刚体的转动

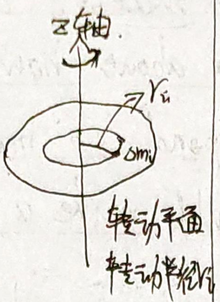
一、刚体运动.

特殊质点系.

每一个质点叫做刚体的质元

刚体平动:

刚体定轴转动.



刚体角速度矢量 (右手定则)

$$\vec{\omega}=\frac{d\theta}{dt}$$

刚体角加速度矢量.

$$\vec{\beta}=\frac{d\vec{\omega}}{dt}$$

二、转动动能. 转动惯量.

$\Delta m_1, \Delta m_2, \cdots, \Delta m_i, \cdots, \Delta m_n \cdots$

$r_1, r_2, \cdots, r_i, \cdots, r_n \cdots$

$v_i=\omega r_i$

$E_{ki}=\frac{1}{2}(\Delta m_i)v_i^2=\frac{1}{2}(\Delta m_i)r_i^2\omega^2$

$E_k=\sum E_{ki}=\sum\frac{1}{2}(\Delta m_i)r_i^2\omega^2=\frac{1}{2}\left(\sum(\Delta m_i)r_i^2\right)\omega^2$

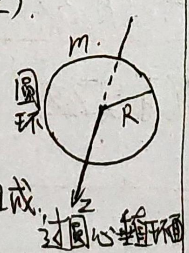
转动惯量 $J=\sum\Delta m_i r_i^2$.

$E_k=\frac{1}{2}J\omega^2$. 恒正的, 可加性.

$dJ=r^2dm$

$J=\int dJ=\int r^2dm$.

$J_0=J_c+md^2$ (平行轴定理).



圆环: $dJ=R^2dm$.

$J=\int dJ=\int R^2dm=mR^2$

圆盘: 无穷多个连续分布的圆环组成.

$ds=2\pi r\,dr$

$dm=\sigma ds=\sigma 2\pi r\,dr$

$dJ=r^2dm=r^2\sigma 2\pi r\,dr$

$J=\int r^2\sigma 2\pi r\,dr=\frac{1}{2}mR^2$

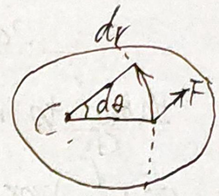
三、刚体转动定理.

合外力矩 $M_{合外}=\sum M_i$

$$\vec{M}_{合外}=J\vec{\beta}=J\frac{d\vec{\omega}}{dt}$$

$$M_{合外}=J\beta=J\frac{d\omega}{dt}$$

四、刚体转动动能定理



元功: $dA=\vec{F}\cdot d\vec{r}$

$=F\,ds\cos\theta$

$=F\sin\varphi\, r\,d\theta$

$=rF\sin\varphi\, d\theta=M\,d\theta$.

$A=\int dA=\int_{\theta_0}^{\theta}M\,d\theta$. 力矩功.

元功之和: $dA=\sum dA_i=\sum M_i\,d\theta$.

$=(\sum M_i)d\theta=M_{合外}d\theta$

$A=\int d\theta=\int_{\theta_0}^{\theta}M_{合外}d\theta=A_1+A_2+\cdots$

$\sum A_{外}+\sum A_{内}=\frac{1}{2}J\omega^2-\frac{1}{2}J\omega_0^2$.

$A=\sum A_{外}=\int_{\theta_0}^{\theta}M_{外}d\theta=\frac{1}{2}J\omega^2-\frac{1}{2}J\omega_0^2$.

刚体重力势能. $E_p=mgh_c$.

刚体的重力矩 $\vec{M}=\vec{r}_c\times m\vec{g}$

☆ $\boxed{\text{刚体的机械能守恒.}}$

六、刚体角动量守恒定律.

$L_{iz}=r_i(\Delta m_i)v_i=r_i(\Delta m_i)r_i\omega=(\Delta m_i)r_i^2\omega$.

$L_z=\sum L_{iz}=\left[\sum(\Delta m_i)r_i^2\right]\omega=J\omega$.

☆ $\boxed{\vec{L}=J\vec{\omega}}$

$\vec{M}_{合外}=J\vec{\beta}=\frac{d\vec{\omega}}{dt}=\frac{d(J\vec{\omega})}{dt}$

$\vec{M}_{合外}=\frac{d\vec{L}}{dt}$. $\vec{M}_{合外}dt=d\vec{L}$

$\int_{t_1}^{t_2}\vec{M}_{合外}dt=\vec{L}_2-\vec{L}_1=J\vec{\omega}_2-J\vec{\omega}_1$

$\int_{t_1}^{t_2}\vec{M}_{合外}dt=\vec{L}_2-\vec{L}_1=J_2\vec{\omega}_2-J_1\vec{\omega}_1$

如果 $M_{合外}=0$. $\Rightarrow \vec{L}=J\vec{\omega}=$ 恒量.

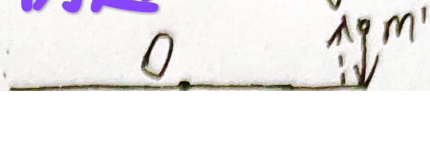
$J_2\vec{\omega}_2=J_1\vec{\omega}_1=$ 恒量.

如果 $M_{iz}=0$. $\Rightarrow \sum L_{iz}=$ 恒量

刚体转动 例题

例一:

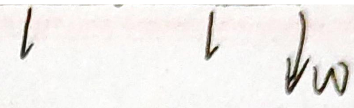
$J m'v=(-(m'v)+J\ldots$



$m'u \cdot l$ [illegible] $J\omega$.

$\frac{1}{2}m'u^2 = \frac{1}{2}m'v^2 + \frac{1}{2}J\omega^2$.

$\Rightarrow \omega . v$.

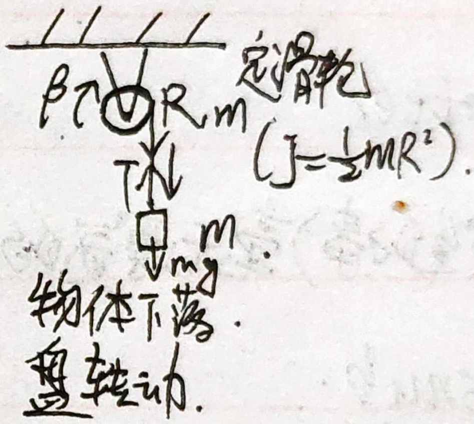


例二：

$$\begin{cases} mg - T = ma. \\ TR = J\beta \quad (\text{对滑轮}). \\ a = \beta R. \end{cases}$$

$\Rightarrow \beta = \frac{2g}{3R}$

$\Rightarrow \theta = \frac{1}{2}\beta t^2 = \frac{1}{2}\frac{2g}{3R}t^2 = \frac{g}{3R}t^2$.



例三：

① $lmv_0 = (\frac{1}{3}ml^2 + ml^2)\omega$.

$\Rightarrow \omega = .$

② 求最大角度 θ.

$\frac{1}{2}(\frac{1}{3}ml^2 + ml^2)\omega^2 = mgl(1-\cos\theta) + mg\frac{l}{2}(1-\cos\theta)$.