

ECE 3337: Test #1

Spring 2024

Last name of student <i>POSTED SOLUTIONS</i>	Student ID number _____
First name of student _____	Email address _____

Do This First:

1. Make sure that you have all the pages of the exam.
2. Fill in the above information.
3. Turn off and put away all electronic gadgets (laptop, tablet, smartphone, calculator, smart watch, ...)

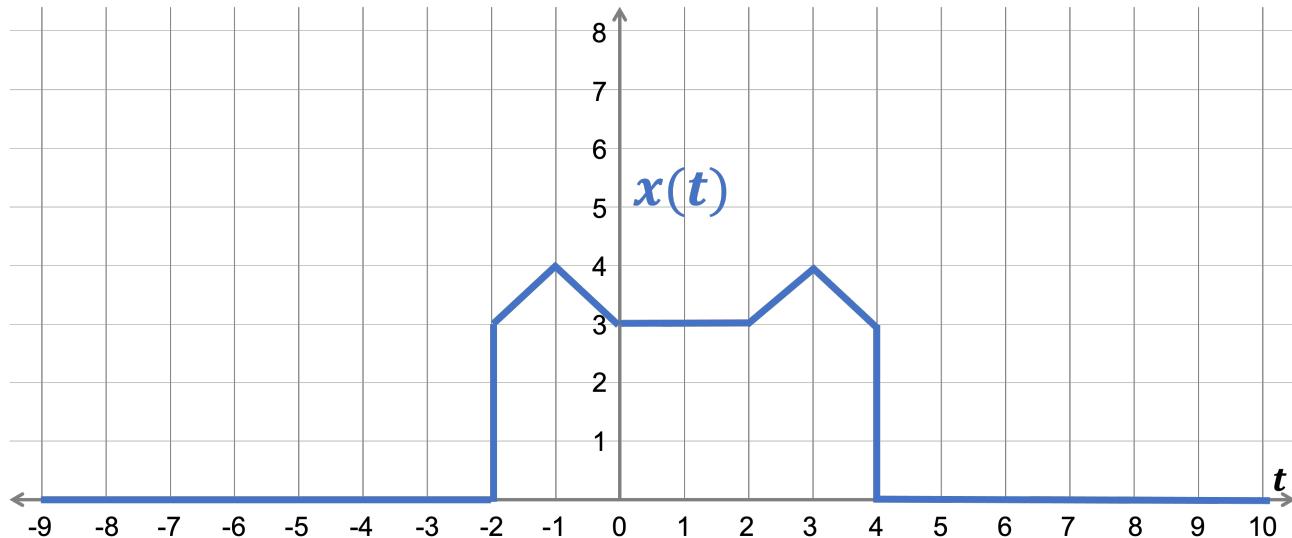
Exam Rules:

1. You are allowed to bring one 8.5" x 11" handwritten crib sheet (double sided).
2. Indicate your answers in the spaces provided. Use the back of the page to carry out calculations.

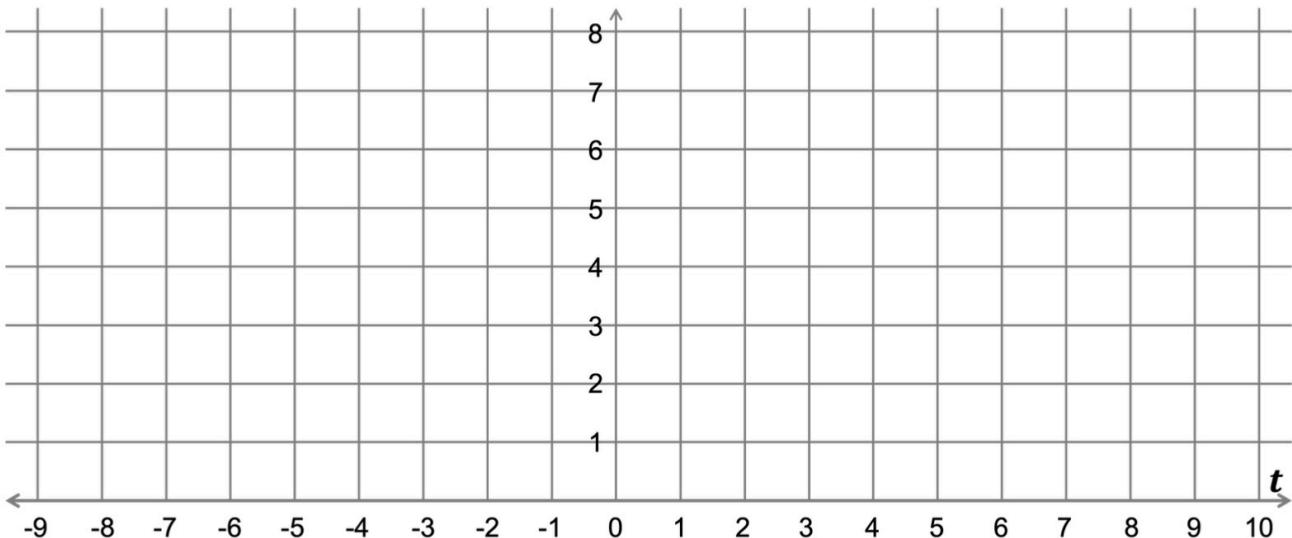
Q1	Q2	Q3	Q4	Q5	Q6	Total
15	15	20	15	15	20	100
Waveforms & Transformations	Signal Properties	System Properties	Convolution Integral	System Response	Frequency Response	_____
_____	_____	_____	_____	_____	_____	_____

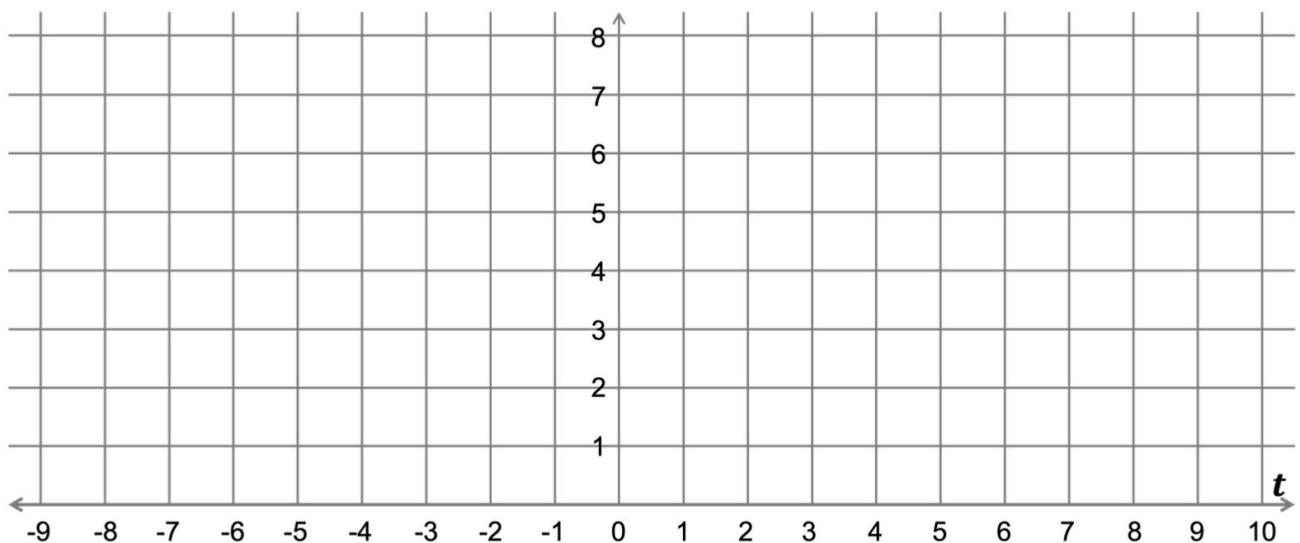
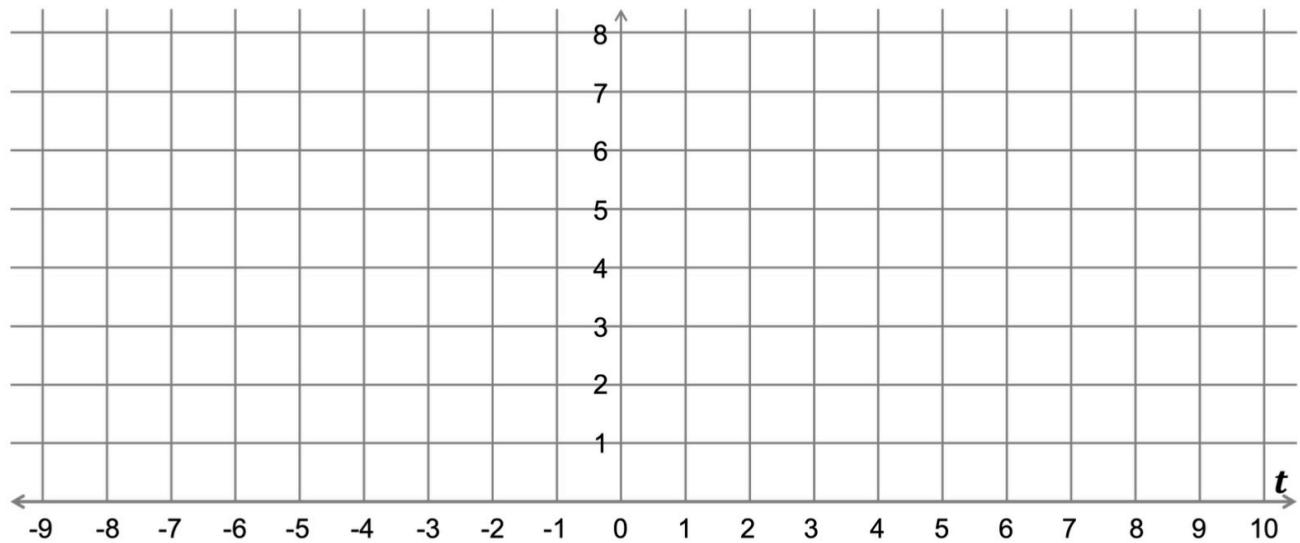
1. (15 points) Waveforms & Signal Transformations

a. (5 points) Write down a formula in terms of steps and ramps describing the signal sketched below.



b. (10 points) Plot $2x\left(\frac{t+3}{2}\right)$ on the graph below. Your final answer should be sketched below.





3. (20 points) System Properties

a. (5 points) Is the system $y(t) = x(t) \sin^2(3t)$ linear? Explain your reasons.

b. (5 points) Is the system $y(t) = \frac{d}{dt}(x(t - 1))$ time invariant? Explain your reasons.

c. (5 points) Is the system $y(t) = \frac{e^{-t^2}}{t^2} x(t + 1)$ BIBO stable? Explain your reasoning.

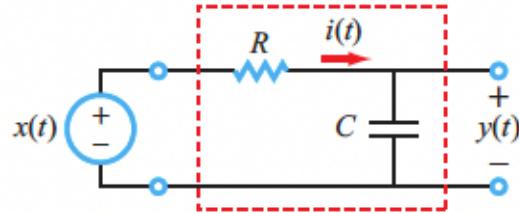
d. (5 points) Is the system defined by $y(t) = 2x(t - 1) + x(t)$ causal? Explain your answer.

4. (15 points) Convolution Integral

a. (6 points) Calculate the output $y(t)$ of an LTI system whose impulse response is $h(t)$ when it is provided the input $x(t)$, where:

$$x(t) = u(t - 2)$$
$$h(t) = e^{1-t}u(t)$$

b. (5 points) Calculate the response of the following circuit with $RC = 1$ for the input $x(t) = (t - 1)u(t - 1)$.



The impulse response is given by $h(t) = \frac{1}{RC} e^{-\frac{t}{RC}} u(t)$, and the initial conditions are zero.

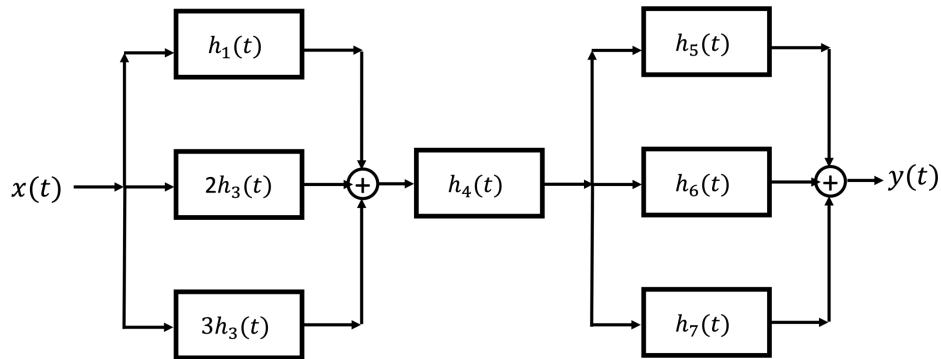
c. (4 points) Compute the convolution $y(t) = u(t - 1) * (\delta(t) + 2u(t) + 4r(t - 1))$ without computing any integrals

5. (15 points) System Response

a. (10 points) Find the step response $y_{step}(t)$ of the system given by the differential equation

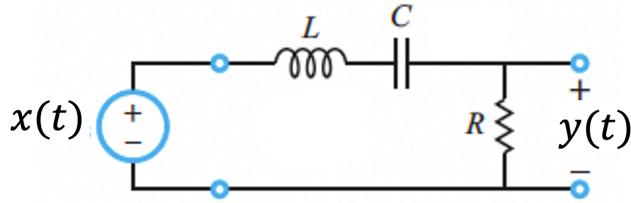
$$2 \frac{dy(t)}{dt} + 4y(t) = 2 \frac{dx(t)}{dt} + 3x(t) . \text{ Assume that } y(0^-) = 2.$$

b. (5 points) Find the overall impulse response of the following system if each of the individual systems $h_1(t) \dots h_7(t)$ are LTI.



6. (20 points) System Frequency Response

Consider the following system with $L = 1H$, $C = 1F$:



- a. (5 points) Show that the following linear differential equation describes the above system.

$$\frac{d^2y(t)}{dt^2} + \frac{R}{L} \frac{dy(t)}{dt} + \frac{1}{LC} y(t) = \frac{1}{LC} x(t)$$

- b. (5 points) Calculate the frequency response function $\hat{H}(\omega)$ for this system.

- c. (2 points) For what values of R is this system's response oscillatory?

- d. (2 points) For what values of R is this system critically damped?
- e. (2 points) For what values of R is this system's response stable?
- f. (4 points) The following input-output pair is observed for a different linear time-invariant system:

$$x(t) = 5 + \sin(2t) + \cos(3t)$$
$$y(t) = 10 + \cos(2t) + \sin(3t - 30^\circ)$$

Calculate the output for the new input $x_2(t) = 1 + \cos(2t) + \sin(3t)$