1.3 system identification

Since $K=1$, we can write the matrix product: $\underline{y}[k] = \underline{\underline{\hat{H}}} \cdot \underline{x}[k] + \underline{w}[k]$, or for every k, we get: $\underline{y} = \underline{\underline{\hat{H}}}\,\underline{x} + \underline{w} \Rightarrow \underset{N \times P-Q}{\underline{x}_{est}} = (\underset{N \times M}{\underline{\underline{\hat{H}}}^T} \underset{M \times M}{\underline{\underline{C}}^{-1}} \underset{M \times N}{\underline{\underline{\hat{H}}}})^{-1} \underset{N \times M}{\underline{\underline{\hat{H}}}} \underset{M \times M}{\underline{\underline{C}}^{-1}} \underset{M \times P-Q}{\underline{y}}$, where

$\underline{\underline{C}} = \begin{cases} 1, & i=j \\ \rho, & i \neq j \end{cases}$ For MMSE was used the ~~same~~ computationally efficient formula: $\theta_{MMSE} = \underline{M}_\theta + (\underline{\underline{H}}^T \underline{\underline{C}}_{ww}^{-1} \underline{\underline{H}} + \underline{\underline{C}}_{\theta\theta}^{-1})^{-1} \underline{\underline{H}}^T \underline{\underline{C}}_{ww}^{-1} (\underline{x} - \underline{\underline{H}}\,\underline{M}_\theta)$.

METRIC: reasoning: • If we use ρ to estim channel - zerro efficiency.

• SNR $\nearrow \Rightarrow$ MSE $\searrow$, $Q \nearrow \Rightarrow$ MSE $\searrow$

$\Rightarrow$

a) $\Downarrow\left(1-\frac{Q}{P}\right)$ ⇐ • the lower the Q, the more data we transmit ⟹ → higher coefficient

• If $Q=P$ (all to estimate) ⟹ zero efficiency

b) $\log_2\left(1+\underbrace{\frac{1}{MSE}}_{\text{scaling factor}}\cdot SNR\right)$ ⇐ • SNR ↗ ⟹ MSE ↘

MSE is in the denominator to show the improvement (small value) for a high SNR and bad performance (high value) for a small SNR