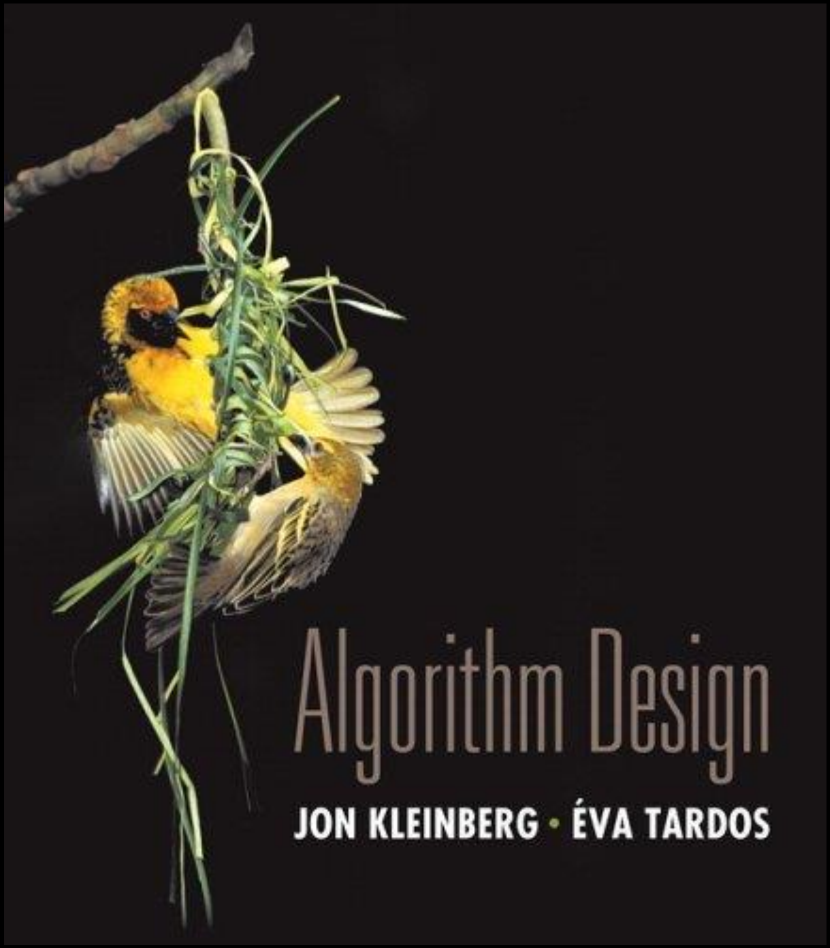


Chapter 7

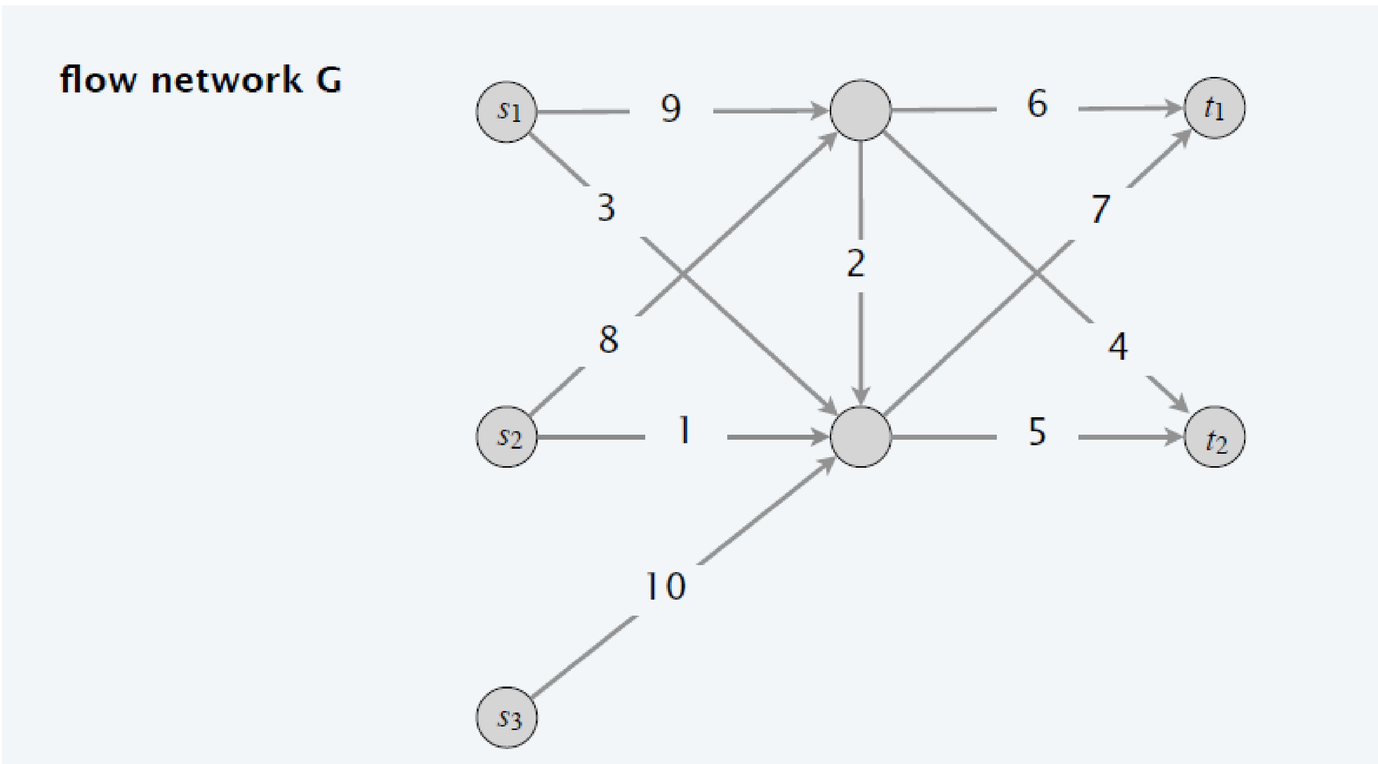
Network Flow

Slides by Kevin Wayne.

7.7 Extensions to Maximum-Flow Problem

Multiple sources and sinks

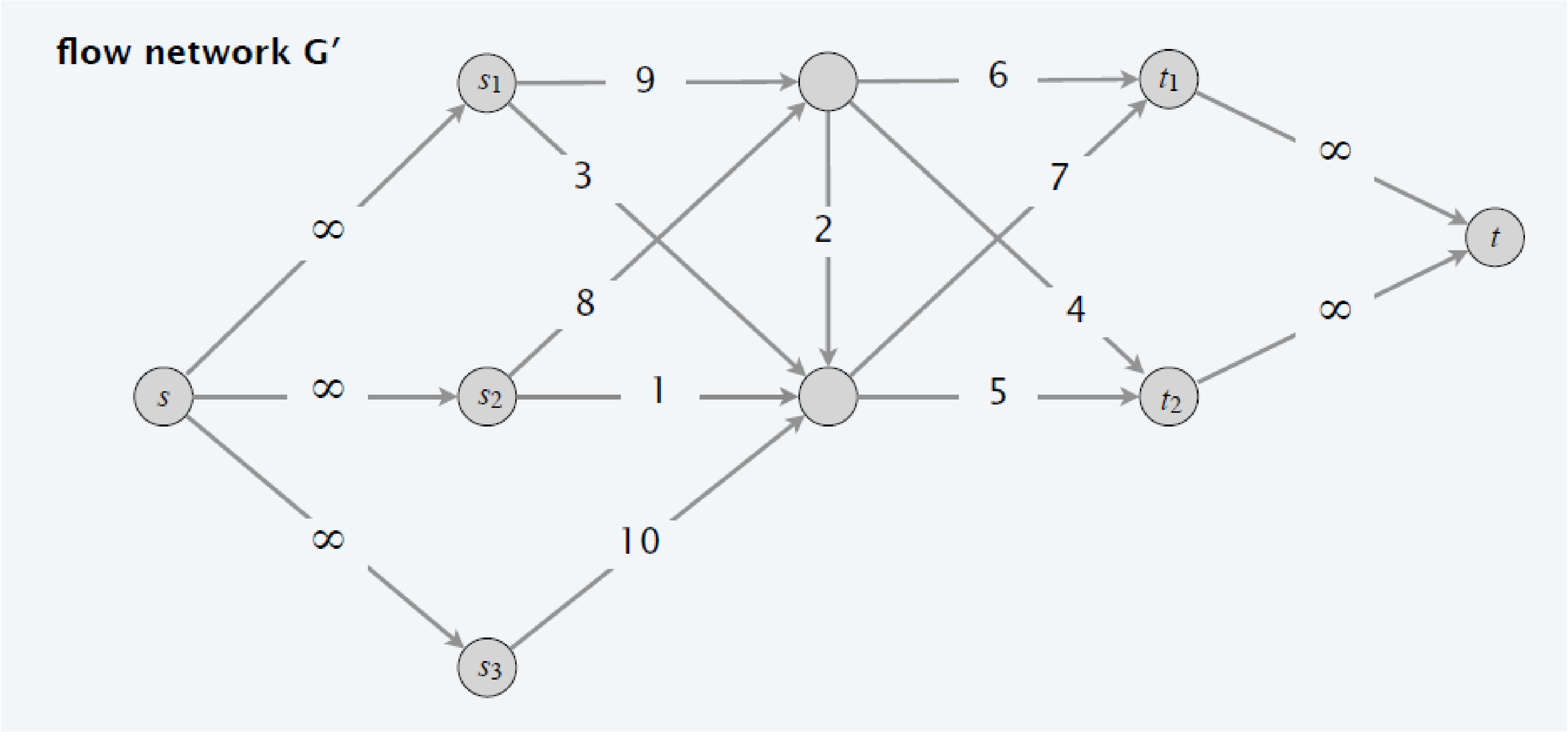
Def. Given a digraph $G = (V, E)$ with edge capacities $c(e) \geq 0$ and multiple source nodes and multiple sink nodes, find max flow that can be sent from the source nodes to the sink nodes.



Multiple sources and sinks: max-flow formulation

- Add a new source node s and sink node t.
- For each original source node s_i add edge (s, s_i) with capacity ∞.
- For each original sink node t_j, add edge (t_j, t) with capacity ∞.

Claim. 1–1 correspondence betweens flows in G and G'.

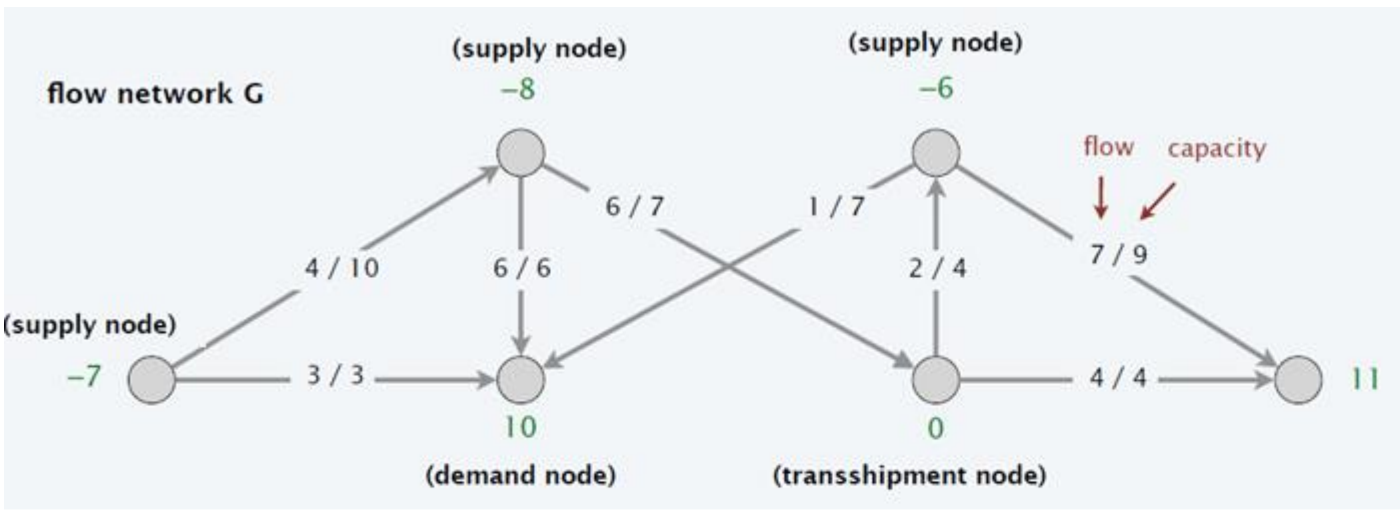


Circulation with supplies and demands

Def. Given a digraph $G = (V, E)$ with edge capacities $c(e) \geq 0$ and node demands $d(v)$, a circulation is a function $f(e)$ that satisfies:

For each $e \in E$: $0 \leq f(e) \leq c(e)$ (capacity)

For each $v \in V$: $\displaystyle\sum_{e \text{ in to } v} f(e) \;-\; \sum_{e \text{ out of } v} f(e) \;=\; d(v)$ (flow conservation)

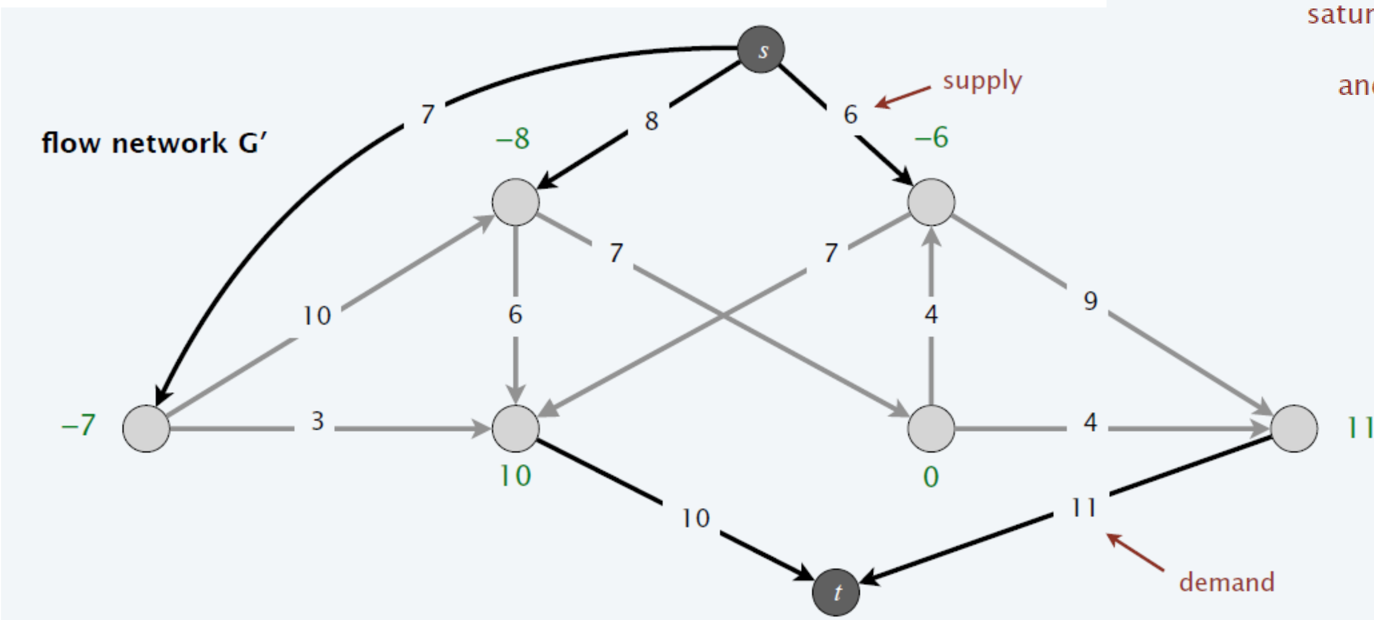


Circulation with supplies and demands: max-flow formulation

- Add new source s and sink t.
- For each v with $d(v) < 0$, add edge (s, v) with capacity $-d(v)$.
- For each v with $d(v) > 0$, add edge (v, t) with capacity $d(v)$.

Claim. G has circulation iff G' has max flow of value $D = \sum_{v:\, d(v)>0} d(v) = \sum_{v:\, d(v)<0} -d(v)$

saturates all edges leaving s and entering t



Circulation with supplies and demands

Integrality theorem. If all capacities and demands are integers, and there exists a circulation, then there exists one that is integer-valued.

Pf. Follows from max-flow formulation + integrality theorem for max flow.

Theorem. Given (V, E, c, d), there does not exist a circulation iff there exists a node partition (A, B) such that $\Sigma_{v \in B} d(v) > cap(A, B)$.

Pf sketch. Look at min cut in G'.

demand by nodes in B exceeds supply of nodes in B plus max capacity of edges going from A to B