

# 能量守恒

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### 一、功和功率

恒力功:  $A = \vec{F} \cdot \vec{s} = F s \cos \alpha$ . 功是标量(过程量)

变力功: 元功:  $dA = \vec{F} d\vec{r}$ .

$$A = \int dA = \int_a^b \vec{F} d\vec{r}.$$

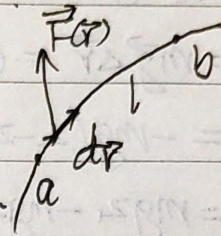
$$A = \int_a^b (\sum \vec{F}) d\vec{r} = \int_a^b \vec{F}_1 d\vec{r} + \dots + \int_a^b \vec{F}_i d\vec{r} \dots$$

$$= A_1 + \dots + A_i + \dots$$

$$= \sum A_i. \text{ (代数和).}$$

$$\text{平均功率: } \bar{N} = \frac{\Delta A}{\Delta t}$$

$$\text{瞬时功率: } N = \lim_{\Delta t \rightarrow 0} \frac{\Delta A}{\Delta t} = \frac{dA}{dt} = \frac{\vec{F} \cdot d\vec{r}}{dt} = \vec{F} \cdot \vec{v}.$$



### 二、质点动能定理

$$\text{元功: } dA = \vec{F} d\vec{r}.$$

$$= F_{\parallel} ds \cos \alpha.$$

$$= \underline{F_{\parallel} \cos \alpha} \cdot ds.$$

$$= (\sum F_{\parallel}) ds.$$

$$= ma_{\parallel} ds.$$

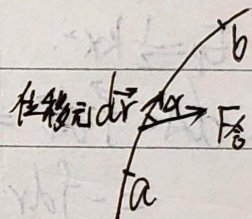
$$= m \frac{dv}{dt} \cdot ds.$$

$$v = \frac{ds}{dt}.$$

$$= m v dv.$$

$$A = \int dA = \int_{v_1}^{v_2} m v dv = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$$\text{动能: } E_k = \frac{1}{2} m v^2. \text{ (状态量)}$$



### 三、物体系的势能

保守力 势能函数.



保守力  $A_{AB} = E_{PA} - E_{PB} = -\Delta E_P$ .

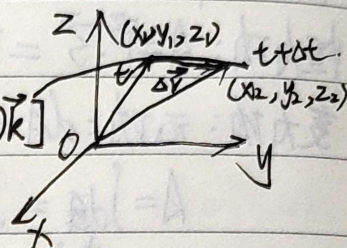
重力: 取  $E_{PB} = 0 \Rightarrow E_{PA} = A_{AB}$ .

重力:  $A = \int_a^b m\vec{g} d\vec{r} = m\vec{g} \int_a^b d\vec{r}$ .

$= m\vec{g} \Delta\vec{r} = (-mg\vec{k})[(x_2-x_1)\vec{i} + (y_2-y_1)\vec{j} + (z_2-z_1)\vec{k}]$

$= -mg(z_2-z_1)$ .

$= mgz_1 - mgz_2$ .



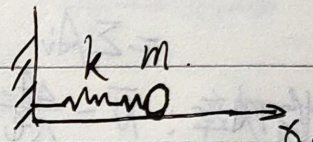
$E_P = mgh \Rightarrow \text{当 } h=0, E_P=0$

弹力:  $f = -kx$ .

$dA = f dx = -kx dx$ .

$A = \int dA = \int_{x_1}^{x_2} -kx dx = -\frac{1}{2}kx^2 + \frac{1}{2}kx_1^2$ .

$E_P = \frac{1}{2}kx^2 \Rightarrow \text{当 } x=0, E_P=0$ .



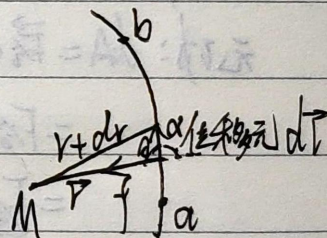
\*3/力:  $dA = \vec{f} \cdot d\vec{r} = f ds \cos\theta = -f ds \cos\alpha$ .

$= -f dr = -G \frac{Mm}{r^2} dr$ .

$A = \int dA = \int_{r_a}^{r_b} -G \frac{Mm}{r^2} dr$

$= (-G \frac{Mm}{r_a}) - (-G \frac{Mm}{r_b})$ .

$E_P = -G \frac{Mm}{r} \Rightarrow \text{当 } r \rightarrow \infty, E_P=0$ .



#### 四. 机械能守恒.

质点系动能定理.

$A_1 = E_{k1} - E_{k01}$ .

$A_2 = E_{k2} - E_{k02}$ .

$\Rightarrow \Sigma A = \Sigma E_k - \Sigma E_{k0}$ .

$A_i = E_{ki} - E_{k0i}$ .

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质点系的内力可以改变质点系的总动能, 而不能改变总动量  
质点系功能原理.

$$\Sigma A = \Sigma A_{\text{外}} + \Sigma A_{\text{内}}$$

$$= \Sigma A_{\text{外}} + \Sigma A_{\text{非保守}} + \Sigma A_{\text{保守}}.$$

$$\Sigma A_{\text{保守}} = -(\Sigma E_p - \Sigma E_{p_0}).$$

$$\Rightarrow \Sigma A_{\text{外}} + \Sigma A_{\text{非保守}} = (\Sigma E_k + \Sigma E_p) - (\Sigma E_{k_0} + \Sigma E_{p_0}) = E - E_0.$$

$$\Rightarrow E = \Sigma E_k + \Sigma E_p. \quad \text{机械能.}$$

$$\text{如果 } \Sigma A_{\text{外}} + \Sigma A_{\text{非保守}} = 0. \Rightarrow E = E_0 = \text{常量.}$$

普遍意义的能量守恒.

守恒律意义

五. 碰撞.

$$\text{冲力: } m_1 \vec{v}_{10} + m_2 \vec{v}_{20} = m_1 \vec{v}_1 + m_2 \vec{v}_2$$

正撞.

$$\text{弹性碰撞: } \begin{cases} m_1 v_{10} + m_2 v_{20} = m_1 v_1 + m_2 v_2. \end{cases}$$

$$\begin{cases} \frac{1}{2} m_1 v_{10}^2 + \frac{1}{2} m_2 v_{20}^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2. \end{cases}$$

$$m_1 = m_2 \Rightarrow v_1 = v_{20}, v_2 = v_{10} \quad \text{两球速度交换.}$$

完全非弹性碰撞.

$$v_1 = v_2 = v = \frac{m_1 v_{10} + m_2 v_{20}}{m_1 + m_2} \quad \text{机械能有损失. 资用能}$$