

Lab9 Q1&Q2

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Lab9 Q1 Solution

- Observe the Sample Input:

```
5
100 100
90 90
60 60
70 70
80 80
```

- If all pairs (A_i, B_i) and (A_j, B_j) satisfy $A_i \leq A_j$ while $B_i \leq B_j$, we know that the sum of the satisfaction of all students = $N*(N-1)/2$
- Let **Total** = $N*(N-1)/2$

Lab9 Q1 Solution

► But actually, if student i and student j , their scores are the following:

(A_i, B_i) and (A_j, B_j) satisfies $A_i \leq A_j$ but $B_i > B_j$

Student i can't give student j satisfaction.

If we can count how many these like student i fail to contribute to student j 's satisfaction, let the number = x , then we can figure out the final satisfaction

sum = **Total - x**

Lab9 Q1 Solution

- ▶ How to count how many pairs (A_i, B_i) and (A_j, B_j) , satisfy $A_i \leq A_j$ but $B_i > B_j$?
- 1. Sort pairs by A_i , generate a new permutation $(A_1, B_1) (A_2, B_2) \dots (A_n, B_n)$
 $A_1 \leq A_2 \leq A_3 \leq A_4 \dots \leq A_n$
- 2. Counting Inversions on sequence $B_1, B_2, B_3 \dots B_n$
- 3. x is the count of inversions. Answer = Total - x

```
Sort-and-Count(L) {  
    if list L has one element  
        return 0 and the list L  
  
    Divide the list into two halves A and B  
     $(r_A, A) \leftarrow \text{Sort-and-Count}(A)$   
     $(r_B, B) \leftarrow \text{Sort-and-Count}(B)$   
     $(r, L) \leftarrow \text{Merge-and-Count}(A, B)$   
  
    return  $r = r_A + r_B + r$  and the sorted list L  
}
```

Lab9 Q2 Hint

VC	len	V number
1	1	1
2	$1*2+1=3$	$1+1=2$
3	$3*2+1=7$	$3+1=4$
4	$7*2+1=15$	$7+1=8$

$VC(1) = V$

$VC(2) = VVC$

$VC(3) = VVCVVCC$

$\rightarrow CCVCCVV$

$VC(4) = VVCVVCCVVCVCVCC$