Lecture 5.1: Gram-Schmidt algorithm

Optimization and Computational Linear Algebra for Data Science

Léo Miolane

Purpose of the algorithm

The Gram-Schmidt process takes as

- **Input:** a *linearly independent* family $(x_1, \dots, x_k)$ of $\mathbb{R}^n$.
- **Output:** an *orthonormal basis* $(v_1, \dots v_k)$ of $\mathrm{Span}(x_1, \dots, x_k)$.

Consequence

Every subspace of $\mathbb{R}^n$ admits an orthonormal basis.

Gram-Schmidt algorithm

The Gram-Schmidt process constructs $v_1, v_2, \ldots, v_k$ in this order, such that for all $i \in \{1, \ldots, k\}$:

$$\mathcal{H}_i : \begin{cases} (v_1, \ldots, v_i) \text{ is an orthonormal family} \\ \mathrm{Span}(v_1, \ldots, v_i) = \mathrm{Span}(x_1, \ldots, x_i). \end{cases}$$

Iterative construction of the v_i's

Iterative construction of the v_i's

Iterative construction of the v_i's