Lecture 3.2: Some properties of the rank

Optimization and Computational Linear Algebra for Data Science

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The rank-nullity theorem

Theorem

Let $L : \mathbb{R}^m \to \mathbb{R}^n$ be a linear transformation. Then

$$\mathrm{rank}(L) + \dim(\mathrm{Ker}(L)) = m.$$

Some inequalities

Proposition

Let $A \in \mathbb{R}^{n \times m}$ and $B \in \mathbb{R}^{m \times k}$. Then the following holds

1. $\operatorname{rank}(A) \leq \min(n, m)$.
2. $\operatorname{rank}(AB) \leq \min(\operatorname{rank}(A), \operatorname{rank}(B))$.

Proof.

$\square$

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