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Q1

* Count the total numbers of symbols of n, it will mean that there are n – 1 operations between them.

*We can break the whole problems to two subproblems

1. How many ways are there to place brackets to make the expression starting from at the l th symbol and ending at r th symbol evaluate to true (T)

2. How many ways are there to place brackets to make the expression starting from at the l th symbol and ending at r th symbol evaluate to false (F)

The base case: T(i,i) == 1 if i == true for example eval (true) can only be true

F(I,i) == 0 if i == false for example eval(false) can only be false

Procedure:

for each subproblem, we are given a string of values of operators(expression), we split the expression around an operator m, so that the whole expression are split into the form of

e1 M e2, and this can be interpreted as (e1) M (e2), when we finish evaluating e1 and e2, then we can combine e1 and e2 togather to evaluate e1 M e2.

Recursion function:

T(l,r) = $\sum_{m=1}^{r-1} \mathrm{TSplit}(l, m, r)$

F(l,r) = $\sum_{m=1}^{r-1} \mathrm{TSplit}(l, m, r)$

TSplit(l,m,r) = $\begin{cases} T(l,m) \times T(m+1,r) \\ \mathrm{T(l,m)} \times \mathrm{F(m+1,r)} + \mathrm{T(l,m)} \times \mathrm{T(m+1,r)} + \mathrm{F(l,m)} \times \mathrm{T(m+1,r)} \\ \mathrm{T(l,m)} \times \mathrm{F(m+1,r)} + \mathrm{F(l,m)} \times \mathrm{F(m+1,r)} + \mathrm{F(l,m)} \times \mathrm{T(m+1,r)} \\ \mathrm{F(l,m)} \times \mathrm{F(m+1,r)} \end{cases}$

1.AND --------> true AND true == true

2.OR ----------> (true OR false || true OR true || false OR false) == true

3.Nand ------> (true NAND false || false NAND true || false NAND false) == true

4.NOR -----> F NOR F = true

FSplit(l,m,r) = $\begin{cases} T(l,m) \times F(m+1,r) + F(l,m) \times F(m+1,r) + F(l,m) \times T(m+1,r) \\ \qquad\qquad\qquad\qquad F(l,m) \times F(m+1,r) \\ \qquad\qquad\qquad\qquad T(l,m) \times T(m+1,r) \\ T(l,m) \times F(m+1,r) + T(l,m) \times T(m+1,r) + F(l,m) \times T(m+1,r) \end{cases}$

1.AND ----------> (false AND false || true AND false || false AND false) == false

2.OR ----------> (false OR false) == false

3.Nand ------> true NAND true == false

4.NOR -----> (true NOR false || true NOR true || false NOR true) == false

Time complexity:

The complexity is O(n 3). There are O(n 2) different ranges that l and r could cover, and each needs the evaluations of TSplit or FSplit at up to n−1 different splitting points.