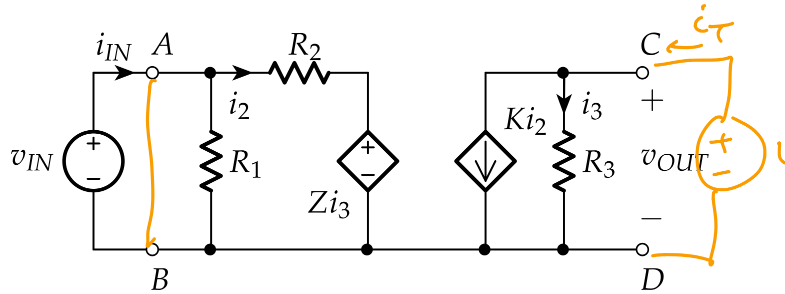
1. (15 points) For the following circuit, find the voltage gain, $\frac{v_{OUT}}{v_{IN}}$, and the input and output resistances.



$v_{OUT} = i_3 R_3 \qquad v_{IN} = i_2 R_2 + Z i_3 \qquad i_3 = -K i_2$

$$\frac{v_{OUT}}{v_{IN}} = \frac{i_3 R_3}{i_2 R_2 + Z i_3} = \frac{-K i_2 R_3}{i_2 R_2 - K i_2 Z} = \frac{-K R_3}{R_2 - KZ} = \boxed{\frac{K R_3}{KZ - R_2}}$$

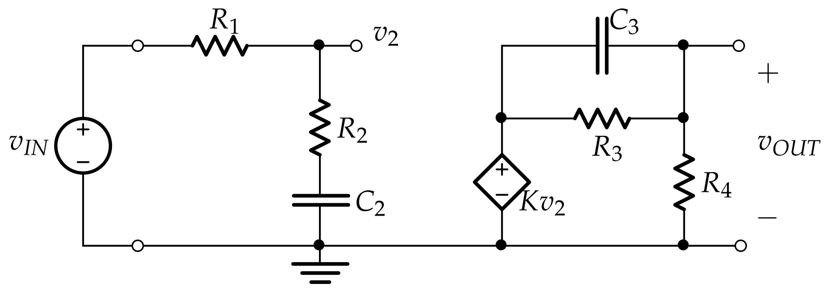
$$R_{in} = \frac{v_{IN}}{i_{IN}} = \frac{v_{IN}}{\frac{v_{IN}}{R_1} + i_2} \qquad v_{IN} = i_2 R_2 - KZ i_2 \Rightarrow i_2 = \frac{v_{IN}}{R_2 - KZ}$$

$$= \frac{v_{IN}}{\frac{v_{IN}}{R_1} + \frac{v_{IN}}{R_2 - KZ}} = R_1 \| (R_2 - KZ) = \boxed{\frac{R_1(R_2 - KZ)}{R_1 + R_2 - KZ}}$$

$$R_{OUT} = \frac{v_T}{i_T} = \frac{v_T}{K i_2 + i_3} \qquad -i_2 R_2 = Z i_3 \qquad v_T = i_3 R_3$$

$$= \frac{i_3 R_3}{K i_2 + i_3} = \frac{i_3 R_3}{K\left(-\frac{Z i_3}{R_2}\right) + i_3} = \boxed{\frac{R_2 R_3}{R_2 - KZ}}$$

2. (10 points) For the following circuit, answer the three questions below.



(a) What is the gain at low frequencies?

C_2 and C_3 are open, so $V_2 = V_{IN}$ and $V_{OUT} = K V_2 \frac{R_4}{R_3+R_4}$

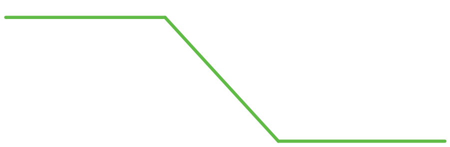
(a) $K \frac{R_4}{R_3+R_4}$

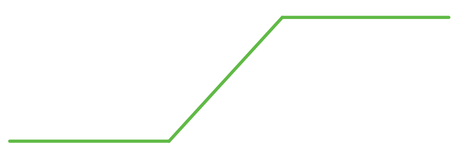
(b) What is the gain at high frequencies?

C_2 and C_3 are shorts, so $V_2 = \frac{R_2}{R_1+R_2} V_{IN}$ and $V_{OUT} = K V_2$

(b) $K \frac{R_2}{R_1+R_2}$

(c) Sketch of the *shape* of the magnitude plot for each of the two stages of the circuit: for $\frac{\overline{V}_2}{\overline{V}_{in}}$ and for $\frac{\overline{V}_{out}}{\overline{V}_2}$ (You do not need to label the x and y axes).

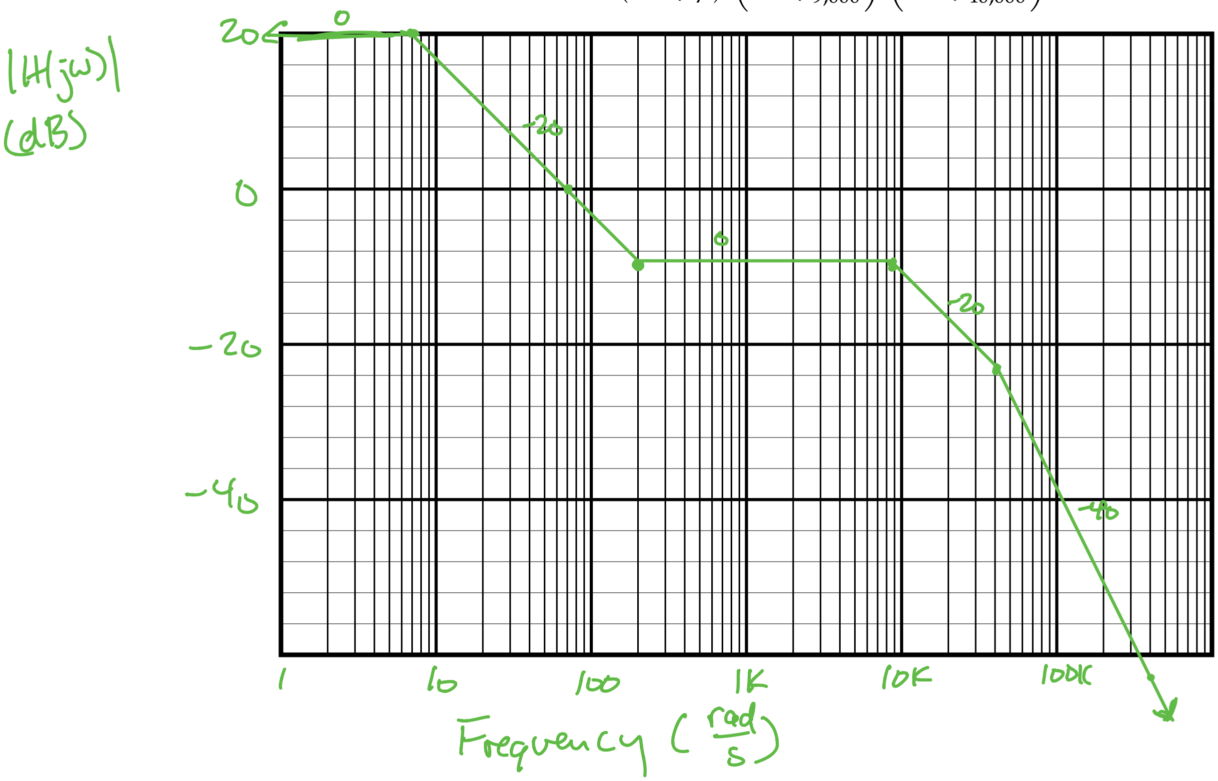




3. (15 points) Bode Plots

(a) Plot a straight-line approximation of the Bode plot on the graph paper provided for the *magnitude only* for the following transfer function (the unit for the values in the denominator of the imaginary terms is rad/sec).

$$H(j\omega) = -10\frac{\left(1+j\frac{\omega}{200}\right)}{\left(1+j\frac{\omega}{7}\right)\left(1+j\frac{\omega}{9{,}000}\right)\left(1+j\frac{\omega}{40{,}000}\right)}.$$



(b) What is the starting phase (i.e., the phase at low frequencies)?

(b) 180° or -180°

(c) What is the ending phase (i.e, the phase at high frequencies)?

(c) 0°

(-360° is OK)

Version B

(b) 0° (or 360°)

(c) -180° (or 180°)

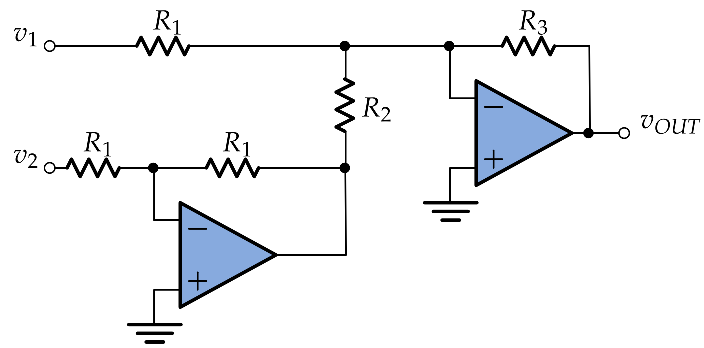
4. (15 points) Difference Amplifier

   (a) Write an expression for the common mode gain (A_{cm}).

   (b) Write an expression for the difference gain (A_d). (*Hint:* if you are using superposition, you may make an important assumption regarding R_1 and R_2.)

   (c) For $R_1 = 3\,\text{k}\Omega$, $R_2 = 2.99\,\text{k}\Omega$, and $R_3 = 30\,\text{k}\Omega$, approximate the CMRR? (*Hint:* you can use the difference gain calculated from Part (b))

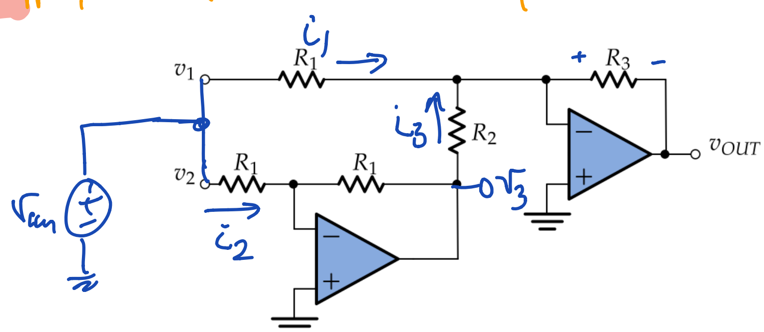
   (d) What is the input resistance seen at each of the input terminals (i.e., at v_1 and v_2)?



There are two ways to solve this problem.

Approach (1) Apply V_d & V_{cm} directly:

(a)



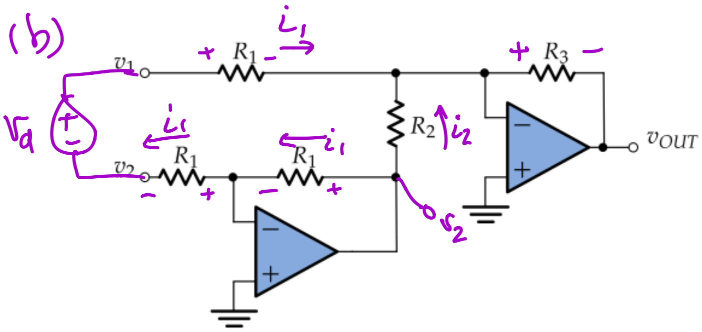
$i_2 = \frac{V_{cm}}{R_1}$ $\quad i_1 = \frac{V_{cm}}{R_1}$ $\quad i_3 = \frac{V_3}{R_2}$ $\quad V_{OUT} = -(i_1 + i_3)R_3$

$V_3 = -i_2 R_1 = -V_{cm}$

so $V_{OUT} = -\left(\frac{V_{cm}}{R_1} + \frac{V_3}{R_2}\right)R_3 = -V_{cm}R_3\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$A_{cm} = \frac{V_{OUT}}{V_{cm}} = -R_3\left(\frac{R_2 - R_1}{R_1 R_2}\right) = \boxed{R_3\left(\frac{R_1 - R_2}{R_1 R_2}\right)}$$

(b)



$$v_{OUT} = -(i_1 + i_2)R_3$$

$$i_1 = \frac{v_d}{2R_1}$$

$$v_2 = i_1 R_1 = \frac{v_d}{2}$$

$$i_2 = \frac{v_2}{R_2} = \frac{v_d}{2R_2}$$

$$v_{OUT} = -\left(\frac{v_d}{2R_1} + \frac{v_d}{2R_2}\right)R_3$$

$$\boxed{A_d = \frac{v_{OUT}}{v_d} = -\left(\frac{R_1 + R_2}{2R_1R_2}\right)R_3}$$

(c) $$CMRR = 20\log_{10}\frac{|A_d|}{|A_{cm}|} = 20\log_{10}\frac{\left|-\frac{R_1+R_2}{2\cancel{R_1R_2}}\cdot\cancel{R_3}\right|}{\left|\cancel{R_3}\left(\frac{R_1-R_2}{\cancel{R_1R_2}}\right)\right|} = 20\log_{10}\left|\frac{-(R_1+R_2)}{2(R_1-R_2)}\right|$$

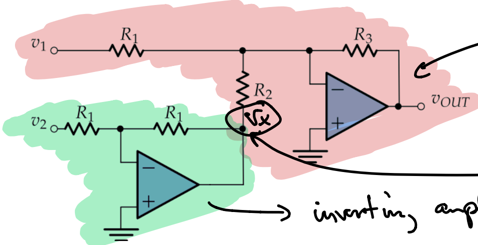
$$= 20\log_{10}\left|\frac{3k\Omega + 2.99k\Omega}{2(0.01k\Omega)}\right| \boxed{= 49.5\,dB}$$

$\boxed{35.8\,dB}$ ← version B

(d) $R_{in} = R_1$ for both v_1 and v_2

Approach (2)

if you recognize the inverting weighted summer and the inverting amplifier,



weighted summer $v_{OUT} = -\frac{R_3}{R_1} v_1 - \frac{R_3}{R_2} v_x$

inverting ampl. $v_x = -\frac{R_1}{R_1} \cdot v_2 = -v_2$

(a) $v_{OUT} = -\frac{R_3}{R_1} \cdot v_1 - \frac{R_3}{R_2} \cdot (-v_2) = -\frac{R_3}{R_1} v_1 + \frac{R_3}{R_2} v_2$

$v_{OUT} = -\frac{R_3}{R_1} v_{cm} + \frac{R_3}{R_2} v_{cm} \Rightarrow \boxed{A_{cm} = \frac{v_{OUT}}{v_{cm}} = R_3\left(\frac{R_1 - R_2}{R_1 R_2}\right)}$

(b) For this to be a difference amplifier, $R_1 = R_2$

So $v_{OUT} = \frac{R_3}{R_1}(v_2 - v_1)$ and $\boxed{A_d = \frac{v_{OUT}}{(v_2 - v_1)} = \frac{R_3}{R_1}}$

$A_d = \frac{30k\Omega}{3k\Omega} = 10$

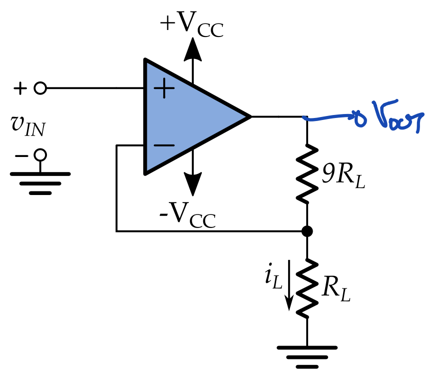
(c) $CMRR = 20\log_{10}\left|\frac{\cancel{\frac{R_3}{R_1}}}{\cancel{R_3}\left(\frac{R_1 - R_2}{\cancel{R_1}R_2}\right)}\right| = 20\log_{10}\left|\frac{R_2}{R_1 - R_2}\right|$

$= 20\log_{10}\left|\frac{2.99k\Omega}{0.01k\Omega}\right| = \boxed{49.5dB}$

version B $= 20\log_{10}\left|\frac{4.92k\Omega}{0.08k\Omega}\right| = 35.8dB$

(d) $R_{in} = R_1$ for both v_1 and v_2

5. (15 points) For the following circuit, the op-amp is powered by a pair of $\pm V_{CC} = \pm 15$ volt supplies (as shown).



(a) Write an expression for i_L in terms of v_{IN} and R_L.

(b) What is the range of v_{IN} that ensures the circuit does not go into voltage saturation?

(c) What range of load resistor values (i.e., R_L) will ensure that the circuit does not go into current saturation for the full input range from above? The maximum current for the op-amp is 15 mA.

(a) $i_L = \frac{V_{IN}}{R_L}$

(b) $V_{OUT} = \left(1 + \frac{9R_L}{R_L}\right) V_{IN} = 10 V_{IN} \Rightarrow$ non-inverting amplifier configuration

since $-15V < V_{OUT} < 15V$

$\boxed{-1.5V < V_{IN} < 1.5V}$

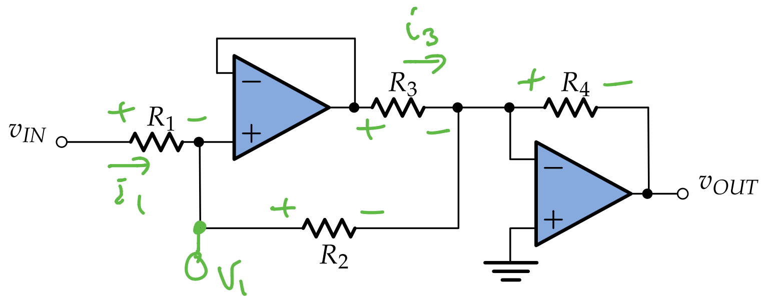
(c) from (a) & (b)

$i_L = \frac{V_{IN}}{R_L} < 15mA$

$R_L > \frac{V_{IN}}{15mA} = \frac{1.5V}{15mA} = 100\Omega$

$\boxed{R_L > 100\Omega}$ $R_L > 50\Omega$ for version B

6. (15 points) For the following circuit, find an expression for the voltage gain, $\frac{v_{OUT}}{v_{IN}}$.



$$v_{OUT} = -(i_1 + i_3) R_4$$

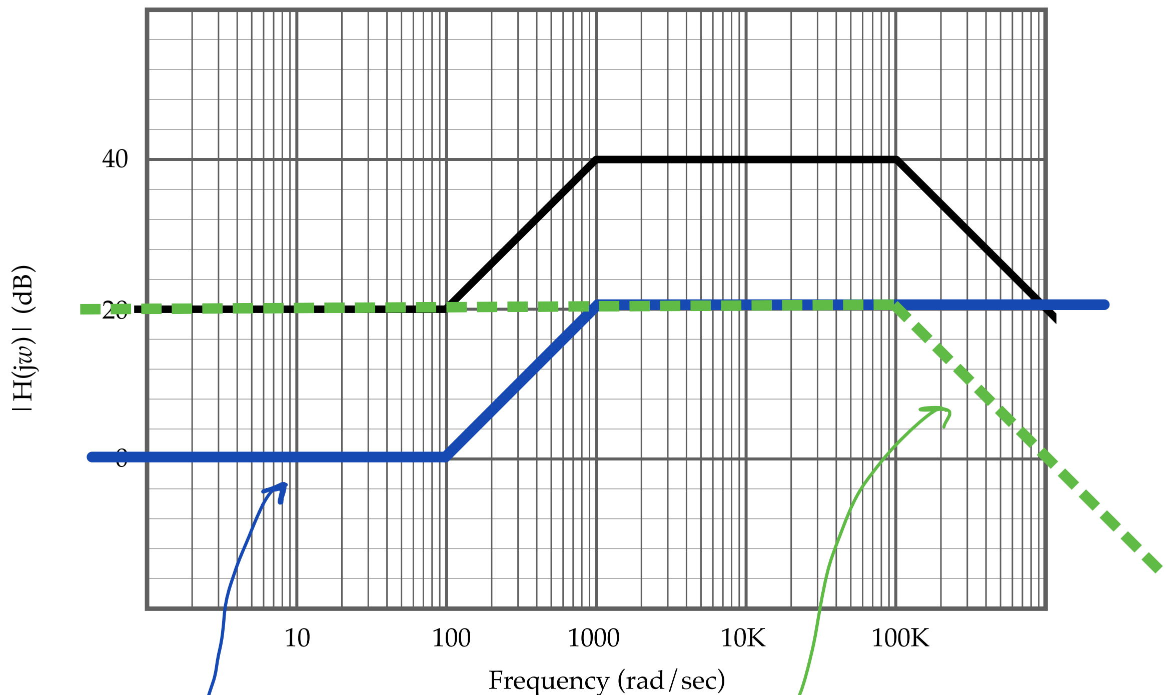
$$i_1 = \frac{v_{IN}}{R_1 + R_2} \qquad v_1 = \frac{R_2}{R_1 + R_2} \cdot v_{IN} \qquad i_3 = \frac{v_1}{R_3}$$

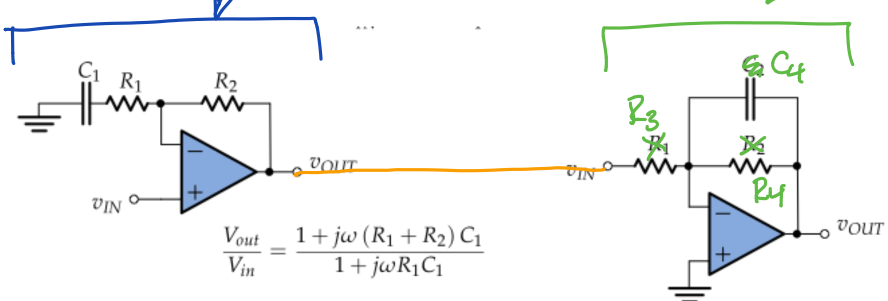
$$v_{OUT} = -\left(\frac{v_{IN}}{R_1 + R_2} + \frac{v_1}{R_3} \right) R_4$$

$$= -\left(\frac{v_{IN}}{R_1 + R_2} + \frac{R_2 v_{IN}}{R_3 (R_1 + R_2)} \right) R_4$$

$$\frac{v_{OUT}}{v_{IN}} = -R_4 \left(\frac{R_3 + R_2}{R_3 (R_1 + R_2)} \right) = -\frac{R_4}{R_3} \left(\frac{R_2 + R_3}{R_1 + R_2} \right)$$

7. (15 points) Design a circuit that has the following straight-line approximation to the Bode plot. Indicate which order the stages should be placed in to ensure that the input resistance does not vary with frequency.





$$\frac{V_{out}}{V_{in}} = \frac{1 + j\omega\,(R_1 + R_2)\,C_1}{1 + j\omega R_1 C_1}$$

$$\frac{V_{out}}{V_{in}} = \frac{-\frac{R_2}{R_1}}{1 + j\omega R_2 C_2}$$

$1 + \frac{R_2}{R_1} = 10$

choose $R_2 = 90\text{k}\Omega$

$R_1 = 10\text{k}\Omega$

$\frac{1}{R_1 C_1} = 1000 \frac{\text{rad}}{\text{s}}$

$C_1 = \frac{1}{10^3 \cdot 10^4} = 10^{-7}\text{F}$

$= 0.1\mu\text{F} = 100\text{nF}$

$\frac{R_4}{R_3} = 10$

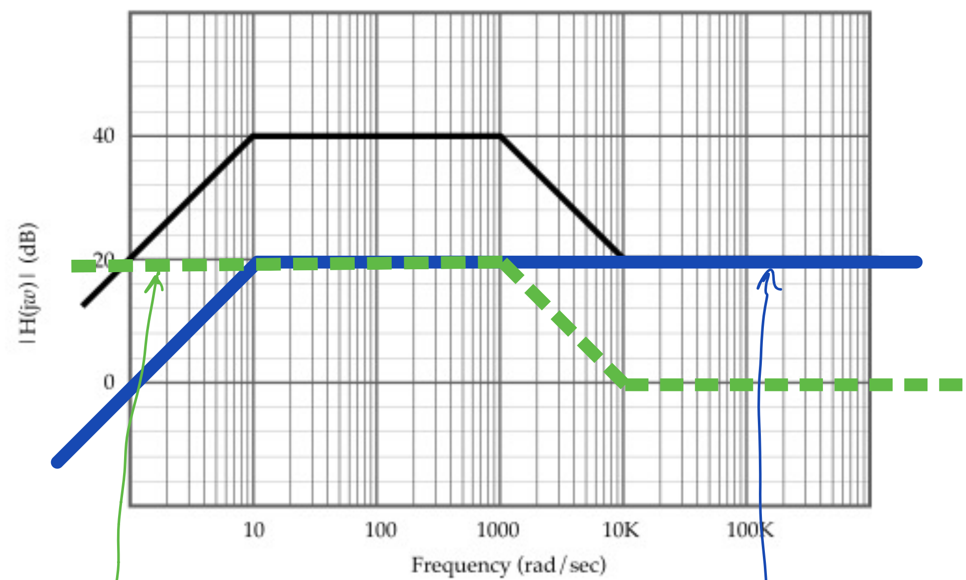
choose

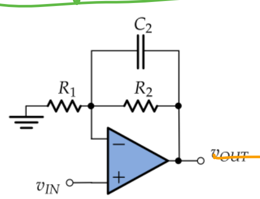
$R_4 = 10\ \text{k}\Omega$

$R_3 = 1\ \text{k}\Omega$

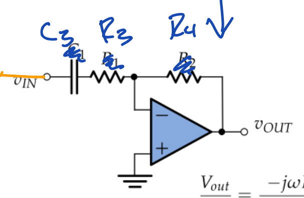
$\frac{1}{R_4 C_3} = 100\ \frac{\text{krad}}{\text{s}}$

$C_3 = \frac{1}{10^5 \cdot 10^4} = 10^{-9}\text{F} = 1\text{nF}$





$$\frac{V_{out}}{V_{in}} = \left[1+\frac{R_2}{R_1}\right]\left[\frac{1+j\omega\left(R_1||R_2\right)C_2}{1+j\omega R_2C_2}\right]$$



$$\frac{V_{out}}{V_{in}} = \frac{-j\omega R_2C_1}{1+j\omega R_1C_1}$$

$$1+\frac{R_2}{R_1} = 10$$

Choose

$R_2 = 90k\Omega$

$R_1 = 10k\Omega$

$$\frac{1}{R_2C_2} = 1000\ \frac{rad}{sec}$$

$$C_2 = \frac{1}{9\times10^4\ 10^3} = 1.11\times10^{-8}F$$

$$= 0.011\mu F = 11.1nF$$

$$\frac{R_4}{R_3} = 10$$

choose

$R_4 = 100k\Omega$

$R_3 = 10k\Omega$

$$\frac{1}{R_3C_3} = 10\ \frac{rad}{sec}$$

$$C_3 = \frac{1}{10^1\cdot10^4} = 10^{-5}F$$

$$= 10\mu F$$