

- Slides to support lectures
- Slides from 2020-2021

## Lecture 1

- Introduction
- Abstract data types and Data structures
- Collection / containers
- Pseudocode
- Algorithm analysis

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Many slides borrowed from: Lect. PhD. Onet-Marian Zsuzsanna

# Course Objectives

- The study of data types
  - The study of the concept of **abstract data types** and the most frequently used container abstract data types.
  - The study of different **data structures** that can be used to implement these abstract data types and their properties (including the complexity of their operations).

What you should learn from this course:

- to work with data stored in different data structures.
- to design and implement different applications starting from the use of abstract data types.
- to choose the abstract data type and data structure best suited for a given application.

Why we should try to create our own implementations for some ADTs if some languages already give us implementations of ADT when we need it?

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- To learn how to “work” with ADTs and understand what’s behind the scene;
- to learn the behaviour of some of the most used DS

# Bibliography

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# Data Type & Abstract Data Type

A data type (DT) is a set of values (domain) and a set of operations on those values. For example:

- built-in data types
- user defined data types.

Examples ?

An Abstract Data Type (ADT) :

- the objects from the domain
  - specified independently of their representation
- the operations
  - procedural abstraction → operation specification
  - specified independently of their implementation

# ADT: Example

Consider the Date ADT

(made of three numbers: year, month, day)

- Elements belonging to this ADT are all the valid dates, as follows:  
year is positive, less than 3000, month is between 1 and 12, day is between 1 and maximum number of possible days for the month

One possible operation for the Date ADT could be:

difference(d1, d2)

- **Descr:** compute the difference in number of days between two dates
- **Pre:** d1 and d2 have to be valid Dates,  $d1 \leq d2$
- **Post:** difference = a natural number and represents the number of days between d1 and d2.
- **Throws:** and exception if d1 is greater than d2

# Working with ADTs ...

- **Abstraction**  
Separation between the specification of an object (its domain and interface) and its implementation
  - **Encapsulation**  
... implementation ?
  - **Localization of change**
  - **Flexibility**
- 
- We have code that uses an ADT. Is it still valid if the ADT implementation changes ?
  - An ADT can be implemented in different ways. How hard is it to switch from one implementation to another ?

# Container

A container is a collection of elements.

- There we can add new elements and/or we can remove elements.

A container should provide at least the following operations:

- creating an empty container
- adding a new element to the container
- removing an element from the container
- provide access to the elements from the container (usually using an iterator )

Different containers are defined based on different properties:

- do the elements need to be unique?
- do the elements have positions assigned?
- can any element be accessed or just some specific ones?
- do we store simple elements or key - value pairs?



# Container vs. Collection

- Python - Collections
- C++ - Containers from STL
- Java - Collections framework and the Apache Collections library
- .Net - System.Collections framework

In the following, in this course we will use the term container.

You are already familiar with: **list** and **dict** from Python ... as ADT!

- Did you know that in Python the following instruction has a different complexity depending on whether **cont** is a **list** or a **dict**?

```
if elem in cont:  
    print("Found")
```

- Implementing these ADT will help us understand better how they work

# Container: ADT , DS , pseudocode

- For every container ADT we will discuss several possible data structures that can be used for the implementation.
- For every possibility we will discuss the advantages and disadvantages of using the given data structure. We will see that, in general, we cannot say that there is one single best data structure for a given container.
- The aim of this course is to give a general description of data structures, one that does not depend on any programming language - so we will use the pseudocode language to describe the algorithms.

See: [Lecture01\\_pseudocode.pdf](#)

# ADT, DS and levels of abstraction

different levels of abstraction between domain & representation

ADT      implementation

## Example:

ADT domain	container with elements	
refinement 0	list of elements	<- ADT List
refinement 1	singly linked list of elements	
refinement 2	singly linked list with dynamically allocated nodes	
DS (representation)	...	

## Remarks

1. Refinements involves (permits) some levels of abstraction. For example, we can study the: “single linked list” regardless of the fact that it is dynamically allocated or it has a semi-static representation.
2. The building blocks for data structures are provided by languages.  
(arrays, records, pointers/references)

# Analyzing algorithm

- Algorithms, like computer hardware, are **technology**.

*Total system performance depends on choosing efficient algorithms as much as on choosing fast hardware. Just as rapid advances are being made in other computer technologies, they are being made in algorithms as well.*

- Predict the resources that the algorithm requires
  - computational running time
  - memory
  - communication bandwidth ...

## ➤ Algorithm efficiency

→ **time**

→ **space**

# Many sorting algorithms - example

- bubble sort - before 1960
- insertion sort - before 1960
- variant of insertion sort:
  - shell sort - Donald Shell, 1959
- QuickSort – Hoare, 1960
- HeapSort
- smoothsort a variation of heapsort developed by Dijkstra in 1981
- ...
- QiSort - Matthew Copeland, 2007

# Algorithm analysis

- Algorithm efficiency → time

how we measure

- asymptotic analysis

***running time complexity***

mathematical analysis of the algorithms

***(~ the limit)***

- empirical analysis

***exact running time***

determine exact running time

- fixed implementation
- fixed input

# Algorithm analysis

We must assume a model of the implementation to use.

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The RAM (random-access machine) model

- It is a hypothetical computer model, a very simplified model
- Under the RAM model we measure the run time of an algorithm by counting the number of steps the algorithm takes on a given input instance. The number of steps is usually a function that depends on the size of the input data.

In the RAM model:

- Each simple operation (+, -, \*, /, =, if, function call) takes one time step/unit.
- Loops and subprograms are not simple operations and we do not have special operations (ex. sorting in one instruction).
- Every memory access takes one time step and we have an infinite amount of memory.
- There are fixed-size integers and floating point data types.

# Algorithm analysis. Example

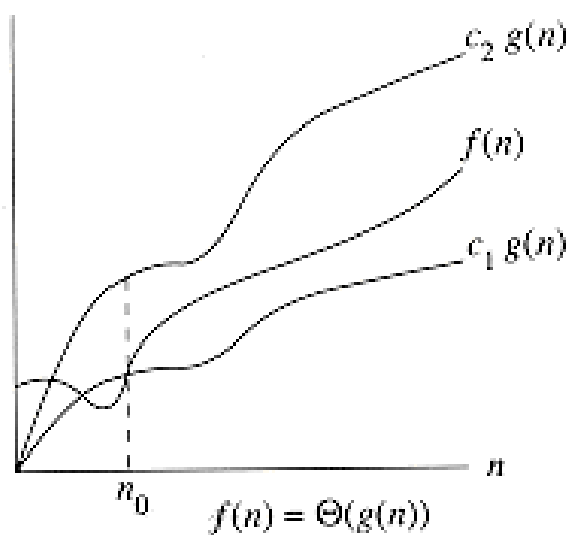
```
subalgorithm something(n) is:  
  //n is an Integer number  
  rez ← 0  
  for i ← 1, n execute  
    sum ← 0  
    for j ← 1, n execute  
      sum ← sum + j  
    end-for  
    rez ← rez + sum  
  end-for  
  print rez  
end-subalgorithm
```

- $T(n) = ?$
- order of growth ?
  - use leading term of  $T(n)$

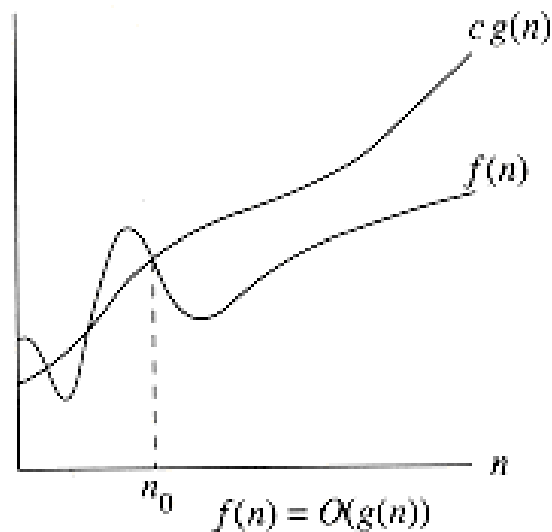
- How many steps does the above subalgorithm take?



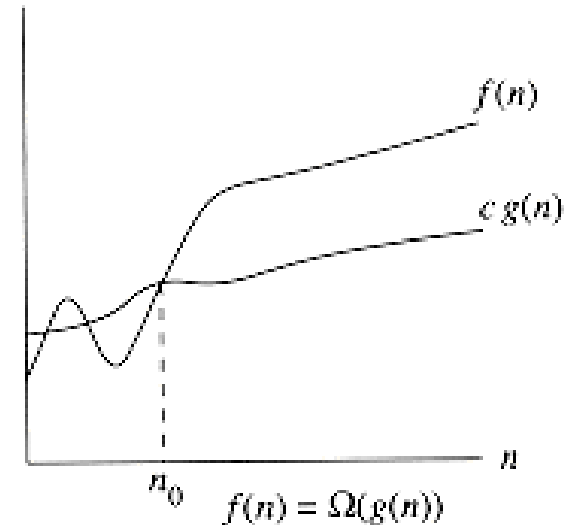
# Asymptotic analysis



(a)

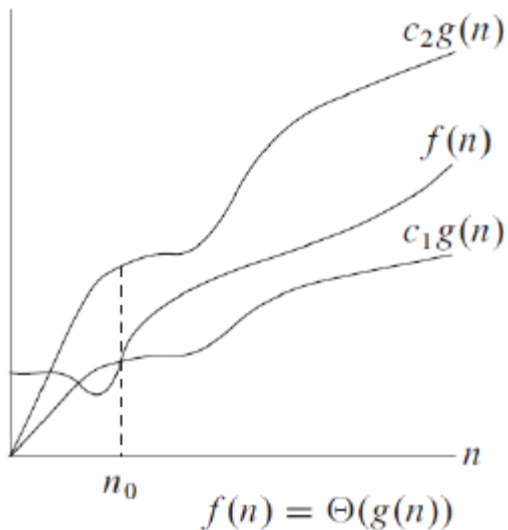


(b)



(c)

# $\Theta$ -notation



(source: Cormen)

## $\Theta$ -notation

For a given function  $g(n)$  we denote by  $\Theta(g(n))$  the set of functions:

$$\Theta(g(n)) = \{f(n) : \text{there exist positive constants } c_1, c_2 \text{ and } n_0 \text{ s. t. } 0 \leq c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n) \text{ for all } n \geq n_0\}$$

- The  $\Theta$ -notation provides an *asymptotically tight bound* for a function: for all values of  $n$  (to the right of  $n_0$ ) the value of the function  $f(n)$  is between  $c_1 \cdot g(n)$  and  $c_2 \cdot g(n)$ .
- We will use the notation  $f(n) = \Theta(g(n))$  or  $f(n) \in \Theta(g(n))$ .

## Alternative definition

$$f(n) \in \Theta(g(n)) \text{ if } \lim_{n \rightarrow \infty} \frac{f(n)}{g(n)}$$

is a nonzero constant (and not  $\infty$ ).

- Consider, for example,  $T(n) = n^2 + 2n + 2$ :
  - $T(n) = \Theta(n^2)$  because  $c_1 \cdot n^2 \leq T(n) \leq c_2 \cdot n^2$  for  $c_1 = 0.5$ ,  $c_2 = 2$  and  $n \geq 3$ .
  - $T(n) = \Theta(n^2)$  because

$$\lim_{n \rightarrow \infty} \frac{T(n)}{n^2} = 1$$

# O-notation

Graphical representation:

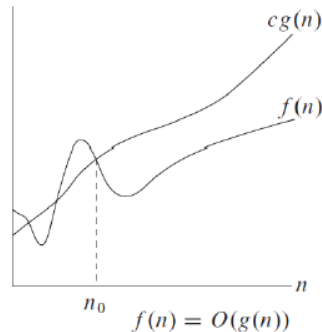


Figure taken from Corman et. al: Introduction to algorithms, MIT Press, 2009

## O-notation

For a given function  $g(n)$  we denote by  $O(g(n))$  the set of functions:

$$O(g(n)) = \{f(n) : \text{there exist positive constants } c \text{ and } n_0 \text{ s. t. } 0 \leq f(n) \leq c \cdot g(n) \text{ for all } n \geq n_0\}$$

- We will use the notation  $f(n) = O(g(n))$  or  $f(n) \in O(g(n))$ .

The O-notation provides an asymptotic upper bound for a function:  
for all values of  $n$  (to the right of  $n_0$ )  
the value of the function  $f(n)$  is on or below  $c \cdot g(n)$ .

## Alternative definition

$$f(n) \in O(g(n)) \text{ if } \lim_{n \rightarrow \infty} \frac{f(n)}{g(n)}$$

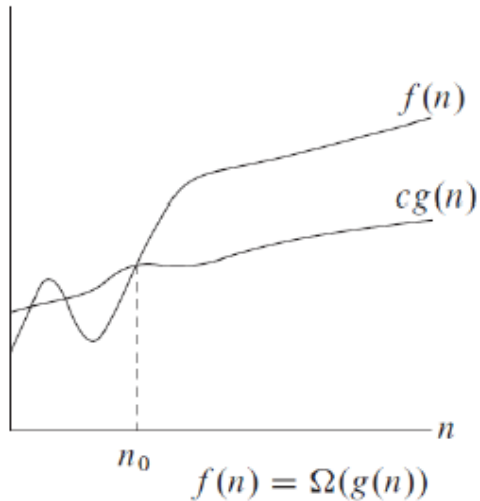
is either 0 or a constant (but not  $\infty$ ).

Consider, for example,  $T(n) = n^2 + 2n + 2$ :

- $T(n) = O(n^2)$  because  $T(n) \leq c \cdot n^2$  for  $c = 2$  and  $n \geq 3$
- $T(n) = O(n^3)$  because

$$\lim_{n \rightarrow \infty} \frac{T(n)}{n^3} = 0$$

# $\Omega$ -notation



(source: Cormen)

## $\Omega$ -notation

For a given function  $g(n)$  we denote by  $\Omega(g(n))$  the set of functions:

$$\Omega(g(n)) = \{f(n) : \text{there exist positive constants } c \text{ and } n_0 \text{ s. t. } 0 \leq c \cdot g(n) \leq f(n) \text{ for all } n \geq n_0\}$$

- The  $\Omega$ -notation provides an *asymptotic lower bound* for a function: for all values of  $n$  (to the right of  $n_0$ ) the value of the function  $f(n)$  is on or above  $c \cdot g(n)$ .
- We will use the notation  $f(n) = \Omega(g(n))$  or  $f(n) \in \Omega(g(n))$ .

## Alternative definition

$$f(n) \in \Omega(g(n)) \text{ if } \lim_{n \rightarrow \infty} \frac{f(n)}{g(n)}$$

is  $\infty$  or a nonzero constant.

Consider, for example,  $T(n) = n^2 + 2n + 2$ :

- $T(n) = \Omega(n^2)$  because  $T(n) \geq c \cdot n^2$  for  $c = 0.5$  and  $n \geq 1$
- $T(n) = \Omega(n)$  because

$$\lim_{n \rightarrow \infty} \frac{T(n)}{n} = \infty$$

# Best Case, Worst Case, Average Case

Think about:

- ... an algorithm that finds the sum of all even numbers in an array.  
How many steps does the algorithm take for an array of length  $n$ ?
  - ... an algorithm that finds the first occurrence of a number in an array.  
How many steps does the algorithm take for an array of length  $n$ ?
- 

**Best - Case** - the best possible case,  
where the number of steps taken by the algorithm is the minimum that is possible

**Worst - Case** - the worst possible case,  
where the number of steps taken by the algorithm is the maximum that is possible

**Average - Case** - the average of all possible cases.

$$\sum_{I \in D} P(I) \cdot E(I)$$

where:

- $D$  is the domain of the problem, the set of every possible input that can be given to the algorithm.
- $I$  is one input data
- $P(I)$  is the probability that we will have  $I$  as an input
- $E(I)$  is the number of operations performed by the algorithm for input  $I$

# Average Case. Example

$$\sum_{I \in D} P(I) \cdot E(I)$$

Think about:

- ... an algorithm that finds the first occurrence of a number in an array.

How many steps does the algorithm take for an array of length  $n$ ?

$D$  would be the set of all possible arrays with length  $n$  ;  $I = ?$

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- Case 1:
  - all the arrays where the first number is the one we are looking for
- Case 2:
  - all the arrays where the first number is not the one we are looking for, but the second is
- ...
- Case  $n$ :
  - all the arrays where the first  $n - 1$  elements are not the one we are looking for, but the last one is
- Case  $n+1$ :
  - all the arrays which do not contain the element we are looking for
- $P(I)$  is usually considered equal for every  $I$

$$\mathbf{AC:} \quad T(n) = \frac{1}{n+1} \sum_{i=1}^n i + \frac{n}{n+1} = \frac{n \cdot (n+1)}{2 \cdot (n+1)} + \frac{n}{n+1} \in \Theta(n)$$

# How to compute complexity in recursion. Example

- In case of a recursive algorithm, complexity computation starts from the recursive formula of the algorithm

- Example:

$$T(n) = \begin{cases} 1, & \text{if } n = 1 \\ 2 * T(\frac{n}{2}) + n, & \text{otherwise} \end{cases}$$

Let  $n = 2^k$

Ignoring the parameter  $x$  we rewrite the recursive branch:

$$T(2^k) = 2 * T(2^{k-1}) + 2^k$$

$$2 * T(2^{k-1}) = 2^2 * T(2^{k-2}) + 2^k$$

$$2^2 * T(2^{k-2}) = 2^3 T(2^{k-3}) + 2^k$$

...

$$2^{k-1} * T(2) = 2^k * T(1) + 2^k$$

---

$$T(2^k) = 2^k * T(1) + k * 2^k$$

$T(1) = 1$  (base case from the recursive formula)

$$T(2^k) = 2^k + k * 2^k$$

Let's go back to the notation with  $n$ .

$$\text{If } n = 2^k \Rightarrow k = \log_2 n$$

$$T(n) = n + n * \log_2 n \in \Theta(n \log_2 n)$$

# How to compute complexity in recursion. Example

Consider the next algorithm.

Decide the recursion formula for it :  $T(n)$

- Assume that  $\text{crossMiddle}(x,a,b)$  has  $T(x, a, b) = b - a$

**function**  $\text{fromInterval}(x, \text{left}, \text{right})$  **is:**

*//x is an array of integer numbers*

*//left and right are the boundaries of the subsequence*

**if**  $\text{left} = \text{right}$  **then**

$\text{fromInterval} \leftarrow x[\text{left}]$

**end-if**

$\text{middle} \leftarrow (\text{left} + \text{right}) / 2$

$\text{justLeft} \leftarrow \text{fromInterval}(x, \text{left}, \text{middle})$

$\text{justRight} \leftarrow \text{fromInterval}(x, \text{middle}+1, \text{right})$

$\text{across} \leftarrow \text{crossMiddle}(x, \text{left}, \text{right})$

$\text{fromInterval} \leftarrow \text{@maximum of justLeft, justRight, across}$

**end-function**



**(Master theorem)**

Let  $a \geq 1$  and  $b > 1$  be constants, let  $f(n)$  be a function, and let  $T(n)$  be defined on the nonnegative integers by the recurrence

$$T(n) = aT(n/b) + f(n) ,$$

where we interpret  $n/b$  to mean either  $\lfloor n/b \rfloor$  or  $\lceil n/b \rceil$ . Then  $T(n)$  has the following asymptotic bounds:

1. If  $f(n) = O(n^{\log_b a - \epsilon})$  for some constant  $\epsilon > 0$ , then  $T(n) = \Theta(n^{\log_b a})$ .
2. If  $f(n) = \Theta(n^{\log_b a})$ , then  $T(n) = \Theta(n^{\log_b a} \lg n)$ .
3. If  $f(n) = \Omega(n^{\log_b a + \epsilon})$  for some constant  $\epsilon > 0$ , and if  $af(n/b) \leq cf(n)$  for some constant  $c < 1$  and all sufficiently large  $n$ , then  $T(n) = \Theta(f(n))$ .

Source: Cormen

**(This is not for the exam)**