Debt valuation

1. Introduction

1.1. Chapter overview

This chapter starts with some calculations to determine the relative attractiveness of bonds.

You will learn the difference between the price you see quoted for a bond, known as the **clean** price, and the price you actually pay if you buy the bond, known as the **dirty** price.

The chapter finishes by taking you through some of the influences on bond prices. You will then be able to identify the different features of bonds that may appeal to different types of investor.

1.2. Learning outcomes

On completion of this module, you will:

Debt valuation: the basics

- 12.2.4 Calculate the price of a fixed income security given its maturity, coupon and yield
- 12.3.10 Define credit risk as it affects bonds
- 12.3.11 Identify the role and drawbacks of the major credit rating agencies
- 12.3.12 Interpret the key classes of rating on the scales published by the major rating agencies
- 12.3.14 Identify key features and financial ratios considered by credit rating agencies in conducting a corporate rating

Prices and yields

- 12.4.1 Define and calculate: flat yield, gross redemption yield (GRY), net redemption yield (NRY), grossed-up NRY
- 12.4.2 Explain when each of the above measures may be appropriate to use
- 17.1.5 Explain how returns are decomposed and attributed to fixed income
- 12.3.3 Identify the two components of interest rate risk (price and reinvestment risk)
- 12.3.4 Identify the nature of the relationship between yield and price
- 12.2.3 Explain clean (quoted) and dirty pricing

Factors affecting bond prices

- 12.3.5 Analyse the factors that affect the sensitivity of a bond's price to a change in required yield
- 12.3.6 Define and calculate the (Macaulay) duration of a bond
- 12.3.7 Define and calculate the modified duration of a bond
- 12.3.8 Calculate, given the duration of a bond, the change in price given a change in required yield

- 12.3.9 Explain the convexity error that arises from using duration to estimate a change in bond price using duration

Debt valuation: yield curves

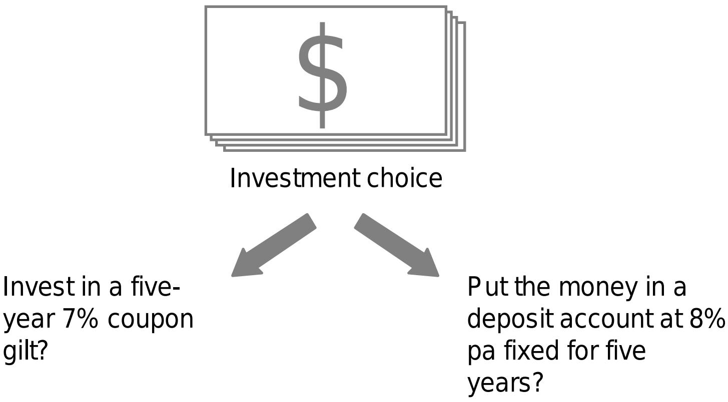
- 12.4.3 Define the yield curve
- 12.4.4 Explain the theories that contribute to explaining the shape of the yield curve
- 12.4.5 Define and calculate forward and spot interest rates
- 12.4.6 Explain the relationship between forward rates, spot rates and the GRY

2. Debt valuation: the basics

2.1. Basic calculations

The value/price of a bond is calculated by comparing the opportunity cost of other investments.

For example, consider an investor who has a choice between investing in a 7% coupon, five-year gilt or an 8%, five-year deposit account:

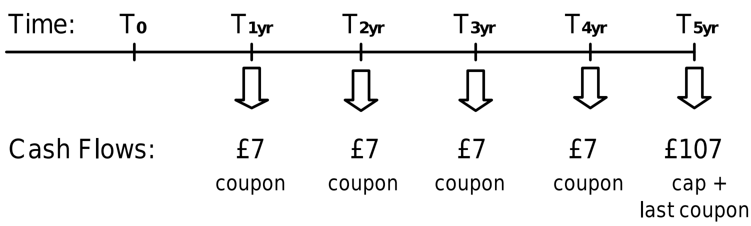


The question is: how much should the investor pay for the gilt (per £100 nominal), given the alternative of the bank deposit?

The investor would want a return of (at least) 8% from his money. Paying the nominal value of £100 would only generate a 7% return (7% coupon).

The value of the bond would therefore be less than £100 - but how much less?

One way to look at a bond is as a stream of predictable cash flows:



If interest rates are 8%, the investor can calculate the present values of the cash flows to arrive at the bond's fair value; in this instance, £96 per £100 nominal.

Time:	T_0	T_{1yr}	T_{2yr}	T_{3yr}	T_{4yr}	T_{5yr}
Present val:		$\frac{£7}{(1+0.08)}$	$\frac{£7}{(1+0.08)^2}$	$\frac{£7}{(1+0.08)^3}$	$\frac{£7}{(1+0.08)^4}$	$\frac{£107}{(1+0.08)^5}$
		£6.48	£6.00	£5.56	£5.14	£72.82

Present value of all cash flows: £96.00
(fair value of bond)

The fair value of a bond is therefore the price that would produce a return of 8%.

A similar but quicker way to value the bond is to look at it as an annuity, the coupons being the annuity payments. The present value of the capital component is calculated separately and added to the total.

$$\text{Bond price} = \underbrace{£\text{coupon} \times \frac{1}{r}\left[1 - \frac{1}{(1+r)^n}\right]}_{\text{Present value of all coupons to the bond}} + \underbrace{\frac{CAP}{(1+r)^n}}_{\text{Present value of the capital amount}}$$

This can be demonstrated using the example above.

$$£7 \times \frac{1}{0.08}\left[1 - \frac{1}{(1+0.08)^5}\right] + \left[\frac{£100}{(1+0.08)^5}\right]$$

$$\left(£7 \times \frac{1}{0.08} \times 0.3194\right) + \left[\frac{£100}{(1+0.08)^5}\right]$$

$$\left(\frac{£7}{0.08} \times 0.3194\right) + £68.06$$

$$£27.94 + £68.06$$

$$£96.00$$

2.2. Credit ratings

Introduction

An investor who purchases a gilt will be confident of achieving the cash flows expected, as the gilt is backed up by the UK Government. They will therefore be willing to achieve a total return in line with other riskless investments. This is known as the risk free rate.

An investor who invests in a corporate bond is taking on the risk that the corporate issuer will not honour its obligations, or default. To cover this risk the investor will demand a higher return than the risk free rate. The total return demanded will therefore consist of the risk free rate plus a premium for the risk taken on.

Different bond issues are associated with differing levels of default risk. A convenient way of assessing risk is by reference to the credit rating of the bond.

Credit rating agencies

Credit rating agencies, such as Moody's and Standard and Poor's, express forward-looking opinions about the credit-worthiness of issuers and issues, and create a relative ranking of credit-worthiness. Those with higher ratings are judged to be more creditworthy than issuers and obligations with lower credit ratings.

Credit-worthiness has many variables and credit ratings attempt to condense all these into a single rating symbol within a simple scale.

The term credit-worthiness refers to the question of whether a bond or other financial instrument will be paid according to its contractual terms.

Credit ratings: Summary

	BB	Ba
Speculative grade	B	B
	CCC	Caa
	CC	Ca
	C	C
	D	D

Explanation

A Moody's rating of Aaa, Aa, A or Baa indicates an investment grade bond; other bonds are regarded as 'speculative' (or non-investment grade).

The ratings of Moody's are also subdivided, this time into 1, 2, and 3. For example Aa1, Aa2, Aa3, A1 are in decreasing order.

A Standard and Poor's (S&P) rating of AAA, AA, A or BBB indicates an investment grade bond; other bonds are regarded as speculative.

The ratings of S&P are typically subdivided into + / -. For example, AA+, AA, AA-, A+ are in decreasing order.

Assessment process

Financial parameters mapped to rating levels based on company-specific business risk. Ratings are forward-looking and take into account current and future business and financial risk often driven by management strategy.

The likelihood of default - encompassing both capacity and willingness to pay - is the single most important factor in any assessment of the credit-worthiness.

Therefore, higher ratings on issuers and obligations reflect an expectation that the rated issuer or obligation should default less frequently than issuers and obligations with lower ratings, all other things being equal.

Other factors

Beyond likelihood of default, there are other factors that may be relevant.

One such factor is the payment priority of an issue following default. Senior secured debt has the highest priority, followed by senior unsecured debt and lastly subordinated debt.

Another factor is the projected recovery that an investor would expect to receive if an obligation defaults. The higher the projected recovery rate, the better the credit rating.

A third factor is financial stability. The more stable a company's finances are, the higher the rating they will achieve.

Summary

When conducting a credit rating assessment, rating agencies typically focus on credit risk and business risk by reviewing the following factors:

- Financial statement analysis, including capital structure, leverage and asset protection
- Macro-economic forecasts
- Industry trends
- Regulatory developments
- Management quality
- The bond's covenant

A number of financial ratios are key to ratings analysis:

- The ability to meet payments, defined as 'profit before interest and tax/interest payable'
- Indebtedness, defined as 'interest-bearing debt/ordinary shareholders' funds'
- Profitability, defined as: operating income/sales

3. Prices and yields

3.1. Introduction

Yields are a way of identifying the return achieved from a particular financial instrument. The two main yield calculations used in the bond market are the Gross Redemption Yield and the Flat Yield.

3.2. Flat yield

The flat yield is also known as the interest yield. The flat yield measures the return achieved from receiving the **coupons** alone.

For example, if £95 were paid for a bond with a 7% coupon, the flat yield would be calculated as follows:

An investor buys a gilt that has a 7% coupon and is trading at a market price of £95 (per £100 nominal value). Calculate the flat yield.

Example

$$\text{Flat yield} = \frac{\text{Gross annual coupon}}{\text{Market price}} \times 100\%$$

$$\frac{£7}{£95} \times 100\%$$

$$= 7.4\%$$

3.3. Redemption yields

Gross redemption yield (GRY)

Also called the Yield-to-Maturity, the GRY represents the **total** return achieved from a bond assuming it is held to redemption.

It therefore takes account of:

- Coupons received
- Coupons re-invested (assuming no re-investment rate risk)
- Capital gain/loss on redemption

The GRY is simply the discount rate that equates the price of a bond to the present value of its future cash flows.

$$\text{Bond price} = £\text{coupon} \times \frac{1}{r}\left[1-\frac{1}{(1+r)^n}\right]+\frac{CAP}{(1+r)^n}$$

Where 'r' is the gross redemption yield

In the previous example where we calculated the price of a five-year, 7% coupon bond to be £96 - the gross redemption yield ('r' in the formula) was 8%.

Another way to calculate this is using a simplified method to estimate the GRY. For the exam, this is an accepted way to achieve the GRY in the questions given.

An investor buys a gilt with the following characteristics.

- 7% coupon
- Trading at a market price of £95 (per £100 nominal value)
- 8 years to redemption

Calculate the gross redemption yield.

$$\text{Flat yield} = \frac{£7}{£95} \times 100\% = 7.4\%$$

+ +

$$\text{Profit yield} = \frac{£5 \text{ profit} / 8 \text{ years}}{£95} \times 100\% = 0.7\%$$

Gross Redemption Yield **8.1%**

Comments

The sum of the flat yield and the profit yield gives the gross redemption yield. The profit yield is spread over the life of the bond in order to annualise the return.

When the bond is trading **below** par (as in the example), i.e. below £100, the GRY will be **greater** than the flat yield.

When the bond is trading **above** par, the GRY will be **less** than the flat yield (because a capital loss is made on redemption).

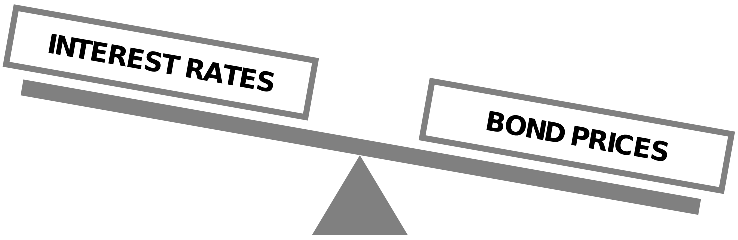
The GRY would be a useful indicator to a non-taxpayer who intends to hold a gilt to redemption (e.g. a charity).

The calculation of GRY requires a known return and a redemption date. It cannot be calculated for securities such as undated stocks and warrants.

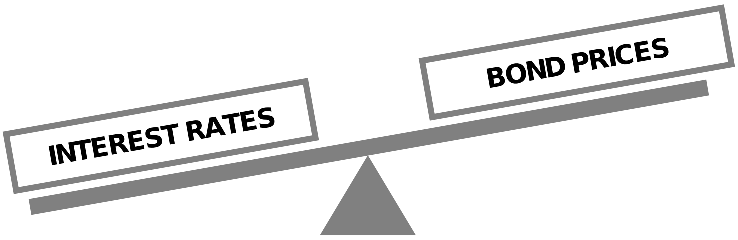
When the GRY of a bond increases (for example, if interest rates rise), the price of the bond decreases and vice-versa, i.e. there is an **inverse** relationship between bond prices and yields/interest rates.

This relationship is illustrated below.

When interest rates rise bond prices fall:



When interest rates fall bond prices rise:



Net redemption yield (NRY)

Net redemption yield considers the tax implication on the bond. These are as follows:

- Income tax payable on the coupon
- No tax payable on the gain

In order to calculate the net redemption yield, the coupon will need to ba adjusted for tax:

- **Net coupon = Gross coupon x (1 – t)**

We can then use the GRY formula, but use the net coupon instead of the gross coupon.

NRY is useful for investors who intend to hold bonds to redemption , but are liable for tax on their income. Example would be collective investment schemes, investment trusts and insurance companies.

Grossed up net redemption yield

This is typically used for zero-coupon bonds, where the total return can be treated as interest income. If the net return on the bond for a higher rate (40%) taxpayer were 5.67%, the grossed up NRY would be:

- **Grossed up NRY = NRY / (1-t)**

In our example:

- **Grossed up NRY = 5.67 / 0.6 = 9.45%**

Decomposing total return on a bond

The gross redemption yield is an estimate of the return that an investor will achieve if they buy the bond and hold to redemption. It assumes that the coupons are reinvested at the prevailing rate and, more importantly, that nothing changes other than time. Things do change, though, so the actual return achieved may differ from the GRY.

The effects of the total return on a bond can be broken into four basic components:

Yield to maturity effect– The yield that we would generate if there were no changes in interest rates or other factors. This is often referred to as the income effect, i.e. the predictable income generated through the bond.

Interest rate effect – this looks at the impact of a movement in the yield curve on the bond returns

Sector/quality effect – Sector effect looks at the impact of a change in the sector from which the issuer come. For example, the economic cycle can have an impact on a whole sector rather than just one company within that sector. The quality effect is the impact on the return of an upgrade or downgrade to the bond's credit rating.

Residual effect – this is what is left over after the other factors have been considered.

In summary:

- Yield to maturity effect
- **PLUS** Interest rate effect
- **PLUS** Sector/quality effect
- **PLUS** Residual effect
- **EQUALS** Total return

3.4. Clean vs. dirty prices

The concept of accrued interest

Quoted bond/gilt prices, such as the prices in the Financial Times, are 'clean' prices. The clean price does not equal the actual ('dirty') price paid in the market.

The difference between the clean and dirty price is accrued interest.

When buying a bond, an investor not only pays the 'true' price of the bond, but also the interest that has accrued to date.

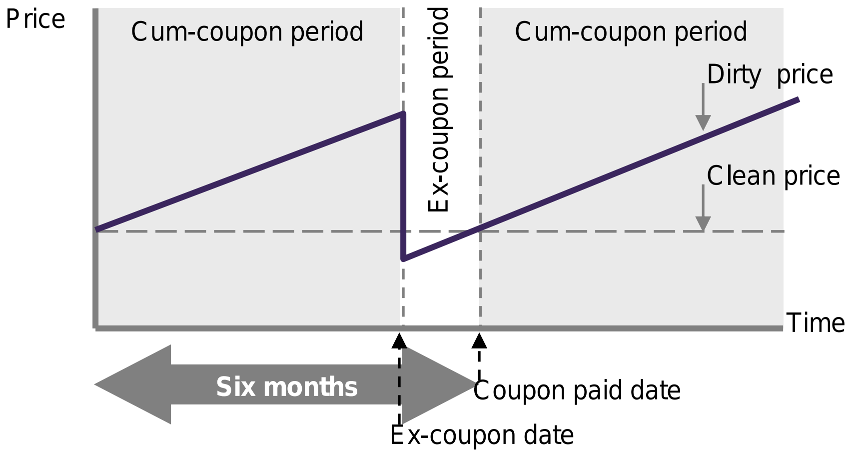
The total amount paid is called the 'dirty price'.

During the cum coupon period - the six-monthly period between coupon payment dates - the dirty price keeps increasing as interest accrues on a daily basis, i.e. clean price + one day's accrued interest, clean price + two days' accrued interest, clean price + three days' accrued interest etc.

The amount actually paid for the bond (the dirty price) consequently increases over time even though the clean price may be constant.

This process will continue until the ex-coupon date - the date after which the buyer is **not** entitled to receive the next coupon payment - when accrued interest is no longer an issue, and the dirty price falls back down to the clean price.

The ex-coupon date



The ex-coupon date is the date after which a buyer of a bond is not entitled to receive the next coupon payment.

Should an investor buy the bond in the ex-coupon period, the dirty price (i.e. the amount paid) will actually be slightly lower than the clean price.

Consider the purchase of a bond in the ex-coupon period five days before the coupon is actually paid.

The purchaser would expect to be entitled to five days' worth of interest when the payment date eventually arrives.

However, because the purchaser bought the bond in the ex-coupon period, they will actually be paid nothing.

To compensate the purchaser for this, the price the purchaser pays for the bond will be the clean price **less** five days' worth of interest.

This is why the dirty price is less than the clean price in the ex-coupon period.

Note: the dirty price will **equal** the clean price on the coupon payment date itself.

Summary

- During the cum coupon period, the dirty price exceeds the clean price
- On the ex-coupon date, the dirty price drops below the clean price
- During the ex-coupon period, the dirty price is less than the clean price
- On the coupon payment date, the dirty and clean prices are equal

Accrued interest on UK Government gilts is calculated on the basis of the actual number of calendar days in the year and the actual number of calendar days in each month of the coupon period. This basis is described as 'actual/actual'.

Clean and dirty price: example

An investor sells a 6% semi-annual bond on the 24 March at a clean price of £107. If the coupon dates are 1 January and 1 July, what would be the dirty price received by the seller?

We saw above that the dirty price is the clean price plus the accrued interest. We have been given the clean price, £107; we now need to calculate the accrued interest.

In the UK, accrued interest is calculated on an actual over actual basis, so the calculation would run as follows:

1. Calculate the number of days held:

   2 January to 25 March

   30 + 28 + 25 = 83 days

Note: we assume it is not a leap year and we include the settlement date.

2. Calculate the number of days in the coupon period:

Note that for the exam we will always assume 182.5 days in a six-month period.

3. Calculate the accrued interest percentage. The proportion the investor is entitled to is 83 / 182.5 of the semi-annual coupon, or £3.

This gives us:

$$\frac{83}{182.5} \text{ x £3} = \text{£1.38}$$

The dirty price is, therefore, the clean price, £107, plus the accrued interest, £1.36.

£107 + £1.36 = £108.36

4. Factors affecting bond prices

4.1. Introduction

We have already seen that there is an inverse relationship between bond prices and interest rates:

However, the degree to which the price moves is determined by three other factors - its coupon, the time to redemption and the required yield. Each of these factors is considered below:

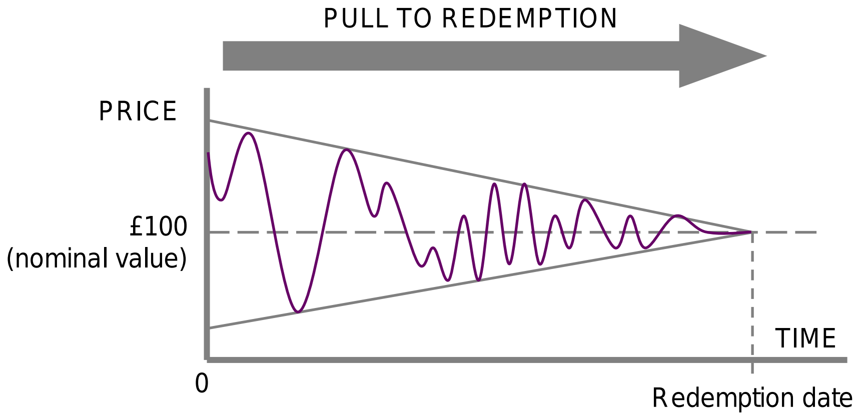
4.2. Coupons

The size of the coupon on the bond will affect the degree of the price movement. The smaller the coupon the more the bond's price will move for a given interest rate change, i.e. low coupon bonds are more volatile than high coupon bonds.

This is because, with a small coupon, more of its return is locked up in the redemption payment that is furthest away in time, and it is therefore more exposed to interest rate changes.

4.3. Remaining life

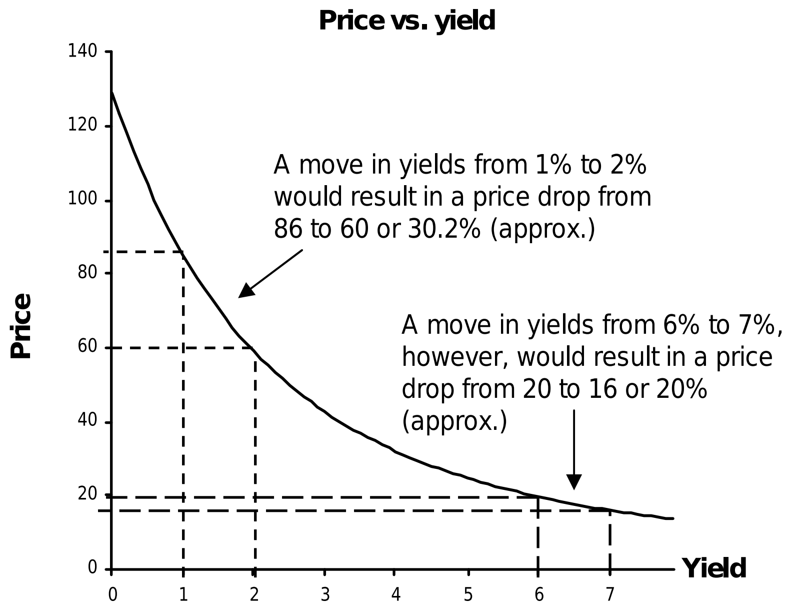
For two bonds with the same coupon, but different redemption dates, it will be the longer dated that is most volatile.



4.4. Required yield

The sensitivity of a bond's price will change as its yield changes.

The lower the yield of a bond, the more sensitive it is to any changes in that yield.



4.5. Macaulay duration

Duration is a measure that reveals how sensitive a bond is.

Macaulay Duration (or the economic life of a bond) is the weighted average maturity of a bond, where the weights are the relative discounted cash flows in each period.

In other words:

$$\text{Duration} = \frac{\sum(\text{Present val of cash flow x time to cash flow})}{\sum \text{Present value of cash flow}}$$

i.e. the bond's price

Duration is expressed in years. The higher the duration, the more sensitive the bond's price.

Time ▶	T0	T1	T2	T3	T4	T5
Cash flows		£7	£7	£7	£7	£107
Present value (PV) of cash flows		$\frac{£7}{1.08}$ ⇩ £6.48	$\frac{£7}{(1.08)^2}$ ⇩ £6.00	$\frac{£7}{(1.08)^3}$ ⇩ £5.56	$\frac{£7}{(1.08)^4}$ ⇩ £5.14	$\frac{£107}{(1.08)^5}$ ⇩ £72.82
Time to cash flows (t)		1yr	2yr	3yr	4yr	5yr
PV x t		£ 6.48yr	£ 12.00yr	£ 16.68yr	£ 20.56yr	£364.10yr

$$\frac{\sum(\text{PV of cash flows x time to cash flows})}{\sum \text{PV cash flows}} = \frac{£419.82\text{yr}}{£96.00} = 4.37\text{yrs}$$

$$\text{Macaulay Duration} = 4.37\text{yrs}$$

For a zero coupon bond the Macaulay duration will be equal to the bonds maturity i.e. a five-year, £100NV, zero coupon bond will have a Macaulay duration of five years.

4.6. Modified duration

Macaulay duration is a **relative** measure. It enables us to determine the riskiest bond from a list of bonds, i.e. a bond with a duration of four years will be more sensitive to changes in interest rates than a bond with a duration of three years.

However, it does not **quantify** the sensitivity. To achieve this, modified duration is required.

The modified duration of a bond estimates how much a bond's price will change if there is a 1% (100bp) change in yield.

Modified duration can be calculated using the following formula:

$$\text{Modified Duration} = \frac{D}{(1+r)}$$

Where D is the bond's duration and r is its present yield.

Applying the formula to the example considered earlier:

Calculate the Macaulay Duration of a five-year 7% coupon bond, with interest rates at 8%.

The modified duration of a bond with a duration of 4.37 and a yield of 8%:

$$\text{Modified Duration} = \frac{D}{(1+r)}$$

$$= \frac{4.37}{1.08}$$

$$= 4.05$$

If the bond’s yield moved up by 0.1% (to 8.1%), the price change can be calculated as follows:

The formula for calculating the price change of a bond given its modified duration is:

$$\Delta P = -MD \times \Delta r \times P$$

Where

ΔP is the change in the bond's price

$-MD$ is the bond's modified duration

Δr is the change in interest rates (or the bond's yield)

P is the bond's current price

Inserting the numbers from the example above using a bond with a value of £96.00:

$= (-4.05) \times 0.001 \times 96.00$

(Notice that a negative value of the bond’s modified duration is used this is to account for the negative price/yield relationship)

$= -0.3888$ or -0.39

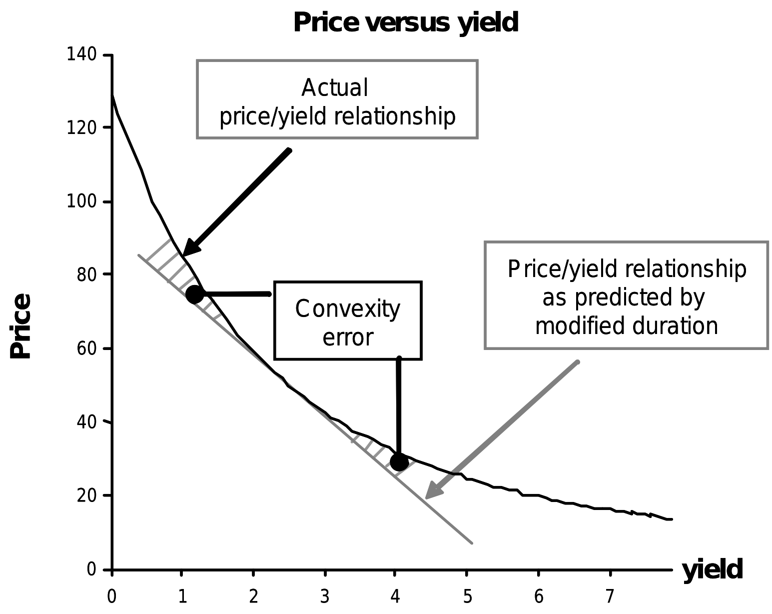
The bond’s price would fall by £0.39, giving a new price of £95.61

Convexity

Modified duration is a linear assumption and does not do a perfect job of predicting the change in price resulting from a change in yield. It over-estimates price falls and under-estimates price rises, when the change in yield is large.

This margin of error is due to the curved, or non-linear, nature of the price yield relationship. The degree of curvature is referred to as the bond's **convexity**. This relationship can clearly be seen in the diagram below.

The convexity of a bond is used to calculate a **convexity adjustment**. This is added to the modified duration calculated price to give a more accurate answer.



Whilst the use of a linear model simplifies the calculation, the approximation leads to a disparity between the calculated value and actual change (convexity error). As the modified duration line is always below the convex line the estimated price for the bond will always be lower than the actual new price after a change in yields. The difference is small for smaller change in yields and increased for larger changes in yields. This means that our estimate of price change using duration will always be less accurate for larger changes in yields.

5. Yield curves

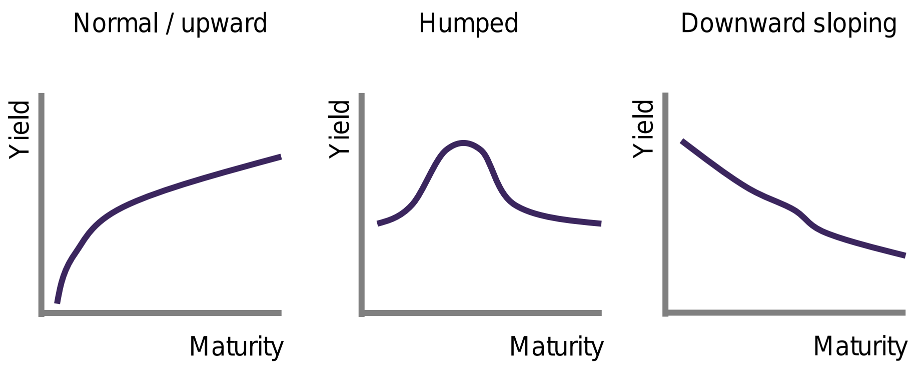
The spot rate is the geometric average of the rate from now until a given maturity. The yield (spot rate) offered by bonds varies according to their maturity dates.

A forward rate is the expected future short term interest rate.

The yield is a weighted average of the spot rates.

The yield curve (or 'term structure of interest rates') illustrates the relationship between maturity dates and yields.

The yield curve for a particular bond market is the result of plotting the yields offered on varying bonds against the maturities of those bonds.



5.1. Yield curve theories

There are three theories that attempt to justify the shape of the yield curve. Each of these is outlined below:

- Liquidity preference theory is based on the assumption that longer dated bonds are more risky than near dated bonds. For this reason investors will require additional yield to compensate them for purchasing a longer dated bond
- Market segmentation theory - the premise here is that there are different appetites for different maturities of bonds. If a particular maturity is popular in the market the bond's price will be higher and therefore its yield would be lower
- Pure expectations theory - this states that 'the long-term rate is a geometric average of expected future short-term rates (forward rates)'. A forward rate is an expected future interest rate i.e. the six months' rate in a year from now. It can be seen as the future interest rate inferred from the term structure of the yield curve. In other words, if the market thinks short-term rates are going to increase, long-term rates will be high. Alternatively, if the view is that short-term rates will fall, longer dated yields will come down

5.2. Forward rate, spot rates and the GRY

Definitions

Spot rates are an interest rate for a period of time starting from today. E.g. a 6 month interest rate available today.

Forward rates are an interest rate for a period of time beginning in the future. E.g. a 3 month interest rate beginning in 3 months time.

The gross redemption yield is the cumulative yield gained from holding a bond to redemption and reinvesting all cash flows.

Relationship between rates and yields

For an **upward sloping** yield curve

- Forward rate > spot rate > GRY

If we create a simple two year annual paying bond and an upward sloping yield curve. One year spot rates are 4% and two year spot rates are 5%.

From this, we can decude that the one year forward rate in one year's time must be (approximately) 6%. The logic being that the two year spot rate must be the average of the one year spot rate and the one year forward rate in one year's time:

- (4% + 6%) / 2 = 5%

The GRY, as we saw previously, is the weighted average return based on the 4% and 5% rates achieved on the cashflows. This will clearly be below 5% (let's say 4.85%).

Hence:

- Forward rate (6%) > spot rate (5%) > GRY (approx 4.85%)

For a **downward sloping** yield curve:

- Forward rate < spot rate < GRY

Estimating forward rates

As a result of these relationships current rates can be used to predict future rates using the following equation:

- (1 + A) x (1 + B) = (1 + AB)

Therefore if the one-year rate (A) is 4% and the two-year rate (AB) is 9%, the one-year rate one year from now (B) is 4.8 %. This can be calculated as a rearrangement of the above equation.

- 1.04 x 1.? = 1.09
- 1.09 / 1.04 = 1.048
- 1.048 – 1 = 4.8%

Note – Often with a multiple choice answer a close approximate answer can be arrived at more simply by using the following equation:

- A + B = AB
- 4% + ? = 9%
- 9% - 4% = 5%

6. Debt valuation: summary

6.1. Key concepts

Debt valuation: the basics

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- 12.3.6 Define and calculate the (Macaulay) duration of a bond
- 12.3.7 Define and calculate the modified duration of a bond
- 12.3.8 Calculate, given the duration of a bond, the change in price given a change in required yield
- 12.3.9 Explain the convexity error that arises from using duration to estimate a change in bond price using duration

Debt valuation: yield curves

- 12.4.3 Define the yield curve
- 12.4.4 Explain the theories that contribute to explaining the shape of the yield curve
- 12.4.5 Define and calculate forward and spot interest rates
- 12.4.6 Explain the relationship between forward rates, spot rates and the GRY

Now you have finished this chapter you should attempt the chapter questions.