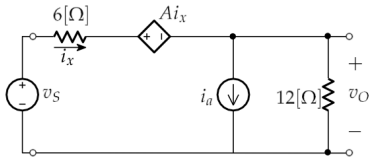
Problem 1

Use superposition to solve this problem. Choose values for i_a and A so that

$$v_0 = 1/2\, v_S - 3[\text{V}]$$

Hint: first find A by using only the voltage source, and then find the value of i_a by using only the current source.



Ans: A = 6Ω; i_a=0.5A

With V_s: + − 6Ω Ai_x i_x KVL 1 i_x 1 V 12Ω V_0' + − $V_0' = 12 i_x$ $i_x = \frac{V_0'}{12}$

KVL: $-V_s + 6i_x + Ai_x + V_0' = 0$

$V_0' = V_s - i_x(6+A)$

$V_0' = V_s - \frac{V_0'}{12}(6+A)$

$V_0' + \frac{V_0'}{12}(6+A) = V_s$

$V_0'\left(1 + \frac{6+A}{12}\right) = V_s$

$V_0'\left(\frac{12+6+A}{12}\right) = V_s$

$V_0' = V_s\left(\frac{12}{18+A}\right)$

From eq.: $\frac{1}{2}V_s = V_s\left(\frac{12}{18+A}\right)$

$18 + A = 24$

$A = 6\,\Omega$

With i_a: V_o Ai_x 6Ω i_x KVL i_a 12Ω V_0'' +

KVL: $6i_x + Ai_x + V_0 = 0$

$V_0'' = -6i_x - Ai_x$

$V_0'' = -i_x(6+A)$

$V_0'' = -i_x(12)$

KCL @ V_0: $-i_x + i_a + \frac{V_0''}{12} = 0$

$i_x = i_a + \frac{V_0''}{12}$

$V_0'' = -12\left(i_a + \frac{V_0''}{12}\right)$

$V_0'' = -12 i_a - V_0''$

$2V_0'' = -12 i_a$

$V_0'' = -6 i_a$

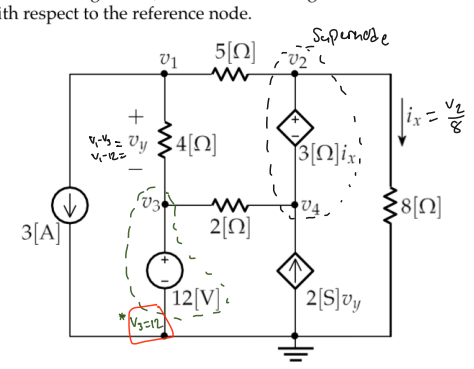
from eq.: $-3 = -6 i_a$

$\frac{1}{2} = i_a$

$0.5\,[\text{A}] = i_a$

Problem 2

For the following circuit, use the node voltage method to determine the values of v_1, v_2, v_3, and v_4 with respect to the reference node.



Ans: v_1=23.5V, v_2=52.9V, v_3=12V, and v_4=33.1V

@ V_1: $\left(3 + \frac{V_1 - V_3}{4} + \frac{V_1 - V_2}{5} = 0\right)^{20}$

$60 + 5V_1 - 5(V_3) + 4V_1 - 4V_2 = 0$

$9V_1 - 4V_2 - 5(12) + 60 = 0$

$9V_1 - 4V_2 = 0$

$V_1 = \frac{4}{9}V_2$

$V_1 = \frac{4}{9}\left(\frac{2592}{49}\right)$

$V_1 = \frac{1152}{49} \approx 23.5$ [V]

Supernode eq:

$V_2 - V_4 = 3i_x = 3\left(\frac{v_2}{8}\right)$

$V_2 - V_4 = \frac{3}{8}V_2$

$\frac{5}{8}V_2 = V_4$

$\frac{5}{8}\left[\frac{2592}{49}\right] = V_4$

$\frac{1620}{49} \approx 33.1$ [V] $= V_4$

Supernode KCL:

$\left(\frac{V_2 - V_1}{5} + \frac{V_2}{8} + \frac{V_4 - V_3}{2} - 2(V_1 - V_3) = 0\right)^{40}$

$8V_2 - 8V_1 + 5V_2 + 20V_4 - 20V_3 - 80(V_1 - V_3) = 0$

$13V_2 - 8V_1 + 20V_4 - 20V_3 - 80V_1 + 80V_3 = 0$

$-88V_1 + 13V_2 + 20V_4 + 720 = 0$

$-88\left(\frac{4}{9}V_2\right) + 13V_2 + 20\left(\frac{5}{8}V_2\right) + 720 = 0$

$-\frac{352}{9}V_2 + 13V_2 + \frac{25}{2}V_2 + 720 = 0$

$V_2\left(-\frac{245}{18}\right) = -720$

$V_2 = \frac{2592}{49} \approx 52.9$ [V]

Problem 3

For the following circuit, you measure an open circuit potential $v_{oc} = 180/11$ [V] between nodes a and b when you remove the load resistor. When you attach a load resistor $R_L = 40[\Omega]$, you

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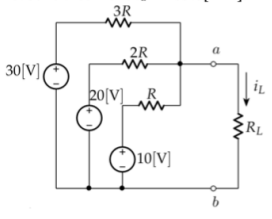
measure a current $i_L = 163.6$[mA]. Find R.



a

$$i_L = \frac{V_{TH}}{R_{TH}+R_L}$$

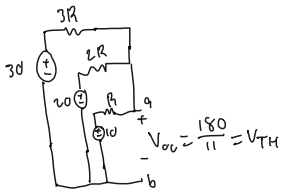
$$R_{TH}+R_L = \frac{V_{TH}}{i_L}$$

$$R_{TH} = \frac{V_{TH}}{i_L} - R_L$$

$$R_{TH} = \frac{\frac{180}{11}}{163.6m} - 40$$

$$R_{TH} = 60.02 \approx 60$$

Ans: 110 Ω



$$V_{oc} = \frac{180}{11} = V_{TH}$$

R_{TH}:

$$3R \,||\, 2R \,||\, R = \frac{1}{\left(\frac{1}{3R}+\frac{1}{2R}+\frac{1}{R}\right)}$$

$$\frac{1}{R}\left(\frac{1}{3}+\frac{1}{2}+1\right)$$

$$\left(\frac{2}{6}+\frac{3}{6}+\frac{6}{6}\right)$$

$$\frac{1}{R}\left(\frac{11}{6}\right)$$

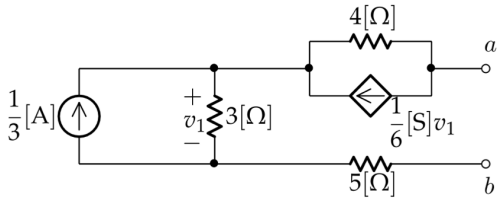
$$= \frac{1}{\frac{11}{6R}} = \frac{6R}{11}$$

$$R_{TH} = \frac{6R}{11} = 60$$

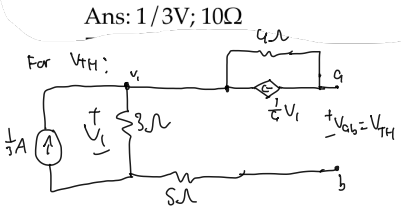
$$R = 110[\Omega]$$

Problem 4

Find the Thevenin equivalent for the following circuit between nodes *a* and *b*.



Ans: 1/3V; 10Ω



Node Volt:

@ a: $\frac{V_a - V_1}{4} + \frac{1}{6}V_1 = 0$

$\frac{V_a}{4} = \frac{V_1}{4} - \frac{V_1}{6}$

$\frac{V_a}{4} = \frac{V_1}{12}$

$V_a = \frac{1}{3}V_1$

$V_a = \frac{1}{3}(1) = \frac{1}{3}$

$\therefore V_{ab} = V_{TH} = \frac{1}{3}(V)$

KCL @ V_1:

$\left(-\frac{1}{3} + \frac{V_1}{3} - \frac{1}{6}V_1 + \frac{V_1 - V_a}{4} = 0\right)12$

$-4 + 4V_1 - 2V_1 + 3V_1 - 3V_a = 0$

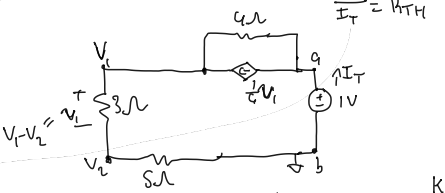
$5V_1 - 3(\frac{1}{3}V_1) = 4$

$4V_1 = 4$

$V_1 = 1$

For R_{TH}

$\frac{1}{I_T} = R_{TH}$



KCL @ a: $-I_T + \frac{V_a - V_1}{4} + \frac{1}{6}V_1 = 0$

$I_T = \frac{V_a - V_1}{4} + \frac{1}{6}V_1$

$I_T = \frac{1 - \frac{4}{5}}{4} + \frac{1}{6}(\frac{4}{5} - \frac{1}{2})$

$I_T = \frac{1}{20} + \frac{1}{8}(\frac{3}{10})$

$I_T = \frac{1}{20} + \frac{1}{20} = \frac{2}{20}$

$I_T = \frac{1}{10}$

$\therefore R_{TH} = \frac{1}{\frac{1}{10}} = 10\Omega$

@ V_2: $\left(\frac{V_2 - V_1}{3} + \frac{V_2}{5} = 0\right)15$

$5V_2 - 5V_1 + 3V_2 = 0$

$8V_2 = 5V_1$

$V_2 = \frac{5}{8}V_1$

$V_2 = \frac{5}{8}(\frac{4}{5})$

$V_2 = \frac{1}{2}$

KCL @ V_1: $\left(\frac{V_1 - V_a}{4} + \frac{V_1 - V_2}{3} - \frac{1}{6}V_1 = 0\right)12$

$3V_1 - 3V_a + 4V_1 - 4V_2 - 2V_1 = 0$

$7V_1 - 3 - 4V_2 - 2(V_1 - V_2) = 0$

$5V_1 - 2V_2 = 3$

$5V_1 = 3 + 2V_2$

$5V_1 = 3 + 2(\frac{5}{8}V_1)$

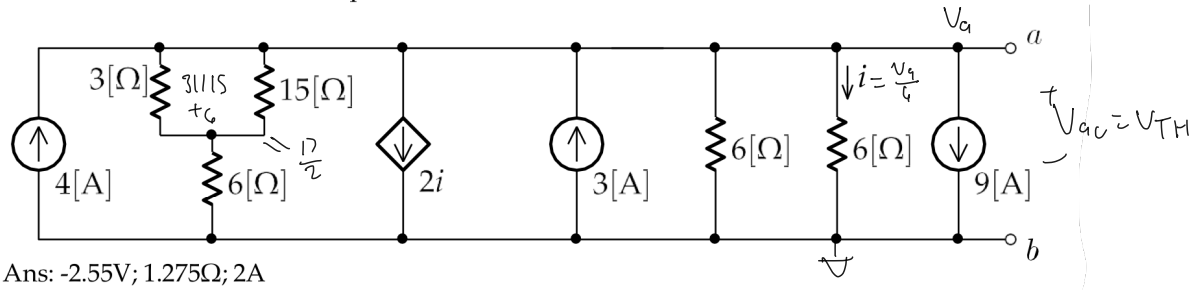
$5V_1 = 3 + \frac{5}{4}V_1$

$\frac{15}{4}V_1 = 3$

$V_1 = \frac{4}{5}$

Problem 5

Find the Thevenin or Norton equivalent for the circuit below between nodes *a* and *b*.



Ans: -2.55V; 1.275Ω; 2A

For V_{TH}:

KCL @ a: $-4 + \frac{V_a}{8.5} + 2i - 3 + \frac{V_a}{6} + \frac{V_a}{6} + 9 = 0$

$2 + \frac{V_a}{8.5} + 2\left(\frac{V_a}{6}\right) + \frac{2}{6}V_a = 0$

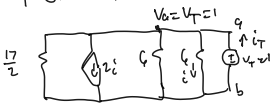
$2 + \frac{V_a}{8.5} + \frac{4}{6}V_a = 0$

$\frac{40}{51}V_a = -2$

$V_a = -2.55[V]$

$\therefore V_{TH} = -2.55[V]$

For R_{TH}



KCL @ a: $\frac{V_a}{\frac{17}{2}} + 2i + \frac{V_a}{6} + \frac{V_a}{6} - i_T = 0$

$\frac{V_a}{8.5} + 2\left(\frac{V_a}{6}\right) + \frac{2}{6}V_a = i_T$

$\frac{4}{6}V_a + \frac{V_a}{8.5} = i_T$

$\frac{40}{51}V_a = i_T$

$\frac{40}{51} = i_T \quad \therefore R_{TH} = \frac{V_T}{I_N} = \frac{1}{\frac{40}{51}}$

$R_{TH} = 1.275[\Omega]$

$\therefore I_N = \frac{V_{TH}}{R_{TH}} = 2[A]$

Problem 6

Find the Thévenin equivalent circuit at terminals (*a*,*b*) in the circuit below.

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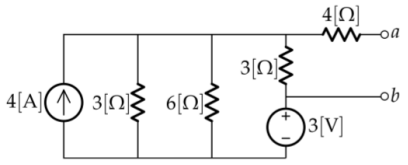
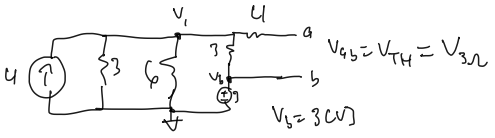


Ans: 3V; 26/5 Ω



Node Volt

@ V_1 : $-4+\frac{V_1}{3}+\frac{V_1}{6}+\frac{V_1-V_b}{3}=0$

$V_1\left(\frac{1}{3}+\frac{1}{6}+\frac{1}{3}\right)-\frac{V_b}{3}=4$

$V_1\left(\frac{5}{6}\right)=5$

$V_1=6[V]$ ∴ $V_{ab}=V_{TH}=V_{3\Omega}=V_1-V_b$

$V_{TH}=6-3=3[V]$

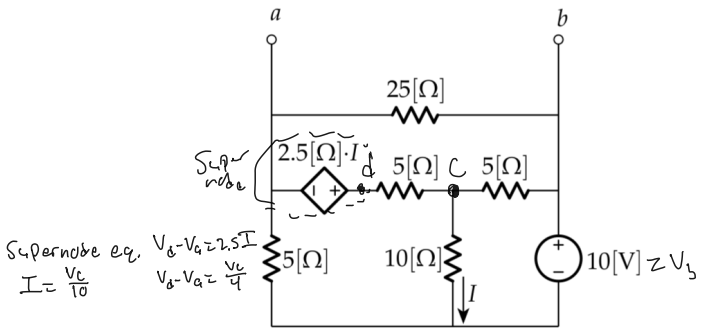
R_{TH}:

$(3\|6\|3)+4=\frac{26}{5}$

$R_{TH}=\frac{26}{5}[\Omega]$

Problem 7

Find the Thévenin equivalent circuit at terminals (a,b) for the circuit below



Ans: -50/7V; =75/28Ω

V_{TH}; Node Volt

@ Supernode: $\left(\frac{V_a - V_b}{25} + \frac{V_a}{5} + \frac{V_d - V_c}{5} = 0\right)25$

$V_a - V_b + 5V_a + 5V_d - 5V_c = 0$

$6V_a - 10 + 5V_d - 5V_c = 0$

$6\left(V_d - \frac{V_c}{4}\right) + 5V_d - 5V_c = 10$

$11V_d - \frac{13}{2}V_c = 10$

@ C: $\left(\frac{V_c - V_d}{5} + \frac{V_c}{10} + \frac{V_c - V_b}{5} = 0\right)10$

$2V_c - 2V_d + V_c + 2V_c - 2V_b = 0$

$5V_c - 2V_d = 20$

$11V_d - \frac{13}{2}V_c = 10 \;\cdot 2 \Rightarrow 22V_d - 13V_c = 20$

$+(-2)V_d + 5V_c = 20 \;\cdot 11 \Rightarrow +(-22V_d + 55V_c = 220)$

$42V_c = 240$

$V_c = \frac{40}{7}$

$\therefore V_d = \frac{5(40/7) - 20}{2}$

$V_d = \frac{30}{7}$

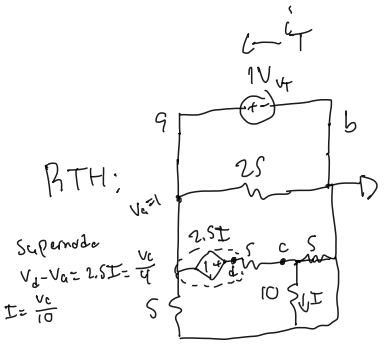
$\therefore V_a = V_d - \frac{V_c}{4}$

$V_a = \frac{30}{7} - \frac{40/7}{4}$

$V_a = \frac{20}{7}$

$V_{TH} = V_{ab} = \frac{20}{7} - 10$

$V_{TH} = -\frac{50}{7}$



R_{TH}:

@ V_a: $\left(-i_T + \frac{V_a}{25} + \frac{V_a}{5} + \frac{V_d - V_c}{5} = 0\right)25$

$V_a + 5V_a + 5V_d - 5V_c = 25i_T$

$6V_a + 5\left(\frac{10}{9}V_a\right) - 5\left(\frac{4}{9}V_a\right) = 25i_T$

$\frac{28}{9}V_a = 25i_T$

$\frac{28}{225}V_a = i_T$

$\frac{28}{225}(A) = i_T \;\therefore\; R_{TH} = \frac{V_a}{i_T} = \frac{1}{\frac{28}{225}}$

$R_{TH} = \frac{225}{28}(\Omega)$

@ V_c: $\left(\frac{V_c - V_d}{5} + \frac{V_c}{10} + \frac{V_c}{5} = 0\right)10$

$2V_c - 2V_d + V_c + 2V_c = 0$

$5V_c = 2V_d$

$V_c = \frac{2}{5}V_d$

Supernode: $V_d = V_a + \frac{\left(\frac{2}{5}V_d\right)}{4}$

$V_d = V_a + \frac{V_d}{10}$

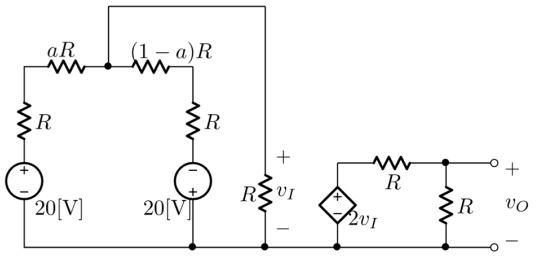
$\frac{9}{10}V_d = V_a$

$V_d = \frac{10}{9}V_a$ or $\frac{5}{2}V_c = \frac{10}{9}V_a$

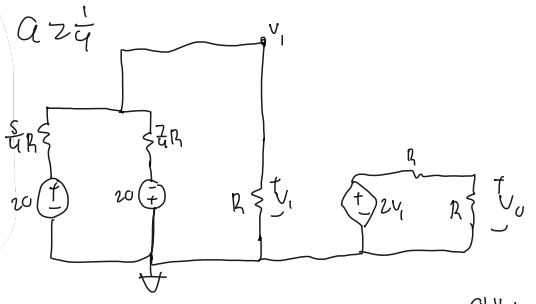
$V_c = \frac{4}{9}V_a$

Problem 8

For the following circuit determine the value of v_o for a=1/2 and a=1/4



$a = \frac{1}{4}$



Node Volt @ V_1: $\frac{V_1-20}{\frac{5}{4}R} + \frac{V_1+20}{\frac{7}{4}R} + \frac{V_1}{R} = 0$

$V_1\left(\frac{4}{5R} + \frac{4}{7R} + \frac{1}{R}\right) - \frac{80}{5R} + \frac{80}{7R} = 0$

$V_1\left(\frac{83}{35R}\right) - \frac{32}{7R} = 0$

$V_1\left(\frac{83}{35R}\right) = \frac{32}{7R}$

$V_1 = \frac{160}{83}$

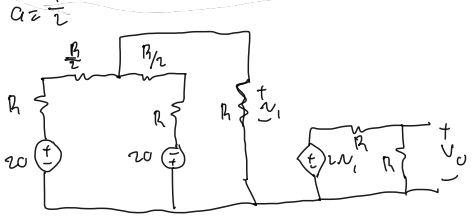
@ V_o: $\frac{V_o-2V_1}{R} + \frac{V_o}{R} = 0$

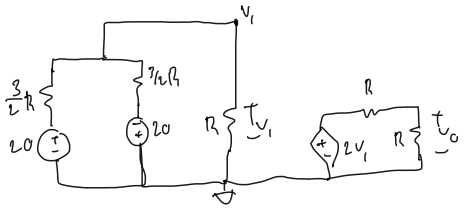
$V_o\left(\frac{1}{R}+\frac{1}{R}\right) = \frac{2(\frac{160}{83})}{R}$

$V_o\left(\frac{2}{R}\right) = \frac{320}{83R}$

$V_o = \frac{160}{83} \approx 1.93[V]$

$a = \frac{1}{2}$





Node V_1: $\frac{V_1-20}{\frac{3}{2}R} + \frac{V_1+20}{\frac{7}{2}R} + \frac{V_1}{R} = 0$

$V_1\left(\frac{2}{3R} + \frac{2}{7R} + \frac{1}{R}\right) - \frac{40}{3R} + \frac{40}{7R} = 0$

$V_1\left(\frac{4}{3R} + \frac{1}{R}\right) = 0$

$V_1 = 0$

@ V_o: $\frac{V_o-2V_1}{R} + \frac{V_o}{R} = 0$ ($2V_1 = 0$)

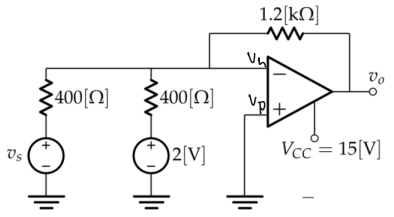
$\frac{V_o}{R} + \frac{V_o}{R} = 0$

$V_o = 0$

Problem 9

a. Determine the output voltage (v_o) in the circuit below in terms of v_s

b. Specify the linear range for v_s



$V_n = V_p = 0$

linear range of V_o:

$-15 \le V_o \le 15$

$\therefore$

$15 \ge -3V_s - 6$ or $-15 \ge -3V_s - 6$

$21 \ge -3V_s$ $\quad -9 \ge -3V_s$

$-7 \ge V_s$ $\quad 3 \ge V_s$

$-7 \le V_s \le 3$ (V)

Ans: $-3v_S$-6[V] for -7[V]< v_S <3[V]

KCL at V_n: $\left(\frac{V_n - V_s}{400} + \frac{V_n - 2}{400} + \frac{V_n - V_o}{1.2K} = 0\right) 1.2K$

$3V_n - 3V_s + 3V_n - 6 + V_n - V_o = 0$

$V_o = 7V_n - 3V_s - 6$

$V_o = -3V_s - 6$