高数 A2 答案

一、填空题

1．2；　2．$-\dfrac{\pi^2}{8}$；3．$\dfrac{ydx+xdy}{1+xy}$；4．　4π　；5．　$y=e^x(C_1\cos 3x+C_2\sin 3x)$．

二、选择题

1．C；　2．B；3．A；4．A ；5．　D．

三、解答题

1．解：$y=e^{-\int\frac{1}{x}dx}(\int e^x e^{\int\frac{1}{x}dx}dx+c)=\dfrac{1}{x}(xe^x-e^x+C)$，由 $y|_{x=1}=3$，得 $C=3$，则

特解 $y=\dfrac{1}{x}(xe^x-e^x+3)$

2．解：$\overrightarrow{OA}=(6,3,2)$,平面 $5x+4y-3z=8$ 的法向量 $n_1=(5,4,-3)$，则所求平面的法向量

$n=\begin{vmatrix} i & j & k \\ 6 & 3 & 2 \\ 5 & 4 & -3 \end{vmatrix}=(-17,28,9)$，则平面方程为 $17x-28y-9z=0$

3．解：交换积分次序，得

$$\int_0^1 \mathrm{d}x\int_{x^2}^1 xe^{-y^2}\mathrm{d}y=\int_0^1 e^{-y^2}\mathrm{d}y\int_0^{\sqrt{y}} x\mathrm{d}x=\frac{1}{2}\int_0^1 ye^{-y^2}\mathrm{d}y=-\frac{1}{4}e^{-y^2}\Big|_0^1=\frac{1}{4}(1-\frac{1}{e})$$

4．解：$\dfrac{2x+1}{x^2+x-2}=\dfrac{1}{x+2}+\dfrac{1}{x-1}$

$$\frac{1}{x+2}=\frac{1}{4}\frac{1}{1+\frac{x-2}{4}}=\sum_{n=0}^{\infty}(-1)^n\frac{(x-2)^n}{4^{n+1}}，\ |x-2|<4$$

$$\frac{1}{x-1}=\frac{1}{1+x-2}=\sum_{n=0}^{\infty}(-1)^n(x-2)^n，\ |x-2|<1$$

$$\frac{2x+1}{x^2+x-2}=\sum_{n=0}^{\infty}(-1)^n\left(1+\frac{1}{4^{n+1}}\right)(x-2)^n，\ |x-2|<1$$

5．$\dfrac{\partial z}{\partial x}=2xf_1'+yf_2'$，$\dfrac{\partial^2 z}{\partial x\partial y}=2x(2yf_{11}''+xf_{12}'')+f_2'+y(2y\,f_{21}''+xf_{22}'')$

$$=4xyf_{11}''+2(x^2+y^2)f_{12}''+xyf_{22}''+f_2'$$

四、解：设 $M(x,y,z)$ 为椭圆上任意点，则原点到 M 的距离满足 $d^2=x^2+y^2+z^2$，

令 $F(x,y,z)=x^2+y^2+z^2+\lambda(x^2+y^2-z)+\mu(x+y+z-1)$

解方程组$\begin{cases} F_x = 2x(1+\lambda)+\mu = 0 \\ F_y = 2y(1+\lambda)+\mu = 0 \\ F_z = 2z-\lambda+\mu = 0 \\ z = x^2+y^2 \\ x+y+z=1 \end{cases}$ 得 $x=\dfrac{-1\pm\sqrt{3}}{2}, y=\dfrac{-1\pm\sqrt{3}}{2}, z=2\mp\sqrt{3}$

$\therefore d^2 = 9\mp 5\sqrt{3}$，即最长距离的点为 $(\dfrac{-1+\sqrt{3}}{2},\dfrac{-1+\sqrt{3}}{2},2-\sqrt{3})$，最短距离的点位 $(\dfrac{-1-\sqrt{3}}{2},\dfrac{-1-\sqrt{3}}{2},2+\sqrt{3})$.

五、解：设球面方程为 $x^2+y^2+z^2=R^2$，转轴为 z 轴，则转动惯量

$$I = k\iiint_{\Omega}(x^2+y^2)\sqrt{x^2+y^2+z^2}dv = k\int_0^{2\pi}d\theta\int_0^{\pi}\sin^3\varphi d\varphi\int_0^R r^5 dr = \frac{4}{9}k\pi R^6$$

六、解：取 $\sum_1$ 为 $z=0, x^2+y^2\le 1$，上侧

$$\begin{aligned} I &= \iint_{\Sigma} x\mathrm{d}y\mathrm{d}z + (z+1)^2\mathrm{d}x\mathrm{d}y \\ &= \oiint_{\Sigma+\Sigma_1} x\mathrm{d}y\mathrm{d}z + (z+1)^2\mathrm{d}x\mathrm{d}y - \iint_{\Sigma_1} x\mathrm{d}y\mathrm{d}z + (z+1)^2\mathrm{d}x\mathrm{d}y \\ &= \iiint_{\Omega}(2z+3)dv - \iint_{x^2+y^2\le 1}\mathrm{d}x\mathrm{d}y \\ &= \int_{-1}^{0}(2z+3)dz\iint_{x^2+y^2\le 1-z^2}dxdy - \pi \\ &= \pi\int_{-1}^{0}(2z+3)(1-z^2)dz - \pi = \frac{3\pi}{2} - \pi = \frac{\pi}{2} \end{aligned}$$

七、解：$\int_L (e^x\sin y - my)\mathrm{d}x + (e^x\cos y - m)\mathrm{d}y$

$$= \oint_{L+\overline{OA}}(e^x\sin y - my)\mathrm{d}x + (e^x\cos y - m)dy - \int_{\overline{OA}}(e^x\sin y - my)\mathrm{d}x + (e^x\cos y - m)\mathrm{d}y$$

$$= m\iint_D dxdy - \int_0^a 0dx = \frac{\pi m a^2}{8}$$

八、解：$\lim\limits_{n\to\infty}\left|\dfrac{u_{n+1}(x)}{u_n(x)}\right| = \lim\limits_{n\to\infty}\left|\dfrac{\frac{2n+1}{2^{n+1}}x^{2n}}{\frac{2n-1}{2^n}x^{2n-2}}\right| = \lim\limits_{n\to\infty}\dfrac{2n+1}{2(2n-1)}|x|^2 = \dfrac{1}{2}|x|^2$

当 $\dfrac{1}{2}|x|^2<1$，即 $|x|<\sqrt{2}$ 时级数收敛，当 $\dfrac{1}{2}|x|^2>1$，即 $|x|>\sqrt{2}$ 时级数发散，所以收敛半

径 $R=\sqrt{2}$ ，且 $x=\pm\sqrt{2}$ 时，级数发散，所以收敛域为 $(-\sqrt{2},\sqrt{2})$

$\forall x\in(-\sqrt{2},\sqrt{2})$

$$\sum_{n=1}^{\infty}\frac{2n-1}{2^n}x^{2n-2}=\sum_{n=1}^{\infty}\frac{1}{2^n}(x^{2n-1})'=(\sum_{n=1}^{\infty}\frac{1}{2^n}x^{2n-1})'=\frac{1}{\sqrt{2}}(\sum_{n=1}^{\infty}(\frac{x}{\sqrt{2}})^{2n-1})'$$

$$=\frac{1}{\sqrt{2}}\left(\frac{\frac{x}{\sqrt{2}}}{1-\frac{x^2}{2}}\right)'=\left(\frac{x}{2-x^2}\right)'=\frac{2+x^2}{(2-x^2)^2}$$