Optimization and Computational Linear Algebra for Data Science Outline

1. Vector spaces

- 1. General definitions
- 2. Linear dependency
- 3. Basis, dimension

2. Linear transformations

- 1. Linear transformations
- 2. Matrix representation
- 3. Kernel and image

3. Rank

- 1. Definition of the rank
- 2. Properties of the rank
- 3. Invertible matrices
- 4. Transpose of a matrix, symmetric matrices

4. NORM AND INNER PRODUCT

- 1. Norm
- 2. Inner product
- 3. Orthogonality
- 4. Orthogonal projection and distance to a subspace

5. Matrices and orthogonality

- 1. Gram-Schmidt orthogonalisation method
- 2. Orthogonal matrices

6. Eigenvalues, eigenvectors and Markov Chains

- 1. Eigenvalues and eigenvectors
- 2. Diagonalizable matrices
- 3. Application to Markov chains
- 4. Example: Google's PageRank algorithm

7. SINGULAR VALUE DECOMPOSITION

- 1. The Spectral Theorem
- 2. Singular value decomposition
- 3. Interpretation and applications of the SVD

8. Graphs and Linear Algebra

- 1. Graphs
- 2. Graph Laplacian
- 3. Spectral clustering with the graph Laplacian
- 4. Spectral clustering as a relaxation
- 5. Spectral clustering beyond graphs

9. Convex functions

- 1. Convex sets
- 2. Convex functions

10. Optimality conditions

- 1. Local and global minimizers
- 2. Constrained optimization
- 3. The Lagrangian and the dual problem
- 4. Kuhn Tucker Theorem

11. Linear regression, matrix completion

- 1. Least squares
- 2. Penalized least squares: Ridge regression and Lasso
- 3. Norms for matrices
- 4. Low-rank matrix estimation and matrix completion

12. Gradient descent

- 1. Gradient descent
- 2. Newton's method