# Recitation 8

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# Midterm review Problem 0.2

True or False: There exists matrices $M \in \mathbb{R}^{2\times 3}$ such that $\dim(\mathsf{Ker}(M)) = 1$ and $\mathsf{rank}(M) = 2$.

# Midterm review Problem 0.3

Let $n > m$ and $A \in \mathbb{R}^{n \times m}$. Assume that $A$ has “full rank”, meaning that rank$(A) = \min(n, m) = m$.

1. Does $Ax = b$ has a solution for all $b \in \mathbb{R}^n$? (Prove or give a counter example).
2. Do there exist two vectors $x \neq x'$ such that $Ax = Ax'$? (Prove or give a counter example).

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# Midterm review Problem 0.8

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# Midterm review Problem 0.9

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Show that $k \leq n$.

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# Extra: 2019 Midterm Problem 6

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