1.2 i (memoryless filter) M=N=4, K=1 (since K=1, convolution becomes the product)

$$\underline{y}_1 = \begin{bmatrix} y_1[1] \\ y_1[2] \\ \vdots \\ y_1[Q] \end{bmatrix} = \begin{bmatrix} x_1[1] \\ x_1[2] \\ \vdots \\ x_1[Q] \end{bmatrix} h_{11} + \dots + \begin{bmatrix} x_4[1] \\ x_4[2] \\ \vdots \\ x_4[Q] \end{bmatrix} h_{14} + \begin{bmatrix} w_1[1] \\ w_1[2] \\ \vdots \\ w_1[Q] \end{bmatrix} \Rightarrow \underline{y}_1 = \underbrace{\begin{bmatrix} \underline{x}_1 & \underline{x}_2 & \underline{x}_3 & \underline{x}_4 \end{bmatrix}}_{\underline{X}} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{14} \end{bmatrix} + \underline{w}_1$$

$$\underset{Q\times 1}{\underline{y}_1} = \underset{Q\times N}{\underline{X}} \underset{N\times 1}{\begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{14} \end{bmatrix}} + \underline{w}_1$$

Generalizing for $\underline{y}_1, \underline{y}_2, \underline{y}_3, \underline{y}_4$ all together:

$$\underset{MQ\times 1}{\underline{y}} = \underset{MQ\times NM}{\begin{bmatrix} \underline{X} & \underline{0} & \underline{0} & \underline{0} \\ \underline{0} & \underline{X} & \underline{0} & \underline{0} \\ \underline{0} & \underline{0} & \underline{X} & \underline{0} \\ \underline{0} & \underline{0} & \underline{0} & \underline{X} \end{bmatrix}} \underset{NM\times 1}{\begin{bmatrix} h_{11} \\ h_{12} \\ \vdots \\ h_{14} \\ h_{21} \\ \vdots \\ h_{24} \\ \vdots \\ h_{44} \end{bmatrix}} + \underset{MQ\times 1}{\begin{bmatrix} w_1[1] \\ \vdots \\ w_1[Q] \\ w_2[1] \\ \vdots \\ w_2[Q] \\ \vdots \\ w_4[Q] \end{bmatrix}}$$

Cov matrix for the noise

| | $w_1[1]$ | $w_1[2]$ | $w_1[Q]$ | $w_2[1]$ | $w_2[Q]$ | $w_3[1]$ | $w_3[Q]$ | $w_4[1]$ |
|---|---|---|---|---|---|---|---|---|
| $w_1[1]$ | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $w_1[2]$ | 0 | 1 | | | 0 | 0 | 0 | 0 |
| $w_1[Q]$ | | | 1 | 0 | 0 | 0 | 0 | 0 |
| $w_2[1]$ | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| $w_2[Q]$ | | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| $w_3[1]$ | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| $w_3[Q]$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| $w_4[1]$ | 0 | 0 | 0 | 0 | 0 | 0 | | |

toeplitz matrix (see the code)

$$\underset{NM\times 1}{\hat{\underline{h}}_{ML}} = (\underset{NM\times MQ}{\underline{X}^T} \underset{MQ\times MQ}{\underline{C}^{-1}} \underset{MQ\times NM}{\underline{X}})^{-1} \underset{NM\times MQ}{\underline{X}^T} \cdot \underset{MQ\times MQ}{\underline{C}^{-1}} \underset{MQ\times 1}{\underline{y}}$$

$\mathrm{kron}(\underline{I}_M, \underline{X})$

1.2 ii (memory filter) M=N=K=4

$$\underset{Q+K-1\times 1}{\underline{y}_1} = \begin{bmatrix} y_1[1] \\ y_1[2] \\ \vdots \\ y_1[Q+K-1] \end{bmatrix} = \underset{Q+K-1\times K\ \text{(conv. matrix)}}{\begin{bmatrix} x_1[1] & 0 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ x_1[Q] & x_1[Q-1] & \dots & x_1[Q-3] \\ 0 & x_1[Q] & \dots & x_1[Q-2] \\ \vdots & \vdots & & x_1[Q-1] \\ 0 & 0 & \dots & x_1[Q] \end{bmatrix}} \underset{K\times 1}{\begin{bmatrix} h_{11}[1] \\ h_{11}[2] \\ h_{11}[3] \\ h_{11}[4] \end{bmatrix}} + \underset{Q+K-1\times K}{\underline{X}_2} \underset{K\times 1}{\begin{bmatrix} h_{12}[1] \\ h_{12}[2] \\ h_{12}[3] \\ h_{12}[4] \end{bmatrix}} + \underline{X}_3 \underline{h}_{13} + \underline{X}_4 \underline{h}_{14} + \begin{bmatrix} w_1[1] \\ w_1[2] \\ \vdots \\ w_1[Q+K-1] \end{bmatrix}$$

$$\underset{Q+K-1\times 1}{\underline{y}_1} = \underset{Q+K-1\times KN}{\begin{bmatrix} \underline{X}_1 & \underline{X}_2 & \underline{X}_3 & \underline{X}_4 \end{bmatrix}} \underset{KN\times 1}{\begin{bmatrix} \underline{h}_{11} \\ \underline{h}_{12} \\ \underline{h}_{13} \\ \underline{h}_{14} \end{bmatrix}} + \underset{Q+K-1\times 1}{\underline{w}_1} \quad ; \quad \underline{y}_i = \begin{bmatrix} \underline{X}_1 & \underline{X}_2 & \underline{X}_3 & \underline{X}_4 \end{bmatrix} \begin{bmatrix} \underline{h}_{i1} \\ \underline{h}_{i2} \\ \underline{h}_{i3} \\ \underline{h}_{i4} \end{bmatrix} + \underline{w}_i$$

Generalazig:

$$\underset{M(Q+K-1)\times 1}{\begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \\ \underline{y}_3 \\ \underline{y}_4 \end{bmatrix}} = \underset{M(Q+K-1)\times KNM}{\begin{bmatrix} X_1X_2X_3X_4 & 0000 & 0000 & 0000 \\ 0000 & X_1X_2X_3X_4 & 0000 & 0000 \\ 0000 & 0000 & X_1X_2X_3X_4 & 0000 \\ 0000 & 0000 & 0000 & X_1X_2X_3X_4 \end{bmatrix}} \underset{KNM\times 1}{\begin{bmatrix} \underset{K\times 1}{h_{11}} \\ h_{12} \\ \vdots \\ h_{14} \\ h_{21} \\ \vdots \\ h_{24} \\ \vdots \\ h_{44} \end{bmatrix}} + \underset{M(Q+K-1)\times 1}{\begin{bmatrix} w_1[1] \\ \vdots \\ w_1[Q+K-1] \\ \vdots \\ \vdots \\ \vdots \\ w_4[Q+K-1] \end{bmatrix}}$$

(N entries for $h_{11}, \dots, h_{14}$; N entries for $h_{21}, \dots, h_{24}$)

$$\underline{X} = \underline{I}_M \otimes \underline{X}_1\underline{X}_2\underline{X}_3\underline{X}_4$$

Similarly to 1.2 i, cov. matrix will have the same structure and the size of $M(Q+K-1) \times M(Q+K-1)$

$$\underset{KNM\times 1}{\hat{\underline{h}}_{ML}} = (\underset{KNM\times M(Q+K-1)}{\underline{X}^T} \underset{M(Q+K-1)\times M(Q+K-1)}{\underline{C}^{-1}} \underset{M(Q+K-1)\times KNM}{\underline{X}})^{-1} \underset{KNM\times M(Q+K-1)}{\underline{X}^T} \underset{M(Q+K-1)\times M(Q+K-1)}{\underline{C}^{-1}} \underset{M(Q+K-1)\times 1}{\underline{y}}$$