Digital Communication II

Adaptive Predistortion

Docente: Prof. Arnaldo Spalvieri

Dipartimento di Elettronica, Informazione e Bioingegneria

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Summary

Non-linear distortion caused by power amplifiers is one of the impairments that limit the performance of digital transmission systems.

In this talk, the power amplifier is modelled as a memoryless non linear system, hence its distortion can be compensated by putting before the amplifier a pre-distorter that, in this case, is a memoryless non linear system that inverts the non liner transformation made by the amplifier. Of course, for inversion to be possible, the input signal must be in the range where the transformation made by the amplifier is invertible.

Adaptivity of the pre-distorter will be discussed. Adaptivity is commonly adopted in the practice because it allows to track slow time variations of amplifier's characteristic.

Outline of the talk

- Amplifier's characterization
- Ideal amplification: the role of predistortion
- Output back off and crest factor
- Characterization of the input signal
- System model and design consideration
- Predistortion and adaptive predistortion
- Results
- Project guidelines

Amplifier's Characterization

Consider baseband equivalent and let x be the complex input to the power amplifier. The output of a memoryless power amplifier can be written as

$$z = xF(|x|) = x\sqrt{F_A(|x|^2)} \cdot e^{jF_\Phi(|x|^2)}, \tag{1}$$

where $F_A(|x|^2) \geq 0$ is the AM-AM power gain and $F_\Phi(|x|^2)$ is the AM-PM gain of the power amplifier. The most important figure of merit of the power amplifier is the saturation output power, which, with non-decreasing input-output amplifier characteristic, is defined as

$$P_{sat} = \lim_{|x|\to\infty} |x|^2 F_A(|x|^2), \tag{2}$$

that is the maximum power that the amplifier can provide at its output. A synthetic model that is often used for the amplifier and that can be found in the RF toolbox of Simulink is the Rapp model.

Amplifier's Characterization

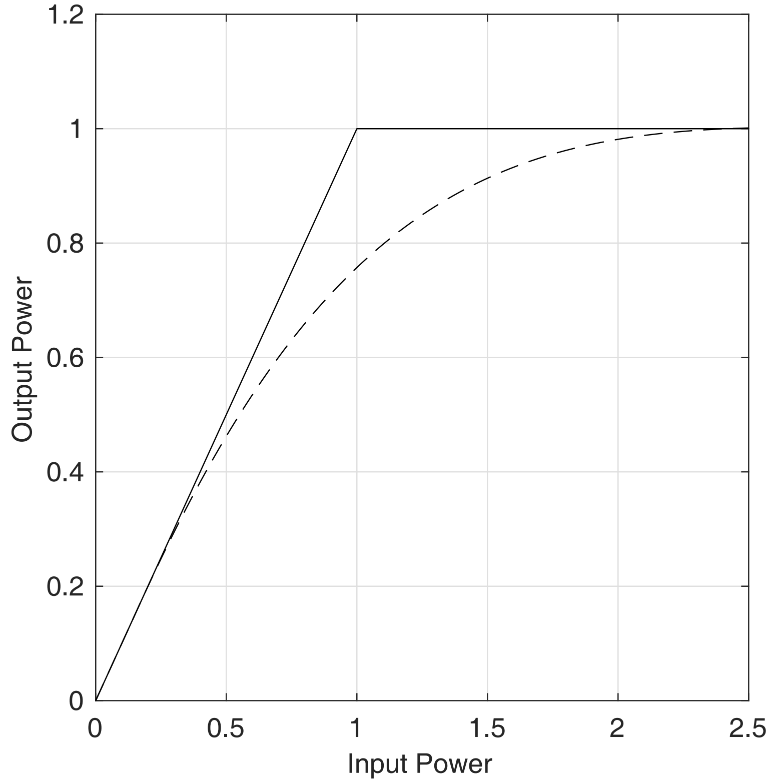

Fig. 1: Solid line: input-output AM-AM characteristic of the soft limiter. Dashed line: characteristic of a typical power amplifier (the one that will be used to derive the results hereafter presented). The suturation power is 1 and the AM-AM gain is close to 1 till about 0.4 times the saturation power.

Ideal Amplification: the Role of Predistortion

Ideal amplification means

$$z = Kx. \tag{3}$$

One can try to compensate the undesired smoothness of the AM-AM of the amplifier by predistortion,that is to impose

$$z = xP(|x|)F(|xP(|x|)|) = Kx,$$

where $xP(|x|)$ is the input-output characteristic of predistortion. Note that a necessary condition for ideal amplification to be possible is that

$$K^2|x|^2_{max} < P_{sat}. \tag{4}$$

When this condition is satisfied, then there exists $P(|x|)$ such that

$$P(|x|)F(|xP(|x|)|) = K$$

in the range $0 \leq |x| \leq |x|_{max}$.

Output Back Off and Crest Factor

Let

$$\text{OBO} = \frac{P_{sat}}{E\{|z|^2\}}, \tag{5}$$

$$\text{CF}_x = \frac{|x|^2_{max}}{E\{|x|^2\}}. \tag{6}$$

The first equation defines the *Output Back Off* (OBO), while the second one defines the *Crest Factor* (CF) or *Peak to Average Power Ratio* (PAPR) of the input signal x. Note that while the crest factor depends only on the input signal, the OBO depends on the output signal through its mean power and on the amplifier through P_{sat}. Imposing ideal amplification we have

$$\frac{\text{OBO}}{\text{CF}_x} = \frac{P_{sat}E\{|x|^2\}}{E\{|z|^2\}|x|^2_{max}} = \frac{P_{sat}}{K^2|x|^2_{max}} > 1. \tag{7}$$

Output Back Off and Crest Factor

With real-world amplifiers it is difficult to measure the saturation power, hence one often takes the OBO at 1 dB from the saturation point to characterize the working point of the amplifier. Here we will use synthetic models, where the saturation power is known, thus allowing to take the OBO from saturation to characterize amplifier's working point.

In order to have high efficiency of the amplifier and high output mean power, that is, in order to exploit the potential of the amplifier, one would like to keep OBO small. However, the condition for ideal amplification imposes OBO$>$ CF$_x$. This means that, fixed the input crest factor, if one tries to impose a too small OBO e.g. by imposing a too big gain K, it will become impossible to guarantee ideal amplification.

Characterization of the Input Signal

The distribution of the squared modulus of Gaussian-like complex signals as OFDM is χ^2 with two degrees of freedom. Normalizing the χ^2 random variable in such a way that the mean value of the squared modulus is $E\{|x|^2\} = 2$, the cumulative distribution results

$$F(\alpha) = 1 - P(|x|^2 > \alpha) = 1 - e^{-\alpha/2}.$$

For instance, for the probability that the squared modulus of the complex signal is 15 dB ($10^{1.5} = 31.6$ in linear scale), that is the probability that the squared modulus is 12 dB above the mean power, we have

$$P(|x|^2 > 31.6) = 1.37 \cdot 10^{-7}.$$

The probability that the squared modulus is 7.9 dB above the mean power (hence the squared modulus is 10.9 dB, corresponding to $10^{1.09} = 12.3$ in linear scale) is

$$P(|x|^2 > 12.3) = 2.13 \cdot 10^{-3}.$$

Characterization of the Input Signal

With real Gaussian input signal with zero mean and unit variance, the cumulative distribution results

$$P(|x|^2 > \alpha) = 2Q\left(\sqrt{\alpha}\right) \approx \frac{2e^{-\alpha/2}}{\sqrt{2\pi\alpha}},$$

To compute the probability of a signal with squared modulus 12 dB above the mean power we put $\alpha = 10^{1.2} = 15.8$, getting

$$P(|x|^2 > 15.8) \approx 7.44 \cdot 10^{-5}.$$

The probability of a signal with squared modulus 7.9 dB above the mean power is obtained with $\alpha = 10^{0.79} = 6.16$, leading to

$$P(|x|^2 > 6.16) \approx 1.48 \cdot 10^{-2}.$$

Hence the real signal is more spiky than the complex signal.

Crest Factor Design

With Gaussian-like signals, the only way to meet the condition OBO $>$ CF$_x$ and, at the same time, to keep OBO reasonably low, is to intentionally reduce the crest factor of the Gaussian source signal. This is done by passing the Gaussian signal through the solid line characteristic of Fig. 1 and taking the crest-reduced signal as input to the predistorter. Crest reduction is subject to the following compromise. Low crest factor is desired to keep OBO low thus better exploiting the amplifier's potential, but this means high non-linear distortion (often called intermodulation), hence large spectral broadening and large performance degradation due to non-linear distortion. On the other hand, with high crest factor there is no problem of spectral broadening and non-linear distortion, but the OBO will be high.

After that distortion has been intentionally introduced before predistortion, the cascade of predistortion and amplification can ideally reproduce the intentionally distorted signal. Compared to the approach where a non-crest-limited signal is distorted by amplifier, the advantage here is that the distortion is intentional and therefore it is known and under control.

System Model

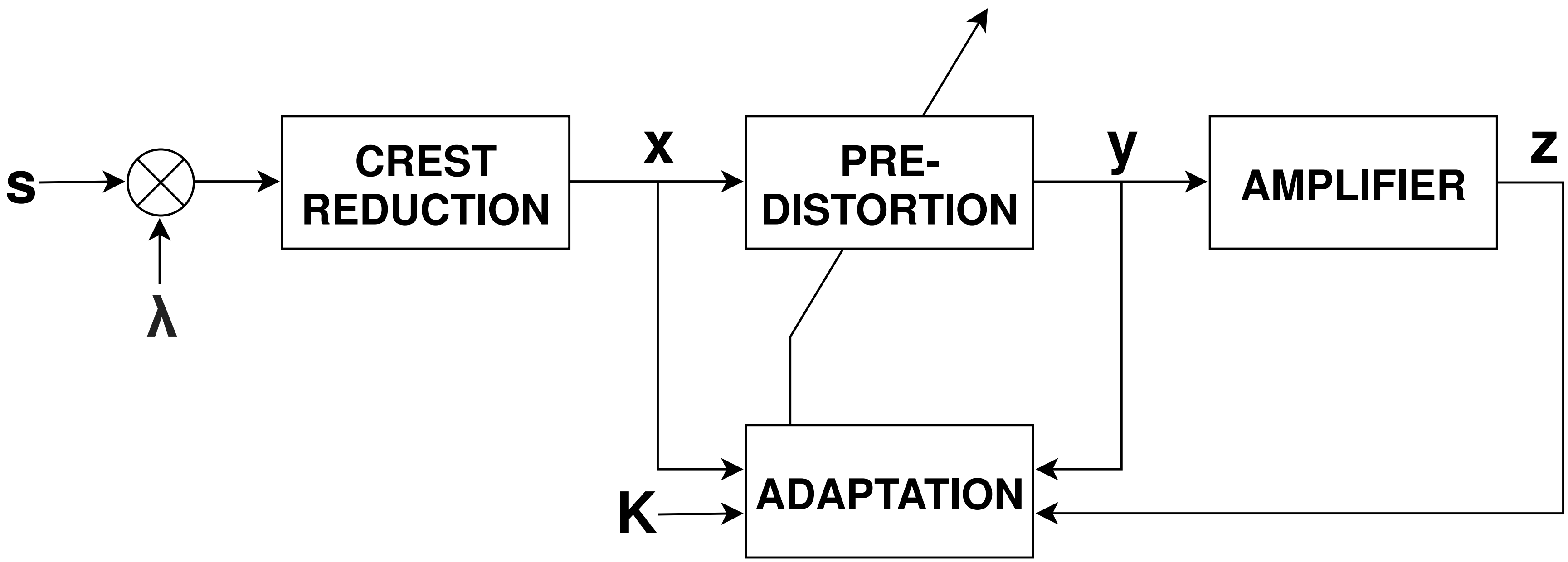

Fig. 2: Block diagram of the system.

The crest reduction block is a soft limiter (the solid line of Fig. 1). The goal of predistortion is to impose $z = Kx$, where K is the desired gain. For Gaussian-like signal s, the crest factor of x is, in practical systems (5 dB $\leq$ CF$_x \leq 10$ dB), approximately proportional to λ^{-2}.

You Can't Desire a Too Big Gain

Let us consider the compatibility between the desired gain K and the condition for ideal cascade of predistorter and amplifier:

$$P_{sat} > K^2 |x|^2_{max}, \tag{8}$$

and suppose that $|x|_{max}$ is fixed by the crest reduction block before the predistorter. If K is too big, so that the inequality is not satisfied and the objective $z = Kx$ cannot be achieved, then the adaptive predistorter is led to increase its gain with the hope that this will bring the gain of the cascade of predistortion and amplifier to the desired gain K. However, there is nothing to do, the objective cannot be achieved. Therefore, if the input x is such that $K^2|x|^2 \geq P_{sat}$, any increase of predistorter's gain has no effect on the actual output power, which cannot go beyond P_{sat}. At the next iteration of the adaptive algorithm, if again the input has a peak that is not reproduced by the amplifier, the adaptive algorithm increases again predistorter's gain and this process continues in a catastrophic way. In other words, the adaptive algorithm drives the predistorter to divergence.

You Can't Desire a Too Big Crest Factor with Fixed OBO

We have already seen that

$$\text{OBO} = \frac{P_{sat}}{K^2 |x|^2_{max}} \text{CF}_x,$$

therefore we could keep the OBO at a fixed level either by acting on K or on CF_x. If, to reduce distortion, we increase the crest factor, then we must increase also the gain K^2 to keep the OBO fixed. However, there is a limit, that is the condition

$$P_{sat} > K^2 |x|^2_{max}. \tag{9}$$

As said before, if K is so big that the above inequality is not satisfied, then the objective of ideal amplification is not achievable. In the attempt of achieving the non-achievable objective, the adaptive algorithm will drive the predistorter to divergence by the mechanism described in the previous slide.

Divergence of Predistorter

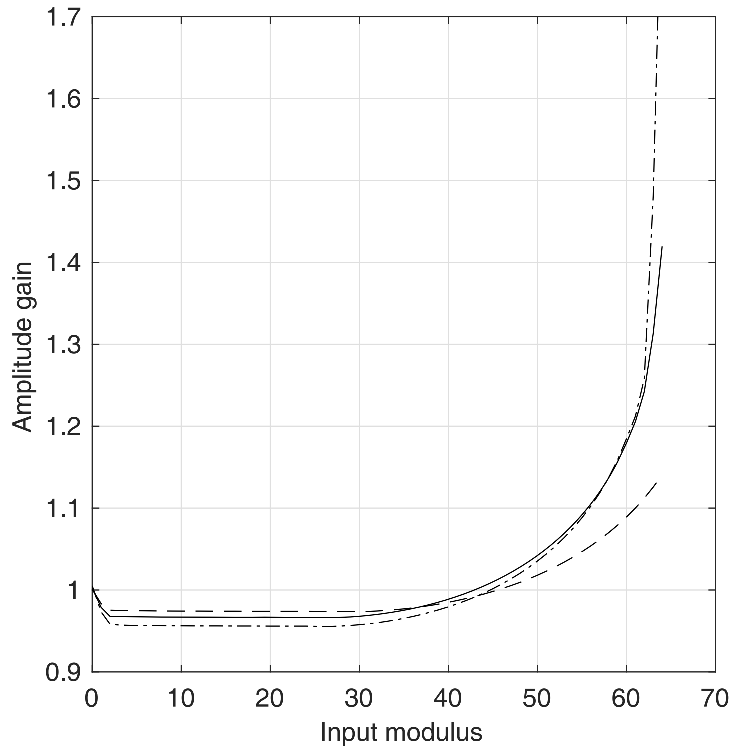

Fig. 3: Predistorter's gain versus input modulus with OBO fixed to 8 dB. Dashed line, $K^2|x|^2_{max}/P_{sat} = 0.79$, CF$_x$= 7 dB. Solid line, $K^2|x|^2_{max}/P_{sat} = 0.977$, CF$_x$=7.9 dB. Dash-dotted line, $K^2|x|^2_{max}/P_{sat} = 1$, CF$_x$= 8 dB: predistorter's gain diverges.

Predistortion

Referring to Fig. 2, the predistorter receives at its input the desired and crest-limited signal x and produces the input to the amplifier

$$y = xP(|x|),$$

where $P(|x|)$ is preditorter's gain, that we assume to be a function of the $N+1$ parameters $\{a_0, a_1, \cdots, a_N\}$. If the necessary condition for ideal amplification is met, then the objective $z = Kx$ can be achieved by minimizing $|z - Kx|^2$:

$$\frac{\partial |z - Kx|^2}{\partial a_n} = 2(z - Kx)\left(\frac{\partial z}{\partial a_n}\right)^* = 0, \;\; \forall x : |x| \le |x|_{max}, \;\; n = 0, 1, \cdots N. \quad (10)$$

The partial derivative can be written as

$$\frac{\partial z}{\partial a_n} = \frac{\partial z}{\partial y}\frac{\partial y}{\partial a_n} = \frac{\partial z}{\partial y} x \frac{\partial P(|x|)}{\partial a_n}. \quad (11)$$

Predistortion

Let us consider the first factor in the right hand side of (11), which is independent of the specific implementation of the predistortion function. The basic concept of derivation in the complex plane is that, when it exists, the derivative of a complex function $z(y)$ w.r.t. the complex variable y is independent of the phase of Δy. This extends to the complex variable the concept of continuity from the left and from the right of the derivative w.r.t. the real variable. For instance,

$$\frac{\partial z}{\partial y} = \lim_{\Delta y \to 0} \frac{z(y+\Delta y) - z(y)}{\Delta y} = \lim_{\Delta \Re\{y\} \to 0} \frac{z(y+\Delta \Re\{y\}) - z(y)}{\Delta \Re\{y\}}, \qquad (12)$$

$$\frac{\partial z}{\partial y} = \lim_{\Delta y \to 0} \frac{z(y+\Delta y) - z(y)}{\Delta y} = \lim_{j\Delta \Im\{y\} \to 0} \frac{z(y+j\Delta \Im\{y\}) - z(y)}{j\Delta \Im\{y\}}, \qquad (13)$$

that is

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial \Re\{y\}} = \frac{\partial z}{j\partial \Im\{y\}}.$$

Predistortion

This principle can be applied also when y is expressed in polar coordinates, leading to

$$\Delta y = |y| \Delta e^{j\angle y} = |y| j e^{j\angle y} \Delta\angle y = jy\Delta\angle y,$$

$$\frac{\partial z}{\partial y} = \frac{1}{jy}\frac{\partial z}{\partial \angle y}. \tag{14}$$

Applying (14) to (1) one has

$$\frac{\partial z}{\partial y} = \frac{1}{jy}\frac{\partial z}{\partial \angle y} = \frac{1}{jy}\frac{\partial y F(|y|)}{\partial \angle y} = \frac{|y|F(|y|)}{jy}\frac{\partial e^{j\angle y}}{\partial \angle y} = \frac{|y|F(|y|)}{jy} j e^{j\angle y} = \frac{z}{y}, \tag{15}$$

that is the complex gain $F(|y|)$ of the amplifier.

Predistortion Based on Piecewise Linear Interpolation

For instance, let the predistortion function be based on uniform quantization of the range $[0, |x|_{max})$ into $2N$ intervals $\{I_0, I_1, \cdots, I_{2N-1}\}$ with interval size $\Delta = \frac{|x|_{max}}{2N}$ (don't quantize signal $|x|$, quantize only the range where it takes its values). Moreover, let I_{2N} be the interval $|x| \geq |x|_{max}$. For

$$|x| \in I_{2n} \cup I_{2n+1}, \quad n = 0, 1, \cdots, N,$$

predistortion is obtained by piecewise linear interpolation as

$$P(|x|) = \frac{(|x| - (2n+1)\Delta)(a_{n+1} - a_n)}{2\Delta} + \frac{a_{n+1} + a_n}{2}, \tag{16}$$

where conventionally we put $a_{N+1} = a_N$. Note that a_n is the result of the interpolation when $|x|$ falls on the border between I_{2n} and I_{2n-1}. In general, a closed form solution of (10) is not available. When this happens, one resorts to the gradient descent method.

Predistortion Based on Piecewise Linear Interpolation

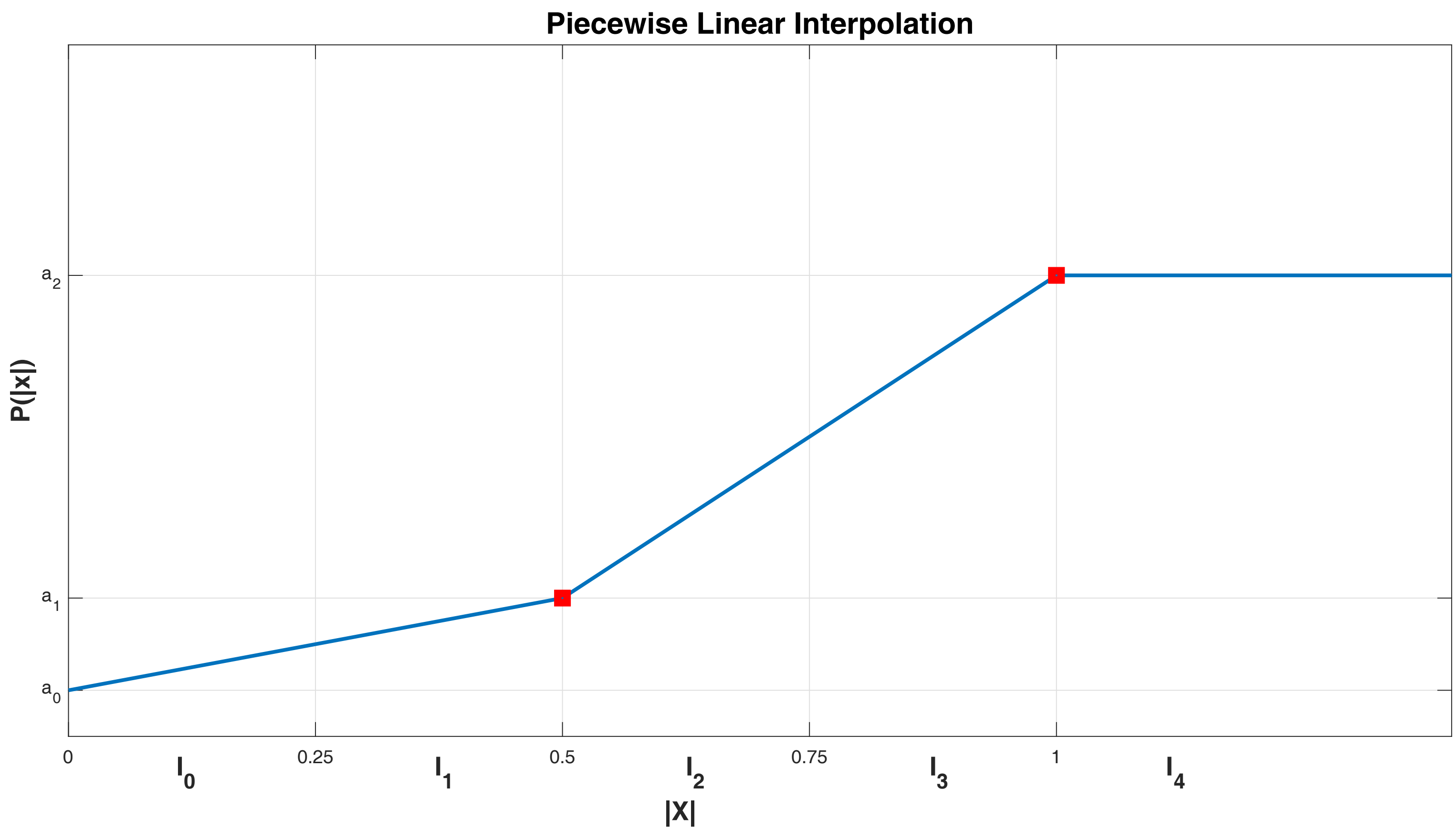

Fig. 4: Piecewise linear predistorter's gain with $N = 2$, 3 parameters $\{a_0, a_1, a_2\}$, and 4 intervals $\{I_0, I_1, I_2, I_3\}$ plus the interval I_4 that is visited by the input signal only when $|x| = |x|_{max} = 1$.

Adaptive Predistortion Based on Piecewise Linear Interpolation

In adaptive predistortion, the points $\{a_n\}$ are found by stochastic gradient search with objective

$$a_n = \arg\min E\{|z(l) - Kx(l)|^2\}, \quad n = 0, 1, \cdots, K, \tag{17}$$

where l is the time index. For

$$|x(l)| \in I_{2n-1} \cup I_{2n}, \quad n = 0, 1, \cdots, N,$$

the adaptive algorithm is

$$a_n(l+1) = a_n(l) - \gamma(z(l) - Kx(l))\frac{z^*(l)}{y^*(l)}x^*(l), \tag{18}$$

$$a_i(l+1) = a_i(l), \quad i \neq n. \tag{19}$$

As noted before, when $|x|$ falls exactly on the border between I_{2n} and I_{2n-1}, the result of the interpolation is just a_n, so the algorithm updates only the value of the parameter corresponding to the abscissa that is closer to $|x|$.

Adaptive Predistortion Based on Piecewise Linear Interpolation

The two equations (18) and (19) are derived as follows. When

$$|x| \in I_{2n} \cup I_{2n+1}, \quad n = 0, 1, \cdots, N,$$

for the derivatives of y w.r.t. a_{n+1} and a_n one has

$$\frac{\partial y}{\partial a_n} = x \frac{\Delta - (|x| - (2n+1)\Delta)}{2\Delta}, \tag{20}$$

$$\frac{\partial y}{\partial a_{n+1}} = x \frac{\Delta + (|x| - (2n+1)\Delta)}{2\Delta}. \tag{21}$$

Adaptive Predistortion Based on Piecewise Linear Interpolation

Note that

$$2\Delta \geq \Delta \pm (|x| - (2n+1)\Delta) \geq 0, \quad |x| \in I_{2n} \cup I_{2n+1}. \tag{22}$$

This observation suggests hard quantization of the term $\Delta \pm (|x| - (2n+1)\Delta)$ in the two values 2Δ and 0, leading to

$$\frac{\partial y}{\partial a_n} = x, \quad |x| \in I_{2n}, \tag{23}$$

$$\frac{\partial y}{\partial a_{n+1}} = x, \quad |x| \in I_{2n+1}, \tag{24}$$

Adaptive Predistortion Based on Piecewise Linear Interpolation

In the end, when

$$|x(l)| \in I_{2n-1} \cup I_{2n}, \quad n = 0, 1, \cdots, N, \quad I_{-1} \equiv \{\oslash\},$$

the adaptive algorithm is

$$\begin{aligned} a_n(l+1) &= a_n(l) - \gamma(z(l) - Kx(l))\frac{z^*(l)}{y^*(l)}x^*(l), \\ a_i(l+1) &= a_i(l), \quad i \neq n. \end{aligned} \tag{25}$$

Since the modulus of the factor $z^*x^*y^{-*}$ is virtually constant inside the quantization region, in the implementation it can be absorbed it into the step size and only the complex exponential of the phase of that term as a factor of the error can be used in the first equation of (25).

FPGA Implementation

We have implemented the predistorter on Xilinx Kintex xc7k160tfbg484-2. In the implementation we used piecewise linear interpolation with $K = 64$ intervals. The sample frequency of the baseband I/Q signal is 40 MHz, which allows for an over-sampling factor equal to 4 signals having maximum single-sided bandwidth of the I and Q signals of 5 MHz (10 MHz bandwidth at RF), a case that occurs, for instance, in digital video broadcasting. The resources used by our design/the total resources on the FPGA are

- LUTs: 13341/101400
- Registers: 12165/202800
- RAM blocks: 18/325
- DSP blocks: 115/600

The probing signal is approximately Gaussian with peak to average power ratio of 12 dB and 3dB single-sided bandwidth of 2.5 MHz and is represented on 18 bits.

Experimental Results

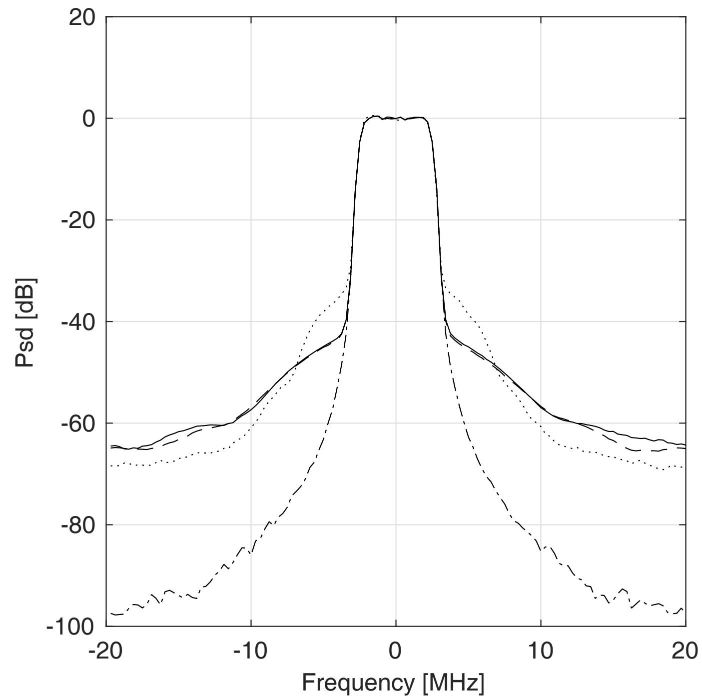

Fig. 5: Power spectral densities of various signals. Dash-dotted line: source OFDM signal. Dashed line: after crest factor reduction to 7.9 dB. Solid line: after the amplifier with crest factor reduction to 7.9 dB, predistortion and $K^2 = 0.977$ (OBO=8 dB). Dotted line: after the amplifier with no predistortion, OBO= 8 dB.

Experimental Results

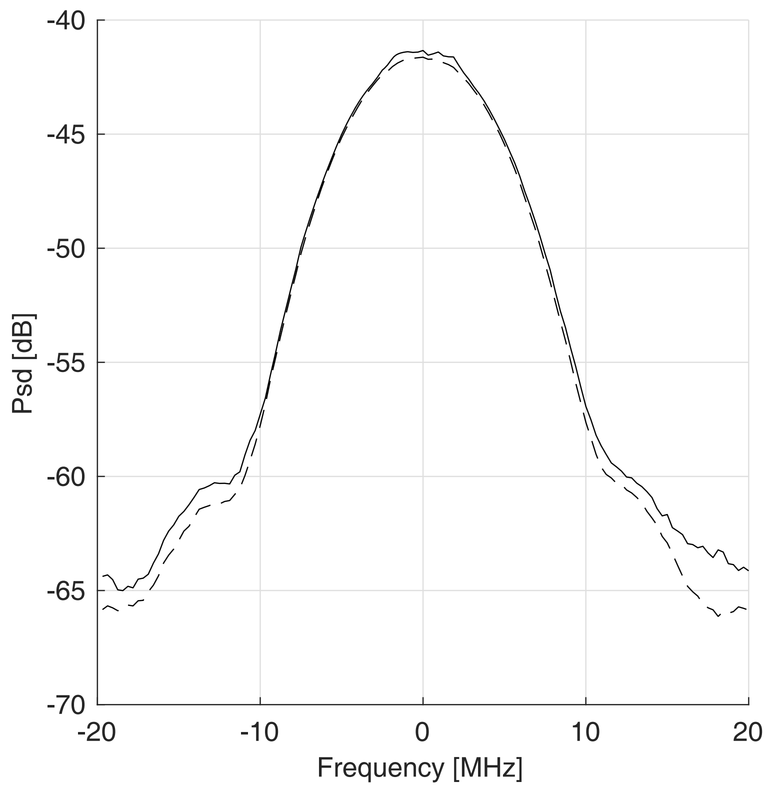

Fig. 6: Power Spectral Density (PSD) of the non-linear distortion with CF$_x$=7.9 dB and OBO= 8 dB ($K^2 = 0.977$). Dashed line: PSD of $\lambda s - x$. Solid line: PSD of $\lambda s - K^{-1} z$.

Experimental Results

A meaningful figure of merit is the Signal-to-Distortion Ratio, defined as

$$\text{SDR} = \frac{\lambda^2 E\{|s|^2\}}{E\{|\lambda s - u|^2\}}, \tag{26}$$

where $\lambda^2 E\{|s|^2\}$ is the power of the distortion free signal before the crest reduction factor. When $u = x$ the distortion $E\{|\lambda s - x|^2\}$ is the integral of the error spectrum of the dashed line of Fig. 6 and it includes only the effect of the crest factor reduction. When $u = K^{-1}z$ the distortion $E\{|\lambda s - K^{-1}z|^2\}$ is the integral of the error spectrum of the solid line of Fig. 6 and it includes the effect of the crest factor reduction and the possibly non ideal cascade of predistortion and amplifier. Also, you can drop the predistortion and feed the amplifier with the output of the crest reduction block. In this case, $u = z$. The results obtained with these three different definitions of u are reported in Fig. 7

Experimental Results

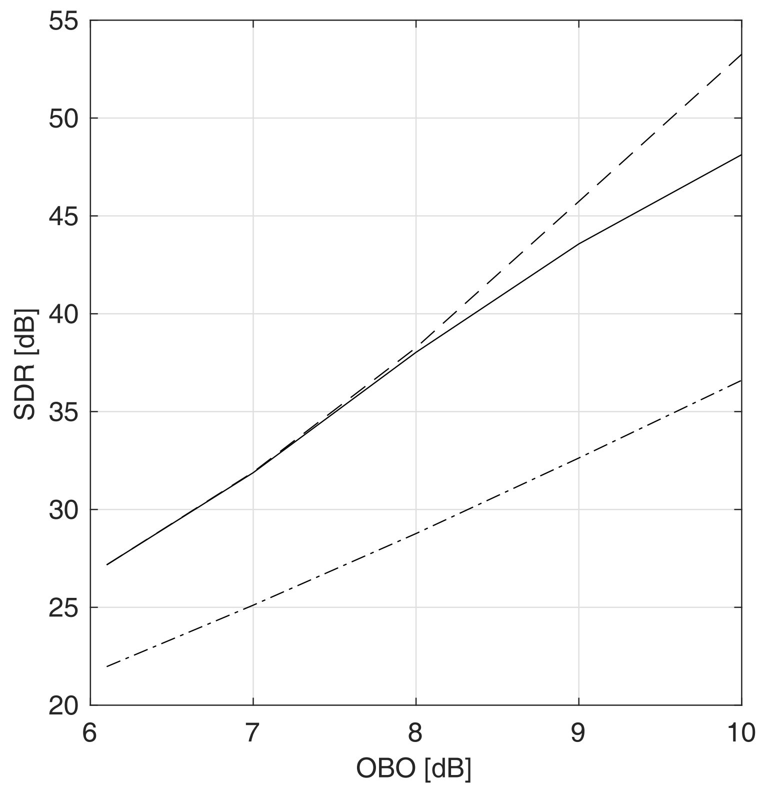

Fig. 7: SDR vs OBO. Dash-dotted line: amplifier alone with OFDM input. Solid line: cascade of crest factor reduction with CF_x= OBO -0.1 dB, predistorter, amplifier. Dashed line: ideal soft limiter.

Simulink/Matlab Project

The system block diagram is that of Fig. 2. Encode two systems, one with real Gaussian input, the other with complex Gaussian input (two Gaussian generators with different seeds or a random source complex Gaussian). The random generators are followed by a square root Nyquist filter with oversampling factor 8. Keep to 1 the saturation and the gain of the crest reduction block and the saturation of the amplifier exactly as in Fig. 1. Produce the results of all the Figures reported in these slides, duplicating them for the real and the complex case, and make a comparison between the real and the complex case. For Fig. 3 fix the CF and increase the desired gain K till you observe divergence. For each K plot $P(|x|)$ and measure the OBO (that is, the output power, the saturation power is fixed to 1). For the other figures set $0.95 < K < 1$. For instance, set $K^2 = 0.977$ to get CF$_x$ $=$ OBO $- 0.1$dB independently of λ. Then manually tune λ to tune CF$_x$ and, with it, OBO. While tuning λ, keep under control the spectrum of the crest-reduced signal. In general, put spectrum analyzers and power meters here and there to help the debug.

Thank You!