UTF(t)$ term, in turn leading to an adjustment of the exposure.

Full Timeline of IMA/CSA Events

1. <u>Details of the IMA/CSA Processes</u>: Loosely speaking, the IMA concerns itself with the events of default, termination, and close out, and the CSA governs the collateral exchanges, including the concrete rules for collateral amount calculations and posting frequencies. While this chapter so far has touched on the workings of the IMA/CSA in the previous sections, model construction hereafter will require more detailed knowledge of certain provisions regarding the normal exchange of collateral, the legal options available in the case of missed payments, and the common dealer policies with respect to availing itself of these options. A

detailed exposition of the IMA and the CSA legal complexities can be found in multiple sources, including at http://www.isda.org; here only a brief summary to the extent necessary to develop the model is provided. The focus is on the development of a plausible timeline of events taking place around the default, and the subsequent portfolio termination.

Events Prior to Default

1. Calculated vs. Actual Collateral Amounts: It is assumed that the dealer D is the Calculation Agent for the computation of the collateral amounts. As before, A_C and A_D denote the *prescribed* collateral amounts for C and D; as discussed they may differ from the *actually* available collateral amounts M_C and M_D if one of the parties fails to make the margin flow or changes the prescribed amount.
2. CSA Specified Margin Process Timelines: The following list describes the complete sequence of events taking place at times $T_0, T_1, \cdots$ The next section simplifies and condenses these into a tractable model.
3. Collateral/Portfolio Valuation Date T_0: The timeline begins at T_0, the as-of-date at which the value of the portfolio and its collateral are measured, for usage in the T_1 evaluation of the formulas for the Credit Support Amount – plainly, the amount of collateral. Typically, T_0 is the close of business on the business day before T_1.
4. Honored Collateral Invocation Date T_1: For the purposes of this treatment, T_1 is used to refer to the last *undisputed and respected* Valuation Date prior to default. At time T_1, besides officially determining $A_c(T_0)$ and $A_C(T_0)$, the dealer D calculates the incremental payment amounts to itself and to C as

$$m_D = A_D(T_0) - M_D(T_0)$$

and

$$m_C = A_C(T_0) - M_C(T_0)$$

respectively. Taking into account any minimum transfer amounts, the transfer amounts m_D and m_C should normally be communicated by D to C prior to a Notification Time (e.g., 1 PM local time).

5. Collateral Transfer Initiation Date T_2: After receiving the notice of the calculated collateral amount, C must initiate the transfers of the sufficient amount of eligible collateral on the payment date T_2. Assuming that D managed to have the collateral amount notification sent to C prior to the Notification Time, T_2 defaults to 1 BD after T_1. If D is late in its notification, T_2 would be 2 BD after T_1. It is assumed here that the required amounts – recalling that they were calculated at T_1 using market data at time T_0 – are all settled without incident at T_2. However, T_2 will be the *last* time that margin flows settle normally before the default takes place.
6. Non honored Collateral Calculation Date T_3: Let T_3 denote the next scheduled valuation date after T_1. If α is the average scheduled time between collateral calculations, one approximately has – ignoring business calendar effects –

$$T_3 \approx T_1 + \alpha$$

At T_3 – hopefully before the Notification Time - D will be able to send payment notice to C, but C has fallen into financial distress and will not be able – or willing – to pay further margin flows. Should C simply fail to pay collateral at the next payment date, a *Credit Support Default* could be triggered shortly thereafter (non-payment of collateral is associated with a 2 BD grace period). To prevent this from happening, it is, as discussed earlier, likely that C would attempt to stall by disputing the result of the T_3 collateral calculation by D.
7. Potential Event of Default τ: Exactly how long the margin dispute is allowed to proceed is a largely a behavioral question that requires some knowledge of D's credit policies and its willingness to risk legal disputes with C. Additionally one needs to consider to what extend C

is able to conceal its position of financial stress by using dispute tactics, or, say, blaming operational issues on its inability to pay collateral. Ultimately, however, D will conclude that C is in default of its margin flows (a Credit Support Default), or C will commit a serious contract violation such as failing to make a trade-related payment. At that point D will conclude that a *Potential Event of Default* (PFD) has occurred. The time of this event is identified as the true *default time* τ.

8. <u>Client PED Communication Date T_4</u>: Once the PED has taken place, D needs to formally communicate it to C, in writing. Taking into account mail/courier delays, legal reviews, and other operational lags, it is likely that the communication time, denoted T_4, takes place at a slight delay to the PED.
9. <u>Event of Default Date T_5</u>: After the receipt of the PED notice, C will be granted a brief period of time to cure the PED. The length of this *cure period* is specified in the IMA and depends on both the type of the PED and the specific IMA. For instance, of the PED in question is Failure to Pay, the default cure period is 3 BD in the 1992 IMA and 1 BD in the 2002 IMA – this may very well be overridden in the actual documents. At the end of the cure period – here denoted T_5 – and *Event of Default* (ED) formally crystallizes. It is emphasized here at the T_5 (the *official* default time) is *not* associated with the true default time τ; instead τ is equated to the time of the actual default (the PED) that, after contractual formalities, will lead to the default of C.
10. <u>ED Communication (T_6) and ETD Designation Dates (T_7)</u>: After the ED has taken place, D will inform C of the ED at time

$$T_6 \geq T_5$$

and may, at time

$$T_7 \geq T_6$$

elect to designate an *Early Termination Date* (ETD).

11. Early Termination Date T_8: The ETD is denoted T_8; per the IMA it is required that $T_8 \in [T_7, T_7 + 20D]$ The ETD constitutes the as-of-date for the termination of C's portfolio and collateral positions. Many dealers will aim for speedy resolution in order to minimize market risk, and will therefore aim to set the ETD as early as possible. There are, however, cases where this may not be optimal, as described in the Section below.
12. Post Client ETD Establishment Events: Once the portfolio claim has been established as of the ETD, the value of any collateral and unpaid trade flows held by C is added to the amount owed to D. Paragraph 8 of the CSA then allows D to liquidate any securities collateral in its possession and to apply the proceeds against the amount it is owed. Should the collateral be insufficient to cover what is owed to D, the residual amount will be submitted as a claim in C's insolvency. The claim is usually challenged by the insolvency representative, and where parties cannot agree, may be referred to court. It can sometimes take a long time before the claim is resolved by the bankruptcy courts and the realized recovery becomes known. The interest on the recovery amount for this time is added to the awarded amount. Note that this chapter focusses exclusively on modeling the magnitude of exposure and bankruptcy claim, and does not challenge the established way of modeling the amount and the timing of the eventual recovery using a loss-given-default (LGD) fraction.

Some Behavioral and Legal Aspects

1. Margin Exposure Modeling Parametrization Components: With the timeline just having been established, it remains for it to be tied with a proper model for exposure. In order to do so, as already mentioned, the timeline needs to be combined with coherent assumptions about the dealer and the client behavior in each sub-period. The assumptions should be determined not only by the rights available under the IMA/CSA, but also by the degree of operational efficiencies in serving notices and getting legal opinions, and also by the level of prudence

injected into the assumptions about the dealer ability and willingness to strictly uphold contractual terms within each client group as it pertains to margin flows and disputes.

2. Issues with Exercising Suspension Rights: From a legal rights perspective, the most important observation is that once notice of a PED has been served (time T_4) the so called *suspension rights* of IMA (Section 2(a)(iii)) and the CSA (Paragraph 4(a)) will allow D to suspend all trade- and collateral- related payments to C until the PED has been cured. The extent to which the suspension rights are actually exercised, however, is quiet situational. A particular danger is that D exercises its suspension rights due to a Potential Event of Default (PED), but that subsequently the PED is ruled to be not valid. Should this happen, the dealer can inadvertently commit a breach of contract which, especially in the presence of cross-default provisions, can have serious consequences for the dealer.
3. Choice of Designating an ETD: Another, somewhat counter-intuitive, reason for D not to enforce its suspension rights is tied to IMA Section 2(a)(iii)which can sometimes make it favorable for D to *never* designate an ETD. Indeed, if D owes C money, it would seem a reasonable course of action for D to simply:
   a. Never designate and ETD, and
   b. Suspend all the payments in the portfolio until the default gets *cured* – which most likely will never happen.

   This tactic basically allows D to walk away from its obligations on the portfolio when C defaults, effectively making D a windfall gain.
4. Jurisdiction Legality of the ETD Delays: The strategy of delaying the ETD is perpetuity has been tested by UK courts and found legal – although contract language has been proposed by ISDA to prevent the issue. In the US, however, local *safe haven* laws have been ruled to prevent ETD's for more than about one year. Still a one-year delay may prove tempting if D has a big negative exposure to C and is unwilling to immediately fund the large cash flow needed to settle. As most large dealers are presumably unlikely to play legal games with the ETD, this topic shall not be considered further here, but note that there is room to make more aggressive model assumptions around the ETD's than is done here.

Simplified Timeline of IMA/CSA Events

1. <u>Motivation for the Timeline Simplification</u>: It should be evident from the preceding section that the full timeline of IMA/CSA reviewed earlier is in many ways different, and more complex, than what is assumed in the Classical- and the Classical+ versions of the classical model. However, it is equally evident that the timeline is too complex to be modeled in every detail. This section offers a simplification of the timeline designed to extract the events most important for exposure modeling. The resulting model offers several important improvements over the classical model, while remaining practical and computationally feasible.

Identification of Key Time Periods

1. <u>Classical MPOR Start/End Dates</u>: To recap, first the classical model only considers two dates in the timeline of default; the start and the end of the MPoR. The start of the MPoR, denoted by $t-\delta$, is defined as the last observation date for which the margin was settled in full (a few days after the observation date). The end of the MPoR, denoted by t, is the observation date on which D's claim is established. Note that t coincides with the IMA's *Early Termination Date* (ETD) discussed earlier.
2. <u>Classical Model Lag Length Error</u>: In the classical model there is no clear distinction between the observation and the payment dates, making it difficult to cleanly capture the trade flow effects. For instance, in the classical version of the model, $t-\delta$ denotes both the last observation date as well as the dates on which all trade flows cease. In reality, the last margin observation date is unlikely to be contentious and trigger stoppage of trade flows, as

the margin payment to which the observation corresponds to will only be missed by C several business days later. Specifically, if the market data is observed on day 0, the valuation is performed in day 1, then only on day 2 (or 3 if the notification was late) is the initiation of the actual payment expected to take place. The length of this lag is of the same order of magnitude as typical assumptions for the length of the MPoR, and can be a source of considerable model error if not handled properly.

3. Delineating Observations and Payment Dates: In the simplified timeline proposed here, care is kept to take care of the distinction between the observation and the payment dates, and also to consider the possibility that D may take the action of stopping a particular type of flow at a different time than C does. Accordingly, the model includes two potentially different observation dates for which D and C later settle their margin flows in full for the last time; and two potentially different dates when they pay their trade flows respectively for the last time. The end of the MPoR is defined as in the same way as in the classical model, to coincide with the ETD. The table below summarizes the notation for the five dates in the simplified timeline.
4. Notation for the Dates in the Simplified Timeline:

Event	**Date Type**	**Notation**
Observation Date for the Last Margin Flow from C	Observation	$t_C = t - \delta_C$
Observation Date for the Last Margin Flow from D	Observation	$t_D = t - \delta_D$
Observation Date for the Last Trade Flow Payment from C	Settlement	$t_C{}' = t - \delta_C{}'$
Observation Date for the Last Trade Flow Payment from D	Settlement	$t_D{}' = t - \delta_D{}'$
ETD	Observation	t

5. Current Scheme MPoR Start Date: The start of the MPoR in the current model is $t - \delta$, which in the notation of table above may be defined symmetrically as

$$\delta = \max(\delta_C, \delta_D)$$

C is always expected to stop posting margin no later than the non-defaulting party D, and therefore on would very likely have

$$\delta_C \geq \delta_D$$

and

$$\delta = \delta_C$$

6. Exposure Model Timeline Lag Choices: The second column in the table above specifies which of the dates is the observation date, and which is the settlement or the payment date. According to the notation established in the table, δ_C and δ_D are the lengths of time preceding the ETD during which changes in the portfolio values no longer result in collateral payments by C and D, respectively. Similarly, $\delta_C{}'$ and $\delta_D{}'$ are the lengths of time preceding the ETD during which the respective parties do not pay trade flows. In, say, a Classical 10 day MPoR model

$$\delta_C = \delta_D = 10\ BD$$

with

$$\delta_C' = \delta_D' = 0$$

for Classical+ and

$$\delta_C' = \delta_D' = 10\ BD$$

for Classical-.

Establishing the Sequence of Events

1. Order of the MPoR Events: *A priori*, the four events in the Table between the start event and the end event of the MPoR can occur in any order. However, this section will now explain why the table very likely shows the proper sequence of events.
2. Time Lag between Margin/Trade Flows: As discussed earlier, missing trade flows are recognized as a more serious breach of contractual obligations than missing margin flows, especially since the latter may take the form of a margin valuation dispute. Therefore, it is reasonable to assume that neither party will stop paying the trade flows before stopping the payment of margin flows. Accounting for the margin settlement lag between the observation date and the payment date, this yields

$$\delta_C' \leq \delta_C - Margin\ Settlement\ Lag$$

$$\delta_D' \leq \delta_D - Margin\ Settlement\ Lag$$

3. Lag between Dealer/Client Events: It is also reasonable to assume that either of the two types of flows is first missed by the defaulting party C, and then only by the non-defaulting party D. This leads to the following additional constraints on the sequence of events within the timeline:

$$\delta_C \geq \delta_D$$

$$\delta_C' \geq \delta_D'$$

4. Client Settlement vs. Dealer Observation: Except in rare and unique situations such as outright operational failures, D would not continue to pay margin flows once C commits a more serious violation by missing a trade flow, resulting in

$$\delta_C' \leq \delta_D - Margin\ Settlement\ Lag$$

Combining these inequalities results in the chronological order of events shown in the table above.

Evaluation of the Client Survival Probability

1. Client Survival at MPoR Start: As was the case for the classical model, the setup anchors the exposure date t at the termination date ETD, at the very end of the MPoR. The ETD is the same for both parties, and constitutes a convenient reference point for aligning the actions of one party against those of the other. It needs to be emphasized that the ETD for which the exposure is evaluated does not coincide with the date at which the survival probability is evaluated, e.g. for the computation of the CVA. In the simplified timeline, the counterparty survival probability should be evaluated for $t - \delta_C'$, the last date when C stops paying trade flows – effectively assuming that the default is due to failure-to-pay. Hence, if $EE(t)$ is the expected exposure anchored at the ETD t, then the incremental contribution to the unilateral CVA from time t is, under suitable assumptions, $EE(t) \cdot \Delta\mathbb{P}_{t-\delta_C}[\cdot]$ where $\mathbb{P}$ is the survival probability under the model's measure – later sections contain concrete examples.
2. Client Survival at MPoR End: Evaluating the default probability at the anchor date t rather than at $t - \delta_C'$ will introduce the slight error in computing the survival probability. While this error is relatively small and is often ignored by practitioners, it takes virtually no effort, and has no impact on model efficiency, to evaluate the survival probability at the right date.

Timeline Calibration

1. Client Customization of IMA/CSA Specifications: As mentioned earlier, the specific IMA/CSA terms for a given counterparty should always be ideally examined in detail, so that any non-standard provisions may be analyzed by their impact on the timeline. For those cases where such bespoke timeline construction is not practical (typically for operational reasons), two standard (*reference*) timelines are proposed here. This will allow the demonstration of the thought process behind the timeline calibration, and will provide some useful base cases for later numerical tests.
2. Parametrizing the Client and the Dealer Timelines: While factors such as portfolio size and dispute history with the counterparty should, of course, be considered in establishing the MPoR, an equally important consideration in calibrating the model is the nature of expected response by B to a missed margin or trade flow by C. Even under plain vanilla IMA/CSA terms, experience shows that the reactions to contract breaches are subject to both human and institutional idiosyncracies, rendering the MPoR quiet variable.
3. Aggressive vs. Conservative Timeline Parametrization: Recognizing that "one size does not fit all", two different calibrations shall therefore be considered; one *aggressive* which assumes the best case scenario for rapidly recognizing the impending default, and taking swift action; and one *conservative*, which takes into consideration not only a likely delay in recognizing that the counterparty default is imminent, but also the possibility that the bank may not aggressively enforce its legal rights afforded under IMA and CSA in order to avoid damaging its reputation. In both scenarios daily re-margining is assumed – if a CSA calls for less frequent margin calls than this, the MPoR must be lengthened accordingly.

Aggressive Calibration

1. Applicability of the Aggressive Calibration: The aggressive calibration applies to trading relationship between two counterparties that both have string operational competence, and where there is little reputational risk associated with swift and aggressive enforcement of the non-defaulting party's legal rights against the defaulting party.
2. Inter-dealer Monitoring and Call-outs: A good example would be trading between two large dealers, both willing to aggressively defend against a possible credit loss. The credit officers here are assumed to be diligent in the monitoring of their counterparties, and generally be able to see a default developing, rather than be caught by surprise.
3. Full Application of Operational Sophistication: Under aggressive calibration, the event of C missing or disputing a margin call by any non-trivial amount will, given C's sophistication, immediately alert D that an impending default is likely. D will not be misled by claims of valuation disputes or other excuses, and will send a Notice of Credit Support Default under the IMA/CSA the next business day after the breach of the margin agreement. At the same time, to protect itself further, D will stop both the margin and the trade flows. The counterparty is assumed to simultaneously stop paying margin and trade flows as well, so that no further payments of any kind are exchanged by the parties.
4. Elimination of Settlement/Herstatt Risk: The simultaneous action by both parties in the Aggressive scenario to stop paying the trade flows at the earliest possible moment results in the elimination of all *settlement* risk – the possibility that the dealer may continue paying on its trade flow obligations while not receiving promised payments in return. In the context of cross-currency trades, this type of settlement risk is frequently referred to as the *Herstatt* risk, after the bank that caused large counterparty losses in this manner (https://en.wikipedia.org/wiki/Settlement_risk). Such risk shall be captured in the Conservative Calibration case below, and shall be discussed in more detail in a later section.
5. Timeline of the corresponding MPoR: Despite D's immediate and aggressive response, the MPoR will still be fairly long due to the way the IMA/CSA operates in practice. In particular,

notice that the first period in the simplified timeline is between the last observation date for which the margin was fully settled, and the first date for which C misses a margin flow.

6. Breakdown of the CSA Steps: As it takes at least 2 business days to settle a margin payment, plus 1 business day between the last margin that was successfully settled and the first margin payment that was not, a minimum of 3 business days will accrue from the start of an MPoR and a margin-related PED. Further, once the margin flow is missed, D must send at least 2 notices and permit a grace period – usually 2 business days – to cure the violation before an event of default (ED) has officially taken place and an ETD has been designated.
7. Comparison with Classical MPoR Timeline: Since an ETD cannot be designated prior to the event of default, it is unlikely that an MPoR can ever be less than 7 business days. It is remarkable that even under the most aggressive set of assumptions, the MPoR is still only 3 business days shorter than the classical 2-week MPoR.
8. Detailed Breakdown of the Aggressive Timeline: The detailed taxonomy of the aggressive timeline is listed in the table down below, and essentially splits the MPoR into two sections; a margin delay period of 3 business days, and a default resolution period of 4 business days. During the latter period, C and D cease paying on the first day, leaving a period of 3 business days where neither party makes any payments. Notice that it is assumed that the ETD is declared to coincide with the ED, i.e., the dealer will terminate as quickly as legally possible.

Conservative Calibration

1. Non-aggressive Enforcement of CSA Rights: The conservative calibration is intended to cover the situation where the dealer's enforcement of its rights under the IMA/CSA is deliberate and cautious, rather than swift. There may be several reasons for such a situation, sometime acing in tandem.
2. Applicable Clients - Less Sophisticated Participants: First, a dealer, if overly trigger-happy, can gain a market-wide reputation as being rigid and litigious, potentially causing clients to

seek other trading partners. In fact, should aggressive legal maneuvers be applied to counterparties that may be considered "unsophisticated", there is even a potential for the dealer to be perceived as predatory by the larger public.

3. "Leakage" of Dealer Positions: Second, there are situations where exercising the legal rights would cause an unattractive leakage of information into the broader market. As indicated earlier, this may happen for instance if the formal collateral dispute methodology of Paragraph 5 of the IDSA CSA is activated; the market poll inherent in the methodology would inevitably reveal the positions held with the counterparty to competing dealers.
4. Ramification of Aggressive Legal Exercise: Third, sometimes an aggressive interpretation of the legal rights can backfire in the form of lawsuits and counter-measures by the counterparty. For example, even when the dealer may have the rights to withhold payments (e.g., under Section 2(a)(iii)), it would often elect to not exercise this right immediately out of concern that a counter-ED would be raised against it or that withholding payments would exacerbate the liquidity situation of the counterparty potentially exposing the dealer to liabilities and lawsuits.
5. Damage from "Improper PED" Rulings: As mentioned, a particular danger is that the dealer exercises its suspension rights due to a Potential Event of Default (PED), but that subsequently the PED is ruled to not be valid. Should this happen, the dealer can inadvertently commit a breach of contract.
6. Limitations with the Dealers' Operational Capacity: Of course, even if a dealer may potentially be willing to aggressively exercise its rights, it may not have the operational capacity to do so quickly. For example, the dealer may not be able to perform the required legal review on a short notice, or may not always have the efficiency to get the notices mailed out at the earliest possible date. On top of this there is always potential for technology related and human errors and oversights.
7. Timeline Incurred by Conservative Calibration: While it is harder to get concrete data to estimate a reasonable timeline for the Conservative case (this case being dependent not only on the IMA/CSA details, but also on the specifics of the dealer's reputational considerations), under a perfectly reasonable set of assumptions the MPoR ends up being more than twice as long as for the Aggressive case above. Under this calibration choice, the Conservative

scenario assumes that the totality of the margin dispute negotiations, operational delays, human errors, legal reviews etc., adds up to 8 business days, yielding an MPoR of a total of 15 business days.

8. Typical Conservative CSA Event Timeline: One plausible scenario with daily re-margining could be:
   a. $t - 15 \Longrightarrow$ D observes the portfolio value as needed for the margin transfer amount #1 as of $t - 15$.
   b. $t - 14 \Longrightarrow$ D sends margin call #1 to C; D observes a margin transfer amount #2.
   c. $t - 13 \Longrightarrow$ D sends margin call #2 to C; C honors margin call #1; D observes a margin transfer amount #3.
   d. $t - 12 \Longrightarrow$ C fails to honor margin call #2 and initiates dispute; D tries to resolve the dispute while still paying and calculating the margin.
   e. $t - 7 \Longrightarrow$ C fails to make a trade payment.
   f. $t - 6 \Longrightarrow$ D stops paying margin and sends a PED notice.
   g. $t - 5 \Longrightarrow$ C receives PED; D keeps making trade payments.
   h. $t - 3 \Longrightarrow$ The PED is not cured.
   i. $t - 2 \Longrightarrow$ D stops making trade payments and sends an ED notice to C, designating t as the ETD.
   j. $t \Longrightarrow$ ETD.
9. Current/Interlacing Outstanding Margin Process: Notice that a number of different margin processes are simultaneously active (denoted #1, #2, and #3), reflecting the interlacing nature of the daily margin calls. Also, unlike the earlier Aggressive Calibration, the above scenario explicitly involves settlement risk, as a time period exists only where D pays trade flows (from $t - 7$ to $t - 3$, both dates inclusive).
10. Dealer/Client Payment/Settlement Lags: To translate the scenario above into the notation of the earlier sections, first notice that

$$\delta_C = 15$$

since the observation date of the last margin call (#1) honored by C is $t-15$. Second, as D makes its last possible margin call at $t-7$ based on an observation at time $t-9$

$$\delta_D = 9$$

Third, as C fails to make a trade payment at $t-7$, C's last payment date is $t-8$, and therefore

$$\delta'_C = 8$$

And finally since D stops its trade payments at $t-2$

$$\delta'_D = 3$$

Summary and Comparison of Timelines

1. Classical+/Classical-/Aggressive/Conservative Parametrizations: Using the notation above, the Aggressive and the Conservative scenarios are presented in the table below. For reference, the Classical+ and the Classical- versions of the classical model are presented in the table as well. Note that the 10 BD assumption of the classical MPoR lies between the two calibration choices proposed, and is closer to the Aggressive scenario.
2. MPoR Periods for CSA's with Daily Re-margining:

Parameter	Conservative	Aggressive	Classical+	Classical-
δ_C	15 BD	7 BD	10 BD	10 BD

δ_D	$9\ BD$	$6\ BD$	$10\ BD$	$10\ BD$
$\delta_C{}'$	$8\ BD$	$4\ BD$	$0\ BD$	$10\ BD$
$\delta_D{}'$	$3\ BD$	$4\ BD$	$0\ BD$	$10\ BD$

3. Caveats over Aggressive/Conservative Parameters: The Aggressive and the Conservative parameter choices represent two opposite types of dealer-client relationships, and may also be used as two limit scenarios for materiality and model risk analysis. Of course, the best approach would always be to set the model parameters based on prudent analysis of the firm's historical default resolution timelines, to the extent that it is practically feasible. The model could also conceivably treat the various time lags as random variables to be simulated as part of the exposure computations; yet it is debatable whether increasing the number of model parameters this way is warranted in practice.

Unpaid Margin Flows and Margin Flow Gap

1. Margin and Trade Flow Gaps: To formulate the model in more precise mathematical terms, this section returns to

$$E(t) = [V(t) + UTF(t) - K(t)]^+$$

and considers how to draw the analysis of the previous sections to reasonably specify both the collateral amount $K(t)$ as well as the value $UTF(t)$ of net unpaid cash flows.
2. Client Last Margin Posting Date: As with the classical model, it is assumed that the MPoR starts at time

$$t_C = t - \delta_C$$

the portfolio observation date associated with the last regular collateral posting by C. Recall that the classical model further assumes that D will stop posting collateral simultaneously with C so that

$$K(t) = c(t_C)$$

where $c(t_C)$ denotes the CSA prescribed collateral support amount calculated from the market data observed at time t_C. This is to be compared with

$$E(t) = [V(t) - K(t)]^+$$

$$K(t) = c(t - \delta)$$

3. <u>Definition of the Margin Flow Gap</u>: In contrast to

$$K(t) = c(t_C)$$

this model assumes that D will continue posting and returning collateral to C for all contractual margin observations dates t_i whenever required by the CSA, after

$$t_C = t - \delta_C$$

and up to and including the observation date

$$t_D = t - \delta_D$$

The presence of an observation period of non-zero length for which D is posting and returning collateral but C is not is referred to as *margin flow gap*.

4. <u>Choosing the Collateral Computation Date</u>: Here, it is always expected that

$$t_D \geq t_C$$

which therefore in effect assumes the possibility of a time interval $(t_D, t_C]$ where only D honors its margin requirements. In this interval, D can match its contractually stipulated amounts $c(t_i)$ only when they involve transfers from D to C. This asymmetry results in D holding at time t the *smallest* collateral computed in the observation interval $[t_C, t_D]$ i.e.

$$K(t) = \underset{t_i \in [t_C, t_D]}{min} c(t_i)$$

5. <u>Implications of the Collateral Date Choice</u>: The *worst case* form of the above provides a less optimistic view on available collateral than on classical modeling, resulting in larger exposure whenever there are multiple collateral observation dates in $[t_C, t_D]$ – it is assumed that one of the observation dates t_i always coincides with the start of the MPoR t_C. All other things being equal, the difference in exposure relative to the classical model will increase with $\delta_C - \delta_D$. If $\delta_C - \delta_D$ is kept constant, the difference will increase with more frequent re-margining. Note that

$$K(t) = \underset{t_i \in [t_C, t_D]}{min} c(t_i)$$

matches

$$K(t) = c(t_C)$$

when

$$\delta_C = \delta_D$$

Unpaid Trade Flows and Trade Flow Gap

1. Origin of the Trade Flow Gap: According to the assumptions in the earlier sections, the last date when C is still paying the trade flows is

$$t_C' = t - \delta_C'$$

and the last date when D is still trade flows is

$$t_D' = t - \delta_D' \geq t_C'$$

The period when D is still paying trade flows while C does not is referred to as the *trade flow gap*.

2. Projecting the Unpaid Trade Flow Value: The value of the net trade flows unpaid by the termination date t can be expressed using the notation established so far as

$$\begin{aligned} UTF(t) &= TF_{C \to D}(t;\ (t_C', t]) + TF_{D \to C}(t;\ (t_D', t]) \\ &= TF_{C \to D}(t;\ (t_C', t_D']) + TF_{NET}(t;\ (t_D', t]) \end{aligned}$$

where an arrow indicates the direction of the trade flows and $C \to D$ ($D \to C$) trade flows have positive (negative) sign.

3. Same Date Dealer/Client Flows: In calculating the above equation, care needs to be taken on how the trade flows are aggregated and accrued to the termination date t. Cash flows of opposite direction scheduled to be paid in the same currency on the same date (for instance, the two legs of an ordinary single-currency interest-rate swap) in the period $(t_D', t]$ are aggregated (netted) at the cash flow date, therefore only the aggregated amount – their

difference – enters in to the above equation. The aggregated amount of the missed cash flows should be accrued to time t at the interest rate of the currency in question, and then converted to D's domestic currency.

4. Different Currency Dealer/Client Flows: Cash flows in opposite direction scheduled to be paid in *different* currencies on the same date (for instance, the two legs of a cross-currency interest rate swap) are *not* netted at the cash flow date. The missed cash flow amounts in each currency should be accrued to time t at the relevant interest rates, and then converted to D's domestic currency.
5. Physically Settled Dealer/Client Flows: The value of each asset flow (for instance, a swap that would result from exercising a physically settled swaption) should be obtained through pricing at time t of the undelivered asset in D's domestic currency. Generally, asset flows are not aggregated.
6. Collateral available at Termination Date: To analyze the impact of these assumptions on the expression for $UTF(t)$ above, consider for simplicity a zero-threshold margin agreement with no MTA/rounding. Then, from

$$K(t) = \min_{t_i \in [t_C, t_D]} c(t_i)$$

the collateral available to D at the termination date can be written as

$$K(t) = V(t_{COL})$$

$$t_{COL} = \min_{t_i \in [t_C, t_D]} V(t_i)$$

7. Corresponding CSA Implied Client Exposure: Substituting

$$UTF(t) = TF_{C \rightarrow D}(t;\ (t_C{}', t_D{}']) + TF_{NET}(t;\ (t_D{}', t])$$

and

$$K(t) = V(t_{COL})$$

$$t_{COL} = \min_{t_i \in [t_C, t_D]} V(t_i)$$

into

$$E(t) = [V(t) + UTF(t) - K(t)]^+$$

yields, for the simple CSA considered

$$E(t) = [V(t) - V(t_{COL}) + TF_{C \to D}(t;\ (t_C', t_D']) + TF_{NET}(t;\ (t_D', t])]^+$$

8. CSA Implied Client Exposure Components: $E(t)$ above implies that trading flows from D to C can occur within the MPoR. Thus trade flows have the potential to generate large spikes in the exposure profiles – especially in the presence of a trade flow gap where only D pays trade flows. To see this, the exposure components of $E(t)$ can be further drilled down as follows. First, ignoring the minor discounting effects inside the MPoR, the portfolio value at time t_{COL} can be represented as a sum of the portfolio's forward value V_F to time t and the value of all the trade flows taking place after t_{COL} and up to and including t:

$$V(t_{COL}) = V_F(t_{COL}; t) + TF_{NET}(t_{COL};\ (t_{COL}, t])$$

9. Trade Flow Post Collateral Date: This may be further re-written as

$$\begin{aligned} TF_{NET}(t_{COL};\ (t_{COL}, t]) &\\ = TF_{NET}(t_{COL};\ (t_{COL}, t_C']) &+ TF_{C \to D}(t_{COL};\ (t_C', t_D']) \\ + TF_{D \to C}(t_{COL};\ (t_C', t_D']) &+ TF_{NET}(t_{COL};\ (t_D', t]) \end{aligned}$$

which, together with

$$V(t_{COL}) = V_F(t_{COL}; t) + TF_{NET}(t_{COL};\ (t_{COL}, t])$$

allows re-stating

$$E(t) = [V(t) - V(t_{COL}) + TF_{C\to D}(t;\ (t_C{}', t_D{}']) + TF_{NET}(t;\ (t_D{}', t])]^+$$

in the following form:

$$\begin{aligned} E(t) = \{V(t) - V_F(t_{COL}; t) + TF_{C\to D}(t;\ (t_C{}', t_D{}']) - TF_{C\to D}(t_{COL};\ (t_C{}', t_D{}']) \\ + TF_{NET}(t;\ (t_D{}', t]) - TF_{NET}(t_{COL};\ (t_D{}', t]) - TF_{NET}(t_{COL};\ (t_{COL}, t_C{}']) \\ - TF_{D\to C}(t_{COL};\ (t_C{}', t_D{}'])\}^+ \end{aligned}$$

10. <u>The Market Driven Portfolio Change $t_{COL} \to t$</u>: The terms in

$$E(t) = [V(t) - V(t_{COL}) + TF_{C\to D}(t;\ (t_C{}', t_D{}']) + TF_{NET}(t;\ (t_D{}', t])]^+$$

have been re-arranged into five separate components, corresponding to the five different contributions to the exposures. The first is

$$V(t) - V_F(t_{COL}; t)$$

- the change of the portfolio forward value to time t driven by the change in the market factors between t_{COL} and t. This term is driven by the volatility of the market factors between t_{COL} and t; it produces no spikes in the expected exposure profile.

11. <u>The Market Driven Trade Flow $t_C{}' \to t_D{}'$</u>:

$$TF_{C\to D}(t;\ (t_C{}', t_D{}']) - TF_{C\to D}(t_{COL};\ (t_C{}', t_D{}'])$$

represents the change of the value of the trade flows scheduled to be paid – but actually unpaid – by C in the interval $(t_C', t_D']$ resulting from the change of the market factors between t_{COL} and t. This term is driven by the volatility of the market factors between t_{COL} and t; it produces no spikes in the expected exposure profiles.

12. <u>The Market Driven Trade Flow $t_D' \rightarrow t$</u>:

$$TF_{NET}(t;\ (t_D', t]) - TF_{NET}(t_{COL};\ (t_D', t])$$

is the change of the value of the net trade flows between C and D scheduled to be paid – but actually unpaid – in the interval $(t_D', t]$ resulting from the change in the market factors between t_{COL} and t. This term likewise produces no spikes in the expected exposure profile.

13. <u>Net Trade Flow between $t_{COL} \rightarrow t_C'$</u>:

$$-TF_{NET}(t_{COL};\ (t_{COL}, t_C'])$$

is the negative value of the net trade flow between C and D scheduled to be paid – and actually paid – in the interval $(t_{COL}, t_C']$. Paths where D is the net payer – so that TF is negative – contribute to the upward spikes in the EE profile.

14. <u>Dealer Trade Flow across $t_C' \rightarrow t_D'$</u>: The negative value of the trade flows scheduled to be paid – and actually paid – by C to D in the interval $(t_C', t_D']$. Whenever such trade flows are present D is always the payer, leading to upward spikes in the EE profile. Furthermore, in some cases, the spikes arising from this term can be of extreme magnitude, e.g., the scheduled notional exchange in a cross-currency swap where D pays the full notional, but receives nothing.

Numerical Examples

1. Illustrative Exposures and CVA Magnitudes: To gain intuition for the model, this section presents exposure profiles and CVA metrics for several trade and portfolio examples, using both the Aggressive and the Conservative calibrations. The focus is on ordinary and cross-currency swaps, as these instruments are the primary sources of exposures in most dealers. For all the numerical examples, the stochastic yield curves are driven by one-factor Hull-White model; for cross-currency swap examples, the FX rate is assumed to follow a Black Scholes model.
2. Single Swap Classical/Conservative Exposures: Andersen, Pykhtin, and Sokol (2017) first examine how the model differs from the classical exposure approach. They use Monte Carlo simulation on a USD 10 MM 1-year par-valued vanilla interest rate swap to compare exposures of the Conservative calibration with those computed against the Classical+ and the Classical- models – see the Table above. To make comparisons more meaningful, they override the default setting of 10 business days for the Classical model, and instead set it equal to 15 business days – the length of the MPoR for the Conservative calibration.
3. Single Swap - Classical Exposure Estimations: As they note in their figures, the Classical- calibration is, of course, the least conservative setting, as it ignores both the effect of trade flows and that of the margin asymmetry. The Classical+ calibration tracks the Classical- calibration at most times, but contains noticeable spikes around the last 3 cash flow dates. No spikes occur in their figure on the first quarterly cash flow date, as they assume that the floating rate is fixed at the fixed rate, making the net cash flow zero in all scenarios.
4. Single Swap Conservative Model - MPoR End: The conservative calibration results also contain spikes around the cash flow dates, although these differ from the Classical+ calibration in several ways. First, the Conservative calibration always recognizes that there will be a part towards the end of the MPoR (after time t_D') where C and D will both have stopped paying margin and coupons; as a result, the spikes of the Conservative calibration start later (here: 3 business days) than those of the Classical+ calibration.
5. Single Swap Conservative Model - Trade Flows: Second, the initial part of the spike – in the period from t_C' to t_D' - is substantially higher for the Conservative calibration due to the assumption of only D paying cash flows in this sub-period. The remainder of the spike is comparable in height to the Classical+ spike.

6. Single Swap - Classical vs. Conservative: *Between* spikes the Conservative calibration produces higher exposures than both the Classical- and the Classical+ methods – by around 40%. This is, of course, a consequence of the *worst case* margin asymmetry mechanism in

$$K(t) = \min_{t_i \in [t_C, t_D]} c(t_i)$$

the effect of which will grow with the diffusion volatility of the rate process. Of course the last coupon period again has no exposure between spikes, since the volatility of swap prices vanishes after the last coupon rate fixing.

7. Single Swap: Aggressive Calibration Exposures: Comparison the Aggressive calibration to the Classical+ and the Classical- calibrations are qualitatively similar. A detailed comparison is therefore skipped here, but Andersen, Pykhtin, and Sokol note that the pick-up in exposure from margin asymmetry falls to about 15%, rather than the 40% observed for the Conservative calibration – a result of the fact that, for Aggressive calibration, the *worst case* margin result is established over much fewer days. Andersen, Pykhtin, and Sokol (2017) demonstrate the comparison of the exposure profiles for the Aggressive and the Conservative calibrations in their graphs; as expected, the Conservative calibration leads to both bigger and wider exposure spikes, as well as to higher exposure levels between spikes.
8. Single Swap - Maturity/Coupon Effect: While instructive, the 1-year vanilla swap example is quiet benign exposure-wise; not only is the instrument very short-dated, it also allows for netting of coupons on trade-flow dates, thereby reducing the effects of trade flow spikes. Andersen, Pykhtin, and Sokol (2017) relax both effects by increasing the maturity of the swap, and by making the fixed and the floating legs pay on different schedules – and illustrate the exposure results in a separate figure.
9. Single Swap - Coupon Payment Mismatch: The upward exposure spikes occur twice per year, whenever the dealer must make a semi-annual fixed payment. On the dates when the counterparty makes a quarterly floating payment that is not accompanied by a fixed payment by the dealer, a narrow *downward* spike emerges, due to the delay in transferring the coupon back to the counterparty through the margin mechanism.

10. Single Swap - Impact of Maturity: The exposure between spikes is also much larger, a consequence of the higher volatility of the 10-year swap compared to the 1-year swap. Of course, as the swap nears its maturity, its duration and volatility die out, so the non-spike exposure profile predictably gets pulled to zero at the 10-year date. Also, as predicted, the Aggressive calibration produces much lower exposures than the Conservative calibration, by nearly a factor of 2.
11. Single Cross-Currency Swap - Herstatt Risk: A more extreme form of trade flow spikes will occur for cross-currency swaps, where neither the coupon nor the final payment can be netted. The notional payment, in particular, can induce a very significant payment exposure spike (the Herstatt Risk), whenever the exposure model allows for a trade flow gap. To recall, the Conservative calibration has a trade flow gap, but the Aggressive calibration does not.
12. Single Cross-Currency Swap - Coupon Mismatch: As confirmed by Andersen, Pykhtin, and Sokol (2017), the exposure for a conservative calibration has a very large spike that is not present in the Aggressive calibration. Like Conservative calibration, Aggressive calibration will, of course, still produce spikes at the cash flow dates, due to margin effects.
13. Single Cross-Currency Swap - Principal Mismatch: As a consequence, the principal exchange is likely fa away from break-even, resulting in a large exposure spike at maturity. Although smaller than for the Conservative calibration, the spike at maturity is also present for the Aggressive calibration; while both C and D pay the principal exchange, C does not make the margin transfer for the balance of the principal payments.

Portfolio Results

1. Single Swap Portfolio: Setup Overview: For individual trades, the presence of localized spikes in the exposure profiles may ultimately have a relatively modest impact on the credit risk metrics, such as the CVA – after all, the likelihood of the counterparty default in a

narrow time interval around quarterly or semi-annual cash flow event is typically low. For a *portfolio* of swaps, however, the spikes will add up and affect the net exposure profile nearly everywhere.

2. Single Swap Portfolio - Draw Algorithm: To illustrate this, Andersen, Pykhtin, and Sokol (2017) picked 50 interest rate swaps with quarterly floating rate payments and semi-annual fixed rate payments of 2%. The terms of the swap were randomized as follows:
    a. Notionals of the swap are sampled uniformly on the interval from 0 to USD 1 MM.
    b. Duration of the fixed leg payments – payer or receiver – is random.
    c. Start date of each swap is subject to a random offset to avoid complete MPoR overlaps.
    d. Swap maturities are scaled uniformly on the interval from 1 to 10 years.
3. Single Swap Portfolio: Aggressive vs Conservative: They also illustrate the resulting exposure profile in a separate figure. Both the Conservative profile, and to a lesser extent, the Aggressive profile include frequent spikes around the trade-flow times above the *baseline* exposure level. As seen in the next section, these spikes make a significant contribution to the CVA metrics. As before, the exposure under the Conservative calibration is twice as large as that under the Aggressive calibration.
4. XCCY Swaps Portfolios Generation Algorithm: To repeat the portfolio results with a cross-currency swap, Andersen, Pykhtin, and Sokol (2017) constructed a 50 deal portfolio by randomization, using the following rules.
    a. EUR notionals are sampled uniformly in the interval from 0 to USD 10 MM
    b. USD notionals are 1.5 times the EUR notionals
    c. EUR leg has a fixed semi-annual coupon of 3%, and the USD leg floating quarterly coupon
    d. Direction of the fixed leg payments (payer or receiver) is random
    e. Start date of each swap is subject to a random offset to avoid complete MPoR overlaps
    f. Swap maturities are sampled uniformly in the interval from 1 to 10 years
5. XCCY Swap Portfolio: Conservative vs. Aggressive: As shown in their figure for an expected exposure for a 10Y cross-currency swap, Andersen, Pykhtin, and Sokol (2017)

generated the swaps within the portfolio such that the principal exchanged and the fixed coupon are not at-the-money, to mimic a typical situation corresponding to a portfolio of seasoned trades. As demonstrated in another figure for the expected exposure of the cross-currency swap portfolio, the exposure for the conservative calibration is, as expected, dominated by a series of Herstatt risk spikes, one per swap in the portfolio.

CVA Results

1. CVA Computation from Expected Exposure: As mentioned earlier, a common use of the expected exposure results is the computation of the CVA. Under suitable assumptions, D's unilateral CVA may be computed from the expected exposure (EE) profile as

$$CVA = (1 - R) \int_{0}^{\infty} P(u + \delta_C{}')EE(u + \delta_C{}')dX(u)$$

   where R is the recovery rate, $P(t)$ is the time-0 discount factor to time t, and $X(t)$ is the time-0 survival probability of C to time t. As discussed before, the exposure profile here is offset by $\delta_C{}'$ to properly align it with the default events.
2. CVA Metrics Dealer/Client Settings: The CVA metric serves as a convenient condensation of the exposure profiles of the previous two sections into single numbers, and Andersen, Pykhtin, and Sokol (2017) tabulate the CVA numbers for the corresponding instruments/portfolios. The CVA integral was discretized using a daily grid, assumed at

$$R = 40\%$$

   and the forward default intensity is left constant at 2.5% such that

$$X(t) = e^{-0.025t}$$

For reference, the table also includes the results of the Classical method, with the MPoR length equal to both that of the Aggressive calibration (7 BD) and the Conservative Calibration (15 BD).

3. CVA Comparison - Classical/Conservative/Aggressive: Their results confirm what was seen earlier. For instance, the CVA for the Aggressive calibration is 50% to 70% smaller than that for the Conservative calibration. In Addition, the CVA of the Conservative calibration is between 50% and 100% larger than that of the Classical+ calibration – at similar MPoR – which in turn is larger than the CVA for the Classical- calibration by around 5% to 25%. Not surprisingly the CVA results for the XCCY portfolio are particularly high in the Conservative calibration due to the Herstatt risk.

Improvement of the Computation Times

1. Computation Speed-Up using Coarse Grids: In exposure calculations for realistic portfolios, horizons can be very long, often exceeding 30 years. For such lengthy horizons, brute-force Monte-Carlo exposures on a daily, or even weekly, time grid will often be prohibitively slow. It is therefore common to use daily simulation steps only for the earliest parts of the exposure profile (e.g., the first month), and then gradually increase the step-length over time to monthly or quarterly, in order to keep the total number of simulation dates manageable. Unfortunately, such a coarsening of the time-grid will inevitably fail to capture both the *worst case* margin effect and the trade spikes that are key to the exposure model.
2. Coarse Grid Lookback Analysis: The next two sections look at ways to capture exposure without having to resort to brute-force daily simulation. A common speed-up technique for the Classical model – the Coarse Grid Look-back Model – is first reviewed, and its

shortcomings and pitfalls are highlighted. An improved practical technique based on Brownian Bridge is the proposed.

The Coarse Grid Lookback Method and its Shortcomings

1. Layout of the Coarse Grid: Assume that the portfolio is not computed daily, but instead on a coarse grid $\{s_j\}$ where j runs from 1 to J. This section uses s rather than t to distinguish the model grid from the daily margin calculation grid.

   Points on the Coarse Grid: In the classical model, the collateral depends only on the portfolio value at the start and at the end of the MPoR, i.e., $s_j - \delta$ and s_j, as is seen from

$$E(t) = [V(t) - K(t)]^{+}$$

$$K(t) = c(t - \delta)$$

$$K(t) = [V(t - \delta) - h_C(t - \delta)]^{+} - [-V(t - \delta) - h_D(t - \delta)]^{+}$$

   where MPoR is usually around

$$\delta = 10\ BD$$

for CSA's with daily margining. To achieve acceptable computational performance, the time step of the coarse model grid $s_j - s_{j-1}$ must be significantly greater than the length of the MPoR. This, however, would preclude one from establishing a portfolio value at $s_j - \delta$.

2. Introducing a Lookback Node: The coarse grid lookback method deals with this issue by simply adding a second *lookback* time point $s_j - \delta$ to all *primary* measurement times s_j, in effect replacing each node on the coarse model grid by a pair of closely spaced nodes. For each simulated portfolio path, the portfolio value at the lookback point is then used to determine the collateral available at the corresponding primary time point.
3. Slowdown due to the Lookback: The Coarse Grid Lookback Scheme causes, at worst, a factor of $\times$ 2 slowdown relative to valuing the portfolio once per node of the Coarse model grid. If even a $\times$ 2 performance loss is not acceptable, a Brownian Bridge constructed between the primary coarse grid nodes can be used to interpolate the value of the portfolio at each lookback point, see, for example, Pykhtin (2009). Notice that the use of the Brownian Bridge for this purpose should not be confused with its use in the next section.
4. Shortcoming of the Model: The Coarse Grid Lookback method is a common way of addressing the mismatch between the long time-step of the coarse model grid and the much shorter MPoR. Similar to the commonly used models of uncollateralized exposure, the method produces accurate – with respect to the underlying assumptions of the Classical model – exposure numbers at the coarse grid time points, but provides no information on the exposure between the grid points.
5. Collateralized vs. Uncollateralized Grid Exposures: For uncollateralized positions, the exposure profiles are reasonably smooth, so one can safely interpolate between the grid points for calculating integral quantities, such as the CVA. In collateralized case, however, one cannot rely on such interpolations because the true exposures, as has been seen above, is likely to have spikes and jumps between the grid points. The Coarse Grid Lookback method has no means to determining the position or the magnitude of the irregularities between the grid points, and thus, is not suitable for CVA or capital calculations.
6. Classical+ Model - Coarse Grid Impact: To briefly expand on this, consider the Classical+ version of the classical model. Here it is assumed that all trade flows are paid within the

MPoR, where, as shown before, trade flows often result in exposure spikes. Exposure profiles computed from daily time steps would consequently show spikes from all trade flows until the maturity of the portfolio.

7. Trade Flows outside the Simulated MPoR's: In contrast, in a typical implementation with sparsely spaced MPoR's, only trade flows that happen to be within a sparsely simulated MPoR's may result in spikes; the exposure profile would then miss all other flows.
8. Simulation Calendar Impact on Exposure: Furthermore, as the location of the simulation point will likely change with the advancement in the calendar time, trade flows would move in and out of the simulated MPoR's, and the exposure profile one report on any given day may very well differ significantly from those that were reported the day before. This in turn causes CVA or risk capital to exhibit significant, and entirely spurious, oscillations.
9. Classical- Model - Coarse Grid Impact: While the Classical- exposure model does not exhibit outright spikes, its exposure profiles still exhibit jumps around significant trade flows. The classical coarse grained implementation would not be able to resolve the position of these jumps, instead only showing the conservative jumps between two exposure measurement points often separated by many months. This creates another source of instability, present in both the Classical- and the Classical+ versions of the classical model.
10. Illustration using the Forward CVA: To illustrate the effects described above, Andersen, Pykhtin, and Sokol (2017) define the concept of time t forward CVA, denoted CVA_t, obtained by
    a. Changing the lower integration limit in

$$CVA = (1 - R) \int_0^\infty P(u + \delta_C{}')EE(u + \delta_C{}')dX(u)$$

    from 0 to t, and
    b. Dividing the result by $P(t)X(t)$.

    Using the same portfolio of 50 EUR-USD cross-currency swaps, they show the t-dependence of CVA_t on a daily grid to portfolio maturity.

11. Spurious Oscillations from Moving Windows: As CVA is an integral of exposures, spikes in exposure profile profiles should result in jumps rather than oscillations in CVA_t. However, when one of the Coarse Grid Lookback method's sparsely located *MPoR window* moves past a large trade flow, the contribution to the CVA temporarily increases only to drop back when the window moves past the large trade flow. As illustrated by Andersen, Pykhtin, and Sokol (2017), such oscillations are spurious and their presence is highly unattractive when CVA is computed and reported as part of daily P&L.

Brownian Bridge Method

1. Brute Force Portfolio Value Simulation: Overcoming the deficiencies outlined in the previous section is, unfortunately, prohibitively expensive for large portfolios, mostly due to the expense of repricing the entire portfolio at each simulation path and each observation date.
2. Daily Simulation of Risk Factors: On the other hand, merely simulating the risk factors at a daily resolution is generally feasible, as the number of the simulated risk factors is typically relatively small (i.e., several hundred) and the equations driving the risk factor dynamics are usually simple.
3. Generation of Daily Trade Flows: Furthermore, having produced risk factors on a daily grid, one can normally also produce all realized trade flows along each path because trade flows, unlike trade prices, are usually simple functions of the realized risk factors.
4. Risk Factors under Daily Resolution: Based on these observations, Andersen, Pykhtin, and Sokol (2017) propose the following algorithm for generating paths of portfolio values and trade flows on a daily time grid. First, simulate paths of market risk factors with daily resolution.
5. Trade Flow under Daily Resolution: For each path m, use the simulated market risk factors to calculate trade flows on the path with daily resolution.

6. Coarse Grid Path Portfolio Valuation: For each path m and each coarse portfolio valuation time point s_j $(j = 1, \cdots, J)$ use the simulated risk factors to calculate portfolio value on the path $V_m(s_j)$
7. Trade Flow Adjusted Forward Value: For each path m and each time point s_j use the trade flows realized on the path between times s_{j-1} and s_j to calculate the *forward* to s_j portfolio value $V_m'(s_{j-1};\ s_j)$:

$$V_m'(s_{j-1};\ s_j) = V_m(s_{j-1}) - TF_{m,NET}(s_j;\ (s_{j-1}, s_j])$$

   Note that $V_m'(s_{j-1};\ s_j)$ is not a true forward value because the realized trade flows are subtracted from the s_{j-1} portfolio value rather than the true forward value being calculated at time s_{j-1}.
8. Portfolio Value Local Variance Estimation: For each path m and each portfolio measurement time point s_j compute the local variance ${\sigma_m}^2(t_{j-1})$ for the portfolio value *diffusion* $V_m(s_j) - V_m'(s_{j-1};\ s_j)$ via a kernel regression estimator – e.g., the Nadaraya-Watson Gaussian kernel estimator (Nadaraya (1964), Watson (1964)) conditional on the realized value of $V_m'(s_{j-1};\ s_j)$. The selection of bandwidth for the kernels is covered in, e.g., Jones, Marron, and Sheather (1996). In their numerical results, Andersen, Pykhtin, and Sokol (2017) use the *Silverman's Rule of Thumb* (Silverman (1986)).The term *diffusion* is used to indicate that the portfolio value change has been defined to avoid any discontinuities resulting from trade flows.
9. Brownian Bridge Local Interpolation Scheme: For each path m and each exposure measurement time point s_j, simulate an independent, daily sampled, Brownian Bridge process (see, for instance, Glasserman (2004)) that starts from the value $V_m'(s_{j-1};\ s_j)$ at time s_{j-1} and ends at the value $V_m(s_j)$ at time s_j. The volatility of the underlying Brownian motion should be set to $\sigma_m(s_{j-1})$.
10. Brownian Bridge Portfolio Value Approximation: For each path m and each exposure measurement time point s_j, the portfolio values for each time u of the daily grid in the

interval (s_{j-1}, s_j) are approximated from the simulated Brownian bridge $BB_m(u)$ by adding the trade flows realized along the path m between the times u and s_j:

$$V_{m,APPROX}(u) = BB_m(u) + TF_{m,NET}(s_j;\ (u, s_j])$$

11. Rational behind Brownian Bridge Methodology: In a nutshell, the algorithm above uses a Brownian bridge process to interpolate portfolio values from a coarse grid in a manner that ensures that intermediate trade flow events are handled accurately. The algorithm produces paths of portfolio values and trade flows in a daily time grid, wherefore exposure can be calculated as described earlier with daily resolution and overlapping MPoR's. Furthermore, daily sampling allows for further refinements of the proposed model by consistently incorporating thresholds, minimum transfer amount, and rounding.
12. Brownian Bridge Portfolio Weiner Increment: A key assumption made by the Brownian Bridge algorithm is that the portfolio value process within the interpolation interval is a combination of an approximately normal *diffusion* overlaid by trade flows. For Weiner process models without risk factor jumps, this approximation is accurate in the limit of infinitesimal interpolation interval, and is often a satisfactory approximation for monthly or even quarterly interpolation steps.
13. Brownian Bridge Approximation Error #1: Nevertheless, the presence of trade flows that depend on the values of the risk factors between the end points introduces two types of errors. Suppose that there is a trade flow at an end point that depends on the risk factor value at the date when it is paid. The independence of the Brownian Bridge process from the risk factor processes that drive that trade flow would result in an error in the expected exposure profile around the trade flow date. This error is largest for trade flows in the middle of the interpolation interval and disappears for trade flows near the ends of the interval.
14. Brownian Bridge Approximation Error #2: Suppose that there is a trade flow that occurs at the end point of an interpolation interval, but whose values depend entirely on the realization of the risk factor within the interpolation interval. A typical example would be a vanilla interest rate swap where the floating leg payment being paid at the end of the interpolation interval depends on the interest rate on a date within the interval. Even in the absence of a

trade flow within the interpolation interval, the volatility of the swap value drops at the floating rate fixing date as some of the uncertainty is resolved. Thus the *true* swap value process has two volatility values; a higher value before the rate fixing date and a lower value after the rate fixing date. In contrast the approximation algorithm assumes a single value of volatility obtained via kernel regression between the end points. Similar to the de-correlation error discussed above, the error resulting from this volatility mismatch is largest for fixing dates in the middle of the interpolation interval and disappears for fixing dates near the end points.

15. <u>Trade Flow at Mid-Interval</u>: To illustrate the two errors above, Andersen, Pykhtin, and Sokol (2017) compute the expected exposure profile for a one year interest rate swap when a monthly grid for full valuation is situated so that the payments/fixing dates sit roughly in the middle of the interpolation interval, thus maximizing the error of the Brownian Bridge algorithm.
16. <u>Unbiased Nature of the Error</u>: While there are, as expected, some error around the trade flow dates, they are acceptable in magnitude and overall unbiased, in the sense that the over-estimation of the exposure is about as frequent as the under-estimation of the exposure. For, say, CVA purposes, the Brownian Bridge results would therefore be quite accurate.
17. <u>Trade Flows at Interval End</u>: Andersen, Pykhtin, and Sokol (2017) also compute the expected exposure profiles when the monthly valuation points are aligned with the rate fixing/payment dates. In this case, Brownian Bridge approximation is nearly exact.
18. <u>Choice of Valuation Grid Location</u>: Of course, in practice such alignment is only possible for a single trade or a small netting set, and not for large portfolios where trade flows will occur daily. Yet, even for large netting sets the calculation accuracy will improve if the interpolation pillars are aligned with the largest trade flows (e.g., principal exchange dates for the largest notional amounts). In practice, errors can be typically expected to be somewhere between the two extremes discussed above.
19. <u>Performance Gains from Brownian Bridge</u>: While the exact speed up provided by the Brownian Bridge method depends on the implementation, for most portfolios the overhead of building the Brownian Bridge at a daily resolution is negligible compared to computing the exposure on the model's coarse grid.

20. Comparison with Coarse Grid Lookback: In this case, the computational effort of the daily Brownian Bridge method is about half the computational effort of the Coarse Grid Lookback method, as the former does not require adding a *lookback* point to each of the primary coarse grids. Thus the Brownian Bridge is both faster and significantly more accurate than the Standard Coarse Grid Lookback method.

Initial Margin

1. Role of IM: Extra Protection: The posting of initial margin (IM), in addition to the regular variation margin collateral (VM), provides dealers with a mechanism to gain additional default protection. The practice of posting IM has been around for many years, typically with IM being computed on trade inception on a trade level basis.
2. Modeling Static Initial Margin Exposure: This type of IM is entirely deterministic and normally either stays fixed over the lifetime of a trade or amortizes down according to a pre-specified schedule. As a consequence, modeling the impact on the exposure is trivial; for the exposure points of interest all trade level IM amounts are summed across the netting set and the total – which is the same for all paths – is subtracted on the portfolio value from each path.
3. Dynamically Refreshed Initial Margin (DIM): A more interesting type of IM is dynamically refreshed to cover portfolio-level close-out risk at some high percentile, often 99%. This type of margin is routinely applied by Clearinghouses (CCPs) and by margin lenders, and will also soon be required by regulators for inter-dealer OTC transactions.
4. BCBS IOSCO Initial Margin Rules: In particular, in 2015 BCBS and IOSCO issued a final framework on margin requirements (BCBS and IOSCO (2015)) under which two covered entities that are counterparties in non-centrally cleared derivatives are required to:
    a. Exchange VM under a zero threshold margin agreement, and
    b. Post IM to each other without netting the amounts.

Covered entities include all financial firms and systematically important non-financial firms. Central banks and sovereigns are not covered entities.

5. Third Party Management of IM: IM must be held in a default remote way, e.g., by a custodian, so that IM posted by the counter-party should be immediately available to it should the other counter-party default.
6. Internal Model/Standardized Schedule IM: Under the BCBS and IOSCO rules, regulatory VM can be calculated by an internal model or by lookup in a standardized schedule.
7. Internal Models Based IM Calculation: If an internal model is used, the calculation must be made at the netting set level as the value-at-risk at the 99% confidence level. The horizon used in this calculation equals 10 business days for daily exchange of VM or 9 business days plus a re-margining period for less frequent exchange of VM.
8. Denial of Cross-Asset Netting: Diversification across distinct asset classes is not recognized, and the IM internal model must be calibrated to a period of stress of each of the asset classes.
9. Handling Adjustments to the IM: The required levels of IM are changed as the cash flows are paid, new trades are booked, or markets move. To accommodate this, dealers would call for more IM or return the excess IM.
10. Complexities Associated with the IM Estimation: For trades done with CCPs or under the new BCBS-IOSCO rules, one must find a way to estimate the future IM requirements for each simulated path. No matter how simple the IM VaR model is, it will likely be difficult to perform such calculations in practice if one wants to incorporate all the restrictions and twists of the IM rules; stress calibration, limited diversification allowance, and, for CCP's, add-ons for credit downgrades and concentration risk.
11. Estimating Simplified Version of IM: However, it is possible to utilize the model in this chapter to calculate the counter-party exposures if one ignores these complications. Note that ignoring such complications is conservative, as it will always lead to a *lower* of IM, and therefore, to a *higher* level of exposure.
12. t_C as IM Delivery Date: To calculate the exposure at time t the assumption here is that the last observation date for which C would deliver VM to D is

$$t_C = t - \delta_C$$

It is reasonable to assume that this date is also the last date at which C would deliver IM to a custodian.

13. Simplified IM Mechanics Timeline: To simplify modeling, it is assumed that the custodian would not return any amount to C for observation dates after $t - \delta_C$. Thus, to calculate exposure at time t, IM on a path has to be estimated from the dynamics of the exposure model as of time $t - \delta_C$
14. t_C IM Estimate using Gaussian Portfolio Evaluation: Assuming, as is common in practice, that the portfolio values are locally Gaussian, it suffices to know the local volatility for the portfolio value for the period $[t - \delta_C, t]$ estimated at $t - \delta_C$. Denoting the IM horizon by δ_{IM} and the local volatility of the portfolio value at time u on path m via $\sigma_m(u)$, the IM available to D at the ETD date t on path m is given by

$$IM_m(t - \delta) = \sigma_m(t - \delta)\sqrt{\delta_{IM}}\Phi^{-1}(q)$$

where q is a confidence level – often 99% - and $\Phi^{-1}(\cdot)$ is the inverse of the standard normal cumulative distribution function.

15. Kernel Regression Based Local Volatility: Estimating the local volatility can be done via kernel regression, as in the previous section. If the portfolio value is simulated at both $t - \delta_C$ and t, the kernel regression for could be run on the $P\&L\ V(t) - V(t - \delta_C) + TF_{NET}(t;\ (t - \delta_C, t])$ conditional on the realization of the portfolio value on path m at the beginning of the $MPoR\ V_m(t - \delta_C)$. If one does not calculate the portfolio value at the beginning of the $MPoR$ but uses the fast approximation outlined earlier instead, $\sigma_m(t - \delta_C)$ can be set equal to the local volatility estimated for the time interval that encloses the given $[t - \delta_C, t]$.
16. Brownian Bridge IM Plus VM: Thus, the Brownian Bridge framework can now produce not only the collateralized exposure under VM alone, but also a reasonable estimate of the collateralized exposure under a combination of VM and IM.
17. IM Timing and Transfer Mechanics: In calculating the IM, an important consideration is the timing and the mechanics of the adjustment to the IM when C misses a margin flow or a trade flow.

18. Assumption - IM Return to the Client: For instance, when a large trade reaches maturity, the portfolio VaR may be reduced, in which case, some of the IM posted by C must be refunded. The issue of whether this refund can be delayed due to an ongoing margin dispute is not yet fully resolved. To simplify the calculations, it is assumed that no part of the IM is returned to C during the $MPoR$.
19. 10Y OTC Swap VM + IM EE: To show some numerical results, Andersen, Pykhtin, and Sokol (2017) consider the individual trades and portfolios of the earlier section. They use the case of a 10Y vanilla swap for which they calculate the impact of IM on exposure.
20. Time Horizon IM Mechanism Impact: As is evident from their calculations, the IM mechanism strongly reduces exposures away from trade flows, but near the trade flow dates the protection gets progressively weaker and disappears almost completely for the last couple of trade flows. The reason for this uneven benefit of IM on this trade is that the 10 day VaR of this trade bears no direct relationship to the size of the trade flows that determines the exposure spikes in the model.
21. Inter/Intra-Spike IM Exposures: The variance of the $P\&L$ reduces as the swap approaches maturity so that the amount of IM on a given path is also reduced. However, the size of the trade flows is not reduced, but can actually grow with simulation time as larger and larger realizations of the floating rates are possible. Thus, when the swap approaches maturity the amount of IM is greatly reduced relative to the trade flows, so exposure spikes grow larger, while the *diffusion* component of the exposure becomes smaller.
22. Cross-Currency Swap VM + IM EE: Andersen, Pykhtin, and Sokol (2017) compute the impact of IM on the vanilla swap and the cross currency swap portfolios described earlier. As can be seen there the IM strongly suppresses the diffusion component of the portfolio value changes, but proves inadequate in reducing the spikes of exposure for both single currency, and especially, cross-currency portfolios.

Conclusion

1. Fully Collateralized Counterparty Exposure: Industry standard models for collateralized credit risk are well-known to produce non-negligible counterparty credit risk exposure, even under full collateralization of the variation margin. This exposure essentially arises due to the inevitable operational an legal delays (margin period of risk, or $MPoR$) that are *baked into* the workings of ISDA contracts that govern OTC trading.
2. Classical Implementations of the $MPoR$: In the most common industry implementation, the length of the $MPoR$, and precisely what transpires inside it, is, however, often treated in a highly stylized fashion. Often the $MPoR$ is set equal to 10 business days for little reason other than tradition, and often counter-parties are assumed to have oddly synchronized behavior inside the $MPoR$.
3. The Classical+ and Classical- Implementations: For instance, one common approach – denoted Classical- - assumes that the $MPoR$ and the trade flows by both counter-parties terminate at the beginning of the $MPoR$, but the trade flows terminate simultaneously at the end of the $MPoR$. Surprisingly, the Classical+ and the Classical- approaches continue to co-exists in the market, and neither has become the sole market practice.
4. Reasons for the Popularity of the Classical Models: One reason for this state of affairs is that the two models correspond to different choices for the trade-off between implementation complexity and the model stability; Classical+ is easier to implement but is prone to spurious spikes in the daily CVA P&L – as demonstrated earlier – whereas Classical- is more difficult to implement, but is free from such spikes.
5. Objectives of *Well* Designed Models: Ultimately, of course, a model should be selected not on the basis of the implementation ease or on the properties of a specific numerical technique, but on the basis of how well the model captures the legal and the behavioral aspects of the events around a counter-party default. The term *well* means different things in different applications of the exposure model. For regulatory capital purposes, prudence and conservatism may, for instance, be as important as outright precision.
6. Inadequacies of the Classical Approach: To this end, even a cursory analysis suggests that the perfect synchronicity of the Classical $\pm$ models cannot be supported in reality. For instance, due to the way the CSA works in practice, the non-defaulting party will need at

least 3 days after a portfolio valuation date to determine for sure that the corresponding margin payment by its counterparty will not be honored.

7. Detailed Analysis of $MPoR$ Timeline: This chapter carefully dissects the $MPoR$ into a full timeline around the default event, starting with the missed margin call and culminating at the post-default valuation date at which the termination value of the portfolio is established.
8. Model Parameters of the Timeline: For modeling purposes, the timeline of the model has been condensed into 4 model parameters, each specified as the number of days prior to the termination for the events below – in contrast the classical model has only one parameter – the full length of the $MPoR$.
9. Dealer/Client Trade/Margin Dates:
   a. The last market data measurement for which the margin flow is received (δ_C) and paid (δ_B) as prescribed.
   b. The last date when the defaulting party ($\delta_C{}'$) and the dealer ($\delta_D{}'$) make the trade payments as prescribed.
10. Legal Operational Basis behind the Parameters: As shown, each of these parameters has a legal and/or operational interpretation, enabling calibration from the CSA and from the operational setup of the dealer. Note that the proposed model parameterization includes the Classical+ and the Classical- models as the limit cases.
11. Aggressive CSA Timeline for the $MPoR$: For indicative purposes, two particular models are described – Aggressive and Conservative. For former assumes that the non-defaulting dealer always operates at an optimal operational level, and will enforce the legal provisions of the ISDA legal contracts as strictly as possible.
12. Conservative CSA for $MPoR$ Timeline: The latter will allow for some slack in the operations of the dealer, to allow for manual checks of calculations, legal reviews, *gaming* behavior of the counterparty, and so forth.
13. Aggressive/Conservative Timeline Exposure Comparison: The Conservative model setting obviously produces higher exposures than the Aggressive setting, for the following reasons.
    a. The Conservative setting has a longer overall length of $MPoR$
    b. The Conservative setting has a margin flow period where the dealer pays, but does not receive, margin flows

c. The Conservative setting, unlike the Aggressive setting, contains a trade flow gap period where the dealer pays, but does not receive, trade flows

14. Comparison of Margin Flow Exposures: In their numerical tests, Andersen, Pykhtin, and Sokol (2017) found that the first two factors of the Conservative setting to have approximately twice the exposure of both the Aggressive and the Classical $\pm$ settings away from the dates of large trade flows.
15. Comparison of Trade Flow Exposures: The last factor, i.e., the presence of a large trade flow gap may cause exposure spikes of extremely large magnitudes under the Conservative calibration. Despite the fairly short duration of these spikes, they may easily add up to very significant CVA contributions, especially for cross-currency trades with principal exchange – the Herstatt risk.
16. Past Realizations of Trade Flow Default: Credit losses due to trade flow gaps materialized in practice during the financial crisis – especially due to the Lehmann Brothers' default – so their incorporation into the model is both prudent and realistic.
17. Impracticality of Daily Simulation Schemes: Detailed tracking of the margin and the trade flow payments requires the stochastic modeling of the trade portfolio on a daily grid. As brute-force simulations at such a resolution are often impractically slow, it is important that numerical techniques be devised to speed up the calculations.
18. Kernel Regression on *Stripped* Cash Flows: While the focus of this chapter was mainly on establishing the fundamental principles for margin exposure, it also proposed an acceleration method based on kernel regression and applied Brownian Bridge to portfolio values *stripped* of cash flows.
19. Impact on Different Product Types: For ordinary and cross-currency swaps this chapter demonstrates that this method is both accurate and much faster than either brute-force simulation or standard acceleration techniques of the desired model. Further improvements in the acceleration techniques, and expansion of applicability into more exotic products, is an area of future research.
20. Initial Margin – Classical- Settings Impact: Under suitable assumptions, kernel regression may also be used to embed risk-based initial margin into exposure simulations. As

demonstrated in the final section of the chapter, initial margin at 99% exposure greatly succeeds in reducing bilateral exposure for the Classical- calibration.

21. Initial Margin Trade Flow Impact: For all other calibration choices, and especially for the Conservative setting, the reduction in counterparty exposure afforded by initial margin fails around the time of large trade flows, when a sudden change of exposure following an initial trade flow exceeds the initial margin level.
22. Initial Margin Maturity Decay Impact: Note that the already inadequate level of IM protection deteriorates around the maturity of the portfolio, where the local volatility of the trade flow value decreases, but the trade flows themselves do not. Overall accurate modeling of the events within the $MPoR$ becomes critically important for portfolios covered by dynamic IM.

References

- Andersen, L., M. Pykhtin, and A. Sokol (2017a): Re-thinking Margin Period of Risk **eSSRN**.
- Basel Committee on Banking Supervision (2015): Margin Requirements for Non-centrally Cleared Derivatives
- Bocker, K., and B. Schroder (2011): Issues Involved in Modeling Collateralized Exposure *Risk Minds Conference* **Geneva**.
- Brigo, D., A. Capponi, A. Pallavicini, and A. Papatheodorou (2011): Collateral Margining in Arbitrage-Free Counterparty Valuation Adjustment including Re-hypothecation and Netting **eSSRN**.
- Gibson, M. (2005): Measuring Counterparty Credit Exposure to a Margined Counterparty, in: *Counterparty Credit Risk Modeling (editor: M. Pykhtin)* **Risk Books**.
- Glasserman, P. (2004): *Monte Carlo Methods in Financial Engineering* **Springer Verlag**.

- Jones, M., J. Marron, and S. Sheather (1996): A Brief Survey of Bandwidth Selection for Density Estimation *Journal of the American Statistical Association* **91 (433)** 401-407.
- Nadaraya, E. A. (1964): On Estimating Regression *Theory of Probability and its Applications* **9 (1)** 141-142.
- Pykhtin, M. (2009): Modeling Credit Exposure for Collateralized Counterparties *Journal of Credit Risk* **5 (4)** 3-27.
- Pykhtin, M. (2010): Collateralized Credit Exposure, in: *Counterparty Credit Risk (editor: E. Canabarro)* **Risk Books**.
- Silverman, B. (1986): *Density Estimation for Statistics and Data Analysis* **Chapman and Hall** London.
- Watson, G. S. (1964): Smooth Regression Analysis *Sankhya: The Indian Journal of Statistics Series A* **26 (4)** 359-372.

ISDA SIMM Methodology Version

Contextual Considerations

1. IM Calculated from Greek Metrics: This chapter includes the initial margin calculations for capturing Delta Risk, Vega Risk, Curvature Risk, inter-curve basis Risk, and Concentration of Risk.

General Provisions

1. ISDA SIMM for Uncleared Trades: This chapter describes the calculations and methodology for calculating the initial margin under the ISDA Standard Initial Margin Model (SIMM) for non-cleared derivatives.
2. SIMM Usage of Risk/Sensitivities: SIMM uses sensitivities as inputs. Risk factors and sensitivities must meet the definition provided in the next section.
3. Aggregation Risk Weights and Correlations: Sensitivities are used as inputs into the aggregation expressions, which are intended to recognize hedging and diversification benefits of position within different risk factors within an asset class. Risk weights and correlations are provided two sections on down.
4. Initial Margin for Complex Trades: This model includes complex trades, which should be handled in the same way as other trades.

Definition of the Interest Rate Risk

1. Interest Rate Risk Factor Vertexes: The *interest rate risk* factors are 12 yields at the following vertexes, one for each currency: 2W, 1M, 3M, 6M, 1Y, 2Y, 3Y, 5Y, 10Y, 15Y, 20Y, and 30Y.
2. Yield Curve of Currency Denomination: The relevant yield curve is the yield curve of the currency in which an instrument is denominated.
3. Index Curves and their Tenors: For a given currency, there are a number of sub yield curves used, named *OIS*, *LIBOR1M*, *LIBOR3M*, *LIBOR6M*, *LIBOR12M*, and – for USD only – *PRIME* and *MUNICIPAL*. Risk should be separately bucketed by currency, tenor, and curve index, expressed as risk to the outright rate of the sub curve. Any sub-curve not given on the above list should be mapped to its closest equivalent.
4. Jurisdiction Inflation Rate Risk Factor: The interest rate risk factors also include a flat inflation rate risk for each currency. When at least one contractual payment obligation depends on the inflation rate, the inflation rate for the relevant currency is used as the risk factor. All sensitivities to the inflation rate for the same currency is fully offset.
5. Treatment of Cross Currency Swaps: For cross-currency swap products whose notional exchange is eligible for exclusion from the margin calculation, the interest-rate risk factors also include a flat cross-currency basis swap for each currency. Cross-currency basis swap spreads should be quoted as a spread to the non-USD LIBOR versus a flat US LIBOR leg. All sensitivities to the basis swap spreads for the same currency are fully offset.
6. The Credit Qualifying Risk Factors: The Credit Qualifying Risk Factors are the five credit spreads for each issuer/seniority pair, separated by the payment currency, at each of the following vertexes: 1Y, 2Y, 3Y, 5Y, and 10Y.
7. Multiple Credit Curves Per Issuer: For a given issuer/seniority, if there is more than one relevant credit spread curve, then the credit spread risk at each vertex should be net sum of

risk at that vertex over all the credit spread curves of that issuer/seniority, which may differ by documentation – such as the seniority clause – but not by currency. Note that delta and vega sensitivities arising from different payment currencies – such as Quanto CDS – are considered different risk factors to the same issuer/seniority from each other.

8. Credit Indexes and Bespoke Baskets: For Credit Qualifying Indexes and bespoke baskets – including securitizations and non-securitizations – delta sensitivities should be compute to the underlying issuer/seniority risk factors. Vega sensitivities to the credit indexes need not be allocated to the underlying risk factors, but rather the entire vega risk should be classed into the appropriate Credit Qualifying Bucket, using the residual bucket for cross-sector indexes.
9. CDX or ITRAXX Index Families: The Credit Qualifying risk factors can also include Base Correlation risks from the CDO tranches from the CDX or the ITRAXX family of Credit indexes. There is one flat risk factor for each index family. Base Correlation risks to the same index family – such as CDX IG, ITRAXX MAIN, and so on – should be fully offset, irrespective of series, maturity, or detachment point.
10. The Credit Non-Qualifying Risk Factors: The *Credit Non-qualifying Risk Factors* are the five credit spreads for each issuer/tranche for each of the following vertexes: 1Y, 2Y, 3Y, 5Y, and 10Y.
11. Sensitivities to the Underlying Tranche: Sensitivities should be computed to the given tranche. For a given tranche, if there is more than one credit spread curve, then the credit spread risk at each vertex should the net sum of risk at that vertex over all the credit spread curves of that tranche. Vertex sensitivities of the credit indexes need not be allocated to the underlying issuer, but rather the entire index vega should be classed into the appropriate non-qualifying bucket, using the residual bucket for cross-sector indexes.
12. Equity Risk Factors Alternative Approaches: The *Equity Risk Factors* are all equity prices; each equity spot price is a risk factor. Sensitivities to equity indexes, funds, and ETF's can be handled in one of two ways: either – the standard preferred approach – the delta can be put into the "Indexes, Funds, ETF's" equity bucket, or – the alternative approach if bilaterally agreed to – the delta can be allocated back to individual equities. The choice between the standard and the alternative approach must be made on a portfolio level basis.

13. Delta and Vega Basket Sensitivities: Delta sensitivities to bespoke baskets should always be allocated to individual equities. Vega sensitivities of equities, funds, and ETF's need not be allocated back to individual equities, but rather the entire vega risk should be classed into "indexes, funds, ETF's" equity bucket. Vega sensitivities to bespoke baskets should be allocated back to the individual equities.
14. Vega and Volatility Index Risk: Note that not all institutions may be able to perform the allocation of vega for equities as described – however – it is the preferred approach. For equity volatility indexes, the index risk should be treated as equity volatility risk and put into the *Volatility Index* bucket.
15. Spot/Forward Commodity Risk Factors: The *Commodity Risk Factors* are all commodity prices; each commodity spot price is a risk factor. Examples include: *Coal Europe*, *Precious Metals Gold*, and *Livestock Lean Hogs*. Risks to commodity forward prices should be allocated back to spot risk prices and aggregated, assuming each commodity forward curve moves in parallel.
16. Standard/Advanced Commodity Index Approaches: Sensitivities to commodity indexes can be handled in one of two ways; either – the standard approach – the entire delta can be put into the *Indexes* bucket, or the advanced approach, where the delta can be allocated back to individual commodities. The choice between standard and advanced approaches should be made on a portfolio-level basis.
17. Delta/Vega Index/Basket Sensitivities: Delta sensitivities to bespoke baskets should always be allocated back to individual commodities. Vega sensitivities of commodities basket should not allocated to individual commodities, but rather the entire index Vega should be classes into the *indexes* bucket.
18. FX Spot and Volatility Risks: The *FX risk factors* are all exchange rates between the calculation currency and any currency, or currency of any FX cross rate, on which the value of the instrument may depend. This excludes the calculation currency itself. The FX vega risk are all the currency pairs to which an instrument has FX volatility risk.

Definition of *Sensitivity* for Delta Margin Calculation

1. Definition of Risk Factor Sensitivity: The following sections define a sensitivity s that should be used as an input into the delta margin calculation. The forward difference is specified in each section for illustration purposes.
2. For Interest Rate and Credit:

$$s = V(x + 1\ bp) - V(x)$$

3. Equities, Commodity, and FX Risk:

$$s = V(x + 1\% \cdot x) - V(x)$$

where s is the sensitivity to the risk factor x, and $V(x)$ is the value of the instrument given the value of the risk factor x.

4. IR/Credit Finite Difference Schemes: However, dealers may make use of central or backward difference methods, or use a smaller shock size and scale up. For Interest Rate and Credit

$$s = V(x + 0.5\ bp) - V(x - 0.5\ bp)$$

$$s = V(x + 1.0\ bp) - V(x)$$

or

$$s = \frac{V(x + \epsilon\ bp) - V(x)}{\epsilon}$$

where

$$0 < |\epsilon| \leq 1$$

5. Equity, Commodity, FX Difference Schemes:

$$s = V(x + 0.5\ bp) - V(x - 0.5\ bp)$$

$$s = V(x + 1.0\ bp) - V(x)$$

or

$$s = \frac{V(x + \epsilon\ bp) - V(x)}{\epsilon}$$

where

$$0 < |\epsilon| \leq 1$$

6. For Interest Rate Risk Factors, the Sensitivity is defined as the PV01: The PV01 of an instrument i with respect to the tenor t of a risk-free curve r - the sensitivity of instrument i with respect to the risk factor r_t - is defined as

$$s(i, r_t) = V_i(r_t + 1bp, cs_t) - V_i(r_t, cs_t)$$

where r_t is the risk-free interest-rate at tenor t, cs_t is the credit spread at tenor t, V_i is the market value of an instrument i as a function of the risk-free interest-rate and the credit spread curve, 1 bp is 1 basis point – 0.0001 or 0.01%.
7. For Credit Non-Securitization Risk Factors, the Sensitivity is defined as CS01: The CS01 of an instrument with respect to tenor t is defined as

$$s(i, cs_t) = V_i(r_t, cs_t + 1bp) - V_i(r_t, cs_t)$$

8. For Credit Qualifying and Non qualifying Securitizations, including n^{th}-to-default Risk Factors, the Sensitivity is defined as CS01: If all of the following criteria are met, the position is deemed to be a qualifying securitization, and the CS01 – as defined by credit non-securitization above – should be computed with respect to the names underlying the securitization or the n^{th}-to-default instrument.
9. Credit Qualifying Securitization Criterion #1: The position should neither be a re-securitization position, nor derivatives of securitization exposures that do not provide a pro-rata proceed in the proceeds of the securitization tranche.
10. Credit Qualifying Securitization Criterion #2: All reference entities are single name products, including single name credit derivatives, for which a liquid two-way market exists – see below – including liquidly traded indexes on these reference entities.
11. Credit Qualifying Securitization Criterion #3: The instrument does not reference an underlying that would be treated as a retail exposure, a residential mortgage exposure, or a commercial mortgage exposure under the standardized approach to credit risk.
12. Credit Qualifying Securitization Criterion #4: The instrument does not reference a claim on a non-special purpose entity.
13. CS01 of the Credit Non Qualifying Instruments: If any of these criteria are not met, the position is deemed to be non-qualifying, and then the CS01 should be calculated to the spread of the instrument rather than the spread of the underlying instruments.
14. Two Way Market Establishment Criterion: A two way market is deemed to exist when there are bonafide independent offers to buy and sell so that a price reasonably related to the last sales price or current competitive bid and offer quotations can be determined within one day and settled at such a price within a relatively short time conforming to trade custom.
15. For Credit Base Correlation Risk Factors, the Sensitivity is defined as BC01: The BC01 is the change in the value for one percentage point increase in the Base Correlation level, that is the sensitivity s_{ik} is defined as

$$s_{ik} = V_i(BC_k + 1\%) - V_i(BC_k)$$

where k is a given credit index family such as CDX IG or ITRAXX MAIN; BC_k is the Base Correlation curve/surface for the index k, with numerical values such as 0.55%; 1% is one percentage point of correlation, that is 0.01; $V_i(BC_k)$ is the value of the instrument i as a function of the Base Correlation for index k.

16. <u>For Equity Risk Factors, the Sensitivity is defined as follows</u>: The change in the value for one percentage point increase in the relative equity price:

$$s_{ik} = V_i(EQ_k + 1\% \cdot EQ_k) - V_i(EQ_k)$$

where k is a given equity; EQ_k is the Market Value for the Equity k; $V_i(EQ_k)$ is the value of the instrument i as a function of the price of equity k.

17. <u>For Commodity Risk Factors, the Sensitivity is defined as follows</u>: The change in the value for one percentage point increase in the relative equity price:

$$s_{ik} = V_i(CTY_k + 1\% \cdot CTY_k) - V_i(CTY_k)$$

where k is a given equity; CTY_k is the Market Value for the commodity k; $V_i(CTY_k)$ is the value of the instrument i as a function of the price of the commodity k.

18. <u>For FX Risk Factors, the Sensitivity is defined as follows</u>: The change in the value for one percentage point increase in the relative FX rate:

$$s_{ik} = V_i(FX_k + 1\% \cdot FX_k) - V_i(FX_k)$$

where k is a given equity; FX_k is the Market Value for the FX rate between the currency k and the calculation currency; $V_i(FX_k)$ is the value of the instrument i as a function of the price of FX rate FX_k.

19. <u>First Order Sensitivity for Options</u>: When computing a first order sensitivity for instruments subject to optionality, it is recommended that the volatility under the bump is adjusted per the prevailing market practice in each risk class.

20. Definition of *Sensitivity* for Vega and Curvature Margin Calculations: The following paragraphs define the sensitivity $\frac{\partial V_i}{\partial \sigma}$ that should be used as input into the vega and the curvature margin calculations shown in the corresponding section. The vega to the implied volatility risk factor is defined as

$$\frac{\partial V_i}{\partial \sigma} = V(\sigma + 1) - V(\sigma)$$

21. Dependence of V_i on σ: Here $V(\sigma)$ is the value of the instrument given the implied volatility σ of the risk factor, while keeping the other inputs – including skew and smile – constant.
22. Type of Implied Volatility σ: The implied volatility σ should be the log-normal volatility, except in the case of interest-rate and credit risks where it should be a normal or a log-normal volatility, or similar, but must match the definition used in the corresponding calculation.
23. σ for Equity/FX/Commodity: For equity, FX, and commodity instruments, the units of σ must be percentages of log-normal volatility, si that 20% is represented as 20. A shock of σ to 1 unit therefore represents an increase in volatility of 1%.
24. σ for Interest Rate/Credit: For interest rate and credit instruments, the units of the volatility σ_{kj} must match that used in the corresponding calculations.
25. Difference Schemes for Sensitivity Calculation: The central or backward difference methods may also be used, or a smaller shock size and scaled up.

$$\frac{\partial V_i}{\partial \sigma} = V(x + 0.5) - V(x - 0.5)$$

$$\frac{\partial V_i}{\partial \sigma} = V(x) - V(x - 1)$$

or

$$\frac{\partial V_i}{\partial \sigma} = \frac{V(x + \epsilon) - V(x)}{\epsilon}$$

where

$$0 < |\epsilon| \leq 1$$

Interest Rate Risk Weights

1. Risk Free Curve within a Currency: The set of risk free curves within each currency is considered to be a separate bucket.
2. Regular Volatility Risk Weight Currencies: The risk weights are set out in the following tables. First there is one table for regular volatility currencies, defined to be: US Dollar (USD), Euro (EUR), British Pound (GBP), Swiss Franc (CHF), Australian Dollar (AUD), New Zealand Dollar (NZD), Canadian Dollar (CAD), Swedish Krona (SEK), Norwegian Krone (NOK), Danish Kroner (DKK), Hong Kong Dollar (HKD), South Korean Won (KRW), Singapore Dollar (SGD), and Taiwanese Dollar (TWD).
3. 2.0 Risk Weights Per Vertex (Regular Currencies):

2W	1M	3M	6M	1Y	2Y	3Y	5Y	10Y	15Y	20Y	30Y
113	113	98	69	56	52	51	51	51	53	56	64

4. 2.1 Risk Weights Per Vertex (Regular Currencies):

2W	1M	3M	6M	1Y	2Y	3Y	5Y	10Y	15Y	20Y	30Y
114	115	102	71	61	52	50	51	51	51	54	62

5. Low Volatility Risk Weight Currency: There is a second table for low volatility currencies, and this currently only contains Japanese Yen (JPY).
6. *2.0* Risk Weights Per Vertex (Low Volatility Currencies):

2W	1M	3M	6M	1Y	2Y	3Y	5Y	10Y	15Y	20Y	30Y
21	21	10	11	15	20	22	21	19	20	23	27

7. *2.1* Risk Weights Per Vertex (Low Volatility Currencies):

2W	1M	3M	6M	1Y	2Y	3Y	5Y	10Y	15Y	20Y	30Y
33	20	10	11	14	20	22	20	20	21	23	27

8. High Volatility Risk Weight Currency: There is a third table for high volatility currencies, which are defined to be all other currencies.
9. Risk Weights Per Vertex (High Volatility Currencies):

2W	1M	3M	6M	1Y	2Y	3Y	5Y	10Y	15Y	20Y	30Y
93	91	90	94	97	103	101	103	102	101	102	101

10. Risk Weights Per Vertex (High Volatility Currencies):

2W	1M	3M	6M	1Y	2Y	3Y	5Y	10Y	15Y	20Y	30Y
91	91	95	88	99	101	101	99	108	100	101	101

11. Cross Currency Inflation Risk Weight:

a. *2.0*: The risk weight for any currency's inflation index is 46. The risk weight for any currency's basis swap rate is 20.
b. *2.1*: The risk weight for any currency's inflation index is 48. The risk weight for any currency's basis swap rate is 21.

12. Interest Rate Vega Risk Weight:
    a. *2.0*: The vega risk weight VRW for the interest rate risk class is 0.21.
    b. *2.1*: The vega risk weight VRW for the interest rate risk class is 0.16.
13. Interest Rate Historical Volatility Ratio:
    a. *2.0*: The historical volatility ratio HVR for the interest rate risk class is 1.00.
    b. *2.1*: The historical volatility ratio HVR for the interest rate risk class is 0.62.
14. *2.0* Interest Rate Tenors Correlation Matrix: The matrix on aggregated weighted sensitivities or risk exposures shown below should be used.

	2W	**1M**	**3M**	**6M**	**1Y**	**2Y**	**3Y**	**5Y**	**10Y**	**15Y**	**20Y**	**30Y**
2W	1.00	1.00	0.79	0.67	0.53	0.42	0.37	0.30	0.22	0.18	0.16	0.12
1M	1.00	1.00	0.79	0.67	0.53	0.42	0.37	0.30	0.22	0.18	0.16	0.12
3M	0.79	0.79	1.00	0.85	0.69	0.57	0.60	0.42	0.32	0.25	0.23	0.20
6M	0.67	0.67	0.85	1.00	0.86	0.76	0.59	0.59	0.47	0.40	0.37	0.32
1Y	0.53	0.53	0.69	0.86	1.00	0.93	0.87	0.77	0.63	0.57	0.54	0.50
2Y	0.42	0.42	0.57	0.76	0.93	1.00	0.98	0.90	0.77	0.70	0.67	0.63
3Y	0.37	0.37	0.50	0.69	0.87	0.98	1.00	0.96	0.84	0.78	0.75	0.77
5Y	0.30	0.30	0.42	0.59	0.77	0.90	0.96	1.00	0.93	0.89	0.86	0.82
10Y	0.22	0.22	0.32	0.47	0.63	0.77	0.84	0.93	1.00	0.98	0.96	0.94
15Y	0.18	0.18	0.25	0.40	0.57	0.70	0.78	0.89	0.98	1.00	0.99	0.98

20Y	0.16	0.16	0.23	0.37	0.54	0.67	0.75	0.86	0.96	0.99	1.00	0.99
30Y	0.12	0.12	0.20	0.32	0.50	0.63	0.71	0.82	0.94	0.98	0.99	1.00

15. *2.1* Interest Rate Tenors Correlation Matrix: The matrix on aggregated weighted sensitivities or risk exposures shown below should be used.

	2W	**1M**	**3M**	**6M**	**1Y**	**2Y**	**3Y**	**5Y**	**10Y**	**15Y**	**20Y**	**30Y**
2W	1.00	0.63	0.59	0.47	0.31	0.22	0.18	0.14	0.09	0.06	0.04	0.05
1M	0.63	1.00	0.79	0.67	0.52	0.42	0.37	0.30	0.23	0.18	0.15	0.13
3M	0.59	0.79	1.00	0.84	0.68	0.56	0.50	0.42	0.32	0.26	0.24	0.21
6M	0.47	0.67	0.84	1.00	0.86	0.76	0.69	0.60	0.48	0.42	0.38	0.33
1Y	0.31	0.52	0.68	0.86	1.00	0.94	0.89	0.80	0.67	0.60	0.57	0.53
2Y	0.22	0.42	0.56	0.76	0.94	1.00	0.98	0.91	0.79	0.73	0.70	0.66
3Y	0.18	0.37	0.50	0.69	0.89	0.98	1.00	0.96	0.87	0.81	0.78	0.74
5Y	0.14	0.30	0.42	0.60	0.80	0.91	0.96	1.00	0.95	0.91	0.88	0.84
10Y	0.09	0.23	0.32	0.48	0.67	0.79	0.87	0.95	1.00	0.98	0.97	0.94
15Y	0.06	0.18	0.26	0.42	0.60	0.73	0.81	0.91	0.98	1.00	0.99	0.97
20Y	0.04	0.15	0.24	0.38	0.57	0.70	0.78	0.88	0.97	0.99	1.00	0.99
30Y	0.05	0.13	0.21	0.33	0.53	0.66	0.74	0.84	0.94	0.97	0.99	1.00

16. Correlation between Sub-Curve Pairs:

a. *2.0*: For sub-curves, the correlation between any two pairs ϕ_{ij} in the same currency is 0.98.

b. *2.1*: For sub-curves, the correlation between any two pairs ϕ_{ij} in the same currency is 0.98.

17. IR/Inflation Rate/Volatility Correlation:

a. *2.0*: For aggregated weighted sensitivities or risk exposures, the correlation between the inflation rate and any yield for the same currency (and the correlation between the inflation volatility and any interest-rate volatility for the same currency) is 29%.

b. *2.1*: For aggregated weighted sensitivities or risk exposures, the correlation between the inflation rate and any yield for the same currency (and the correlation between the inflation volatility and any interest-rate volatility for the same currency) is 33%.

18. IR/Cross Currency/Inflation Volatility Correlation:

a. *2.0*: For aggregated weighted sensitivities or risk exposures, the correlation between the cross-currency basis swap spread and any yield or inflation rate for the same currency is 20%.

b. *2.1*: For aggregated weighted sensitivities or risk exposures, the correlation between the cross-currency basis swap spread and any yield or inflation rate for the same currency is 19%.

19. Correlation used for Different Currencies:

a. *2.0*: The parameter

$$\gamma_{bc} = 23\%$$

should be used for aggregating across different currencies.

b. *2.1*: The parameter

$$\gamma_{bc} = 21\%$$

should be used for aggregating across different currencies.

Credit Qualifying: Risk Weights

1. Credit Quality/Sector Risk Exposure: Sensitivities or risk exposures to an issuer/seniority should first be assigned to a bucket according to the following table:

Bucket Number	Credit Quality	Sector
1	Investment Grade (IG)	Sovereigns including Central Banks
2		Financials including Government-backed Financials
3		Basic Materials, Energy, Industrials
4		Consumer
5		Technology, Telecommunications
6		Health-care, Utilities, Local Governments, Government-backed Corporates (non-financial)
7	High Yield (HY) and Non-rated (NR)	Sovereigns including Central Banks
8		Financials including Government-backed Financials
9		Basic Materials, Energy, Industrials
10		Consumer
11		Technology, Telecommunications
12		Health-care, Utilities, Local Governments, Government-backed Corporates (non-financial)

2. Position/Sensitivities under different Currencies: Sensitivities must be distinguished depending upon the payment currency of the trade – such as Quanto CDS and non-quanto CDS. No initial netting or aggregation is applied between position sensitivities from different currencies – except as described for the situation below.
3. *2.0* Vertex Risk Weight by Bucket: The risk weights should be used for all vertexes (1Y, 2Y, 3Y, 5Y, 10Y) according to bucket, as set out in the following table.

Bucket	**Risk Weight**
1	85
2	85
3	73
4	49
5	48
6	43
7	161
8	238
9	151
10	210
11	149
12	102
Residual	238

4. *2.1* Vertex Risk Weight by Bucket: The risk weights should be used for all vertexes (1Y, 2Y, 3Y, 5Y, 10Y) according to bucket, as set out in the following table.

Bucket	Risk Weight
1	69
2	107
3	72
4	55
5	48
6	41
7	166
8	187
9	177
10	187
11	129
12	136
Residual	187

5. Base Correlation/Vega Risk Weight:
   a. *2.0*: The vega risk weight VRW for the credit risk class is 0.27. The Base Correlation risk weight is 20 for all index families.
   b. *2.1*: The vega risk weight VRW for the credit risk class is 0.27. The Base Correlation risk weight is 19 for all index families.

Credit Qualifying: Correlations

1. *2.0* Same Bucket Risk Factor Correlation: The correlation parameter ρ_{kl} applicable to sensitivity or risk exposure pairs within the same bucket are set out in the following table:

	Same Issuer/Seniority, different Vertex or Currency	**Different Issuer/Seniority**
Aggregate Sensitivities	97%	45%
Residual Bucket	50%	50%

2. *2.1* Same Bucket Risk Factor Correlation: The correlation parameter ρ_{kl} applicable to sensitivity or risk exposure pairs within the same bucket are set out in the following table:

	Same Issuer/Seniority, different Vertex or Currency	**Different Issuer/Seniority**
Aggregate Sensitivities	96%	39%
Residual Bucket	50%	50%

3. Quanto-Currency/Base Correlation Values: Here *currency* refers to the payment currency of sensitivity if there are sensitivities to multiple payment currencies – such as Quanto CDS and non-Quanto CDS – which will not be fully offset.

a. *2.0*: The correlation parameter ρ_{kl} applying to the Base Correlation risks across different indexes/families is 10%.

b. *2.1*: The correlation parameter ρ_{kl} applying to the Base Correlation risks across different indexes/families is 5%.

4. *2.0* Different Bucket Risk Factor Calculations: The correlation bucket parameters applying to sensitivities of risk exposure pairs across different non-residual buckets is set out in the following table:

Bucket	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	1.00	0.42	0.39	0.39	0.40	0.38	0.39	0.34	0.37	0.39	0.37	0.31
2	0.42	1.00	0.44	0.45	0.47	0.45	0.33	0.40	0.41	0.44	0.43	0.37
3	0.39	0.44	1.00	0.43	0.45	0.43	0.32	0.35	0.41	0.42	0.40	0.36
4	0.39	0.45	0.43	1.00	0.47	0.44	0.40	0.34	0.39	0.43	0.39	0.36
5	0.40	0.47	0.45	0.47	1.00	0.47	0.31	0.35	0.40	0.44	0.42	0.37
6	0.38	0.45	0.43	0.44	0.47	1.00	0.30	0.34	0.38	0.40	0.39	0.38
7	0.39	0.33	0.32	0.40	0.31	0.30	1.00	0.28	0.31	0.31	0.30	0.26
8	0.34	0.40	0.35	0.34	0.35	0.34	0.28	1.00	0.34	0.35	0.33	0.30
9	0.37	0.41	0.41	0.39	0.40	0.38	0.31	0.34	1.00	0.40	0.37	0.32
10	0.39	0.44	0.42	0.43	0.44	0.40	0.31	0.35	0.40	1.00	0.40	0.35
11	0.37	0.43	0.40	0.39	0.42	0.39	0.30	0.33	0.37	0.40	1.00	0.34
12	0.31	0.37	0.36	0.36	0.37	0.38	0.26	0.30	0.32	0.35	0.34	1.00

5. *2.1* Different Bucket Risk Factor Calculations: The correlation bucket parameters applying to sensitivities of risk exposure pairs across different non-residual buckets is set out in the following table:

Bucket	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	1.00	0.38	0.36	0.36	0.39	0.35	0.34	0.32	0.34	0.33	0.34	0.31
2	0.38	1.00	0.41	0.41	0.43	0.40	0.29	0.38	0.38	0.38	0.38	0.34
3	0.36	0.41	1.00	0.41	0.42	0.39	0.30	0.34	0.39	0.37	0.38	0.35
4	0.36	0.41	0.41	1.00	0.43	0.40	0.28	0.33	0.37	0.38	0.38	0.34
5	0.39	0.43	0.42	0.43	1.00	0.42	0.31	0.35	0.38	0.39	0.41	0.36
6	0.35	0.40	0.39	0.40	0.42	1.00	0.27	0.32	0.34	0.35	0.36	0.33
7	0.34	0.29	0.30	0.28	0.31	0.27	1.00	0.24	0.28	0.27	0.27	0.26
8	0.32	0.38	0.34	0.33	0.35	0.32	0.24	1.00	0.33	0.32	0.32	0.29
9	0.34	0.38	0.39	0.37	0.38	0.34	0.28	0.33	1.00	0.35	0.35	0.33
10	0.33	0.38	0.37	0.38	0.39	0.35	0.27	0.32	0.35	1.00	0.36	0.32
11	0.34	0.38	0.38	0.38	0.41	0.36	0.27	0.32	0.35	0.36	1.00	0.33
12	0.31	0.34	0.35	0.34	0.36	0.33	0.26	0.29	0.33	0.32	0.33	1.00

Credit Non-Qualifying Risk

1. Non-Qualifying Credit Risk Spread: Sensitivities to credit-spread risk arising from non-qualifying securitization positions are treated according to the risk weights and the correlations as specified in the following paragraphs.
2. Credit Non-Qualifying Bucket Classifications: Sensitivities or risk exposures should first be assigned to a bucket according to the following table:

Bucket	Credit Quality	Sector
1	Investment Grade (IG)	RMBS/CMBS
2	High Yield (HY) and Not-Rated (NR)	RMBS/CMBS
Residual		

3. *2.0* Credit Non-Qualifying Risk Weights: The risk weights are set out in the following table:

Bucket Number	Risk Weight
1	140
2	2000
Residual	2000

4. *2.1* Credit Non-Qualifying Risk Weights: The risk weights are set out in the following table:

Bucket Number	Risk Weight
1	150
2	1200
Residual	1200

a) *2.0*: The vega risk weight VRW for Credit Non-qualifying is 0.27.

b) *2.1*: The vega risk weight VRW for Credit Non-qualifying is 0.27.

Credit Non-Qualifying – Correlations

1. *2.0* Non-Qualifying Correlation – Same Bucket: For other buckets, the correlation parameter ρ_{kl} applicable to sensitivity or risk exposure pairs within the same bucket is set out in the following table:

	Same Underlying Names (more than 80% Overlap in Notional Terms)	**Different Underlying Names (less than 80% Overlap in Notional Terms)**
Aggregate Sensitivities	57%	27%
Residual Bucket	50%	50%

2. *2.1* Non-Qualifying Correlation – Same Bucket: For other buckets, the correlation parameter ρ_{kl} applicable to sensitivity or risk exposure pairs within the same bucket is set out in the following table:

	Same Underlying Names (more than 80% Overlap in Notional Terms)	**Different Underlying Names (less than 80% Overlap in Notional Terms)**
Aggregate	57%	20%

Sensitivities		
Residual Bucket	50%	50%

3. *2.0* Non-Qualifying Correlations: Different Buckets: The correlation parameter γ_{bc} applicable to sensitivity or risk exposure pairs across different buckets is set out in the following table:

	Correlation
Non-residual Bucket to Non-residual Bucket	0.21

4. *2.1* Non-Qualifying Correlations: Different Buckets: The correlation parameter γ_{bc} applicable to sensitivity or risk exposure pairs across different buckets is set out in the following table:

	Correlation
Non-residual Bucket to Non-residual Bucket	0.16

Equity Risk Weights

1. Sector-Based Equity Risk Classification: Sensitivities or risk exposures should first be assigned to a bucket according to the definitions below in the following table:

Bucket Number	Size	Region	Sector
1	Large	Emerging Markets	Consumer Goods and Services, Transportation and Storage, Administrative and Service Activities, Utilities
2			Telecommunications and Industrials
3			Basic Materials, Energy, Agriculture, Manufacturing, Mining, and Quarrying
4			Financials including Government-backed Financials, Real Estate Activities, Technology
5		Developed Markets	Consumer Goods and Services, Transportation and Storage, Administrative and Service Activities, Utilities
6			Telecommunications and Industrials
7			Basic Materials, Energy, Agriculture, Manufacturing, Mining, and Quarrying
8			Financials including Government-backed Financials, Real Estate Activities, Technology
9	Small	Emerging Markets	All Sectors
10		Developed Markets	All Sectors
11	All	All	Indexes, Funds, and ETF’s
12	All	All	Volatility Indexes

2. Large vs. Small Market Capitalization: *Large* is defined as a market capitalization equal to or greater than USD 2 billion and *small* is defined as a market capitalization of less than USD 2 billion.
3. Global Aggregate of Market Cap: *Market Capitalization* is defined as the sum of the market capitalizations of the same legal entity or a group of legal entities across all stock markets globally.
4. Jurisdictions of the Developed Markets: The developed markets are defined as: Canada, US, Mexico, the Euro area, the non-Euro area Western European countries – the UK, Norway, Denmark, Sweden, and Switzerland – Japan, Oceania – Australia and New Zealand – Singapore, and Hong Kong.
5. Determination of the Allocation Bucket: The sectors definition is the one generally used in the market. When allocating an equity position in a particular bucket, the bank must prove that the equity issuer's most material activity indeed corresponds to the bucket's definition. Acceptable proof may be external provider's information, or internal analysis.
6. Multi-national Cross-Sector Issuers: For multinational multi-sector equity issuers, the allocation to a particular bucket must be done according to the most material region and the sector the issuer operates in.
7. *2.0* Sector Based Risk Weight Assignment: If it is not possible to allocate a position to one of these buckets – for example because data on categorical variables is not available – the position must then be allocated to a *residual bucket*. Risk weights should be assigned to each notional position as in the following table:

Bucket	**Risk Weight**
1	25
2	32
3	29
4	27

5	18
6	21
7	25
8	22
9	27
10	29
11	16
12	16
Residual	32

8. *2.1* Sector Based Risk Weight Assignment: If it is not possible to allocate a position to one of these buckets – for example because data on categorical variables is not available – the position must then be allocated to a *residual bucket*. Risk weights should be assigned to each notional position as in the following table:

Bucket	**Risk Weight**
1	24
2	30
3	31
4	25
5	21
6	22

7	27
8	24
9	33
10	34
11	17
12	17
Residual	34

9. *2.0* Equity Risk Historical Volatility Ratio: The historical volatility ratio HVR for equity risk class is 0.65.
10. *2.1* Equity Risk Historical Volatility Ratio: The historical volatility ratio HVR for equity risk class is 0.59.
11. *2.0* Equity Class Vega Risk Weight: The vega risk weight VRW for the equity risk class is 0.28 for all buckets except bucket 12 for which the vega risk weight is 0.64.
12. *2.1* Equity Class Vega Risk Weight: The vega risk weight VRW for the equity risk class is 0.28 for all buckets except bucket 12 for which the vega risk weight is 0.63.

Equity Correlations

1. *2.0* Correlation within a Single Equity Bucket: The correlation parameter ρ_{kl} applicable to sensitivity or risk exposure pairs within the same bucket are set out in the following table:

Bucket	Correlation
1	14%
2	20%
3	19%
4	21%
5	24%
6	35%
7	34%
8	34%
9	20%
10	24%
11	62%
12	62%
Residual	0%

2. *2.1* Correlation within a Single Equity Bucket: The correlation parameter ρ_{kl} applicable to sensitivity or risk exposure pairs within the same bucket are set out in the following table:

Bucket	Correlation
1	14%
2	20%

3	25%
4	23%
5	23%
6	32%
7	35%
8	32%
9	17%
10	16%
11	51%
12	51%
Residual	0%

3. *2.0* Correlations across Equity Buckets: The correlation parameters ρ_{kl} applicable to sensitivity or risk exposure pairs across different non-residual buckets are set out in the following table:

Bucket	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	1.00	0.15	0.14	0.16	0.10	0.12	0.10	0.11	0.13	0.09	0.17	0.17
2	0.15	1.00	0.16	0.17	0.10	0.11	0.10	0.11	0.14	0.09	0.17	0.17
3	0.14	0.16	1.00	0.19	0.14	0.17	0.18	0.17	0.16	0.14	0.25	0.25
4	0.16	0.17	0.19	1.00	0.15	0.18	0.18	0.18	0.18	0.15	0.28	0.28

5	0.10	0.10	0.14	0.15	1.00	0.28	0.23	0.27	0.13	0.21	0.35	0.35
6	0.12	0.11	0.17	0.18	0.28	1.00	0.30	0.34	0.16	0.26	0.45	0.45
7	0.10	0.10	0.18	0.18	0.23	0.30	1.00	0.29	0.16	0.24	0.41	0.41
8	0.11	0.11	0.17	0.18	0.27	0.34	0.29	1.00	0.16	0.26	0.44	0.44
9	0.13	0.14	0.16	0.18	0.13	0.16	0.16	0.16	1.00	0.13	0.24	0.24
10	0.09	0.09	0.14	0.15	0.21	0.26	0.24	0.26	0.13	1.00	0.33	0.33
11	0.17	0.17	0.25	0.28	0.35	0.45	0.41	0.44	0.24	0.33	1.00	0.62
12	0.17	0.17	0.25	0.28	0.35	0.45	0.41	0.44	0.24	0.33	0.62	1.00

4. *2.1* Correlations across Equity Buckets: The correlation parameters ρ_{kl} applicable to sensitivity or risk exposure pairs across different non-residual buckets are set out in the following table:

Bucket	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	1.00	0.16	0.16	0.17	0.13	0.15	0.15	0.15	0.13	0.11	0.19	0.19
2	0.16	1.00	0.20	0.20	0.14	0.16	0.16	0.16	0.15	0.13	0.20	0.20
3	0.16	0.20	1.00	0.22	0.15	0.19	0.22	0.19	0.16	0.15	0.25	0.25
4	0.17	0.20	0.22	1.00	0.17	0.21	0.21	0.21	0.17	0.15	0.27	0.27
5	0.13	0.14	0.15	0.17	1.00	0.25	0.26	0.23	0.14	0.17	0.32	0.32
6	0.15	0.16	0.19	0.21	0.25	1.00	0.30	0.31	0.16	0.21	0.38	0.38
7	0.15	0.16	0.22	0.21	0.23	0.30	1.00	0.29	0.16	0.21	0.38	0.38

8	0.15	0.16	0.19	0.21	0.26	0.31	0.29	1.00	0.17	0.21	0.39	0.39
9	0.13	0.15	0.16	0.17	0.14	0.16	0.16	0.17	1.00	0.13	0.21	0.21
10	0.11	0.13	0.15	0.15	0.17	0.21	0.21	0.21	0.13	1.00	0.25	0.25
11	0.19	0.20	0.25	0.27	0.32	0.38	0.38	0.39	0.21	0.25	1.00	0.51
12	0.19	0.20	0.25	0.27	0.32	0.38	0.38	0.39	0.21	0.25	0.51	1.00

Commodity Risk Weights

1. *2.0* Risk Weights for Commodity Buckets: The risk weights depend on the commodity type; they are set out in the following table:

Bucket	**Commodity**	**Risk Weight**
1	Coal	19
2	Crude	20
3	Light Ends	17
4	Middle Distillates	18
5	Heavy Distillates	24
6	North American Natural Gas	20
7	European Natural Gas	25
8	North American Power	41

9	European Power	24
10	Freight	91
11	Base Metals	20
12	Precious Metals	19
13	Grains	16
14	Softs	15
15	Livestock	10
16	Other	91
17	Indexes	17

2. *2.1* Risk Weights for Commodity Buckets: The risk weights depend on the commodity type; they are set out in the following table:

Bucket	**Commodity**	**Risk Weight**
1	Coal	19
2	Crude	20
3	Light Ends	17
4	Middle Distillates	19
5	Heavy Distillates	24
6	North American Natural Gas	22
7	European Natural Gas	26

8	North American Power	50
9	European Power	27
10	Freight	54
11	Base Metals	20
12	Precious Metals	20
13	Grains	17
14	Softs	14
15	Livestock	10
16	Other	54
17	Indexes	16

3. *2.0* Commodity Class Historical Volatility Ratio: The historical volatility ratio HVR for the commodity risk class is 0.80.
4. *2.1* Commodity Class Historical Volatility Ratio: The historical volatility ratio HVR for the commodity risk class is 0.80.
5. *2.0* Commodity Class Vega Risk Weight: The vega risk weight VRW for the commodity risk class is 0.38.
6. *2.1* Commodity Class Vega Risk Weight: The vega risk weight VRW for the commodity risk class is 0.38.

Commodity Correlations

1. *2.0* <u>Commodity Correlations within the same Bucket</u>: The correlation parameters ρ_{kl} applicable to sensitivity or risk exposure pairs within the same bucket are set out in the following table:

Bucket	**Correlation**
1	0.30
2	0.97
3	0.93
4	0.98
5	0.97
6	0.92
7	1.00
8	0.58
9	1.00
10	0.10
11	0.55
12	0.64
13	0.71
14	0.22
15	0.29
16	0.00

17	0.21

2. *2.1* Commodity Correlations within the same Bucket: The correlation parameters ρ_{kl} applicable to sensitivity or risk exposure pairs within the same bucket are set out in the following table:

Bucket	**Correlation**
1	0.27
2	0.97
3	0.92
4	0.97
5	0.99
6	1.00
7	1.00
8	0.40
9	0.73
10	0.13
11	0.53
12	0.64
13	0.63
14	0.26

15	0.26
16	0.00
17	0.38

3. *2.0* Commodity Correlations among Different Buckets: The correlation parameters γ_{bc} applicable to sensitivity or risk exposure pairs across different buckets are set out in the following table:

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**
1	1.00	0.18	0.15	0.20	0.25	0.08	0.09	0.20	0.27	0.00	0.15	0.02	0.06	0.07	-0.04	0.00	0.06
2	0.18	1.00	0.89	0.94	0.93	0.32	0.22	0.27	0.24	0.09	0.45	0.21	0.32	0.28	0.17	0.00	0.37
3	0.15	0.89	1.00	0.87	0.88	0.25	0.16	0.19	0.12	0.10	0.26	-0.01	0.19	0.17	0.10	0.00	0.27
4	0.20	0.94	0.87	1.00	0.92	0.29	0.22	0.26	0.19	0.00	0.32	0.05	0.20	0.22	0.13	0.00	0.28
5	0.25	0.93	0.88	0.92	1.00	0.30	0.26	0.22	0.28	0.12	0.42	0.23	0.28	0.19	0.17	0.00	0.34
6	0.08	0.32	0.25	0.29	0.30	1.00	0.13	0.57	0.05	0.14	0.57	-0.02	0.13	0.17	0.01	0.00	0.26
7	0.09	0.22	0.16	0.22	0.26	0.13	1.00	0.07	0.80	0.19	0.16	0.05	0.17	0.18	0.00	0.00	0.18
8	0.20	0.27	0.19	0.26	0.22	0.57	0.07	1.00	0.13	0.06	0.16	0.03	0.10	0.12	0.06	0.00	0.23
9	0.27	0.24	0.12	0.19	0.28	0.05	0.80	0.13	1.00	0.15	0.17	0.05	0.15	0.13	-0.03	0.00	0.13
10	0.00	0.09	0.10	0.00	0.12	0.14	0.19	0.06	0.15	1.00	0.07	0.07	0.17	0.10	0,02	0.00	0.11
11	0.15	0.45	0.26	0.32	0.42	0.57	0.16	0.16	0.17	0.07	1.00	0.34	0.20	0.21	0.16	0.00	0.27
12	0.02	0.21	-0.01	0.05	0.23	-0.02	0.05	0.03	0.05	0.07	0.34	1.00	0.17	0.26	0.11	0.00	0.14

13	0.06	0.32	0.19	0.20	0.28	0.13	0.17	0.10	0.15	0.17	0.20	0.17	1.00	0.35	0.09	0.00	0.22
14	0.07	0.28	0.17	0.22	0.19	0.17	0.18	0.12	0.13	0.10	0.27	0.26	0.35	1.00	0.06	0.00	0.20
15	-0.04	0.17	0.10	0.13	0.17	1.00	0.00	0.06	-0.03	0.02	0.16	0.11	0.09	0.06	1.00	0.00	0.16
16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
17	0.06	0.37	0.27	0.28	0,34	0.26	0.18	0.23	0.13	0.11	0.21	0.14	0.22	0.20	0.16	0.00	1.00

4. *2.1* <u>Commodity Correlations among Different Buckets</u>: The correlation parameters γ_{bc} applicable to sensitivity or risk exposure pairs across different buckets are set out in the following table:

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**
1	1.00	0.16	0.11	0.19	0.22	0.12	0.22	0.02	0.27	0.08	0.11	0.05	0.04	0.06	0.01	0.00	0.10
2	0.16	1.00	0.89	0.94	0.93	0.32	0.24	0.19	0.21	0.06	0.39	0.23	0.39	0.29	0.13	0.00	0.66
3	0.11	0.89	1.00	0.87	0.88	0.17	0.17	0.13	0.12	0.03	0.24	0.04	0.27	0.19	0.08	0.00	0.61
4	0.19	0.94	0.87	1.00	0.92	0.37	0.27	0.21	0.21	0.03	0.36	0.16	0.27	0.28	0.09	0.00	0.64
5	0.22	0.93	0.88	0.92	1.00	0.29	0.26	0.19	0.23	0.10	0.40	0.27	0.38	0.30	0.15	0.00	0.64
6	0.12	0.32	0.17	0.37	0.29	1.00	0.19	0.60	0.18	0.09	0.22	0.09	0.14	0.16	0.10	0.00	0.37
7	0.22	0.24	0.17	0.27	0.26	0.19	1.00	0.06	0.68	0.16	0.21	0.10	0.24	0.25	-0.01	0.00	0.27
8	0.02	0.19	0.13	0.21	0.19	0.60	0.06	1.00	0.12	0.01	0.10	0.03	0.02	0.07	0.10	0.00	0.21
9	0.27	0.21	0.12	0.21	0.23	0.18	0.68	0.12	1.00	0.05	0.16	0.03	0.19	0.16	-0.01	0.00	0.19
10	0.08	0.06	0.03	0.03	0.10	0.09	0.16	0.01	0.05	1.00	0.08	0.04	0.05	0.11	0.02	0.00	0.00

11	0.11	0.39	0.24	0.36	0.40	0.22	0.21	0.10	0.16	0.08	1.00	0.34	0.19	0.22	0.15	0.00	0.34
12	0.05	0.23	0.04	0.16	0.27	0.09	0.10	0.03	0.03	0.04	0.34	1.00	0.14	0.26	0.09	0.00	0.20
13	0.04	0.39	0.27	0.27	0.38	0.14	0.24	0.02	0.19	0.05	0.19	0.14	1.00	0.30	0.16	0.00	0.40
14	0.06	0.29	0.19	0.28	0.30	0.16	0.25	0.07	0.16	0.11	0.22	0.26	0.30	1.00	0.09	0.00	0.30
15	0.01	0.13	0.08	0.09	0.15	0.10	-0.01	0.10	-0.01	0.02	0.15	0.09	0.16	0.09	1.00	0.00	0.16
16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
17	0.10	0.66	0.61	0.64	0.64	0.37	0.27	0.21	0.19	0.00	0.34	0.20	0.40	0.30	0.16	0.00	1.00

Foreign Exchange Risk

1. *2.0* Foreign Exchange Risk Weights: A unique risk weight equal to 8.2 applies to all FX sensitivities or risk exposures. The historical volatility ratio HVR for the FX risk class is 0.60. The vega risk weight VRW for the FX volatility 0.33.
2. *2.1* Foreign Exchange Risk Weights: A unique risk weight equal to 8.1 applies to all FX sensitivities or risk exposures. The historical volatility ratio HVR for the FX risk class is 0.63. The vega risk weight VRW for the FX volatility 0.30.
3. *2.0* Foreign Exchange Correlations: A unique correlation ρ_{kl} equal to 0.5 applies to all pairs of FX sensitivities or risk exposures.
4. *2.1* Foreign Exchange Correlations: A unique correlation ρ_{kl} equal to 0.5 applies to all pairs of FX sensitivities or risk exposures.
5. Single Bucket Foreign Exchange Sensitivities: All foreign exchange sensitivities are considered to be within a single bucket within the FX risk class, so no inter-bucket

aggregation is necessary. Note that the cross-bucket curvature calculations are still required on a single bucket.

Concentration Thresholds

1. Asset Class/Bucket Concentration Thresholds: The concentration thresholds in this section are defined for the asset-class-specific buckets specified earlier. For those cases where the same concentration threshold applies to the related range of buckets, the tables in this section specify a precise range of applicable buckets in the Bucket column and give a narrative description of that group of buckets in the Risk Group column.
2. *2.0* Interest Rate Risk - Delta Concentration Thresholds: The delta concentration thresholds for interest rate risk – inclusive of inflation risk – are given by the currency group:

Currency Risk Group	Concentration Threshold (USD mm/bp)
High Volatility	8
Regular Volatility, Well Traded	230
Regular Volatility, less Well Traded	28
Low Volatility	82

3. *2.1* Interest Rate Risk - Delta Concentration Thresholds: The delta concentration thresholds for interest rate risk – inclusive of inflation risk – are given by the currency group:

Currency Risk Group	Concentration Threshold (USD mm/bp)

High Volatility	12
Regular Volatility, Well Traded	210
Regular Volatility, less Well Traded	27
Low Volatility	170

4. Concentration Threshold Currency Risk Group: The Currency Risk Groups used in establishing concentration thresholds for Interest Rate Risk are as follows:
   a. High Volatility => All other currencies
   b. Regular Volatility, Well Traded => USD, EUR, GBP
   c. Regular Volatility, Less Well Traded => AUD, CAD, CHF, DKK, HKD, KRW, NOK, NZD, SEK, SGD, TWD
   d. Low Volatility => JPY
5. *2.0* Credit Spread Risk – Delta Concentration Thresholds: The delta concentration thresholds for credit spread risk are given by credit spread risk are given by the Credit Risk Group and Bucket.

Bucket (s)	**Credit Risk Group**	**Concentration Threshold (USD mm/bp)**
	Qualifying	
1, 7	Sovereigns including Central Banks	0.95
2-6, 8-12	Corporate Entities	0.29
Residual	Non-Classified	0.29
	Non-Qualifying	
1	IG (RMBS and CMBS)	9.50
2	HY/non-rated (RMBS and CMBS)	0.50

Residual	Not Classified	0.50

6. *2.1* Credit Spread Risk – Delta Concentration Thresholds: The delta concentration thresholds for credit spread risk are given by credit spread risk are given by the Credit Risk Group and Bucket.

Bucket (s)	Credit Risk Group	Concentration Threshold (USD mm/bp)
Qualifying		
1, 7	Sovereigns including Central Banks	1.00
2-6, 8-12	Corporate Entities	0.24
Residual	Non-Classified	0.24
Non-Qualifying		
1	IG (RMBS and CMBS)	9.50
2	HY/non-rated (RMBS and CMBS)	0.50
Residual	Not Classified	0.50

7. *2.0* Equity Risk – Delta Concentration Thresholds: The delta concentration thresholds for equity risk are given by bucket.

Bucket (s)	Equity Risk Group	Concentration Threshold (USD mm/bp)
1-4	Emerging Markets – Large Cap	3.3
5-8	Developed Markets – Large Cap	30.0
9	Emerging Markets – Small Cap	0.6

10	Developed Markets – Small Cap	2.3
11-12	Indexes, Funds, ETF's, and Volatility Indexes	900.0
Residual	Not Classified	0.6

8. *2.1* Equity Risk – Delta Concentration Thresholds: The delta concentration thresholds for equity risk are given by bucket.

Bucket (s)	**Equity Risk Group**	**Concentration Threshold (USD mm/bp)**
1-4	Emerging Markets – Large Cap	8.4
5-8	Developed Markets – Large Cap	26.0
9	Emerging Markets – Small Cap	1.8
10	Developed Markets – Small Cap	1.9
11-12	Indexes, Funds, ETF's, and Volatility Indexes	540.0
Residual	Not Classified	1.8

9. *2.0* Commodity Risk – Delta Concentration Thresholds: The delta concentration thresholds for commodity risk are given by:

Bucket (s)	**Commodity Risk Group**	**Concentration Threshold (USD mm/bp)**
1	Coal	1,400
2	Crude Oil	20,000

3-5	Oil Fractions	3,500
6-7	Natural Gas	6,400
8-9	Power	2,500
10	Freight – Dry or Wet	300
11	Base Metals	2,900
12	Precious Metals	7,600
13-15	Agricultural	3,900
16	Other	300
17	Indexes	12,000

10. *2.1* Commodity Risk – Delta Concentration Thresholds: The delta concentration thresholds for commodity risk are given by:

Bucket (s)	**Commodity Risk Group**	**Concentration Threshold (USD mm/bp)**
1	Coal	700
2	Crude Oil	3,600
3-5	Oil Fractions	2,700
6-7	Natural Gas	2,600
8-9	Power	1,900
10	Freight – Dry or Wet	52
11	Base Metals	2,000

12	Precious Metals	3,200
13-15	Agricultural	1,100
16	Other	52
17	Indexes	5,200

11. <u>*2.0* FX Risk – Delta Concentration Thresholds</u>: The delta concentration thresholds for FX risk are given by the FX Risk Group:

FX Risk Group	**Concentration Threshold (USD mm/bp)**
Category 1	8,400
Category 2	1,900
Category 3	560

12. <u>*2.1* FX Risk – Delta Concentration Thresholds</u>: The delta concentration thresholds for FX risk are given by the FX Risk Group:

FX Risk Group	**Concentration Threshold (USD mm/bp)**
Category 1	9,700
Category 2	2,900
Category 3	450

13. <u>FX Risk Concentration Threshold Classifications</u>: Currencies were placed in three categories as those for delta risk weights, constituted as follows:
    a. Category 1 => Significantly Material – USD, EUR, JPY, GBP, CAD, AUD, CHF

b. Category 2 => Frequently Traded – BRL, CNY, HKD, INR, KRW, MXN, NOK, NZD, RUB, SEK, SGD, TRY, ZAR
c. Category 3 => Others – All other currencies

14. <u>*2.0* Interest Rate Risk - Vega Concentration Thresholds</u>: The Vega Concentration thresholds for Interest Rate Risk are:

Currency Risk Group	**Concentration Threshold (USD mm/bp)**
High Volatility	110
Regular Volatility, Well Traded	2,700
Regular Volatility, less Well Traded	150
Low Volatility	960

15. <u>*2.1* Interest Rate Risk - Vega Concentration Thresholds</u>: The Vega Concentration thresholds for Interest Rate Risk are:

Currency Risk Group	**Concentration Threshold (USD mm/bp)**
High Volatility	120
Regular Volatility, Well Traded	2,200
Regular Volatility, less Well Traded	190
Low Volatility	770

The currency risk groups used in establishing the concentration thresholds correspond to the *Concentration Threshold Currency Risk Group* above.

16. <u>*2.0* Credit Spread Risk - Vega Concentration Thresholds</u>: The Vega Concentration thresholds for Credit Spread Risk are:

Credit Risk Group	Concentration Threshold (USD mm/bp)
Qualifying	290
Non Qualifying	65

17. *2.1* Credit Spread Risk - Vega Concentration Thresholds: The Vega Concentration thresholds for Credit Spread Risk are:

Credit Risk Group	Concentration Threshold (USD mm/bp)
Qualifying	250
Non Qualifying	54

18. *2.0* Equity Risk - Vega Concentration Thresholds: The Vega Concentration thresholds for Equity Risk are:

Bucket (s)	Equity Risk Group	Concentration Threshold (USD mm/bp)
1-4	Emerging Markets – Large Cap	700
5-8	Developed Markets – Large Cap	7,300
9	Emerging Markets – Small Cap	70
10	Developed Markets – Small Cap	300
11-12	Indexes, Funds, ETF’s, and Volatility Indexes	21,000
Residual	Not Classified	70

19. *2.1* Equity Risk - Vega Concentration Thresholds: The Vega Concentration thresholds for Equity Risk are:

Bucket (s)	**Equity Risk Group**	**Concentration Threshold (USD mm/bp)**
1-4	Emerging Markets – Large Cap	20
5-8	Developed Markets – Large Cap	2,300
9	Emerging Markets – Small Cap	43
10	Developed Markets – Small Cap	250
11-12	Indexes, Funds, ETF's, and Volatility Indexes	8,100
Residual	Not Classified	43

20. *2.0* Commodity Risk - Vega Concentration Thresholds: The Vega Concentration thresholds for Commodity Risk are:

Bucket (s)	**Commodity Risk Group**	**Concentration Threshold (USD mm/bp)**
1	Coal	250
2	Crude Oil	2,000
3-5	Oil Fractions	510
6-7	Natural Gas	1,900
8-9	Power	870
10	Freight – Dry or Wet	220
11	Base Metals	450

12	Precious Metals	740
13-15	Agricultural	370
16	Other	220
17	Indexes	430

21. *2.1* Commodity Risk - Vega Concentration Thresholds: The Vega Concentration thresholds for Commodity Risk are:

Bucket (s)	**Commodity Risk Group**	**Concentration Threshold (USD mm/bp)**
1	Coal	250
2	Crude Oil	1,800
3-5	Oil Fractions	320
6-7	Natural Gas	2,200
8-9	Power	780
10	Freight – Dry or Wet	99
11	Base Metals	420
12	Precious Metals	650
13-15	Agricultural	570
16	Other	99
17	Indexes	330

22. *2.0* FX Risk - Vega Concentration Thresholds: The Vega Concentration thresholds for FX Risk are:

FX Risk Group	**Concentration Threshold (USD mm/bp)**
Category 1 – Category 1	4,000
Category 1 – Category 2	1,900
Category 1 - Category 3	320
Category 2 – Category 2	120
Category 3 – Category 3	110
Category 3 - Category 3	110

23. *2.1* FX Risk - Vega Concentration Thresholds: The Vega Concentration thresholds for FX Risk are:

FX Risk Group	**Concentration Threshold (USD mm/bp)**
Category 1 – Category 1	2,000
Category 1 – Category 2	1,000
Category 1 - Category 3	320
Category 2 – Category 2	410
Category 3 – Category 3	210
Category 3 - Category 3	150

The Currency categories used in establishing Concentration Thresholds for FX are identified under *FX Risk – Delta Concentration Thresholds*.

24. *2.0* Correlation between Risk Classes within Products: The correlation parameters ψ_{rs} applying to initial margin risk classes within a single product class are set out in the following table.

Class	Interest Rate	Credit Qualifying	Credit Non-Qualifying	Equity	FX	Commodity
Interest Rate	1.00	0.28	0.18	0.18	0.30	0.22
Credit Qualifying	0.28	1.00	0.30	0.66	0.46	0.27
Credit Non-Qualifying	0.18	0.30	1.00	0.23	0.25	0.18
Equity	0.18	0.66	0.23	1.00	0.39	0.24
FX	0.30	0.46	0.25	0.39	1.00	0.32
Commodity	0.22	0.27	0.18	0.24	0.32	1.00

25. *2.1* Correlation between Risk Classes within Products: The correlation parameters ψ_{rs} applying to initial margin risk classes within a single product class are set out in the following table.

Class	Interest Rate	Credit Qualifying	Credit Non-	Equity	FX	Commodity

			Qualifying			
Interest Rate	1.00	0.25	0.15	0.19	0.30	0.26
Credit Qualifying	0.25	1.00	0.26	0.65	0.45	0.24
Credit Non-Qualifying	0.15	0.26	1.00	0.23	0.25	0.18
Equity	0.19	0.65	0.23	1.00	0.39	0.24
FX	0.30	0.45	0.25	0.39	1.00	0.32
Commodity	0.26	0.24	0.18	0.24	0.32	1.00

Additional Initial Margin Expressions

1. <u>Additional Initial Margin – Standardized Expressions</u>: Standardized formulas for calculating additional initial margin are as follows:

$$\begin{aligned} AdditionalInitialMargin &= AddOnIM + (MS_{RATESFX} - 1)SIMM_{RATESFX} \\ &+ (MS_{CREDIT} - 1)SIMM_{CREDIT} + \left(MS_{EQUITY} - 1\right)SIMM_{EQUITY} \\ &+ (MS_{COMMODITY} - 1)SIMM_{COMMODITY} \end{aligned}$$

where $AddOnIM$ is defined as follows:

$$AddOnIM = AddOnFixed + AddOnFactor_P Notional_P$$

where $AddOnFixed$ is a fixed add-on amount, $AddOnFactor_P$ is the add-on factor for each affected product P expressed as a percentage of the notional (e.g., 5%), and $Notional_P$ is the total notional of the product – sum of the absolute trade notionals. In such use – where a variable notional is involved – current notional amount should be used.

2. Multiplicative Scales for Product Classes: The four variables - $MS_{RATESFX}$, MS_{CREDIT}, MS_{EQUITY}, and $MS_{COMMODITY}$ are the four *multiplicative scales* for the four product classes RatesFX, Credit, Equity, and Commodity. Their values can be individually specified to be more than 1.0 – with 1.0 being the default and the minimum value.

Structure of the Methodology

1. Six Classes of Risk Factors: There are six risk classes – Interest Rate, Credit (Qualifying), Credit (Non-Qualifying), Equity, Commodity, and FX – and the margin for each risk class is defined to be the sum of the Delta Margin, the Vega Margin, the Curvature Margin, and the Base Correlation Margin – if applicable – for that risk class.
2. Four Classes of Sensitivity Margins: That is

$$IM_X = DeltaMargin_X + VegaMargin_X + CurvatureMargin_X + BaseCorrelationMargin_X$$

for each risk class X, where the $BaseCorrelationMargin_X$ is only present in the Credit Qualifying risk class.
3. Four Classes of Products Needing Margin: There are four product classes:

a. Interest Rates and Foreign Exchange (RatesFX)
b. Credit
c. Equity
d. Commodity

4. Product Class per Marginable Trade: Every trade is assigned to an individual product class and SIMM is considered separately for each product class.
5. Isolating Risk Factors across Products: Buckets are still defined in risk terms, but within each product class the risk class takes its component risks only from trades of that product class. For example, equity derivatives would have risk only in the interest rate risk class as well as from the equity risk class; but all those risks are kept separate from the risks of the trades in the RatesFX product class.
6. Product SIMM from Risk Factor IM: Within each product class, the initial margin (IM) for each of the risk classes is calculated as above. The total margin for that product class is given by

$$SIMM_{PRODUCT} = \sqrt{\sum_{r} {IM_r}^2 + \sum_{r} \sum_{s \neq r} \psi_{rs} IM_r IM_s}$$

   where $PRODUCT$ is one of the four product classes above, and the sums on r and s are taken over the six risk classes. The correlation matrix ψ_{rs} of the correlations between the risk classes is given earlier.
7. Portfolio SIMM as Linear Sum: The total SIMM is the sum of these four product class SIMM values:

$$SIMM = SIMM_{RATESFX} + SIMM_{CREDIT} + SIMM_{EQUITY} + SIMM_{COMMODITY}$$

8. Product Specific SIMM Add-On: The SIMM equation can be extended to incorporate notional based add-ons for specified products and/or multipliers to the individual product class SIMM values. The section on SIMM add-ons contains the modified version of SIMM in that case.

Interest Rate Risk Delta Margin

1. Approach for IR Delta Margin: The following step-by-step approach to capture delta risk should be applied to capture delta risk for the interest-rate risk class only.
2. Sensitivity to Tenor/Risk Factor: Find a net sensitivity across instruments to each risk factor (k, i) where k is the rates tenor and i is the index name of the sub-yield curve as defined in the sections that outline the interest rate risk class.
3. Risk Weight applied to the Sensitivity: Weight the net sensitivity $s_{k,i}$ to each risk factor (k, i) by the corresponding risk weight RW_k according to the vertex structure laid out in the section on Interest Rate Risk Weights.
4. Risk Weighted Vertex Sensitivity Expression:

$$WS_{k,i} = RW_k s_{k,i} CR_b$$

   is where CR is the concentration risk factor defined as

$$CR_b = \max\left(1, \sqrt{\frac{\sum_{k,i} s_{k,i}}{T_b}}\right)$$

   for the concentration threshold T_b defined for each currency b.
5. Treating Inflation/Cross Currency Sensitivities: Note that inflation sensitivities to currency b are included in $\sum_{k,i} s_{k,i}$ but cross-currency basis swap sensitivities are not. Neither should the cross-currency basis swap sensitivities be scaled by the concentration factor.

6. Single Curve Sensitivity Roll Up: The weighted sensitivities should then be aggregated within each currency. The sub-curve correlations ϕ_{ij} and the tenor correlation parameters ρ_{kl} are set out in the Section on Interest Rate Risk Correlations.
7. Single Curve Composite Sensitivity - Expression:

$$K = \sqrt{\sum_{i,k} {WS_{k,i}}^2 + \sum_{i,k} \sum_{(j,l) \neq (i,k)} \phi_{ij} \rho_{kl} WS_{k,l} WS_{i,j}}$$

8. Multiple Currency Weighted Roll Up: Delta amounts should then be aggregated across currencies within the risk class. The correlation parameters γ_{bc} applicable are set out in the Section on Interest Rate Risk factor correlations.
9. Multiple Currency Delta Margin Expression:

$$DeltaMargin = \sqrt{\sum_{b} {K_b}^2 + \sum_{b} \sum_{c \neq b} \gamma_{bc} g_{bc} S_b S_c}$$

where

$$S_b = \max\left(\min\left\{ \sum_{i,k} WS_{i,k}, K_b \right\}, -K_b \right)$$

and

$$g_{bc} = \frac{\min(CR_b, CR_c)}{\max(CR_b, CR_c)}$$

for all currencies b and c.

Non Interest Rate Risk Classes

1. Non-IR Delta Margin Approaches: The following step-by-step approach to capture delta risk should be applied separately to each risk class other than interest rates.
2. Sensitivity to Tenor/Risk Factor: Find the net sensitivity across instruments to each risk factor k, which are defined in the sections for each risk class.
3. Risk Weight Applicability to Sensitivity: Weight the net sensitivity s_k to each risk factor k by the corresponding risk weight RW_k according to the bucketing structure for each risk class set out in the Section *Credit Qualifying Risk*.
4. Risk Weighted Vertex Sensitivity Expression:

$$WS_k = RW_k s_k CR_k$$

is where CR_k is the concentration risk factor defined as

$$CR_k = \max\left(1, \sqrt{\frac{\left|\sum_j s_j\right|}{T_b}}\right)$$

for credit spread risk with the sum j taken over all the issuers and seniorities as the risk factor k irrespective of the tenor of the payment currency, and

$$CR_k = \max\left(1, \sqrt{\frac{|s_k|}{T_b}}\right)$$

for equity, commodity, and FX risk where T_b is the concentration threshold for the bucket – or FX category - b as given in the appropriate section.

5. Incorporating the Base Correlation Risk: Note that the base correlation sensitivities are not included in the concentration risk, and the concentration risk for these factors should be taken as 1.
6. Roll Up within Risk Factor: Weighted sensitivities should then be aggregated within each bucket. The buckets and the correlation parameters applicable to each risk class are set out in the section on *Credit Qualifying Risk*.
7. Single Risk Factor Composition Expression:

$$K = \sqrt{\sum_k {WS_k}^2 + \sum_k \sum_{l \neq k} \rho_{kl} f_{kl} WS_k WS_l}$$

where

$$f_{kl} = \frac{\min(CR_k, CR_l)}{\max(CR_k, CR_l)}$$

8. Roll Up across Risk Factors: Delta margin amounts should then be aggregated across buckets in each risk class. The correlation parameters γ_{bc} applicable to each risk class are set out earlier.
9. Cross Risk Factor Composition - Expression:

$$DeltaMargin = \sqrt{\sum_b {K_b}^2 + \sum_b \sum_{c \neq b} \gamma_{bc} S_b S_c} + K_{RESIDUAL}$$

where

$$S_b = \max\left(\min\left\{\sum_{i,k} WS_{i,k}, K_b\right\}, -K_b\right)$$

for all risk factors in bucket b.

10. Margin Requirements for Volatility Instruments: Instruments that are option or include an option – including pre-payment – or have volatility sensitivity – instruments subject to optionality – are subject to additional margin risks such as vega and curvature risk – as described down below. Instruments not subject to optionality with no volatility sensitivity are not subject to vega risk and curvature risk.
11. Approaches for Vega Risk Exposure: The following step-by-step to capture vega risk exposure should be applied separately to each risk class.
12. Risk Factor/Tenor ATM Volatility: For interest-rate and credit instruments, the volatility σ_{kj} for risk-factor k and maturity j is defined to be the implied at-the-money (ATM) volatility of the swaption with expiry time equal to the tenor k at some swap maturity j. The volatility can be quoted as normal, log-normal, or something similar.
13. Inflation Risk Factor ATM Volatility: In the case where k is the inflation risk factor, the inflation volatility σ_{kj} of the inflation swaption of type j is defined to the at-the-money volatility of the swaption, where the type j comprises an initial inflation observation date and a final inflation observation date.
14. Buckets Mirroring Interest-Rate Factors: The option expiry date shall be defined to be the final inflation observation date, and risk should be defined on a set of option expiries equal to the same-tenor bucket as the interest rate delta. The volatility can be quoted as normal, log-normal, or something similar.
15. Volatility for Equity/FX/Commodity: For Equity, FX, and Commodity instruments, the volatility σ_{kj} of the risk factor k at each vol-tenor j is given by the expression

$$\sigma_{kj} = \frac{RW_k\sqrt{365/14}}{\alpha}$$

where

$$\alpha = \Phi^{-1}(0.99)$$

RW_k *vol-tenor* j is the option expiry index, which should use the same tenor-buckets as the interest-rate delta risk: 2W, 1M, 3M, 6M, 1Y, 2Y, 3Y, 5Y, 10Y, 15Y, 20Y, and 30Y. Here $\Phi^{-1}(0.99)$ is the 99th percentile of the standard normal distribution.

16. Commodity Index Volatility Risk Weights: For commodity index volatilities, the risk weight to use is that of the *Indexes* bucket.
17. FX Delta Sensitivity Risk Weight: For FX vega – which depends upon a pair of currencies – the risk weight to use here is the common risk weight for the FX delta sensitivity given explicitly in the corresponding section.
18. Instrument Level Vega Risk Expression: The vega risk for each instrument i to the risk factor k is estimated using the expression

$$VR_{ik} = HVR_c \sum_j \sigma_{jk} \frac{\partial V_{ij}}{\partial \sigma}$$

19. Product Specific Vega Value applicable: Here σ_{jk} is the volatility defined over the last five clause points.
20. Instrument-Specific Price-Vega Sensitivity: $\frac{\partial V_{ij}}{\partial \sigma}$ is the sensitivity of the price of the instrument i with respect to the implied at-the-money volatility – vega – as defined later, but must match the definition above.
21. Historical Volatility Risk Class Correction: HVR_c is the historical volatility ratio for the risk class concerned c as set out in the section on Equity Risk, which corrects for the inaccuracy in the volatility estimate σ_{jk}. The historical volatility ratio for the interest rate and the credit risk classes is fixed at 1.0.
22. 5Y Interest Rate Swap Vega: For example, the 5Y interest rate vega is the sum of all the vol-weighted interest rate caplet and swaption vegas which expire in 5 years' time; the USDJPY FX vega is the sum of all the vol-weighted USD/JPY FX vegas.
23. Gross Vega for Inflation Products: For inflation, the inflation vega is the sum of all vol-weighted inflation swaption vegas in the particular currency.

24. Instrument Net Vega Risk Exposure: Find a net vega risk exposure VR_k across instruments i to each risk factor k – which are defined in the later sections – as well as the vega concentration risk factor.

25. Portfolio/Factor IR Vega Risk: For interest-rate vega risk, these are given by the formulas

$$VR_k = VRW\left(\sum_i VR_{ik}\right)VCR_k$$

where

$$VCR_b = \max\left(1, \sqrt{\frac{\left|\sum_{i,k} VR_{ik}\right|}{VT_b}}\right)$$

where b is the bucket which contains the risk factor k.

26. Portfolio/Factor Credit Risk Vega: For credit spread vega risk, the corresponding formulas are

$$VR_k = VRW\left(\sum_i VR_{ik}\right)VCR_k$$

where

$$VCR_k = \max\left(1, \sqrt{\frac{\left|\sum_{i,j} VR_{ij}\right|}{VT_b}}\right)$$

where the sum j is taken over tenors of same issuer/seniority curve as the risk factor k, irrespective of the tenor or the payment currency.

27. Equity/FX/Commodity Vega Risk: For equity, FX, and commodity vega risks, the corresponding formulas are

$$VR_k = VRW\left(\sum_i VR_{ik}\right)VCR_k$$

where

$$VCR_k = \max\left(1, \sqrt{\frac{|\sum_k VR_{ik}|}{VT_b}}\right)$$

28. Vega Weights for the Risk Class: Here VRW is the vega risk weight for the risk class concerned as set out in the corresponding sections, and VT_b is the vega concentration threshold for the bucket – or FX category b - as given in the corresponding section.
29. Index Volatilities for Risk Classes: Note that there is a special treatment for index volatilities in Credit Qualifying, Equity, and Commodity Risk Classes.
30. Vega Exposure across Risk Class: The vega risk exposure should then be aggregated within each bucket. The buckets and the correlation parameters applicable to each risk class are set out in the Sections on Risk Weights and Correlations.
31. Cross Factor Vega Margin Expression:

$$VegaMargin = \sqrt{\sum_b {K_b}^2 + \sum_b \sum_{c \neq b} \gamma_{bc} g_{bc} S_b S_c} + K_{RESIDUAL}$$

where

$$S_b = \max\left(\min\left\{\sum_{k=1}^{K} VR_k, V_b\right\}, -K_b\right)$$

for all risk factors in bucket b.

32. Outer Correlation Adjustment Factors g_{bc}: The outer correlation adjustment factors g_{bc} are identically 1.0 for all risk classes other than interest rates, and for interest rates they are defined as

$$g_{bc} = \frac{\min(VCR_b, VCR_c)}{\max(VCR_b, VCR_c)}$$

33. Approach underlying Tension/Curvature Risk: The following step-by-step approach to capture curvature risk exposure should be applied to each risk class.
34. Expression for Tension/Curvature Risk: The curvature risk exposure for each instrument i to risk factor k is estimated using the expression

$$CVR_{ik} = \sum_j SF(t_{jk}) \sigma_{jk} \frac{\partial V_i}{\partial \sigma}$$

35. Pairwise Volatility and Vega: Here σ_{jk} and $\frac{\partial V_i}{\partial \sigma}$ are the volatility and the vega defined in the items above.
36. Incorporating Standard Option Expiry Time: t_{jk} is the expiry time in calendar days from the valuation date until the expiry of the standard option corresponding to this volatility and vega.
37. Scaling Function Vega/Gamma Linkage: is the value of the scaling function obtained from the linkage between vega and gamma for vanilla options:

$$SF(t) = 0.5 \min\left(1, \frac{14\ days}{t\ days}\right)$$

38. Scaling Function Dependence on Expiry: The scaling function is a function of expiry only, is independent of both the vega and the vol, and is show in the table below.

2w	**1m**	**3m**	**6m**	**12m**	**2y**	**3y**	**5y**	**10y**

0.500	0.230	0.077	0.038	0.019	0.010	0.006	0.004	0.002

39. Conversion of Tenors to Days: Here the tenors are converted to calendar days using the convention that *12m* equals 365 calendar days, with pro-rata scaling for other tenors so that

$$1m = \frac{365}{12}\ days$$

and

$$5y = 365 \times 5\ days$$

40. Cross Factor Curvature Risk Aggregation: The curvature risk should then be aggregated within each bucket using the following expression:

$$K_b = \sqrt{\sum_k CVR_k{}^2 + \sum_k \sum_{l \neq k} \rho_{kl}{}^2 CVR_{bk} CVR_{bl}}$$

where ρ_{kl} is the correlation applicable to each risk class as set out in the section on risk weights and correlations. Note the use of $\rho_{kl}{}^2$ rather than ρ_{kl}.

41. Instrument Risk Factor Curvature Exposure: The curvature risk exposure CVR_{ik} can then be netted across instrument i to each risk factor k, which are defined in the Sections on Risk Factors and Sensitivities. Note that the same special treatment as for vega applies for indexes in credit, equity, and commodity risk exposures.

42. The Non-Residual Curvature Standard Derivation Scaler: Margin should then be aggregated across buckets within each risk class:

$$\theta = \min\left(\frac{\sum_{b,k} CVR_{bk}}{\sum_{b,k} |CVR_{bk}|}, 0\right)$$

and

$$\lambda = [\Phi^{-1}(0.995)^2 - 1](1+\theta) - \theta$$

where the sums are taken over all the non-residual buckets in the risk class, and $\Phi^{-1}(0.995)$ is the 99.5th percentile of the standard normal distribution.

43. Non-Residual Curvature Margin Expression: Then the non-residual curvature margin is

$$CurvatureMargin_{NON-RESIDUAL} = \max\left(\sum_{b,k} CVR_{bk} + \lambda\sqrt{\sum_b {K_b}^2 + \sum_b \sum_{c \neq b} {\gamma_{bc}}^2 S_b S_c}\,, 0\right)$$

where

$$S_b = \max\left(\min\left\{\sum_{k=1}^{K} VR_k\,, V_b\right\}, -K_b\right)$$

44. The Residual Curvature Margin Expression: Similarly the residual equivalents are defined as

$$\theta_{RESIDUAL} = \min\left(\frac{\sum_k CVR_{RESIDUAL,k}}{\sum_k \left|CVR_{RESIDUAL,k}\right|}, 0\right)$$

and

$$\lambda_{RESIDUAL} = [\Phi^{-1}(0.995)^2 - 1](1+\theta_{RESIDUAL}) - \theta_{RESIDUAL}$$

$$CurvatureMargin_{RESIDUAL} = \max\left(\sum_k CVR_{RESIDUAL,k} + \lambda_{RESIDUAL} K_{RESIDUAL}, 0\right)$$

45. Applying the Risk Factor Correlations: Here the correlation parameters γ_{bc} applicable to each risk class are set out in the Sections on Risk Weights and Correlations. Note the use of ${\gamma_{bc}}^2$ rather than γ_{bc}.

46. Expression for Composite Curvature Margin: The total curvature margin is defined to be the sum of the two curvature terms

$$CurvatureMargin = CurvatureMargin_{RESIDUAL} + CurvatureMargin_{NON-RESIDUAL}$$

47. Interest Rate Curvature Margin Scaler: For the interest rate risk class only, the *Curvature Margin* must be multiplied by a scaler of 2.3. This provisional adjustment addresses a known weakness in the expression that converts gamma into curvature, which will be properly addressed in a later version of the model.

48. Base Correlation Model Credit Charge: Credit Qualifying Only – Instruments whose prices is sensitive to the correlation between the defaults of different credits within an index or a basket – such as CDO tranches – are subject to Base Correlation margin charges described below. Instruments not sensitive to base correlation are not subject to Base Correlation margin requirements.

49. Base Correlation Risk Exposure Approach: The following step-by-step approach to capture the Base Correlation risk exposure should be applied to the Credit Qualifying Risk Class.

50. Base Correlation Risk Factor Sensitivity: The net sensitivity across instruments to each Base Correlation risk factor k is calculated, where k is the index family such as CDX/IG.

51. Risk Weight applied to Sensitivity: Weight the net sensitivity s_k to each risk factor k by the corresponding risk weight RW_k specified in the section on Credit Qualifying Risk:

$$WS_k = RW_k s_k$$

52. Aggregation of the Weighted Sensitivities: Weighted sensitivities should then be aggregated to calculate the Base Correlation Margin as follows:

$$BaseCorrelationMargin = \sqrt{\sum_{k} {WS_k}^2 + \sum_{k} \sum_{l \neq k} \rho_{kl} WS_k WS_l}$$

The correlation parameters are set out in the section on Credit Qualifying Risk Weights and Correlations.

References

- International Swaps and Derivatives Association (2016): ISDA SIMM 2.0 Methodology
- International Swaps and Derivatives Association (2017): ISDA SIMM 2.1 Methodology

Dynamic Initial Margin Impact on Exposure

Abstract

1. VM and IM Collateralized Positions: This chapter leverages the new framework for collateralized exposure modeling introduced by Andersen, Pykhtin, and Sokol (2017b) to analyze credit risk positions collateralized with both initial and variation margin. Special attention is paid to the dynamics BCBS-IOSCO uncleared margin rules soon to be mandated for bilateral inter-dealer trading in OTC derivatives markets.
2. Insufficiency of BCBS IOSCO Rules: While these rules set the initial margin at 99^{th} 2-week percentile level and aim to all but eliminate portfolio close-out risk, this chapter demonstrates that the trade flow effects can result in exposures being reduced significantly less than expected.
3. Efficient IM Simulation on an MC Path: The analysis is supplemented with several practical schemes for estimating IM on a simulation path, and for improving the speed and the stability of the exposure simulation.
4. Handling Trade Flow Exposure Spikes: This chapter also briefly discusses potential ways to adjust the margin framework to more effectively deal with exposures arising from trade flow events.

Introduction

1. Collateralization based on Variation Margin: Collateralization has long been a way of mitigating counterparty risk in OTC bilateral trading. The most common collateral mechanism is *variation margin* (VM) which aims to keep the threshold gap between portfolio value and posted collateral below a certain, possibly stochastic, threshold.. While it is
2. Imperfect Collateralization under VM Schemes: Even when the thresholds for the VM are set to zero, however, there remains residual exposure to the counterparty default resulting from a sequence of contractual and operational time lags, starting from the last snapshot of the market for which the counterparty would post in full the required VM to the termination date after the counterparty's default. The various collateral mechanisms, including the precise definition of the variation margin thresholds, are typically captured in the ISDA Credit Support Annex (CSA) – the portfolio level legal agreement that supplements the ISDA Master Agreement.
3. MPoR - Margin Period of Risk: The aggregation of these lags results in a time period called the *Margin Period of Risk* (MPoR) during which the gap between the portfolio value and collateral can widen. The length of the MPoR is a critical input to any model of collateral exposure.
4. IM Supplementing the VM Collateral: Posting of *initial margin* (IM) to supplement VM provides the dealers with a mechanism to reduce the residual exposure resulting from market risk over MPoR. While it is often believed that the IM is posted strictly in addition to the VM, many CSAs intermingle the two types of collateral by letting IM affect the threshold computation of VM.
5. Genesis and Structure of IM: Historically, IM in bilateral trading has been mostly reserved for dealer counterparties deemed as high-risk – e.g., hedge funds – and typically done as a trade level calculation, established in term sheets at the transaction time of each trade. This type of IM posting is normally deterministic and either stays fixed over the life of a trade or amortizes down according to a pre-specified schedule.
6. Ne Basel Rules for IM: In the inter-dealer bilateral OTC world, changes to the long-standing VM and IM collateral practices are now imminent. BCBS and IOSCO proposed (Basel

Committee on Banking Supervision (2013)) and later finalized (Basel Committee on Banking Supervision (2015)) new uncleared margin rules for bilateral trading.

7. Key Features of the UMR: Under UMR, VM thresholds are forced to zero, and IM must be posted bilaterally into segregated accounts at the netting set level, by either using an internal model or by a lookup in a standardized schedule.
8. IM as a Horizon-Specific VaR: If an internal model is used, IM must be calculated as a netting set Value-at-Risk (VaR) for a 99% confidence level. The horizon used in this calculation equals $9 + a$ business days, where a is the re-margining period - 1 business day under US rules.
9. No Cross Asset Class Diversification: In these calculations, diversification across distinct asset classes is not recognized, and calibration of the IM internal model for each asset class must include a period of stress for that asset class. To reduce the potential for margin disputes and to increase the overall market transparency, ISDA has proposed a standardized sensitivity-based IM calculator known as SIMM (Standard Initial Margin Model) (International Swaps and Derivatives Association (2016)). As a practical matter it is expected that virtually all dealers will use SIMM for their day-to-day IM calculations.
10. Dynamic Nature of Initial Margin: Under UMR required levels of IM continuously change as trade cash flows are paid, new trades are booked, or markets move, and dealers regularly need to call for more IM or to return excess IM. This dynamic aspect of IM requirements makes the modeling of the future exposures a challenge.
11. Modeling under Dynamics IM and VM: This chapter discusses modeling credit exposure in the presence of dynamic IM and questions the conventional wisdom that IM essentially eliminates counterparty risk. Leaning on the recent results from Andersen, Pykhtin, and Sokol (2017a), it starts by formulating a general model of exposure in the presence of VM and/or IM.
12. Simple Case - No Trade Flows: The resulting framework is first applied to the simple case where no trade flows take place within the MPoR. For processes with Gaussian increments – e.g., an Ito process – a limiting scale factor that converts the IM free expected exposure (EE) to IM-protected EE is derived, for sufficiently small MPoR.

13. IM vs no IM EE Ratio: The universal value depends only on the IM confidence level and the ratio of the IM horizon to the MPoR; it equals 0.85% at the BCBS-IOSCO confidence level of 99%, provided the IM horizon equals the MPoR. While conceptually the IM and the MPoR horizons are identical, a prudent MPoR for internal calculations may differ from the regulatory minimum IM horizon.
14. No-Trade-Flow Exposure Reduction: While some deviations from this universal limit due to a non-infinitesimal MPoR are to be expected, the reduction of EE by about 2 orders of magnitude is, as will be demonstrated below, generally about right when no trade flows are present within the MPoR.
15. Exposure Spikes from Trade Flows: For those periods for which trade flows do take place within the MPoR, however, any trade payment flowing away from the dealer will result in a spike in the EE profile. Without IM these spikes can make a fairly significant contribution to the Credit Valuation Adjustment (CVA) – say, 20% of an interest rate swap's total CVA may originate with spikes – but the CVA would still mostly be determined by the EE level between the spikes.
16. Exposure Spikes vs. Dynamic IM: This chapter shows that while IM is effective in suppressing the EE *between* spikes, it will often fails to significantly suppress the spikes themselves. As a result the relative contribution of the spike to CVA is greatly increased in the presence of IM – e.g., for a single interest rate swap, the spike's contribution to the CVA can be well about 90% for a position with IM.
17. Corresponding Impact on the CVA: Accounting for the spikes, the IM reduces the CVA by much less than two orders of magnitude one might expect, with the reduction for the interest rate swaps often being less than a factor of 10.
18. Estimating the Path-wise IM: The final part of this chapter discusses the practical approaches to calculating the EE profiles in the presence of IM. The first step in this calculation is the estimation of IM on simulation paths, which can be done by parametric regression or by kernel regression.
19. IM Covering Few Netting Trades: When IM covers an insignificant number of trades in the netting set, IM calculated on the path can be subtracted from the no-IM exposure realized on that path to generate EE profiles.

20. IM covering most Netting Trades: However, when most trades of the netting set are covered by the IM, this approach can be problematic because of excessive simulation noise and other errors. An alternative approach that dampens the noise is proposed, and is generally more accurate.
21. Suggested Alterations to the Exposure Rules: This chapter concludes by summarizing the results and briefly discussing the possible modifications to trade and collateral documentation that would make IM more effective in reducing residual counterparty risk.

Exposure in the Presence of IM and VM

1. VM/IM over single Netting Set: Consider a dealer D that has a portfolio of OTC derivatives contracts traded with a counterparty C. Suppose for simplicity that the entire derivatives portfolio is covered by a single netting agreement, which is supported by a margin agreement that includes VM and may include IM on a subset of the portfolio.
2. Exposure of Client to Dealer: Quiet generally the exposure of D to the default of client C measured at time t – assumed to be the early termination time after C's default – is given by

$$E(t) = [V(t) - VM(t) + U(t) - IM\ (t)]^{+}$$

   where $V(t)$ is the time t portfolio value from D's perspective; $VM(t)$ is the VM available to D at time t; $U(t)$ is the value of the trade flows scheduled to be paid by both D (negative) and C (positive) up to time t, yet unpaid as of time t; $IM(t)$ is the value of IM available to D at time t.
3. Sign of VM and IM: Notice that VM can be positive – C posts VM – or negative – D posts VM – from D's perspective. On the other hand IM is always positive as IM for both counterparties is kept in segregated accounts, whereby IM posted by D does not contribute to D's exposure to the default of C.

4. Modeling Individual Terms in the Exposure: The above equation for $E(t)$ specifies the exposure of D to C in a generic, model-free way. To add modeling detail, this chapter assumes that D and C both post VM with zero-threshold and are required to post BCBS-IOSCO compliant IM to a segregated account. The modeling of each of these terms VM, U, and IM are dealt with turn by turn.

Modeling VM

1. Concurrent Dealer/Client VM Stoppage: The length of the $MPoR$ denoted by δ_C defines the last portfolio valuation date

$$t_0 = t - \delta_C$$

prior to the termination date t - after C's default – for which C delivers VM to D. A common assumption – denoted here as the *Classical Model* – assumes that D stops paying VM to C at the exact same time C stops posting to D.

2. Expression for Classical Model VM: That is, the VM in the equation above is the VM prescribed for the margin agreement for the portfolio valuation date

$$t_0 = t - \delta_C$$

Ignoring minimum transfer amount and rounding, the prescribed VM in the Classical Model is thus simply

$$VM_{CLASSICAL}(t) = V(t_c) = V(t - \delta_C)$$

3. Advanced Model Incorporating Operational Details: In the *Advanced Model* of Andersen, Pykhtin, and Sokol (2017a), operational aspects and gamesmanship of margin disputes are considered in more detail, leading to the more realistic assumption that D may continue to post VM to C for some period of time, even after C stops posting.
4. Non-Concurrent Dealer/Client VM Stoppages: The model introduces another parameter

$$\delta_D \geq \delta_C$$

that specifies that last portfolio valuation date

$$t_D = t - \delta_D$$

for which D delivers VM to C. For the portfolio valuation dates

$$T_i \in [t_C, t_D]$$

D would post VM to C when the portfolio value decreases, but will receive no VM from C when the portfolio value increases.
5. Expression for Advanced Model VM: This results in VM of

$$VM_{ADVANCED}(t) = \min_{T_i \in [t_C, t_D]} V(T_i)$$

This equation of course reduces to

$$VM_{CLASSICAL}(t) = V(t_c) = V(t - \delta_C)$$

when one sets

$$\delta_D = \delta_C$$

Modeling *U*

1. <u>The Classical+ Version Trade Flow *Currentness*</u>: In the most conventional version of the Classical model – denoted here *Classical*+ - it is assumed that all trade flows are paid by both C and D for the entire $MPoR$ up to and including the termination date, i.e., in the time interval $[t_C, t]$ - time here is measured in discrete business days, so that $[u, s]$ is equivalent to $[u + 1\ BD, s]$. This assumption simply amounts to setting

$$U_{CLASSICAL+}(t) = 0$$

2. <u>Trade Flow Exposure Profile Spikes</u>: One of the prominent features of the Classical+ model is that the time 0 expectation of $E(t)$ – denoted $EE(t)$ – will contain upward spikes whenever there is a possibility of trade flows from D to C within the interval $[t_C, t]$.
3. <u>Absence of Client Margin Flows</u>: These spikes appear because, by the classical model's assumption, C makes no margin payments during the $MPoR$ and would consequently fail to post an offsetting VM to D after D makes a trade payment to C. In the Classical model, D will also not post VM to C in the event of trade payment from C to D, which results in a negative jump in the exposure. However, these scenarios do not fully offset the scenarios where D makes a trade payment because the zero floor in the exposure definitions effectively limits the size of the downward exposure jump.
4. <u>Sparse Fixed Time Exposure Grid</u>: For dealers having a sparse fixed time exposure grid, the alignment of grid nodes relative to trade flows will add numerical artifacts to genuine spikes, causing EE exposure to appear and disappear as the calendar date moves. As a consequence an undesirable instability in the EE and the CVA is introduced.

5. Classical- Version – No Trade Flows: An easy way to eliminate exposure spikes is to assume that neither C nor D make any trade payment inside the $MPoR$. The resulting model – here denoted Classical- - consequently assumes that

$$U_{CLASSICAL-}(t) = TF_{NET}(t;\ (t_C, t])$$

where $TF_{NET}(t;\ (s, u])$ denotes the time t of all net trade flows payable in the interval $(s, u]$.
6. Implicit Simplifications in Classical- and Classical+: It should be evident that neither the Classical- nor the Classical+ assumptions on trade flows are entirely realistic; in the beginning of the $MPoR$ both C and D are likely to make trade payments, while at the end of the $MPoR$ neither C no D are likely making any trade payments.
7. Last Dealer/Client Trade Flows: To capture this behavior, Andersen, Pykhtin, and Sokol (2017a) add two more parameters in the model ${\delta_C}'$ and

$${\delta_D}' \leq \delta_C$$

that specify the last dates

$${t_C}' = t - {\delta_C}'$$

and

$${t_D}' = t - {\delta_D}'$$

for D at which trade payments are made prior to closeout at t.
8. Advanced Model Trade Flow Expansion: This results in unpaid trade flow terms of

$$U_{ADVANCED}(t) = TF_{C \rightarrow D}(t;\ ({t_C}', {t_D}']) + TF_{NET}(t;\ ({t_D}', t])$$

where an arrow indicates the direction of the trade flows and $C \rightarrow D$ $(D \rightarrow C)$ trade flows have positive (negative) sign.

9. Advanced Model Exposure Profile Structure: The EE profiles obtained with the Advanced Model contain spikes that are typically narrower and have a more complex structure than spikes under the Classical+ model. Rather than being unwelcome noise, it is argued in Andersen, Pykhtin, and Sokol (2017a) that spikes in EE profiles are important features that represent actual risk.
10. Approximating the Trade Flow Spikes: To eliminate any numerical instability associated with the spikes an approximation was proposed in Andersen, Pykhtin, and Sokol (2017a) for calculation of the EE on a daily time grid without daily re-evaluation of the portfolio.

Modeling IM

1. Expression for Netting Set IM: Following the BCBS-IOSCO restrictions on diversification, the IM is defined for the netting set as a sum of the IM's over K asset classes as

$$IM(t) = \sum_{k=1}^{K} IM_k(t)$$

   Current netting rules have

$$K = 4$$

   k can be one of Rates/FX, Credit, Equity, and Commodity.
2. Netting Set Granularity IM Value: Let $V_k(t)$ denote the value at time t of all trades in the netting set to asset class k and are subject to IM requirements. Note that only trades executed after UMR go-live will be covered by the IM. The case where only a subset of the netting set is covered by the IM will therefore be common in the near future.

3. Horizon/Confidence Based IM Definition: For an asset class k we define IM as the quantile at the confidence level q of the *clean* portfolio value increment over BCBS-IOSCO IM horizon δ_{IM} - which may or may not coincide with δ_C - conditional on all the information available at t_C.
4. Mathematical Expression for the IM: That is

$$IM_k(t) = Q_q\left[V_k(t_C + \delta_{IM}) + TF_{NET,k}(t_C + \delta_{IM};\ (t_C, t_C + \delta_{IM}]) - V_k(t_C) | \mathcal{F}_{t_C}\right]$$

where $Q_q(\cdot \mid \mathcal{F}_s)$ denotes the quantile of confidence level q conditional on information available at time s. Note that the above expression assumes that C stops posting IM at the same time it stops posting VM; hence IM is calculated as of

$$t_C = t - \delta_C$$

Summary and Calibration

1. Classical+/Classical-/Advanced Model Exposures: To summarize, three different models have been outlined in the generic exposure calculation

$$E(t) = [V(t) - VM(t) + U(t) - IM(t)]^+$$

for the Classical-, Classical+, and Advanced Models. Collecting results, one has

$$E_{CLASSICAL+}(t) = [V(t) - V(t - \delta_C) - IM(t)]^+$$

$$E_{CLASSICAL-}(t) = [V(t) - V(t - \delta_C) + TF_{NET}(t;\ (t_C, t]) - IM(t)]^+$$

$$E_{ADVANCED}(t) = \left[V(t) - \min_{T_i \in [t_C, t_D]} V(T_i) + TF_{C \to D}(t;\ (t_C', t_D']) + TF_{NET}(t;\ (t_D', t]) - IM(t)\right]^+$$

where for all the three models IM is computed as

$$IM_k(t) = Q_q\left[V_k(t_C + \delta_{IM}) + TF_{NET,k}(t_C + \delta_{IM};\ (t_C, t_C + \delta_{IM}]) - V_k(t_C) |\ \mathcal{F}_{t_C}\right]$$

2. IMA/CSA Dealer/Client Suspension Rights: In practice, the calibration of the time parameters of the Advanced Model should be informed by the dealers' legal rights and its aggressiveness in pursuing them. For the former, Andersen, Pykhtin, and Sokol (2017a) contain a full discussion of ISDA Master Agreements. But note that once a notice of Potential Event of Default (PED) has been served, the *suspension rights* of ISDA Master Agreement 2 (a) (iii) and its accompanying Credit Support Annex Paragraph 4 (a) allow a bank to suspend all trade-related and collateral-related payments to its counterparty until the PED has been cured.
3. Risks of Enforcing the Rights: The extent to which the suspension rights are actually exercised in practice, however, depends on the dealer and its operating model, as well as the specifics of each of the PED. One particular danger in the aggressive and immediate enforcement of the suspension rights is that the original PED might be ruled unjustified. Should this happen, the dealer can inadvertently commit a breach of contract, which, especially in the presence of the cross-default provisions, can have serious consequences for the dealer.
4. Calibrating the MPoR Lag Parameter: With the four time parameters, the Advanced Model allows for a greater flexibility in modeling its risk tolerance and procedures for exercising its suspension rights. A dealer may, in fact, calibrate these parameters differently for different counter-parties to reflect its risk management practices towards counterparties of a given type. One may make the time parameters stochastic, e.g., by making the legs a function of the

exposure magnitude. This way one can, say, model the fact that a dealer might tighten its operational controls when the exposures are high.

5. Prototypical Aggressive vs. Conservative Timelines: Additional discussions can be found in Andersen, Pykhtin, and Sokol (2017a), which also provides some prototypical parameter settings; the Aggressive Calibration (D can always sniff out financial distress in its clients and is swift and aggressive in enforcing its legal rights) and the Conservative Calibration (D is deliberate and cautious in enforcing its rights, and acknowledges potential for operational errors and for rapid, unpredictable deterioration in client credit). For the numerical results in this chapter, the values of the time parameters are mostly set in between the Aggressive and the Conservative.

The Impact of IM: No Trade Flows within the MPoR

1. No Trade Flow IM EE Impact: This section examines the impact of IM on EE when there are no trade flows within the MPoR. For simplicity the Classical Model is considered, and the entire netting set is assumed to be covered by the IM and comprised of all trades belonging to the same asset class. Results for the Advanced Models are similar, as shown in the later sections.
2. Estimating the *IM Efficiency Ratio*: In the absence of trade flows on $(t_C, t]$

$$E_{CLASSICAL+}(t) = [V(t) - V(t - \delta_C) - IM(t)]^+$$

$$E_{CLASSICAL-}(t) = [V(t) - V(t - \delta_C) + TF_{NET}(t;\ (t_C, t]) - IM(t)]^+$$

show that the Classical Model computes the expected exposure as

$$EE(t) = \mathbb{E}\left[\left\{V(t) - V(t_C) - Q_q\left[V_R(t_C + \delta_{IM}) | \mathcal{F}_{t_C}\right]\right\}^+\right]$$

$$t_C = t - \delta_C$$

where $\mathbb{E}[\cdot]$ is the expectation operator. In the absence of IM this expression would be

$$EE_0(t) = \mathbb{E}[\{V(t) - V(t_C)\}^+]$$

One of the objectives is to establish meaningful estimates of the IM *Efficiency Ratio*

$$\lambda(t) \triangleq \frac{EE(t)}{EE_0(t)}$$

Local Gaussian Approximation

1. Portfolio Value following Ito Process: Suppose that the portfolio value $V(t)$ follows an Ito process:

$$\Delta V(t) = \mu(t)\Delta t + s^T(t)\Delta W(t)$$

where $W(t)$ is a vector of independent Brownian motions, and $\mu(t)$ and $s^T(t)$ are well-behaved processes – with $s^T(t)$ being vector-valued – adapted to $W(t)$. Notice that both μ and s^T may depend on the evolution of multiple risk factors prior to time t. For convenience denote

$$\sigma(t) = |s^T(t)|$$

2. Portfolio Increment as a Gaussian: Then, for a sufficiently small horizon δ, the increment of the portfolio value over $[t_C,\ t_C + \delta]$ conditional on $\mathcal{F}_{t_C}$ is well-approximated by a Gaussian distribution with a mean $\mu(t_C)\delta$ and a standard deviation $\frac{\sigma(t_C)}{\sqrt{\delta}}$.
3. Approximation for the Expected Exposure: Assuming $\sigma(t_C) > 0$ the drift term may be ignored for small δ. Under the Gaussian approximation above it is then straightforward to approximate the expectation in

$$EE(t) = \mathbb{E}\left[\left\{V(t) - V(t_C) - Q_q\left[V_R(t_C + \delta_{IM}) \middle| \mathcal{F}_{t_C}\right]\right\}^+\right]$$

$$t_C = t - \delta_C$$

in the closed form

$$EE(t) \approx \mathbb{E}[\sigma(t_C)]\sqrt{\delta_C}\left[\phi\big(z(q)\big) - z(q)\Phi\big(-z(q)\big)\right]$$

$$z(q) \triangleq \sqrt{\frac{\delta_{IM}}{\delta_C}}\Phi^{-1}(q)$$

where ϕ and Φ are the standard Gaussian PDF and CDF, respectively.
4. Expression for *IM Efficiency Ratio*: Similarly

$$EE_0(t) \approx \mathbb{E}[\sigma(t_C)]\sqrt{\delta_C}\phi(0)$$

so that λ in

$$\lambda(t) \triangleq \frac{EE(t)}{EE_0(t)}$$

is approximated by

$$\lambda(t) \approx \frac{\phi\big(z(q)\big) - z(q)\Phi\big(-z(q)\big)}{\phi(0)}$$

5. λ Dependence: $q/MPoR$ Ratio: Interestingly, the multiplier $\lambda(t)$ above is independent of t and depends only on two quantities – the confidence level q used for specifying the IM, and the ratio of the IM horizon to the $MPoR$. It is emphasized that the above expression for $\lambda(t)$ was derived under weak conditions; relying on the local normality assumptions for portfolio value increments. Otherwise, the ratio is model-free; no further assumptions are made on the distribution of the portfolio value or on the dependence of the local volatility on risk factors.
6. λ Exact under Brownian Motion: The special case for which the above expression for λ becomes exact is when the portfolio process follows a Brownian motion, a case discussed in Gregory (2015). The point here is that the $\lambda(t)$ above constitutes a small- δ limit, and a useful approximation, for a much broader class of processes. In fact it can be extended to jump-diffusion processes, provided that the portfolio jumps are approximately Gaussian.
7. λ for the BCBS-IOSCO Parameters: Andersen, Pykhtin, and Sokol (2017b) illustrate the value of $\lambda(t)$ for the case

$$\delta_{IM} = \delta_C$$

graphed as a function of q. For the value of the confidence level

$$q = 99\%$$

specified by the BCBS-IOSCO framework

$$\lambda(t) \triangleq \frac{EE(t)}{EE_0(t)}$$

results in a value of

$$\lambda = 0.85\%$$

i.e., IM is anticipated to reduce the expected exposure by a factor of 117.

Numerical Tests

1. Portfolio following Geometric Brownian Motion: Recall that the approximation

$$\lambda(t) \triangleq \frac{EE(t)}{EE_0(t)}$$

hinges on the $MPoR$ being small, but it is ex-ante unclear if, say, the commonly used value of $10\ BD$ is small enough. To investigate the potential magnitude of errors introduced by the non-infinitesimal $MPoR$, assume now – in a slight abuse of the notation – that $V(t)$ follows a geometric Brownian motion with a constant volatility σ such that

$$\frac{\Delta V(t)}{V(t)} \cong \mathcal{O}(\Delta t) + \sigma \Delta W(t)$$

where $W(t)$ is one-dimensional Brownian motion.

2. Left/Right Portfolio Value Behavior: Compared to a Gaussian distribution, the distribution of $V(t)$ over a finite time interval is skewed left for

$$V(0) < 0$$

and right for

$$V(0) > 0$$

with V never crossing the origin. While this specification may seem restrictive, its only purpose is to test the accuracy of the local Gaussian approximation.

3. <u>Lognormal Portfolio Dynamics λ Estimate</u>: Assuming for simplicity that

$$\delta_{IM} = \delta_C$$

applying the calculations of the previous section to the lognormal setup, and neglecting terms of order δ or higher, one gets

$$\lambda(t) = \psi \frac{1 - \Phi\left(\Phi^{-1}(q) - \psi\sigma\sqrt{\delta_C}\right) - (1-q)e^{\psi\sigma\sqrt{\delta_C}\Phi^{-1}(q)}}{2\Phi\left(\frac{\sigma\sqrt{\delta_C}}{2}\right) - 1}$$

$$\psi \triangleq sign\big(V(0)\big)$$

4. <u>λ Dependence for Lognormal Portfolios</u>: While the multiplier in

$$\lambda(t) \approx \frac{\phi\big(z(q)\big) - z(q)\Phi\big(-z(q)\big)}{\phi(0)}$$

depends only on the confidence level q when

$$\delta_{IM} = \delta_C$$

the multiplier in

$$\lambda(t) = \psi \frac{1 - \Phi\left(\Phi^{-1}(q) - \psi\sigma\sqrt{\delta_C}\right) - (1 - q)e^{\psi\sigma\sqrt{\delta_C}\Phi^{-1}(q)}}{2\Phi\left(\frac{\sigma\sqrt{\delta_C}}{2}\right) - 1}$$

$$\psi \triangleq sign\big(V(0)\big)$$

additionally depends on the product $\sigma\sqrt{\delta_C}$ and on the sign of the portfolio exposure; the limit for δ_C - or σ - approaching zero can be verified to be equal to

$$\lambda(t) \approx \frac{\phi\big(z(q)\big) - z(q)\Phi\big(-z(q)\big)}{\phi(0)}$$

Andersen, Pykhtin, and Sokol (2017b) compare

$$\lambda(t) \approx \frac{\phi\big(z(q)\big) - z(q)\Phi\big(-z(q)\big)}{\phi(0)}$$

to

$$\lambda(t) = \psi \frac{1 - \Phi\left(\Phi^{-1}(q) - \psi\sigma\sqrt{\delta_C}\right) - (1 - q)e^{\psi\sigma\sqrt{\delta_C}\Phi^{-1}(q)}}{2\Phi\left(\frac{\sigma\sqrt{\delta_C}}{2}\right) - 1}$$

$$\psi \triangleq sign\big(V(0)\big)$$

for various levels of σ.

5. Long/Short Portfolio Volatility Impact: As expected, Andersen, Pykhtin, and Sokol (2017b) show that the deviation of the local lognormal multiplier from the Gaussian one increases with the volatility σ - more precisely, with the product $\sigma\sqrt{\delta_C}$. Furthermore the multiplier for the case

$$\psi = 1$$

is always greater – and the multiplier for the case

$$\psi = -1$$

is always lesser – than the limit case of 0.85% - an easily seen consequence of the fact that the relevant distribution tail for the cases

$$\psi = 1$$

is thicker – and

$$\psi = -1$$

is thinner – than the tail of the Gaussian distribution, as discussed earlier.

6. No Trade Flow λ Estimate: Yet, overall the results illustrate that, for the case of the $10\ BD$ horizon, the local Gaussian approximation produces reasonable values of the scaling multiplier at most levels of relative volatility. Certainly, the results support the idea that introducing IM at the level of

$$q = 99\%$$

should result in about two orders of magnitude reduction in the EE, when no trade flows occur within the $MPoR$.

The Impact of IM: Trade Flows within the MPoR

1. Impact of Trade Flows – Introduction: This section considers the efficacy of the IM in the more complicated case where trade flows are scheduled to take place inside the $MPoR$. In practice, many portfolios produce trade flows every business day, so this case is of considerable relevance. The introduction of cash flows will make the computation of EE both more complex and more model-dependent, so additional care is needed here to distinguish between the Classical+, the Classical- and the Advanced Models.
2. Trade Flows in the Classical- Model: Unless one operates under the assumption of the Classical- model assumptions of no trade flows paid by either C or D within the $MPoR$, any possibility of D making a trade payment to C in the future results in a spike in the EE profile.
3. Origin of Trade Flow Spikes: These spikes will appear because the portfolio value will jump following D's payment, but C would fail to post or return the VM associated with the jump. By way of example, suppose there is a trade payment due at future time u where D is a net payer.
4. Range of Trade Flow Spikes: A spike in the EE profile originating with this cash flow will appear for a range of termination flows t such that
    a. u lies within the $MPoR$ – mathematically

    $$t \in [u, u + \delta_C)$$

    b. D would actually make the payment.
5. Trade Flows in the Classical+ Model: The assumption in the Classical+ model is that D – as well as C – would make contractual trade payments for the entire $MPoR$ so a spike of width δ_C will appear in $EE(t)$ for the range

$$t \in [u, u + \delta_C)$$

6. Trade Flows in the Advanced Model: Here it is assumed that D would make the contractual trade payments from the beginning $t - \delta_C$ of the $MPoR$ to the time $t - t_D{}'$ so a spike of width $\delta_C - \delta_D{}'$ would appear in $EE(t)$ for the range

$$t \in [u + \delta_D{}', u + \delta_C)$$

7. Advanced vs. Classical Spike Width: While spikes produced by the Advanced model would always be narrower than those produced by the Classical+ model, the former is often taller. In particular, the Advanced Model, unlike the Classical+ Model, contains a range of trade payment times

$$t \in [t - \delta_D{}', t - \delta_C)$$

   where D would make a contractual trade payment but C would not.
8. Advanced vs Classical Spike Peak: This creates a narrow peak for the range

$$t \in [u + \delta_D{}', u + \delta_C)$$

   at the left edge of the spike. This peak can be very high when C's and D's payments do not net – e.g., because of payments in different currencies – and makes an extra contribution to the EE that is not present in the Classical+ model.
9. IM Impact on the Margin Exposure: IM is now introduced to the exposure computation. More specifically assume that the netting set is composed of trades belonging to a single asset class, and in addition to the VM, is fully covered by dynamic IM as defined by UMR in

$$IM_k(t) = Q_q\left[V_k(t_C + \delta_{IM}) + TF_{NET,k}(t_C + \delta_{IM};\ (t_C, t_C + \delta_{IM}]) - V_k(t_C) \middle| \mathcal{F}_{t_C}\right]$$

10. IM Impact on Exposure Spikes: As just discovered, in the areas *between* the EE spikes, IM will reduce EE by a factor given approximately by

$$\lambda(t) \approx \frac{\phi\big(z(q)\big) - z(q)\Phi\big(-z(q)\big)}{\phi(0)}$$

The question remains on how the exposure spikes will be affected by IM. The answer to that question is: *It depends*.

11. Payment Count Decrease with Time: As the simulation time passes, the portfolio will closer to maturity, and fewer and fewer trade payments will remain. Because of this *amortization* effect, the width of the distribution of the portfolio value increments on each path becomes smaller, and being the VaR of increments, the IM is reduced.
12. VM vs Trade Spike Ratio: On the other hand, trade payments do not generally become smaller as the trade approaches its maturity, and in fact can often become larger due to risk factor diffusion. The effectiveness of the IM in reducing the EE spikes depends on the size of the trade payments relative to the portfolio value increment distribution and can therefore in many cases decline towards the maturity of the portfolio.

Expected Exposure – Numerical Example #1

1. Fix-Float Two-Way CSA: As an example consider a 2Y interest rate swap under a two-way CSA with zero threshold daily VM, but without IM. Assume that D pays a fixed rate of 2% semi-annually, and receives a floating rate quarterly. Setting the initial interest rate curve at 2% - quarterly compounding – with 50% lognormal volatility, Andersen, Pykhtin, and Sokol (2017b) illustrate the exposure profiles for the Classical+, the Classical-, and the Advanced models.
2. Spikes in both Classical+/Advanced: As expected both the Classical+ and the Advanced models produce spikes around the quarterly dates when the payment takes place. The nature of the spikes depends upon whether a fixed payment is made or not.

3. Fixed Payment Time Points - Twice a Year: As C pays quarterly on its semi-annual fixed payments, D will pay on average twice as much as C, resulting in high upward spikes in the exposure profile. As discussed above, the width of the Classical+ model spikes is

$$\delta_C = 10\ BD$$

while the width of the Advanced model spikes is

$$\delta_C - \delta_D{}' = 6\ BD$$

4. Floating Only Payment Points - Twice a Year: On quarterly payment dates when no fixed payment is due by D, C still pays a floating rate for the quarter. This results in downward spikes in the EE profile – C makes a trade payment and defaults before D must return the VM. The Classical+ model assumes that the trade payments are made by C (D) and the margin payments are not made by D (C) for the entire $MPoR$. Under these unrealistic assumptions the downward spike width is

$$\delta_C = 10\ BD$$

The Advanced model, on the other hand, assumes that C would make the trade payments over the time interval $\delta_C - \delta_D$ from which the interval $\delta_C - \delta_D{}'$ over which D would pay VM to C would be subtracted. In the aggregate, the width of the downward spikes in the Advanced model is therefore

$$\delta_C - \delta_D{}' = 2\ BD$$

5. Intra-Spike Change in VM: Between the spikes, the EE profile produced by the Advanced model is about 22% higher than the one produced by the Classical model. The difference originates with the VM specifications – the Classical models assume that C and D stop paying at the beginning of the $MPoR$ – see

$$VM_{CLASSICAL}(t) = V(t_c) = V(t - \delta_C)$$

while the Advanced model assumes that D would post VM for some time after D has stopped doing so – see

$$VM_{ADVANCED}(t) = \min_{T_i \in [t_C, t_D]} V(T_i)$$

6. Impact of IM Numerical Setting: Let us consider the impact of IM on the EE profile for this interest rate swap. All the assumptions about the $MPoR$ and the CSA are retained, and the IM horizon is set at

$$\delta_{IM} = \delta_C = 10\ BD$$

Since the dynamics of the swap value are modeled with a single risk factor, the IM can be conveniently calculated on each path exactly by stressing the risk factor to the 99% level over the IM horizon each path. Andersen, Pykhtin, and Sokol (2017b) illustrate the EE profile on the same scale as the no-IM result – in fact, the only part of the EE profile visible in this scale is the upward spikes of D's semi-annual payments of the fixed rate.

7. Numerical Comparison - Exposure between Spikes: To analyze the IM results, the exposure profile is broken down into two categories. First, between the spikes the EE produced by the Classical model with IM is 1.06% of the EE produced by the classical model without IM. Since the interest rate is modeled by a lognormal process, this number is closer to the value of 1.10% predicted by

$$\lambda(t) = \psi \frac{1 - \Phi\left(\Phi^{-1}(q) - \psi\sigma\sqrt{\delta_C}\right) - (1 - q)e^{\psi\sigma\sqrt{\delta_C}\Phi^{-1}(q)}}{2\Phi\left(\frac{\sigma\sqrt{\delta_C}}{2}\right) - 1}$$

$$\psi \triangleq sign\big(V(0)\big)$$

which is the lognormal case, than the value of 0.85% given by

$$\lambda(t) \approx \frac{\phi\big(z(q)\big) - z(q)\Phi\big(-z(q)\big)}{\phi(0)}$$

which is the local Gaussian approximation. Under the Advanced model, the IM scales down EE to 1.00% of its value under VM alone, which is very similar to the Classical model case.

8. Numerical Comparison - Impact of Spikes: As discussed, the degree of suppression for the spikes decreases with the simulation time, as the IM on all simulation paths shrinks with time. Because the payments at the end of a period are determined at the beginning of the period, the final payment is known exactly at time 1.75 years. So, there is in fact no IM requirement in the period from 1.75 years to 2 years. As a result the final spike is not reduced at all.
9. Impact of Mismatch on Exposure: To examine the extent to which the unequal payment is responsible for the spike dominance in the presence of IM, Andersen, Pykhtin, and Sokol (2017b) move on to a 2Y IRS with quarterly payment on both legs, with all other model assumptions being the same as in the previous example.
10. Numerical Analysis of Spike Reduction: For this swap, D is a net payer on the payment dates only on approximately half of the scenarios, so the spike height will necessarily be reduced. This is confirmed by Andersen, Pykhtin, and Sokol (2017b) where they compute the EE profile for the quarterly-quarterly swap under the Classical $\pm$ and the Advanced models. As is seen there, while the upward EE spikes are present around the payment dates in the absence of IM, the height of the spikes is, as expected, significantly lower than before. Nevertheless, the IM remains incapable of completely suppressing the spikes.

Expected Exposure: Numerical Exposure 2

1. Mismatched Fix-Float Pay Frequencies: To some extent, the spikes in the presence of IM are particularly pronounced because of unequal payment frequency on the fixed and the floating legs on the swap (e.g., semi-annual fixed against quarterly floating is the prevailing market standard in the US).
2. Matched Fix-Float Pay Frequencies: Indeed, on the semi-annual payment dates, the fixed leg pays, on average, twice as much as the floating leg, so D is a net payer on the vast majority of the scenarios, which in turn results in sizeable semi-annual upward spikes when no IM is present.

The Impact of IM on CVA

1. CVA as Period EE Proxy: CVA constitutes a convenient condensation of EE profiles into a single number, of considerable practical relevance.
2. Numerical Estimation of the CVA: To demonstrate the impact of IM on CVA for the swap examples above, Andersen, Pykhtin, and Sokol (2017b) show the CVA calculated from the expected exposure profiles. To illustrate the impact of the VM on the uncollateralized exposure, all CVA numbers are shown relative to the CVA of an otherwise uncollateralized swap.
3. CVA in the Semi-Annual Case: Since the Classical- model does not have spikes, CVA in this model is reduced by about 2 orders of magnitude by the IM, as one would expect from

$$\lambda(t) \approx \frac{\phi\big(z(q)\big) - z(q)\Phi\big(-z(q)\big)}{\phi(0)}$$

The presence of spikes, however, reduces the effectiveness of the IM significantly; for the case of semi-annual fixed payments, the CVA with IM is about 24% of the CVA without the IM for the Classical+ model and 15% of the Advanced model.

4. CVA in Quarterly Float Case: For the case of quarterly fixed payments, the reduced height of the spikes renders the IM noticeably more effective in reducing the CVA; the CVA with IM here is about 9% of the CVA with IM for the Classical+ model and about 5% for the Advanced model.
5. EE Spikes still dominate CVA: Nonetheless, when M is present, EE spikes still dominate the CVA, as can be verified for the Classical+ or Advanced model to the CVA for the Classical- model.
6. Importance of the Advanced Model: Overall the swap flows demonstrate that when the trade flows within the MPoR are properly modeled, the IM at 99% VaR may not be sufficient to achieve even a one order of magnitude reduction in the CVA. Also since spikes dominate CVA in the presence of IM, accurately modeling the trade flows within the MPoR is especially important when the IM is present. Hence the Advanced model is clearly preferable since neither the Classical+ nor the Classical- models produce reasonable CVA numbers for portfolios with IM.
7. IM CVA Advanced Model Calibration: Finally it should be noted that even if one uses only the Advanced model, the impact of IM on CVA may vary significantly, depending on the trade/portfolio details and the model calibration. In particular the following general observation can be made.
8. Payment Frequency: Higher frequency of trade payments results in more EE spikes, thus reducing the effectiveness of the IM.
9. Payment Size: Higher payment size relative to the trade/portfolio volatility results in higher EE spikes thus reducing the effectiveness of the IM.
10. Payment Asymmetry: EE spikes are especially high for payment dates where only B pays or where B is almost always the net payer. The presence of such payment dates reduces the effectiveness of the IM.
11. Model Calibration: In the Advanced Model, the width of these spikes is determined by the time interval within the $MPoR$ where D makes trade payments, i.e., $\delta_C - {\delta_D}'$. Thus larger the

δ_C – i.e., the $MPoR$ – and/or smaller the $\delta_D{}'$ - i.e., the time interval within the $MPoR$ where D does not make trade payments – would result in wider spikes, and thus reduced effectiveness of IM.

12. Illustrating the Advanced Model Effects: Andersen, Pykhtin, and Sokol (2017b) illustrate briefly the last point using additional calculation. In the swap examples above they use the *baseline* timeline calibration of the Advanced model –

$$\delta_C = 10\ BD$$

$$\delta_D = 8\ BD$$

$$\delta_C{}' = 6\ BD$$

$$\delta_D{}' = 4\ BD$$

with $MPoR$ being equal to the BCBS-IOSCO mandated IM horizon of

$$\delta_{IM} = 10\ BD$$

Their table shows CVA numbers and their ratios for two alternative calibrations of the Advanced Model mentioned earlier.

Numerical Techniques – Daily Time Grid

1. EE Estimation under Daily Resolution: The discussion so far has made obvious the importance of accurately capturing exposure spikes from trade flows. To achieve this one

would need to calculate the EE with a daily resolution, something that, if done by brute force methods, likely will not be feasible for large portfolios.

2. Coarse Grid Portfolio - First Pass: In Andersen, Pykhtin, and Sokol (2017a), the authors discuss a fast approximation that produces a reasonably accurate EE profile without a significant increase in the computation time relative to the standard, coarse-grid calculations. The method requires simulation of risk factors and trade flows with a daily resolution, but the portfolio valuations – which are normally the slowest part of the simulation process – can be computed on a much coarser time grid.
3. Second Pass using Brownian Bridge: Portfolio values on the daily grid are then obtained by Brownian Bridge interpolation between the values at the coarse grid points, with careful accounting for trade flows.

Calculation of the Path-wise IM

1. Need for Numerical IM Calculations: As per

$$IM(t) = \sum_{k=1}^{K} IM_k(t)$$

and

$$IM_k(t) = Q_q\left[V_k(t_C + \delta_{IM}) + TF_{NET,k}(t_C + \delta_{IM};\ (t_C, t_C + \delta_{IM}]) - V_k(t_C) \middle| \mathcal{F}_{t_C}\right]$$

the calculation of the IM requires dynamic knowledge of the portfolio value increments ($P\&L$) across K distinct asset classes. Since conditional distributions of the $P\&L$ are generally not known, one must rely on numerical methods to calculate IM.

2. Path-wise IM using Regression Approach: Regression approaches are useful for this, although the selection of regression variables for large diverse portfolios where IM would depend on an impracticably large number of risk factors can be difficult.
3. Regression Variable - Value of$V_k(t_C)$: To simplify the regression approach, here the portfolio value $V_k(t_C)$ is chosen as the single regression variable for the asset class k. Mathematically, the conditioning on $\mathcal{F}_{t_C}$ in

$$IM_k(t) = Q_q\left[V_k(t_C + \delta_{IM}) + TF_{NET,k}(t_C + \delta_{IM};\ (t_C, t_C + \delta_{IM}]) - V_k(t_C) |\ \mathcal{F}_{t_C}\right]$$

is replaced with the conditioning on $V_k(t_C)$ with the hope that the projection model would not have a material impact on the result.
4. Initial Margin Exposure Quantile Re-formulation: For Monte Carlo purposes these assumptions approximate

$$IM_k(t) = Q_q\left[V_k(t_C + \delta_{IM}) + TF_{NET,k}(t_C + \delta_{IM};\ (t_C, t_C + \delta_{IM}]) - V_k(t_C) |\ \mathcal{F}_{t_C}\right]$$

with

$$IM_{k,m}(t_C) \approx Q_q\left[V_k(t_C + \delta_{IM}) + TF_{NET,k}(t_C + \delta_{IM};\ (t_C, t_C + \delta_{IM}]) - V_k(t_C) |\ V_k(t_C) = V_{k,m}(t_C)\right]$$

where m designates the m^{th} simulation path and $TF_{NET,k}(\cdot)$ is the time $t_C + \delta_{IM}]$ value of all net trade flows scheduled to be paid on the interval $(t_C, t_C + \delta_{IM}]$ realized along the simulation path m.
5. Portfolio Value Process for IM: Following Andersen, Pykhtin, and Sokol (2017a) the discounting effects are ignored, and the P&L under the quantile in the right-hand side of

$$IM_{k,m}(t_C) = Q_q\left[V_k(t_C + \delta_{IM}) + TF_{NET,k}(t_C + \delta_{IM};\ (t_C, t_C + \delta_{IM}]) - V_k(t_C) |\ V_k(t_C) = V_{k,m}(t_C)\right]$$

conditional on

$$V_k(t_C) = V_{k,m}(t_C)$$

is assumed to be Gaussian with zero drift such that simply

$$IM_{k,m}(t_C) \approx \sigma_{k,m}(t_C)\sqrt{\delta_{IM}}\Phi^{-1}(q)$$

$$\sigma_{k,m}{}^2(t_C) \triangleq \frac{1}{\delta_{IM}}\mathbb{E}\left[\left\{V_k(t_C + \delta_{IM}) + TF_{NET,k}(t_C + \delta_{IM};\ (t_C, t_C + \delta_{IM}]) - V_k(t_C)\right\}^2 \mid V_k(t_C) = V_{k,m}(t_C)\right]$$

6. <u>Non Gaussian Portfolio Value Process</u>: Andersen and Pykhtin (2015) explored non-Gaussian assumptions, using kernel regressions to estimate the first four conditional moments of the $P\&L$. As kernel regression for the third and the fourth moments is prone to instability, this approach was deemed insufficiently robust.
7. <u>Conditional Variance Estimation Parameters Regression</u>: Estimation of the conditional expectation in

$$\sigma_{k,m}{}^2(t_C) \triangleq \frac{1}{\delta_{IM}}\mathbb{E}\left[\left\{V_k(t_C + \delta_{IM}) + TF_{NET,k}(t_C + \delta_{IM};\ (t_C, t_C + \delta_{IM}]) - V_k(t_C)\right\}^2 \mid V_k(t_C) = V_{k,m}(t_C)\right]$$

may be computed by, say, parametric regression or kernel regression. In parametric regression, the conditional variance is approximated as a parametric function – e.g., a polynomial – of $V_k(t_C)$ with function parameters estimated by least squares regression; see, for instance, Anfuso, Aziz, Giltinan, and Loukopoulos (2017).
8. <u>Local Standard Deviation Calculation Details</u>: In defining the local standard deviation on path m, Andersen, Pykhtin, and Sokol (2017b) follow Pykhtin (2009) and scale the

unconditional standard deviation of $V_k(t_C)$ by a ratio of two probability densities; in the denominator is the probability density of $V_k(t_C)$ on path m; in the numerator is the probability density that a normally distributed random variable with the same mean and standard deviation as $V_k(t_C)$ would have on the path m. The actual calculation of the local standard deviation is performed as presented in Pykhtin (2009).

9. Usage in Conjunction with SIMM: It is t be noted in passing that if a dealer uses an out-of-model margin calculator – e.g., the SIMM method in International Swaps and Derivatives Association (2016) – an adjustment will be needed to capture the difference between an in-model IM computed as above by regression methods and the out-of-model margin calculator actually used. Many possible methods could be contemplated here - e.g. a multiplicative adjustment factor that aligns the two margin calculations at time 0.

Calculation of Path-wise Exposure

1. IM Adjusted Path-wise Exposure Estimation: Once the IM on a path is calculated, the exposure on that path, in principle, can be obtained by directly subtracting the calculated IM value from the no-IM exposure. While not of central importance to this chapter, it is to be noted that the path-wise IM results allow for a straightforward computation of the Margin Value Adjustment (MVA) – see, for e.g. Andersen, Duffie, and Song (2017).
2. Dominance of IM Covered Trades: This approach works well when all the trades covered by an IM represent a reasonably small fraction of the netting set. However, when the netting set is dominated by the IM covered trades, this approach suffers from two issues that have an impact on the accuracy of the EE – and therefore CVA – calculations.
3. Simulation Noise: Suppose that all trades of the netting set are covered by IM and that

$$\delta_{IM} = \delta_C$$

If all trades belong to the same asset class, then the non-zero exposure will be realized on average only 1% of the time between the exposure spikes, as IM by design covers 99% of the $P\&L$ increase. If multiple asset classes are present in the netting set, the percentages of non-zero realizations will be even less because of the disallowed diversification across asset classes. Thus EE calculated by direct exposure simulation will be extremely noisy between spikes.

4. Non-Normality: IM is calculated under the assumption that, conditional on a path, the $P\&L$ over the $MPoR$ is Gaussian. When all – or most – of the netting is covered by IM, the EE between the spikes calculated by a direct of the exposure is very sensitive to deviations from local normality. If the conditional $P\&L$ distribution has a heavier (lighter) upper tail than the Gaussian distribution, IM between the spikes will be understated (overstated) and EE will therefore be overstated (understated).
5. Time t_C Path-wise Expected Exposure: Bothe of these issues can be remedied by calculating a time t_C path-wise *expected* exposure for a time t default, rather than the exposure itself. If our target exposure measure is the unconditional time-0 EE, this substitution is valid, of course, by the Law of Iterated Expectations.
6. Simplification of the Advanced Model: To proceed with this idea, one simplifies the Advanced Model slightly - and assumes – as the Classical Model does – that D and C stop posting margin simultaneously – i.e. that

$$\delta_C = \delta_D$$

Then

$$E_{ADVANCED,m}(t) = \left[V_m(t) - V_m(t_C) + U_{ADVANCED,m}(t) - \sum_{k=1}^{K} IM_k(t)\right]^+$$

where $U_{ADVANCED,m}(t)$ is the realization of the right hand side of

$$U_{ADVANCED,m}(t) = TF_{C \rightarrow D}(t;\ (t_C', t_D']) + TF_{NET}(t;\ (t_D', t])$$

on path m.

7. <u>Expected Exposure Conditional on $V_k(t_C) = V_{k,m}(t_C)$</u>: The expectation of this exposure conditional on

$$V_k(t_C) = V_{k,m}(t_C)$$

is, in the Advanced model

$$EE_m(t; t_C) \triangleq \mathbb{E}[E_{ADVANCED}(t) \mid V(t_C) = V_m(t_C)]$$

Averaging

$$E_{ADVANCED,m}(t) = \left[V_m(t) - V_m(t_C) + U_{ADVANCED,m}(t) - \sum_{k=1}^{K} IM_k(t)\right]^+$$

and

$$EE_m(t; t_C) \triangleq \mathbb{E}[E_{ADVANCED}(t) \mid V(t_C) = V_m(t_C)]$$

over all Monte Carlo paths will lead to the same result for $EE(t; t_C)$ up to Monte Carlo sample noise.

8. <u>Evolution of the Underlying Portfolio</u>: To calculate the right hand side of

$$EE_m(t; t_C) \triangleq \mathbb{E}[E_{ADVANCED}(t) \mid V(t_C) = V_m(t_C)]$$

analytically, the mismatch of the portfolio value over the $MPoR$ is assumed to be Gaussian. Specifically, given

$$V(t_C) = V_m(t_C)$$

it is assumed that $TF_m(t;\ (t_C, t])$ is known at time t_C and equal to its realization on the path, and that

$$V(t_C) = V_m(t_C) \sim \mathcal{N}\left(TF_m(t;\ (t_C, t]), \sigma_m(t_C)\sqrt{\delta_C}\right)$$

where $TF_m(t;\ (t_C, t])$ represents the sum of all scheduled portfolio payments $(t_C, t]$ on that path m, and ${\sigma_m}^2(t_C)$ is defined as

$${\sigma_m}^2(t_C) \triangleq \frac{1}{\delta_C}\mathbb{E}[\{V(t) + TF_m(t;\ (t_C, t]) - V_m(t_C)\}^2 \mid V(t_C) = V_m(t_C)]$$

9. <u>Netting Set Level Portfolio Volatility</u>: It is emphasized that the expression for ${\sigma_m}^2(t_C)$ above differs from

$${\sigma_{k,m}}^2(t_C) \triangleq \frac{1}{\delta_{IM}}\mathbb{E}\left[\left\{V_k(t_C + \delta_{IM}) + TF_{NET,k}(t_C + \delta_{IM};\ (t_C, t_C + \delta_{IM}]) - V_k(t_C)\right\}^2 \mid V_k(t_C) = V_{k,m}(t_C)\right]$$

in several important ways. First the portfolio used in

$${\sigma_m}^2(t_C) \triangleq \frac{1}{\delta_C}\mathbb{E}[\{V(t) + TF_m(t;\ (t_C, t]) - V_m(t_C)\}^2 \mid V(t_C) = V_m(t_C)]$$

spans the entire netting set rather than just the sub-portfolio covered by the IM and associated with the asset class k. Second the length of the time horizon is δ_C which may be different from δ_{IM}. As before, ${\sigma_m}^2(t_C)$ can be calculated by parametric or kernel regression.

10. <u>Unconditional Brownian Bridge Expected Exposure</u>: Using this, one can calculate

$$EE_m(t; t_C) \triangleq \mathbb{E}[E_{ADVANCED}(t) \mid V(t_C) = V_m(t_C)]$$

to obtain, in the Advanced model

$$EE_m(t) = \sigma_m(t_C)\sqrt{\delta_C}\left[d_m(t)\Phi\left(d_m(t)\right)\right] + \phi\left(d_m(t)\right)$$

$$d_m(t) \triangleq \frac{-PTF_{ADVANCED,m}(t;\ (t_C, t]) - \sum_{k=1}^{K} IM_{k,m}(t)}{\sigma_m(t_C)\sqrt{\delta_C}}$$

where – omitting arguments –

$$PTF_{ADVANCED,m} = TF_m - U_{ADVANCED,m}$$

are the net trade flows on the $MPoR$ *actually* paid, according to the Advanced model.

11. <u>Relaxing the $\delta_D = \delta_C$ Simplification</u>: Recall that

$$EE_m(t) = \sigma_m(t_C)\sqrt{\delta_C}\left[d_m(t)\Phi\left(d_m(t)\right)\right] + \phi\left(d_m(t)\right)$$

$$d_m(t) \triangleq \frac{-PTF_{ADVANCED,m}(t;\ (t_C, t]) - \sum_{k=1}^{K} IM_{k,m}(t)}{\sigma_m(t_C)\sqrt{\delta_C}}$$

was derived using the Advanced Model for the simplifying case

$$\delta_D = \delta_C$$

$$\delta_D < \delta_C$$

would, however, result in a significant understatement of the EE between spikes – e.g. for the earlier swap sample, the understatement would be around 22%.

12. <u>Handling the $\delta_D < \delta_C$ Case using Scaling</u>: Directly extending

$$EE_m(t) = \sigma_m(t_C)\sqrt{\delta_C}\left[d_m(t)\Phi\left(d_m(t)\right)\right] + \phi\left(d_m(t)\right)$$

$$d_m(t) \triangleq \frac{-PTF_{ADVANCED,m}(t;\ (t_C, t]) - \sum_{k=1}^{K} IM_{k,m}(t)}{\sigma_m(t_C)\sqrt{\delta_C}}$$

to cover

$$\delta_D < \delta_C$$

is, however, not straightforward. Therefore Andersen, Pykhtin, and Sokol (2017b) propose a simple scaling solution. Here the paths of the exposure *without* the IM are simulated first for the Advanced model, i.e. for

$$E_{ADVANCED}(t) = \left[V(t) - \min_{T_i \in [t_C, t_D]} V(T_i) + TF_{C\to D}(t;\ (t_C', t_D']) + TF_{NET}(t;\ (t_D', t]) - \sum_{k=1}^{K} IM_k(t)\right]^+$$

with

$$IM(t) = 0$$

Then for each exposure simulation time t and each measurement path m the exposure value is multiplied by the ratio of $\frac{EE_m(t)}{EE_{0,m}(t)}$ where $EE_m(t)$ is computed from

$$EE_m(t) = \sigma_m(t_C)\sqrt{\delta_C}\left[d_m(t)\Phi\left(d_m(t)\right)\right] + \phi\left(d_m(t)\right)$$

$$d_m(t) \triangleq \frac{-PTF_{ADVANCED,m}(t;\ (t_C,t]) - \sum_{k=1}^{K} IM_{k,m}(t)}{\sigma_m(t_C)\sqrt{\delta_C}}$$

and $EE_{0,m}(t)$ is the special case of $EE_m(t)$ where

$$IM(t) = 0$$

Numerical Example

1. EE Computations using Different Approaches: To illustrate the benefits of the conditional EE simulation method described above, Andersen, Pykhtin, and Sokol (2017b) turn to the 2Y IRS with unequal payment frequencies considered earlier. They illustrate EE profile comparisons using several computational approaches.
2. Expected Exposures at Spike - Comparison: First they focus their attention on the upward EE spikes produced by trade payments at the 1 year point. Both the unconditional and the conditional EE estimators here produce an almost identical spike, exceeding the benchmark spike height – the consequence of using kernel regression and a Gaussian distribution to estimate the IM.
3. Expected Exposure between Spikes - Comparison: They then show the EE exposure at a fine exposure scale, allowing for a clear observation of the EE between spikes. There the advantages of the conditional EE approach can be clearly seen.
4. Reduction in Simulation Noise: While both methods use Monte Carlo simulation with 5,000 paths, the simulation noise in the conditional EE approach is substantially less than that in the unconditional EE approach. In fact, the conditional EE noise is even less than in the benchmark EE results that were calculated using 50,000 paths.
5. Reduced Error from Non-Gaussian Dynamics: Andersen, Pykhtin, and Sokol (2017b) have used a high-volatility lognormal interest rate model in their example, to produce significant

deviations in a $10\ D$ horizon. This non-normality is the main reason for the deviation of the conditional and the unconditional EE curves from the benchmark. In estimating

$$EE(t) = \mathbb{E}\left[\left\{V(t) - V_k(t_C) - Q_q\left(V_k(t_C + \delta_{IM}) \middle| \mathcal{F}_{t_C}\right)\right\}^+\right]$$

$$t_C = t - \delta_C$$

the conditional estimator uses a Gaussian distribution to approximate *both* the IM and the portfolio increment, resulting in partial error cancellation. Such error cancellation does not take place for the unconditional estimator which uses the empirical – here lognormal – distribution for the portfolio increment, yet estimates IM from a Gaussian distribution. Between the spikes the EE errors for the unconditional estimator are therefore significantly larger here than for the conditional estimator.

Conclusion

1. <u>Consequences from the BCBS-IOSCO IM Rules</u>: There is universal agreement that the new BCBS-IOSCO IM rules will lead to very substantial postings of margin amounts into segregated accounts accompanied by inevitable increase in the funding costs (MVA) that the dealers will face when raising funds for the IM. According to conventional wisdom, these postings, while expensive, should effectively eliminate counter party risk.
2. <u>Handling the Trade Flow Spikes</u>: This chapter examines the degree to which the bilateral IM required by the BCBS-IOSCO margin rules suppresses counter party exposure. As shown by Andersen, Pykhtin, and Sokol (2017a) any trade flow to the defaulting party for which it does not return margin during the MPoR causes a spike in the exposure profile.

3. Ignoring the Trade Flow Spikes: These spikes are often ignored by dealers a being *spurious* or as being part of *settlement risk*. In reality these spikes are integral part of the exposure profile and represent real risk that has previously materialized in many well-documented incidents, notably the non-payment of Lehman of reciprocal margin to trade payments that arrived at around the time of the bankruptcy filing.
4. IM reduces Expected Exposure Considerably: The chapter shows that, under very general assumptions the BCBS IOSCO IM specified as the 99% 10 $D\ VaR$ reduces the exposure between the spikes by a factor of over 100 but fails to suppress the spikes by a comparable degree. This happens because IM is calculated without reference to trade payments, and is based only on the changes to the portfolio value resulting from the risk factor variability. As an example, Andersen, Pykhtin, and Sokol (2017b) show that IM reduces the CVA of a 2Y IRS with VM by only a factor of 7.
5. Impracticality of Increasing the IM: While VaR based IM fails to fully suppress the contribution of exposure spikes to CVA and EAD, increasing the IM to always exceed the peak exposure would be impractical, and would require moving large amounts of collateral back and forth in a matter of days.
6. IM CVA dominated by Spikes: Another important property of CVA under full IM coverage is that it is dominated by exposure spikes; in their 2Y IRS example, Andersen, Pykhtin, and Sokol (2017b) find that spike contribution to the CVA is about 95% in the presence of IM (compared to about 20% without IM).
7. Modeling CSA Time-line in IM: Thus, in the presence of IM, the focus of exposure modeling should be on capturing the impact of trade payments, which involves making realistic assumptions on what the dealer and the client are expected to make contingent on the client's default.
8. Computationally Feasible Daily Portfolio Value: Furthermore, to accurately calculate the CVA mostly produced by narrow exposure spikes, one needs to produce exposure on a daily time grid. A method for producing daily exposures without daily portfolio re-evaluations was discussed above, along with other useful numerical techniques.
9. Approach taken by the CCP: A natural question to ask is why similar payment effects have not been recognized in trading through central counterparties (CCPs), which also require IM

posting that is typically based on 99% VaR over the $MPoR$. As it turns out, CCPs already use a mechanism that amounts to netting of trade and margin payments.

10. Infeasibility of the CCP Approach: Unfortunately the same approach cannot be adopted in bilateral trading as it would require changing all of the existing trade documentation, which is a practical impossibility.
11. Nettability of Trade/Margin Payments: While a trade payment and its reciprocal margin payment cannot be netted in bilateral trading, this lag can be eliminated and the two payments made to fall on the same day by making a simple change in the CSA.
12. CSA Amendment for Trade Payments: Specifically, if the CSA is amended to state that known trade payments due to arrive prior to the scheduled margin payment date must be subtracted from the portfolio valuation for the purposes of margin – technically this amendment effectively sets the VM based on a 2-day portfolio forward value – then the call for the reciprocal margin will happen ahead of time, and it will on the same day as the trade payment – a *no lag margin settlement*.
13. Reduction in the $MPoR$ Duration: From an IT and a back office perspective, this change in the CSA is relatively easy to align with existing mark-to-market and cash-flow processes, and is beneficial in several ways. First it shortens the duration of the exposure spikes and the $MPoR$ overall, reducing counter-party risk.
14. Margin vs MTM $2D$ Lag: Second it makes margin follow MTM without a $2D$ lag, thereby eliminating the need to use outside funding to fund hedging during this $2D$ period.
15. Trade/Margin Payments Reciprocal Concurrence: Finally, with the reciprocal trade and the margin payments falling on the same day, payment-versus-payment services (PvP) such as CLS Bank (Galati (2002), Lindley (2008), Brazier (2015)) may be able to settle trade and margin payments together, reducing residual counterparty risk even further.

References

- Andersen, L., D. Duffie, and Y. Song (2017): Funding Value Adjustment **eSSRN**
- Andersen, L. and M. Pykhtin (2015): *Accounting for Dynamic Initial Margin in Credit Exposure Models* **Risk Minds International**
- Andersen, L., M. Pykhtin, and A. Sokol (2017a): Re-thinking Margin Period of Risk **eSSRN**
- Andersen, L., M. Pykhtin, and A. Sokol (2017b): Credit Exposure in the Presence of Initial Margin **eSSRN**
- Anfuso, C., D. Aziz, K. Loukopoulos, and P. Giltinan (2017): A Sound Modeling and Backtesting Framework for Forecasting Initial Margin **eSSRN**
- Basel Committee on Banking Supervision (2013): Basel III: The Liquidity Coverage Ratio and Liquidity Risk Monitoring Tools
- Basel Committee on Banking Supervision (2015): Margin Requirements for Non-centrally Cleared Derivatives
- Brazier, J. (2015): CLS, Markit Launch Cross-Currency FX Settlement Service **Waters Technology**
- Galati, G. (2002): Settlement Risk in Foreign Exchange Markets and CLS Bank *BIS Quarterly Review* **December** 55-66
- Gregory, J. (2015): *The XVA Challenge: Counterparty Credit Risk, Funding, Collateral, and Capital, 3rd Edition* **Wiley**
- International Swaps and Derivatives Association (2016): ISDA SIMM Methodology
- Lindley, R. (2008): Reducing Foreign Exchange Settlement Risk *BIS Quarterly Review* **September** 53-65
- Pykhtin, M. (2010): Collateralized Credit Exposure, in: *Counterparty Credit Risk (editor: E. Canabarro)* **Risk Books**

Initial Margin Backtesting Framework

Abstract

1. Mandatory Margins for OTC Transactions: The introduction of mandatory margining for bilateral OTC transactions is significantly affecting the derivatives market, particularly in light of the additional funding costs that financial institutions could face.
2. Initial Margin Forecast Models Backtest: This chapter details the approach by Anfuso, Aziz, Loukopoulos, and Giltinan (2017) for a consistent framework, applicable equally to cleared and non-cleared portfolios, to develop and backtest forecasting models for initial margin.

Introduction

1. BCBS-IOSCO Mandatory Margining Guidelines: Since the publication of the new Basel Committee on Banking Supervision and the International Organizations of Securities Commissions (BCBS-IOSCO) guidance for mandatory margining for non-cleared OTC derivatives (Basel Committee on Banking Supervision (2015)) there has been a growing interest in the industry regarding the development of dynamic initial margin models (DIM) – see, for example, Green Kenyon (2015), Andersen, Pykhtin, and Sokol (2017b). By *DIM model* this chapter refers to any model that can be used to forecast portfolio initial margin requirements.

2. Protection Afforded by BCBS-IOSCO: The business case for such a development is at least two fold. First, the BCBS-IOSCO IMR (B-IMR) rules are expected to protect against potential future exposure at a high-level of confidence (99%) and will substantially affect funding costs, XVA, and capital.
3. IM and VM Based Margining: Second, B-IMR has set a clear incentive for clearing; extensive margining in the form of variation margin (VM) and initial margin (IM) is the main element of the central counter-party (CCP) risk management as well.
4. IMR Impact on Bilateral + Cleared: Therefore, for both bilateral and cleared derivatives, current and future IMR significantly affects the probability and the risk profile of a given trade.
5. B-IMR Case Study - Performance Evaluation: This chapter considers B-IMR as a case study, and shows how to include a suitably parsimonious DIM model on the exposure calculation. It also proposes an end-to-end framework and also defines a methodology to backtest model performance.
6. Organization of this Chapter: This chapter is organized as follows. First, the DIM model for forecasting future IMR is presented. Then methodologies for two distinct levels of back-testing analysis are presented. Finally, conclusions are drawn.

How to Construct a DIM Model

1. Applications of the DIM Model: A DIM model can be used for various purposes. In the computation of the counter-party credit risk (CCR), capital exposure, or credit valuation adjustment (CVA), the DIM model should forecast, in a path-by-path basis, the amount of posted and received IM at any revaluation point.
2. Path Specific IMR Estimation: For this specific application, the key ability of the model is to associate a realistic IMR to any simulated market scenario based on a mapping that makes use of a set of characteristics of the path.

3. RFE Dependence on the DIM: The DIM model is *a priori* agnostic to the underlying risk factor evolution (RFE) models to generate the exposure paths (as shall be seen, dependencies may arise, if for example, the DIM is computed on the same paths that are generated for the exposure).
4. Cross-Probability Measure IMR Distribution: It is a different story if the goal is to predict the IMR distribution (IMRD) at future horizons, either in the real-world P or the market-implied Q measures.
5. IMRD Dependence on the RFE: In this context, the key feature of the model is to associate the right probability weight with a given IMR scenario; hence the forecast IMRD also becomes a measure of the accuracy if the IMRD models (which ultimately determine the likelihood of different market scenarios).
6. P vs. Q Measure IMRD: The distinction between the two cases will become clear later on, in the discussion of how to assess model performance.
7. ISDA SIMM BCBS IOSCO IM: The remainder of this chapter considers the BCBS-IOSCO IM as a case study. For the B-IMR, the current industry proposal is the International Swaps and Derivatives Association Standard Initial Margin Model (SIMM) – a static aggregation methodology to compute the IMR based on first-order delta-vega trade sensitivities (International Swaps and Derivatives Association (2016)).
8. Challenges with SIMM Monte Carlo: The exact replication of SIMM in a capital exposure or an XVA Monte Carlo framework requires in-simulation portfolio sensitivities to a large set of underlying risk factors, which is very challenging in most production implementations.
9. Andersen-Pykhtin-Sokol IM Proposal: Since the exposure simulation provides the portfolio mark-to-market (MTM) on the default (time t) and closeout (time $t + MPoR$, where $MPoR$ is the *margin period of risk*) grids, Andersen, Pykhtin, and Sokol (2017b) have proposed using this information to infer path-wise the size of any percentile of the local $\Delta MTM(t, t + MPoR, Path_i)$ distribution, based on a regression that uses the simulated portfolio $MTM(t)$ as a regression variable.
10. Andersen-Pykhtin-Sokol Proposal Assumptions: The

$$\Delta MTM(t, t + MPoR) = MTM(t + MPoR) - MTM(t)$$

distributed is constructed assuming that no cash flow takes place between the default and the closeout. For a critical review of this assumption, see Andersen, Pykhtin, and Sokol (2017a).

11. Enhancing the Andersen-Pykhtin-Sokol Model: This model can be further improved by adding more descriptive variables to the regression, e.g., values at the default time of the selected risk factors of the portfolio.
12. Optimization: Re-using Exposure Paths: For the DIM model, the following features are desirable. First the DIM should consume the same number of paths as the exposure simulation, to minimize the computational burden.
13. DIM Optimization – B-IMR SIMM Reconciliation: Second, the output of the DIM model should reconcile with the known IMR value for

$$t = 0$$

i.e.

$$IM(Path_i, 0) = IMR_{SIMM}(0)$$

for all i.

14. Key Aspects of IOSCO/SIMM: Before proceeding, this section notes some of the key aspects of the BCBS-IOSCO margining guidelines, and, consequently, of the ISDA SIMM Model (International Swaps and Derivatives Association (2016)).
15. Andersen-Pykhtin-Sokol Proposal Assumptions: First, the $MPoR$ for the IM calculation of a daily margined counter-party is 10 BD. This may differ from the capital exposure calculation, in which, for example

$$MPoR = 20\ BD$$

if the number of trades in the portfolio exceeds $5{,}000$.

16. <u>No Netting across the Asset Classes</u>: Second, the B-IMR in the Basel Committee on Banking Supervision (2015) prescribes calculating the IM by segregating trades from different asset classes. Tis feature is reflected in the SIMM model design.
17. <u>SIMM Methodology Market Volatility Independence</u>: Finally, the SIMM methodology consumes trade sensitivities as its only inputs and has a static calibration that is not sensitive to market volatility.
18. <u>Regression on the ΔMTM Distribution</u>: For the IM calculation, the starting point is similar to that of Andersen, Pykhtin, and Sokol (2017a), i.e.
    a. A regression methodology based on path's $MTM(t)$ is used to compute the moments of the local $\Delta MTM(t, t + MPoR, Path_i)$ distribution, and
    b. $\Delta MTM(t, t + MPoR, Path_i)$ is assumed to be a given probability distribution that can be fully characterized by its first two moments – the drift and the volatility. Additionally, since the drift is immaterial over the $MPoR$ horizon, it is not computed and set to 0.
19. <u>Quadratic Regressor for Local Volatility</u>: There are multiple regression schemes that can be used to determine the local volatility $\sigma(i, t)$ The present analysis follows the standard American Monte Carlo literature (Longstaff and Schwartz (2001)) and uses a least squares method (LSM) with a polynomial basis:

$$\sigma^2(i, t) = \mathbb{E}[\Delta MTM^2(i, t) \mid MTM(i, t)] = \sum_{k=0}^{n} a_{\sigma k} MTM^k(i, t)$$

$$IM_{R/P,U}(i, t) = \Phi^{-1}\big(0.99/0.01, \mu = 0, \sigma = \sigma(i, t)\big)$$

where R/P indicates received and poste, respectively. In this implementation, the n in

$$\sigma^2(i, t) = \mathbb{E}[\Delta MTM^2(i, t) \mid MTM(i, t)] = \sum_{k=0}^{n} a_{\sigma k} MTM^k(i, t)$$

is set equal to 2, i.e., a polynomial regression of order 2 is used.

20. Calculating the Unnormalized IM Value: The unnormalized posted and received $IM_{R/P,U}(i,t)$ and calculated analytically in

$$\sigma^2(i,t) = \mathbb{E}[\Delta MTM^2(i,t) \mid MTM(i,t)] = \sum_{k=0}^{n} a_{\sigma k} MTM^k(i,t)$$

$$IM_{R/P,U}(i,t) = \Phi^{-1}\big(0.99/0.01, \mu = 0, \sigma = \sigma(i,t)\big)$$

by applying the inverse of the cumulative distribution $\Phi^{-1}(x,\mu,\sigma)$ to the appropriate quantiles; $\Phi(x,\mu,\sigma)$ being the probability distribution that models the local $\Delta MTM(t, t + MPoR, Path_i)$.

21. Note on the Distributional Assumptions: The precise choice of Φ does not play a crucial role, since the difference in the quantiles among the distribution can be compensated in calibration be applying the appropriate scaling factors (see the $\alpha_{R/P}(t)$ functions below). For simplicity, in the below Φ is assumed to be normal.

22. Comparative Performance of the LSM: It is observed that the LSM method performs well compared to the more sophisticated kernel methods such as Nadaraya-Watson, which is used in Andersen, Pykhtin, and Sokol (2017a), and it has the advantage of being parameter free and cheaper from a computational stand-point.

23. Applying $t = 0, MPoR$ and SIMM Reconcilers: The next step accounts for the

$$t = 0$$

reconciliation as well as the mismatch between SIMM and the exposure model calibrations – see the corresponding items above.

24. De-normalizing using IM Scaling Parameters: These issues can be tackled by scaling $IM_{R/P,U}(i,t)$ with suitable normalization functions $\alpha_{R/P}(t)$:

$$IM_{R/P}(i,t) = \alpha_{R/P}(t) \times IM_{R/P,U}(i,t)$$

$$\alpha_{R/P}(t) = [1 - h_{R/P}(t)] \times \sqrt{\frac{10\ BD}{MPoR}} \times \left[\alpha_{R/P,\infty} + (\alpha_{R/P,0} - \alpha_{R/P,\infty})e^{-\beta_{R/P}(t)t}\right]$$

$$\alpha_{R/P,0} = \sqrt{\frac{MPoR}{10\ BD}} \times \frac{IMR_{R/P,SIMM}(t=0)}{q\left(0.99/0.01\,, \Delta MTM(0, MPoR)\right)}$$

25. Differential Calibration for Posted/Received IM: In

$$\alpha_{R/P}(t) = [1 - h_{R/P}(t)] \times \sqrt{\frac{10\ BD}{MPoR}} \times \left[\alpha_{R/P,\infty} + (\alpha_{R/P,0} - \alpha_{R/P,\infty})e^{-\beta_{R/P}(t)t}\right]$$

$$\beta_{R/P}(t) > 0$$

and

$$h_{R/P}(t) < 1$$

with

$$h_{R/P}(t = 0) = 0$$

are four functions to be calibrated – two for received and two for posted IM's. As will become clearer later in this chapter, the model calibration generally differs for received and posted DIM models.

26. Scaling IM using RFE MPoR: In

$$IM_{R/P}(i,t) = \alpha_{R/P}(t) \times IM_{R/P,U}(i,t)$$

$$\alpha_{R/P}(t) = \left[1 - h_{R/P}(t)\right] \times \sqrt{\frac{10\ BD}{MPoR}} \times \left[\alpha_{R/P,\infty} + \left(\alpha_{R/P,0} - \alpha_{R/P,\infty}\right)e^{-\beta_{R/P}(t)t}\right]$$

$MPoR$ indicates the $MPoR$ relevant for the Basel III exposure. The ratio of $MPoR$ to 10 BD accounts for the VM vs. IM margin period, and it is taken as a square root because the underlying models are typically Brownian, at least for short horizons.

27. <u>Components of the $\alpha_{R/P}(t)$ Term</u>: In

$$\alpha_{R/P,0} = \sqrt{\frac{MPoR}{10\ BD}} \times \frac{IMR_{R/P,SIMM}(t = 0)}{q\left(0.99/0.01\,, \Delta MTM(0, MPoR)\right)}$$

$IMR_{R/P,SIMM}(t = 0)$ are the $IMR_{R/P}$ computed at

$$t = 0$$

using SIMM; $\Delta MTM(0, MPoR)$ is the distribution of the MTM variations over the first $MPoR$; and $q(x, y)$ is a function that gives quantile x for the distribution y.

28. <u>$t = 0$ chosen to match SIMM</u>: The values of the normalization functions $\alpha_{R/P}(t)$ at

$$t = 0$$

are chosen in order to reconcile $IM_{R/P}(i, t)$ with the starting SIMM IMR.

29. <u>Mean-reverting Nature of the Volatility</u>: The functional form of $\alpha_{R/P}(t)$ at

$$t > 0$$

is dictated by what is empirically observed, as is illustrated by Anfuso, Aziz, Loukopoulos, and Giltinan (2017); accurate RFE models, in both P and Q measures, have either a volatility term structure or an underlying stochastic volatility process that accounts for the mean-reverting behavior to the normal market conditions generally observed from extremely low or high volatility.

30. Reconciliation with Static SIMM Methodology: Since the SIMM calibration is static (independence of market volatility for SIMM), the

$$t = 0$$

reconciliation factor is not independent of the market volatility, and thus not necessarily adequate for the long-term mean level.

31. Volatility Reducing Mean-reversion Speed: Hence, $\alpha_{R/P}(t)$ is an interpolant between the

$$t = 0$$

scaling driven by $\alpha_{R/P,0}$ and the long-erm scaling driven by $\alpha_{R/P,\infty}$, where the functions $\beta_{R/P}(t)$ are the mean-reverting speeds.

32. Estimating from the Long-End: The values of $\alpha_{R/P,\infty}$ can be inferred by a historical analysis of a group of portfolios, or it can be *ad hoc* calibrated, e.g., by computing a different $\Delta MTM(0, MPoR)$ distribution in

$$\alpha_{R/P,0} = \sqrt{\frac{MPoR}{10\ BD}} \times \frac{IMR_{R/P,SIMM}(t = 0)}{q\big(0.99/0.01\,, \Delta MTM(0, MPoR)\big)}$$

using the long-end of the risk-factor implied volatility curves and solving the equivalent scaling equations for $\alpha_{R/P,\infty}$.

33. Interpreting the Haircut $h_{R/P}(t)$ Term: As will be seen below, the interpretation of $h_{R/P}(t)$ can vary depending on the intended application of the model.

34. $h_{R/P}(t)$ for Capital/Risk Models: For capital and risk models, $h_{R/P}(t)$ are two capital and risk functions that can be used to reduce the number of back-testing exceptions (see below) and ensure that the DIM model is conservatively calibrated.
35. $h_{R/P}(t)$ for the XVA Models: For XVA pricing, $h_{R/P}(t)$ can be fine-tuned – together with $\beta_{R/P}(t)$ - to maximize the accuracy of the forecast based on historical performance.
36. Lack of Asset Class Netting: Note that owing to the *No netting across Asset Classes* clause, the $IM_{R/P,x}(i,t)$ can be computed on a stand-alone basis for every asset class x defined by SIMM (IR/FX, equity, qualified and non-qualified credit, commodity) without any additional exposure runs. The total $IM_{R/P}(i,t)$ is then given by the sum of the $IM_{R/P,x}(i,t)$ values.
37. Historical vs. Computed IM Calibrations: A comparison between the forecasts of the DIM model defined in

$$\sigma^2(i,t) = \mathbb{E}[\Delta MTM^2(i,t) \mid MTM(i,t)] = \sum_{k=0}^{n} a_{\sigma k} MTM^k(i,t)$$

$$IM_{R/P,U}(i,t) = \Phi^{-1}\big(0.99/0.01, \mu = 0, \sigma = \sigma(i,t)\big)$$

$$IM_{R/P}(i,t) = \alpha_{R/P}(t) \times IM_{R/P,U}(i,t)$$

$$\alpha_{R/P}(t) = \left[1 - h_{R/P}(t)\right] \times \sqrt{\frac{10\ BD}{MPoR}} \times \left[\alpha_{R/P,\infty} + \left(\alpha_{R/P,0} - \alpha_{R/P,\infty}\right)e^{-\beta_{R/P}(t)t}\right]$$

$$\alpha_{R/P,0} = \sqrt{\frac{MPoR}{10\ BD}} \times \frac{IMR_{R/P,SIMM}(t=0)}{q\big(0.99/0.01\,, \Delta MTM(0, MPoR)\big)}$$

and the historical IMR realizations computed with the SIMM methodology is shown in Anfuso, Aziz, Loukopoulos, and Giltinan (2017) where alternative scaling approaches are considered.

38. Criteria Utilized in the Comparison: A comparison is performed at different forecasting horizons using 7 years of historical data, monthly sampling, and averaging among a wide representation of single-trade portfolios for the posted and the received IM cases.
39. $\mathcal{L}_1$ Error Metric Choice: For a given portfolio/horizon, the chosen error metric is given by $\mathbb{E}_{t_k}\left[\frac{|F_{R/P}(t_k+h)-G_{R/P}(t_k+h)|}{G_{R/P}(t_k+h)}\right]$ where $\mathbb{E}_{t_k}[\cdot]$ indicates an average across historical sampling dates – the definitions of $F_{R/P}$ and $G_{R/P}$ are contained below. Here and throughout this chapter, t_k is used in place of t whenever the same quantity is computed at multiple sampling dates.
40. Comparison of the Tested Universe: The tested universe is made up of 102 single-trade portfolios. The products considered, always at-the-money and of different maturities, include cross-currency swaps, IR swaps, FX options, and FX forwards – approximately 75% of the population is made up of

$$\Delta = 1$$

trades.
41. Calibrated Estimates of the Parameters: As is made evident by Anfuso, Aziz, Loukopoulos, and Giltinan (2017), the proposed term structure of $\alpha_{R/P}(t)$ improves the accuracy of the forecast by a significant amount – they also provide the actual calibration used for their analysis.
42. Conservative Calibration of the Haircut Function: Below contains further discussions on the range of values that the haircut functions $h_{R/P}(t)$ are expected to take for a conservative calibration of DIM to be used for regulatory exposure.
43. Comparison with CCP IMR: Finally, as an outlook, Anfuso, Aziz, Loukopoulos, and Giltinan (2017) show the error metrics for the case of CCP IMR where the Dim forecasts are compared against the Portfolio Approaches to Interest Rate Scenarios (Pairs: LCH.ClearNet) and historical value-at-risk (HVaR; Chicago Mercantile Exchange) realizations.
44. Prototype Replications of CCP Methodologies: The realizations are based on prototype replications of the market risk components of the CCP IM methodologies.

45. Universe Used for the CCP Tests: The forecasting capability of the model is tested separately for Pairs and HVaR IMR as well as for 22 single-trade portfolios (IRS trades of different maturities and currencies). The error at any given horizon is obtained by averaging among 22×2 cases.
46. Accuracy of the Proposed Scaling: Without fine tuning the calibration any further, the time-dependent scaling $\alpha_{R/P}(t)$ drives a major improvement in the accuracy of the forecasts with respect to the alternative approaches.

How to Backtest a DIM Model

1. Assessing Model for Different Applications: The discussion so far has focused on a DIM model for B-IMR without being too specific about how to assess the model performance for different applications, such as CVA and margin valuation adjustment (MVA) pricing, liquidity coverage ratio/net stable funding ratio (LCR/NSFR) monitoring (Basel Committee on Capital Supervision (2013)), and capital exposure.
2. Estimating the IMR Distribution Accurately: As mentioned above, depending upon which application one considers, it may or may not be important to have an accurate assessment of the distribution of *the simulated IM requirements* value (IMRD).
3. Backtesting to measure DIM Performance: This chapter introduces two distinct levels of backtesting that can measure the DIM model performance in two topical cases:
    a. DIM applications that do not depend directly on the IMRD (such as capital exposure and the CVA), and
    b. DIM applications that directly depend on the IMRD (such as MVA calculation and LCR/NSFR monitoring).

    The methodologies are presented below, with a focus on the P-measure applications.

Backtesting DIM Mapping Functions (for Capital Exposure and CVA)

1. Review of the Monte-Carlo Framework: In a Monte-Carlo simulation framework, the exposure is computed by determining the MTM values of a given portfolio on a large number of forward looking risk-factor scenarios.
2. Adequacy of Forecasts across Scenarios: To ensure that a DIM model is sound, one should verify that the IM forecasts associated with the future simulation scenarios are adequate for a sensible variety of forecasting horizons as well as initial and terminal market conditions.
3. Setting up a Suitable Backtesting Framework: A suitable historical backtesting framework so as to statistically assess the performance of the model by comparing the DIM forecast with the realistic exact IMR, e.g., in the case of B-IMR calculated according to the SIMM methodology – for a representative sample of historical dates as well as market conditions and portfolios.
4. Generic IMR of a Portfolio: Let us first define generic IMR of a portfolio p as

$$IMR = g_{R/P}\left(t = t_\alpha, \Pi = \Pi\big(p(t_\alpha)\big), \vec{M}_g = \vec{M}_g(t_\alpha)\right)$$

   The terms are as follows.
5. Posted/Received IMR Computation Algorithm: The functions g_R and g_P represent the exact algorithm used to compute the IMR for the posted and the received IM's, respectively (e.g., such as SIMM for B-IMR, or in the case of the CCP's, IM methodologies such as Standard Portfolio Analysis of Risk (SPAN), Pairs, or HVaR).
6. Date of the IMR Valuation:

$$t = t_\alpha$$

   is the time at which the IMR portfolio p is determined.

7. Portfolio Trade Population at t_{α}: $\Pi\big(p(t_{\alpha})\big)$ is the trade population of portfolio p at time t_{α}.
8. Market State Information at t_{α}: $\vec{M}_g(t_{\alpha})$ is a generic state variable that characterizes all of the $T \le t_{\alpha}$ market information required for the computation of the IMR.
9. DIM Forecast of the Portfolio: Similarly, the DIM forecast for the future IMR of a portfolio p can be defined as

$$DIM = f_{R/P}\left(t_0 = t_k, t = t_k + h, \vec{r}, \qquad \Pi = \Pi\big(p(t_k)\big), \vec{M}_{DIM} = \vec{M}_{DIM}(t_k)\right)$$

   The terms are as follows.
10. Posted/Received DIM Computation Algorithm: The functions f_R and f_P represent the DIM forecast for the posted and the received IM's, respectively.
11. Date of the DIM Forecast:

$$t_0 = t_k$$

   is the date time at which the DIM forecast is computed.
12. Horizon of the DIM Forecast:

$$t = t_k + h$$

   is the time for which the IMR is forecast – over a forecasting horizon

$$h = t - t_0$$

13. Predictor Set of Market Variables: $\vec{r}$ - the *predictor* – is a set of market variables whose forecasted values on a given scenario are consumed by the DIM models as input to infer the IMR.
14. $\vec{r}$ as Simulated Portfolio MTM: The exact choice of $\vec{r}$ depends on the DIM model. For the one considered previously, $\vec{r}$ is simply given by the simulated MTM of the portfolio.

15. Market State Information at t_k: $\vec{M}_{DIM}(t_k)$ is the generic state variable characterizing all the

$$T \leq t_k$$

market information required for the computation of the DIM forecast.
16. Portfolio Trade Population at t_k: $\Pi(\cdot)$ is defined as before.
17. Caveats around f_R and f_P: Despite being computed using the stochastic RFE models, f_R and f_P are not probability distributions, as they do not carry any information regarding the probability weight of a given received/posted IM value. $f_{R/P}$ are instead mapping functions between the set $\vec{r}$ chosen as predictor and the forecast value for IM.
18. Confidence Level Based DIM Calibration: In terms of $g_{R/P}$ and $f_{R/P}$ one can define exception counting tests. The underlying assumption is that the DIM model is calibrated at a given confidence level (CL); therefore it can be tested as a $VaR(CL)$ model.
19. Model Conservatism Linked to CL: This comes naturally in the context of real-world P applications, such as capital exposure or liquidity monitoring, where a notion of model conservatism, and hence of exception, is applicable, since the model will be conservative whenever it understates (overstates) posted (received) IM.
20. The Portfolio Backtesting Algorithm Steps: For a portfolio p, a single forecasting day t_k, and a forecasting horizon h, one can proceed as follows.
21. t_k Estimate of the Forecast Functions: The forecast functions $f_{R/P}$ computed at time t_k are $f_{R/P}\left(t_0 = t_k, t = t_k + h, \vec{r},\ \Pi = \Pi\big(p(t_k)\big), \vec{M}_{DIM} = \vec{M}_{DIM}(t_k)\right)$ Note that $f_{R/P}$ depends exclusively on the predictor $\vec{r}$ –

$$\vec{r} = MTM$$

for the case considered above.
22. Impact of the Horizon on Predictor/Portfolio: The realized value of the predictor

$$\vec{r} = \vec{R}$$

is determined. For the model considered above, $\vec{R}$ is given by the portfolio value $p(t_k + h)$ where the trade population $\Pi\big(p(t_k + h)\big)$ at $t_k + h$ differs from t_k only because of portfolio aging. Aside from aging, no other portfolio adjustments are made.

23. Forecast Received/Posted IMR Estimate: The forecast values for the received and the posted IM's are computed as

$$F_{R/P}(t_k + h) = f_{R/P}\left(t_0 = t_k, t = t_k + h, \vec{r}, \qquad \Pi = \Pi\big(p(t_k)\big), \vec{M}_{DIM} = \vec{M}_{DIM}(t_k)\right)$$

24. Forecast of the Received/Posted IM Estimate: The realized values for the received and the posted IM's are computed as

$$G_{R/P}(t_k + h) = g_{R/P}\left(t = t_k + h, \Pi = \Pi\big(p(t_k + h)\big), \vec{M}_g = \vec{M}_g(t_k + h)\right)$$

25. Exception Case: F/G Mismatch Conservatism: The forecast and the realized values are then compared. The received and the posted DIM models are considered independently, and a backtesting exception occurs whenever $F_R(F_P)$ is larger (smaller) than $G_R(G_P)$. As discussed above, this definition of exception follows from the applicability of a notion of model conservatism.
26. Detecting the Backtesting Exception History: Applying the above steps to multiple sampling points t_k one can detect back-testing exceptions for the considered history.
27. Dimensionality Reduction for the Comparison: The key step is the estimate of the posted/received IMR forecast, where the dimensionality of the forecast is reduced – from a function to a value – making use of the realized value of the predictor, and, hence, allowing for a comparison with the realized IMR.
28. Determining the Test p-value using TVS: The determination of the test p-value requires additional knowledge of the Test Value Statistics (TVS), which can be derived numerically if the forecasting horizons are overlapping (Anfuso, Karyampas, and Nawroth (2017)).

29. Caveats behind Blind TVS Usage: In the latter situation, it can happen that a single change from one volatility regime to another may trigger multiple correlated exceptions; hence the TVS should adjust the beck-testing assessments for the presence of false positives.
30. Accuracy of the $\alpha_{R/P}(t)$ Scaling: The single trade portfolios seen earlier have been tested by Anfuso, Aziz, Loukopoulos, and Giltinan (2017) using the SIMM DIM models with the three choices of scaling discussed earlier. The results confirm the greater accuracy of the term structure scaling of $\alpha_{R/P}(t)$.
31. Accuracy in the Presence of Haircut: In fact, for the same level of the haircut function

$$h_{R/P}(t > 0) = \pm 0.25$$

positive/negative for posted/received – a much lower number of exceptions is detected.
32. Realistic Values for the Haircut: Anfuso, Aziz, Loukopoulos, and Giltinan (2017) also observe that, in this regard, for realistic diversified portfolios and calibration targets of

$$CL = 95\%$$

the functions $h_{R/P}(t)$ take values typically in the range of $10 - 40\%$.
33. Assumptions Underlying the Haircut Assumption: The range of values for $h_{R/P}(t)$ has been calibrated using

$$\beta_{R/P}(t) = 1$$

and

$$\alpha_{R/P,\infty}(t) = 1$$

Both assumptions are broadly consistent with historical data.

34. IOSCO results in Over-collateralization: Note also that the goal of the BCBS-IOSCO regulations is to ensure that the netting sets are largely over-collateralized as a consequence of:
    a. The high confidence level at which the IM is computed, and
    b. The separate requirements for IM and VM.
35. Impact of Over-collateralization: Hence, the exposure generating scenarios are tail events, and the effect on capital exposure of a conservative haircut applied to the received IM is rather limited in absolute terms.
36. Over-collateralization Impact on Exposure: This issue is demonstrated by Anfuso, Aziz, Loukopoulos, and Giltinan (2017) where the expected exposure (EE) at a given horizon t is shown as a function of $h_R(t)$ – the haircut to be applied to the received IM collateral – for different distributional assumptions on $\Delta MTM(t, t + MPoR)$.
37. Distribution Dependence on Haircut Functions: In particular, they compute the expected exposure for

$$h_R(t) = 0$$

and

$$h_R(t) = 1$$

indicating full IM collateral benefit or no benefit at all – and take the unscaled IM as the 99^{th} percentile of the corresponding distribution. For different classes of the ΔMTM distribution, the exposure reduction is practically unaffected up to haircuts of $\approx 50\%$.

Backtesting the IMRD for MVA and LCR/NSFR

1. MC Based DIM IMR Distributions: The same Monte Carlo framework can be used in combination with a DIM model to forecast the IMD at any future horizon – implicit here are the models in which the DIM is not always constant across the scenarios. The applications of the IMRD are multiple.
2. Some Applications using the IMRD: The following are two examples that apply equally to the cases of B-IMR and CCP IMR:
    a. Future IM funding costs in the P measure, i.e., the MVA
    b. Future IM funding costs in the Q measure, e.g., in relation to LCR and NSFR regulations (Basel Committee on Banking Supervisions (2013))
3. Numerically Forecasting the IMR Distributions: The focus here is on the forecasts on the P-measure – tackling the case of the Q-measure may require a suitable generalization of Jackson (2013). The main difference with the backtesting approach discussed above is that the new model forecasts are the numerical distributions of the simulated IMR values.
4. Scenario-specific IM Forecasting: These can be obtained for a given horizon by associating every simulated scenario with its corresponding IMR forecast, computed according to the given DIM model.
5. Posted/Received IMR Density CDF: Using the notation introduced previously, the numerical representations of the received/posted IMRD cumulative density functions (CDF's) of a portfolio p for a forecasting day t_k and a horizon h are given by

$$CDF_{R/P}(x, t_k, h) = \frac{\#\{v \in \mathbb{V} \mid v \leq x\}}{N_{\mathbb{V}}} \; \forall\, \vec{r}_\omega \in \Omega$$

$$\mathbb{V} = \left\{ f_{R/P} \left(t_0 = t_k, t = t_k + h, \vec{r}, \qquad \Pi = \Pi\big(p(t_k)\big), \vec{M}_{DIM} = \vec{M}_{DIM}(t_k) \right) \right\}$$

6. Terms of the CDF Expression: In

$$CDF_{R/P}(x, t_k, h) = \frac{\#\{v \in \mathbb{V} \mid v \leq x\}}{N_{\mathbb{V}}}$$

$N_{\mathbb{V}}$ is the total number of scenarios. In

$$\mathbb{V} = \left\{ f_{R/P}\left(t_0 = t_k, t = t_k + h, \vec{r}, \quad \Pi = \Pi\big(p(t_k)\big), \vec{M}_{DIM} = \vec{M}_{DIM}(t_k)\right) \forall\, \vec{r}_\omega \in \Omega \right\}$$

$f_{R/P}$ are the functions computed using the DIM model, $\vec{r}_\omega$ are the scenarios for the predictor – the portfolio MTM values in the case originally discussed, and Ω is the ensemble of $\vec{r}_\omega$ spanned by the Monte Carlo simulation.

7. Suitability of IMRD for Backtesting: The IMRD in this form is directly suited for historical backtesting using the Probability Integral Transformation (PIT) framework (Diebold, Gunther, and Tay (1998)).
8. Forecasting Horizon PIT Time Series: Referring to the formalism described in one can derive the PIT time series $\tau_{R/P}$ for a portfolio p for a given forecasting horizon h and backtesting history $\mathcal{H}_{BT}$ as:

$$\tau_{R/P} = CDF\left(g_{R/P}\left(t = t_k + h, \Pi = \Pi\big(p(t_k + h)\big), \vec{M}_g = \vec{M}_g(t_k + h)\right), t_k, h\right) \forall\, t_k \in \mathcal{H}_{BT}$$

9. Samples from the Actual IMR Algorithm: In the expression for $\tau_{R/P}$ above, $g_{R/P}$ is the exact IMR algorithm for the IMR methodology that is to be forecast – defined as

$$IMR = g_{R/P}\left(t = t_\alpha, \Pi = \Pi\big(p(t_\alpha)\big), \vec{M}_g = \vec{M}_g(t_\alpha)\right)$$

and t_α are the sampling points in $\mathcal{H}_{BT}$.

10. Probability of t_k-realized IMR: Every element of the PIT time series $\tau_{R/P}$ corresponds to the probability of the realized IMR at time $t_k + h$ according to the DIM forecast built at t_k.
11. Backtesting of the Portfolio Models - Variations: As discussed extensively in Anfuso, Karyampas, and Nawroth (2017) one can backtest $\tau_{R/P}$ using uniformity tests. In particular, analogous to what was shown in Anfuso, Karyampas, and Nawroth (2017) for portfolio backtesting in the context of capital exposure models, one can use test metrics that do not

penalize conservative modeling – i.e., models overstating/understating posted/received IM. In all cases the appropriate TVS can be derived using numerical Monte Carlo simulations.

12. Factors affecting the Backtesting: In this setup the performance of a DIM is not done in isolation. The backtesting results will be mostly affected by the following.
13. Impact of $\vec{r}$ on Backtesting: As discussed earlier, $\vec{r}$ is the predictor used to associate an IMR with a given scenario/valuation time point. If $\vec{r}$ is a poor indicator for the IMR, the DIM forecast will consequently be poor.
14. Mapping of $\vec{r}$ to IMR: If the mapping model is not accurate, then the IMR associated with a given scenario will be inaccurate. For example, the models defined in

$$\sigma^2(i,t) = \mathbb{E}[\Delta MTM^2(i,t) \mid MTM(i,t)] = \sum_{k=0}^{n} a_{\sigma k} MTM^k(i,t)$$

$$IM_{R/P,U}(i,t) = \Phi^{-1}\big(0.99/0.01, \mu = 0, \sigma = \sigma(i,t)\big)$$

$$IM_{R/P}(i,t) = \alpha_{R/P}(t) \times IM_{R/P,U}(i,t)$$

$$\alpha_{R/P}(t) = \big[1 - h_{R/P}(t)\big] \times \sqrt{\frac{10\ BD}{MPoR}} \times \big[\alpha_{R/P,\infty} + \big(\alpha_{R/P,0} - \alpha_{R/P,\infty}\big)e^{-\beta_{R/P}(t)t}\big]$$

$$\alpha_{R/P,0} = \sqrt{\frac{MPoR}{10\ BD}} \times \frac{IMR_{R/P,SIMM}(t=0)}{q\big(0.99/0.01\,, \Delta MTM(0, MPoR)\big)}$$

include scaling functions to calibrate the calculated DIM to the observed

$$t = 0$$

IMR. The performance of the model is therefore dependent on the robustness of this calibration at future points in time.

15. RFE Models used for $\vec{r}$: The models ultimately determine the probability of a given IMR scenario. It may so happen that the mapping functions $f_{R/P}$ are accurate but the probabilities for the underlying scenarios for $\vec{r}$ are misstated, and, hence, cause backtesting failures.
16. Differential Impact of Backtesting Criterion: Note that
    a. The choice of $\vec{r}$, and
    b. The mapping

$$\vec{r} \rightarrow IMR$$

are also relevant to the backtesting methodology discussed earlier in this chapter. RFE models used for $\vec{r}$, however, are particular to this backtesting variance, since it concerns the probability weights of the IMRD.

Conclusion

1. Framework to Develop/Backtest DIM: This chapter has presented a complete framework to backtest and develop DIM models. The focus has been on B-IMR and SIMM, and the chapter has shown how to obtain forward-looking IM's from the simulated exposure paths using simple aggregation methods.
2. Applicability of the Proposed Model: The proposed model is suitable for both XVA pricing and capital exposure calculations; the haircut functions in

$$\alpha_{R/P}(t) = \left[1 - h_{R/P}(t)\right] \times \sqrt{\frac{10\ BD}{MPoR}} \times \left[\alpha_{R/P,\infty} + \left(\alpha_{R/P,0} - \alpha_{R/P,\infty}\right)e^{-\beta_{R/P}(t)t}\right]$$

can be used to either improve the accuracy (pricing) or to ensure the conservatism of the forecast (capital).

3. CCR Capital using DIM Models: If a financial institution were to compute CCR exposure using internal model methods (IMM), the employment of a DIM could reduce the CCR capital significantly, even after the application of a conservative haircut.
4. Over-collateralization inherent in Basel SA-CCR: This should be compared with the regulatory alternative SA-CCR, where the benefits from over-collateralization are largely curbed (Anfuso and Karyampas (2015)).
5. Backtesting Methodology to Estimate Performance: As part of the proposed framework, this chapter introduced a backtesting methodology that is able to measure model performance for different applications of DIM.
6. Agnosticity of DIM to the Underlying IMR: The DIM model and the backtesting methodology presented are agnostic to the underlying IMR algorithm, and they can be applied in other contexts such as CCP IM methodologies.

References

- Andersen, L., M. Pykhtin, and A. Sokol (2017a): Re-thinking Margin Period of Risk **eSSRN**.
- Andersen, L., M. Pykhtin, and A. Sokol (2017b): Credit Exposure in the Presence of Initial Margin **eSSRN**.
- Anfuso, C., D. Aziz, K. Loukopoulos, and P. Giltinan (2017): A Sound Modeling and Backtesting Framework for Forecasting Initial Margin **eSSRN**.
- Anfuso, C., and D. Karyampas (2015): Capital Flaws *Risk* **27 (7)** 44-47
- Anfuso, C., D. Karyampas, and A. Nawroth (2017): A Sound Basel III Compliant Framework for Backtesting Credit Exposure Models **eSSRN**.

- Basel Committee on Banking Supervision (2013): Basel III: The Liquidity Coverage Ratio and Liquidity Risk Monitoring Tools
- Basel Committee on Banking Supervision (2015): Margin Requirements for Non-centrally Cleared Derivatives
- Diebold, F. X., T. A. Gunther, and A. S. Tay (1998): Evaluating Density Forecasts with Applications to Financial Risk Management *International Economic Review* **39 (4)** 863-883
- Green, A. D., and C. Kenyon (2015): MVA by Replication and Regression **arXiV**
- International Swaps and Derivatives Association (2016): ISDA SIMM Methodology
- Jackson, L. (2013): Hedge Backtesting for Model Validation *Risk* **25 (9)** 64-67
- Longstaff, F., and E. Schwartz (2001): Valuing American Options by Simulation: A Simple Least-Squares Approach *Review of Financial Studies* **14 (1)** 113-147

CCP and SIMM Initial Margin

Initial Margin

1. Initial Margin as Portfolio VaR: Initial margin is initiated as VaR for the derivative portfolio. VM represents P and IM uses P&L in some holding period.
2. Bilateral IM as Parametric VaR: Bilateral IM uses the parametric VaR where the parameters were calibrated to the historical data. Bilateral IM is called the Standard Initial Margin Model (SIMM).
3. CCP IM using Historical VaR: On the other hand, the Central Counter-party (CCP) uses historical VaR. CCP also demands other extra pool of assets to cover losses in the event of multiple members' default.
4. Bilateral CCP VM/IM Methodology:

	Metric	**Bilateral**	**CCP**
Variation Margin	MTM	-	-
Initial Margin	VaR	SIMM	VaR/CVaR
Default Fund	Stress VaR	-	Systemic Risk

5. Bilateral CCP/Holding Period Horizon: The holding period is defined as 10 days for bilateral and 5 days for CCP.

6. VaR Estimation Time Period Horizon: For SIMM, the scenario considered is the recent 3 years plus the common stress period (30 Aug 2008 to 29 November 2008), and the parameter called Risk Weight is calibrated. CCP directly uses the rate shift or the spread shift data for the reference period. For IRS cases, LCH uses 10 years (2500 days), JSCC uses 5 years (1250 days), and CME uses similar reference period (1260 + stress).
7. SIMM/ES VaR Confidence Level: The confidence level used for SIMM is 99% (i.e., its equivalent calibrated level), 99.7% expected shortfall (ES) for LCH, 99% ES for JSCC, and 99.7% VaR for CME.
8. Expression for VaR/Confidence Shortfall: VaR:

$$\mathbb{P}[V(t + \Delta t) - V(t) \leq Threshold] = ConfidenceLevel$$

Expected Shortfall (CVaR):

$$\mathbb{E}[V(t + \Delta t) - V(t) \leq Threshold] = ConfidenceLevel$$

CCP IM

1. Regulation Mandated Clearing of Derivatives: At the September 2009 Pittsburgh summit, it was agreed that G20 should clear certain OTC derivatives transactions at CCP's by the end of 2012 at the latest.
2. IRS/FX - LCH/JSCC/CME: LCH SwapClear, Japanese Securities Clearing Corporation (JSCC), and Chicago Mercantile Exchange (CME) clear not only Interest Rate Swaps, but also other FX products.
3. CDS: ICE/JSCC - US/EU Sovereigns: In the CDS market, ICE Clearing is dominant for clearing US, European, and Sovereign CDS names.

4. Bond/Repo/Equity/Futures: Other securities such as Bond and Repo or Bond Futures and Equity Futures are also clearable.
5. Components of the IRS IM: Base IM is ES, and Liquidity Add-On is what is embedded as the additional factor of concentration for the DV01. Other add-ons are the CCP-specific model costs (e.g., LCH specific).
6. CDS - ICE and JSCC:

$$IM = BaseIM + BidOffer\ \ AddOn + ShortCharge\ AddOn + Other\ AddOn$$

7. Components of the CDS IM: $BidOffer\ \ AddOn$ is the transaction cost on a specific name's CS01. Short Charge is the jump-to-default (JTD) charge for selling the position, which is typically applied on the one shortest name only. Other Add-On includes such factor as Recovery Rate Add-On.
8. Bond/Equity and Futures - LCH, JSCC, and CME: There is a standard method the CME developed in 1988 called SPAN@ (Standard Portfolio Analysis of Risks). It uses basic representative scenarios – 16 of them – and the netting among intra-month in the same security and the security futures and the netting among the different securities are considered.

Interest Rate Swap Methodology

1. Base IM: Base IM has 1250 to 2500 scenarios in which the rate absolute/relative shift $R(t)$ is used. The absolute shift causes the rate to behave like a normal distribution. Relative shift, however, results in a log-normal distribution.
2. Adjusting Historical Shifts using EWMA: The relative shift was originally used in all CCP's, but after the onset of the negative interest rates, those shifts originally taken from historical data were adjusted by exponentially weighted moving average (EWMA).

$$R_N(t) = R_N + S(t)$$

3. Calculation of Rate Shift Adjustment: Suppose that the rate shift at t is calculated using the LCH method below. It is adjusted by the ratio of volatility.

$$N = Today$$

$$t = Scenario\ Date$$

$$R(t) = \{Z(t+5) - Z(t) : t \in T\} \Longrightarrow S(t) = \left\{R(t) \cdot \left(\frac{\sigma_N}{\sigma_t} + 1\right) \cdot \frac{1}{2} : t \in T\right\}$$

4. Application of EWMA Decay Factor: EWMA uses the decay factor λ to calculate the volatility. It represents how much the older volatilities affect the next volatility.

$$\sigma^2(t+1) = \lambda[\sigma^2(t) + (1-\lambda)^2 R^2(t+1)]$$

5. CCP Rate Shift Generation Settings:

	JSCC	**LCH**	**CME**
IR Change Measure	Absolute	Absolute	Relative (+4%)
FX Change Measure	Relative	Relative	Relative
EWMA Decay Factor	0.985	0.992	0.970
Scenario	1250	2500	1260
Confidence Level	ES of Worst 12	ES of Worst 6	99.70%

6. Rate Shift Methodology used by CME: A different approach is taken by CME, which is to shift the rate by some offset α.

$$R(t) = \left\{ \ln \frac{Z(t+5)+\alpha}{Z(t)+\alpha} \ : t \in T \right\}$$

7. Liquidity Margin: Liquidity Margin is the add-on term for the concentration position on a specific tenor bucket b and instrument i - $PV01(b,i)$ The correlation among the instruments and the tenor buckets are handled by using either a correlation matrix or a similar methodology – the function below is denoted as LM in the later sections

$$LM = Function\left(PV01(b,i)\right)$$

Interest Rate Swap Calculation

1. Scenario-Specific Trade Level-IM: The base IM can be calculated with re-gridded delta multiplied by the scenario's ω – official CCP is based on full revaluation, but the gammas are typically not large.
2. Scenario P&L Linear in Delta: As scenario P&L is a linear function of delta, the Base IM numbers can be incremented in the accumulator once the scenarios are fixed.
3. Liquidity Margin Dependent on $PV01(b,i)$: On the other hand, Liquidity Margin uses re-gridded delta flows called $RepFlows$ with which the portfolio's $PV01(b,i)$ is calculated.
4. Portfolio Base IM/Liquidity Margin:

$$Base\ IM(Portfolio, \omega) = \sum_j Base\ IM\left(Trade_j, \omega\right)$$

$$Liquidity\ Margin\ (Portfolio) = LM\left(\sum_j RepFlows_j\right)$$

Credit Default Swap Methodology

1. Base IM: Base IM is called Spread Response Requirement in ICE's Terminology, and it represents the log returns of the EOD credit spread resulting from the time series analysis from April 2007.
2. Scaling/Flattening/Steepening/Tightening/Inverting/Widening: The result of the log returns are molded into 6 shapes, scaling/flattening/steepening/tightening or scaling/flattening/inverting/widening scenarios.

$$R_{T,SCENARIO} = R_T \cdot e^{RiskFactor \cdot ShapeFactor(T)}$$

3. Base IM as the Greatest Loss Scenario: Tightening and widening risk factors are described for each reference credit. From those spread responses, the greatest loss scenario is chosen as the base IM.
4. JSCC CDS Clearing IM Methodology: JSCC uses the relative shift without any EWMA adjustment from the last 750 days.
5. Bid-Offer Margin: Bid-offer margin is the transaction cost associated with unwinding CDS trades. It is surveyed on tenor buckets and reference credits.
6. Short Charge (JtD Charge): Short charge is the loss given default risk if the reference credit of the CDS trades defaults at the same time as the Clearing Member's (CM) default.
7. Notional Decomposition for Index Trades: For the index trades, the notional is decomposed into each constituent's amount.

$$Short\ Charge = \max(JtD_j)$$

8. RR, Basis, and IR Margins: ICE provides other factors including Recovery Rate Risk, Basis Risk, and IRS Risk Margins.

SIMM

1. SIMM Bilateral Initial Margin Specifications: Bilateral Initial Margin was introduced in BCBS 226 and 261, and SIMM is used as the ISDA agreed methodology. The final rule was published for JFSA, CFTC, USPR, and ESA in 2015 and 2016. The calculation method is updated quiet often as new risk factors are introduced.
2. SIMM Structure: SIMM is the summation of the product IM's, ranging from IR&FX, Credit, Equity, and Commodity.

$$SIMM = SIMM_{RATESFX} + SIMM_{CREDIT} + SIMM_{EQUITY} + SIMM_{COMMODITY}$$

   For a product class IM, six risk classes are used: Interest Rate, Credit (Qualifying), Credit (Non-qualifying), Equity, Commodity, and FX. There is a correlation matrix among the risk factors.
3. Delta Margin:

$$DeltaMargin = \sqrt{\sum_b K_b{}^2 + \sum_b \sum_{c \neq b} \gamma_{bc} S_b S_c} + K_{RESIDUAL}$$

   where K_b is the bucket margin, S_b has the sign with floored and the weighted sensitivity ($WS_{k,i}$). $WS_{k,i}$ is the delta multiplied by the risk weight.

$$WS_{k,i} = RW_k s_{k,i} CR_b$$

$$K = \sqrt{\sum_k WS_k{}^2 + \sum_k \sum_{l \neq k} \rho_{kl} f_{kl} WS_k WS_l}$$

where

$$f_{kl} = \frac{\min(CR_k, CR_l)}{\max(CR_k, CR_l)}$$

and

$$S_b = \max\left(\min\left\{\sum_{i,k} WS_{i,k}, K_b\right\}, -K_b\right)$$

4. <u>Vega Margin</u>: Vega Margin uses the similar calculation as Delta Margin. The vega risk is computed as the product of the volatility and the vega.

$$VR_{ik} = HVR_c \sum_j \sigma_{jk} \frac{\partial V_{ij}}{\partial \sigma}$$

5. <u>Curvature Margin</u>: Curvature margin uses the gamma inferred from the vega. It can be represented by the scaling function $SF(t_{jk})$ multiplied by the volatility and the vega.

$$CVR_{ik} = \sum_j SF(t_{jk}) \sigma_{jk} \frac{\partial V_i}{\partial \sigma}$$

MVA

1. Margin Value Adjustment (MVA) Definition: The formulation of the IM cost can be cast as the equation below:

$$MVA = \int f(t) IM(t) D_f(t) dt$$

where $f(t)$ is the funding spread and $IM(t)$ – the forward IM – uses forward risks where the future delta and vega are implied from the spot IM with an approximation for the future delta as

$$Delta(T) = Delta(t) \times \frac{T-t}{T}$$

Vega works as

$$Vega(T) = Vega(t) \times \sqrt{\frac{T-t}{T}}$$

2. Funding Rate for the MVA: If the collateral rate is the repo rate, then the funding spread is the spread between the funding rate and the repo rate.
3. Swaption: As an example, for a swaption using a normal Black Scholes, the formula can be written as a function of the forward (F), strike (K), volatility (σ), and expiry (t_{ex}). Forward Risk (delta):

$$\Phi\left(\frac{F-k}{\sqrt{\sigma(t_{ex}-t)}}\right) \sim Constant\ if\ t\ would\ not\ change$$

Volatility Risk (Vega):

$$\sqrt{(t_{ex}-t)}\, f\left(\frac{F-k}{\sqrt{\sigma(t_{ex}-t)}}\right) \sim Constant \times \sqrt{\frac{t_{ex}-t}{t_{ex}}}$$

4. <u>5Y - 10Y 10 billion Yen ATM Payer Swaption</u>:

$$Risk_{IRCurve} \sim ExerciseProbability \times DV01 \times 20$$

for 5Y risk weight or

$$Risk_{IRCurve} \sim ExerciseProbability \times DV01 \times 22$$

for 15Y risk weight.

$$Risk_{IRVol} \sim Vega \times Normal\,Vol \times 21$$

and the forward profile is based on the above approximation.

Summary

1. <u>CCP IM and SIMM</u>: CCP IM uses a historical scenario in its VaR – or CVaR – calculation, while SIMM uses parametric VaR. Other add-ons such as the Liquidity Margin are not small in the CCP IM.

2. MVA Estimation: Margin Valuation Adjustment can be done by taking two factors – the funding cost and the forward IM profile. The forward IM profile could be further enhanced depending on the trade activity and the optimization activity.