Lecture 4

- List
  - ADT
    List, IteratedList, IndexedList
    SortedList
  - Singly Linked List
  - Doubly Linked List

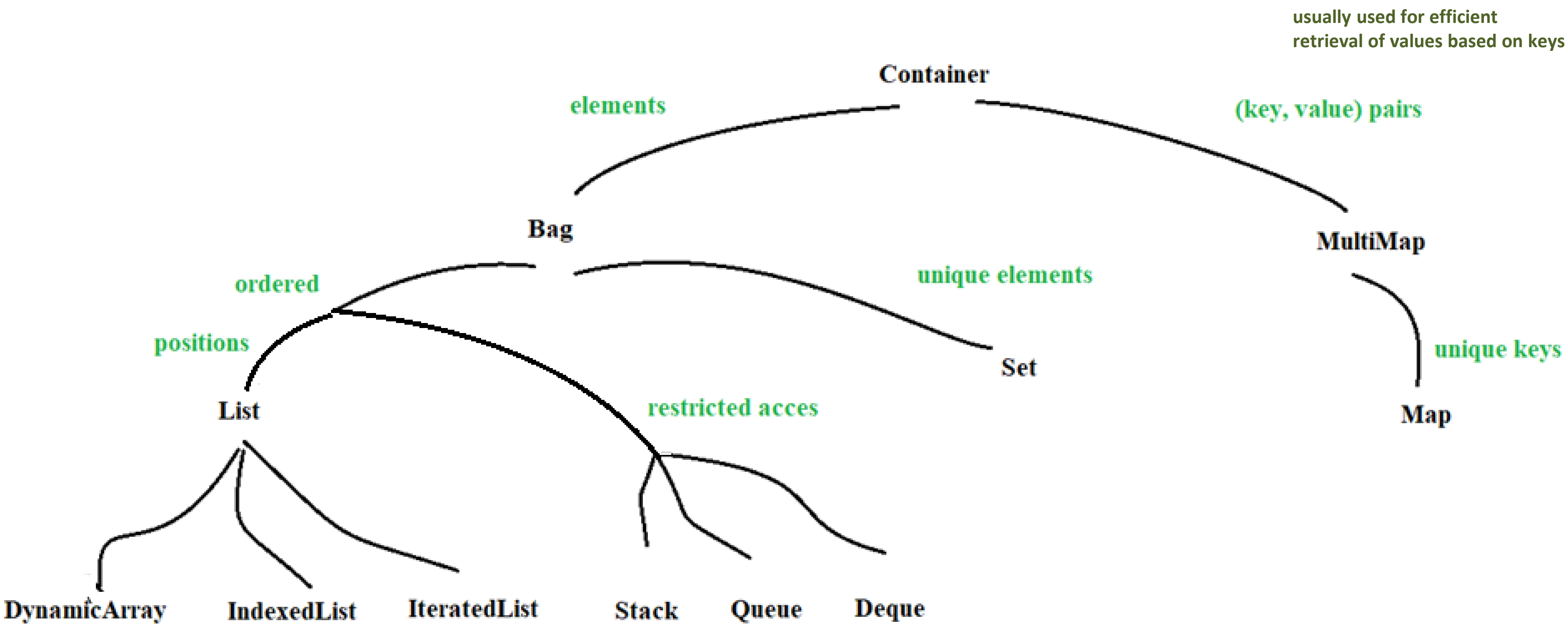
Lect. PhD. Lupsa Dana
Babes - Bolyai University
Computer Science and Mathematics Faculty
2022 - 2023

Previously, in Lecture 3

- Containers
  - Bag
  - Set
  - Map
  - Multimap
  - Stack
  - Queue
  - Deque
  - PriorityQueue

Previously, in Lecture 3
Overview

Unsorted containers:



List (or linear list)

Specific:
- elements are arranged in a strict linear order

A List is a container which is either empty or
- it has a unique first element
- it has a unique last element
- for every element (except for the last) there is a unique successor element
- for every element (except for the first) there is a unique predecessor element

Terminology:	**Sequence containers**	(C++ STL)
	List	(Java.util)

List. Positions

Element - Position

- Every element from a list has a unique position in the list.
  The position of an element identifies the element from the list.
- Valid/invalid position
  - For an invalid position we will use the following notation: $\perp$ (in pseudocode)
  - A position p will be considered valid if it denotes the position of an actual element from the list
- Positions are used to “determine” the position of the successor and predecessor element (if they exist).

See: ADT List.pdf

Many operations of ADT List have the next parameters:

i. element (of type TElem)
ii. elements' positions (of type TPosition)

List

ADTList:

- open to many possible instantiations of Position

ADT IndexedList

e.g. Python, Java

- TPosition is an Integer.
- There are less operations in the interface of the IndexedList.
  - Operations first, last, next, previous, valid do not exist.

    …

ADT IteratedList

e.g. STL C++

- TPosition is (given by) an Iterator.
- In case of an IteratedList the operations that take as parameter a position use an Iterator (and the position is the current element from the Iterator)
- There are less operations
  - Operations valid, next, previous no longer exist in the interface of the List (they are operations of the Iterator).

  … *add, remove ?*

SortedList

ADT SortedList

The interface of the ADT SortedList is (very) similar to that of List

Some differences:

- The **init** function takes as parameter a relation that is going to be used to order the elements
- We no longer have several **add** operations (addToBeginning, addToEnd, addToPosition), we have one single add operation, which takes as parameter only the element to be added (and adds it to the position where it should go based on the relation)
- We no longer have a **setElement** operation (might violate ordering)

Linked Lists

Basic idea:

Order is determined by position information stored with each element

Node:

- elements are stored in nodes
- each node "knows" the position of the next (/previous) node in the list

Information to store in nodes (choices)

- one link ➔ singly linked list

  SLLNode = {***(e; n)*** | ***e***:TElem and ***n*** : TP**osition**)}

  next, link

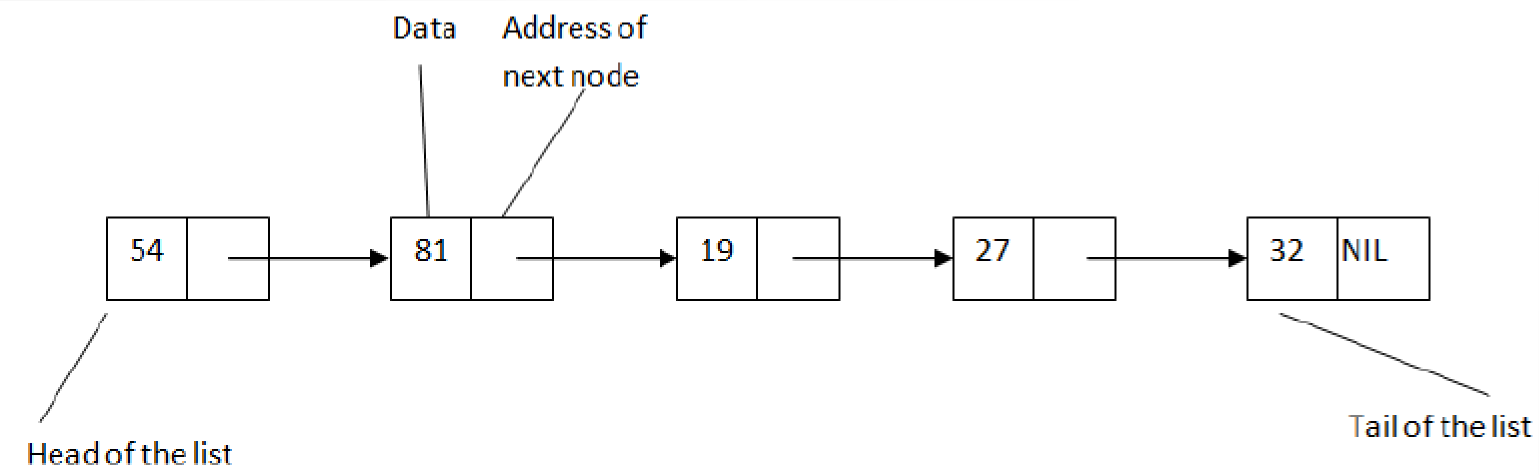
- two links ➔ doubly linked list

  DLLNode = {(***e; n, p)*** | ***e***:TElem and ***n*** : TP**osition** *next*(DLLNode))

  and ***p*** : TP**osition** (*prev*(DLLNode))

  } ***prev***

Singly Linked Lists - SLL



- In a SLL each node from the list contains the data and the address of the next node.
- The first node of the list is called head of the list and the last node is called tail of the list.
- The tail of the list contains the special value as the address of the next node (which does not exist).

Singly Linked Lists - Representation

Terminology (*often*) : dynamic linked storage

- using dynamic memory allocation
  - for each node individually

```
SLLNode:
    info: TElem //the actual information
    next: ↑ SLLNode //address of the next node
```

```
SLL:
    head: ↑ SLLNode //address of the first node
```

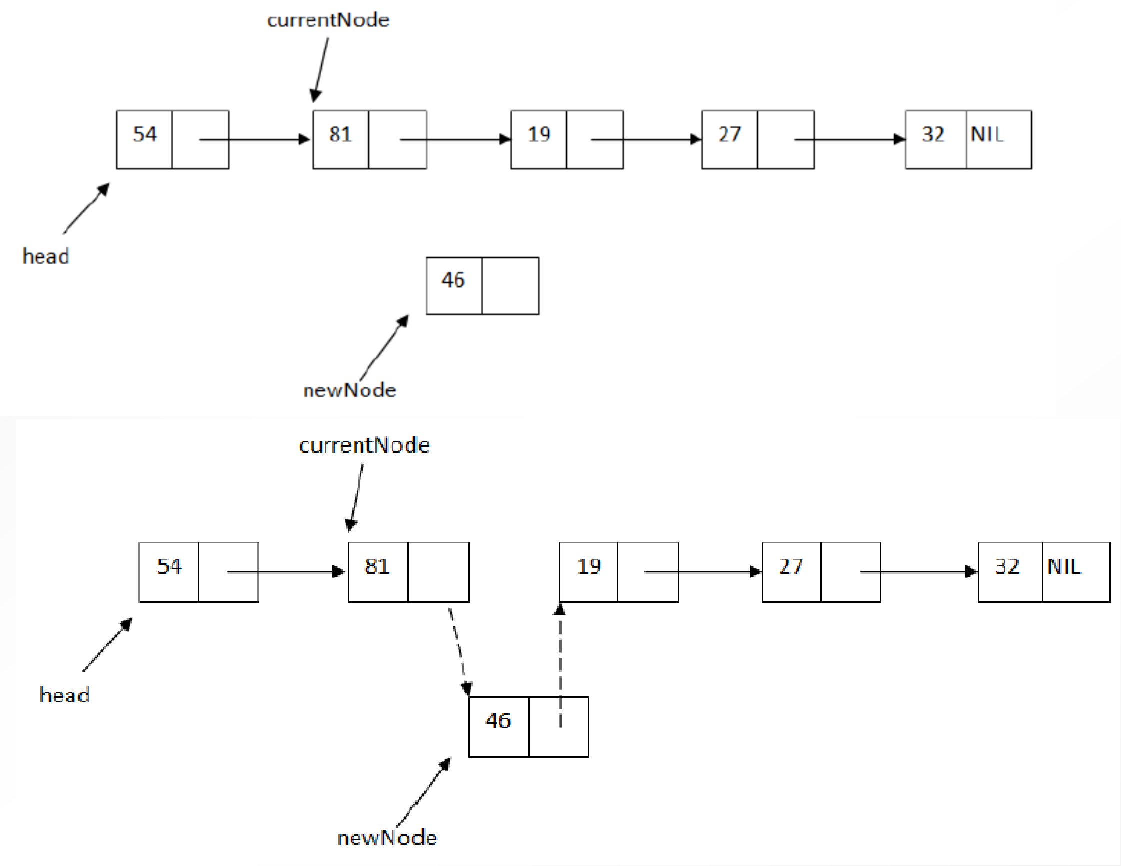
- represented over an array

(We will discuss it next time.)

\- *space management in the array !!*

SLL – Operations

e.g.: Insert after a node



SLL – Operations. Insert

<u>e.g.</u>: Insert after a node

subalgorithm insertAfter(sll, currentNode, elem) **is:**
//pre: sll is a SLL; currentNode is an SLLNode from sll;
//elem is a TElem
//post: a node with elem will be inserted after node currentNode
  newNode ← allocate() *//allocate a new SLLNode*
  [newNode].info ← elem
  [newNode].next ←[currentNode].next
  [currentNode].next ← newNode
end-subalgorithm

Complexity: ?

SLL – Operations. Insert

Think about:

(specification of the operation, pseudocode, complexity)

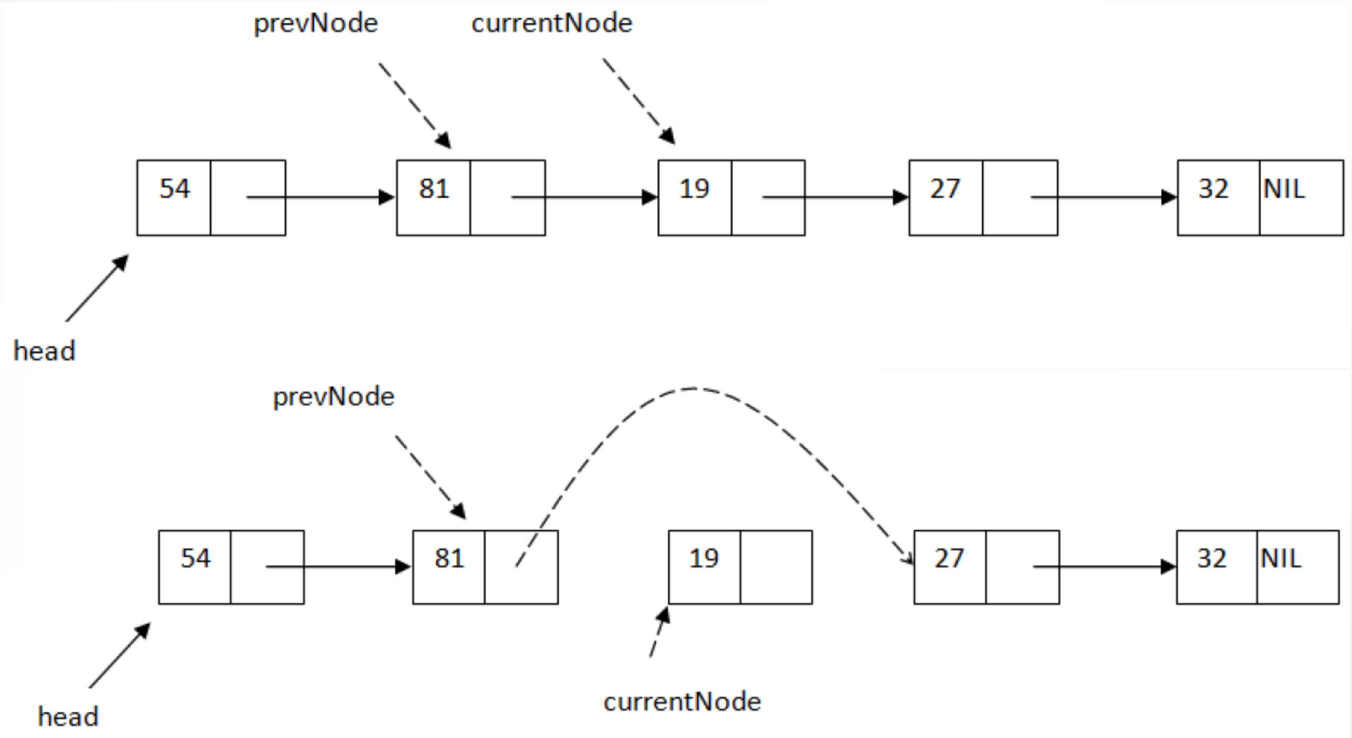
- Insert into the head of a SLL
- Design only one insert operation to insert into the head of a SLL or after a position
- Insert an element on a given indexed position
- Insert an element before a given node

Can it be done in $\Theta(1)$? Is there a cost to be paid?

SLL – Operations. Delete

e.g.: Delete a given element

- Move with the two pointers until currentNode is the node we want to delete.
- Delete currentNode by jumping over it



SLL – Operations. Delete

e.g.: Delete a given element

```
function deleteElement(sll, elem) is:
//pre: sll is a SLL, elem is a TElem
//post: returns true if elem was removed, false otherwise
    currentNode ← sll.head
    prevNode ← NIL
    while currentNode ≠ NIL and [currentNode].info ≠ elem execute
        prevNode ← currentNode
        currentNode ← [currentNode].next
    end-while
    if currentNode ≠ NIL then
        if prevNode = NIL then //we delete the head
            sll.head ← [sll.head].next
        else
            [prevNode].next ← [currentNode].next
        end-if
         free(currentNode)
         deleteElement← True
    else
         deleteElement← False
    end-if
end-function
```

Complexity: O(n)

SLL - Iterator

Read-only, uni-directional iterator:

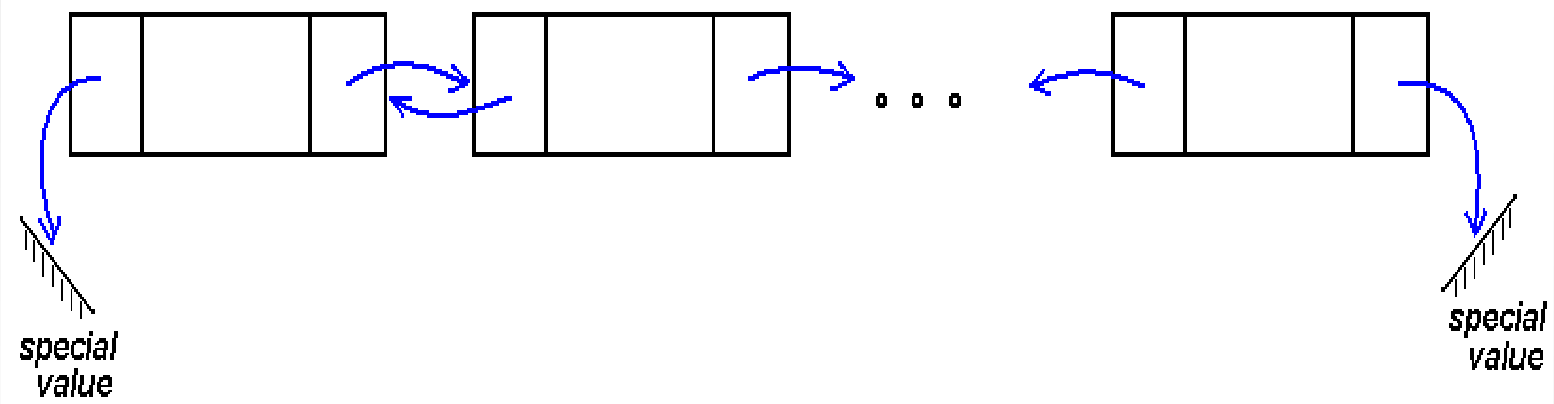
- In case of a SLL, the current element from the iterator is actually a node of the list.

```
SLLIterator:
    list: SLL
    currentElement: ↑ SLLNode
```

- What would be the complexity of the operations?

- How could we define a bi-directional iterator for a SLL? What would be the complexity of the previous operation?
- How could we define a bi-directional iterator for a SLL if we know that the previous operation will never be called twice consecutively (two consecutive calls for the previous operation will always be preceded by at least one call to the next operation)? What would be the complexity of the operations?

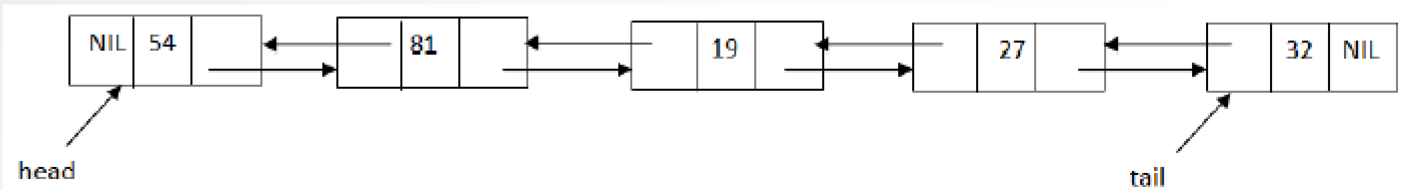
Doubly-Linked list



- can access elements if we know one of the two ends
- *but*: keep Position s of both ends

    reason: access in both direction

Doubly-Linked List. Representation



- using dynamic memory allocation
  - for each node individually

DLLNode:
  info: TElem
  next: ↑ DLLNode
  prev: ↑ DLLNode

DLL:
  head: ↑ DLLNode
  tail: ↑ DLLNode

- represented over an array
  - over an array (We will discuss it later.)

Doubly-Linked List

Inserting/deleting a new element at a given position

- It is similar to the operations for SLL.
- The main difference is that we need to set more links (we have the prev links as well) and we have to check whether we modify the tail of the list.

Doubly-Linked List. Example: operation insertLast

- Inserting a new element at the end of a DLL is in **Θ(1)**, because we have the tail of the list

```
subalgorithm insertLast(dll, elem) is:
//pre: dll is a DLL, elem is TElem
//post: elem is added to the end of dll
    newNode ← allocate() //allocate a new DLLNode
    [newNode].info ← elem
    [newNode].next ← NIL
    [newNode].prev ← dll.tail
    if dll.head = NIL then //the list is empty
        dll.head ← newNode
        dll.tail ← newNode
    else
        [dll.tail].next ← newNode
        dll.tail ← newNode
    end-if
end-subalgorithm
```

Complexity: Θ(1)

DLL. Operation deleteElement

e.g.: Delete a given element

```
function deleteElement(dll, elem) is:
    currentNode ← dll.head
    while currentNode ≠ NIL and [currentNode].info ≠ elem execute
        currentNode ← [currentNode].next
    end-while
    deletedNode ← currentNode
    if currentNode ≠ NIL then
        if currentNode = dll.head then
            if currentNode = dll.tail then
                dll.head ← NIL ; dll.tail ← NIL
            else
                dll.head ← [dll.head].next ; [dll.head].prev ← NIL
            end-if
        else if currentNode = dll.tail then
            dll.tail ← [dll.tail].prev ; [dll.tail].next ← NIL
        else
            [[currentNode].next].prev ← [currentNode].prev
            [[currentNode].prev].next ← [currentNode].next
        end-if
         free(currentNode)
         deleteElement ← True
    else
         deleteElement ← False
    end-if
end-function
```

Complexity: O(n)

DLL vs. SLL

Double-linked lists …

- require more space per node … than SLL
- sequential access in both directions is “easier”
- Some operations are “less expensive”
  - one can insert or delete a node in a constant number of operations given only that node's address, compared with singly-linked lists, which require the previous node's address in order to insert or delete

Specific iterators

- SLL - forward iterator
- DLL – bidirectional iterator

Linked List vs. Array

- **Extra-storage & linked-list**

  ? Total overhead, overhead per element

- **Memory allocation**:
  - allocate memory separately for each new element - slow
  - operation resize for array - an expensive operation

- **Operations**

  In case of linked lists
  - new items can be added or deleted anywhere in the list, just by managing the links between the nodes
  - elements are never moved (important if copying an element takes a lot of time).
  - access to the element based on index is difficult (complexity of linear time)

When to use linked-list?

- if you have elements that are frequently added and deleted
- when the dimension of an elements is significant

Think about:

1. Consider ADT IndexedList and list traversal by using:

a) by using the iterator

```
while (valid(it)) execute
        e ← getElement(it)
        @process e
        next(it)
endwhile
```

b) by using indexing

```
for i ← 1 , n execute
        e ←  getElement(l, i)
        @process e
endwhile
```

Analyze traversal time complexity, when:

i) List is represented over an array

ii) List is represented over a linked list

2. Consider ADT SortedList and search operation.

Which is the total time complexity that we can achieve

by choosing an appropriate algorithm for the next 3 representations ?

i) Array

ii) SLList

iii) DLList

Linked Lists

- **Circularly-linked list**

  the first and final nodes are linked together

- **Sentinel nodes**

  node with default value for *info* : sentinel

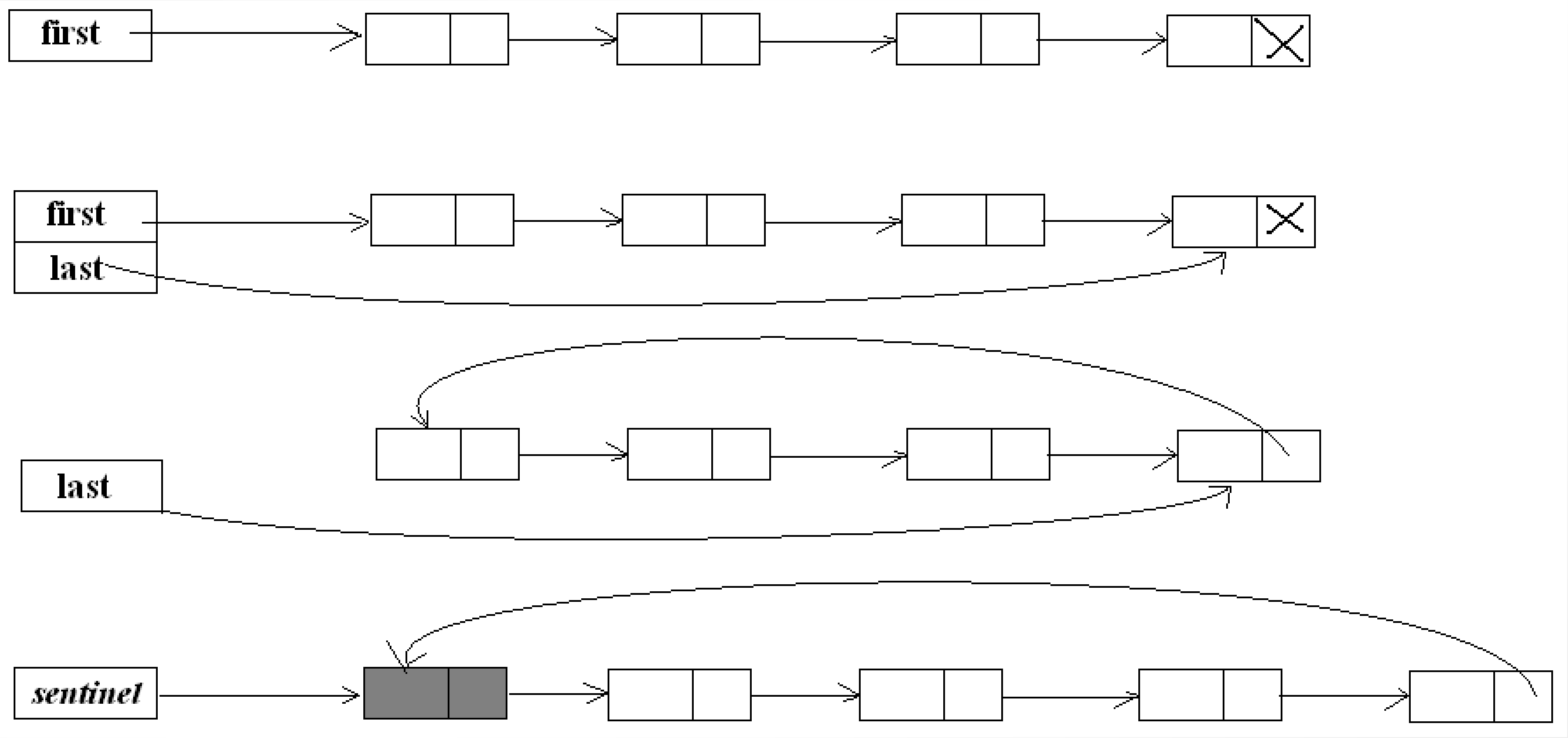
  → avoid special value ⊥ for position

  – may simplify certain operations

    ensure that next and/or previous nodes exist for any element

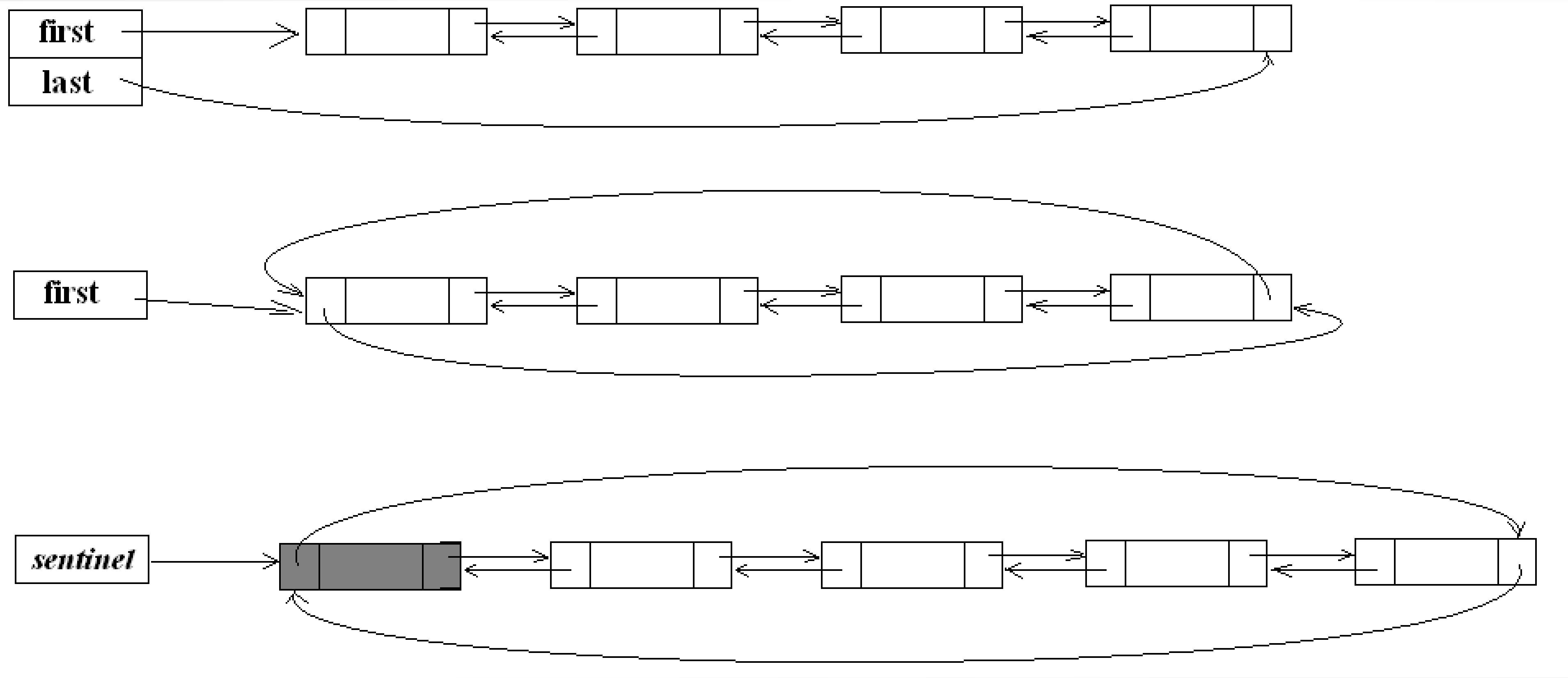
  – use extra space

Singly-linked list (representation choices/variations)



- ? SLList with fast addLast

Doubly-linked list (representation choices/variations)



DLL with sentinel. Operation deleteElement

DLL:
    sentinel: ↑ DLLNode

e.g.: Delete a given element

```
function deleteElement(dll, elem) is:
    currentNode ← dll.sentinel.next
    while currentNode ≠ NIL and [currentNode].info ≠ elem execute
        currentNode ← [currentNode].next
    end-while
    deletedNode ← currentNode
    if currentNode ≠ NIL then
        [[currentNode].next].prev ← [currentNode].prev
        [[currentNode].prev].next ← [currentNode].next
         free(currentNode)
         deleteElement ← True
    else
         deleteElement ← False
    end-if
end-function
```

mplexity: O(n)

Think about it

- Find the n^{th} node from the end of a SLL. Can we do it in one while?
- Reverse a SLL non-recursively in linear time using $\Theta(1)$ extra storage.
- Choose an efficient linked representation for:
  - Stack
  - Queue
  - Deque