Optimization and Computational Linear Algebra for Data Science
Lecture 3: Rank

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Warning: *This material is not meant to be lecture notes. It only gathers the main concepts and results from the lecture, without any additional explanation, motivation, examples, figures...*

1 Rank of a matrix

Definition 1.1 (*Rank*)

The rank of a matrix $M \in \mathbb{R}^{n\times m}$ is defined as the dimension of the image of M:

$$\mathrm{rank}(M) = \dim(\mathrm{Im}(M)).$$

Proposition 1.1

Let $L : \mathbb{R}^m \to \mathbb{R}^n$ and $M : \mathbb{R}^n \to \mathbb{R}^k$, two linear applications. Then the following holds

(i) $\mathrm{rank}(L) \leq \min(n, m)$.

(ii) $\mathrm{rank}(ML) \leq \min(\mathrm{rank}(L), \mathrm{rank}(M))$.

Theorem 1.1 (*Rank-nullity theorem*)

Let $L : \mathbb{R}^m \to \mathbb{R}^n$ be a linear transformation. Then

$$\mathrm{rank}(L) + \dim(\mathrm{Im}(L)) = m.$$