

Lecture 5.2: Orthogonal matrices

Optimization and Computational Linear Algebra for Data Science

Orthogonal matrices

Definition

A matrix $A \in \mathbb{R}^{n \times n}$ is called an *orthogonal matrix* if its columns are an orthonormal family.

A proposition

Proposition

Let $A \in \mathbb{R}^{n \times n}$. The following points are equivalent:

1. A is orthogonal.
2. $A^T A = \text{Id}_n$.
3. $AA^T = \text{Id}_n$

Orthogonal matrices & norm

Proposition

Let $A \in \mathbb{R}^{n \times n}$ be an orthogonal matrix. Then A preserves the dot product in the sense that for all $x, y \in \mathbb{R}^n$,

$$\langle Ax, Ay \rangle = \langle x, y \rangle.$$

In particular if we take $x = y$ we see that A preserves the Euclidean norm: $\|Ax\| = \|x\|$.