

Q2

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The problem can be solved by solving the following subproblem.

Subproblem: When moving from a square to the other 2 squares always choose the square with the minimal elevation numbers.

Recursion function finds the minimum value of the elevation numbers:

$$\begin{aligned}\text{MinEle}(c,r) &= \min\{\text{MinEle}(c+1,r) + 1, \text{MinEle}(c,r-1)\}, \text{ if } A[c][r] < A[c+1][r] \ \&\& \ A[c][r] \geq A[c][r-1] \\ &= \min\{\text{MinEle}(c+1,r), \text{MinEle}(c,r-1) + 1\}, \text{ if } A[c][r] \leq A[c+1][r] \ \&\& \ A[c][r] > A[c][r-1] \\ &= \min\{\text{MinEle}(c+1,r) + 1, \text{MinEle}(c,r-1)+1\}, \text{ if } A[c][r] \leq A[c+1][r] \ \&\& \ A[c][r] \leq A[c][r-1]\end{aligned}$$

BaseCase: $\text{MinEle}(c,r) = 0$ if $c == C$ and $r == 1$,

$\text{MinEle}(c,r) = \text{INF}$ if $c > C$ or $r < 1$ ----> if (c,r) is out of boundary

Recursion function that finds the path according to the MinEle

$$\begin{aligned}\text{From}(i) &= \text{args min}\{\text{MinEle}(c+1,r) + 1, \text{MinEle}(c,r-1)\}, \text{ if } A[c][r] < A[c+1][r] \ \&\& \ A[c][r] \geq A[c][r-1] \\ &= \text{args min}\{\text{MinEle}(c+1,r), \text{MinEle}(c,r-1) + 1\}, \text{ if } A[c][r] \leq A[c+1][r] \ \&\& \ A[c][r] > A[c][r-1] \\ &= \text{args min}\{\text{MinEle}(c+1,r) + 1, \text{MinEle}(c,r-1)+1\}, \text{ if } A[c][r] \leq A[c+1][r] \ \&\& \ A[c][r] \leq A[c][r-1]\end{aligned}$$

The args from the From(i) function will return the pair (c,r) which will obtain smallest elevation numbers from $(1,R)$ to $(1,C)$

The TimeComplexity is $O((R*C)^2)$, because there are about $R*C/2$ subproblems, and each of these subproblems are calling 2 subproblems which makes the overall time complexity $((R*C)/2)^2$.