Lecture 3.1: The rank

Optimization and Computational Linear Algebra for Data Science

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Rank of a family of vectors

Definition

We define the rank of a family $x_1, \dots, x_k$ of vectors of $\mathbb{R}^n$ as the dimension of its span:

$$\operatorname{rank}(x_1, \dots, x_k) \stackrel{\text{def}}{=} \dim(\operatorname{Span}(x_1, \dots, x_k)).$$

Rank of a matrix

Definition

Let $M \in \mathbb{R}^{n \times m}$. Let $c_1, \ldots, c_m \in \mathbb{R}^n$ be its columns. We define

$$\operatorname{rank}(M) \stackrel{\text{def}}{=} \operatorname{rank}(c_1, \ldots, c_m) = \dim(\operatorname{Im}(M)).$$

Example

« Rank of columns = rank of rows »

Proposition

Let $M \in \mathbb{R}^{n\times m}$. Let $r_1, \dots, r_n \in \mathbb{R}^m$ be the rows of M and $c_1, \dots, c_m \in \mathbb{R}^n$ be its columns. Then we have

$$\mathrm{rank}(r_1, \dots, r_n) = \mathrm{rank}(c_1, \dots, c_m) = \mathrm{rank}(M).$$

Computing the rank in practice

Computing the rank in practice