1. a) Label node after R_1 as A and node after C_2 as B

17 pt
3 digits

$$Z_1 = \frac{1}{j\omega C_1} \qquad Z_2 = \frac{1}{j\omega C_2}$$

Impedance seen from node A

$$Z_f = Z_2 + R_2 = \frac{1}{j\omega C_2} + R_2$$

Shunt at node A

$$Z_p = Z_1 \| Z_f = \frac{Z_f \cdot Z_1}{Z_f + Z_1}$$

3pt

$$V_A = V_i \frac{Z_p}{R_1 + Z_p}$$

$$V_B = V_o = V_A \frac{R_2}{R_2 + Z_2}$$

4pt

$$H(\omega) = \frac{R_2}{R_2 + \frac{1}{j\omega C_2}} \cdot \frac{\frac{1}{j\omega C_1}\left(R_2 + \frac{1}{j\omega C_2}\right)}{\left[\frac{1}{j\omega C_1} + \left(R_2 + \frac{1}{j\omega C_2}\right)\right]\left[R_1 + \left(\frac{1}{j\omega C_1} \| R_2 + \frac{1}{j\omega C_2}\right)\right]}$$

7

$$H(\omega) = \frac{j\omega C_2 R_2}{1 + j\omega(R_1 C_1 + R_1 C_2 + R_2 C_2) - \omega^2 C_1 C_2 R_1 R_2}$$

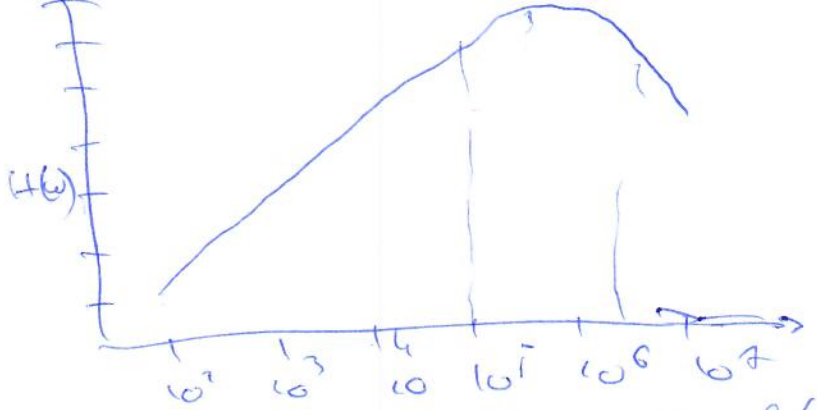
5) Magnitude and phase plots

17 pt
3 digits

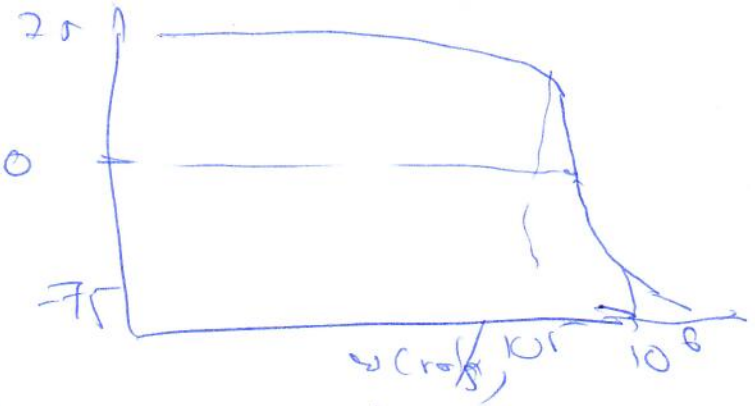
$$\omega_{C1} = \frac{1}{1 \cdot 10^{-6}} = 10^6 \text{ rad/s}$$

$$\omega_{C2} = \frac{1}{R_2 C_2} = 2.5 \cdot 10^5 \text{ rad/s}$$

6 pt



4pt



4pt

c) cut off and pole/slope 1 pt

first pole $\omega_c = \frac{1}{R_1 C_2} = 2.5 \cdot 10^5$ rad/s 3 trying

$\omega < \omega_c$ $|H(\omega)| \approx$ cost

$\omega >> \omega_c$ but less than 10^6 rad/s

slope -20 dB/dec 6pt

$\omega >> 10^6$ -40 dB/dec 7pt

7° $Z_L = j\omega L$ $Z_C = \frac{1}{j\omega C}$ $Z_R = R$ 3pt

2 integrators $Z_{11} = Z_2 \parallel Z_C = \frac{R}{1 + j\omega RC}$

Voltage divider 3pt

$V_o = V_i \frac{Z_{11}}{Z_{11} + Z_L} \Rightarrow |H(\omega)| < \frac{Z_{11}}{Z_L + Z_{11}}$

$H(\omega) = \frac{R}{R + j\omega L - \omega^2 RLC}$ 7pt

ii freq at which $H(\omega)$ is purely real

5 pt

$12 \quad j\omega L - \omega^2 LRC$

imaginary part

16 pt

3 + 3 + 2

zero @ $\omega = 0$ because $L > 0$

8 pt

iii)

$Z_j \to j\infty$ open $\quad H(\omega) = 0$

$Z_c \to 0$ short

4 pt

17 pt

straight line at $\omega = 0$

$Z_c = j\omega L \to 0$ short

$Z_c \to \infty$ open 4 pt

$$H(0) = \frac{R}{R} = 1$$

This filter rejects high freq 6 pt

and passes DC Low pass