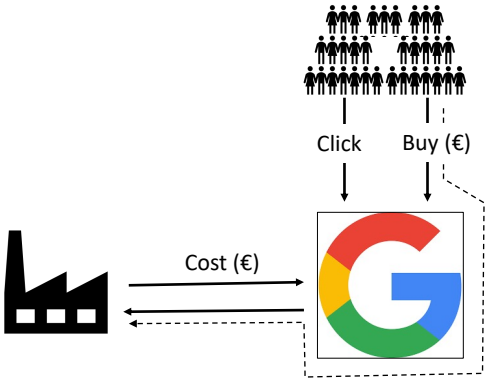
Homework #3, AA2021-2022: Causal impact on e-marketing data (Google)

In e-marketing the evaluation of the benefit from digital advertising is evaluated in term of cost and benefit tradeoff. This means that, as a result of some actions, one expects some benefits in terms of a set of clicks and/or selling. These are referred as Key Performance Indicators (KPI), that are numerical indexes to quantify, in this case, the beenfits of a marketing campain.



The figure illustrates the set of actions in e-marketing. One specific company requires an advertising campain to Google paying a fee (**cost**), and Google sets an online advertising campaing. Google is enabled to monitor anonymously the set of actions by the customers: the interest to a certai advertising (**click**), up to a **buy** of the product. The three set of variables **cost**, **click** and **buy** are typically called Key Performance Indicators (KPIs). The monitoring of the 3 KPIs before or after the beginning of the advertising should have a change of the stochastic properties. To strenghten the quantitative evaluation of the benefit of the advertising, we will extent the usage of the KPIs from other affine products referred as **control** KPIs.

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Let the individual evolution of one KPI (**cost** or **click** or **buy**) vs time for the kth company be $x_k(t)$, and the number of company tracks be $k = 1, 2, ..., K = 20$, where t is a discrete variable that denotes the time samples in days (day 1, day 2, ...). Each of the KPI track is affected by seasonal activity, e.g., summer or winter is more or less active. Furthermore, each trace can change its behavior compared to others as consequence of marketing instances (e.g., product or brand promotion, etc) occurring at the time T_k, the behavour of $x_k(t)$ is a combination of these instances:

$$x_k(t) = \bar{x}_k(t) + g_k(t - T_k)$$

where $\bar{x}_k(t)$ represents the KPI without any marketing istances (i.e., no advertising, no Google adv, etc..), and the behavior $g_k(t)$ is a causal signature, dependent on the specific instance. Signatures are typically linearly growing as $g_k(t) \simeq \alpha_k t$ for a certain time interval after the instance in T_k, and dropping to zero after a while. The marketing instances make each signature $x_k(t)$ be non stationary random sequence, even if $\bar{x}_k(t)$ can be considered as stationary.

The tracks $x_0(t), x_1(t), ..., x_K(t)$ are somewhat mutually correlated as likely selected to belong to same retail sectors. This means that, on the average, one can state that $E[x_k(t)x_\ell(\tau)] \neq 0$, and thus the values of each tracks are predictable from the other up to a certain degree of confidence.

Data can be ordered into $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_K]$ of dimension $T \times K$ (here $K = 20$), where each KPI time series are columnwise ordered $T \times 1$:

$$\mathbf{x}_k = \begin{bmatrix} x_k(t=1) \\ x_k(t=2) \\ \vdots \\ x_k(t=T) \end{bmatrix}.$$

Different KPI are active, such as $\mathbf{X}$ (cost for advertising), $\mathbf{Y}$ (clicks), and $\mathbf{Z}$ (conversion: how many customer actions). Notice that numerical values of costs and conversions are are not exactly in € but their values are properly scaled by unknown values.

Goal is to consider $x_K(t)$ as track of interest (test time series), and all the other tracks ($k < K = 20$) are called control time series. The control time series (or simply the control). Notice that in questions 1,2, the KPI $x_k(t)$ denotes cost, and clicks, and conversions.

1) Derive an instantaneous estimator of every value $\hat{x}_k(t)$ using linear prediction from $x_k(t)$, by increasing the prediction length and the prediction step. Evaluate quantitatively their properties recalling that for $K = 20$

one should expect a strong variation for $t > T_K = T_{20}$. Define a proper metric for performance illustrationn (what and how to plot, values, etc..), and comment the results.

2) Consider the problem of making a set of instantaneous linear estimators $f[.]$ to estimate

$$\hat{x}_i(t) = f_i[x_1(t), ...x_k(t), ..., x_K(t)|k \neq i]$$

2a) define proper metrics to evaluate their accuracy, illustrate and comment the results;

2b) are all control traces useful? Spot if any can be removed with improvement of the estimated KPI;

2c) evaluate if one can estimate the value of advertisinng T_K (assuming not known) from the analysis of the prediction error.

3) repeat the exercises from 2a,2b,2c) (instantaneous linear estimator) when using all KPI at the same time (cost, and clicks, and conversions).

4) repeat the exercisess from 2a, 2b, 2c) using memory estimators (could be causals, anticausals, or both) with unknown length to be evaluated as the one that minimized the MSE till the time$T_K = T_{20}$, or using data $t < T_K = T_{20}$

5) Define and compute one or more metrics that highlight the benefits from the action at time $T_K = T_{20}$ for the test trace, using all the other control traces.

Matlab GoogleDatset is composed by 3 table, data matrix `Cost`, `Click`, and `Conversion` is ordered in progressive time, and the **control KPI** traces are 1:19 columns (`Cost(:,1:19)`, `Click(:,1:19)`, `Conversion(:,1:19)`), and test of interest is the column 20 (more specifically `Cost(:,20)`, `Click(:,20)`, `Conversion(:,20)`). The instance time from data is $T_{20} = 397$ sample.

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