

# Transaction Cost Analytics in DRIP

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**Optimal Execution of Portfolio Transactions**

**Overview, Scope, and Key Results**

1. Portfolio Transactions under Market Impact: Almgren and Chriss (2000) consider the execution of portfolio transactions with the aim of minimizing a combination of volatility risk and transaction costs arising from temporary and permanent market impact.
2. Efficient Frontier under Linear Cost: For a simple linear cost model, they explicitly construct an *efficient frontier* in the space of time-dependent liquidation strategies, which have the minimum expected cost for a given level of uncertainty.
3. Choice of the Utility Function: This enables one to select optimal strategies either by minimizing a quadratic utility function, or by minimizing the Value-at-Risk.
4. Liquidity Adjusted Value at Risk: The latter choice leads to the concept of liquidity-adjusted VaR, or L-VaR, that explicitly considers the best trade-off between the volatility risk and the liquidity costs.

**Motivation Background, and Synopsis**

1. Transactions Changing the Portfolio Composition: Almgren and Chriss (2000) consider the optimal execution of portfolio transactions that move a portfolio from a given staring composition to a specified final composition within a specified period of time.
2. The Bertsimas and Lo Approach: Bertsimas and Lo (1998) define the best execution as the dynamic trading strategy that provides the minimum cost of trading over a fixed period of time, and they also show that under a variety of circumstances one can find such a strategy by employing a dynamic optimization procedure; but they ignore the volatility of revenues of different trading strategies.
3. Maximization of Expected Trading Revenue: Almgren and Chriss (2000) work in the more general framework of maximizing the *expected revenue* – or equivalently minimizing the costs – with a suitable penalty for the *uncertainty* of revenue (or cost).
4. Market Microstructure Framework: This general framework arises in the market microstructure theory, but with a different purpose in mind. The *uninformed discretionary trader* trades an exogenous endowment over an exogenously specified amount of time to maximize the profits (Admati and Pfleiderer (1988)); the informed strategic trader trades over multiple periods on information not widely available, again to maximize profits (Kyle (1985)). In both cases the literature focuses on the link between the trader and the market maker, and a theory is produced to predict the market clearing price of the security at each period. Thus a trader’s optimal strategy is used as a means to study the price formation in the markets, not as an object of interest in itself.
5. Variance of the Trading Cost: Almgren and Chriss (2000) study the variance of the trading cost in optimal execution because it fits in with the intuition that the trader’s utility should figure in the definition of *optimal* in “optimal execution”.
6. Example: Trading Illiquid Volatile Securities: For example, in trading a highly illiquid, volatile security, there are two extreme outcomes; trade everything now at a known, but high, cost, or trade in equal sized packets over a fixed time at a relatively lower cost. The latter strategy has a lower expected, but this comes at the expense of greater uncertainty in the final revenue.
7. Estimation of the Trading Uncertainty: How to evaluate the above uncertainty is partly subjective, and is a function of the trader’s tolerance for risk. All that can be done is to insist that for a given level of uncertainty that the cost be minimized. This idea extends to a complete theory of optimal execution that includes an efficient frontier of optimal execution strategies.
8. Consistency with Expectations from Intuition: The framework of risk in execution yields several results that are consistent with the intuition. For example, it is evident that all else equal, a trader will choose to execute a block of illiquid security less rapidly than a liquid security.
9. Models Lacking Consistency with Intuition: While this seems obvious, Almgren and Chriss (2000) demonstrate that a model that ignores risk does not have this property; without enforcing a strictly positive penalty for risk one cannot produce models that trade differently across the spectrum of liquidity.
10. Arithmetic Brownian Motion Price Dynamics: The incorporation of risk into optimal execution does not come without cost. First, in order to be able to produce tractable analytical results, Almgren and Chriss (2000) are forced to work in largely in the framework of price dynamics that are an arithmetic walk with independent increments.
11. Use of Static Optimization Procedures: They obtain results using *static optimization* procedures which they show lead to globally optimal trading trajectories. That is, optimal trading paths may be determined in advance of trading. Only the composition of the portfolio and the trader’s utility function figure on the trading path.
12. Why does Static Optimization Work? The fact that the static strategy can be optimal even when the trader has the option to dynamically change his trading mid-course is a direct result of the assumptions of independence of returns and symmetry for the penalty functions for risk.
13. Using Non-Symmetric Penalty Functions: An interesting deviation from the symmetric penalty function was communicated by Ferstenberg, Karchmer, and Malamut at ITG Inc. They argue that the opportunity is a subjective quantity and is measures differently by different traders. Using a trader defined cost function , they define opportunity costs as the expected costs of applied to the average execution price obtained by the trader relative a benchmark price. They assume that the risk-averse traders will use a convex function that is not symmetric in the sense that there is a strictly greater penalty for underperformance than for the same level of outperformance. They show that in this setting, the optimal strategy relative to not only depends on the time remaining, but also on the performance of the strategy up to the present time, and the present price of the security. In particular, this means that in their setting, optimal strategies are dynamic.
14. Serial Correlations among Price Movements: As it is well known that price movements exhibit some serial correlations across various time horizons (Lo and MacKinlay (1988)), that market conditions change, and that some participants possess private information (Bertsimas and Lo (1998)), one may question the usefulness of results that obtain strictly in an independent-increment framework.
15. The Dynamic Nature of Trading: Moreover, as trading is known to be a dynamic process, the conclusion that optimal trading strategies can be statically determined calls for critical examination. Almgren and Chriss (2000) examine what quantitative gains are available that incorporate all the relevant information.
16. Impact of the Serial Correlations: First they consider short term serial correlations in price movements. They demonstrate that the marginal improvements available by explicitly incorporating this information into trading strategies is small, and more importantly, independent of the portfolio sizes; as portfolio sizes increase, the percentage gains possible decrease proportionately.
17. Combining “Correlated” and “Shifting” Strategies: The above is precisely true for linear transaction cost models, and is approximately true for more general models. The results of Bertsimas and Lo (1998) suggest that trading a strategy built to take advantage of serial correlation will essentially be a combination of a “correlation free” strategy and a “shifting strategy” that moves from one trade period to the next based on the information available in the last period’s return. Therefore Almgren and Chriss (2000) argue that by ignoring serial correlation, they a) preserve the main interesting features of their analysis, and b) introduce virtually no bias away from “truly optimal” solutions.
18. Impact of Scheduled News Events: Second, Almgren and Chriss (2000) examine the impact of scheduled new events on optimal execution strategies. There is ample evidence that anticipated news announcements, depending on their outcome, can have a significant temporary impact on the parameters governing price movements.
19. Scheduled News Events - Literature Review: For a theoretical treatment see Brown, Harlow, and Tinic (1988), Kim and Verrecchia (1991), Easterwood and Nutt (1999), and Ramaswami (1999). For empirical studies concerning earnings announcements, see Patell and Wolfson (1984) for changes in mean and variance of intra-day prices, and Lee, Mucklow, and Ready (1993) and Krinsky and Lee (1996) for changes in the bid-ask spread. For additional studies concerning news announcements, see Charest (1978), Morse (1981), and Kalay and Loewentstein (1985).
20. Model Incorporation of Scheduled Events: Almgren and Chriss (2000) work in a simple extension of their static framework by assuming that the security again follows an arithmetic random walk, but at a time known at the beginning of trading, an uncorrelated event will cause a material shift in price dynamics, e.g., an increase or decrease of volatility.
21. Combining Piece-Wise Static Strategies: In this context they show that optimal strategies are piece-wise static. To be precise, they show that an optimal strategy entails following a static strategy up to the moment of the event, followed by another static strategy that can only be determined once the outcome of the event in known.
22. Variation from the Original Static Strategy: It is interesting to note that the static strategy that one follows in the first leg is in general not the same strategy one would follow in the absence of information concerning the event.
23. Accommodating Unanticipated External “Sudden’ Events: Finally Almgren and Chriss (2000) note that any optimal execution strategy is vulnerable to *unanticipated events*. If such an event occurs during the course of trading and causes a material shift in the parameters of the price dynamics, then indeed a shift in the optimal trading trajectory must also occur.
24. Adaptation at Parameter Shift Edges: However if one makes a simplifying assumption that all events are either “scheduled” or “unanticipated” one then concludes that optimal execution is always a game of static trading punctuated by shifts in the trading strategies that adapt to material changes in the price dynamics.
25. Pre-determined vs. Active Approaches: If shifts are caused by events that are known ahead of time, then optimal execution benefits from a precise knowledge of the possible outcomes of the event. If not, the best approach is to be actively “watching” the market for such changes and react swiftly should they occur.
26. Simple Proxy for Unexpected Uncertainty: One approximate way to include such completely unexpected uncertainty into the model is to artificially raise the value of the volatility parameter.
27. Risk Averse Optimal Trading Strategies: As a first step, Almgren and Chriss (2000) obtain closed form solutions for trading strategies for any level of risk aversion.
28. Efficient Frontier of Optimal Strategies: They then show that this leads to an efficient frontier of optimal strategies, where an element of the frontier is represented by a strategy with a minimal level of cost for its level of variance of the cost.
29. Graphical Structure of the Frontier: The structure of the frontier is of some interest. It is a smooth convex function differentiable at its minimal point. The minimal point is what Bertsimas and Lo (1998) call the naïve strategy because is corresponds to trading equally sized packets using all available trading time equally.
30. Differential at the Minimum Point: The differentiability of the frontier at its minimum point indicates that one can obtain a first order reduction in the variance of the trading cost at the expense of only a second order in cost by trading a strategy slightly away from the globally minimal strategy.
31. Curvature at the Minimal Point: The curvature of the frontier at its minimum point is a measure of the liquidity of the security.
32. Half-Life of Optimal Execution: Another ramification of the Almgren and Chriss (2000) study is that for all levels of risk aversion except risk neutrality, optimal execution trades have a “half-life” that fall out of the calculations.
33. Independence form the Time to Complete Execution: A trade’s half-life is independent of the actual specified time to liquidation, and is a function of the security’s liquidity and volatility, and the trader’s level of risk aversion.
34. Half-Life as Execution Time: As such Almgren and Chriss (2000) regard the half-life as an idealized time to execution, and perhaps a guide to the proper amount of time over which to execute a transaction.
35. Time Lesser than Half Life: If the specified time to liquidation is short relative to the trade’s half-life, once can expect the cost of trading to be dominated by transaction costs.
36. Time Greater than Half Life: If the time to trade is long relative to the half-life, one can then expect most of the liquidation to take place well in advance of the limiting time.

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# Algorithmic Market Making

### Symbology Glossary

1. Tight Skew: 
2. Loose Skew: 
3. Tight Width: 
4. Loose Width: 
5. Algorithmically generated Ideal Mid Cash Price: 
6. Position: (expressed in cumulative net position per unit under consideration – firm/desk/trader)
7. Position Pivot: . Dimensionless ontological view of the scaling position metric – roughly equivalent to the Reynolds’ number of market making position units. Expressed in currency units.
8. Risk: (expressed in cumulative net risk per unit under consideration – firm/desk/trader)
9. Risk Pivot: . Dimensionless ontological view of the scaling risk metric – roughly equivalent to the Reynolds’ number of market making risk units. Expressed in PV01 currency units.

### Framework Glossary

1. Equilibrium quantity: Quantity that only changes with the macro drivers/factors, and not the technical factors. Typically stable, but jumpy and undergoes changes when drivers shift – and introduces perturbations on the disequilibrium quantities.
2. Disequilibrium quantity: Quantity that changes with the technical, transient factors.

## Width/Skew/Size Estimation Models

1. Tight Models:

* Tight models estimate the market making quantities on a trader/firm/desk independent manner.
* They estimate the “secular” market making parameters – width, skew, and size for either the Market Making Outputs or the Axe Outputs – estimate them based on classes of input parameters.
* For each input parameter class, the following are needed:
  + - * 1. A proxy that serves as a quantitative estimate of the desired parameter class.
        2. Segmentation of the proxy over the sub-classified parameter set.

1. Input Class => Risk Profile:

* Captures all the cumulative risk components => the credit/solvency, market, and liquidity risk behind the issue.
* Proxy => CDS Spread, rating, bond basis
* Sub-classification => Issue, issuer, and sector.

1. Input Class => Liquidity:

* Captures the frequency and volume of the trade flow of a given issue, and the ease of getting in and getting out at the given side.
* Proxy:
  + 1. Aggregated periodic (e.g., daily) volume for each side (buy/sell).
    2. Aggregated periodic (e.g., daily) notional for each side (buy/sell).
* Sub-classification => Issue, issuer, sector, and the instrument universe.

1. Firm/Desk/Trader level parameters: These provide aggregated controls for trading.

* Net Position => vital metric for inventory control.
* Risk limits => to control/manage exposure to specific granules – issue, issuer, tenor, sector, unit etc.

1. Monitor Mobility: Certain measures such as PV01 based risk, inventory, etc. are more easily human-monitored, so they are done daily. Others (such as tenor 01s) are less easily monitored, so they are done infrequently.

## Market Making System SKU

* 1. Intra day Curve Generation Scheme
  2. Mid Price Estimation Models
     1. Accommodate different mid price estimation models, and their respective parameters
  3. Algorithmic Quote Construction => used for generating venue/ECN independent width/skew/size [composed of tight/loose components]. Broadly speaking achieves the following:
     1. Specific parameters to control skew for targeted alpha generation strategies
     2. Accommodate different width and size estimation models, and their respective parameters
     3. Venue-independent base quote synthesis/construction
     4. Circuit breaker heuristics
     5. Policy driven/policy enforcement/policy control applied at this level
  4. Quote Management: Publishing/tailoring the constructed quote towards specific venues (possibly with order routing applied at this stage).
     1. Venue specific rules (and thereby external vendor incorporations, like Broadway etc. at this stage.

## Market Making Parameter Types

1. Model Parameters: Parameters for generation of algorithmic generation of width, skew, and size.
2. Quote Generation Control Parameters
3. Quote Heuristics Control
4. Quote Management Control

## Intra-day Pricing Curve Generation Schemes

1. Issue Benchmark Bonds: The following set of threshold criteria are used to determine the issuer specific benchmark bonds:

* + 1. Threshold of daily TRACE volume/number of trades
    2. Threshold of outstanding notional
    3. Only senior obligations
    4. Some combination of the following threshold of the ratios:
* 
* 

1. Benchmark bonds basis tracking: Track the bid side and ask side credit basis of the benchmark bonds from each TRACE print, using EMA VWAP/TWAP from the intra-day rates/credit curves. This will be the attempt to estimate the mid credit basis for the, and it is generally well behaved.

* Need to find a way to accommodate the institutional closing CDS mid marks and the benchmark bonds into the credit curve construction – these are highly valid points.

1. Liquid vs. illiquid: Typical liquid securities’ quote may be proxied out of print (or at least EMA’d). Intra-day quote generation, however, is materially important for illiquid securities.
2. Intra-day credit curve generation inputs: Need a way to generate the credit curve from

The CDS marks

The basis-adjusted benchmark bonds

It always needs to be used in conjunction with tension splines.

Also need intra-day TRACE series to update the basis (direct or EMA) – will use this to establish the intra-day relationship between the CDS nodes and the TRACE cut-off threshold).

1. Intra-day credit curve updating:
   * + - 1. Use the relationship grid between CDS 5Y, the off-tenors, and the benchmark bonds
         2. Any change in any of them automatically re-adjusts using the set relationships.
         3. CDS Curves are trader set; bond basis are EMA’d from the TRACE series using the prior credit curve
         4. Relationships are either reviewed daily EOD
2. Live updating of bond prices: Use the live curve (either pure CDS, or a mixture of CDS/bond instruments) to extract the basis of each print, and then EMA that to generate the bond live prices.

## Mid Price Models

* + 1. Definition: Computed theoretical mid-price, as to where the next print should be – assuming zero transaction costs, zero position/risk constraints, and infinite liquidity. Mid Price is an ***Equilibrium Quantity***.
    2. Estimation parameters: Typical mid price estimation parameters are: the IR curve, the survival curve, and the recovery curve. The other possible drivers are: funding curve – typically for long position, and repo curve – typically for shorts.

## Width Models

1. Tight Width: Computed theoretical width, after accounting for the issue liquidity and the issue riskiness. Tight width is the first in the set of disequilibrium quantities. Tight width is:
   1. Proportional to issue risk (combination of credit and market risk – not counter party risk).
   2. Inversely proportional to liquidity

## Skew Models

* 1. Tight Skew: This measure how far the last print has been OFF from the theoretical mid price. Thus Tight Skew is representative of the alpha potential – for a theoretical mid price that chases the print in a sequence, the tight skew is zero.
  2. Tight Bid Skew and Tight Ask Skew: This is an alternative SKU – instead of tight width and tight skew cognitive view, tight bid/ask skew parameters are determined only from their corresponding liquidity and flow metrics (i.e., bid/ask liquidity metrics).
  3. Loose Skew: Simply put, loose skew is:



* 1. Heuristic Checks on Loose Skew: Following checks applied to round out quoting:

1. Ceiling/floor applied
2. Maximum cutoff for width
3. Best right skew – bid becomes ask.
4. Best left skew – ask becomes bid.

## Size Models

1. Tight bid size/tight ask size: Basically, tight size is inversely proportional to tight width, to within normalized bounds.

## Heuristics Control

* 1. Can Buy/Can Short: Can But/Can Short => whether the bid/ask stays within the LONGABLE/SHORTABLE cutoff.
  2. ECN Threshold Cross: Check to see if there is a cross between the published bid/ask and a given ECN’s bid/ask.

## Published Market Quote Picture

1. Bid/Ask Sizes: Truncated to their appropriate rounding.
2. Bid Price: 
3. Ask Price: 
4. Bid/Ask Prices rounded downward/upward to their appropriate increments.

## Flow Analysis

1. Dimensionless flow classifier: If the metric (ADV etc.) is greater than a specific threshold, then the flow becomes “turbulent”, else it is “laminar”.
2. Flow Potential: Skew of all kinds is related to the flow driver/equilibration strength.