UNIVERSITY of HOUSTON | ECE

ECE 3337: Test #1

Fall 2024

Last name of student	Student ID number
First name of student	Email address

Do This First:

1. Make sure that you have all the pages of the exam.
2. Fill in the above information.
3. Turn off and put away <u>all</u> electronic gadgets (laptop, tablet, smartphone, calculator, smart watch, …)

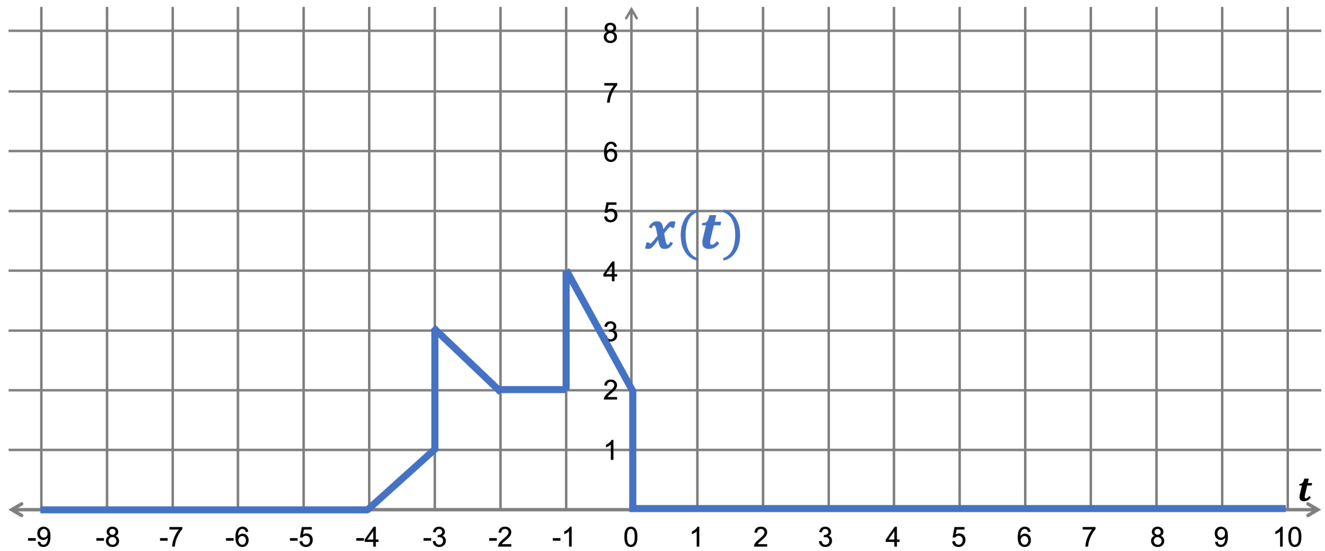
Exam Rules:

1. You are allowed to bring one 8.5” x 11” handwritten crib sheet (double sided).
2. Indicate your answers in the spaces provided. Use the back of the page to carry out calculations.

Q1	Q2	Q3	Q4	Q5	Q6	Total
15	20	15	15	15	20	100
Waveforms & Transformations	Signal Properties	System Properties	Convolution Integral	System Response	Frequency Response	

1. (15 points) Waveforms & Signal Transformations

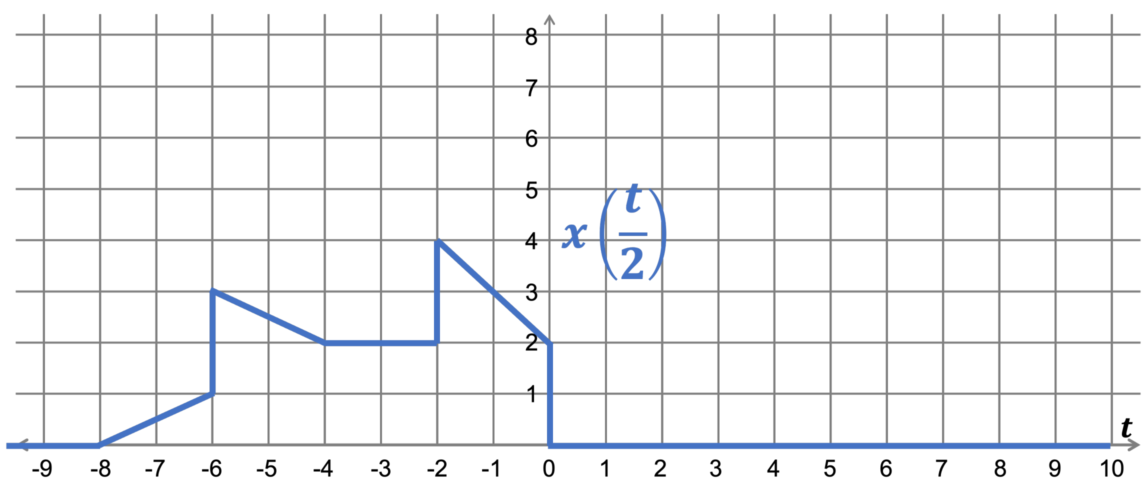
a. (5 points) Write down a formula in terms of steps and ramps describing the signal sketched below.

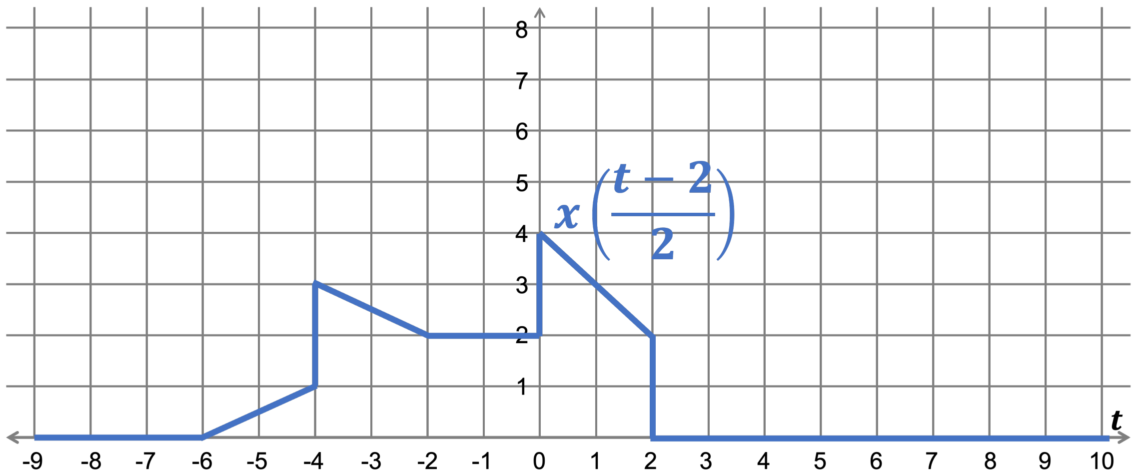


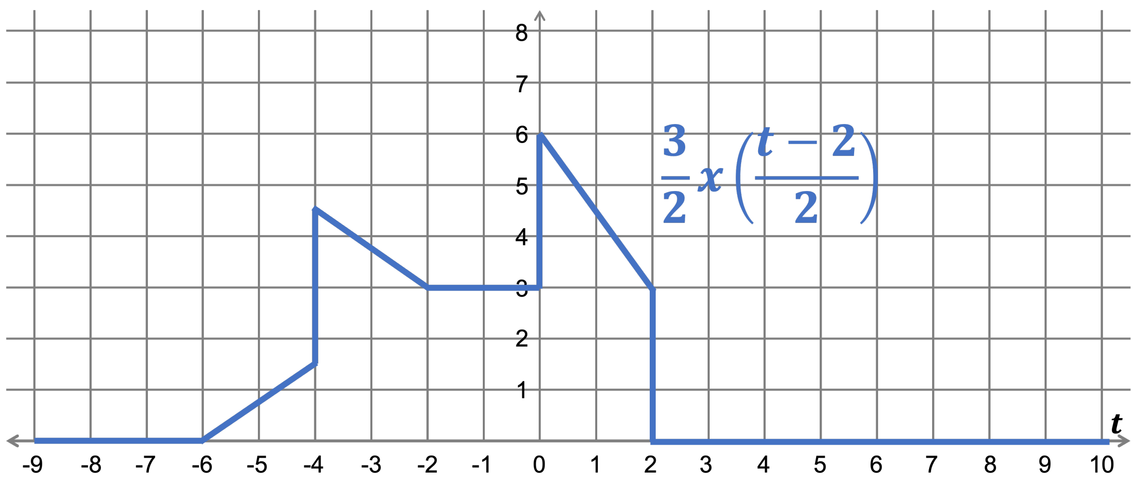
1 point for trying, 0.5 point for each term.

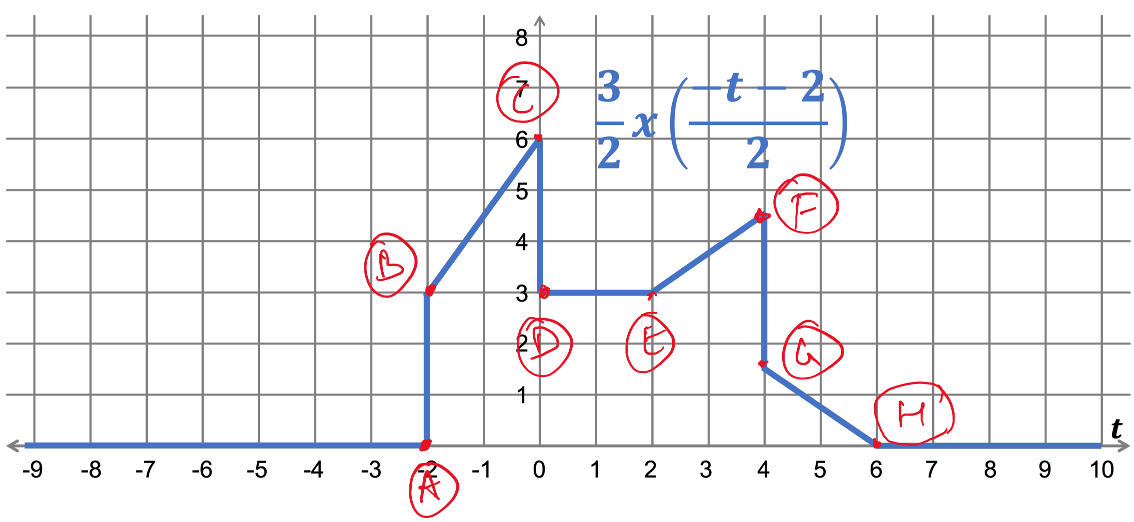
$$x(t) = r(t+4) + 2u(t+3) - 2r(t+3) + r(t+2) + 2u(t+1) - 2r(t+1) + 2r(t) - 2u(t)$$

b. (10 points) Plot $\frac{3}{2}x\left(\frac{-t-2}{2}\right)$ on the graph below. Your final answer should be sketched below. You may use the blank graphs at the end of the exam book for your work (they will not be graded).









2 point for trying

1 point each for A-H.

2. (20 points) Signal Properties

a. (4 points) Determine the time period of the signal $x_1(t) = |\, 2\cos(2\pi t)\, |$, where | | represents absolute value.

The time period is the *shortest* time after which $x(t) = x(t+T)$.

A

$$x\left(t+\frac{1}{2}\right) = \left|2\,cos\left(2\pi(t+\frac{1}{2}\right)\right|$$

$$= |2\cos(2\pi t + \pi)|$$ B

$$= |-2\cos(2\pi t)|$$

$$= |2\cos(2\pi t)| = x(t)$$

Therefore, the time period is $T = \frac{1}{2}$ seconds.

C

Full for final correct answer

1 point for trying, 1 each for A-C.

b. (6 points) Calculate the results of sampling the signal $x_2(t) = u(t) + 2r(t)$ with $s(t) = \delta\left(\frac{t}{3} - 1\right)$

Since $s(t)$ contains a $\delta()$-function, we can use the snapshot property.

The result of sampling is: y=$\int_{-\infty}^{\infty} x_2(t)s(t)dt$ A

$$= \int_{-\infty}^{\infty} [u(t) + 2r(t)][\delta\left(\frac{t}{3} - 1\right)]dt$$ B

Using the property $\delta(at) = \frac{1}{|a|}\delta(t)$, C

$$= 3\int_{-\infty}^{\infty} [u(t) + 2r(t)]\delta(t-3)dt$$

Using the snapshot property:

$$= 3[u(3) + 2r(3)]$$ D

Noting that $u(3) = 1$ and $r(3) = 3$,

$$= 3[1+6] = 21$$ E

1 point for trying, 1 each for A-E.

c. (5 points) If the odd and even components of the signal $x(t)$ are $x_o(t)$ and $x_e(t)$, respectively, then prove that the signal $y(t) = x_e(t)\, x_o(t)$ is odd symmetric.

A

Using the definitions of even and off components of a signal: $x_e = \frac{x(t)+x(-t)}{2}$ and $x_o = \frac{x(t)-x(-t)}{2}$.

$$y(t) = x_e(t)\, x_o(t) = \left(\frac{x(t)+x(-t)}{2}\right)\left(\frac{x(t)-x(-t)}{2}\right)$$

B

$$= \frac{1}{4}(x^2(t) - x^2(-t) - x(t)x(-t) + x(-t)x(t))$$

$$= \frac{1}{4}\left(x^2(t) - x^2(-t)\right) \quad \text{C}$$

$$y(-t) = \frac{1}{4}\left(x^2(-t) - x^2(t)\right)$$

$$= -\frac{1}{4}\left(x^2(t) - x^2(-t)\right)$$

$$y(-t) = -y(t) \quad \text{D}$$

Therefore, the signal $y(t) = x_e(t)\, x_o(t)$ is odd symmetric.

1 point for trying, 1 each for A-D.

d. (5 points) If the average power of a signal $x(t)$ is P, what is the average power of the signal $\frac{1}{2}x(4t-2)$?

$$P[x(t)] = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \; dt \quad \text{A}$$

$$P\left[\frac{1}{2}x(4t-2)\right] = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \left|\frac{1}{2}x(4t-2)\right|^2 \; dt \quad \text{B}$$

$$= \frac{1}{4} \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(4t-2)|^2 \; dt$$

Substitute $4t - 2 = u$, $4dt = du$

$$= \frac{1}{4}\left\{ \lim_{T\to\infty} \frac{1}{T} \int_{-4T/2-2}^{4T/2-2} |x(u)|^2 \frac{du}{4} \right\} \quad \text{C}$$

$$= \frac{1}{4}\left\{ \lim_{T\to\infty} \frac{1}{4T} \int_{-2T-2}^{2T-2} |x(u)|^2 \, du \right\}$$

$$P\left[\frac{1}{2}x(4t-2)\right] = \frac{1}{4} P[x(t)] \quad \text{D}$$

1 point for trying, 1 each for A-D.

3. (15 points) System Properties

a. (5 points) Is the system $y(t) = 2x(t)\mathrm{e}^{-j2t}$ linear? Explain your reasons.

Let $y_1(t) = 2x_1(t)\mathrm{e}^{-j2t}$ and $y_2(t) = 2x_2(t)\mathrm{e}^{-j2t}$ A

If the input is $x(t) = c_1x_1(t) + c_2x_2(t)$, then the output is

B

$$y(t) = 2x(t)\mathrm{e}^{-j2t} = 2[c_1x_1(t) + c_2x_2(t)]\mathrm{e}^{-j2t}$$
$$= c_1 2x_1(t)e^{-2jt} + c_2 2x_2(t)e^{-2jt}$$
$$= c_1y_1(t) + c_2y_2(t)$$

C

The system has the additivity and scaling properties, and is therefore linear.

D

1 point for trying, 1 each for A-D.

b. (5 points) Is the system defined by $y(t) = x\left(\frac{t+1}{2}\right)$ causal? Explain your reasons.

The output of the system at $t = 0$ depends on the input at time $t = ½$ seconds in the future. The output at time t depends on the future values of the input. Therefore, the system is NOT causal.

A B

1 point for trying, 2 each for A-B.

c. (5 points Is the system $y(t) = \frac{x(t-1)}{1-e^{-t}}$ BIBO stable? Explain your reasoning.

A

If the input is a (bounded) step function, at time $t = 0$, the denominator becomes $1 - e^0 = 1 - 1 = 0$.

The output becomes ∞ for a bounded input, and therefore, the system is NOT BIBO stable.

B

1 point for trying, 2 each for A-B.

4. (15 points) Convolution Integral

a. (5 points) Calculate the output $y(t)$ of an LTI system whose impulse response is $h(t)$ when it is provided the input $x(t)$, where:

$$x(t) = u(t)$$
$$h(t) = [t + \cos(t)]u(t)$$

$$y(t) = x(t) * h(t) = h(t) * x(t) = = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau \quad \text{— A}$$

$$= \int_{-\infty}^{\infty} [\tau + \cos(\tau)]\, u(\tau)u(t-\tau)d\tau \quad \text{B}$$

The functions $u(\tau) = 1$ for $\tau > 0$ and
$u(t-\tau) = 1$ for $t - \tau > 0$.
This implies, $0 < \tau < t$, and $t > 0$

C

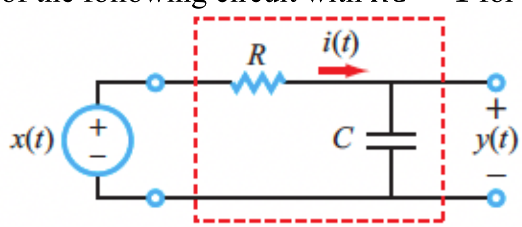
$$y(t) = u(t)\int_{0}^{t} [\tau + \cos(\tau)]\, d\tau$$

$$= u(t)\left[\frac{\tau^2}{2} + \sin(\tau)\right]_0^t$$

$$= u(t)\left(\frac{t^2}{2} + \sin\,(t)\right) \quad \text{D}$$

1 point for trying, 1 each for A-D.

b. (5 points) Calculate the response of the following circuit with $RC = 1$ for the input $x(t) = e^{j3t}u(t)$.



The impulse response is given by $h(t) = \frac{1}{RC}e^{-\frac{t}{RC}}u(t)$, and the initial conditions are zero.

For $RC = 1, h(t) = e^{-t}u(t)$.
Input $x(t) = e^{j3t}u(t)$.
Output $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$

$$y(t) = \int_{-\infty}^{\infty} e^{j3\tau}u(\tau)e^{-(t-\tau)}u(t-\tau)d\tau \quad \text{— A}$$

The functions $u(\tau) = 1$ for $\tau > 0$ and

$u(t-\tau) = 1$ for $t - \tau > 0$.
This implies, $0 < \tau < t$, and $t > 0$

B

$$y(t) = u(t)\int_0^t e^{j3\tau}e^{-(t-\tau)}d\tau$$ — C

$$= u(t)\int_0^t e^{j3\tau}e^{-t}e^{\tau}d\tau$$

$$= u(t)e^{-t}\int_0^t e^{(j3+1)\tau}d\tau$$

$$= u(t)e^{-t}\left.\frac{e^{(3j+1)\tau}}{(3j+1)}\right]_0^t$$

$$= u(t)e^{-t}\left[\frac{e^{(3j+1)t}-1}{(3j+1)}\right]$$ — D

$$= u(t)\left[\frac{e^{3jt}-e^{-t}}{(3j+1)}\right]$$

1 point for trying, 1 each for A-D.

c. (5 points) Compute the convolution $y(t) = u(t-2) * \{u(t-3) + 3r(t+1) + 4\delta(t-1)\}$ without computing any integrals. You can utilize the following results: $u(t) * u(t) = r(t)$ and $u(t) * r(t) = \frac{t^2}{2}u(t)$

$$y(t) = u(t-2) * \{u(t-3) + 3r(t+1) + 4\delta(t-1)\}$$

Using the distributive property of convolutions:

$$= u(t-2) * u(t-3) + u(t-2) * 3r(t+1) + u(t-2) * 4\delta(t-1)$$ — A

$$u(t-2) * u(t-3) = u(t) * \delta(t-2) * u(t) * \delta(t-3)$$
$$= u(t) * u(t) * \delta(t-2) * \delta(t-3)$$
$$= u(t) * u(t) * \delta(t-5)$$
$$= r(t) * \delta(t-5)$$
$$= r(t-5)$$

$$u(t-2) * 3r(t+1) = u(t) * \delta(t-2) * 3r(t) * \delta(t+1)$$
$$= u(t) * 3r(t) * \delta(t-2) * \delta(t+1)$$
$$= u(t) * 3r(t) * \delta(t-1)$$

$$= u(t) * 3r(t) * \delta(t-1)$$

$$= 3\frac{t^2}{2}u(t) * \delta(t-1)$$

$$= 3\frac{(t-1)^2}{2}u(t-1)$$

$$u(t-2) * 4\delta(t-1) = u(t) * \delta(t-2) * \delta 4(t-1)$$

$$= 4u(t) * \delta(t-3)$$

$$= 4u(t-3)$$

$$y(t) = \underbrace{r(t-5)}_{B} + \underbrace{3\frac{(t-1)^2}{2}u(t-1)}_{C} + \underbrace{4u(t-3)}_{D}$$

1 point for trying, 1 each for A-D.

5. (15 points) System Response

a. (10 points) Find the ramp response $y_{ramp}(t)$ of the system given by the differential equation

$\frac{dy(t)}{dt} + 3y(t) = \frac{d^2x(t)}{dt^2} + 2\frac{dx(t)}{dt}$. Assume zero initial conditions.

A

To find the ramp response $y_{ramp}(t)$, let is input a ramp signal $r(t)$ and find the response.

$$x(t) = r(t)$$

$$\frac{dx(t)}{dt} = u(t) \quad \text{B}$$

$$\frac{d^2x(t)}{dt^2} = \delta(t) \quad \text{C}$$

$$\frac{dy(t)}{dt} + 3y(t) = \frac{d^2x(t)}{dt^2} + 2\frac{dx(t)}{dt} = \delta(t) + 2u(t) \quad - \text{D}$$

The integrating factor is $IF = e^{\int 3dt} = e^{3t}$ E

$$\frac{e^{3t}dy(t)}{dt} + 3y(t)e^{3t} = e^{3t}\delta(t) + 2u(t)e^{3t}$$

$$\frac{d[y(t)e^{3t}]}{dt} = \delta(t) + 2u(t)e^{3t} \quad \text{F}$$

$$y(t)e^{3t}|_{0-}^{t} = \int_{0-}^{t} [\delta(t) + 2u(t)e^{3t}]dt$$

Assuming zero initial condition,

$$y(t)e^{3t} = u(t) + \frac{2}{3}e^{3t}|_{0-}^{t}$$

$$y(t)e^{3t} = u(t) + \frac{2}{3}[e^{3t} - 1] \quad \text{G}$$

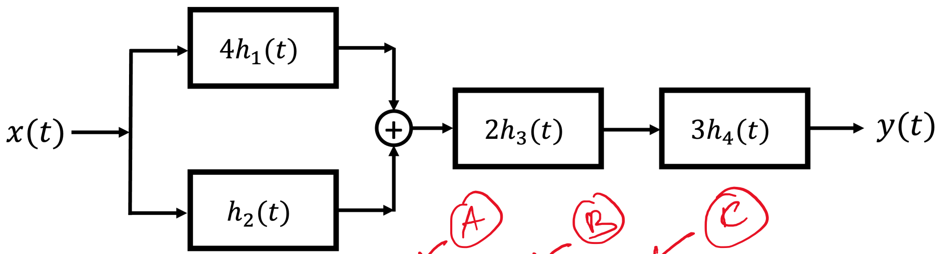
Noting that the result is nonzero for $t > 0$

$$y(t) = \left(e^{-3t} + \frac{2}{3}[1 - e^{-3t}]\right)u(t)$$

$$y(t) = \left(\frac{2}{3} + \frac{1}{3}e^{-3t}\right)u(t) \quad \text{H}$$

2 points for trying, 1 each for A-H.

b. (5 points) Find the overall impulse response of the following system if each of the individual systems $h_1(t) \dots h_4(t)$ are LTI.



A B C

$$h(t) = [4h_1(t) + h_2(t)] * 2h_3(t) * 3h_4(t)$$
$$= 6[4h_1(t) + h_2(t)] * h_3(t) * h_4(t)$$

1 point for trying, 1 each for A-D. D

6. (20 points) System Frequency Response

a. (10 points) Find the frequencies at which the LTI system described by the following linear constant coefficient differential equation introduces a zero phase change to an input signal $x(t) = \cos \omega t$. In other words find all values of ω_0 for which, $\angle \widehat{\mathbf{H}}(\omega_0) = 0$.

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 4y(t) = \frac{dx(t)}{dt}$$

Let $x(t) = e^{j\omega t}$ and $y(t) = \widehat{\mathbf{H}}(\omega)e^{j\omega t}$

A B

The differential equation simplifies to

$$\widehat{\mathbf{H}}(\omega)e^{j\omega t}[(j\omega)^2 + 3(j\omega) + 4\,] = e^{j\omega t}[j\omega]$$ C

$$\widehat{\mathbf{H}}(\omega) = \frac{j\omega}{(j\omega)^2 + 3(j\omega) + 4}$$

$$\widehat{\mathbf{H}}(\omega) = \frac{1}{j\omega + 3 + \frac{4}{j\omega}}$$ D

$$\widehat{\mathbf{H}}(\omega) = \frac{1}{3 + j(\omega - \frac{4}{\omega})}$$

E

For , $\angle \widehat{\mathbf{H}}(\omega_0) = 0$, we need to find the value of ω at which $\widehat{\mathbf{H}}(\omega)$ is purely real. F

In other words, the imaginay part of $\widehat{\mathbf{H}}(\omega)$ should be zero. G

Therefore, $j\left(\omega - \frac{4}{\omega}\right) = 0$ at $\omega = \omega_0$

$$\omega_0^2 = 4$$
$$\omega_0 = \pm 2$$ H

2 points for trying, 1 each for A-H.

b. A system is defined by the following differential equation:

$$\frac{d^2y(t)}{dt^2} + B\frac{dy(t)}{dt} + 9y(t) = 2\frac{dx(t)}{dt} + 3\frac{d^2x(t)}{dt^2}$$

i. (4 points) Determine the value of B that will make the following system critically damped:

The characteristic equation is

$$s^2 + Bs + 9 = 0 \quad \text{A}$$

For the system to be critically dampled, the characteristic equation must have equal roots. Therefore,

$$B^2 - 4 \cdot 9 = 0$$

$$B^2 = 2 \cdot 2 \cdot 3 \cdot 3 = 2^2 \cdot 3^2$$

$$B^2 = 6^2$$

$$B = \pm 6 \quad \text{B}$$

For BIBO stability, $B > 0$

Therefore, $B = +6$ C

1 point for trying, 1 each for A-C

ii. (3 points) For $B = 7$, find the roots of the characteristic equation.

The characteristic equation is

$$s^2 + 7s + 9 = 0 \quad \text{A}$$

The roots are

$$s = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 9}}{2} = \frac{-7 \pm \sqrt{49 - 36}}{2} = \frac{-7 \pm \sqrt{13}}{2} \quad \text{B}$$

1 point for trying, 1 each for A, B

iii. (3 points) For $B = 5$, is the system BIBO stable? Why or Why not?

The characteristic equation is

$$s^2 + 5s + 9 = 0$$

The coefficients $a_1 = 5$ and $a_2 = 9$ are both > 0 and therefore, the system is BIBO stable.

A B

1 point for trying, 1 each for A, B

USE THESE FOR YOUR WORK IF NEEDED.

ONLY YOUR FINAL ANSWER ON PAGE 2 WILL BE GRADED.