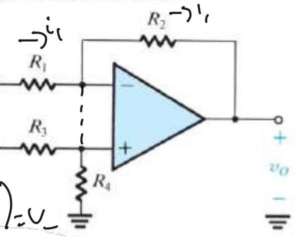
1. Find the voltage gain v_O/v_{Id} for the difference amplifier below for the case $R_1 = R_3 = 5\ k\Omega$ and $R2 = R4 = 100\ k\Omega$. What is the differential input resistance R_{id}? If the two key resistance ratios (R_2/R_1) and (R_4/R_3) are different from each other by 1%, what do you expect the common-mode gain A_{cm} to be? Also, find the CMRR in this case. Neglect the effect of the ratio mismatch on the value of A_d.



$$\frac{R_4}{R_3} = (1+.01)\frac{R_2}{R_1}$$

$$R_4 = (1.01)\frac{R_2R_3}{R_1}$$

$$V_+ = V_{I2}\left(\frac{R_4}{R_3+R_4}\right) = V_-$$

$$\frac{V_{I1}-V_-}{R_1} = \frac{V_- - V_0}{R_2}$$

$$\frac{V_0}{R_2} = V_-\left(\frac{1}{R_1}+\frac{1}{R_2}\right) - V_{I1}\left(\frac{1}{R_1}\right)$$

$$\frac{V_0}{R_2} = V_{I2}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) - V_{I1}\left(\frac{1}{R_1}\right)$$

$$V_0 = V_{I2}\left(\frac{R_2}{R_1}+1\right)\left(\frac{R_4}{R_3+R_4}\right) - V_{I1}\left(\frac{R_2}{R_1}\right)$$

$$V_0 = V_{I2}\left(\frac{R_4}{R_3+R_4}\right)\left(\frac{R_2R_4}{R_1(R_3+R_4)}\right) + V_{I1}\left(-\frac{R_2}{R_1}\right)$$

$$V_o = V_{I2}\left(\frac{R_4}{R_3+R_4}\right)\left(1-\frac{R_2R_3}{R_1R_4}\right) + V_{I1}\left(-\frac{R_2}{R_1}\right), \quad \frac{R_2}{R_1}=\frac{R_4}{R_3}, \quad \frac{R_2R_3}{R_1R_3}=1$$

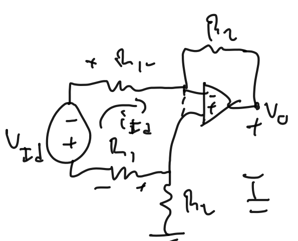
$$V_0 = -\frac{R_2}{R_1}V_{I1}$$

$$\frac{V_0}{V_{I1}} = -\frac{R_2}{R_1}$$

$$V_0 = V_I\frac{R_2}{R_1} \qquad V_{Id} = V_{I2} - V_{I1}$$

$$V_{Id} = -V_{I1}$$

$$\frac{V_o}{-V_{I1}} = -\frac{R_2}{R_1} = \frac{100K}{5K} = 20 = A_d$$



$$V_{Id} - i_{Id}R_1 - i_{Id}R_1 = 0$$

$$V_{Id} = 2i_{Id}R_1$$

$$\frac{V_{Id}}{i_{Id}} = 2R_1 = R_{Id} \therefore 2(5K) = R_{Id}$$

$$R_{Id} = 10K\Omega$$

$$\frac{V_0}{V_{I1}} = A_{I1} = -\frac{R_2}{R_1} \qquad \frac{V_0}{V_{cm}} = A_{I1}+A_{I2}$$

$$\frac{V_0}{V_{I2}} = A_{I2} = \left(1+\frac{R_2}{R_1}\right)\frac{R_4}{R_4+R_3}$$

$$A_{cm} = A_{I1} + A_{I2} \quad \text{(superposition)}$$

$$= -\frac{R_2}{R_1} + \frac{R_4}{R_4+R_3} + \frac{R_2R_4}{R_1(R_4+R_3)}$$

$$= \frac{R_2}{R_1}\left(-1+\frac{R_4}{R_4+R_3}\right) + \frac{R_4}{R_4+R_3}$$

$$A_{cm} = \frac{R_2}{R_1}\left(\frac{-\cancel{R_4}-R_3+\cancel{R_4}}{R_4+R_3}\right) + \frac{R_4}{R_4+R_3}$$

$$= -\frac{R_2}{R_1}\frac{R_3}{R_4+R_3} + \frac{R_4}{R_4+R_3}$$

$$= \frac{R_4 - \frac{R_2R_3}{R_1}}{R_4+R_3}$$

$$= \frac{R_4\left(1-\frac{1}{1.01}\right)}{R_4+R_3}$$

$$A_{cm} = \frac{R_4}{R_4+R_3}\left(1-\frac{R_2R_3}{R_1R_4}\right)$$

$$= \frac{1.01\frac{R_2\cancel{R_3}}{R_1}}{1.01\frac{R_2R_3}{R_1}+R_3}\left(1-\frac{R_2\cancel{R_3}}{\cancel{R_1}}\cdot\frac{1}{1.01\frac{R_2\cancel{R_3}}{\cancel{R_1}}}\right)$$

$$\cancel{R_3}\left(1.01\frac{R_2}{R_1}+1\right)$$

$$A_{cm} = \frac{1.01\frac{R_2}{R_1}}{1+1.01\frac{R_2}{R_1}}\left(1-\frac{1}{1.01}\right), \quad \frac{R_2}{R_1} = A_d = 20$$

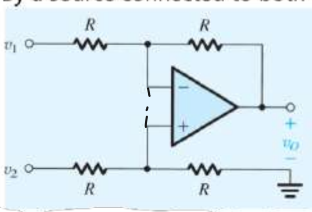
$$= \frac{1.01(20)}{1+1.01(20)}\left(1-\frac{1}{1.01}\right)$$

$$A_{cm} = .0094$$

$$CMMR = 20\log\left(\frac{A_d}{A_{cm}}\right)$$

$$CMMR = 66.53\ dB$$

2. For the circuit shown below, express v_O as a function of v_1 and v_2. What is the input resistance seen by v_1 alone? By v_2 alone? By a source connected between the two input terminals? By a source connected to both input terminals simultaneously?



$V_+ = V_2\left(\frac{R}{R+R}\right) = \frac{V_2}{2} = V_-$ KCL @ V_-: $\frac{V_1 - V_-}{R} = \frac{V_- - V_O}{R}$

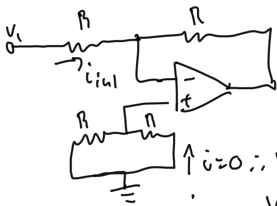
$\frac{V_1}{R} - \frac{2V_-}{R} = -\frac{V_o}{R}$

$2V_- + V_1 = V_o$

$2\left(\frac{V_2}{2}\right) + V_1 = V_o$

$V_o = V_1 + V_2$

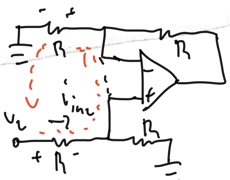
R_{in1}: $= \frac{V_1}{i_{in1}}$



$i = 0 \therefore V_+ = 0 = V_-$

$i_{in1} = \frac{V_1 - V_-}{R} = \frac{V_1}{R}$

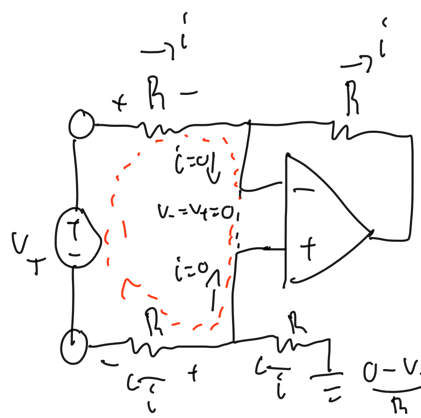
$R_{in1} = \frac{V_1}{\frac{V_1}{R}} = R$



$R_{in2} = \frac{V_2}{i_{in2}}$ KVL 1: $-V_2 + 2 i_{in2} R = 0$

$V_2 = 2 i_{in2} R$

$\frac{V_2}{i_{in2}} = 2R = R_{in2}$



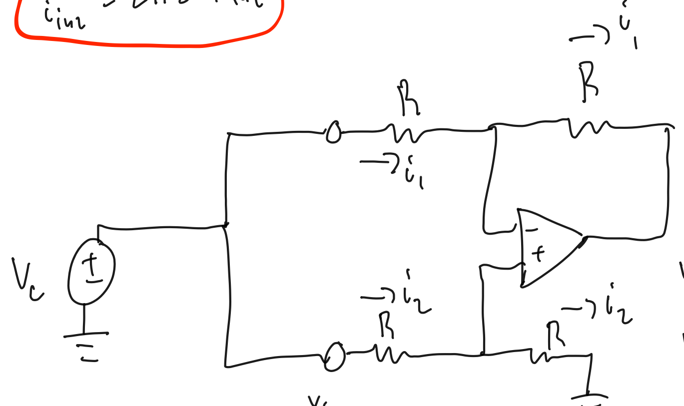
$\frac{0 - V_+}{R} = i$

$R_T = \frac{V_T}{i}$ $i \to V_+ = 0 \therefore V_+ = 0 = V_-$

KVL: $-V_T + iR + Ri = 0$

$V_T = 2iR$

$\frac{V_T}{i} = 2R = R_T$



$V_+ = V_c\left(\frac{R}{2R}\right)$

$V_+ = \frac{V_c}{2} = V_-$

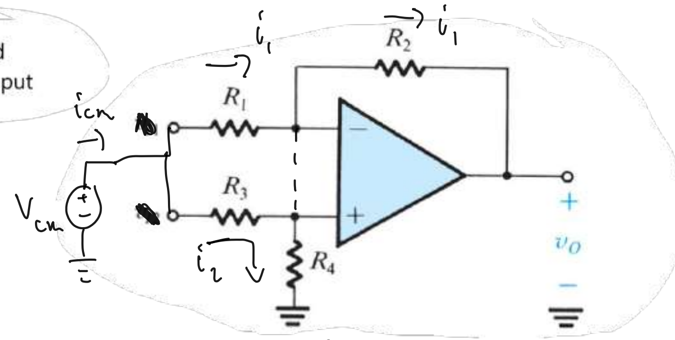
$i_1 = \frac{V_c - V_-}{R} = \frac{V_c - \frac{V_c}{2}}{R} = \frac{V_c}{2R}$

$i_2 = \frac{V_c - V_+}{R} = \frac{V_c - \frac{V_c}{2}}{R} = \frac{\frac{2V_c}{2} - \frac{V_c}{2}}{R} = \frac{\frac{V_c}{2}}{R} = \frac{V_c}{2R}$

$R_c = \frac{V_c}{i_1 + i_2} = \frac{V_c}{\frac{V_c}{2R} + \frac{V_c}{2R}} = \frac{V_c}{\frac{2V_c}{2R}} = R$

3. Consider the difference amplifier in problem 1 with the two input terminals connected together to an input common-mode signal source. For $R_2/R_1 = R_4/R_3$, show that the input common-mode resistance is $(R_3+R_4)||(R_1+R_2)$.

$R_2 = R_4 \quad R_1 = R_3$



$$V_+ = V_{cm}\left(\frac{R_4}{R_3+R_4}\right) = V_-$$

$$i_1 = \frac{V_{cm} - V_-}{R_1} = \frac{V_{cm} - V_{cm}\left(\frac{R_4}{R_3+R_4}\right)}{R_1}$$

$$= \frac{R_3 V_{cm} + V_{cm} R_4 - R_4 V_{cm}}{R_1(R_3+R_4)}$$

$$= V_{cm}\frac{R_3}{R_1(R_3+R_4)}$$

$$i_2 = \frac{V_{cm} - V_{cm}\left(\frac{R_4}{R_3+R_4}\right)}{R_3}$$

$$i_2 = \frac{V_{cm} R_3}{R_3(R_3+R_4)}$$

$$i_2 = \frac{V_{cm}}{R_3+R_4}$$

$$R_{cm} = \frac{V_{cm}}{i_{cm}} = \frac{V_{cm}}{i_1 + i_2} = \frac{V_{cm}}{\quad}$$

$$i_{cm} = i_1 + i_2$$

$$= V_{cm}\left(\frac{R_3}{R_1(R_3+R_4)} + \frac{1}{R_3+R_4}\right)$$

$$= V_{cm}\left(\frac{1}{R_1}\left(\frac{R_3}{R_3+R_4}\right) + \frac{1}{R_3+R_4}\right), \quad R_1 = R_3, \; R_2 = R_4$$

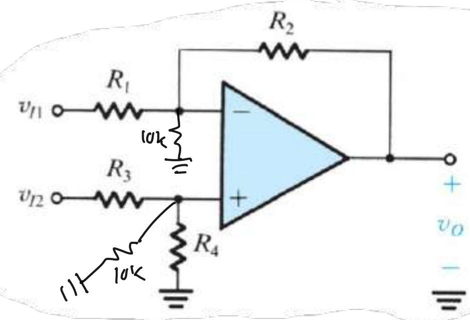
$$i_{cm} = V_{cm}\left(\frac{1}{\cancel{R_1}}\left(\frac{\cancel{R_1}}{R_1+R_2}\right) + \frac{1}{R_3+R_4}\right)$$

$$= V_{cm}\left(\frac{1}{R_1+R_2} + \frac{1}{R_3+R_4}\right)$$

$$\frac{1}{\left(\frac{1}{R_1+R_2} + \frac{1}{R_3+R_4}\right)} = (R_1+R_2)||(R_3+R_4) = \frac{V_{cm}}{i_{cm}} = R_{cm}$$

def of parallel

4. (a) Find A_d and A_{cm} for the difference amplifier circuit in Problem 1 with $R_1=R_2=R_3=R_4=100k\Omega$.
(b) If the op amp is specified to operate properly as long as the common-mode voltage at its positive and negative inputs falls in the range ±2.5 V, what is the corresponding limitation on the range of the input common-mode signal v_{Icm}? (This is known as the *common-mode range* of the differential amplifier.)
(c) The circuit is modified by connecting a 10-kΩ resistor between inverting input and ground, and another 10-kΩ resistor between non-inverting input and ground. What will now be the values of A_d, A_{cm}, and the input common-mode range?



a) From 1. $A_{cm} = \left(\frac{R_4}{R_3+R_4}\right)\left(1 - \frac{R_2 R_3}{R_1 R_4}\right)$

$R_1=R_2=R_3=R_4=100k \Rightarrow A_{cm} = \left(\frac{1}{2}\right)(1-1) = 0$

$A_{cm} = 0$

From 1, $A_d = \frac{R_2}{R_1}$

$A_d = 1$

b) $V_{Icm} = V_{I1} = V_{I2}$

$V_+ = V_{Icm} \frac{R_4}{R_3+R_4} = V_-$

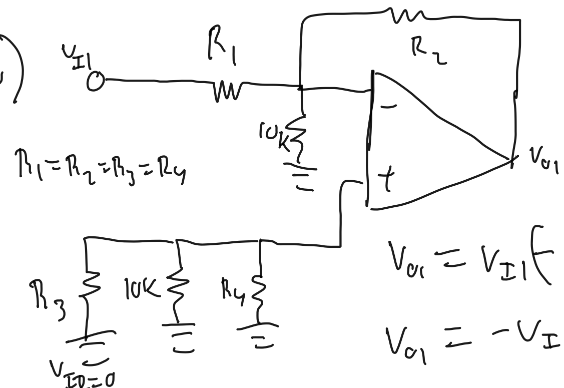
$V_+ = \frac{1}{2} V_{Icm} = V_-$

$V_- = -2.5 = \frac{1}{2} V_{Icm}$

$V_+ = 2.5 = \frac{1}{2} V_{Icm}$

$-5[V] \le V_{Icm} \le 5[V]$

c)

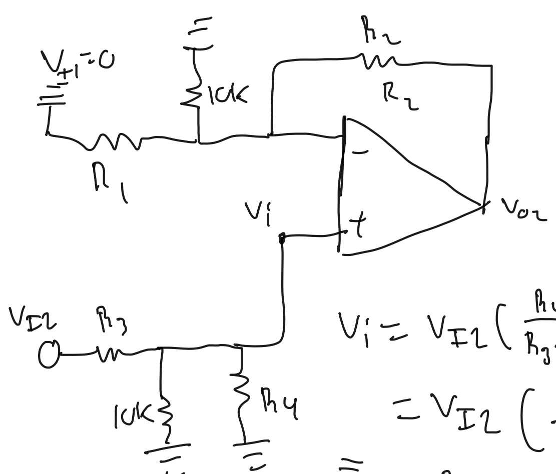


$R_1=R_2=R_3=R_4$

$V_{o1} = V_{I1}\left(-\frac{R_2}{R_1}\right)$

$V_{o1} = -V_{I1}$

$\frac{V_{o1}}{V_{I1}} = -1 = A_1$



$\frac{V_o}{V_i} = 1 + \frac{R_f}{R_g}$

$R_g = 10k \| R_1 = 10k \| 100k$

$V_i = V_{I2}\left(\frac{R_4 \| 10k}{R_3 + (R_4 \| 10k)}\right)$

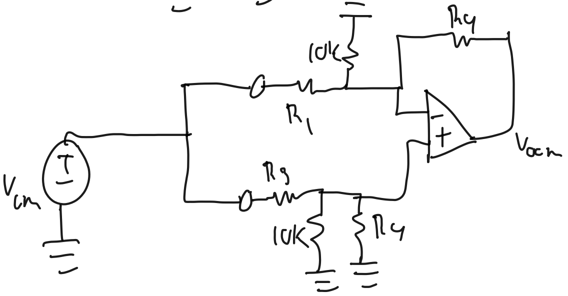
$= V_{I2}\left(\frac{100k \| 10k}{R_3 + (100k \| 10k)}\right)$

$V_{o2} = V_{I2}\left(\frac{100k \| 10k}{100k + (100k \| 10k)}\right)\left(1 + \frac{100k}{10k \| 100k}\right)$

$\frac{V_{o2}}{V_{I2}} = \frac{1}{12}(12) = 1 = A_2$

$A_d = \frac{A_2 - A_1}{2} = \frac{1-(-1)}{2} = 1$

$A_d = 1$



$A_{cm} = A_1 + A_2 \Rightarrow A_{cm} = -1 + 1$

$A_{cm} = 0$

$V_+ = V_- = V_{cm}\left(\frac{10k \| 100k}{100k + (10k \| 100k)}\right) = \frac{1}{12} V_{cm}$

$-2.5[V] \le \frac{1}{12} V_{cm} \le 2.5[V]$

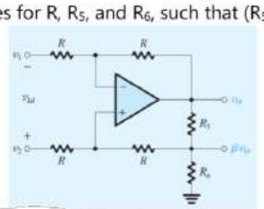
$-30[V] \le V_{cm} \le 30[V]$

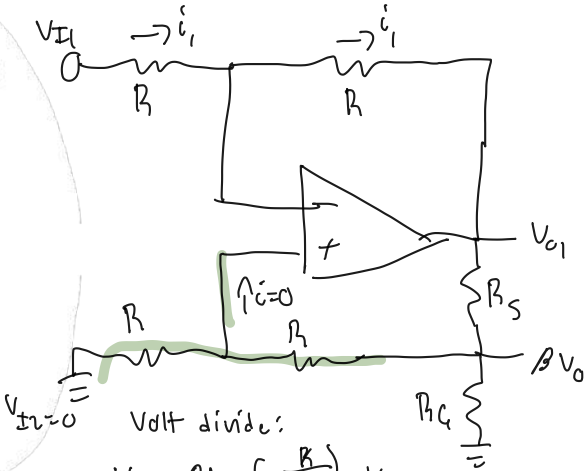
5. To obtain a high-gain, high-input-resistance difference amplifier, the circuit below employs positive feedback, in addition to the negative feedback provided by the resistor R connected from the output to the negative input of the op amp. Specifically, a voltage divider (R_5, R_6) connected across the output feeds a fraction β of the output, that is, a voltage βv_O, back to the positive-input terminal of the op amp through a resistor R. Assume that R_5 and R_6 are much smaller than R so that the current through R is much lower than the current in the voltage divider, with the result that $\beta \sim R_6/(R_5+R_6)$. Show that the differential gain, A_d, is given by

$$\frac{v_O}{v_{Id}} = \frac{1}{1-\beta}$$

(Hint: Use superposition.)

Design the circuit to obtain a differential gain of 10 V/V and differential input resistance of 2MΩ. Select values for R, R_5, and R_6, such that $(R_5+R_6) \le R/100$.





$V_{Id} = V_{I2} - V_{I1}$

$$i_1 = \left(\frac{V_{I1}-V_-}{R} = \frac{V_- - V_{o1}}{R}\right) R$$

Volt divide:

$$V_+ = \beta V_{o1}\left(\frac{R}{2R}\right) = V_-$$

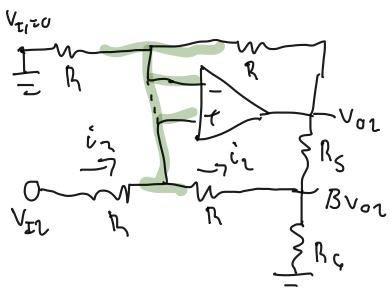
$$V_+ = \frac{\beta V_{o1}}{2} = V_-$$

$$V_{o1} - 2V_- = -V_{I1}$$

$$V_{o1} - 2\frac{\beta V_{o1}}{2} = -V_{I1}$$

$$V_{o1}(1-\beta) = -V_{I1}$$

$$V_{o1} = \frac{-V_{I1}}{1-\beta}$$



$$V_- = V_{o2}\left(\frac{R}{2R}\right) = V_+$$

$$V_- = V_+ = \frac{V_{o2}}{2}$$

$$i_2 = \frac{V_{I2}-V_+}{R} = \frac{V_+ - \beta V_{o2}}{R}$$

$$\beta V_{o2} - 2V_+ = -V_{I2}$$

$$\beta V_{o2} - 2\frac{V_{o2}}{2} = -V_{I2}$$

$$V_{o2}(\beta - 1) = -V_{I2}$$

$$V_{o2} = \frac{V_{I2}}{1-\beta}$$

$$V_o = V_{o1} + V_{o2}$$

$$= -\frac{V_{I1}}{1-\beta} + \frac{V_{I2}}{1-\beta}$$

$$V_o = \underbrace{(V_{I2} - V_{I1})}_{V_{Id}} \frac{1}{1-\beta}$$

$$\boxed{\frac{V_o}{V_{Id}} = \frac{1}{1-\beta} = A_d}$$

$$A_d = 10 = \frac{1}{1-\beta}$$

$$1-\beta = .1$$

$$\beta = .9 \quad , \quad \beta \approx \frac{R_6}{R_5+R_6}$$

$$R_{id} = R + R = 2R$$

$$2M\Omega = 2R$$

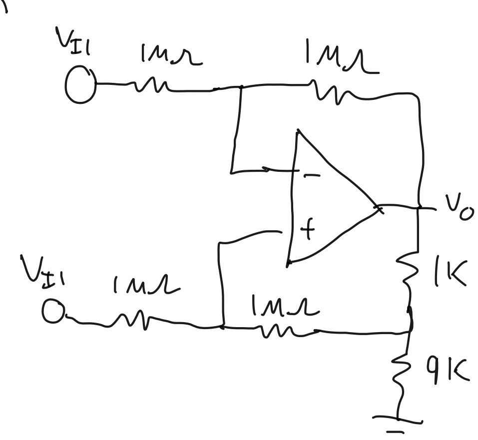
$$\boxed{1M\Omega = R}$$

$$R_5 + R_6 \le \frac{1M}{100}$$

$$R_5 + R_6 \le 10K$$

$$.9 \approx \frac{R_6}{10K}$$

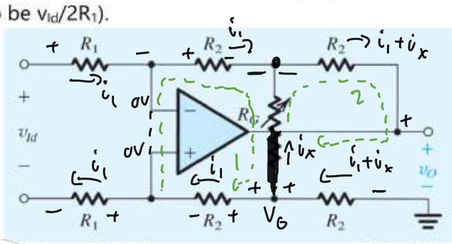
$$\boxed{9K \approx R_6 \therefore R_5 \approx 1K}$$



6. 2Figure below shows a modified version of the difference amplifier. The modified circuit includes a resistor RG, which can be used to vary the gain. Show that the differential voltage gain is given by

$$\frac{v_O}{v_{Id}} = -2\frac{R_2}{R_1}\left[1+\frac{R_2}{R_G}\right]$$

(Hint: The virtual short circuit at the op-amp input causes the current through the R1 resistors to be $v_{Id}/2R_1$).



$$i_1 = \frac{V_{Id}}{2R_1}$$

$$V_{Id} = 2R_1 i_1$$

$$V_G = i_x R_G$$

$$i_x = \frac{V_G}{R_g} = \frac{2 i_1 R_2}{R_g}$$

KCL (1): $i_1 R_2 + 0 + i_1 R_2 - V_G = 0$

$$V_G = 2 i_1 R_2$$

KVL (2):

$$V_G - (-R_2(i_1+i_x)) - (-R_2(i_1+i_x)) + V_o = 0$$

$$V_o = -V_G - 2R_2\left(i_1 + 2i_1\frac{R_2}{R_g}\right)$$

$$V_o = -2R_2 i_1 - 2i_1 R_2\left(1 + 2\frac{R_2}{R_g}\right)$$

$$V_o = -2R_2 i_1\left(1 + \left(1 + 2\frac{R_2}{R_g}\right)\right)$$

$$= -2R_2 i_1\left(2 + 2\frac{R_2}{R_g}\right)$$

$$V_o = -4R_2 i_1\left(1 + \frac{R_2}{R_g}\right)$$

$$\frac{V_o}{V_{Id}} = \frac{-4R_2 \cancel{i_1}\left(1+\frac{R_2}{R_g}\right)}{2R_1 \cancel{i_1}}$$

$$= -2\frac{R_2}{R_1}\left(1+\frac{R_2}{R_g}\right)$$

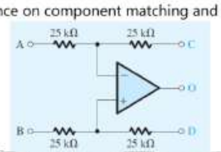
7. The circuit shown below is a representation of a versatile, commercially available IC, the INA105, manufactured by Burr-Brown and known as a differential amplifier module. It consists of an op amp and precision, laser-trimmed, metal-film resistors. The circuit can be configured for a variety of applications by the appropriate connection of terminals A, B, C, D, and O.

(a) Show how the circuit can be used to implement a difference amplifier of unity gain.

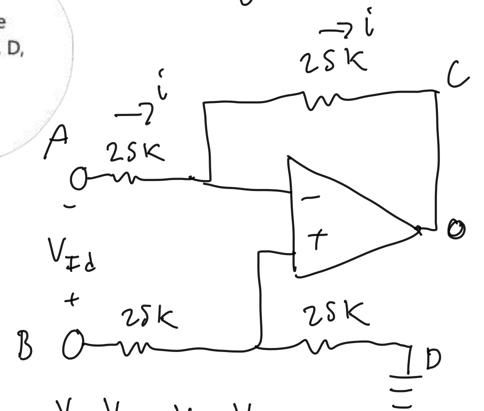
(b) Show how the circuit can be used to implement single-ended amplifiers with gains:

(i) −1 V/V
(ii) +1 V/V
(iii) +2 V/V
(iv) +1/2 V/V

Avoid leaving a terminal open-circuited, for such a terminal may act as an "antenna," picking up interference and noise through capacitive coupling. Rather, find a convenient node to connect such a terminal in a redundant way. When more than one circuit implementation is possible, comment on the relative merits of each, taking into account such considerations as dependence on component matching and input resistance.



a) Unity gain = 1 V/V



$V_+ = V_B\left(\frac{1}{2}\right) = V_-$

$i = \frac{V_A - V_-}{25K} = \frac{V_- - V_o}{25K}$

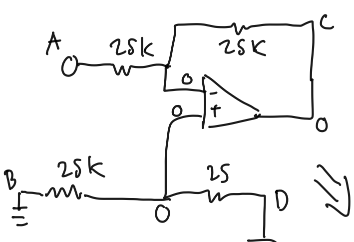
$V_o = 2V_- - V_A$

$V_o = 2\left(\frac{V_B}{2}\right) - V_A$

$V_o = V_B - V_A$

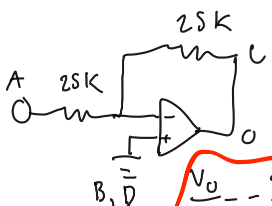
$V_o = V_{Id} \Rightarrow \boxed{\frac{V_o}{V_{Id}} = 1}$

i) $\frac{V_o}{V_i} = -\frac{R_f}{R_i} = -1 \Rightarrow \frac{R_f}{R_i} = 1$



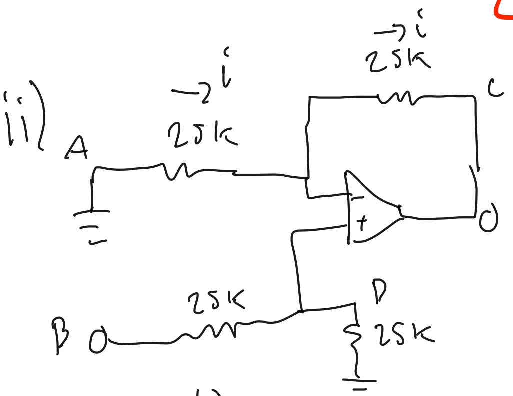
$V_+ = 0\left(\frac{1}{2}\right) = 0 = V_-$ ∴

$\Rightarrow$



$\boxed{\frac{V_o}{V_A} = -\frac{25K}{25K} = -1}$

ii)



$V_+ = V_B\left(\frac{1}{2}\right) = V_-$

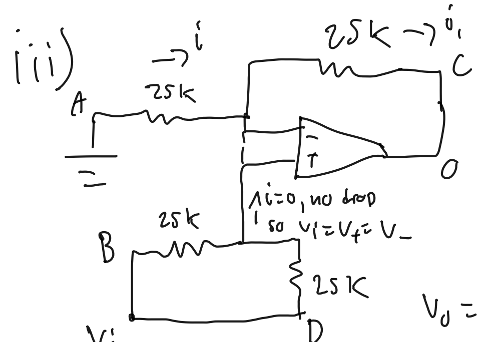
$i = -\frac{V_-}{25} = \frac{V_- - V_o}{25K}$

$V_o = 2V_-$

$V_o = 2\left(\frac{V_B}{2}\right)$

$\boxed{\frac{V_o}{V_B} = 1}$

iii)



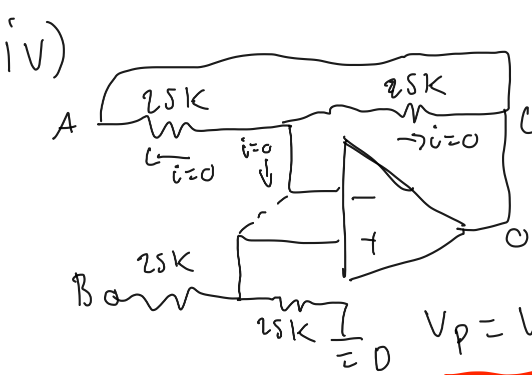
$i = \frac{-V_-}{25K} = \frac{V_- - V_o}{25K}$

$V_o = 2V_-$

$V_o = 2V_i$

$\boxed{\frac{V_o}{V_i} = 2}$

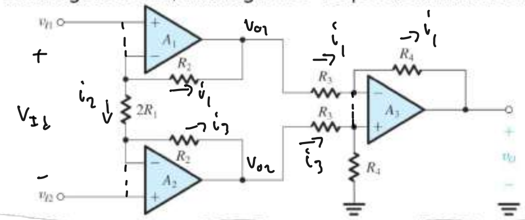
iv)



$V_p = V_B\left(\frac{1}{2}\right) = V_- = V_o$

$\boxed{\frac{1}{2} = \frac{V_o}{V_B}}$

8. Design the instrumentation-amplifier circuit below to realize a differential gain, variable in the range 2 to 100, utilizing a 100-kΩ pot as variable resistor.



$$\frac{V_{I1}-V_{I2}}{2R_1}+\frac{V_{I1}-V_{o1}}{R_2}=0$$

$$\frac{V_{I}-V_{I2}}{2R_1}+\frac{V_{I2}-V_{o2}}{R_2}=0$$

$$\frac{V_{I1}-V_{I2}}{2R_1}=\frac{V_{o1}-V_{I1}}{R_2}$$

$$\frac{V_{I2}-V_{I1}}{2R_1}=\frac{V_{o2}-V_{I2}}{R_2}$$

$$\frac{V_{o1}}{R_2}=V_{I1}\left(\frac{1}{2R_1}+\frac{1}{R_2}\right)-V_{I2}\left(\frac{1}{2R_1}\right)$$

$$\frac{V_{o2}}{R_2}=V_{I2}\left(\frac{1}{R_2}+\frac{1}{2R_1}\right)-V_{I1}\left(\frac{1}{2R_1}\right)$$

$$V_{o1}=V_{I1}\left(\frac{R_2}{2R_1}+1\right)-V_{I2}\left(\frac{R_2}{2R_1}\right)$$

$$V_{o2}=V_{I2}\left(1+\frac{R_2}{2R_1}\right)-V_{I1}\left(\frac{R_2}{2R_1}\right)$$

$$V_{o1}=\frac{R_2}{2R_1}(V_{I1}-V_{I2})+V_{I1}$$

$$V_{o2}=V_{I2}-\frac{R_2}{2R_1}(V_{I1}-V_{I2})$$

$$V_{o1}-V_{o2}=\frac{R_2}{2R_1}\left[V_{I1}-V_{I2}-\left(-(V_{I1}-V_{I2})\right)\right]+V_{I1}-V_{I2}$$

$$=\frac{R_2}{\cancel{2}R_1}\left[\cancel{2}V_{I}-\cancel{2}V_{I2}\right]+V_{I1}-V_{I2}$$

$$V_{o1}-V_{o2}=2(V_{I1}-V_{I2})\left(\frac{R_2}{R_1}+1\right)$$

$$\frac{V_{o1}-V_{o2}}{V_{I1}-V_{I2}}=A_d=\left(1+\frac{R_2}{R_1}\right)$$

For A_3: $V_+=V_{o2}\left(\frac{R_4}{R_3+R_4}\right)=V_-$

$$i_1=\frac{V_{o1}-V_-}{R_3}=\frac{V_--V_o}{R_4}$$

$$=\frac{V_o}{R_4}=V_-\left(\frac{1}{R_3}+\frac{1}{R_4}\right)-V_{o1}\left(\frac{1}{R_3}\right)$$

$$V_o=V_{o2}\left(\frac{R_4}{R_3+R_4}\right)\left(\frac{R_4}{R_3}+1\right)-V_{o1}\left(\frac{R_4}{R_3}\right)$$

$$=V_{o2}\left(\frac{R_4}{\cancel{R_3+R_4}}\right)\left(\frac{\cancel{R_4+R_3}}{R_3}\right)-V_{o1}\left(\frac{R_4}{R_3}\right)$$

$$V_o=\frac{R_4}{R_3}(V_{o2}-V_{o1})$$

$$V_o=\frac{R_4}{R_3}(V_{I1}-V_{I2})\left(\frac{R_2}{R_1}+1\right)$$

$$\frac{V_O}{V_{Id}}=\left(1+\frac{R_2}{R_1}\right)\frac{R_4}{R_3}=A_d$$

$$2R_1=R_{pot}+R_o$$

$$R_1=\frac{R_{pot}+R_o}{2}$$

Assume

$R_4=100k$ $R_3=100k$

$$A_3=\frac{R_4}{R_3}=1$$

$$A_d=2=\left(1+\frac{R_2}{R_1}\right)\frac{R_4}{R_3}$$

$$2=1+\frac{R_2}{\frac{100k+R_o}{2}}$$, low ∴ pot=max

$$1=\frac{2R_2}{100k+R_o}$$

$$100k+R_o=2R_2$$

$$A_d=50=1+\frac{R_2}{\frac{R_o}{2}}$$ high ∴ pot=0

$$49=\frac{2R_2}{R_o}$$

$$49R_o=2R_2$$

$$49R_o=100k+R_o$$

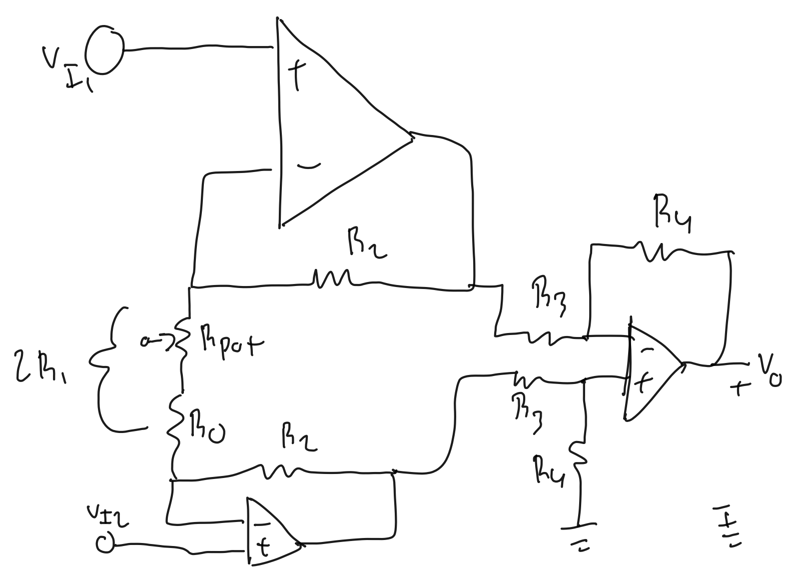
$$R_o=2.083\ k\Omega$$

$$49R_o=2R_2$$

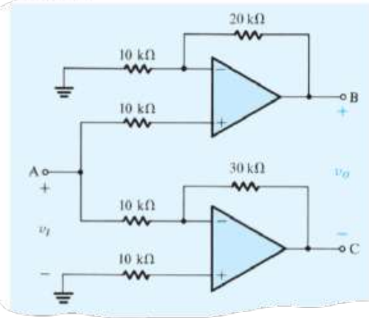
$$51.042\ k\Omega=R_2$$

$$\frac{R_o}{2}\le R_1\le\frac{100k+R_o}{2}$$

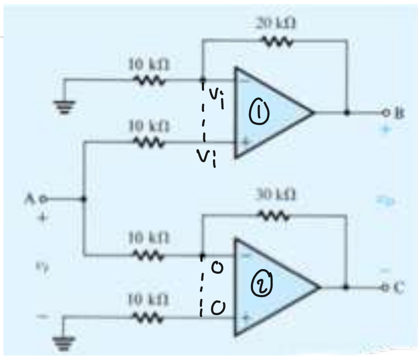
$$1.042\ k\Omega\le R_1\le 51.042\ k\Omega$$



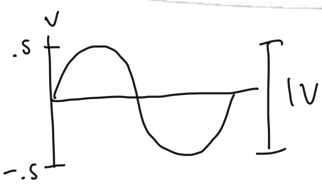
9. The circuit shown below is intended to supply a voltage to floating loads (those for which both terminals are ungrounded) while making greatest possible use of the available power supply.
   (a) Assuming ideal op amps, sketch the voltage waveforms at nodes B and C for a 1-V peak-to-peak sine wave applied at A. Also sketch v_O.
   (b) What is the voltage gain v_O/v_I?
   (c) Assuming that the op amps operate from ±15-V powersupplies and that their output saturates at ±14 V (in the manner shown in Fig. 1.14), what is the largest sine-wave output that can be accommodated? Specify both its peak-to-peak and rms values.



a)



$V_i = 1\,V_{pp}$



$V_o = V_B - V_c = 3V_i - (-3V_i) = 6V_i$

① $\frac{V_i}{10K} + \frac{V_i - V_B}{20K} = 0$

$\frac{V_B}{20K} = V_i\left(\frac{1}{10K} + \frac{1}{20K}\right)$

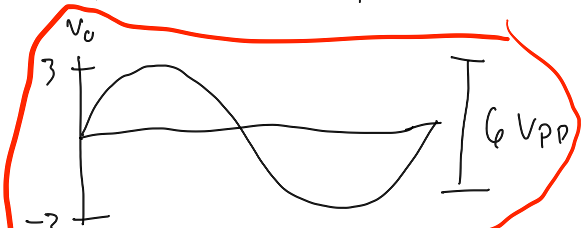
$V_B = V_i(2+1) = 3V_i$

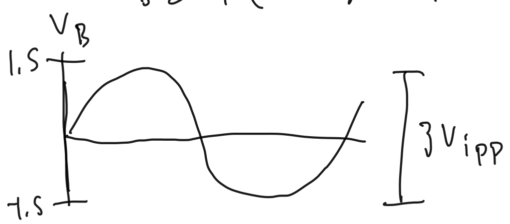
② $-\frac{V_i}{10K} - \frac{V_c}{30K} = 0$

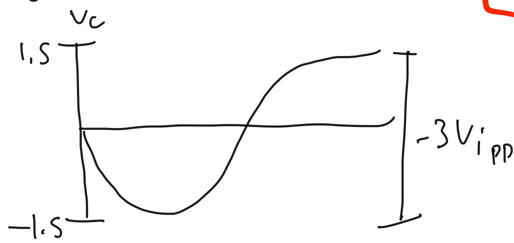
$\frac{V_c}{30K} = -\frac{V_i}{10K}$

$V_c = -3V_i$

$\boxed{\frac{V_o}{V_i} = 6 \therefore}$







c) output saturation ±14 [V]

$3V_{i,peak} \le 14$

$V_{i,peak} \le \frac{14}{3}$

$V_{o\,pk} = 6\,V_{i,pk} = 6\left(\frac{14}{3}\right)$

$V_{o,pk} = 28 \therefore$

$$\boxed{\begin{aligned} V_{o,pp} &= 56\ [V] \\ V_{o,rms} &= \frac{V_{pk}}{\sqrt{2}} = 19.8\ [V] \end{aligned}}$$

10. A particular op amp using ±15-V supplies operates linearly for outputs in the range −14 V to +14 V. If used in an inverting amplifier configuration of gain −100, what is the rms value of the largest possible sine wave that can be applied at the input without output clipping?

$V_o = A_{mp} \sin(\omega t)$

$V_o = 14 \sin(\omega t)$

$A_v = \frac{V_o}{V_i} = 100$

$V_i = \frac{14}{100} \sin(\omega t)$

$V_i = .14 \sin(\omega t)$ [V]

$V_{i,rms} = \frac{.14}{\sqrt{2}}$

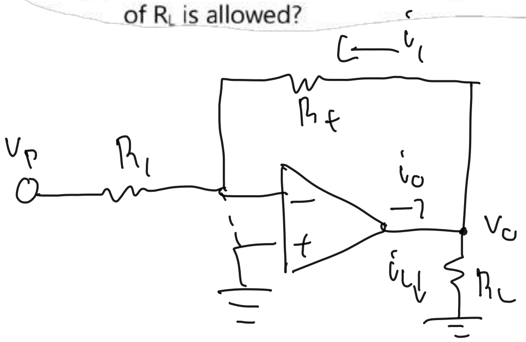
$= .0989\overline{9}$ [V]

11. Consider an op amp connected in the inverting configuration to realize a closed-loop gain of −100 V/V utilizing resistors of 1 kΩ and 100 kΩ. A load resistance R_L is connected from the output to ground, and a low-frequency sine-wave signal of peak amplitude V_p is applied to the input. Let the op amp be ideal except that its output voltage saturates at ±10V and its output current is limited to the range ±20 mA.

(a) For R_L = 1 kΩ, what is the maximum possible value of V_p while an undistorted output sinusoid is obtained?

(b) Repeat (a) for R_L = 200 Ω.

(c) If it is desired to obtain an output sinusoid of 10-V peak amplitude, what minimum value of R_L is allowed?



$A = -\frac{R_f}{R_i} = -100$

$R_f = 100K$, $R_i = 1K$

$\pm 20mA = i_o \geq i_L + i_1$

a) $\frac{V_o}{V_p} = -100$, $R_L = 1K$

$V_o = -100 V_p \sin(\omega t)$

$\left(\frac{V_o}{R_L} + \frac{V_o}{100K} \leq 20mA\right) 100k$

$100V_o + V_o \leq 2K$

$101 V_o \leq 2K$

$V_o \leq \pm 19.8$ [V] but saturates at

$V_o = \pm 10$ [V]

$10 = 100 V_p$

$.1$ [V] $= V_{p,max}$

for undistorted V_o

b) $\frac{V_o}{200} + \frac{V_o}{100k} \leq 20mA$

$500V_o + V_o \leq 2K$

$501 V_o \leq 2K$

$V_o \leq \pm 3.99$ [V] $\therefore$

$V_o = 100 V_p$

$3.99 = 100 V_p$

39.9 [mV] $= V_{p,max}$

for $R_L = 200$

c) $V_o = 10 = 100 V_p$

$\frac{V_o}{R_L} + \frac{V_o}{100k} = 20mA$

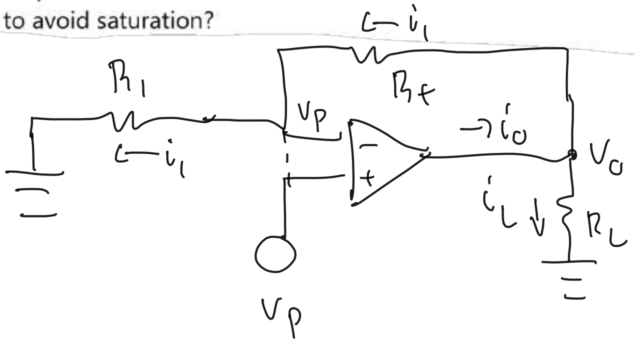
$\frac{10}{R_L} + .1m = 20mA$

$\frac{10}{19.9m} = R_L$

$502.5\Omega = R_{L,min}$

for $V_o = 10$

12. Repeat problem 11 from above assuming this time that it is a non-inverting configuration amplifier and that it utilizes 1k and 99k resistors. What is the minimum output resistor value to avoid saturation?



$$\frac{V_o}{V_p} = 1 + \frac{R_f}{R_1} = 100$$

$$\frac{R_f}{R_1} = 99$$

$$R_f = 99K, \; R_1 = 1K$$

$$R_L = 1K$$

$$V_o = 100 V_p \sin(\omega t)$$

$$i_o = i_1 + i_L$$

$$i_o = \frac{V_o}{R_f + R_1} + \frac{V_o}{R_L}$$

$$\frac{10}{100K} + \frac{10}{R_L} = 20mA$$

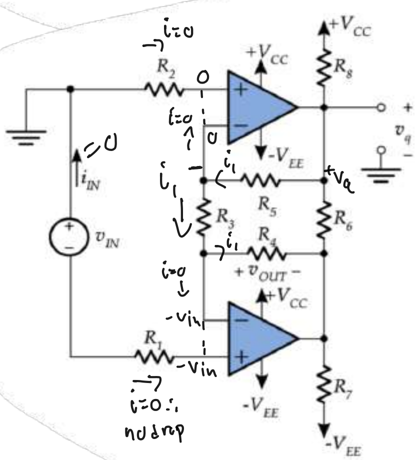
$$\frac{10}{R_L} = 19.9m$$

$$\boxed{R_{L,min} = 502.5\Omega \text{ to avoid saturation}}$$

13. Assuming ideal op-amps,
- Find the voltage gain, v_{OUT}/v_{IN}
- Find input resistance, v_{IN}/i_{IN}
- If v_{IN} = 2 V, find v_q

Use R_1=38 kΩ, R_2=15 kΩ, R_3=1.2 kΩ, R_4=5.6 kΩ, R_5=18 kΩ, R_6=2.2 kΩ, R_7=4.7 kΩ, R_8=3.3 kΩ, $+V_{CC}$=+15 V, and $-V_{EE}$= −15 V.

$$i_1 = \frac{0-(-V_{in})}{R_3} = \frac{V_{in}}{R_3}$$



$$V_{out} = i_1 R_4$$

$$V_{out} = \frac{V_{in}}{R_3} R_4$$

$$\frac{V_q - 0}{R_5} = i_1$$

$$\frac{V_q}{R_5} = \frac{2}{R_3}$$

$$\boxed{V_q = \frac{2R_5}{R_3} = 30[V]}$$

$$\frac{V_{out}}{V_{in}} = \frac{R_4}{R_3} = \frac{5.6K}{1.2K}$$

$$\boxed{\frac{V_{out}}{V_{in}} = 4.6\overline{6}}$$

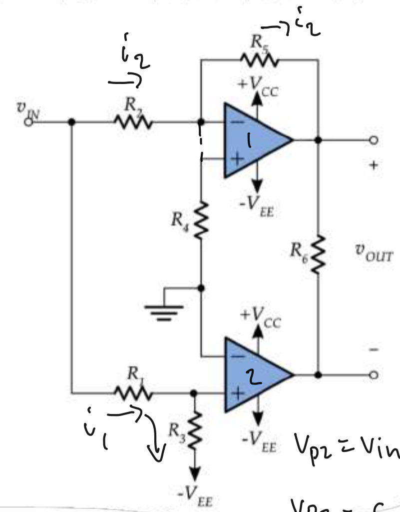
$$i_{in} = 0 \therefore R_{in} = \frac{V_{in}}{i_{in}}$$

$$= \frac{V_{in}}{0}$$

$$\boxed{R_{in} \rightarrow \infty}$$

14. Assuming ideal op-amps, find v_{OUT}

Use R_1=7.2 kΩ, R_2=47 kΩ, R_3=4.7 kΩ, R_4=3.3 kΩ, R_5=56 kΩ, R_6=8.2 kΩ, v_{IN}=+5V, $+V_{CC}$=+15 V, and $-V_{EE}$=−15 V.



$$V_{o1} = V_{in} - \left(\frac{R_5}{R_2}\right)$$

$$= -5\left(\frac{56K}{47K}\right)$$

$$V_{o1} = -\frac{280}{47}\ [V]$$

$$\boxed{\begin{aligned} V_{out} &= V_{o1} - V_{o2} \\ &= -\frac{280}{47} - (-15) \\ &= 9.043\ [V] \end{aligned}}$$

$$V_{p2} = V_{in} - \left((V_{in} - V_{EE})\left(\frac{R_1}{R_1+R_3}\right)\right)$$

$$V_{p2} = 5 - (5-(-15))\left(\frac{7.2K}{7.2K+4.7K}\right)$$

$$V_{p2} = -7.1\ [V]$$

$$V_{n2} = 0$$

$$V_{o2} = A_2(V_+ - V_-)$$

$$V_+ < V_- \therefore V_o \to -V_{EE}$$