Workshop #4, AA2021-2022: Array Processing

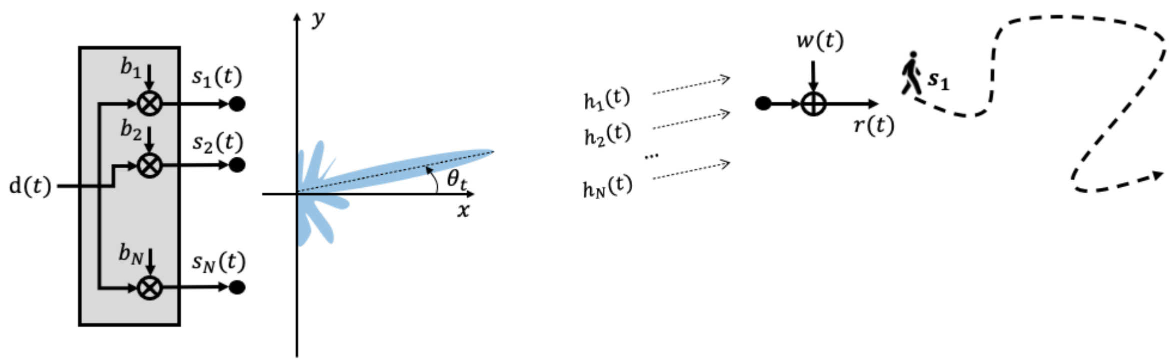

Figure 1: Reference scenario

Array processing techniques are used in multiple-input multiple-output (MIMO) systems to efficiently send signals to a specific direction in space, e.g., using beamforming. The key idea is to appropriately delay the signals emitted by each sources so that they combine constructively in one direction and destructively in others.

Let us consider the system depicted in Fig. 1, where an array of emitting sources is sending a signal $d(t)$ to a moving measuring sensor (receiver). Each emitting source i applies a weighting b_i to the signal $d(t)$ such that

$$s_i(t) = b_i \; d(t), \tag{1}$$

where $\mathbf{b} = \{b_1, b_2, \dots, b_N\}$ are called beamforming weights. The measured signal by the receiver can be modeled as

$$r(t) = \sum_{i=1}^{N} h_i(t) * s_i(t) + w(t) \tag{2}$$

where $h_i(t)$ is the filter response between the ith source and the receiving sensor, $w(t)$ is an additive noise.
The processing unit (grey box) estimates the beamforming weights $\mathbf{b}$ that maximize the received signal energy E_r:

$$\mathbf{b}_{opt} = \max_{\mathbf{b}} E_r = \max_{\mathbf{b}} \int_{-\infty}^{\infty} |r(t)|^2 dt \tag{3}$$

The moving sensor can measure the received energy $E_r(t)$ and send it to the emitting array as feedback. Since the receiving sensor is moving, the weights $b_i(t)$ need to be adapted over time to track its motion. Assuming that all the emitting sources of the arrat are placed along the y-axis, i.e., $\{y_1, y_2, y_3, ..., y_N\} = \{0, f_o/2c, 2f_o/2c, ..., (N-1)f_o/2c\}$, and for a source signal that is narrowband (see Sect. 19.1 of the book), the filter response is

$$h_i(t) = \frac{1}{\rho_i^2(t)} e^{j \frac{2\,\pi\, f_o}{c}\, y_i \; \sin(\theta_i(t))} \tag{4}$$

where $\rho_i(t)$ and $\theta_i(t)$ are the distance and angle between the ith source of the array and the receiving sensor, which are changing over time due to the motion of receiver. $fo = 2.9GHz$ is the carrier frequency, $c = 3 \times 10^8 m/s$ is the speed of light, and y_i is the position of the ith sensor on the y-axis such that $|y_i - y_{i+1}| = f_o/2c$. To simplify the model, one can assume that $\rho_i \approx \rho_j = \rho$ and $\theta_i \approx \theta_j = \theta_t \; \forall i, j$ being $i \neq j$. This assumption, known as far field assumption (Sect. 19.1 of the book), holds when the distance between the sources and array is sufficiently large.

The noise is complex normal (see Sect. 3.7 of the book), temporally and spatially white, i.e., $\mathbb{E}[w_i^*(t)w_i(t-\tau)] = \sigma_w^2\delta(\tau)$, and

$$\mathbf{w}(t) \sim \mathcal{CN}(\mathbf{0}, \sigma_w^2\mathbf{I}_N) \tag{5}$$

The input signal $d(t)$ can be generated from uncorrelated and white random processes (see Sect. 3.7 of the book):

$$d(t) \sim \mathcal{CN}(0, \sigma_d^2). \tag{6}$$

1. Trajectory Generation

Generate a trajectory for the receiving sensor using the constant velocity (CV) model. The discrete time CV model is:

$$\begin{aligned}
x[k+1] &= x[k] + v[k]\ \cos(\phi[k])\ dT + u_x[k] \\
y[k+1] &= y[k] + v[k]\ \sin(\phi[k])\ dT + u_y[k] \\
v[k+1] &= v[k] + u_v[k] \\
\phi[k+1] &= \phi[k] + u_\phi[k]
\end{aligned}$$

where (x, y) is the 2D position of the receiving sensor, v is the absolute speed, ϕ is the direction of motion, $\mathbf{u}[k] = [u_x[k], u_y[k], u_v[k], u_\phi[k]]^{\mathrm{T}}$ is the excitation process, and $dT = 0.5s$ is the time sampling rate.

2. Optimal beamformer

Assuming that the receiver position is unknown to the emitting array, define an algorithm to estimate the optimal beamformer in equation 3. Evaluate the performance of the proposed algorithm for different $SNR = \frac{\sigma_d^2}{\sigma_w^2} = \mathtt{SNR} = \mathtt{5:5:40\ dB}$, with $\sigma_d^2 = 1$.

Hints: two algorithms must be implemented: one for the initial optimal beamformer and one for updating the previous optimal beamformer. The second algorithm should use previous information for a faster convergence.