1. Given the transfer function

initial slope 20dB/dec

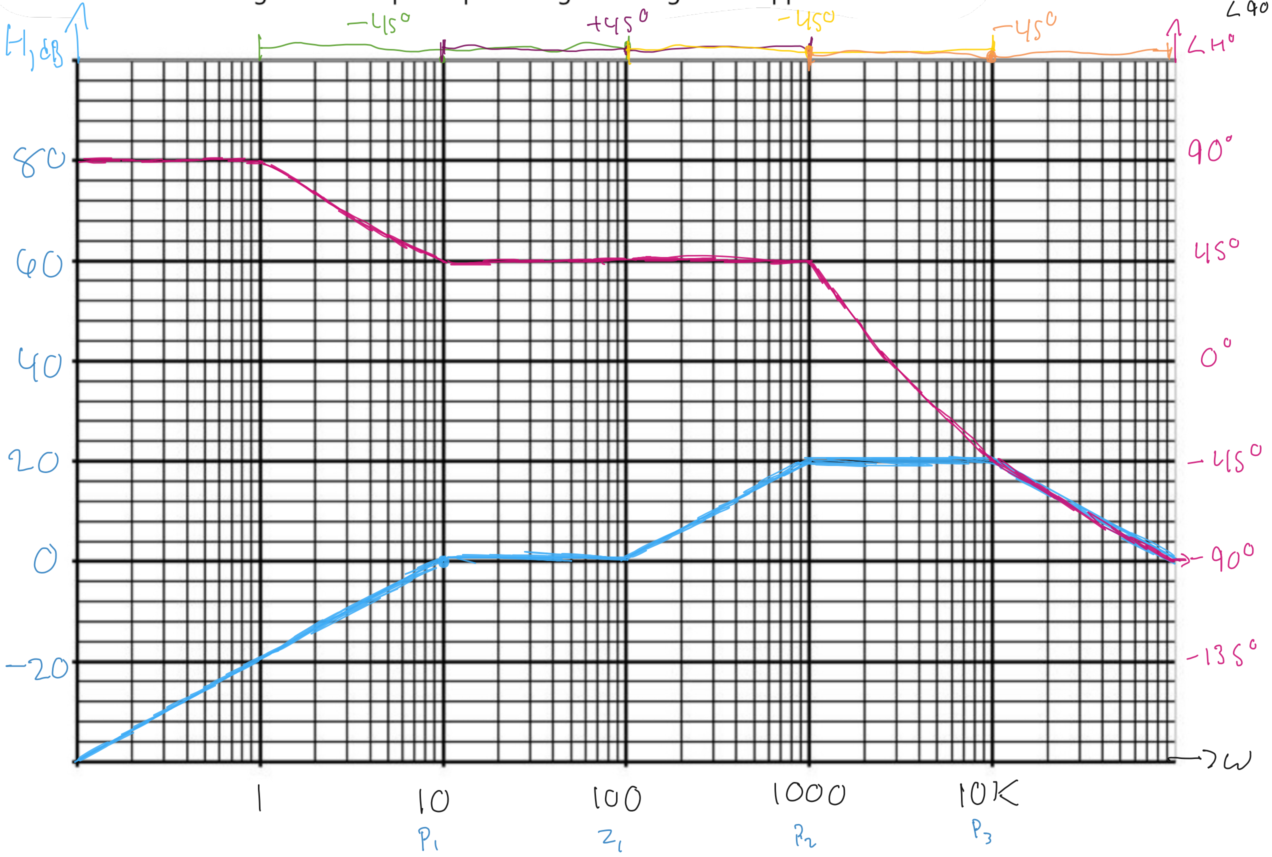
$$H(j\omega) = \frac{j\frac{\omega}{10}\left(1+j\frac{\omega}{100}\right)}{\left(1+j\frac{\omega}{10}\right)\left(1+j\frac{\omega}{10,000}\right)\left(1+j\frac{\omega}{1,000}\right)}$$

sketch the Bode magnitude and phase plot using the straight-line approximations.

$Z_1 = 100$ +20dB/dec

$P_1 = 10$ $P_2 = 1K$ $P_3 = 10K$ -20dB/dec

$H(j\cdot 10) \approx j$ $|H(j\omega)| \approx j\omega$ $\angle 90$



2. Given the transfer function

slope 20dB/dec initial

$$H(j\omega) = \frac{60\cdot j\frac{\omega}{30}}{\left(1+j\frac{\omega}{30}\right)\left(1+j\frac{\omega}{20,000}\right)^2}$$

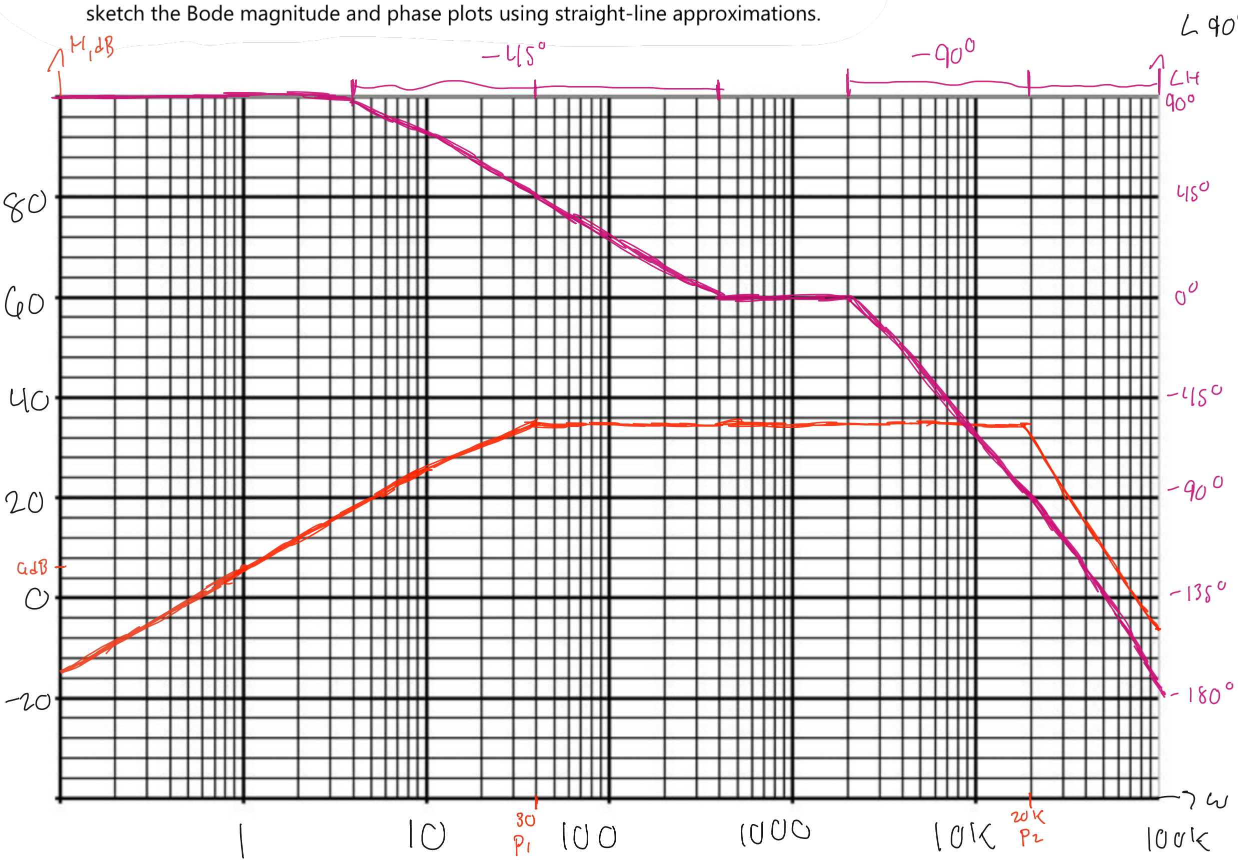
sketch the Bode magnitude and phase plots using straight-line approximations.

$P_1 = 30$ -20dB/dec

$P_2 = 20K$ -40dB/dec

$H(j\cdot 1) \approx 2j$ $|H(j\cdot 1)| \approx 2$ $20\log(2) \approx 6\,dB$

$|H(j\omega)| = 2j\omega$ $\angle 90°$

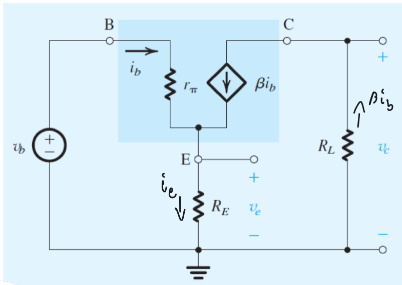


3. For the circuit below show

$$\frac{v_c}{v_b} = \frac{-\beta R_L}{r_\pi + (\beta+1)R_E}$$

and

$$\frac{v_e}{v_b} = \frac{R_E}{R_E + [r_\pi/(\beta+1)]}$$



$V_c = -R_L \beta i_b$

$$\frac{V_c}{V_b} = \frac{-R_L \beta \cancel{i_b}}{\cancel{i_b}(r_\pi + R_E(1+\beta))}$$

$$\frac{V_c}{V_b} = -\frac{R_L \beta}{r_\pi + R_E(1+\beta)}$$

KCL @ E: $i_e = i_b + \beta i_b$

$i_e = i_b(1+\beta)$

$V_e = R_E i_e = R_E i_b (1+\beta)$

$\frac{V_b - V_e}{r_\pi} = i_b$

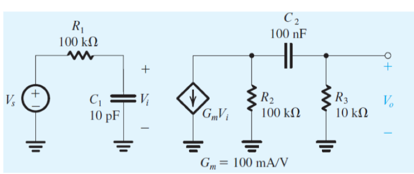
$V_b - V_e = i_b r_\pi$

$V_b = i_b r_\pi + i_b R_E(1+\beta)$

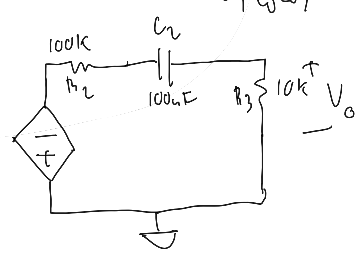
$V_b = i_b(r_\pi + R_E(1+\beta))$

$$\frac{V_e}{V_b} = \frac{R_E \cancel{i_b}(1+\beta)}{\cancel{i_b}(r_\pi + R_E(1+\beta))} \cdot \frac{\frac{1}{(1+\beta)}}{\frac{1}{(1+\beta)}} = \frac{R_E}{\frac{r_\pi}{(1+\beta)} + R_E}$$

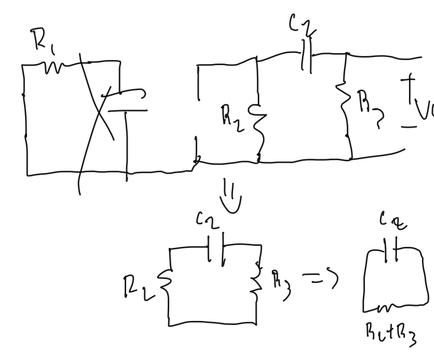
4. For the circuit shown below, first evaluate $H_i(j\omega) = V_i(j\omega)/V_s(j\omega)$ and the corresponding cutoff (corner) frequency. Second, evaluate $H_o(j\omega) = V_o(j\omega)/V_i(j\omega)$ and the corresponding cutoff frequency. Put each of the transfer functions in the standard form discussed in class, and combine them to form the overall transfer function, $H(j\omega) = H_i(j\omega) \times H_o(j\omega)$. Provide a Bode magnitude plot for $|H(j\omega)|$. What is the bandwidth between 3-dB cutoff points?



$H_o(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$ high pass



Req:



$H_i(j\omega) = \frac{V_i(j\omega)}{V_s(j\omega)}$ low pass

$V_i = V_s\left(\frac{C_1}{R_1 + C_1}\right)$

$$\frac{V_i(j\omega)}{V_s(j\omega)} = \frac{\frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}} = \frac{1}{1 + j\omega R_1 C_1}$$

$V_o = (-100m\, V_i)\left(\frac{R_2 R_3}{R_2 + R_3 + \frac{1}{j\omega C_2}}\right)$

$$\frac{V_o}{V_i} = -100m\left(\frac{j\omega R_2 R_3 C_2}{j\omega C_2(R_2+R_3)+1}\right)$$

$\tau_1 = R_1 C_1 = 1\mu$

$\omega_1 = \frac{1}{\tau_1} = 1 \cdot 10^6$

$f_1 \approx 159[kHz]$

$\tau_2 = (R_2+R_3)C_2 = 11m$

$\omega_2 = \frac{1}{11m} \approx 90$

$f_2 \approx 14.5\ [Hz]$

band pass

$H(j\omega) = H_i(j\omega)\, H_o(j\omega)$

$$= \left(\frac{1}{1+j\omega R_1 C_1}\right)\left(-100m\left(\frac{j\omega R_2 R_3 C_2}{j\omega C_2(R_2+R_3)+1}\right)\right) \cdot \frac{R_2+R_3}{R_2+R_3}$$

midband: $\omega_2 << \omega << \omega_1$

$\omega_1 << 1$: $\frac{1}{1+j\omega R_1 C_1} \approx 1$

$$H(j\omega) = \left(\frac{1}{1+j\omega R_1 C_1}\right)\left(\frac{-100m\, R_3}{R_2+R_3}\right)\left(\frac{j\omega C_2(R_2+R_3)}{j\omega C_2(R_2+R_3)+1}\right)$$

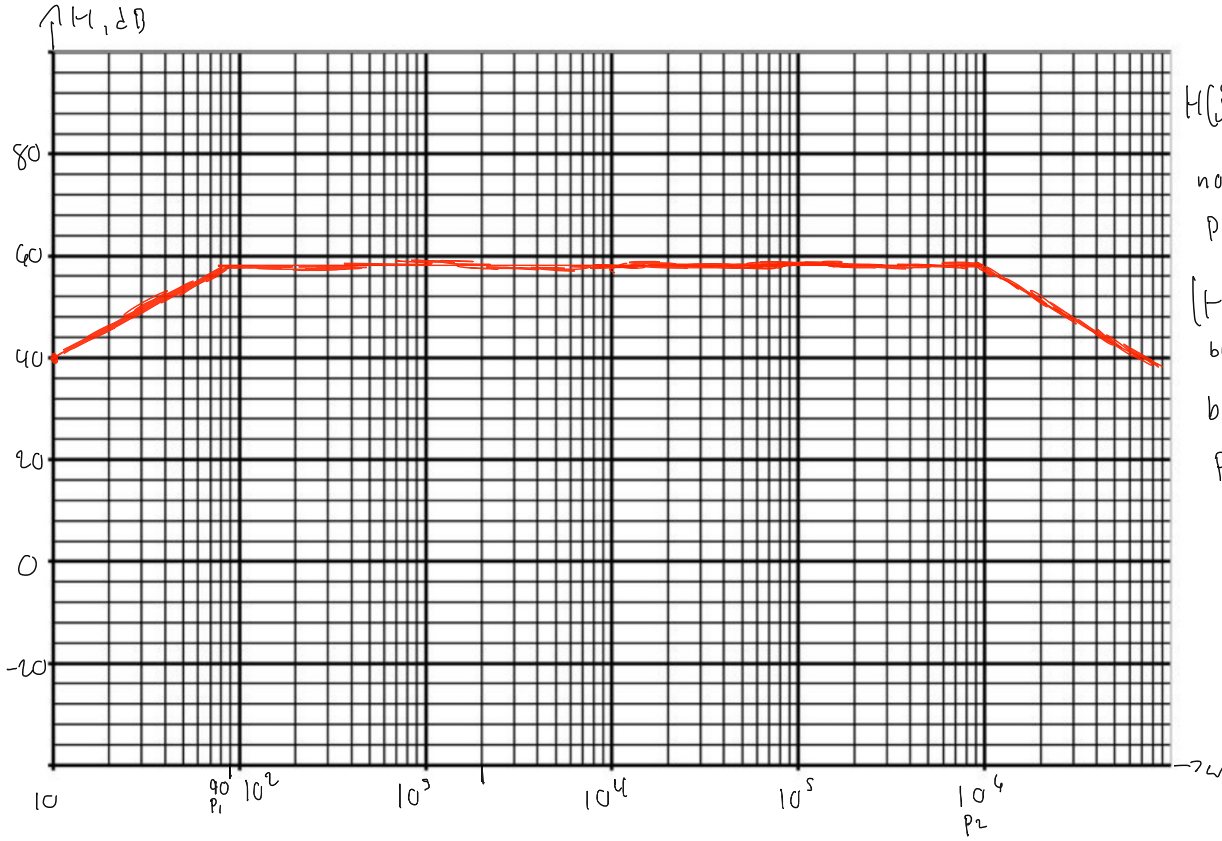
$\omega_2 >> 1$: $-100m \frac{R_2 R_3}{R_2 + R_3 + \frac{1}{j\omega C_2}} \approx -100m \frac{R_2 R_3}{R_2 + R_3} \approx -909$ ∴

$\frac{1}{j\omega C_2} \to 0$

$$H(j\omega) = \left(\frac{1}{1+j\omega\tau_1}\right)\left(-909\left(\frac{j\omega\tau_2}{j\omega\tau_2+1}\right)\right)$$

$$H(j\omega) = \left(\frac{1}{1+j\frac{\omega}{\omega_1}}\right)\left(-909\left(\frac{j\frac{\omega}{\omega_2}}{1+j\frac{\omega}{\omega_2}}\right)\right) = -909\frac{j\frac{\omega}{90}}{\left(1+j\frac{\omega}{1\cdot 10^6}\right)\left(1+j\frac{\omega}{90}\right)}$$

since high pass + low pass gives band pass



$$H(j\omega) = -909\,\frac{j\frac{\omega}{90}}{\left(1+j\frac{\omega}{1\cdot10^6}\right)\left(1+j\frac{\omega}{90}\right)}$$

no 0's

$P_1 \approx 90 \quad P_2 \approx 1\cdot10^6$

$|H(j\cdot10)| \approx -909\,\frac{10}{90} \approx 100 = 40\,dB$

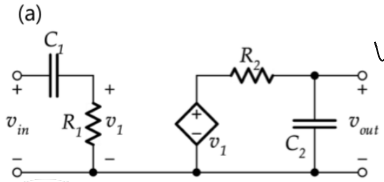
below midband

bandwidth between 3dB cutoff points: $f_1 - f_2 \approx 159K - 14.5$

bandwidth ≈ 159.14 [KHz]

5. Find the transfer function, $\frac{v_{out}}{v_{in}}$, for the following circuits. For (a) and (b), $R_1 = R_2 = 1\ K\Omega$, $C_1 = 1\ \mu F$, and $C_2 = 100\ \mu F$. For (c), $R_1 = 1\ K\Omega$, $R_2 = 100\ \Omega$, $C_1 = C_2 = 1\ \mu F$, and $K = 60$.

(a)



$$V_1 = V_{in}\left(\frac{R_1}{R_1 + \frac{1}{j\omega C_1}}\right) \qquad V_{out} = V_1\left(\frac{\frac{1}{j\omega C_2}}{R_2 + \frac{1}{j\omega C_2}}\right)$$

$$V_{out} = V_{in}\left(\frac{j\omega R_1 C_1}{1 + j\omega R_1 C_1}\right)\left(\frac{1}{1 + j\omega R_2 C_2}\right)$$

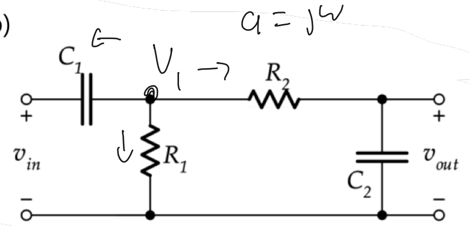
$$\frac{V_{out}}{V_{in}} = \frac{j\omega R_1 C_1}{1 + j\omega R_2 C_2 + j\omega R_1 C_1 - \omega^2 R_1 R_2 C_1 C_2}$$

$$\frac{V_{out}}{V_{in}} = \frac{j\omega R_1 C_1}{j\omega(R_2C_2 + R_1C_1) - \omega^2 R_1 R_2 C_1 C_2 + 1}$$

$$\frac{V_{out}}{V_{in}} = \frac{j\omega 0.001}{j\omega(0.101) - \omega^2(0.0001) + 1}$$

(b)

$a = j\omega$



$$V_0 = V_1\,\frac{\frac{1}{aC_2}}{\left(R_2 + \frac{1}{aC_2}\right)} = V_1\left(\frac{1}{1 + aR_2C_2}\right)$$

$$\frac{V_1 - V_{in}}{\frac{1}{aC_1}} + \frac{V_1}{R_1} + \frac{V_1 - V_0}{R_2} = 0$$

$$aC_1(V_1 - V_{in}) + \frac{V_1}{R_1} + \frac{V_1}{R_2}\left(1 - \frac{1}{1 + aR_2C_2}\right) = 0$$

$$aC_1(V_1 - V_{in}) + \frac{V_1}{R_1} + \frac{V_1}{\cancel{R_2}}\left(\frac{\cancel{1} + a\cancel{R_2}C_2 - \cancel{1}}{1 + aR_2C_2}\right) = 0$$

$$aC_1(V_1 - V_{in}) + \frac{V_1}{R_1} + V_1\,\frac{aC_2}{1 + aR_2C_2} = 0$$

$$V_1\left(aC_1 + \frac{1}{R_1} + \frac{aC_2}{1 + aR_2C_2}\right) = aC_1 V_{in}$$

$$\frac{V_1}{V_{in}} = \frac{aC_1}{aC_1 + \frac{1}{R_1} + \frac{aC_2}{1 + aR_2C_2}}$$

$$\frac{\cancel{V_1}}{V_{in}} = \frac{aC_1}{aC_1 + \frac{1}{R_1} + \frac{aC_2}{1 + aR_2C_2}} \cdot V_0 = \cancel{V_1}\left(\frac{1}{1 + aR_2C_2}\right)$$

$$\frac{V_0}{V_{in}} = \frac{aC_1}{\left(aC_1 + \frac{1}{R_1} + \frac{aC_2}{1 + aR_2C_2}\right)(1 + aR_2C_2)}$$

$$\frac{V_0}{V_{in}} = \frac{aC_1}{(1 + aR_2C_2)\left(aC_1 + \frac{1}{R_1}\right) + aC_2}$$

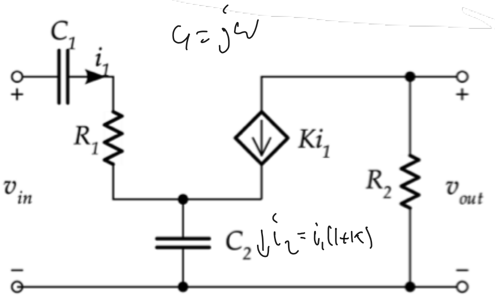
$$\frac{V_0}{V_{in}} = \frac{aC_1}{aC_1 + \frac{1}{R_1} + a^2R_2C_1C_2 + a\frac{R_2C_2}{R_1} + aC_2}$$

$$\frac{V_0}{V_{in}} = \frac{aR_1C_1}{aR_1C_1 + 1 + a^2R_1R_2C_1C_2 + aR_2C_2 + aR_1C_2}$$

$$\frac{V_o}{V_{in}} = \frac{aR_1C_1}{a(R_1C_1+R_2C_2+R_1C_2)+1+a^2R_1R_2C_1C_2} = \frac{j\omega 0.001}{j\omega(0.201)+1+(j\omega)^2 0.0001}$$

(c)

$a = j\omega$



$V_o = -Ki_1R_2$

$$V_o = -K\left[\frac{V_{in}\, aC_1C_2R_2}{C_2+C_1(1+K)+aC_1C_2R_1}\right]$$

$$\frac{V_o}{V_{in}} = -\frac{j\omega\, 6\cdot10^{-9}}{62\cdot10^{-6}+j\omega 10^{-9}}$$

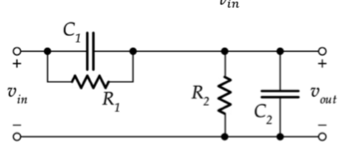
$$V_{in} = i_1\left(\frac{1}{aC_1}+R_1+\frac{1+K}{aC_2}\right)$$

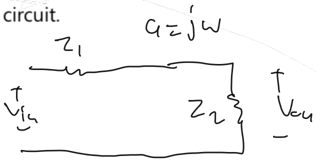
$$V_{in} = i_1\left(\frac{C_2+C_1(1+K)+aC_1C_2R_1}{aC_1C_2}\right)$$

$$i_1 = \frac{V_{in}\, aC_1C_2}{C_2+C_1(1+K)+aC_1C_2R_1}$$

6. Find the transfer function, $\frac{v_{out}}{v_{in}}$, for the following circuit.

$a = j\omega$





By proper choice of the components C_1, R_1, C_2, and R_2, the transfer function of this circuit can be made independent of frequency. In that case, the circuit is called a compensated attenuator and can be used as an oscilloscope probe. In this application, the signal from a circuit being viewed on an oscilloscope is attenuated, and the input impedance of the oscilloscope is compensated by the probe so that it does not distort the measurement. Assume that this circuit, R_1 and C_1 represent the probe, while R_2 and C_2 represent the oscilloscope input. For an oscilloscope with an input of 1 MΩ and 30 pF, design a 10:1 probe, that is, a probe which attenuates the signal by a factor of 10. R_2 C_2

$$Z_1 = \frac{\frac{R_1}{aC_1}}{R+\frac{1}{aC_1}} = \frac{R_1}{1+aR_1C_1}$$

$$Z_2 = \frac{\frac{R_2}{aC_2}}{R_2+\frac{1}{aC_2}} = \frac{R_2}{1+aR_2C_2}$$

$$V_o = V_{in}\left(\frac{Z_2}{Z_1+Z_2}\right)$$

$$= V_{in}\left[\frac{R_2}{(1+aR_2C_2)\left(\frac{R_1}{1+aR_1C_1}+\frac{R_2}{1+aR_2C_2}\right)}\right]$$

$$\frac{V_o}{V_{in}} = \frac{R_2}{\cancel{(1+aR_2C_2)}\left(\frac{R_1+aR_1R_2C_2+R_2+aR_1R_2C_1}{(1+aR_1C_1)\cancel{(1+aR_2C_2)}}\right)}$$

$$\frac{V_o}{V_{in}} = \frac{R_2(1+aR_1C_1)}{R_1+R_2+aR_1R_2(C_1+C_2)}$$

$$\frac{V_o}{V_{in}} = \frac{R_2}{R_1+R_2}\cdot\frac{1+aR_1C_1}{\left(1+\frac{aR_1R_2(C_1+C_2)}{R_1+R_2}\right)}$$

frequency independent $\therefore$ $j\omega$ must cancel (a)

$$1+aR_1C_1 = 1+\frac{aR_1R_2(C_1+C_2)}{R_1+R_2}$$

$$R_1C_1 = \frac{R_1R_2(C_1+C_2)}{R_1+R_2}$$

$$(R_1+R_2)C_1 = R_2(C_1+C_2)$$

$$R_1C_1+\cancel{R_2C_1} = \cancel{R_2C_1}+R_2C_2$$

$$R_1C_1 = R_2C_2$$

but for frequency independence and 10x attenuation $\Rightarrow$ $\frac{V_o}{V_{in}} = 0.1 = \frac{R_2}{R_1+R_2}$

$$(R_1+R_2)0.1 = R_2$$

$$.1R_1+.1R_2 = R_2$$

$$0.1R_1 = 0.9R_2$$

$$R_1 = 9R_2$$

$$R_1 = 9(1M) = 9\,[M\Omega]$$

$$\therefore R_1C_1 = R_2C_2$$

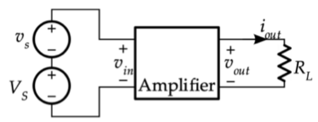
$$C_1 = \frac{1M(30p)}{9M}$$

$$C_1 = 3.\overline{33}\,[pF]$$

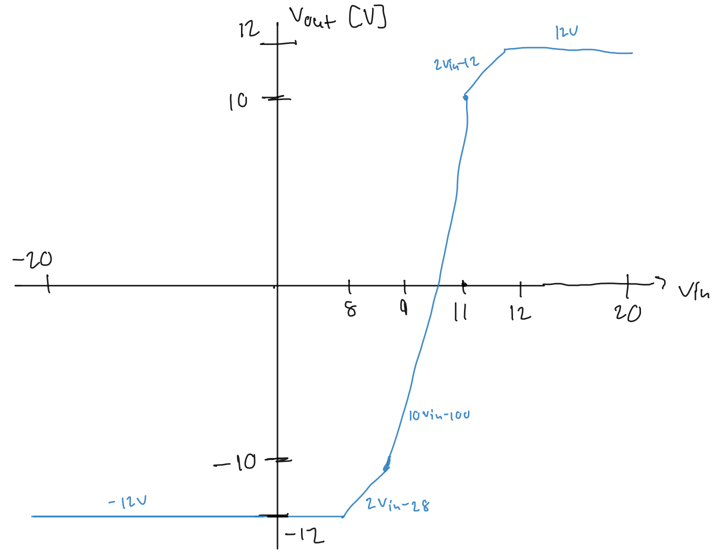
7. An amplifier has a transfer characteristic that can be reasonably approximated by the expression

$$v_{OUT} = \begin{cases} -12\,V & -20\,V \le v_{IN} \le 8\,V \\ 2\,v_{IN} - 28\,V & 8\,V < v_{IN} \le 9\,V \\ 10\,v_{IN} - 100\,V & 9\,V < v_{IN} \le 11\,V \\ 2\,v_{IN} - 12\,V & 11\,V < v_{IN} \le 12\,V \\ 12\,V & 12\,V < v_{IN} \le 20\,V \end{cases}$$

a. Sketch this transfer characteristic.
b. Assume that V_S = 10.5 V and v_s = A sin(ω t), where A = 0.3 V and ω = 60 rad/s, as shown in the figure below. For this signal level and DC bias value, what is the voltage gain, A_v? Assume that $A_v = v_{out}/v_{in}$.
c. Assume the above values for V_S and v_s and find an expression for $i_{out}(t)$ when R = 100Ω.
d. What are the largest values for v_s that will yield an undistorted output?



a)



b)
$V_{in} = V_S + v_s = 10.5 + 0.3\sin(\omega t)$

$V_{in,min}$: $\omega t = \frac{3\pi n}{2}$ $10.5 - 0.3$

max ; $\omega t = \frac{\pi n}{2}$ $10.5 + 0.3$

$10.2 \le V_{in} \le 10.8$ $\quad 9 < V_{in} < 11$, $V_{out} = 10V_{in} - 100$

$\frac{V_{out}}{V_{in}} = A_v = 10$

c) $V_{out}(t) = i_{out}(t) \cdot R$

$10(10.5 + 0.3\sin(60t)) - 100 = i_{out}(t) \cdot R$

$\frac{5 + 3\sin(60t)}{100} = i_{out}(t)$

$i_{out}(t) = 50 + 30\sin(60t)$ [mA]

d) $v_s = A\sin(60t)$

undistorted $\quad 9 \le v_{in} \le 11$

$9 \le 10.5 + A\sin(60t) \le 11$

largest v_s value∴ $10.5 + A\sin(60t) \le 11$

$A\sin(60t) \le 0.5$

∴ largest v_s value when $A = 0.5$ to be undistorted