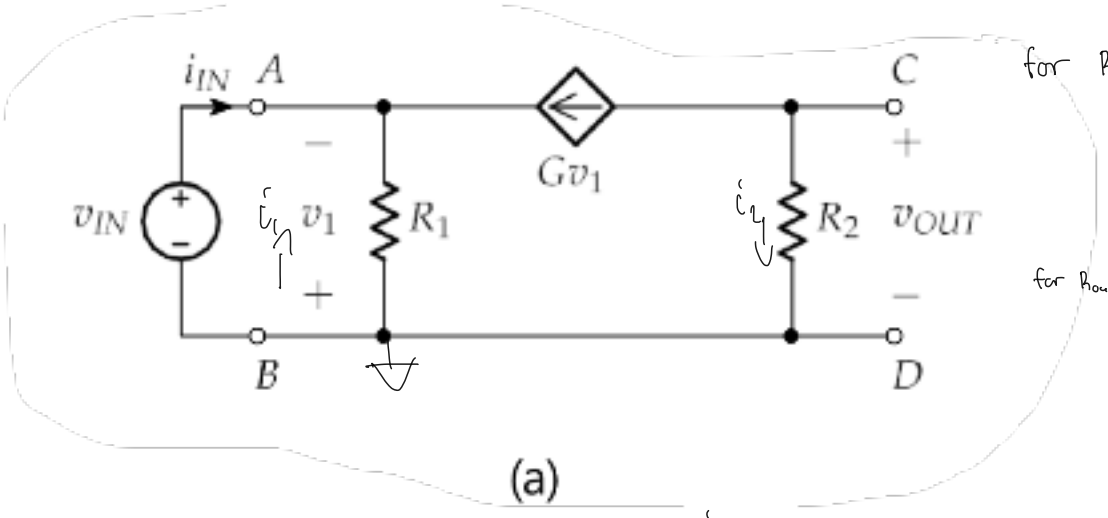


(a)

for R_{IN}:

KCL @ A: $-i_{IN} - Gv_1 - i_1 = 0$

$i_{IN} = -Gv_1 - \frac{v_1}{R_1}$

$i_{IN} = -Gv_1 - \frac{(-v_{IN})}{R_1}$

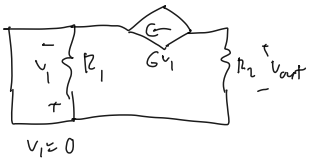
$i_{IN} = -G(-v_{IN}) + \frac{v_{IN}}{R_1}$

$i_{IN} = G v_{IN} + \frac{v_{IN}}{R_1}$

$i_{IN} = v_{IN}(G + \frac{1}{R_1})$

$R_{IN} = \frac{v_{IN}}{i_{IN}} = \frac{1}{(G + \frac{1}{R_1})}$

for R_{out}: since v_{out} is across R_2, $R_{out} = R_2$



$v_1 = 0$

$\therefore R_2 \{ I_{v_{out}}$

$R_{out} = R_2$

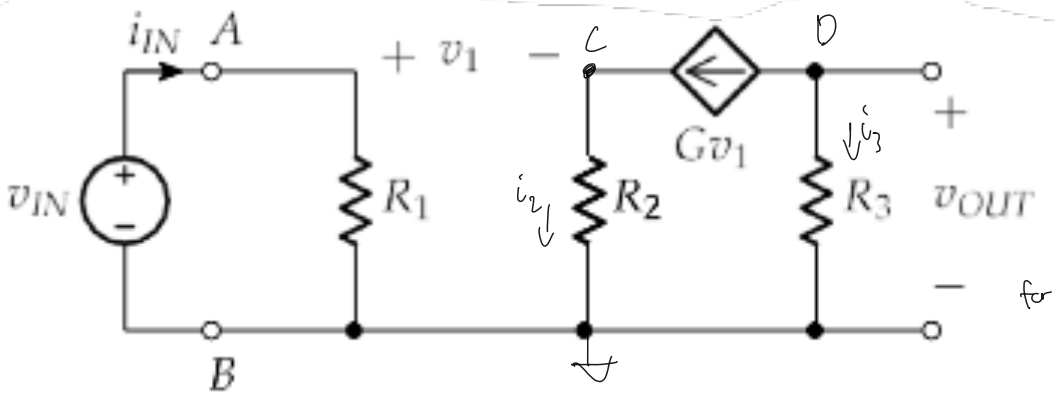
$V_1 = -V_{IN}$ KCL @ C: $Gv_1 + i_2 = 0$

$\frac{V_{out}}{R_2} = -GV_1$

$V_{out} = -GV_1 \cdot R_2$

$V_{out} = -G(-V_{IN}) \cdot R_2$

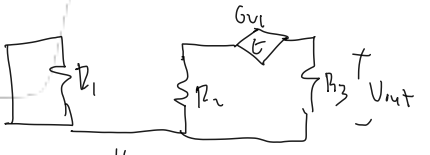
$\frac{V_{out}}{V_{IN}} = GR_2 = A_v$ (gain)

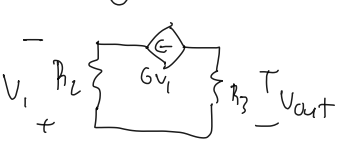


(b)

for R_{IN}: $R_{IN} = R_1$ since current from V_{IN} flows in loop and doesn't contribute to rest of circuit

for R_{out}: V_{out} defined across R_3 so $R_{out} = R_3$





KCL @ C: $-Gv_1 + i_2 = 0$

$i_2 = Gv_1$

$\frac{v_c}{R_2} = Gv_1$

$v_c = Gv_1 R_2$

KCL @ D: $Gv_1 + i_3 = 0$

$i_3 = -Gv_1$

$\frac{v_{out}}{R_3} = -Gv_1$

$v_{out} = -Gv_1 R_3$

$V_1 = V_{IN} - V_c$

$V_1 = V_{IN} - GV_1 R_2$

$V_1 + GV_1 R_2 = V_{IN}$

$V_{IN} = V_1(1 + GR_2)$

$A_v = \frac{v_{out}}{v_{IN}} = \frac{-GV_1 R_3}{V_1(1+GR_2)}$

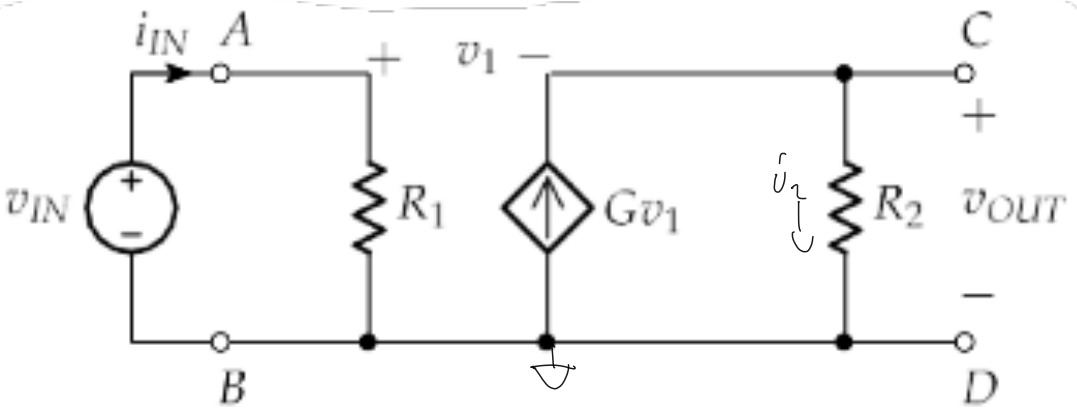
$A_v = -\frac{GR_3}{1+GR_2}$ (gain)

$V_1 = -GV_1 R_2$

$V_1(GR_2) = 0$

$V_1 = 0$ ∴ R_3 + V_{out} −

$= R_{out}$



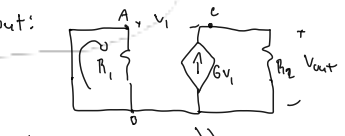
$R_1 = R_{IN}$ since i_{IN} flows only in loop and $V_{IN} = i_{IN} \cdot R_{IN}$

$\frac{V_{IN}}{i_{IN}} = R_{IN}$

and i_{IN} only flows thru R_1 $\therefore R_1 = R_{IN}$

(c)

for R_{out}:

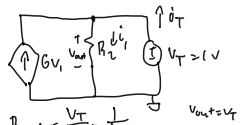


$V_A = 0$

$V_c = V_{out}$

$V_1 = V_A - V_c = 0 - V_c = -V_{out}$

$V_1 = -V_{out}$



$R_{out} = \frac{V_T}{i_T} = \frac{1}{i_T}$

KCL: $-Gv_1 - i_T + i_1 = 0$

$i_T = i_1 - Gv_1$

$i_T = \frac{V_T}{R_2} - G(-V_{out})$

$i_T = \frac{V_T}{R_2} + GV_T$

$i_T = V_T\left(\frac{1}{R_2} + G\right)$

$\left(\frac{1}{R_2} + G\right) = \frac{V_T}{i_T} = R_{out}$

KCL @ C: $-Gv_1 + \frac{V_c}{R_2} = 0$ $\quad * V_c = V_{out}$

$\frac{V_{out}}{R_2} = Gv_1$

$V_1 = V_{IN} - V_c = V_{IN} - V_{out}$

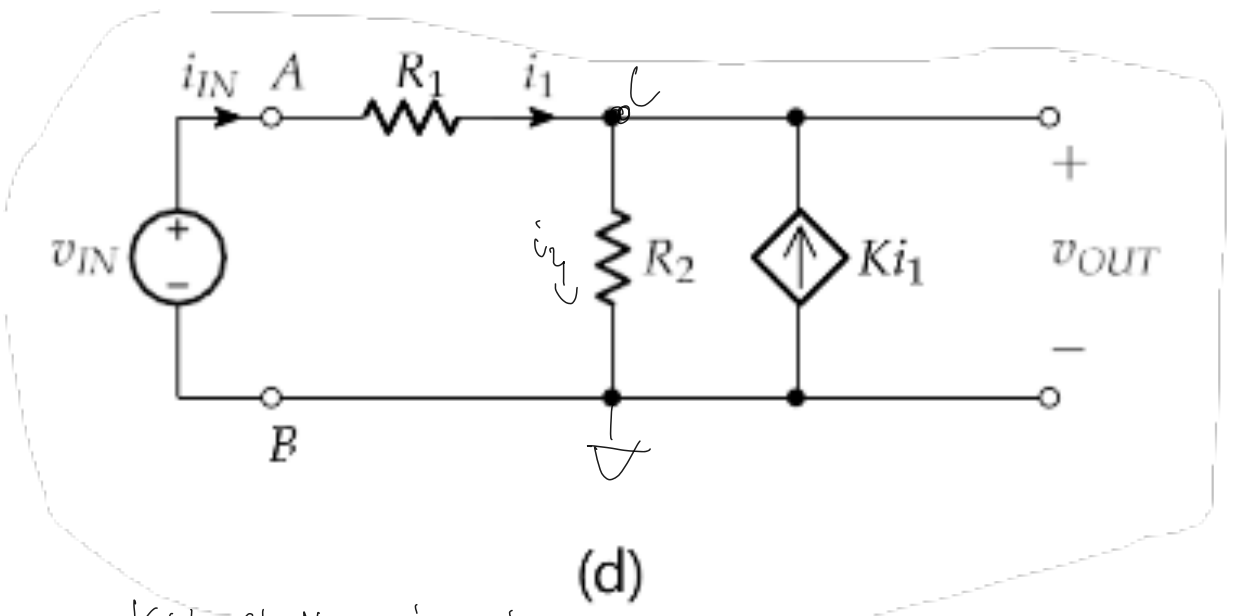
$\frac{V_{out}}{R_2} = G(V_{IN} - V_{out})$

$\frac{V_{out}}{R_2} = GV_{IN} - GV_{out}$

$\frac{V_{out}}{R_2} + GV_{out} = GV_{IN}$

$V_{out}\left(\frac{1}{R_2} + G\right) = GV_{IN}$

$\frac{V_{out}}{V_{IN}} = \frac{G}{\left(\frac{1}{R_2} + G\right)}$



(d)

KCL @ V_A: $-i_{IN} + i_1 = 0$

$i_1 = i_{IN}$

$\frac{V_A - V_C}{R_1} = i_{IN}$

$\frac{V_{IN} - V_{out}}{R_1} = i_{IN} = i_1$

KCL @ C: $-i_1 + i_2 - K i_1 = 0$

$-\frac{V_A - V_C}{R_1} + \frac{V_C}{R_2} - K\left(\frac{V_A - V_C}{R_1}\right) = 0$

$\frac{V_{out} - V_{IN}}{R_1} + \frac{V_{out}}{R_2} - K\left(\frac{V_{IN} - V_{out}}{R_1}\right) = 0$

$\frac{V_{out} - V_{IN}}{R_1} + \frac{V_{out}}{R_2} + K\frac{V_{out} - V_{IN}}{R_1} = 0$

$V_{out}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{K}{R_1}\right) + V_{IN}\left(-\frac{1}{R_1} - \frac{K}{R_1}\right) = 0$

$V_{out}\left(\frac{1}{R_2} + \frac{1+K}{R_1}\right) = -V_{IN}\left(\frac{-1-K}{R_1}\right)$

$\frac{V_{out}}{V_{IN}} = \frac{\left(\frac{1+K}{R_1}\right)}{\left(\frac{1}{R_2} + \frac{1+K}{R_1}\right)} = \frac{1+K}{\left(\frac{R_1}{R_2} + 1 + K\right)} = A_v$ (gain)

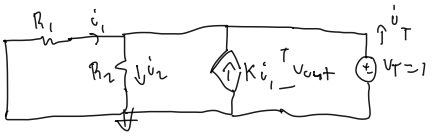
For R_{IN}: $R_{IN} = \frac{V_{IN}}{i_{IN}}$

$i_{IN} = i_1 = \frac{V_{IN} - V_{out}}{R_1}$

$R_{IN} = \frac{V_{IN}}{\frac{V_{IN} - V_{out}}{R_1}}$

$R_{IN} = \frac{V_{IN} R_1}{V_{IN} - V_{out}}$

For R_{out}:



KCL @ top node: $-i_1 - Ki_1 - i_T + i_2 = 0$

$i_T = i_2 - i_1 - Ki_1$

$i_T = \frac{V_T}{R_2} - \frac{0 - V_T}{R_1} - K\left(\frac{0 - V_T}{R_1}\right)$

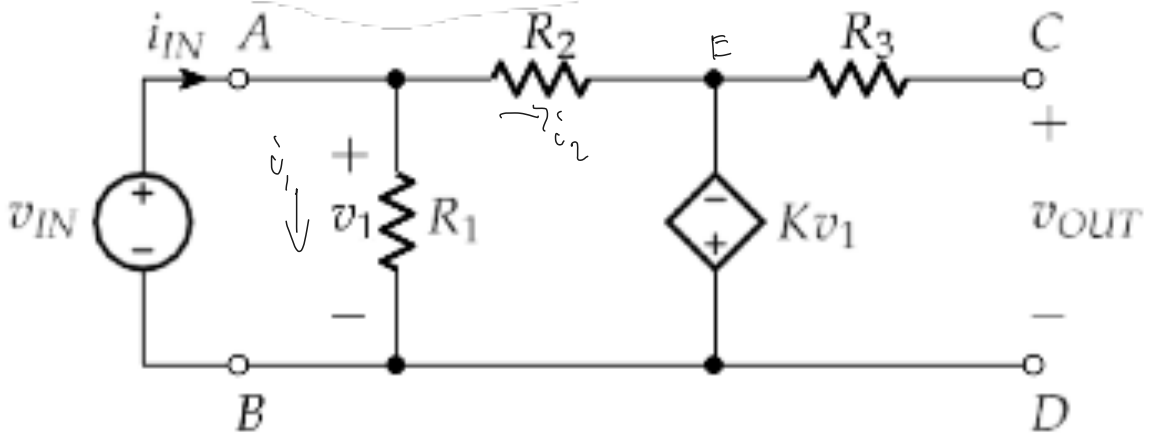
$i_T = \frac{V_T}{R_2} + \frac{V_T}{R_1} + K\frac{V_T}{R_1}$

$i_T = V_T\left(\frac{1}{R_2} + \frac{1}{R_1} + \frac{K}{R_1}\right)$

$R_{out} = \frac{V_{out}}{i_{out}} = \frac{V_T}{i_T}$

$R_{out} = \frac{V_T}{V_T\left(\frac{1}{R_2} + \frac{1+K}{R_1}\right)}$

$R_{out} = \frac{1}{\frac{1}{R_2} + \frac{1+K}{R_1}}$



(e)

$V_{IN} = V_1 \qquad V_{out} = -Kv_1$

$V_{out} = -K V_{IN}$

$\frac{V_{out}}{V_{IN}} = -K = A_v$ (gain)

for $R_{IN} = \frac{V_{IN}}{i_{IN}}$

KCL at A: $i_{IN} + i_1 + i_2 = 0$

$i_{IN} = i_1 + i_2$

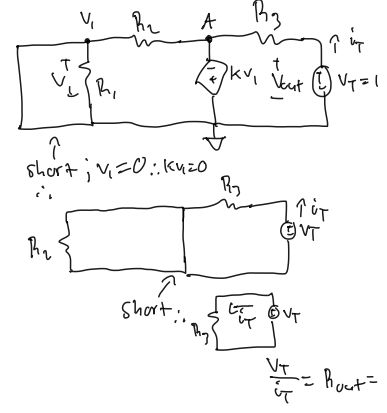
$i_{IN} = \frac{V_{IN}}{R_1} + \frac{V_{IN} - V_E}{R_2}$ $\qquad V_E = 0 - Kv_1 = -KV_{IN}$

$i_{IN} = \frac{V_{IN}}{R_1} + \frac{V_{IN} - (-K V_{IN})}{R_2}$

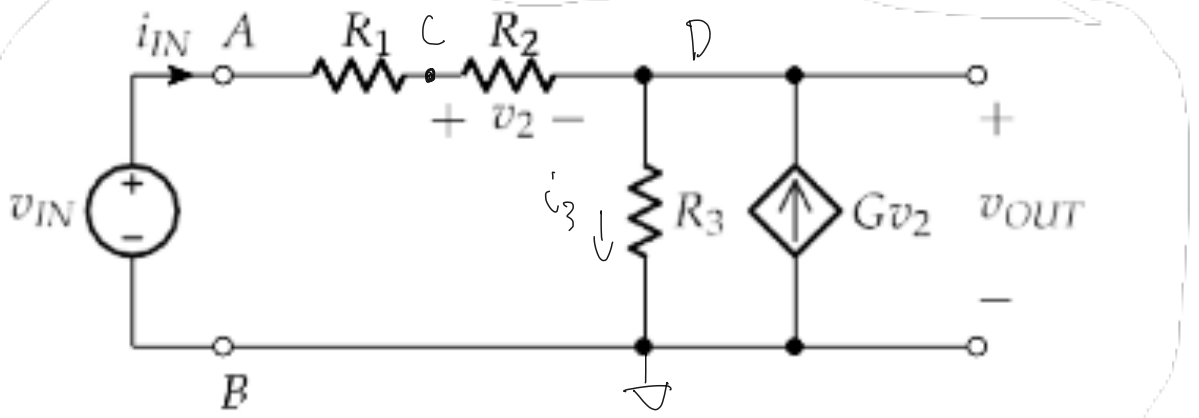
$i_{IN} = V_{IN}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{K}{R_2}\right)$

$\frac{V_{IN}}{i_{IN}} = R_{IN} = \frac{1}{\left(\frac{1}{R_1} + \frac{1+K}{R_2}\right)}$

For $R_{out} = \frac{V_{out}}{i_{out}} = \frac{V_T}{i_T} = \frac{1}{i_T}$



$\frac{V_T}{i_T} = R_{out} = R_3$



$V_2 = i_{IN} R_2$ $\quad V_D = V_{out}$

$= \left(\frac{V_{IN}-V_{out}}{R_1+R_2}\right) R_2$ (f)

KCL @ D: $-i_N + i_3 - Gv_2 = 0$

$$-\left(\frac{V_{IN}-V_D}{R_1+R_2}\right) + \frac{V_D}{R_3} - Gv_2 = 0$$

$$\frac{V_{out}-V_{IN}}{R_1+R_2} + \frac{V_{out}}{R_3} - G\left(\left(\frac{V_{IN}-V_{out}}{R_1+R_2}\right)R_2\right) = 0$$

$$V_{out}\left(\frac{1}{R_1+R_2} + \frac{1}{R_3} + \frac{GR_2}{R_1+R_2}\right) + V_{IN}\left(-\frac{1}{R_1+R_2} - \frac{GR_2}{R_1+R_2}\right) = 0$$

$$V_{out}\left(\frac{1}{R_1+R_2} + \frac{1}{R_3} + \frac{GR_2}{R_1+R_2}\right) = -V_{IN}\left(-\frac{1}{R_1+R_2} - \frac{GR_2}{R_1+R_2}\right)$$

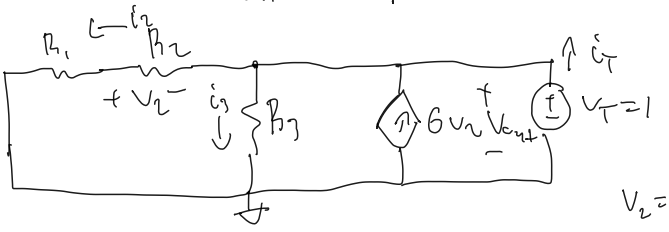
$$\frac{V_{out}}{V_{IN}} = \frac{\left(\frac{1+GR_2}{R_1+R_2}\right)}{\frac{1+GR_2}{R_1+R_2} + \frac{1}{R_3}} = \frac{1+GR_2}{1+GR_2+\frac{R_1+R_2}{R_3}} = A_v \text{ (gain)}$$

For $R_{IN} = \frac{V_{IN}}{i_{IN}}$

$$i_{IN} = \frac{V_{IN}-V_{out}}{R_1+R_2}$$

$$R_{IN} = \frac{V_{IN}}{\frac{V_{IN}-V_{out}}{R_1+R_2}} = \frac{V_{IN}(R_1+R_2)}{V_{IN}-V_{out}}$$

For $R_{out} = \frac{V_{out}}{i_{out}} = \frac{V_T}{i_T}$



$V_2 = -i_2 R_2$

$V_2 = -\left(\frac{V_T}{R_1+R_2}\right) R_2$

KCL @ top: $-i_T - Gv_2 + i_3 + i_2 = 0$

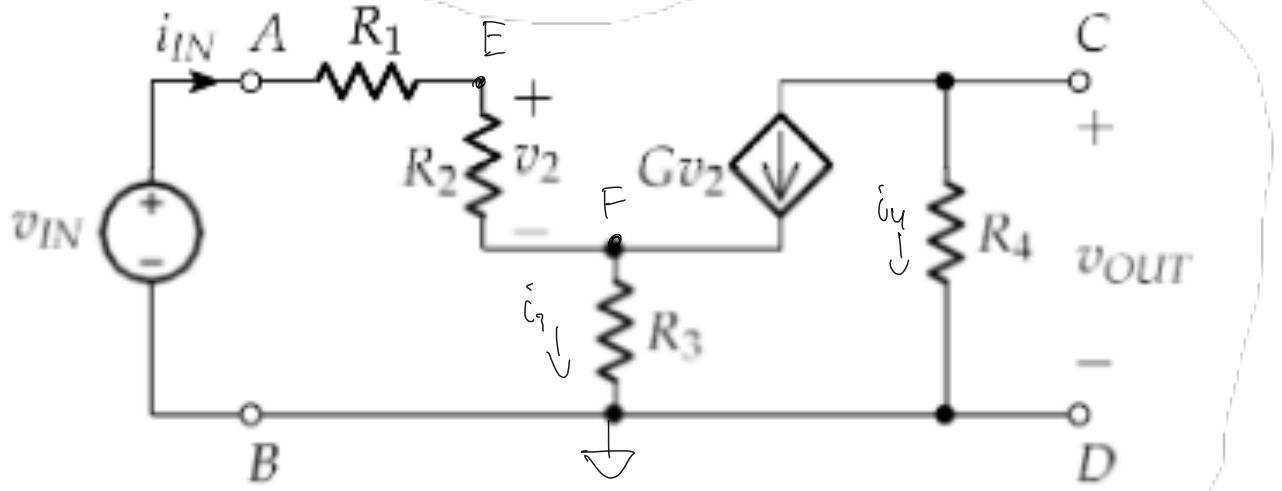
$$-i_T - Gv_2 + \frac{V_T}{R_3} + \frac{V_T}{R_1+R_2} = 0$$

$$-i_T - G\left(-\left(\frac{V_T}{R_1+R_2}\right)R_2\right) + \frac{V_T}{R_3} + \frac{V_T}{R_1+R_2} = 0$$

$$i_T = V_T\left(\frac{GR_2}{R_1+R_2} + \frac{1}{R_3} + \frac{1}{R_1+R_2}\right)$$

$$i_T = V_T\left(\frac{GR_2+1}{R_1+R_2} + \frac{1}{R_3}\right)$$

$$\frac{V_T}{i_T} = R_{out} = \frac{1}{\frac{GR_2+1}{R_1+R_2} + \frac{1}{R_3}}$$



(g)

$\frac{V_{IN}-V_F}{R_1+R_2} = i_{IN}$ $\quad V_2 = V_E - V_F = i_N R_2 = \frac{V_{IN}-V_F}{R_1+R_2} R_2$

KCL @ F: $-i_{IN} + i_3 - GV_2 = 0$

$-\left(\frac{V_{IN}-V_F}{R_1+R_2}\right) + \frac{V_F}{R_3} - GV_2 = 0$

$\frac{V_F - V_{IN}}{R_1+R_2} + \frac{V_F}{R_3} - G\left(\frac{V_{IN}-V_F}{R_1+R_2}\right)R_2 = 0$

$V_F\left(\frac{1}{R_1+R_2} + \frac{1}{R_3} + \frac{GR_2}{R_1+R_2}\right) + V_{IN}\left(\frac{-1}{R_1+R_2} - \frac{GR_2}{R_1+R_2}\right) = 0$

$V_F = V_{IN} \dfrac{\left(\frac{1+GR_2}{R_1+R_2}\right)}{\frac{1+GR_2}{R_1+R_2} + \frac{1}{R_3}}$

$V_F = \dfrac{1+GR_2}{1+GR_2 + \frac{R_1+R_2}{R_3}}$

$V_F = \left(\dfrac{R_3(1+GR_2)}{R_3(1+GR_2)+R_1+R_2}\right) V_{IN}$ $\Rightarrow$

KCL @ C: $i_4 + GV_2 = 0$

$\frac{V_{out}}{R_4} + GV_2 = 0$

$\frac{V_{out}}{R_4} = -G\left(\frac{R_2}{D} V_{IN}\right)$

$\frac{V_{out}}{V_{IN}} = -\frac{R_2 R_4}{D} G$

$A_v(\text{gain}) = -G \frac{R_2 R_4}{R_3(1+GR_2)+R_1+R_2}$

$V_2 = \dfrac{R_2}{R_1+R_2}\left(V_{IN} - \left[V_{IN}\left(\dfrac{R_3(1+GR_2)}{R_3(1+GR_2)+R_1+R_2}\right)\right]\right)$ $\quad D = R_3(1+GR_2)+R_1+R_2$

$V_2 = \frac{R_2}{R_1+R_2}\left[V_{IN}\left(1 - \frac{R_3(1+GR_2)}{D}\right)\right]$

$= \frac{R_2}{R_1+R_2}\left[V_{IN}\left(\frac{D - R_3(1+GR_2)}{D}\right)\right]$

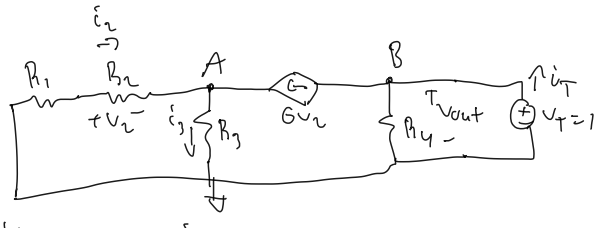
$V_2 = \frac{R_2}{R_1+R_2}\left[V_{IN}\left(\frac{\cancel{R_3(1+GR_2)} + R_1 + R_2 - \cancel{R_3(1+GR_2)}}{D}\right)\right]$

$V_2 = R_2 \frac{V_{IN}}{D}$

For $R_{IN} = \frac{V_{IN}}{i_{IN}} = \frac{V_{IN}}{\frac{V_2}{R_2}} = \frac{V_{IN}}{\frac{V_{IN}}{D}} = D$

$R_{IN} = R_3(1+GR_2)+R_1+R_2$

R_{out}:



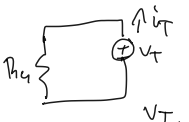
KCL @ A: $-i_2 + i_3 - GV_2 = 0$

$-\left(\frac{0-V_A}{R_1+R_2}\right) + \frac{V_A}{R_3} - GV_2 = 0$

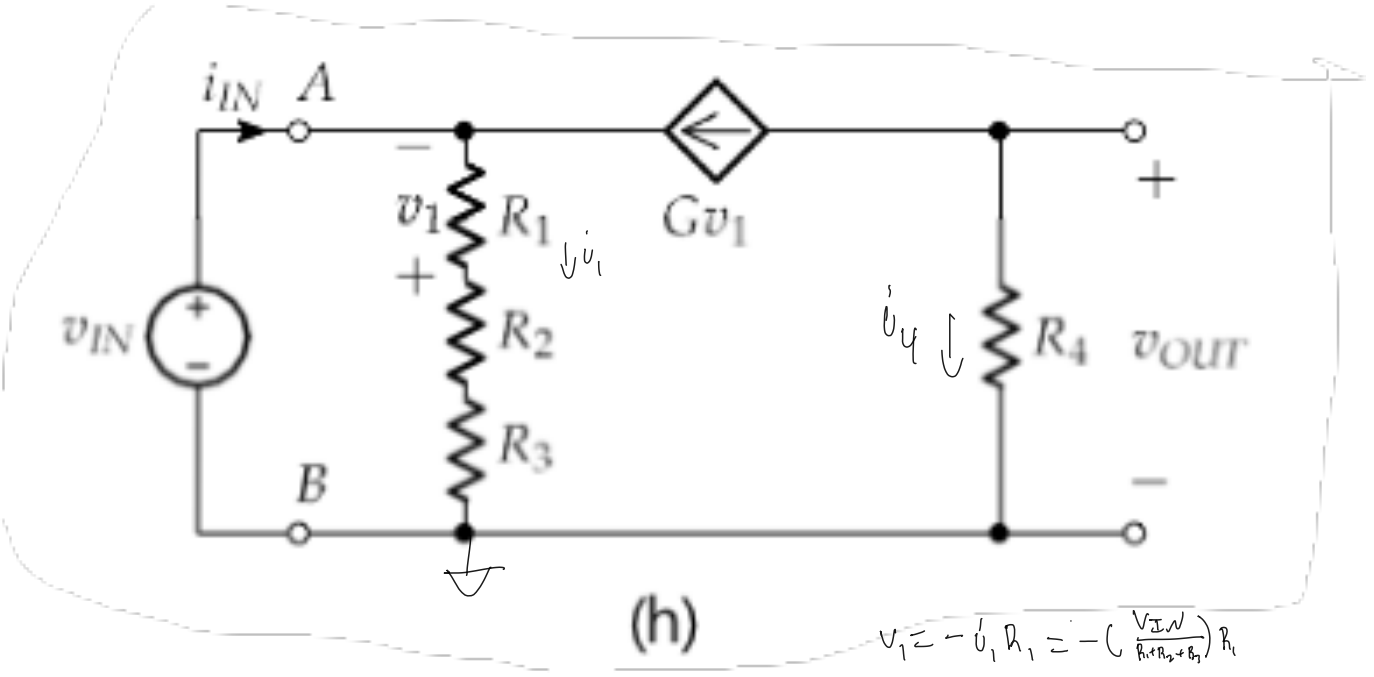
$\frac{V_A}{R_1+R_2} + \frac{V_A}{R_3} - G\left(R_2\left(\frac{-V_A}{R_1+R_2}\right)\right) = 0$

$V_A\left(\frac{1}{R_1+R_2} + \frac{1}{R_3} + \frac{GR_2}{R_1+R_2}\right) = 0$

$V_A = 0 \therefore V_A = 0$ on $V_2 = 0 - V_A = 0 - 0 \therefore V_2 = 0$



$\Rightarrow$ $R_{out} = \frac{V_T}{i_T} = R_4$



$v_1 = -i_1 R_1 = -\left(\frac{V_{IN}}{R_1+R_2+R_3}\right) R_1$

KCL @ C: $Gv_1 + i_4 = 0$

$$G\left(-\left(\frac{V_{IN}}{R_1+R_2+R_3}\right)R_1\right) + \frac{V_{out}}{R_4} = 0$$

$$V_{IN}\left(-\frac{GR_1}{R_1+R_2+R_3}\right) = -\frac{V_{out}}{R_4}$$

$$\frac{V_{out}}{V_{IN}} = \frac{GR_1R_4}{R_1+R_2+R_3} = A_v \text{ (gain)}$$

For $R_{IN} = \frac{V_{IN}}{i_{IN}}$

KCL @ A: $-i_{IN} + i_1 - Gv_1 = 0$

$$\frac{V_{IN}}{R_1+R_2+R_3} = i_{IN} + G\left(-\left(\frac{V_{IN}}{R_1+R_2+R_3}\right)R_1\right)$$

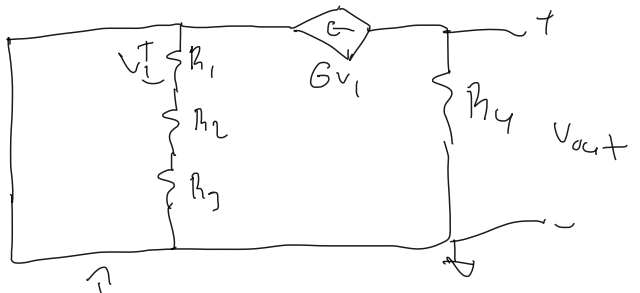
$$V_{IN}\left(\frac{1}{R_1+R_2+R_3} + \frac{GR_1}{R_1+R_2+R_3}\right) = i_{IN}$$

$$i_{IN} = V_{IN}\left(\frac{1+GR_1}{R_1+R_2+R_3}\right)$$

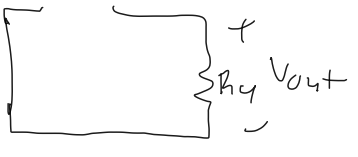
so $R_{IN} = \dfrac{V_{IN}}{V_{IN}\left(\frac{1+GR_1}{R_1+R_2+R_3}\right)}$

$$R_{IN} = \frac{R_1+R_2+R_3}{1+GR_1}$$

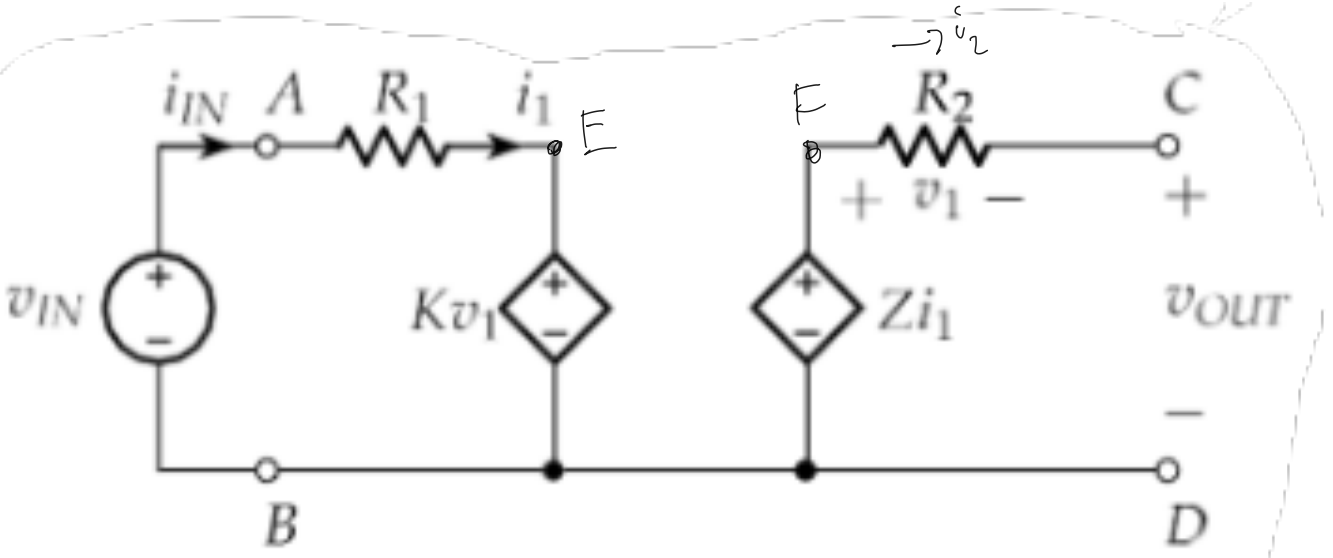
For R_{out}



short ⇒ $v_1 = 0$



$\therefore R_{out} = R_4$



(i)

$$i_N = i_1 = \frac{V_{IN} - Kv_1}{R_1}$$

$$i_1 = \frac{V_{IN} - 0}{R_1} = i_{IN}$$

$$V_1 = Z i_1 - V_{out}$$

$$i_2 = 0 = \frac{Z i_1 - V_{out}}{R_2}$$

$$Z i_1 = V_{out} \quad \therefore V_1 = 0$$

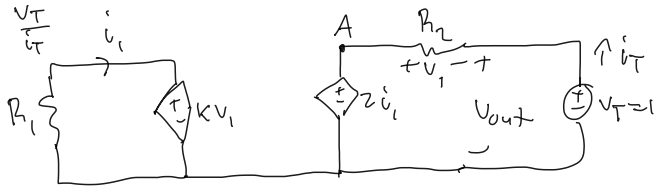
$$Z\left(\frac{V_{IN}}{R_1}\right) = V_{out}$$

$$\frac{V_{out}}{V_{IN}} = \frac{Z}{R_1} = A_V \text{ (gain)}$$

For $R_{IN} = \frac{V_{IN}}{i_{IN}} = \frac{V_{IN}}{\frac{V_{IN}}{R_1}} = R_1$

$$R_{IN} = R_1$$

For $R_{out} = \frac{V_T}{i_T}$



$$\frac{V_T - V_A}{R_2} = i_T \qquad i_1 = -\left(\frac{KV_1}{R_1}\right) \qquad V_1 = V_A - V_T \qquad V_A = Z i_1 = -\frac{KZ}{R_1} V_1$$

$$V_A = -\frac{KZ}{R_1}(V_A - V_T)$$

$$V_A\left(1 + \frac{KZ}{R_1}\right) = V_T \frac{KZ}{R_1}$$

$$\frac{V_T - \frac{KZ}{R_1 + KZ}}{R_2} = i_T$$

$$V_A = V_T \frac{\frac{KZ}{R_1}}{1 + \frac{KZ}{R_1}}$$

$$\frac{\frac{R_1 + KZ - KZ}{R_1 + KZ}}{R_2} = i_T$$

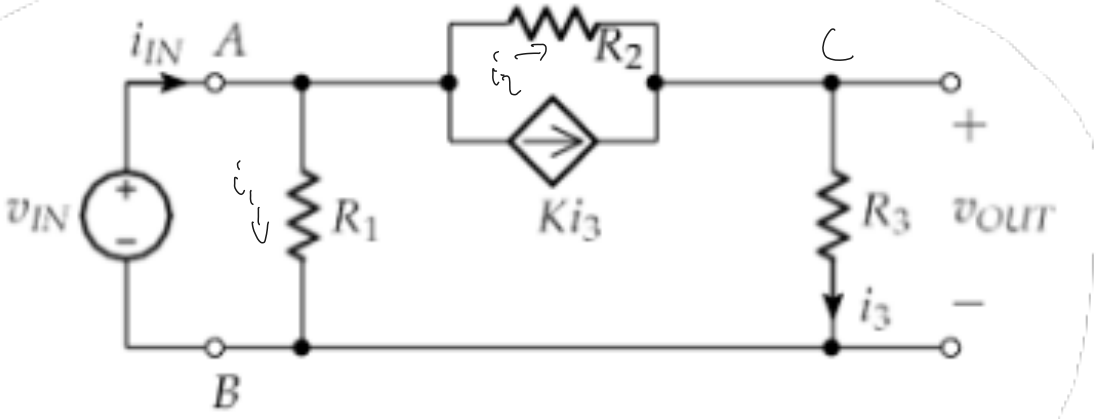
$$V_A = V_T \frac{KZ}{R_1 + KZ}$$

$$\frac{R_1}{R_2(R_1 + KZ)} = i_T$$

$$V_A = \frac{KZ}{R_1 + KZ}$$

$$R_{out} = \frac{V_T}{i_T} = \frac{1}{\frac{R_1}{R_2(R_1 + KZ)}}$$

$$R_{out} = \frac{R_2(R_1 + KZ)}{R_1}$$



(j)

KCL @ C: $-i_2 - Ki_3 + i_3 = 0$

$$-\left(\frac{V_{IN}-V_{out}}{R_2}\right) - K\left(\frac{V_{out}}{R_3}\right) + \frac{V_{out}}{R_3} = 0$$

$$V_{out}\left(\frac{1}{R_2} - \frac{K}{R_3} + \frac{1}{R_3}\right) + V_{IN}\left(-\frac{1}{R_2}\right) = 0$$

$$V_{out}\left(\frac{1}{R_2} + \frac{1-K}{R_3}\right) = V_{IN}\cdot\frac{1}{R_2}$$

$$\frac{V_{out}}{V_{IN}} = \frac{1}{R_2}\cdot\frac{1}{\frac{1}{R_2}+\frac{1-K}{R_3}}$$

$$= \frac{1}{1+\frac{R_2(1-K)}{R_3}}$$

$$= \frac{R_3}{R_3+R_2(1-K)} = A_v \text{ (gain)}$$

For $R_{IN} = \frac{V_{IN}}{i_{IN}}$

from KCL at C: $\frac{V_{out}}{R_3} = \frac{V_{IN}-V_{out}}{R_2} + K\left(\frac{V_{out}}{R_3}\right)$

$V_{out} = A_v V_{IN}$

$$\frac{A_v V_{IN}}{R_3} = \frac{V_{IN} - A_v V_{IN}}{R_2} + A_v V_{IN}\frac{K}{R_3}$$

$$\frac{A_v}{R_3} = \frac{1-A_v}{R_2} + A_v\frac{K}{R_3}$$

$$\frac{1-A_v}{R_2} = A_v\left(\frac{1-K}{R_3}\right)$$

$$A_v\left(\frac{1}{R_2}+\frac{1-K}{R_3}\right) = \frac{1}{R_2}$$

$$A_v\left(\frac{R_3+R_2(1-K)}{R_2R_3}\right) = \frac{1}{R_2}$$

$$A_v = \left(\frac{R_3}{R_3+R_2(1-K)}\right)$$

KCL @ A: $-i_{IN} + i_1 + i_2 + Ki_3 = 0$

$$i_{IN} = \frac{V_{IN}}{R_1} + \frac{V_{IN}-V_{out}}{R_2} + K\left(\frac{V_{out}}{R_3}\right)$$

$$i_{IN} = \frac{V_{IN}}{R_1} + \frac{V_{IN}-A_vV_{IN}}{R_2} + K\frac{A_vV_{IN}}{R_3}$$

$$i_{IN} = V_{IN}\left(\frac{1}{R_1} + \frac{1-A_v}{R_2} + \frac{KA_v}{R_3}\right)$$

$$= V_{IN}\left(\frac{1}{R_1} + \frac{A_v(1-K)+KA_v}{R_3}\right)$$

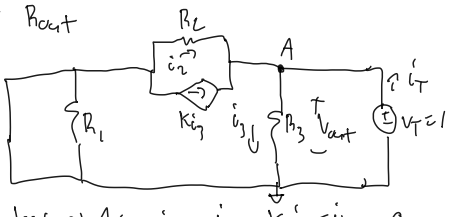
$$i_{IN} = V_{IN}\left(\frac{1}{R_1} + \frac{A_v}{R_3}\right)$$

$$i_{IN} = V_{IN}\left(\frac{1}{R_1} + \frac{R_3}{R_3(R_3+R_2(1-K))}\right)$$

$$i_{IN} = V_{IN}\left(\frac{1}{R_1} + \frac{1}{R_3+R_2(1-K)}\right)$$

$$\frac{V_{IN}}{i_{IN}} = R_{IN} = \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_3+R_2(1-K)}\right)}$$

For R_{out}



KCL @ A: $-i_T + i_3 - Ki_3 - i_2 = 0$

$$i_T = i_3(1-K) - i_2$$

$$i_T = \frac{V_T}{R_3}(1-K) - \frac{0-V_T}{R_2}$$

$$i_T = \frac{1-K}{R_3} + \frac{1}{R_2} = \frac{R_2(1-K)+R_3}{R_2R_3}$$

$$R_{out} = \frac{V_T}{i_T} = \frac{1}{\frac{R_2(1-K)+R_3}{R_2R_3}}$$

$$R_{out} = \frac{R_2R_3}{R_2(1-K)+R_3}$$