

静电场中的电势

2.1 静电场具有保守性.

$$\oint \vec{E} d\vec{l} = 0.$$

2.2 静电势能、电势

$$A_{PQ} = W_P - W_Q = q_0 \int_P^Q \vec{E} d\vec{l}$$

$$W_P = A_{PQ} = q_0 \int_P^Q \vec{E} d\vec{l} \quad (W_Q = 0) \text{ 取 } Q \text{ 点为零电势.}$$

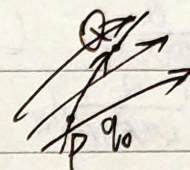
$$W_P = A_{P\infty} = q_0 \int_P^\infty \vec{E} d\vec{l} \quad (W_\infty = 0) \text{ 取无穷远处为零电势.}$$

$$\frac{W_P}{q_0} = \frac{A_{PQ}}{q_0} = \int_P^Q \vec{E} d\vec{l} = U_P \quad (U_Q = 0)$$

$$\frac{W_P}{q_0} = \frac{A_{P\infty}}{q_0} = \int_P^\infty \vec{E} d\vec{l} = U_P \quad (U_\infty = 0).$$

$$\text{电势差: } U_{PQ} = U_P - U_Q = \int_P^Q \vec{E} d\vec{l} - \int_Q^\infty \vec{E} d\vec{l} = \int_P^Q \vec{E} d\vec{l}$$

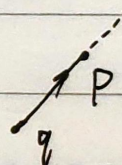
$$\text{电势做功 } A_{PQ} = q_0 \int_P^Q \vec{E} d\vec{l} = q_0 U_{PQ} = q_0 (U_P - U_Q) = W_P - W_Q$$



2.3 电势的计算.

$$U_P = \int_P^\infty \vec{E} d\vec{l} = \int_r^\infty \frac{q}{4\pi\epsilon_0 r^2} dr$$

$$U_P = \frac{q}{4\pi\epsilon_0 r}$$



$$\textcircled{1} q > 0, U_P > 0, r \uparrow, U_P \downarrow, r \rightarrow \infty, U_P = 0$$

$$\textcircled{2} q < 0, U_P < 0, r \uparrow, U_P \downarrow, r \rightarrow \infty, U_P = 0$$

$$U_P = \int_P^\infty \vec{E} d\vec{l} = \int_P^\infty (\sum \vec{E}_i) d\vec{l} = \int_P^\infty \vec{E}_1 d\vec{l} + \int_P^\infty \vec{E}_2 d\vec{l} + \dots + \int_P^\infty \vec{E}_n d\vec{l}$$

$$= \frac{q_1}{4\pi\epsilon_0 r_1} + \frac{q_2}{4\pi\epsilon_0 r_2} + \dots + \frac{q_n}{4\pi\epsilon_0 r_n}$$

$$\therefore U_P = \sum_{i=1}^n \frac{q_i}{4\pi\epsilon_0 r_i} = \sum_{i=1}^n U_i \quad (\text{代数数和}).$$

$$dU = \frac{dq}{4\pi\epsilon_0 r}$$

$$U = \int dv = \int \frac{dq}{4\pi\epsilon_0 r}$$

2.4 电势梯度 (略).

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$$\text{例 } E = \begin{cases} 0, & r < R \\ \frac{Q}{4\pi\epsilon_0 r^2}, & r > R \end{cases}$$

$$U_P = \int_P^\infty \vec{E} \cdot d\vec{r} = \int_r^R \vec{E} \cdot d\vec{r} + \int_R^\infty \vec{E} \cdot d\vec{r}$$

$$= \int_R^\infty \frac{Q}{4\pi\epsilon_0 r^2} dr$$

$$= \frac{Q}{4\pi\epsilon_0 R} \quad (r < R).$$

$$U_P = \int_r^\infty \frac{Q}{4\pi\epsilon_0 r^2} dr = \frac{Q}{4\pi\epsilon_0 r} \quad (r > R).$$

$$\text{例. } dq = \lambda dx.$$

$$U = \int du = \int_d^{d+l} \frac{\lambda dx}{4\pi\epsilon_0 x} = \frac{\lambda}{4\pi\epsilon_0} \ln \frac{d+l}{d}.$$

