Lecture 5.2: Eigenvalues & Eigenvectors

Optimization and Computational Linear Algebra for Data Science

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Intro

Definition

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Let $A \in \mathbb{R}^{n\times n}$. A **non-zero** vector $v \in \mathbb{R}^n$ is said to be an *eigenvector* of A is there exists $\lambda \in \mathbb{R}$ such that

$$Av = \lambda v.$$

The scalar λ is called the eigenvalue (of A) associated to v.

Matrix with no eigenvalues/vectors

Eigenspaces

Definition

If $\lambda \in \mathbb{R}$ is an eigenvalue of $A \in \mathbb{R}^{n\times n}$, the set

$$E_\lambda(A) = \{x \in \mathbb{R}^n \,|\, Ax = \lambda x\} = \mathrm{Ker}(A - \lambda \mathrm{Id})$$

is called the eigenspace of A associated to λ. The dimension of $E_\lambda(A)$ is called the multiplicity of the eigenvalue λ.