Lecture 1.1: Introduction

Optimization and Computational Linear Algebra for Data Science

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Contents

Linear algebra

Why linear algebra?

« Linear algebra $\simeq$ geometry in arbitrary dimension »

Why do we need to do geometry ?

- In many case, our data is a collection of « data points » that are points $(x_1, \ldots, x_n)$
- To understand the structure of our data, we have to investigate the geometry of our data points: are they divided into clusters? are they «aligned» ?
- When $n = 1, 2, 3$, one can easily plot our data, but what about $n = 10000$?

Applications

You will learn linear algebra, while studying applications for data science such as:

- Data compression

  You will compress images using wavelets
- Principal component analysis

  Find directions along which the variance of the data is maximal
- Dimensionality reduction

  Reduce the dimension of a dataset while preserving its structure
- Linear regression
- Google's Page Rank and Markov chains

  Ranking any objects that can be compared!
- Clustering on networks
- Matrix completion

Optimization

Optimization

In machine learning, we often have to minimize functions

$$f(\theta) = \mathrm{Loss}(\mathrm{data}, \mathrm{model}_\theta) \quad \text{with respect to} \quad \theta \in \mathbb{R}^n.$$

- For $n = 1, 2$, one could plot f to find the minimizer.
- This is intractable for larger dimension.

We will

- focus on convex cost functions f.
- study gradient descent algorithms to minimize f.

Overview of the lectures

Outline

1. Vectors and vector spaces
2. Linear transformations and matrices
3. The rank
4. Norm and inner product
5. Eigenvalues, eigenvectors and Markov chains
6. The spectral theorem and PCA
7. Graphs and Linear Algebra
8. Convex functions
9. Linear regression
10. Optimality conditions
11. Gradient descent

Further informations

Course's webpage:

`leomiolane.github.io/linalg-for-ds.html`