

## Exotic hadron states

Wei Chen\*, J. Ho and T. G. Steele

*Department of Physics and Engineering Physics, University of Saskatchewan, Saskatoon,  
Saskatoon, SK, S7N 5E2, Canada*

*\*E-mail: wec053@mail.usask.ca*

R. T. Kleiv, B. Bulthuis, D. Harnett and T. Richards

*Department of Physics, University of the Fraser Valley,  
Abbotsford, BC, V2S 7M8, Canada*

Shi-Lin Zhu

*School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking  
University,*

*Beijing 100871, China*

*Collaborative Innovation Center of Quantum Matter,*

*Beijing 100871, China*

*Center of High Energy Physics, Peking University,*

*Beijing 100871, China*

Many charmonium-like and bottomonium-like  $XYZ$  resonances have been observed by the Belle, Babar, CLEO and BESIII collaborations in the past decade. They are difficult to fit in the conventional quark model and thus are considered as candidates of exotic hadrons, such as multi-quark states, meson molecules, and hybrids. In this talk, we first briefly introduce the method of QCD sum rules and then provide a short review of the mass spectra of the quarkonium-like tetraquark states and the heavy quarkonium hybrids in the QCD sum rules approach. Possible interpretations of the  $XYZ$  resonances are briefly discussed.

*Keywords:* Exotic states; Tetraquarks; Hybrids.

### 1. Introduction

In the conventional quark model, mesons are composed of a pair of quark and antiquark ( $q\bar{q}$ ) while baryons are composed of three quarks ( $qqq$ ). Hadrons with different quark contents from  $q\bar{q}$  or  $qqq$  are called exotic states, such as glueballs, hybrids, multiquarks and so on. Although these exotic hadron configurations are allowed in QCD, there was no significant evidence of their existence until recently.

Since 2003, many new charmonium-like and bottomonium-like states were observed experimentally (see Refs.<sup>2-4</sup> for recent reviews). These new states were observed through either the hidden-charm/hidden-bottom or open-charm/open-bottom final states and thus contain a heavy quark-antiquark pair. However, some of these states do not fit in the conventional quark model and are considered as candidates for exotic states. To explore the underlying structures of these  $XYZ$

states, many theoretical speculations have been proposed such as the molecular states, quarkonium-like tetraquark states, quarkonium hybrids and conventional quarkonium states.

Hadronic molecules are loosely bound states composed of two heavy mesons  $(Q\bar{q})(\bar{Q}q)$ , in which  $Q$  represents a heavy quark (charm or bottom quark) and  $q$  a light quark (up, down or strange quark). They are probably bound by the long-range color-singlet pion exchange. The masses of the molecules are slightly lower than some open-flavor thresholds. Some XYZ states, such as  $X(3872)$  and the charged  $Z_c, Z_b$  states, were discovered to be very close to the open-charm/bottom thresholds. They were naturally considered as candidates of the molecular states. Tetraquarks are composed of a pair of diquark and antidiquark  $(Qq)(\bar{Q}\bar{q})$ , which are bound by the colored force between quarks. They can decay into a pair of heavy mesons or one charmonium/bottomonium plus a light meson through rearrangement process. Thus the tetraquarks are expected to be very broad resonances<sup>5</sup>. Hybrids are bound states of a quark-antiquark pair and an excited gluon. The  $Y(4260)$  meson was considered as a compelling candidate of the charmonium hybrid<sup>6-8</sup>. The spectroscopy of these exotic hadrons should be studied systematically to improve our understanding of the relations between the exotic hadrons and the newly observed XYZ states.

In this talk, we focus on the quarkonium-like tetraquark states and the quarkonium hybrid states. We briefly introduce our recent results of the mass spectra of these systems. We then try to understand the nature of some XYZ states using the formalisms of tetraquark states and charmonium hybrids.

## 2. QCD sum rules

In the past several decades, QCD sum rules have been proven to be a very powerful non-perturbative tool to study the hadron structures<sup>2,9-11</sup>. Considering the two-point correlation function induced by two hadronic currents

$$\Pi(q^2) = i \int d^4x e^{iq \cdot x} \langle 0 | T[J(x)J^\dagger(0)] | 0 \rangle, \quad (1)$$

in which  $J(x)$  is an interpolating current carrying the same quantum numbers as the hadrons we want to study. The basic idea of QCD sum rule approach is that this two-point function can be achieved at both the hadron level and the quark-gluon level. A fundamental assumption is the quark-hadron duality, which ensures the equivalence of the correlation functions obtained at these two levels.

At the hadron level, the correlation function is described via the dispersion relation

$$\Pi(q^2) = (q^2)^N \int_{4m_c^2}^{\infty} \frac{\rho(s)}{s^N(s - q^2 - i\epsilon)} ds + \sum_{n=0}^{N-1} b_n (q^2)^n, \quad (2)$$

where  $b_n$  is the unknown subtraction constant which can be removed by taking the Borel transform.  $\rho(s)$  is the spectral function in the narrow resonance approxima-

tion

$$\begin{aligned}\rho(s) &\equiv \sum_n \delta(s - m_n^2) \langle 0 | J | n \rangle \langle n | J^\dagger | 0 \rangle \\ &= f_X^2 \delta(s - m_X^2) + \text{continuum}.\end{aligned}\quad (3)$$

The two-point function  $\Pi(q^2)$  can also be calculated at the quark-gluonic level via the operator product expansion (OPE)

$$\Pi(q) = i \int d^4x e^{iq \cdot x} \langle 0 | T[J(x) J^\dagger(0)] | 0 \rangle = \sum_n C_n(Q^2) O_n, \quad Q^2 = -q^2, \quad (4)$$

in which  $C_n(Q^2)$  are the Wilson coefficients and  $O_n$  are the various QCD condensates, including the quark condensate  $\langle \bar{q}q \rangle$ , gluon condensate  $\langle g_s^2 GG \rangle$ , quark-gluonmixed condensate  $\langle \bar{q}g_s \sigma \cdot Gq \rangle$ , tri-gluon condensate  $\langle g_s^3 fGGG \rangle$ , four quark condensate  $\langle \bar{q}q \rangle^2$ , and dimension-8 condensate  $\langle \bar{q}q \rangle \langle \bar{q}g_s \sigma \cdot Gq \rangle$ . In general, the Wilson coefficients  $C_n(Q^2)$  can be calculated in perturbation theory and expressed in terms of the QCD parameters such as the quark mass and the strong coupling constant  $\alpha_s$ . The long distance non-perturbative effects are included in the various condensates  $O_n$ , which are ordered by increasing dimension in the expansion.

We then can obtain information on hadron properties by equating the two-point correlation functions at these two levels. The mass sum rules are usually established after performing Borel transform of both sides

$$f_X^2 m_X^{2k} e^{-m_X^2/M_B^2} = \int_{4m_Q^2}^{s_0} ds e^{-s/M_B^2} \rho(s) s^k = \mathcal{L}_k(s_0, M_B^2), \quad (5)$$

where  $s_0, M_B$  are the continuum threshold and Borel mass respectively and  $\rho(s) = \frac{1}{\pi} \text{Im}\Pi(s)$ . The lowest lying hadron mass is then extracted as

$$m_X(s_0, M_B^2) = \sqrt{\frac{\mathcal{L}_1(s_0, M_B^2)}{\mathcal{L}_0(s_0, M_B^2)}}. \quad (6)$$

In the next two sections, we will introduce the QCD sum rule study of the mass spectra of the quarkonium-like tetraquark states and quarkonium hybrid states.

### 3. Quarkonium-like tetraquark states

The diquark ( $qq$ ) concept plays a very important role in the study of the tetraquark states. They are the bricks used to construct a tetraquark field ( $qq\bar{q}\bar{q}$ ). The properties of diquark fields, including their spins, parities, flavor, color and Lorentz structures were studied in Refs.<sup>1,12</sup> Tetraquarks ( $qq\bar{q}\bar{q}$ ) are then composed of diquarks and antidiquarks. The low-lying scalar mesons below 1 GeV have been considered as good candidates of the light tetraquark states<sup>13,14</sup>. In the heavy quark sector, some of the recently observed quarkonium-like states were suggested to be candidates of hidden charm/bottom  $Q\bar{Q}q\bar{q}$ -type tetraquark states<sup>15–18,20–22</sup>.

### 3.1. $X(3872)$ and the charged $Z_c, Z_b$ states

$X(3872)$  was the first charmonium-like state discovered in B-factories. It was reported in 2003 by Belle Collaboration in the  $J/\psi\pi^+\pi^-$  final states on the process of  $B^+ \rightarrow K^+ J/\psi\pi^+\pi^-$ <sup>23</sup>. Ten years after that, the LHCb Collaboration determined its quantum numbers as  $J^{PC} = 1^{++}$ <sup>24</sup>.

Recently, the family of the charged quarkonium-like states has become more abundant. The first charged state  $Z(4430)^+$  was observed in the  $\psi(2S)\pi^+$  invariant mass spectrum in the process  $\bar{B}^0 \rightarrow \psi(2S)\pi^+K^-$  by the Belle Collaboration<sup>25</sup> and confirmed recently by the LHCb Collaboration<sup>26</sup>. Later, The BESIII Collaboration reported  $Z_c(3900)^+$ <sup>27</sup>, which was confirmed quickly by the Belle Collaboration<sup>28</sup> and by using CLEO data<sup>29</sup>. The BESIII Collaboration also observed  $Z_c(4025)^{\pm}$ <sup>30</sup> and  $Z_c(4020)^{\pm}$ <sup>31</sup> later. This year, the Belle Collaboration reported two new charged states  $Z_c(4200)^+$ <sup>32</sup> and  $Z_c(4050)^+$ <sup>33</sup>. Moreover, the Belle Collaboration also observed two charged bottomonium-like states  $Z_b(10610)$  and  $Z_b(10650)$ <sup>34</sup>. All these resonances have the exotic flavor content  $c\bar{c}u\bar{d}$  for  $Z_c$  states and  $b\bar{b}u\bar{d}$  for  $Z_b$  states. They are isovector axial-vector states with positive  $G$ -parities. Thus their neutral partners carry the quantum numbers  $I^G J^{PC} = 1^+ 1^{+-}$ .

Table 1. Mass spectra for the  $qc\bar{q}\bar{c}$  and  $qb\bar{q}\bar{b}$  tetraquark states with  $J^{PC} = 1^{++}$ .

	Currents	$s_0(\text{GeV}^2)$	$[M_{\min}^2, M_{\max}^2](\text{GeV}^2)$	$m_X(\text{GeV})$	PC(%)
$qc\bar{q}\bar{c}$ system	$J_{3\mu}$	$4.6^2$	$3.0 - 3.4$	$4.19 \pm 0.10$	47.3
	$J_{4\mu}$	$4.5^2$	$3.0 - 3.3$	$4.03 \pm 0.11$	46.8
$qb\bar{q}\bar{b}$ system	$J_{4\mu}$	$10.8^2$	$8.5 - 9.2$	$10.22 \pm 0.11$	44.6
	$J_{7\mu}$	$10.7^2$	$7.8 - 8.4$	$10.14 \pm 0.10$	44.8
	$J_{8\mu}$	$10.7^2$	$7.8 - 8.4$	$10.14 \pm 0.09$	44.8

Table 2. Mass spectra for the  $qc\bar{q}\bar{c}$  and  $qb\bar{q}\bar{b}$  tetraquark states with  $J^{PC} = 1^{+-}$ .

	Currents	$s_0(\text{GeV}^2)$	$[M_{\min}^2, M_{\max}^2](\text{GeV}^2)$	$m_X(\text{GeV})$	PC(%)
$qc\bar{q}\bar{c}$ system	$J_{3\mu}$	$4.6^2$	$3.0 - 3.4$	$4.16 \pm 0.10$	46.2
	$J_{4\mu}$	$4.5^2$	$3.0 - 3.3$	$4.02 \pm 0.09$	44.6
	$J_{5\mu}$	$4.5^2$	$3.0 - 3.4$	$4.00 \pm 0.11$	46.0
	$J_{6\mu}$	$4.6^2$	$3.0 - 3.4$	$4.14 \pm 0.09$	47.0
$qb\bar{q}\bar{b}$ system	$J_{3\mu}$	$10.6^2$	$7.5 - 8.5$	$10.08 \pm 0.10$	45.9
	$J_{4\mu}$	$10.6^2$	$7.5 - 8.5$	$10.07 \pm 0.10$	46.2
	$J_{5\mu}$	$10.6^2$	$7.5 - 8.4$	$10.05 \pm 0.10$	45.3
	$J_{6\mu}$	$10.7^2$	$7.5 - 8.7$	$10.15 \pm 0.10$	47.6

Could these charged  $Z_c, Z_b$  states and  $X(3872)$  be quarkonium-like tetraquark states? In Refs.<sup>21,35</sup>, we constructed all the tetraquark interpolating currents with  $J^{PC} = 1^{++}, 1^{+-}$  in a systematic way. Then we calculated the two-point correlation

functions and performed QCD sum rules analyses of these systems. The mass spectra of the charmonium-like and bottomonium-like tetraquark states with  $J^{PC} = 1^{++}$  are given in Table 1. For the current  $J_{4\mu}$ , the mass of the  $qc\bar{q}\bar{c}$  state was extracted as  $m_X = 4.03 \pm 0.11$  GeV, which is slightly higher than the mass of  $X(3872)$ . In Table 2, we give the mass spectra for the  $qc\bar{q}\bar{c}$  and  $qb\bar{q}\bar{b}$  tetraquark states with  $J^{PC} = 1^{+-}$ . It was shown that the masses for the charmonium-like tetraquark states were  $m_X = 3.9 - 4.3$  GeV. These masses are consistent with the mass regions of the charged  $Z_c$  states. In other words, our result supports these charged resonances, such as  $Z_c(3900)$ ,  $Z_c(4025)$ ,  $Z_c(4050)$  and  $Z_c(4200)$ , to be the candidates of the charmonium-like tetraquark states. However, it is difficult to differentiate these charged states in the two-point sum rule analysis. Instead, one needs to calculate the three-point functions to study the hadronic three-point interaction vertices<sup>36</sup>. The coupling constants and hadronic decay widths of these states can then be determined.

For the bottomonium-like  $qb\bar{q}\bar{b}$  systems with  $J^{PC} = 1^{++}$  and  $1^{+-}$ , their masses were both extracted as  $m_{X^b} = 10.0 - 10.3$  GeV, which were much lower than the masses of  $Z_b(10610)$  and  $Z_b(10650)$ .

The numerical results for the other channels can be found in Refs.<sup>12,18,20,21,37</sup>.

#### 4. Charmonium and bottomonium hybrid states

The heavy quarkonium hybrids were firstly studied in Refs.<sup>38-40</sup> by using QCD sum rule approach, in which only the leading-order contributions of the perturbative terms and the dimension four gluonic condensate were considered. The mass sum rules of some  $J^{PC}$  channels were unstable and thus these mass predictions were unreliable. Recently, the  $J^{PC} = 1^{--}$ <sup>41</sup>,  $1^{++}$ <sup>42</sup> and  $0^{-+}$ <sup>43</sup> channels have been re-analyzed by including the dimension six tri-gluon condensate. The dimension six contributions have proven to be very important because they stabilize the hybrid sum rules. In Ref.<sup>44</sup>, we have extended the calculations to the remaining channels. After performing the QCD sum-rule analysis, we updated the mass spectra of charmonium and bottomonium hybrids with exotic and non-exotic quantum numbers in Table 3 and Table 4, respectively.

It was shown that the negative-parity states with  $J^{PC} = (0, 1, 2)^{-+}, 1^{--}$  form the lightest hybrid supermultiplet while the positive-parity states with  $J^{PC} = (0, 1)^{+-}, (0, 1, 2)^{++}$  belong to a heavier hybrid supermultiplet, confirming the supermultiplet structure found in the other approaches<sup>45</sup>. The hybrid with  $J^{PC} = 0^{--}$  has a much higher mass which may suggest a different excitation of the gluonic field compared to the other channels. Consistent with the previous results, we found that the  $J^{PC} = 1^{++}$  charmonium hybrid is substantially heavier than the  $X(3872)$ , which seems to preclude a pure charmonium hybrid interpretation for this state. The open-flavour bottom-charm  $\bar{b}Gc$  hybrid systems were also studied in Ref.<sup>46</sup>.

Table 3. Mass spectrum of the charmonium hybrid states.

$J^{PC}$	$s_0(\text{GeV}^2)$	$[M_{\min}^2, M_{\max}^2](\text{GeV}^2)$	$m_X(\text{GeV})$	PC(%)
$1^{--}$	15	2.5 – 4.8	$3.36 \pm 0.15$	18.3
$0^{-+}$	16	5.6 – 7.0	$3.61 \pm 0.21$	15.4
$1^{-+}$	17	4.6 – 6.5	$3.70 \pm 0.21$	18.8
$2^{-+}$	18	3.9 – 7.2	$4.04 \pm 0.23$	26.0
$0^{+-}$	20	6.0 – 7.4	$4.09 \pm 0.23$	15.5
$2^{++}$	23	3.9 – 7.5	$4.45 \pm 0.27$	21.5
$1^{+-}$	24	2.5 – 8.4	$4.53 \pm 0.23$	33.2
$1^{++}$	30	4.6 – 11.4	$5.06 \pm 0.44$	30.4
$0^{++}$	34	5.6 – 14.6	$5.34 \pm 0.45$	36.3
$0^{--}$	35	6.0 – 12.3	$5.51 \pm 0.50$	31.0

Table 4. Mass spectrum of the bottomonium hybrid states.

$J^{PC}$	$s_0(\text{GeV}^2)$	$[M_{\min}^2, M_{\max}^2](\text{GeV}^2)$	$m_X(\text{GeV})$	PC(%)
$1^{--}$	105	11 – 17	$9.70 \pm 0.12$	17.2
$0^{-+}$	104	14 – 16	$9.68 \pm 0.29$	17.3
$1^{-+}$	107	13 – 19	$9.79 \pm 0.22$	20.4
$2^{-+}$	105	12 – 19	$9.93 \pm 0.21$	21.7
$0^{+-}$	114	14 – 19	$10.17 \pm 0.22$	17.6
$2^{++}$	120	12 – 20	$10.64 \pm 0.33$	19.7
$1^{+-}$	123	10 – 21	$10.70 \pm 0.53$	28.5
$1^{++}$	134	13 – 27	$11.09 \pm 0.60$	27.7
$0^{++}$	137	13 – 31	$11.20 \pm 0.48$	30.0
$0^{--}$	142	14 – 25	$11.48 \pm 0.75$	24.1

## Acknowledgments

This project was supported by the Natural Sciences and Engineering Research Council of Canada (NSERC) and the National Natural Science Foundation of China under Grant NO. 11261130311.

## References

1. R. L. Jaffe, *Phys. Rept.* **409**, 1 (2005).
2. Wei Chen, T. G. Steele and Shi-Lin Zhu, *Universe* **2**, NO. 1, 13 (2014).
3. G. T. Bodwin *et al.*, arXiv:1307.7425.
4. R. Faccini, A. Pilloni and A. D. Polosa, *Mod. Phys. Lett. A* **27**, 1230025 (2012).
5. Wei Chen, *et al.*, in *Proceedings of the XV International Conference on Hadron Spectroscopy-Hadron 2013*, arXiv:1311.3763.
6. Shi-Lin Zhu, *Phys. Lett. B* **625**, 212 (2005).
7. E. Kou and O. Pene, *Phys. Lett. B* **631**, 164 (2005).
8. F. E. Close and P. R. Page, *Phys. Lett. B* **628**, 215 (2005).
9. M. A. Shifman, A. I. Vainshtein and V. Zakharov, *Nucl. Phys. B* **147**, 385

- (1989).
10. L. J. Reinders, H. Rubinstein and S. Yazaki, *Phys. Rept.* **127**, 1 (1985).
  11. P. Colangelo and A. Khodjamirian, *Frontier of Particle Physics* **3** (2000).
  12. Meng-Lin Du, Wei Chen, Xiao-Lin Chen and Shi-Lin Zhu, *Phys. Rev. D* **87**, 014003 (2013).
  13. Hua-Xing Chen, Atsushi Hosaka and Shi-Lin Zhu, *Phys. Lett. B* **650**, 369 (2007).
  14. Hua-Xing Chen, Atsushi Hosaka and Shi-Lin Zhu, *Phys. Rev. D* **76**, 094025 (2007).
  15. R. D. Matheus, S. Narison, M. Nielsen and J. A. Richard, *Phys. Rev. D* **75**, 014005 (2007).
  16. L. Maiani, A. D. Polosa and V. Riquer, *Phys. Rev. Lett.* **99**, 182003 (2007).
  17. D. Ebert, R. N. Faustov and V. O. Galkin, *Phys. Lett. B* **634**, 214 (2006).
  18. Wei Chen and Shi-Lin Zhu, *Phys. Rev. D* **81**, 105018 (2010).
  19. Wei Chen and Shi-Lin Zhu, *Phys. Rev. D* **83**, 034010 (2011).
  20. Meng-Lin Du, Wei Chen, Xiao-Lin Chen and Shi-Lin Zhu, *Chin. Phys. C* **37**, 033104 (2013).
  21. Wei Chen and Shi-Lin Zhu, *Phys. Rev. D* **83**, 034010 (2011).
  22. R. T. Kleiv, T. G. Steele, A. Zhang and I. Blokland, *Phys. Rev. D* **87**, 125018 (2013).
  23. S. Choi *et al.*, *Phys. Rev. Lett.* **91**, 262001 (2003).
  24. R. Aaij *et al.*, *Phys. Rev. Lett.* **110**, 222001 (2013).
  25. S. Choi *et al.*, *Phys. Rev. Lett.* **100**, 142001 (2008).
  26. R. Aaij *et al.*, *Phys. Rev. Lett.* **112**, 222002 (2014).
  27. M. Ablikim *et al.*, *Phys. Rev. Lett.* **110**, 252001 (2013).
  28. Z. Q. Liu *et al.*, *Phys. Rev. Lett.* **110**, 252002 (2013).
  29. T. Xiao, *et al.*, *Phys. Lett. B* **727**, 366 (2013).
  30. M. Ablikim *et al.*, *Phys. Rev. Lett.* **112**, 132001 (2014).
  31. M. Ablikim *et al.*, *Phys. Rev. Lett.* **111**, 242001 (2013).
  32. K. Chilikin, *et al.*, arXiv:1408.6457.
  33. X. L. Wang, *et al.*, arXiv:1410.7641.
  34. I. Adachi, *et al.*, arXiv:1105.4583.
  35. Wei Chen and Shi-Lin Zhu, in *Proceedings of the Hadron Nuclear Physics 2011*.
  36. Wei Chen, T. G. Steele, Hua-Xing Chen and Shi-Lin Zhu, *in preparation*.
  37. Wei Chen, T. G. Steele and Shi-Lin Zhu, *Phys. Rev. D* **89**, 054037 (2014).
  38. J. Govaerts, *et al.*, *Nucl. Phys. B* **258**, 215 (1985).
  39. J. Govaerts, L. J. Reinders and J. Weyers, *Nucl. Phys. B* **262**, 575 (1985).
  40. J. Govaerts, *et al.*, *Nucl. Phys. B* **284**, 674 (1987).
  41. Cong-Feng Qiao, *et al.*, *J. Phys. G* **39**, 015005 (2012).
  42. D. Harnett, *et al.*, *J. Phys. G* **39**, 125003 (2012).
  43. R. Berg, *et al.*, *Phys. Rev. D* **86**, 034002 (2012).
  44. Wei Chen, *et al.*, *JHEP* **09**, 019 (2013).

45. Liuming Liu, *et al.*, *JHEP* **1207**, 126 (2012).
46. Wei Chen, T. G. Steele and Shi-Lin Zhu, *J. Phys. G* **41**, 025003 (2014).