

# Optimization and Computational Linear Algebra for Data Science

## OUTLINE

### 1. VECTOR SPACES

1. General definitions
2. Linear dependency
3. Basis, dimension

### 2. LINEAR TRANSFORMATIONS

1. Linear transformations
2. Matrix representation
3. Kernel and image

### 3. RANK

1. Definition of the rank
2. Properties of the rank
3. Invertible matrices
4. Transpose of a matrix, symmetric matrices

### 4. NORM AND INNER PRODUCT

1. Norm
2. Inner product
3. Orthogonality
4. Orthogonal projection and distance to a subspace

### 5. MATRICES AND ORTHOGONALITY

1. Gram-Schmidt orthogonalization method
2. Orthogonal matrices

### 6. EIGENVALUES, EIGENVECTORS AND MARKOV CHAINS

1. Eigenvalues and eigenvectors
2. Diagonalizable matrices
3. Application to Markov chains
4. Example: Google's PageRank algorithm

### 7. THE SPECTRAL THEOREM AND PCA

1. The Spectral Theorem
2. Application: Principal Component Analysis (PCA)
3. Singular value decomposition
4. Interpretations of the SVD

### 8. GRAPHS AND LINEAR ALGEBRA

1. Graphs
2. Graph Laplacian
3. Spectral clustering with the graph Laplacian
4. Spectral clustering as a relaxation
5. Spectral clustering beyond graphs

### 9. CONVEX FUNCTIONS

1. Convex sets
2. Convex functions

### 10. LINEAR REGRESSION

1. Least squares
2. Penalized least squares: Ridge regression and Lasso
3. Norms for matrices, application to matrix completion

### 11. OPTIMALITY CONDITIONS

1. Local and global minimizers
2. Constrained optimization
3. The Lagrangian and the dual problem
4. Kuhn Tucker Theorem

## 12. GRADIENT DESCENT

1. Gradient descent
2. Newton's method
3. Stochastic gradient descent