

Optimization and Computational Linear Algebra for Data Science

OUTLINE

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 3. Basis, dimension
2. LINEAR TRANSFORMATIONS
 1. Linear transformations
 2. Matrix representation
 3. Kernel and image
3. RANK
 1. Definition of the rank
 2. Properties of the rank
 3. Invertible matrices
 4. Transpose of a matrix, symmetric matrices
4. NORM AND INNER PRODUCT
 1. Norm
 2. Inner product
 3. Orthogonality
 4. Orthogonal projection and distance to a subspace
5. MATRICES AND ORTHOGONALITY
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 2. Orthogonal matrices
6. EIGENVALUES, EIGENVECTORS AND MARKOV CHAINS
 1. Eigenvalues and eigenvectors
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 4. Example: Google's PageRank algorithm
7. THE SPECTRAL THEOREM AND PCA
 1. The Spectral Theorem
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4. Low-rank matrix estimation and matrix completion

12. GRADIENT DESCENT

1. Gradient descent
2. Newton's method