MST

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MST

Implementation: Prim's Algorithm

Implementation. Use a priority queue ala Dijkstra.

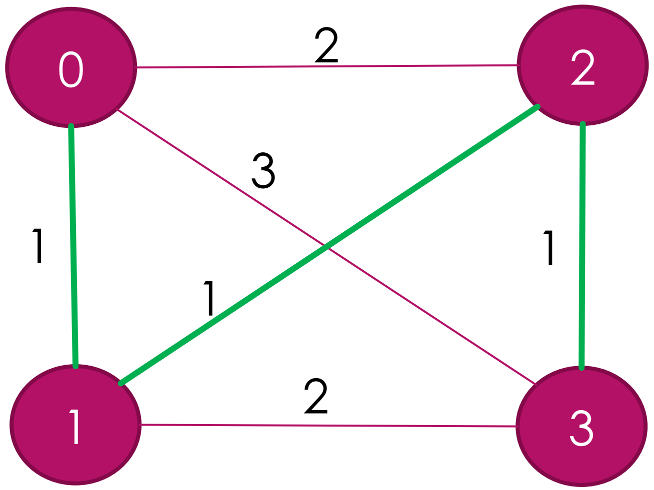
- Maintain set of explored nodes S.
- For each unexplored node v, maintain attachment cost a[v] = cost of cheapest edge v to a node in S.
- $O(n^2)$ with an array; $O(m \log n)$ with a binary heap.

Lab7 Q2

- Why array faster than heap?(Prim)
- Observe the graph is a completely connected graph

$m = n(n-1)/2$

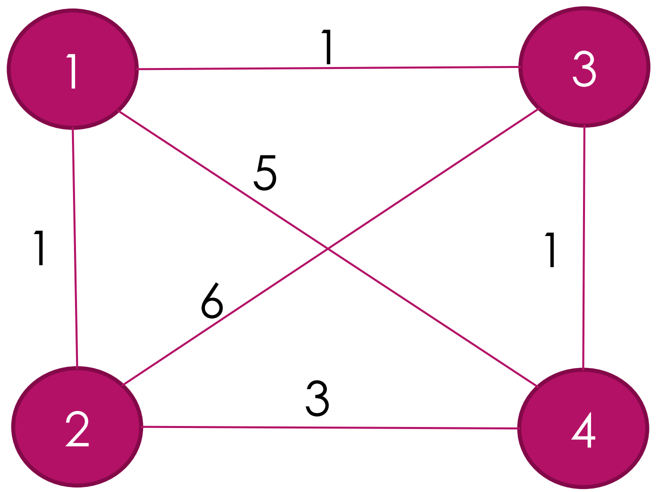
$O(m\log n) \rightarrow O(n^2 \log n) > O(n^2)$



Adjacency matrix

	0	1	2	3
0	-	1	2	3
1	1	-	1	2
2	2	1	-	1
3	3	2	1	-

Prim vs Dijkstra



	1	2	3	4
1	-	1	1	5
2	1	-	6	3
3	1	6	-	1
4	5	3	1	-

Prim

index	1	2	3	4
loop1	0	1	1	5
loop2	0	1	1	3
loop3	1	1	1	1

Visited 1 (1,2)next 2
Visited 2 (1,3) next 3
Visited 3 (3,4) end

Dijkstra

index	1	2	3	4
loop1	0	1	1	5
loop2	0	1	1	4
loop3	0	1	1	2
loop4	0	1	1	2

Visited 1 next 2
Visited 2 next 3
Visited 3 next 4
Visited 4 end

Implementation: Kruskal's Algorithm

Implementation. Use the union-find data structure.

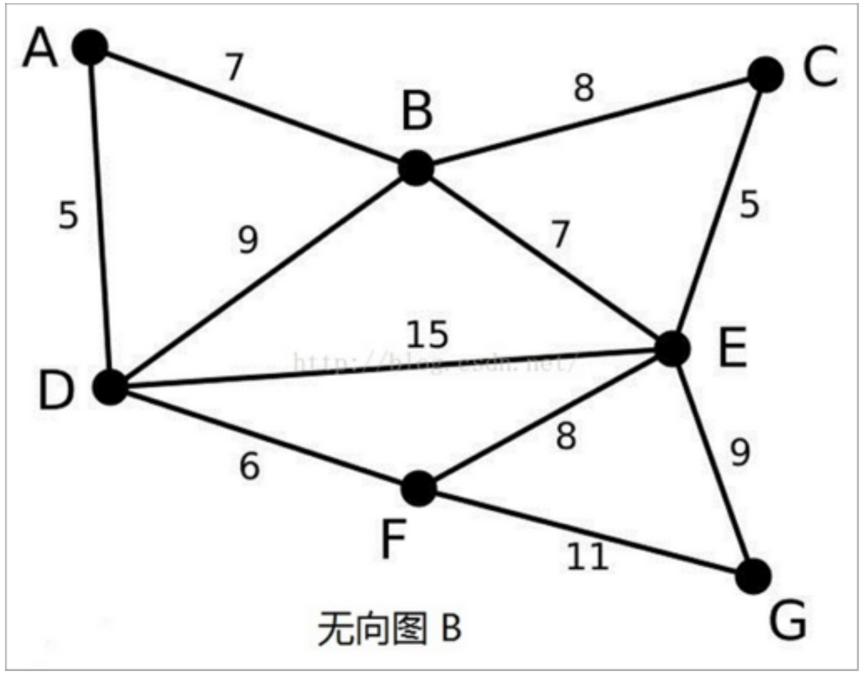
- Build set T of edges in the MST.
- Maintain set for each connected component.
- $\boxed{O(m \log n)}$ for sorting and $\boxed{O(m\ \underbrace{\alpha\,(m, n)}_{\text{essentially a constant}})}$ for union-find.

$m \le n^2 \Rightarrow \log m$ is $O(\log n)$

O(m) create heap

O(mlogm α (m,n))

Kruskal



Kruskal:

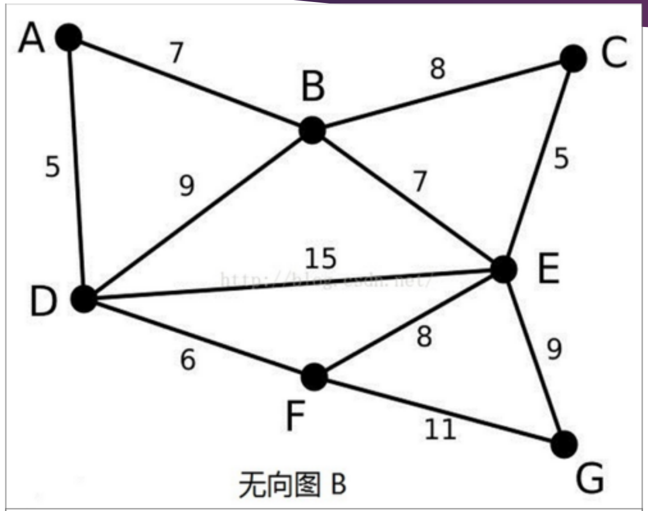
1. Sorting all the sides
2. Finding smallest bridge (n, m)
3. Whether node n and node m are in a same tree?
   If yes, skip
   If no, merge two trees
4. If the number of node is N, we should merge N-1 times.
5. When merge two trees, add the w value

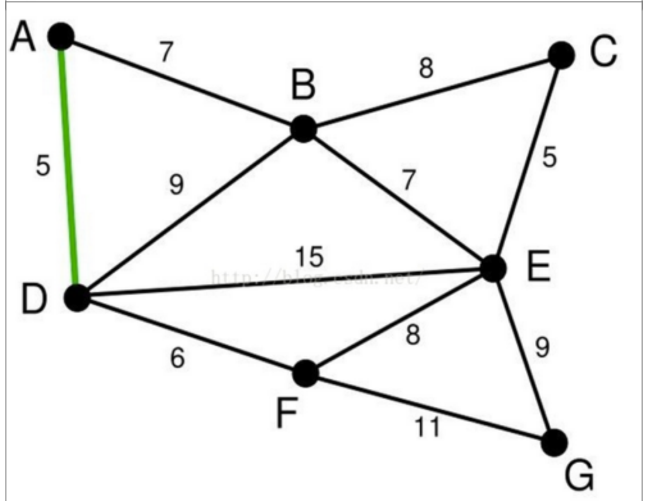
How to merge two trees (n, m)? Disjoint Set

1. Find root of n and m respectively
2. If root of n equals to root of m, n and m is in a same tree. Skip
3. Get the height of root n and root m

```
if(rootN.height > rootM.height) rootM.parent =rootN
else if(rootN.height<rootM.height) rootN.parent=rootM
else  rootM.parent=rootN   rootN.height++;
```

Sample Kruskal-1

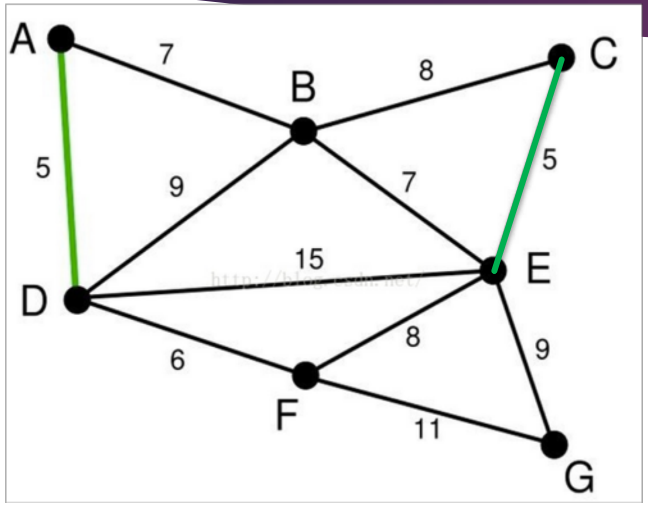


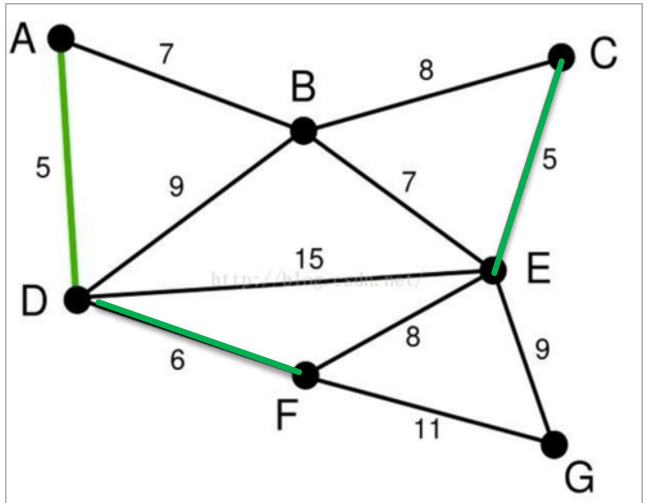


index	1	2	3	4	5	6	7
node	A	B	C	D	E	F	G
parent	0	0	0	0	0	0	0
weight	0	0	0	0	0	0	0

index	1	2	3	4	5	6	7
node	A	B	C	D	E	F	G
parent	0	0	0	1	0	0	0
weight	1	0	0	0	0	0	0

Sample Kruskal-2



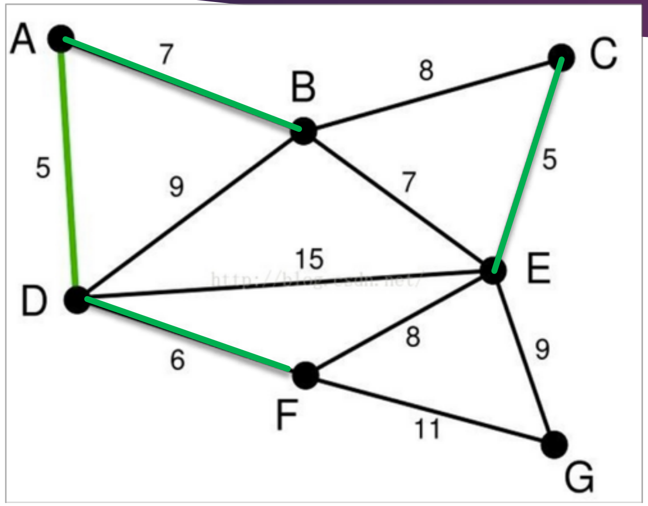


index	1	2	3	4	5	6	7
node	A	B	C	D	E	F	G
parent	0	0	0	1	3	0	0
weight	1	0	1	0	0	0	0

f(root).weigth<d(root).weight

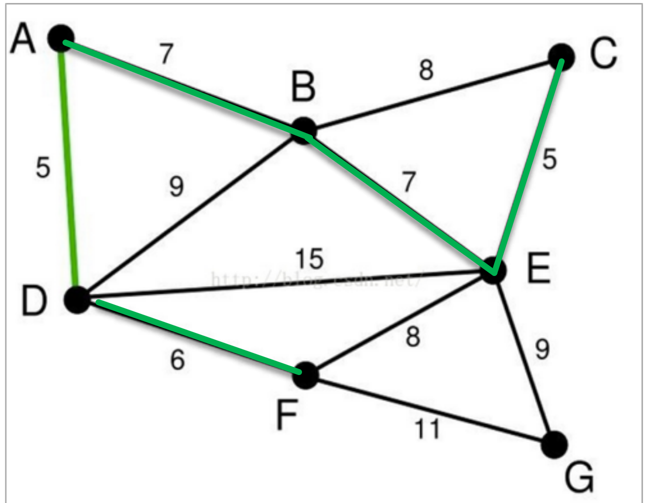
index	1	2	3	4	5	6	7
node	A	B	C	D	E	F	G
parent	0	0	0	1	3	1	0
weight	1	0	1	0	0	0	0

Sample Kruskal-3



b(root).weigth<a(root).weight
b.parent =a index

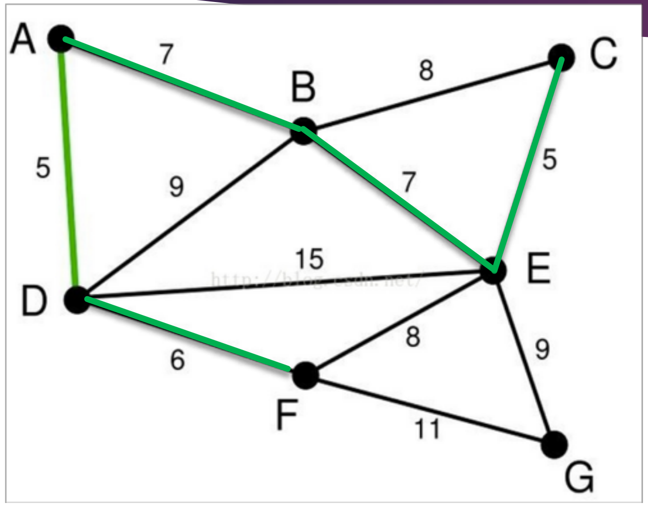
index	1	2	3	4	5	6	7
node	A	B	C	D	E	F	G
parent	0	1	0	1	3	1	0
weight	1	0	1	0	0	0	0

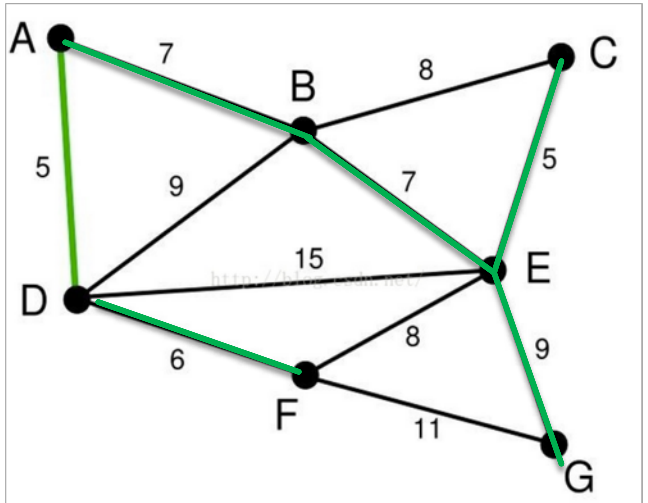


e(root).weigth==b(root).weight
c.parent =a index
a.weight++

index	1	2	3	4	5	6	7
node	A	B	C	D	E	F	G
parent	0	1	1	1	3	1	0
weight	2	0	1	0	0	0	0

Sample Kruskal-4





B (root)==C (root) skip
F (root) ==E (root) skip

e(root).weigth>g (root).weight
g.parent=a

index	1	2	3	4	5	6	7
node	A	B	C	D	E	F	G
parent	0	1	1	1	3	1	1
weight	2	0	1	0	0	0	0