POLITECNICO MILANO 1863

DIPARTIMENTO DI ELETTRONICA INFORMAZIONE E BIOINGEGNERIA

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Lab 1
STATIC LOCALIZATION

Where you should be…

Measurement categories and typical accuracies

Measurement	Model $h_i(\mathbf{u})$	Typical measurement accuracy $\sigma_{n,i}$
TOA	$d_i = \lvert\mathbf{u}-\mathbf{s}_i\rvert$	5-100m outdoor <1m indoor 3G cellular: B=4MHz, σ_t=T_c/2=125ns → 38m 4G cellular, WiFi: B=20MHz, σ_t=T_c/2=50ns→ Ultra wideband: B=500MHz, σ_t=T_c/2=1ns → GPS: 5-20m
TDOA	$d_i - d_j = \lvert\mathbf{u}-\mathbf{s}_i\rvert - \lvert\mathbf{u}-\mathbf{s}_j\rvert$	10-60m outdoor <1m indoor
AOA	$\angle(\mathbf{u}-\mathbf{s}_i)$	5°–10° (depends on antenna array aperture)
RSS	$P_0 - 10 n_P \log_{10}\frac{\lvert\mathbf{u}-\mathbf{s}_i\rvert}{d_0}$	4-12dB
RSS digital map	$P_{\mathrm{map},i}(\mathbf{u})$	3dB

$\boldsymbol{u} = [u_x \quad u_y]$ USER

Measurement error model

- Each measurement (TOA, TDOA, AOA, RSS, digital map),

$$\rho_i = h_i(\mathbf{u}) + n_i$$

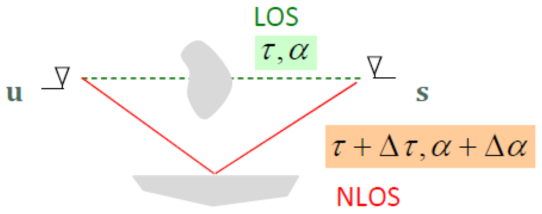
is affected by an error n_i for which a Gaussian distribution is often assumed, motivated by asymptotic arguments:

$$p(n_i) = \mathcal{N}(n_i, 0, \sigma^2_{los})$$ LOS conditions

$$p(n_i) = \mathcal{N}(n_i, \mu, \sigma^2_{nlos})$$ NLOS conditions

μ>0

σ_{nlos}>σ_{los}



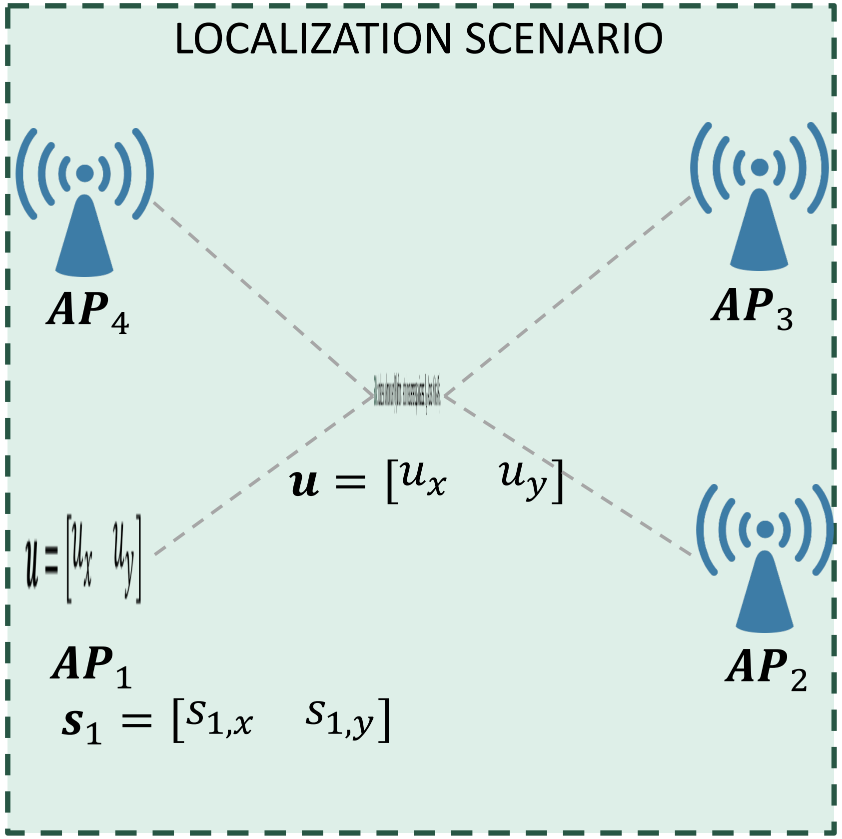
$$p(n_i) = P_{los}\mathcal{N}(n_i, 0, \sigma^2_{los}) + (1-P_{los})\mathcal{N}(n_i, \mu, \sigma^2_{nlos})$$ Mixed LOS/NLOS conditions

Gaussian mixture

- Bias is due to multipath, fading and syncronization errors (TOA).
- The Gaussian model is useful to derive performance bounds. The Gaussian pdf, however, is the least informative for a given variance. If measurements are non-Gaussian, algorithms should take into account any a-priori information on the statistics to improve performance.

2D localization: problem formulation

GOAL: localize an unknown user (UE) $\boldsymbol{u}$ from a set of measurements $\boldsymbol{\rho}$ available at N_{AP} Access Points (APs).



$$\boldsymbol{\rho} = [\rho_1 \ \rho_2 \ \rho_3 \ \rho_4]$$

The single measurement $\rho_i = h_i(\boldsymbol{u}, \boldsymbol{s}_i) + n_i$ is a non-linear function of the AP/UE states, corrupted by noise

non-linear function   states   noise

Maximum likelihood - ML

We want to estimate the position $\hat{\boldsymbol{u}}$ from a set of measurements $\boldsymbol{\rho} = \boldsymbol{h}(\boldsymbol{u}) + \boldsymbol{n}$ of N_{AP}

Non-Bayesian estimation: maximum likelihood (ML) 13

- The estimate is based on the likelihood function p(ρ| **u**), which indicates how likely it is that the measurements ρ has been generated by the model parameter **u**.
- Maximum likelihood estimate:

$$\hat{\mathbf{u}}_{\mathrm{ML}} = \arg\max_{\mathbf{u}} p(\boldsymbol{\rho} \mid \mathbf{u}) = \arg\max_{\mathbf{u}} \mathcal{L}(\boldsymbol{\rho} \mid \mathbf{u})$$

$\mathcal{L}(\boldsymbol{\rho} \mid \mathbf{u}) = \ln p(\boldsymbol{\rho} \mid \mathbf{u})$ Log-likelihood

- Since h(·) is a deterministic function, the likelihood function can be expressed as:

$$p(\boldsymbol{\rho} \mid \mathbf{u}) = p_{\mathbf{n}}(\boldsymbol{\rho} - \mathbf{h}(\mathbf{u}))$$

- If the measurements at different reference stations are independent:

$$p(\boldsymbol{\rho} \mid \mathbf{u}) = \prod_{i=1}^{N} p_{n_i}(\rho_i - h_i(\mathbf{u}))$$

$$\hat{\mathbf{u}}_{\mathrm{ML}} = \arg\max_{\mathbf{u}} \prod_{i=1}^{N} p_{n_i}(\rho_i - h_i(\mathbf{u}))$$

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Maximum Likelihood

$$\hat{\boldsymbol{u}}_{\mathrm{ML}} = argmax \quad p_n(\,\boldsymbol{\rho} \mid \boldsymbol{u}\,)$$

since $\boldsymbol{h}(\boldsymbol{u})$ is deterministic

$$= argmax \quad p_n(\,\boldsymbol{\rho} - \boldsymbol{h}(\boldsymbol{u}) | \boldsymbol{u}\,)$$

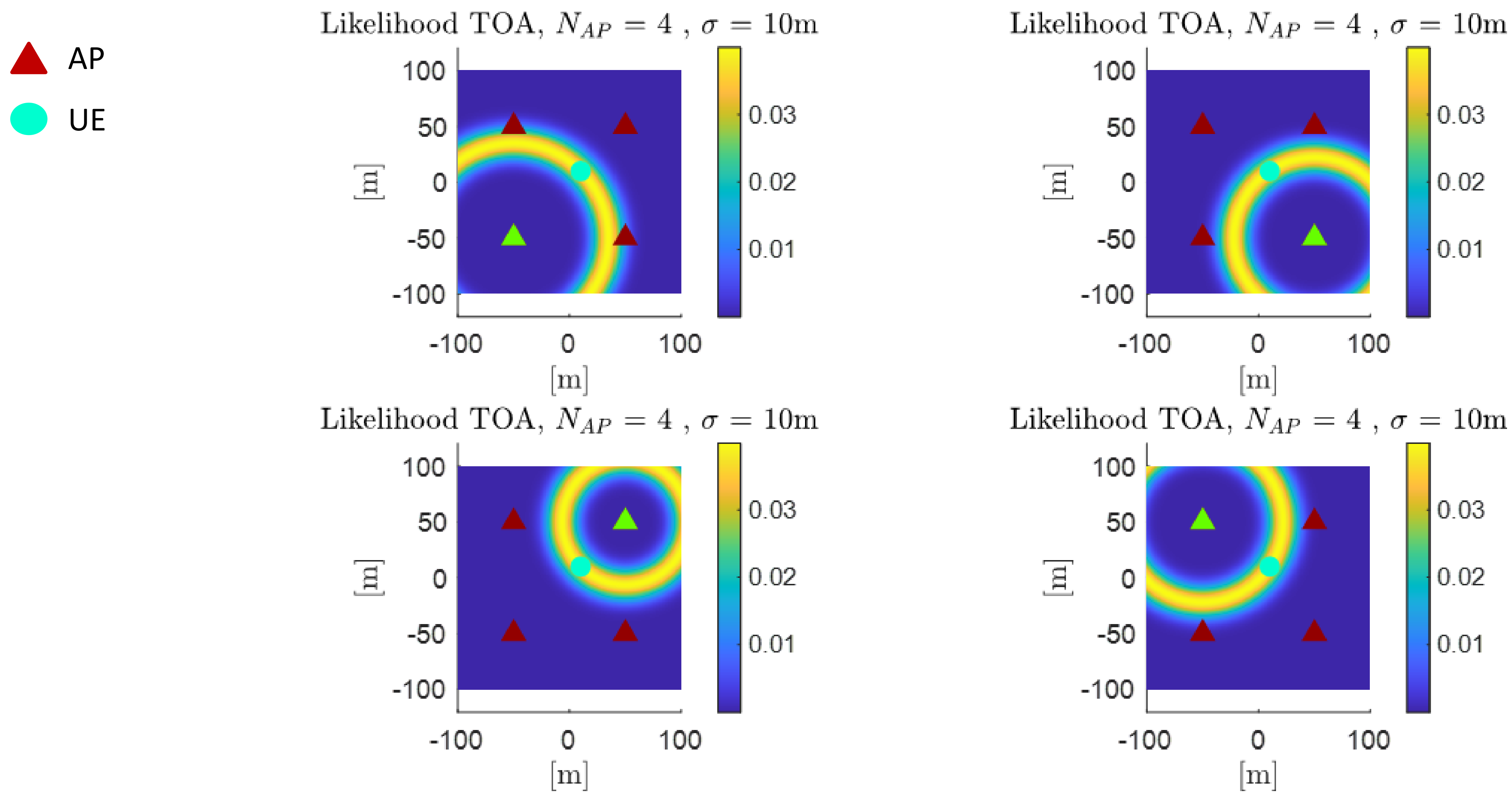
since measurements are independent across APs

$$= argmax \quad \prod_{i=1}^{N_{AP}} p_{n_i}(\,\rho_i - h_i(\boldsymbol{u}) | \boldsymbol{u}\,)$$

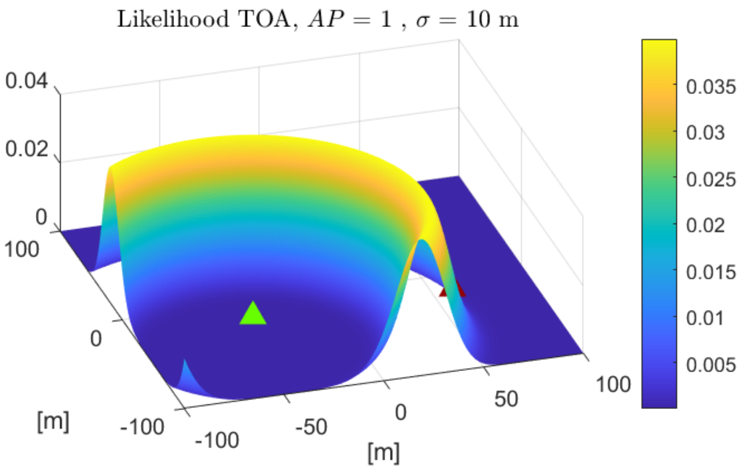
with Gaussian measurement errors

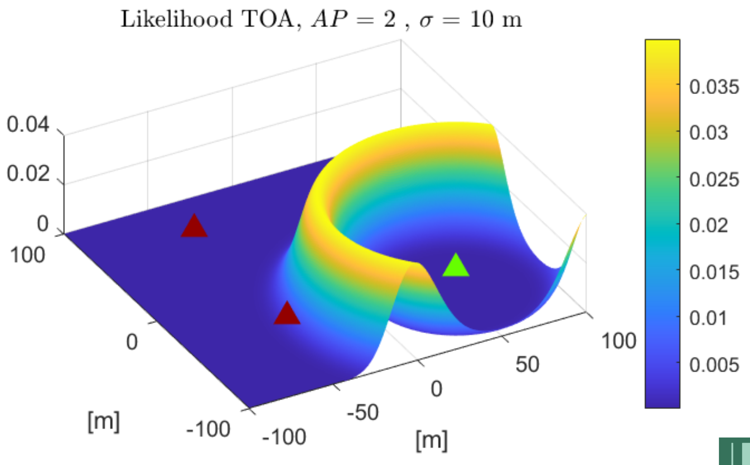
$$= argmax \quad \prod_{i=1}^{N_{AP}} \frac{1}{\sqrt{2\pi{\sigma_i}^2}} e^{\left(-0.5\left(\frac{\rho_i - h_i(\boldsymbol{u})}{\sigma_i}\right)^2\right)}$$

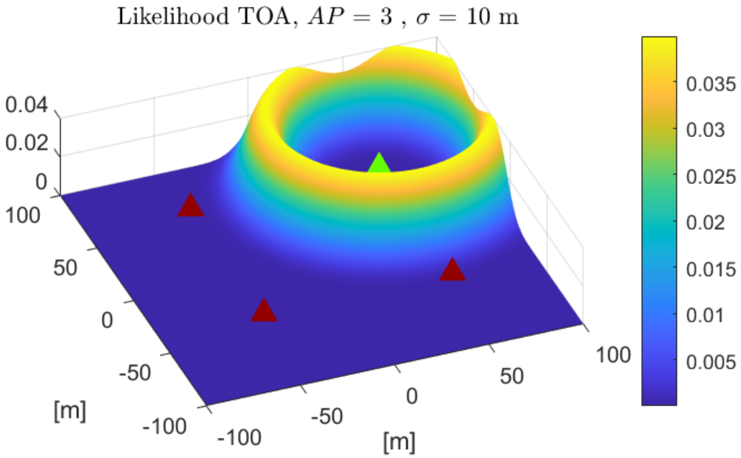
Examples of $p(\rho_i - h_i(\boldsymbol{u})|\boldsymbol{u})$ in 2D

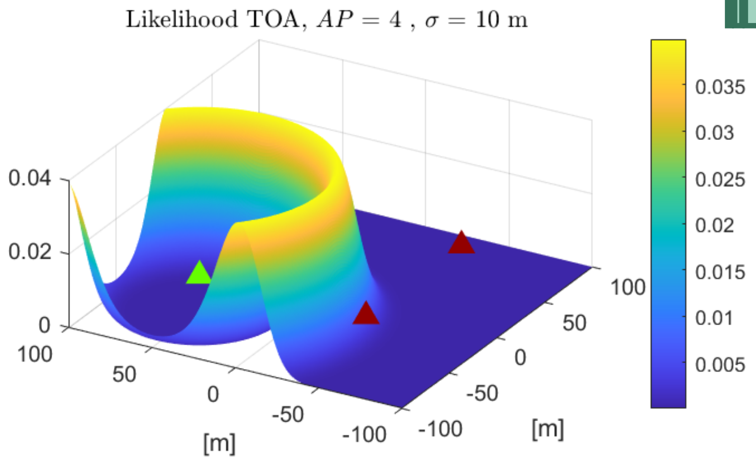


Examples of $p(\,\rho_i - h_i(\boldsymbol{u})|\boldsymbol{u}\,)$ in 3D

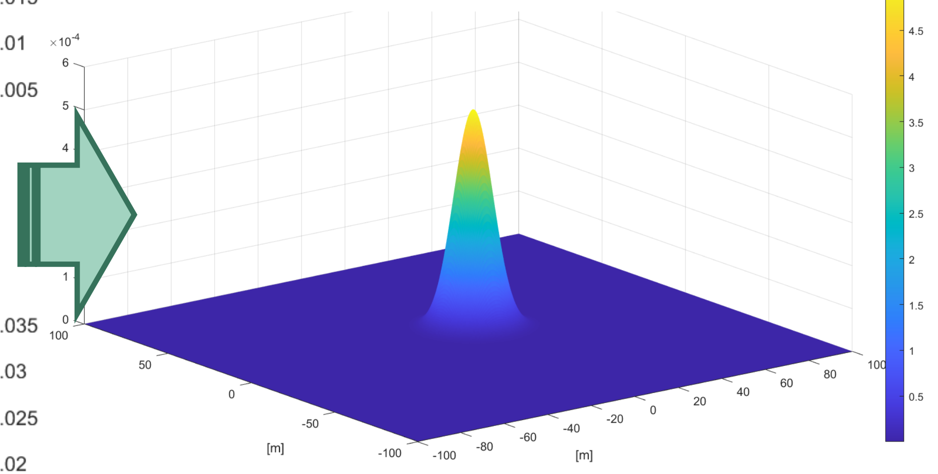








Maximum Likelihood

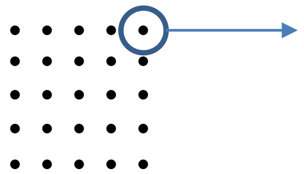


Matrix of the model

Method	Measurement $h_i(\boldsymbol{u})$	Jacobian $[\mathbf{H}(\boldsymbol{u})]_i = \dfrac{\partial h_i(\boldsymbol{u})}{\partial \boldsymbol{u}}$
TOA	$d_i = \lvert \boldsymbol{u} - \boldsymbol{s}_i \rvert$	$\dfrac{u_x - s_{i,x}}{d_i}, \dfrac{u_y - s_{i,y}}{d_i}$
AOA	$\tan^{-1}\left(\dfrac{u_y - s_{i,y}}{u_x - s_{i,x}}\right)$	$-\dfrac{u_y - s_{i,y}}{{d_i}^2}, \dfrac{u_x - s_{i,x}}{{d_i}^2}$
RSS	$P_0 - 10n_p \log_{10} \dfrac{\lvert \boldsymbol{u} - \boldsymbol{s}_i \rvert}{d_0}$	$-\dfrac{10n_p}{\ln 10}\dfrac{u_x - s_{i,x}}{{d_i}^2}, -\dfrac{10n_p}{\ln 10}\dfrac{u_y - s_{i,y}}{{d_i}^2}$
TDOA	$d_i - d_j = \lvert \boldsymbol{u} - \boldsymbol{s}_i \rvert - \lvert \boldsymbol{u} - \boldsymbol{s}_j \rvert$	$\dfrac{u_x - s_{i,x}}{d_i} - \dfrac{u_x - s_{j,x}}{d_j}, \dfrac{u_y - s_{i,y}}{d_i} - \dfrac{u_y - s_{j,y}}{d_j}$

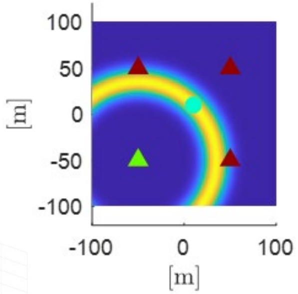
To do

1. Define the localization scenario
2. Create a grid of evaluation points to sample the scenario

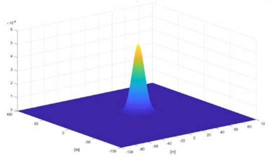


$\boldsymbol{p} = [p_x \; p_y]$

3. Compute the likelihood of the "ideal" measurement $\rho_i = h_i(\boldsymbol{u})$ (no noise now) with respect to each evaluation point $\boldsymbol{p}$: $p_{n_i}(\, \rho_i - h_i(\boldsymbol{p})\,)$
4. Plot the likelihood for each AP to show the "heatmap"

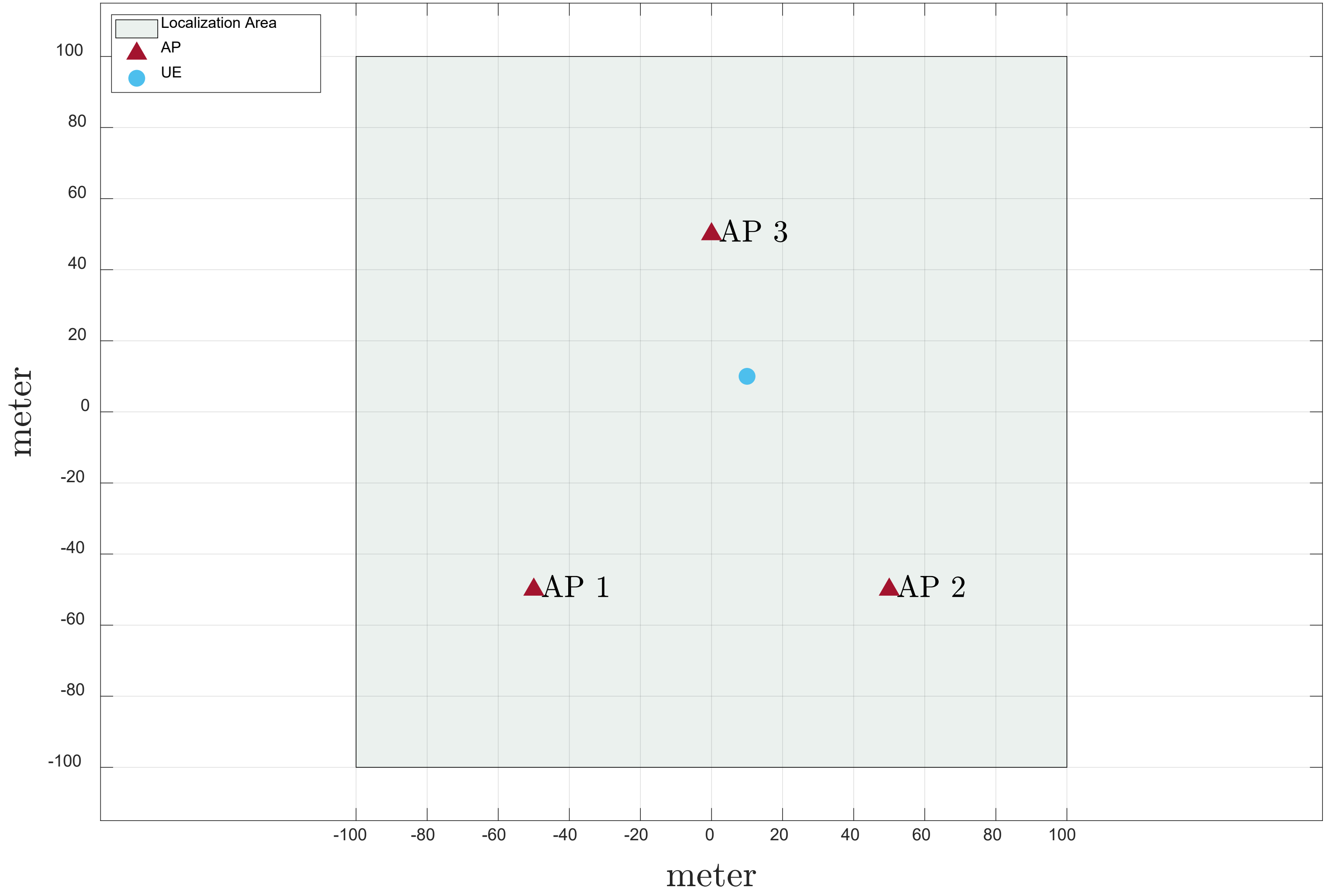


5. Compute the maximum likelihood and plot it

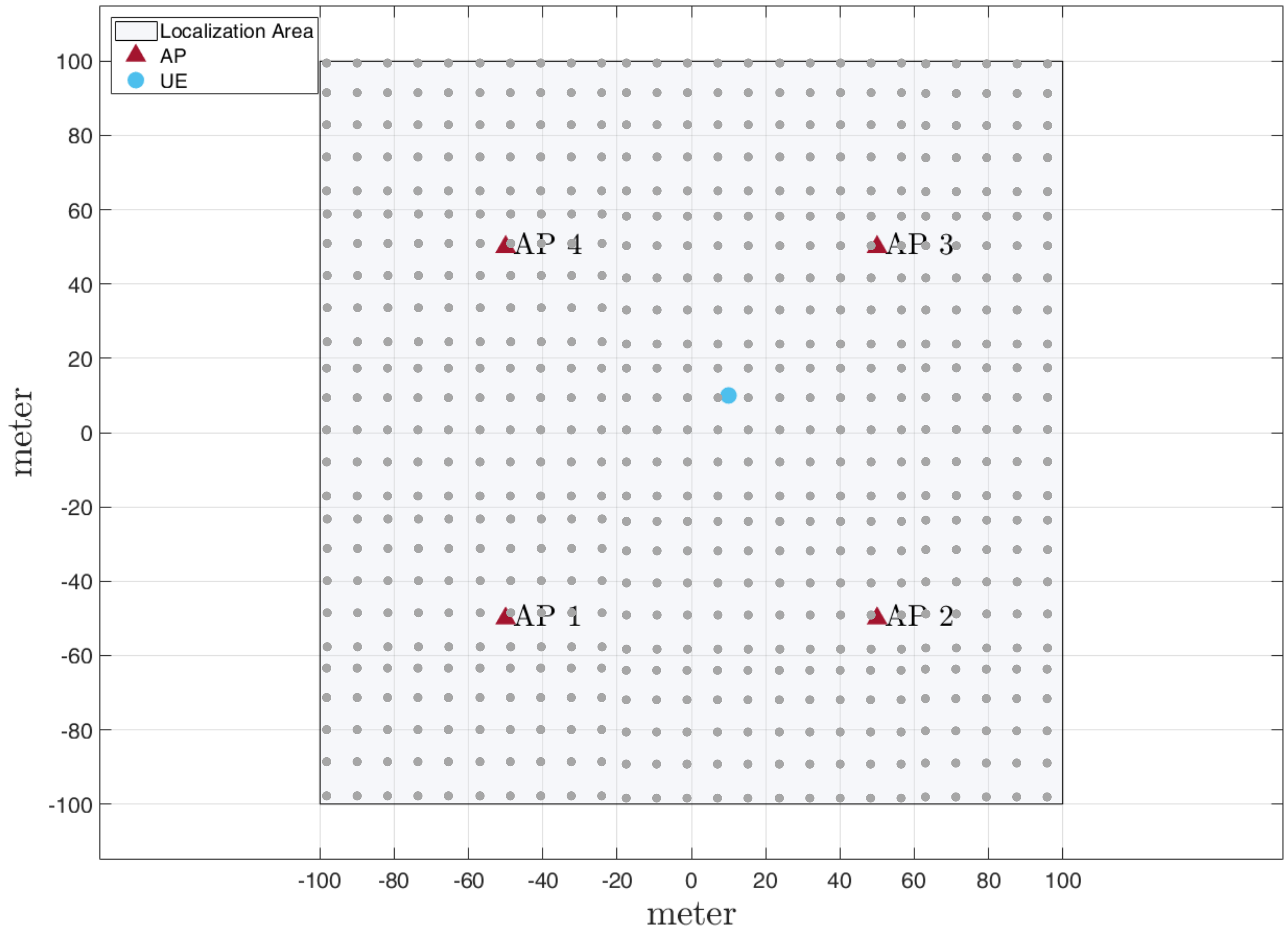


6. Evaluate $\hat{\boldsymbol{u}}_{\mathrm{ML}}$ (without noise it should coincide with the UE location)
7. Add Gaussian noise to measurements and see what happens *(home)*

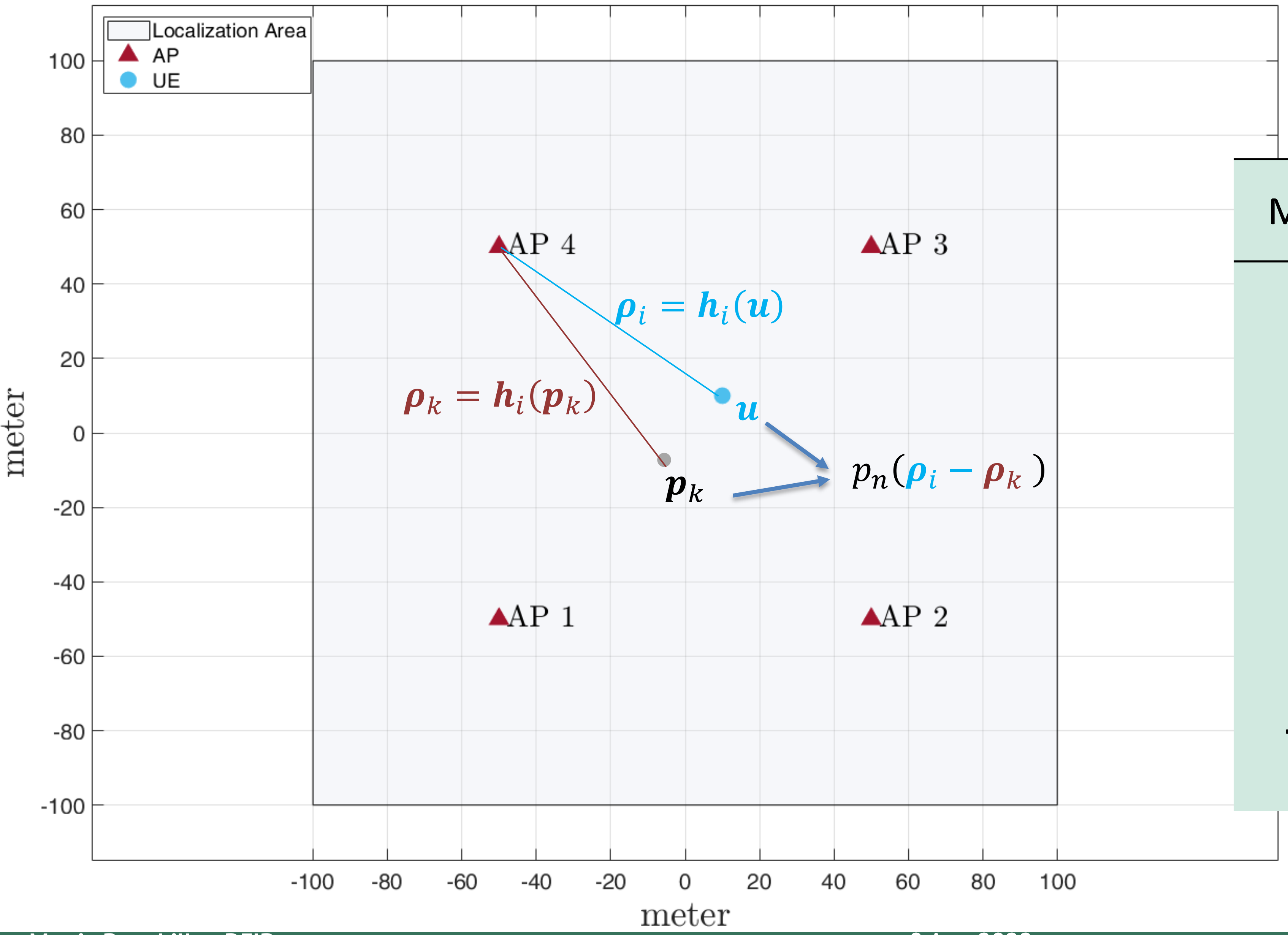
1. Define the localization scenario



2. Create a grid of evaluation points to sample the scenario

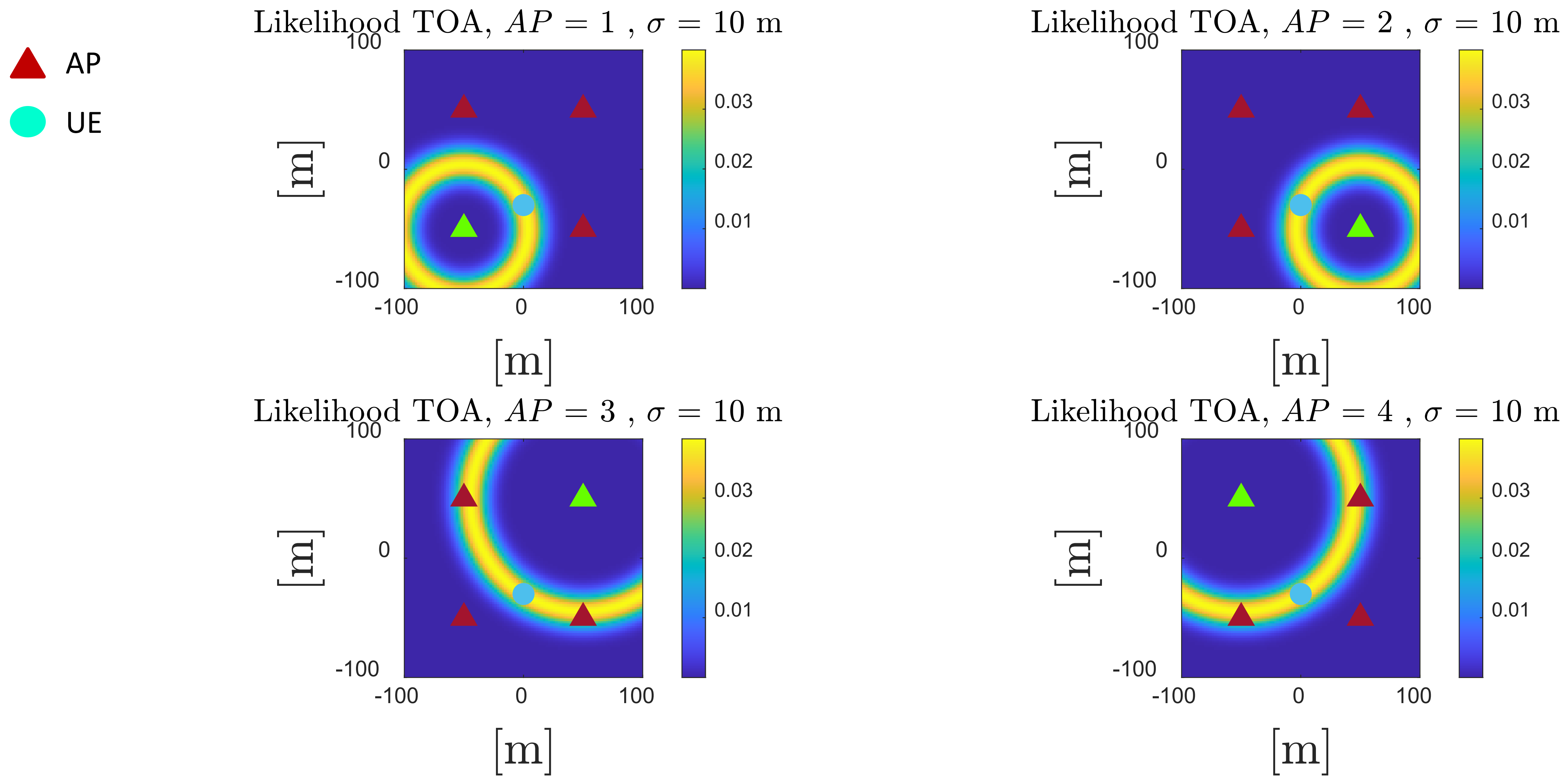


3. Compute the likelihood of each evaluation point p_k



	Measurement
Method	$h_i(\boldsymbol{p}_k)$
TOA	$d_i = \lvert \boldsymbol{p}_k - \boldsymbol{s}_i \rvert$
AOA	$\tan^{-1}\left(\frac{p_{k,y} - s_{i,y}}{p_{k,x} - s_{i,x}}\right)$
RSS	$P_0 - 10 n_p \log_{10} \frac{\lvert \boldsymbol{p}_k - \boldsymbol{s}_i \rvert}{d_0}$
TDOA	$d_i - d_j = \lvert \boldsymbol{p}_k - \boldsymbol{s}_i \rvert - \lvert \boldsymbol{p}_k - \boldsymbol{s}_j \rvert$

4. Plot the likelihood for each AP to show the “heathmap”



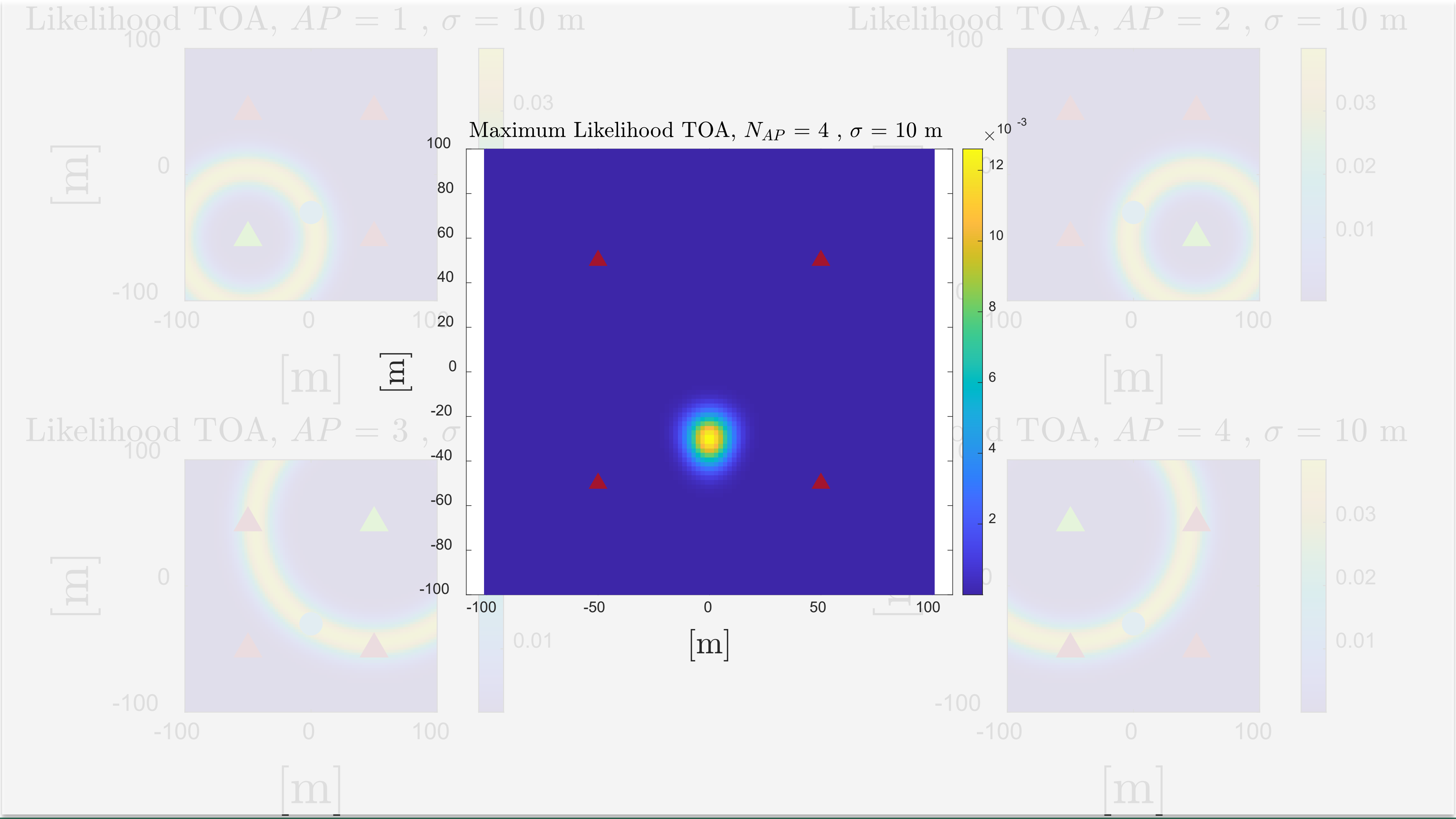
5. Compute the maximum likelihood and plot it

$$\prod_{i=1}^{N_{AP}} p_{n_i}(\,\rho_i - h_i(\boldsymbol{u})|\boldsymbol{u}\,)$$

5. Compute the maximum likelihood and plot it

AP

UE



6. Evaluate $\hat{\boldsymbol{u}}_{ML}$

$$\hat{\boldsymbol{u}}_{ML} = argmax \prod_{i=1}^{N_{AP}} p_{n_i}(\,\rho_i - h_i(\boldsymbol{u})|\boldsymbol{u}\,)$$

7. Add noise to measurements

To do at home…