Q5

1.

The differentiate of f(n) = log2 (n) is

f(n)' = $\frac{1}{ln(2) * n}$

The differentiate of g(n) = 10√ n is

g(n)' = $\frac{1}{-10 * n^{11/10}}$

If we calculate

$$\lim_{n \to \infty} \frac{f(n)'}{g(n)'} = \frac{-10 * n^{11/10}}{ln(2) * n}$$

$$= \frac{-10 * n^{1/10}}{ln(2)}$$

We also knew

$\lim_{n \to \infty}$ n$n^{1/10}$ = ∞, ln(2) is a constant

Therefore $\lim_{n \to \infty} \frac{f(n)'}{g(n)'}$ = ∞, This result indicates

$\lim_{n \to \infty} \frac{f(n)}{g(n)}$ = ∞(according to L'Hˆopital's rule)

we can also calculate

$$\lim_{n \to \infty} \frac{g(n)'}{f(n)'} = \frac{ln(2) * n}{-10 * n^{11/10}}$$

$$= \frac{ln(2)}{-10 * n^{1/10}}$$

Recall

$\lim_{n \to \infty}$ n$n^{1/10}$ = ∞, ln(2) is a constant

Thus

$\lim_{n \to \infty} \frac{g(n)}{f(n)}$= 0

From the above two limits and According to the limit asymptotic theorem, we can come to the conclusion that g(n) = O(f(n).

2.

For g(n) = $2^{n*log_2 n^2}$ can be simplified by using the log identity($2^{log_2 n^2}$ = n^2).Based on the log identity, we can reduce g(n) to 2^n* n^2.

For f(n) = n^n, this can be rewritten to f(n) = $e^{n*ln(\ n)}$ which always grows faster than e^n, and therefore it will be faster than 2^n as well(e > 2).

therefore we have f(n) = 2^n* n^2<= n^n for n >= 4, c = 1.Thus f(n) = O(n)

3.

For

f(n) = n^(1 + sin(pi * n)), for the sub function 1 + sin(pi * n), n ∈ N it is worthy to note that this function could has maximum and minimum values

When n mod 2 != 0(n is odd number)

    1 + sin(pi * n) = 0

When n == ½ + 2*pi

    1 + sin(pi * n) = 2

Thus if n is an odd number

f(n) = n^0 = 1

g(n) = n

We know that g(n) grows faster than f(n)

However if n == ½ + 2*pi

f(n) = n^2

g(n) = n

We know that n^2 is definitely growing faster than n, therefore f(n) grows faster than g(n). Thus, neither f(n) = O(g(n)) nor g(n) = O(f(n)).