
6-10 questions

**Answer to the questions on the previous slide:**

B
50 + 0.3 x 130 = 89

C
? + 0.38 x 140 = 100
100 - 0.38 x 140 =  46.8

## 2. Discounting

**The time value of money**

£100 today is worth £100, but what is it worth today if you don't receive it for three years?

- We use compounding to find a terminal value:
  - **Compounding** is used to convert a present value to a terminal value
- We use discounting to find a present value:
  - **Discounting** is used to convert a terminal (or 'future') value to a present value



## Hints

£100 to spend today is typically considered to have greater purchasing power than £100 in three years' time. To encourage us not to spend the money today, banks offer us interest on our deposits and companies offer us interest on bonds and dividends on equity.

## 2. Simple vs. Compound Interest

**Example**: $1,000 is deposited for three years at 6% pa. Calculate the final amount after the three years (**terminal value**).

**Simple** (take interest out):

- $1,000 x 0.06 = $60
- $60 x 3 = $180
- $1,000 + $180 = $1,180

**Compound** (leave interest in):

- $1,000 x 1.06 = $1,060
- $1,060 x 1.06 = $1,123.60
- $1,123.60 x 1.06 = $1,191.016



## Further information

*Reinvestment return*

The reinvestment return is the return an investor achieves after reinvesting their income from an investment. In the example on the slide, this would be:

$1,191.016 - $1,000 = $191.016

## 3. Discounting

**Compounding formula**

$$TV = PV(1+r)^n$$

Where:

TV is the terminal value of the deposit (how much capital and compounded interest there will be in total).

PV is the amount of money to be deposited, or the present value of the deposit.

n is the number of periods the deposit is to run for (the usual period is a year)

r is the rate of interest on the deposit per period.

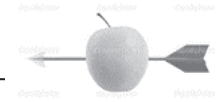
**Example**: Calculate the terminal value of $20,000 invested for 14 years at 5.16% pa.



## Keeping on target

Manvir has deposited £30,000 into a bank account that pays interest quarterly. The interest rate on the account is fixed at 7% p.a., the value after 5 years would be closest to:

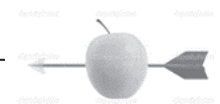
A. £30,525
B. £32,100
C. £42,077
D. £42,443



## Keeping on target

Vilchez has opened a savings account by paying in £10,000. If the interest is paid semi-annually at a rate of 3% p.a. and the account is closed after 2 years and 6 months, the reinvestment return will be closest to:

A. £614
B. £773
C. £10,614
D. £10,767



## Further information

Where the frequency of compounding increases, ***n*** increases and ***r*** reduces in proportion. Where compounding is continuous, it becomes impractical to do this, so the natural exponent ***e*** is used.

$$TV = PV \times e^{(r \times n)}$$

## 3. Discounting

**Calculating present values**

$$PV = \frac{TV}{(1+r)^n}$$

Where:

TV is the amount of money to be received in the future.

PV is the present value of the amount (how much TV is worth now)

n is the number of periods until the amount is received (the usual period is a year)

r is the rate of interest on the deposit per period.

**Example**: Calculate the present value of $20,000 to be paid in five years' time at a discount rate of 4%.



## Keeping on target

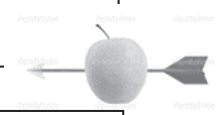
The present value of £100,000 received in 7 years' time at a quarterly compound rate of 4% p.a. is closest to?

A. £83,521

B. £75,991

C. £75,684

D. £73,521



## Keeping on target

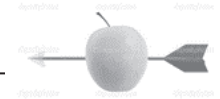
If the interest rate is 3% p.a. calculate the present value of £30.000 to be received in one year time and £40,000 to be received in two years' time?

A. £57,404

B. £55,203

C. £66,830

D. £67,691



**Answer to the questions on the previous slide:**

D

£30,000 x $1.0175^{20}$ = £42,443

B

£10,000 x $1.015^{5}$ = £10,772.84

£10,772.84 - £10,000 = £772.84

## 3. Discounting

**Annuities**

An annuity is a series of equal payments received at the end of each year for a fixed number of years:

E.g.

**A £5,000 3 year annuity with interest rates at 5% p.a.**

| $T_0$ | $T_1$ | $T_2$ | $T_3$ |
|---|---|---|---|
| | £5,000 | £5,000 | £5,000 |
| Present val | $\frac{£5,000}{(1.05)}$ | $\frac{£5,000}{(1.05)^2}$ | $\frac{£5,000}{(1.05)^3}$ |
| | ↓ | ↓ | ↓ |
| | £4,761.90 | £4,535.15 | £4,319.19 |

£13,616.24



## Hint

An annuity is:

- Equal cash payments
- Received or made at regular intervals
- Over a specified period of time.

**Answer to the questions on the previous slide:**

C

£100,000 / $1.01^{28}$ = £75,683.56

C

£30,000 / 1.03 = £29,126.21

£40,000 / $1.03^{2}$ = £37,703.84

£29,126.21 + £37,703.84 = £66,830.05

## 3. Discounting

**Annuities**

Formula for present valuing an annuity:

$$\text{PV annuity} = £X \times \frac{1}{r}\left[1 - \frac{1}{(1+r)^n}\right]$$

Where:

**£*X*** is the annuity payment each year; paid at the end of the year.

**r** is the interest rate (normally annual) over the life of the annuity.

**n** is the number of periods (normally years) that the annuity will run for



## Keeping on target

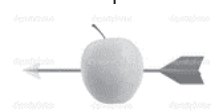
If the investor's rate of return is 8% p.a., and a 10 year £5,000 annuity is required, how much will the annuity cost?

A. £101,500

B. £33,550

C. £31,514

D. £31,224



## Keeping on target

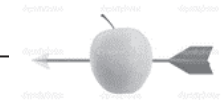
If the investor's rate of return is 4% p.a., and a 25 year quarterly annuity of £2,000 p.a. is required, how much will the annuity cost?

A. £126,057

B. £33,550

C. £31,514

D. £31,244



## 3. Discounting

**Mortgages**

The annuity formula may be used to calculate mortgages or **amortised loans**.

**Example**: A company arranges a £80,000 mortgage over 25 years at 7% pa. Calculate the annual re-payments:

$$\text{£80,000} = \text{Annual payment} \times \frac{1}{r}\left[1 - \frac{1}{(1+r)^n}\right]$$

$$= \text{Annual payment} \times \frac{1}{0.07}\left[1 - \frac{1}{(1.07)^{25}}\right]$$

$$= \text{Annual payment} \times 14.28571 \times 0.81575$$

$$= \text{Annual payment} \times 11.6536$$

So :

$$\frac{\text{£80,000}}{11.6536} = \text{£6,864.84}$$



## Keeping on target

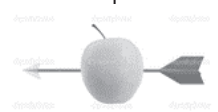
A repayment mortgage of £150,000 is taken out over 25 years. What is the annual repayment required at the end of each year if the rate of interest is fixed at 7% p.a.?

A. £11,872

B. £12,872

C. £13,872

D. £14,872



## Keeping on target

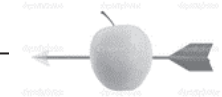
A repayment mortgage of £300,000 is taken out over 20 years. What is the monthly repayment required if the rate of interest is fixed at 6% p.a.?

A. £80.43

B. £139.58

C. £229.45

D. £2149.29



**Answer to the questions on the previous slide:**

B

£5,000 x 1 / 0.08 x (1- 1 / $1.08^{10}$) = £33,550

C

£500 x 1 / 0.01 x (1- 1 / $1.01^{100}$) = £31,514

## 3. Discounting

**Perpetuity**

$$PV \text{ perpetuity} = \frac{£x}{r}$$

Calculate the present value of a 2.5% consolidated stock, assuming a nominal value of £100 and a discount rate of 5%.



## Keeping on target

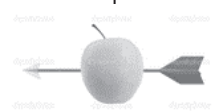
Calculate the present value of a £100 perpetuity at a discount rate of 2% p.a.?

A. £2,000.00

B. £3,000.00

C. £5,000.00

D. £7,000.00



## Keeping on target

A perpetuity of £250 p.a. is valued at £8333.33 the current discount rate is closest to:

A. 7%

B. 5%

C. 3%

D. 1%

**Answer to the questions on the previous slide**

*B*

$$\frac{£150{,}000}{1 / 0.07\ x\ (1 - 1 / 1.07^{25})} = £12{,}871.58$$

D

$$\frac{£300{,}000}{1 / 0.005\ x\ (1 - 1 / 1.005^{240})} = £2149.29$$

## 4. Annualised Rates

**Cost of debt: APR or EAR**

$$APR = (1 + \text{Period rate})^{n} - 1$$

Where :

n = Number of periods per year

Period rate = Annual rate/n

Number of periods per year (n) = 4

Annual rate = 12%

Period rate = Annual rate/n = 12%/4 = 3%

APR = $(1 + 0.03)^4 - 1 = 12.6\%$



## Keeping on target

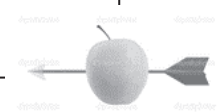
The nominal rate on a loan is 10% p.a. What is the annual percentage rate, if interest is compounded quarterly?

A. 9.87%

B. 10.22%

C. 10.38%

D. 10.66%



**Answer to the questions on the previous slide:**

C

£100 / 0.02 = £5000

C

£250 / ? = £8,333.33

£250 / £8,333.33 = 0.03 (or 3%)

## 5. Investment Appraisal Techniques

**Net present value**

NPV = PVinflows - PVoutflows

NPV $\geq$ 0 then accept the project

Positive NPV increases shareholder value

Mutually exclusive projects

- Accept project with highest NPV



**Answer to the questions on the previous slide:**

C

$$1.025^4 - 1 = 10.38\%$$

## 5. Investment Appraisal Techniques

**Example**:

| Time | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Project A | (2000) | 1500 | 700 | 400 |
| Project B | (2000) | 800 | 1500 | 500 |

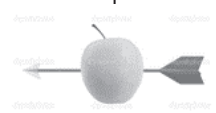
Calculate the NPV of both projects assuming a cost of capital of 10%. Assess which is the best if they are mutually exclusive projects.



## Keeping on target

A-Tech has borrowed at a rate of 3% p.a. to fund a project that costs £10,000 and produces returns of £2,000, £3,000 and £6,000 at the end of each subsequent year. The NPV and project decision is:

A. £260.39 Accept
B. £260.39 Reject
C. (£260.39) Accept
D. (£260.39) Reject



## Keeping on target

B-Tech has a project that costs £37,219.42 at T0 with a further cost of £5,000 at the end of year 2. What is the NPV, if the project returns £19,002 at the end of the first year, £17,000 at the end of the second year, and £11,000 at the end of the third year? Use a cost of capital of 7% p.a.

A. (£4,367)
B. £0
C. £4,267
D. £10,000

## 5. Investment Appraisal Techniques

**Internal rate of return (IRR)**

Given a set of cash flows and an NPV, the IRR is the rate of return that when used to discount the cash flows leaves the NPV equal to zero.

Interpretation of IRR

- Discount rate where NPV = 0
- If IRR > required rate then accept project
- If IRR < required rate then reject project



**Answer to the questions on the previous slide:**

A

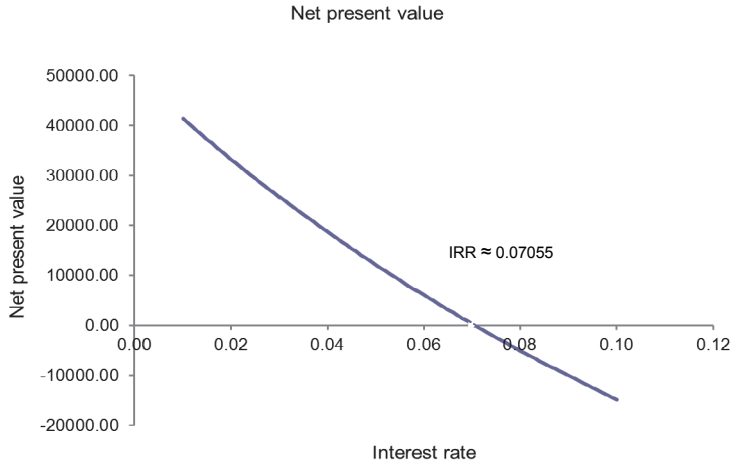
$$\frac{£2,000}{1.03}+\frac{£3,000}{1.03^2}+\frac{£6,000}{1.03^3}-£10,000=£260.39$$

B

$$\frac{£19,002}{1.07}+\frac{(£17,000-£5,000)}{1.07^2}+\frac{£11,000}{1.07^3}-£37,219.42=£0.00$$

## 5. Investment Appraisal Techniques

**Net present value (NPV) and the internal rate of return (IRR)**



## 5. Investment Appraisal Techniques

**Calculating IRR**

**Example**: What is the IRR of a project costing £188.61 today and generating £100 at the end of the first year and a further £100 at the end of the second year?

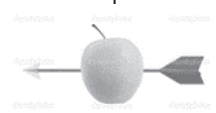
A. 3%
B. 4%
C. 5%
D. 6%



## Keeping on target

A-Tech has a project that costs £8,858.22 that returns £1,000, £3,000 and £6,002 at the end of each year. The IRR is closest to:

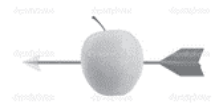
A. 3%
B. 4%
C. 5%
D. 6%



## Keeping on target

B-Tech has a project that costs £39,682.92 at T0 with a further cost of £5,000 at the end of year 1. What is the IRR if the project returns £19,000, £17,000 and £11,000 at the end of years 1, 2 and 3 respectively?

A. 7%
B. 5%
C. 3%
D. 1%



## Further information

Problems with IRR include:

- Ignores quality of earnings. For example:
  - IRR = 5% profit =£10,000
  - IRR = 4.5% profit = £20,000
- Cannot be used where there is a variability in rates


1-2 questions

**Answer to the question on the previous classroom slide**

*Try each in turn. B, 4%, gives an NPV of zero, and hence the IRR is 4%.*

**Answers to the questions on the previous slide:**

C

$$\frac{£1{,}000}{1.05}+\frac{£3{,}000}{1.05^2}+\frac{£6002}{1.05^3}-£8{,}858.22=£0$$

C

$$\frac{£14{,}000}{1.03}+\frac{£17{,}000}{1.03^2}+\frac{£11{,}000}{1.03^3}-£39{,}682.92=£0$$