Session 1: Vector spaces

Optimization and Computational Linear Algebra for Data Science

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Questions ?

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Subspaces

What are the subspaces of $\mathbb{R}^2$?

The span is always a subspace

Proposition

Let $x_1, \dots, x_k \in V$. Then, $\mathrm{Span}(x_1, \dots, x_k)$ is a subspace of V.

Linear dependency

A useful lemma

Lemma

Let $v_1, \ldots, v_n \in V$ and let $x_1, \ldots, x_k \in \mathrm{Span}(v_1, \ldots, v_n)$.
Then, if $k > n$, $x_1, \ldots, x_k$ are linearly dependent.

Abuse of language: Instead of saying «$x_1, \ldots, x_k$ are linearly dependent», we should have said «the family $(x_1, \ldots, x_k)$ is linearly dependent».

Basis, dimension

Proving the theorem

Theorem

If V admits a basis $(v_1, \dots, v_n)$, then every basis of V has also n vectors. We say that V has dimension n and write $\dim(V) = n$.

Proof.

$\square$

A bit of vocabulary

Definition

Let S be a subspace of $\mathbb{R}^n$.

- We call S a *line* if $\dim(S) = 1$.
- We call S an *hyperplane* if $\dim(S) = n - 1$.

Overview of the lectures

Outline

1. Vectors and vector spaces
2. Linear transformations and matrices
3. The rank
4. Norm and inner product
5. Eigenvalues, eigenvectors and Markov chains
6. The spectral theorem and PCA
7. Graphs and Linear Algebra
8. Convex functions
9. Linear regression
10. Optimality conditions
11. Gradient descent

Further informations

Course's webpage:

`leomiolane.github.io/linalg-for-ds.html`