ASB
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Program : Bachelors of Computer Applications

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Student's Name	Vipul Giri
Student's Enrollment No	ASB/BCA/20/051
Student's Signature	*Vipul*
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Faculty Name	Prof Soma Das
Signature	

Assignment - 2

Q.1. Use Runge-Kutta method of fourth order, to find $y(0.2)$ for the equation.

$$\frac{dy}{dx} = \frac{y-x}{y+x} \quad y(0)=1, \text{ take } h=0.2.$$

Given, $\frac{dy}{dx} = f(x,y) = \frac{y-x}{y+x}$

$x_0 = 0$; $y_0 = y(x_0) = y(0) = 1$; $h = 0.2$

$x_1 = x_0 + h = 0 + 0.2 = 0.2$

$y_1 = y(x_1) = y(0.2) = ?$

$$K_1 = h\, f(x_0, y_0)$$
$$= 0.2\, f(0, 1)$$
$$= 0.2 \left[\frac{1-0}{1+0}\right]$$

$$\boxed{K_1 = 0.2}$$

$$K_2 = h f\left[x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right]$$
$$= (0.2)\, f\left[0 + \frac{0.2}{2}, 1 + \frac{0.2}{2}\right]$$
$$= (0.2)\, f(0.1, 1.1)$$

$$\cancel{(0.2)\left[\frac{1.1 + 0.1}{1.1 - 0}\right]} = (0.2)\left[\frac{1.1 - 0.1}{1.1 + 0.1}\right]$$

$$\boxed{K_2 = 0.1667}$$

$$K_3 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$$

$$= 0.2 f\left(0 + \frac{0.2}{2}, 1 + \frac{0.1667}{2}\right)$$

$$= 0.2 f(0.1, 1.08335)$$

$$= 0.2 \left[\frac{1.08335 - 0.1}{1.08335 + 0.1}\right]$$

$$= 0.2 \left[\frac{0.98335}{1.18335}\right]$$

$$\boxed{K_3 = 0.1662}$$

$$K_4 = h f(x_0 + h, y_0 + K_3)$$

$$= 0.2 f(0 + 0.2, 1 + 0.1662)$$

$$= 0.2 f(0.2, 1.1662)$$

$$= 0.2 \left[\frac{1.1662 - 0.2}{1.1662 + 0.2}\right]$$

$$= 0.2 \left[\frac{0.9662}{1.3662}\right]$$

$$\boxed{K_4 = 0.1414}$$

$$K = \frac{1}{6}\left[K_1 + 2K_2 + 2K_3 + K_4\right]$$

$$= \frac{1}{6}\left[0.2 + 2(0.1667) + 2(0.1662) + 0.1414\right]$$

$$= \frac{1}{6}\left[1.0072\right]$$

$$\boxed{K = 0.1679}$$

$\therefore y_1 = y(x_1) = y(0.2) = y_0 + k$

$y_1 = y(0.2) = 1 + 0.1679$

$$\boxed{y_1 = y(0.2) = 1.1679}$$

Q. (2) From the give table find $\frac{dy}{dx}$ at $x = 1.2$

x	1.0	1.2	1.4	1.6	1.8	2.0
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891

Soln: $\frac{dy}{dx}$ at $x = 1.2$; $h = 0.2$; $x_0 = 1.2$

$$u = \frac{x - x_0}{h} = \frac{1.2 - 1.2}{0.2} = 0$$

$$\boxed{u = 0}$$

The difference table of given data is,

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
$x_1 =$	1.0	$2.7183_{y_{-1}}$					
			$0.6018_{\Delta y_{-1}}$				
$x_0 =$	1.2	3.3201_{y_0}		$0.1333_{\Delta^2 y_{-1}}$			
			$0.7351_{\Delta y_0}$		$0.0294_{\Delta^3 y_{-1}}$		
$x_1 =$	1.4	4.0552_{y_1}		$0.1627_{\Delta^2 y_0}$		$0.0067_{\Delta^4 y_{-1}}$	
			$0.8978_{\Delta y_1}$		$0.0361_{\Delta^3 y_0}$		$0.0013_{\Delta^5 y_{-1}}$
$x_2 =$	1.6	4.9530_{y_2}		$0.1988_{\Delta^2 y_1}$		$0.0080_{\Delta^4 y_0}$	
			$1.0966_{\Delta y_2}$		$0.0441_{\Delta^3 y_1}$		
$x_3 =$	1.8	6.0496_{y_3}		$0.2429_{\Delta^2 y_2}$			
			$1.3395_{\Delta y_3}$				
$x_4 =$	2.0	7.3891_{y_4}					

Here, $x_0 = 1.2$, $y_0 = 3.3201$, $h = 0.2$, $\Delta y_0 = 0.7351$

$\Delta^2 y_0 = 0.1627$, $\Delta^3 y_0 = 0.0361$, $\Delta^4 y_0 = 0.0080$

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Then,

first derivate

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \left(\frac{dy}{dx}\right)_{u=0}$$

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0\right]$$

$$= \frac{1}{0.2}\left[(0.7351) - \frac{1}{2}(0.1627) + \frac{1}{3}(0.0361) - \frac{1}{4}(0.0080)\right]$$

$$= \frac{1}{0.2}\left[0.7351 - 0.0812 + 0.0120 - 0.002\right]$$

$$= \frac{0.6639}{0.2}$$

$$= 3.319$$

Second derivate

$$\cancel{\frac{d^2}{d}}\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \left(\frac{d^2y}{dx^2}\right)_{u=0}$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0\right]$$

$$= \frac{1}{(0.2)^2}\left[(0.1627) - (0.0361) + \frac{11}{12}(0.0080)\right]$$

$$= \frac{1}{0.04}\left[0.1339\right] \simeq 3.3475$$

Third derivate

$$\left(\frac{d^3y}{dx^3}\right)_{x=x_0} = \left(\frac{d^3y}{dx^3}\right)_{u=0}$$

$$\left(\frac{d^3y}{dx^3}\right)_{x=x_0} = \frac{1}{h^3}\left[\Delta^3 y_0 - \frac{3}{2}\Delta^4 y_0\right]$$

$$= \frac{1}{(0.2)^3}\left[(0.0361) - \frac{3}{2}(0.0080)\right]$$

$$= \frac{1}{0.008}\left[0.0241\right]$$

$$= 3.0125$$

Q. 3. Find the value of the integral $\int_0^1 \frac{dx}{1+x}$ by using Simpson's $\frac{1}{3}$ and $\frac{3}{8}$ Rule.

Solun (3) Given; $\int_0^1 \frac{dx}{1+x}$

$y = \frac{1}{1+x}$, $a=0$, $b=1$

Let, $n=6$

$h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$

$\boxed{h = \frac{1}{6}}$

$x_0 = 0$

$x_1 = x_0 + h = 0 + \frac{1}{6} = \frac{1}{6}$

$x_2 = x_1 + h = \frac{1}{6} + \frac{1}{6} = \frac{\cancel{2}}{\cancel{6}_3} = \frac{1}{3}$

$x_3 = \frac{1}{3} + \frac{1}{6} = \frac{2+1}{6} = \frac{\cancel{3}}{\cancel{6}} = \frac{1}{2}$

$x_4 = \frac{1}{2} + \frac{1}{6} = \frac{3+1}{6} = \frac{\cancel{4}}{\cancel{6}} = \frac{2}{3}$

$x_5 = \frac{2}{3} + \frac{1}{6} = \frac{4+1}{6} = \frac{5}{6}$

$x_6 = x_5 + h = \frac{5}{6} + \frac{1}{6} = \frac{\cancel{6}}{\cancel{6}} = 1$

$y_i = \frac{1}{1+x_i}$

$y_0 = \frac{1}{1+x_0} = \frac{1}{1+0} = 1$

$$y_1 = \frac{1}{1+x_1} = \frac{1}{1+\frac{1}{6}} = \frac{1}{\frac{6+1}{6}} = \frac{1}{\frac{7}{6}} = \frac{6}{7}$$

$$\boxed{y_1 = 0.8571}$$

$$y_2 = \frac{1}{1+x_2} = \frac{1}{1+\frac{1}{3}} = \frac{1}{\frac{3+1}{3}} = \frac{1}{\frac{4}{3}} = \frac{3}{4}$$

$$\boxed{y_2 = 0.75}$$

$$y_3 = \frac{1}{1+x_3} = \frac{1}{1+\frac{1}{2}} = \frac{1}{\frac{2+1}{2}} = \frac{2}{3}$$

$$\boxed{y_3 = 0.6667}$$

$$y_4 = \frac{1}{1+x_4} = \frac{1}{1+\frac{2}{3}} = \frac{1}{\frac{3+2}{3}} = \frac{3}{5}$$

$$\boxed{y_4 = 0.6}$$

$$y_5 = \frac{1}{1+x_5} = \frac{1}{1+\frac{5}{6}} = \frac{1}{\frac{6+5}{6}} = \frac{6}{11}$$

$$\boxed{y_5 = 0.5455}$$

$$y_6 = \frac{1}{1+x_6} = \frac{1}{1+1} = \frac{1}{2}$$

$$\boxed{y_6 = 0.5}$$

Simpson's $\frac{1}{3}$ ~~for~~ Rule formula.

$$\int_0^1 \frac{dx}{1+x} = \frac{h}{3}\left[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)\right]$$

$$\cancel{= \frac{1/6}{3}\left[[1+0.5]+4[0.8571+0.6667+0.6]+2[0.75+0.6]\right]}$$

$$= \frac{1/6}{3}\left[(1+0.5)+4(0.8571+0.6667+0.5455)+2(0.75+0.6)\right]$$

$$= \frac{1}{18}\left[(1.5)+4(2.0693)+2(1.35)\right]$$

$$= \frac{(12.4772)}{18} = 0.69318$$

$$\boxed{\int_0^1 \frac{dx}{1+x} = 0.69318}$$

Simpson's $\frac{3}{8}$ Rule formula.

$$\int_0^1 \frac{dx}{1+x} = \frac{3}{8}h\left[(y_0+y_n)+3(y_1+y_2+y_4+y_5)+2(y_3+y_6)\right]$$

$$= \frac{3}{8}\times\frac{1}{6}\left[(1+0.5)+3(0.8571+0.75+0.6+0.5455)+2(0.6667+0.5)\right]$$

$$= \frac{1}{16}\left[1.5+8.2578+2.3334\right]$$

$$= \frac{(12.0912)}{16}$$

$$\boxed{\int_0^1 \frac{dx}{1+x} = 0.7557}$$

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Q. 4. The following value of the function $f(x)$ for the values of x are given:
$f(1) = 4,\ f(2) = 5,\ f(7) = 5,\ f(8) = 4.$

Find the value of $f(6)$ and also the value of x for which $f(x)$ is maximum or minimum.

Soluⁿ 4. Given:
$$f(1) = 4$$
$$f(2) = 5$$
$$f(7) = 5$$
$$f(8) = 4$$

We write,
$$f(x) = ax^3 + bx^2 + cx + d$$

Substituting the values in $f(1)$, $f(2)$, $f(7)$ & $f(8)$

$$f(1) = a(1)^3 + b(1)^2 + c(1) + d$$
$$f(1) = a + b + c + d$$

$$f(2) = a(2)^3 + b(2)^2 + c(2) + d$$
$$= 8a + 4b + 2c + d$$

$$f(7) = a(7)^3 + b(7)^2 + c(7) + d$$
$$= 343a + 49b + 7c + d.$$

$$f(8) = a(8)^3 + b(8)^2 + c(8) + d$$
$$= 512a + 64b + 8c + d$$

Solving above equⁿ we get,

$$a = 0,\quad b = \frac{-1}{6},\quad c = \frac{3}{2},\quad d = \frac{8}{3}$$

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Thus, $f(x) = \frac{-1}{6}x^2 + \frac{3}{2}x + \frac{8}{3}$

differentiating, $f'(x) = \frac{-\cancel{2}}{\cancel{6}_3}x + \frac{3}{2}$

$f'(x) = -\frac{x}{3} + \frac{3}{2}$

To find maximum or minimum

$f'(x) = 0 \Rightarrow -\frac{x}{3} + \frac{3}{2} = 0$

$\cancel{+}\frac{x}{3} = \cancel{+}\frac{3}{2}$

$\boxed{x = \frac{9}{2}}$

value of, $f(6) = -\frac{1}{\cancel{6}}(\cancel{6})^{2} + \frac{3}{\cancel{2}}(\cancel{6})^{3} + \frac{8}{3}$

$= -6 + 9 + \frac{8}{3}$

$= \frac{-18 + 27 + 8}{3}$

$= \frac{17}{3}$

$\boxed{f(6) = 5.667}$

Q.5. Using Euler's method, find an approximate value of y corresponding to $x = 1.4$, given $\frac{dy}{dx} = x\sqrt{y}$ and $y = 1$ when, $x = 1$.

Solⁿ Let, $f(x,y) = x\sqrt{y}$

$y_0 = 1$, $x_0 = 1$, $x = 1.4$, Let, $h = 0.1$

$x_0 = 1$	$y_0 = 1$
$x_1 = 1.1$	$y_1 = y_0 + hf(x_0, y_0)$
$x_2 = 1.2$	$y_2 = y_1 + hf(x_1, y_1)$
$x_3 = 1.3$	$y_3 = y_2 + hf(x_2, y_2)$
$x_4 = 1.4$	$y_4 = y_3 + hf(x_3, y_3)$
	$y_5 = y_4 + hf(x_4, y_4)$

$y_1 = 1 + (0.1) f(1,1) = 1 + (0.1)[1\sqrt{1}] = 1 + 0.1$

$\boxed{y_1 = 1.1}$

$y_2 = 1.1 + (0.1) f(1.1, 1.1) = 1.1 + 0.1(1.1\sqrt{1.1}) = 1.1 + 0.1049$

$\boxed{y_2 = 1.2049}$

$y_3 = 1.2049 + 0.1 f(1.2, 1.2049) = 1.2049 + 0.1(1.2\sqrt{1.2049})$

$\boxed{y_3 = 2.5222}$

$y_4 = 2.5222 + 0.1 f(1.3, 2.5222) = 2.5222 + 0.1(1.3\sqrt{2.5222})$

$\boxed{y_4 = 2.7289}$

$\cancel{y_5 = 2.7289 + 0.1 f(1.4, 2.7289) = 2.7289}$

$y_5 = 2.7289 + 0.1 f(1.4, 2.7289) = 2.7289 + 0.1(1.4\sqrt{2.7289})$

$\boxed{y_5 = 2.9602}$ Ans.

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Q. 6. Using Picard's method, find a solu^n of ~~dy~~ $\frac{dy}{dx} = 1+xy$ upto the third approximation, when $x_0 = 0$, $y_0 = 0$

Solu^n (6) $f(x,y) = \frac{dy}{dx} = 1+xy$; $x_0 = 0$; $y_0 = 0$

By Picard's method of successive approximation we have,

$$y = y_0 + \int_{x_0}^{x} f(x,y) \cdot dx$$

First approximation:

$$y_1 = y_0 + \int_{x_0}^{x} f(x, y_0)\, dx$$

$$\therefore\ y_1 = 0 + \int_0^x [1 + x(0)]\, dx$$

$$= \int_0^x dx$$

$$= |x|_0^x$$

$$\boxed{y_1 = x}$$

Second approximation:

$$y_2 = y_0 + \int_{x_0}^{x} f(x, y_1)\, dx$$

$$= 0 + \int_0^x (1 + x \cdot x)\, dx$$

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$$= \int_0^x (1+x^2)\,dx$$

$$= \left| x + \frac{x^3}{3} \right|_0^x$$

$$\boxed{y_2 = x + \frac{x^3}{3}}$$

Third approximation:

$$y_3 = y_0 + \int_{x_0}^{x} f(x, y_2)\,dx$$

$$= 0 + \int_0^x \left\{1 + x\left(x + \frac{x^3}{3}\right)\right\} dx$$

$$= \int_0^x \left(1 + x^2 + \frac{x^4}{3}\right) dx$$

$$= \left| x + \frac{x^3}{3} + \frac{x^5}{(3)5} \right|_0^x$$

$$\boxed{y_3 = x + \frac{x^3}{3} + \frac{x^5}{15}}$$