ECE 3337: Signals & Systems Analysis, Class Worksheet 9

1. (100 points) Consider the LTI system specified by the following LCCDE:

$$\frac{d^2y(t)}{dt^2} + B\frac{dy(t)}{dt} + 5y(t) = 4\frac{dx(t)}{dt} + 25x(t)$$

a. Write down the characteristic equation for this LTI system.

b. For $B = 6$, calculate the frequency response function $\hat{\mathbf{H}}(\omega)$.

c. For what value(s) of *B* does this equation have real roots p_1 and p_2 ? What kind of behavior does the system exhibit when the characteristic equation has real roots?

d. For what value(s) of *B* does this equation have complex conjugate roots? What kind of behavior does the system exhibit when the characteristic equation has complex conjugate roots?

e. For what value(s) of *B* does this equation have equal roots? What kind of behavior does the system exhibit when the characteristic equation has equal roots?

f. For $B = 6$, calculate the attenuation coefficient α.

g. For $B = 6$, calculate the undamped natural frequency ω_0.

h. For $B = 6$, calculate the damping coefficient ξ.

i. For $B = 6$, calculate the step response $y_{step}(t)$

j. When $B = 6$, compute $\lim_{t\to\infty} y_{step}(t)$. To what final value does the system converge?