Digital Communication 2 Project: Predistortion and Adaptation

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Abstract

This abstract will discuss the implementation in MATLAB/Simulink of an adaptive algorithm to distortion in order to compensate for the distortion caused by a non-linearity of an amplifier. The main idea is to use a pre-distorter before an amplifier which will compensate for its non-linearity and use the cascade of them to linearize the signal within a certain region. Adaptation is implemented in order to achieve the proper pre-distorter gain.

Keywords: Adaptation, Rapp model, Predistortion, Crest Factor

1. Introduction to the Amplifier

Non-linear distortion caused by the power amplifiers is one of the impairments that limit the performance of digital transmission systems.

The power amplifier is modeled as a memoryless non-linear system, so that its distortion can be compensated using a pre-distortion block before the amplifier to invert the nonlinearity of the power amplifier.

The Rapp model has been used in this project as the power amplifier model. Keeping in mind that the desired signal is,

$$z = Ky$$

where z – output signal; y – input signal, K – desired gain, the formula for the model is the following:

$$z = \frac{K_a y}{\left(1 + \left(\frac{K_a|y|}{\sqrt{P_{sat}}}\right)^{2s}\right)^{\frac{1}{2s}}}$$

where s- smoothing factor. As it is seen, the higher the s value, the closer our model to linear until the saturation point $|z^2| = P_{sat}$ with the slope K_a.

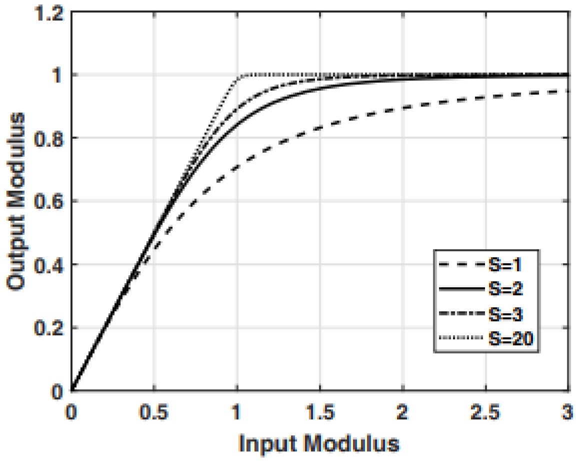

Figure 1. **The Rapp Model for different values of the smoothing parameter**

In order to characterize the amplifier, in this project we use OBO (Output Back Off) and IBO (Input Back Off)

$$OBO = \frac{P_{sat}}{E\{|y|^2\}},$$

$$IBO = \frac{\alpha P_{sat}}{K_a^2 E\{|y|^2\}},$$

which are referred to as the output and input signals of the amplifier respectively.

2. Predistortion

Due to the non-linearity of the amplifier, the power is observed at the frequencies where there is no power at the input. In order to regulate the output one can

- Lower the input power to stay within a linear region
- apply predistortion to bring the amplifier with it to the linear region

In the second case, it can be written as

$$z = yG(|y|) = xP(|x|)G(|xP(|x|)|) = Kx,$$

where *P* – predistorter gain, *x* – signal that is wanted to amplify. This is true if signal *z* is lower than the saturation power of the amplifier, since it is impossible to exceed physical limitations. The idea is to find the compromise between the maximum input value and the gain, meaning that $K^2 x_{max}^2 < P_{sat}$. In case of equality, the divergence of the predistorter gain can be observed. Since in this formula x_{max} is fixed, increasing the value K leads to $K^2 x_{max}^2 > P_{sat}$, which causes a divergence as the amplifier is limited to the peak.

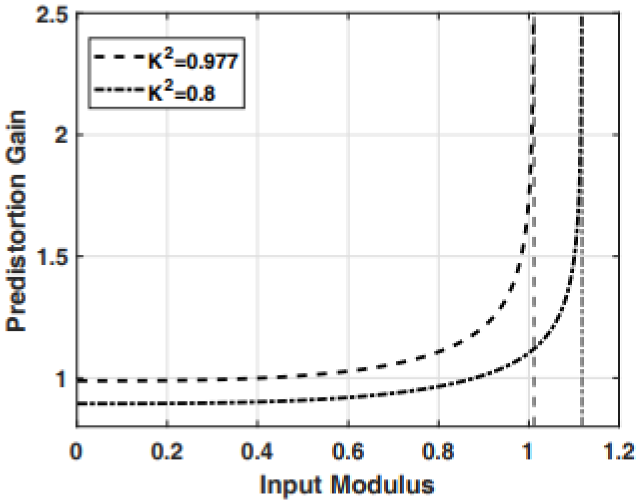

Figure 2. **Limits of predistortion gain**

3. Crest Factor Reduction

In order to meet the condition discussed previously, it is needed to intentionally cut the power of the signal to a specific upper limit (tails of the PSD of the Gaussian signal are problematic for the system). This operation is called crest factor reduction, and the Crest Factor (CF) is also known as *PAPR*, it can be written as

$$CF_x = \frac{|x_{max}|^2}{E\{|x|^2\}}$$

In order to make crest reduction one needs to find a compromise between

-low CF (low OBO) means that there is high mean power as well as high non-linear distortion (broadening of the spectrum)

-high CF (high OBO) means that there is low spectral broadening as well as low average power.

4. Adaptive Predistortion and Non-Parametric Amplifier Model

In real life cases, there are no parametric models like the Rapp Model, instead only the results of characteristics measurements. Therefore, there are no closed forms for the predistorter and it needs to be presented numerically. In that case it is assumed that predistorter's gain is the function of (a_0,..,a_N) parameters. In that case it is needed only the formulas for the input and the output and the knowledge of MSE.

In this project piecewise linear interpolation is applied by uniformly quantizing the input signal range into 2N intervals in order to find the predistortion

$$P(|x|) = \frac{(|x| - (2n+1)\Delta)(a_{n+1} - a_n)}{2\Delta} + \frac{a_{n+1} + a_n}{2}.$$

In that case the adaptation is applied in order to approximate the gain iteratively. By minimizing the error, using Stochastic Gradient Search and applying derivatives, it can be written as an adaptive algorithm using

$$a_n(l+1) = a_n(l) - \gamma(z(l) - Kx(l))\frac{z^*(l)}{y^*(l)}x^*(l),$$

$$a_i(l+1) = a_i(l), i \neq n.$$

5. System Model

The block diagram of the system can be represented as follows

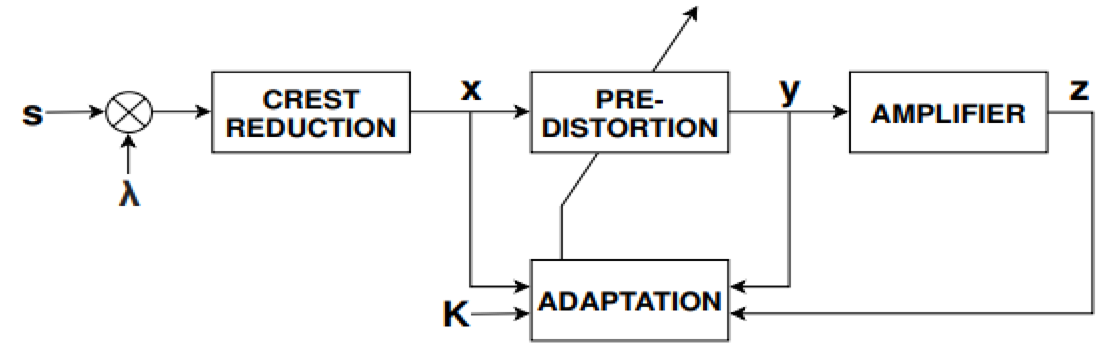

Figure 3. **Block diagram of the system**

The input signal, adaptation and predistortion parts were implemented using MATLAB functions, whereas the crest reduction and the amplifier - using Simulink.

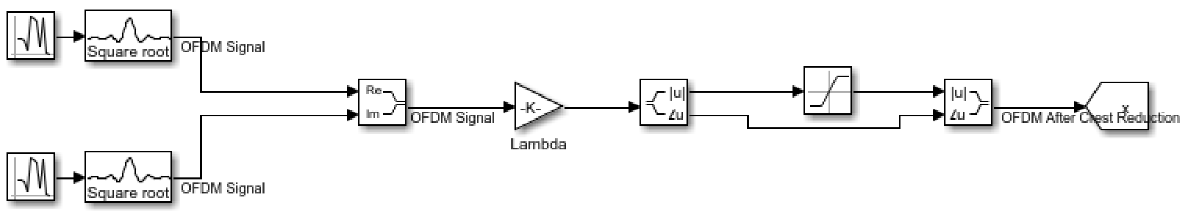

Figure 4. **Implementation of the input signal and the crest reduction**

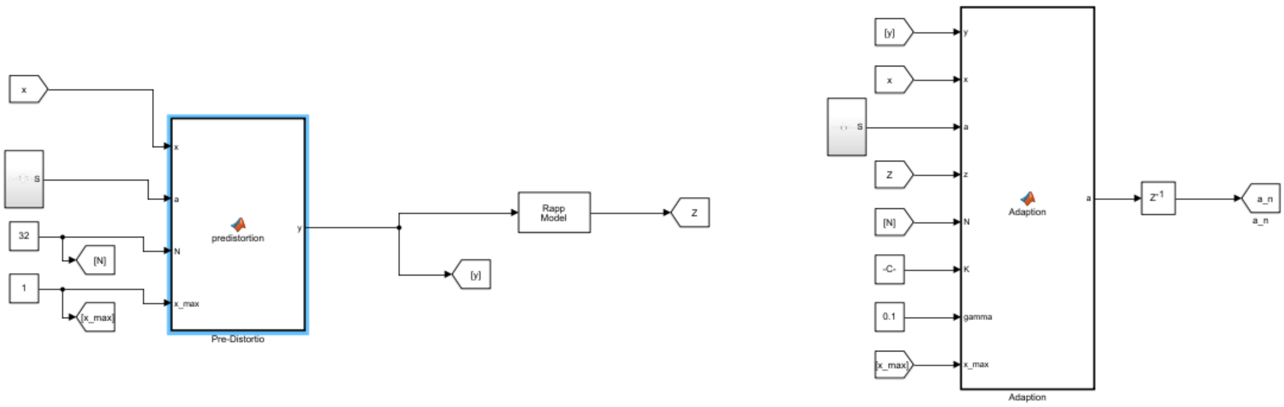

Figure 5. **Implementation of the adaptation and predistortion**

```
function y  = predistortion(x,a,N,x_max)
    %Interval detection
    delta = abs(x_max) / (2*N);
    % All intervals
    I = [delta*2:delta*2:x_max inf];
    % Find the Parameter Index where the abs(x) falls
    n = 1:(N+1);
    n = n(abs(x) < I);
    n = n(1);
    % Piecewise Linear Interpolation
    P_x = ((abs(x) - (2*n-1)*delta) * (a(min(n+1,N+1)) - a(n)) /...
           (2*delta)) + ((a(min(n+1,N+1)) + a(n)) / 2);
    % Output
    y_old = complex( x * P_x);
    y = y_old;
end
```

Algorithm for Predistortion Block

```
function a = Adaption(y, x, a, z, N, K, gamma, x_max)

    %Re-do the same because of causality of the first initialization
    delta = abs(x_max) / (2*N);
    % All Intervals
    I = [delta:delta*2:x_max inf];
    n = 1:(N+1);
    n = n(abs(x) < I);
    n = n(1);
    % Update Coefficients
    a(n) = (a(n) - gamma * (z - K * x) * conj(z) / conj(y) * conj(x));
end
```

Algorithm for Adaptation Block

6. Results

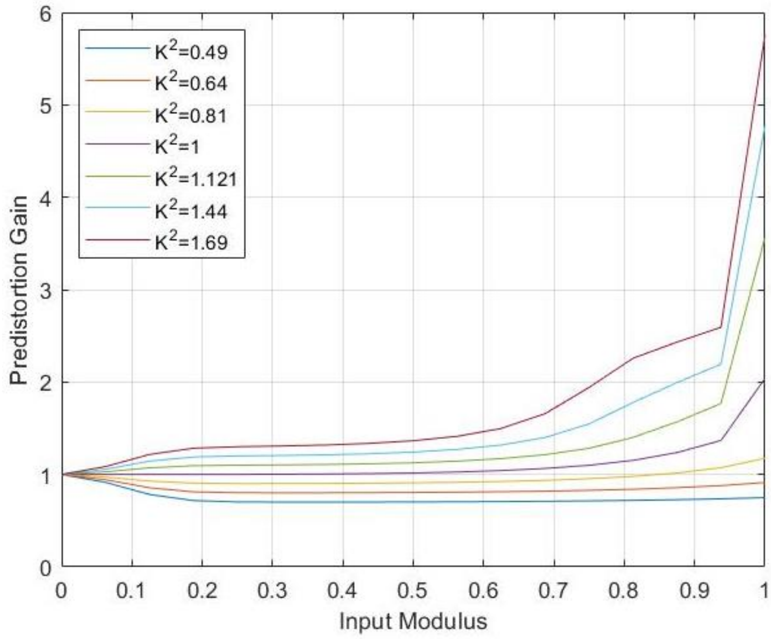

Figure 6. **Limits of predistortion gain** ***(Figure 5 in th.)***

As seen on the Figure 6, experimental results for increasing the gain K causes an increase in nonlinearity around the x_{max}. Since the system should meet the criteria $K^2|x_{max}| \leq P_{sat}$ and the saturation power and x_{max} is fixed to 1 in this project, there should be a K value smaller than 1. For K values that are too small the amount of nonlinearity can be ignored as can be seen on the figure. Hence, there is no point of using a predistortion system. The K^2 value chosen as 0.977 in this project to provide a decent nonlinearity.

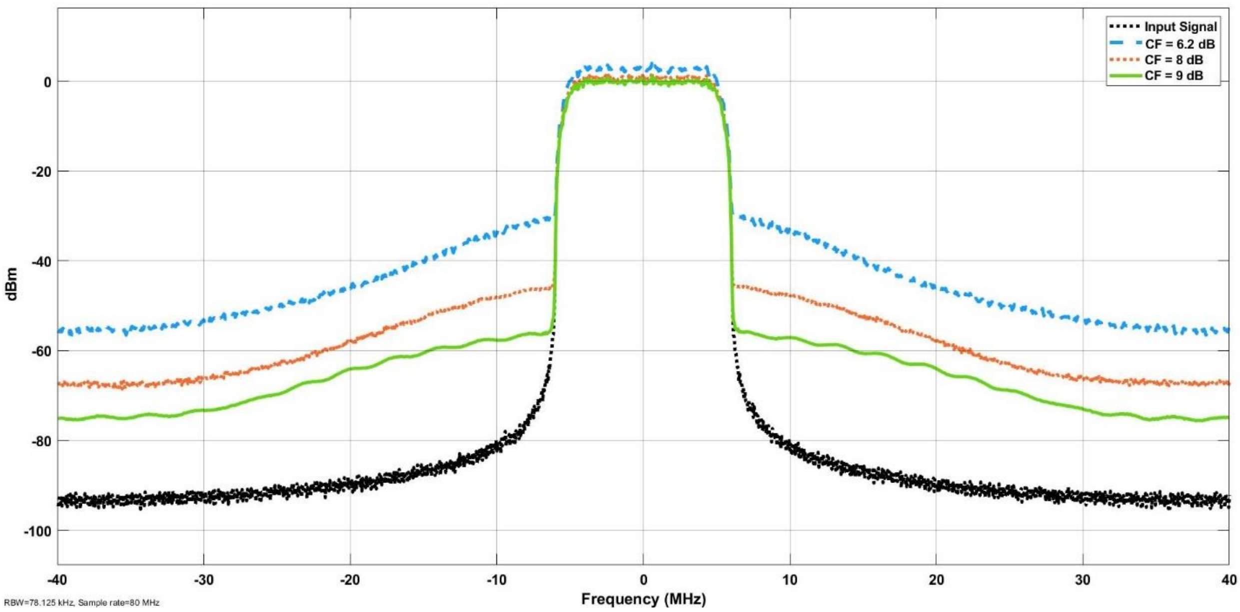

Figure 7. **Spectral broadening due to crest reduction for the complex signal** ***(Figure 7 in th.)***

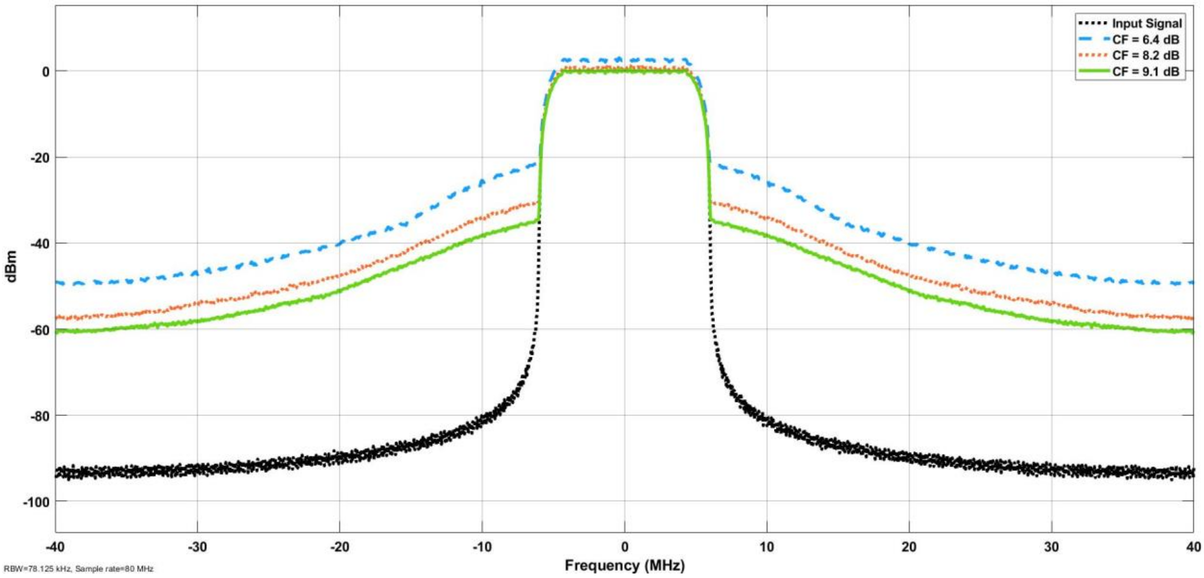

Figure 8. **Spectral broadening due to crest reduction for the real signal** ***(Figure 7 in th.)***

Crest factor reduction is used to decrease the PAPR ratio by clipping the signal over a certain value, hence increasing the efficiency of power amplifier. Clipping causes a distortion on the main lobe spectrum and broadening on the rest of the spectrum. The clipping causes a nonlinear distortion, but it is a controlled and desired distortion that is introduced to the system in order to increase efficiency.

In this project x_{max} is fixed therefore the crest factor is tuned by changing values of λ. λ values and crest factor is inversely proportional as mentioned in the theory. During the crest reduction process there is spectral broadening in every case compared to original OFDM signal as the result of non-linearity caused by crest factor reduction. As the crest factor decreases average power and spectral broadening increases as expected from the theory. At the same time, the higher value of CF increases the average power, increases the spectral broadening, meaning that for the lower CF values (higher λ) the more the signal is cut. Since the real signal is "spikier", as mentioned in the theory part, it is clipped more, hence subjected to more nonlinear distortion, causing a higher spectral broadening and more distortion in our OFDM message.

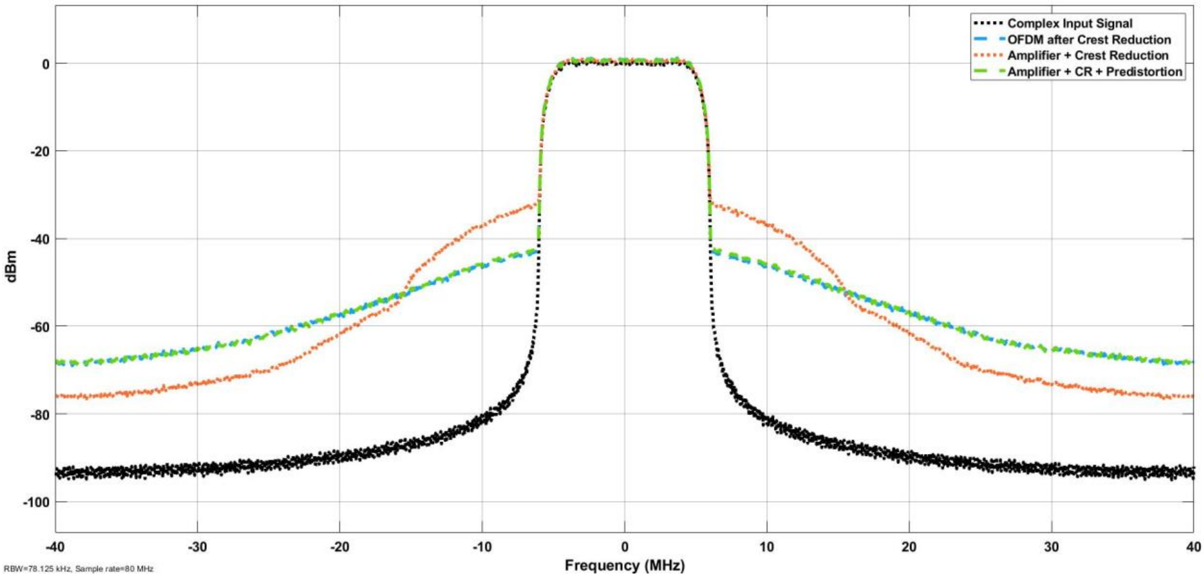

Figure 9. **Power spectral densities of various *Complex* signals *(Figure 10 in th.)***

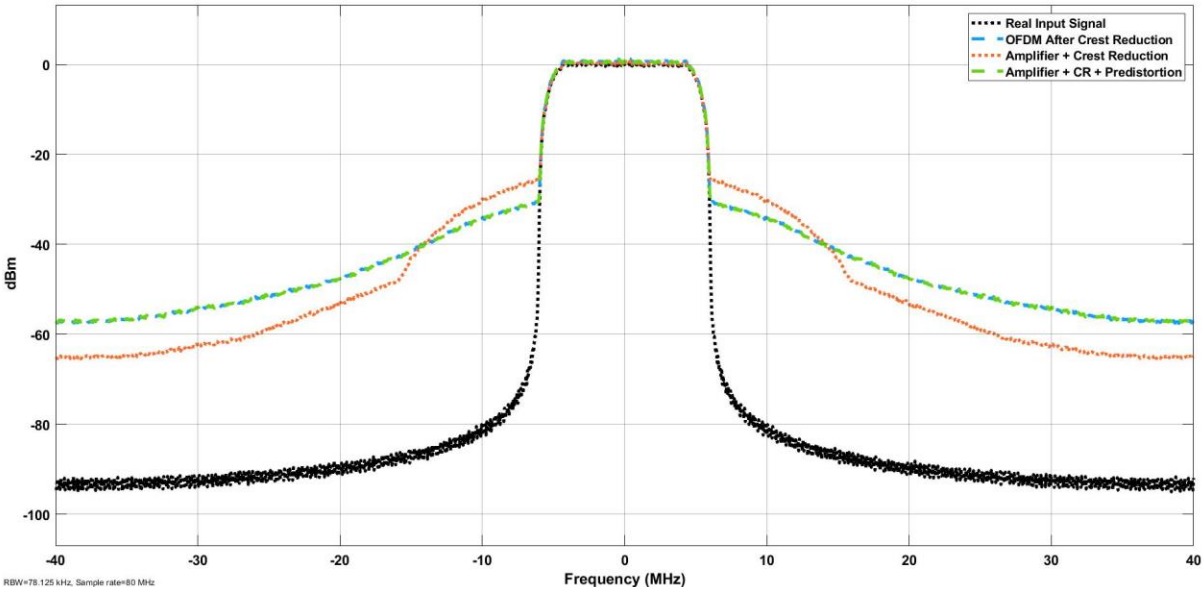

Figure 10. **Power spectral densities of various real signals. *(Figure 10 in th.)***

Comparing the effects of different operations on the spectrum, after CF = 7.9 dB, (blue dashed line) one can observe the spectral broadening and increase of average power. Sending this signal directly into the amplifier without predistortion (orange dotted line) reduces the spectral broadening and increases the average power around the main lobe. Using predistortion block, the spectrum stays nearly the same with the one after crest reduction, proving that the efficiency of the predistortion and adaptation block is sufficient. Again, since the real signal is subjected more nonlinear distortion by the crest factor reduction block, the spectral broadening is higher than the complex signal for each case.

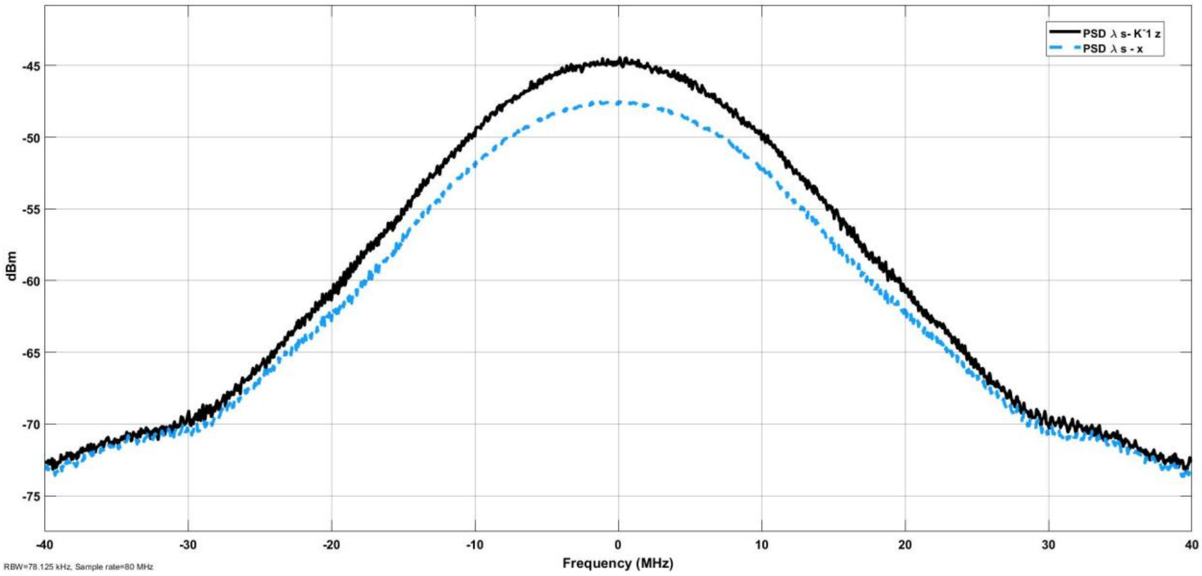

Figure 11. **Power Spectral Density (PSD) of the non-linear distortion of a complex signal (*Figure 11 in th.*)**

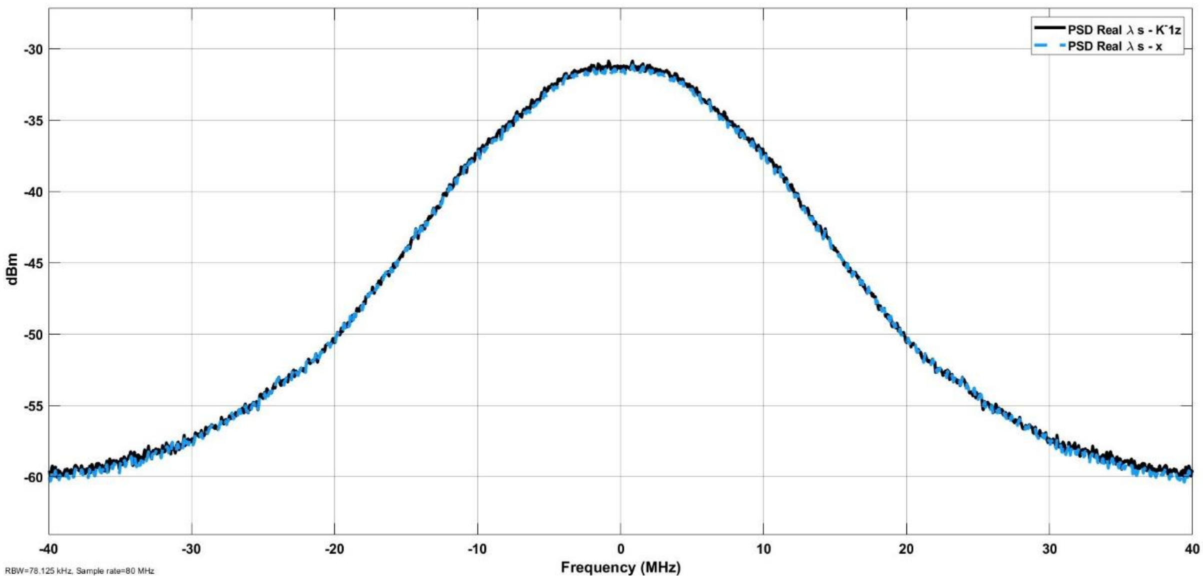

Figure 12. **Power Spectral Density (PSD) of the non-linear distortion of a real signal (*Figure 11 in th.*)**

Figure 11 and Figure 12 are the PSD's of the nonlinear distortion with CF = 7.9 dB and OBO = 8 dB. Solid line represents the distortion of the output while dashed line represents the distortion after the crest factor reduction. In the complex case, output has more distortion compared to the signal after the crest reduction (input to the predistorter), but still following closely. Compared to real signal case, complex signal has much lower nonlinear distortion PSD values while the distortion caused by predistorter is less for the real case due to the higher probability of reaching large values of power in compared to the complex one (real signal is more). The reason behind the gap between the signals in Figure 11 is likely to be due to the usage of AM-AM instead of AM-PM and not taking into account the phase, which is not the case for the real signal (Figure 12).

Usually, it is important to measure the ratio between the power of the distortion free signal before the CF and the distortion of the signal, which is referred as Signal-to-Distortion-Ratio

$$SDR = \frac{\lambda^2 E\{|s|^2\}}{E\{|\lambda s - u|^2\}}$$

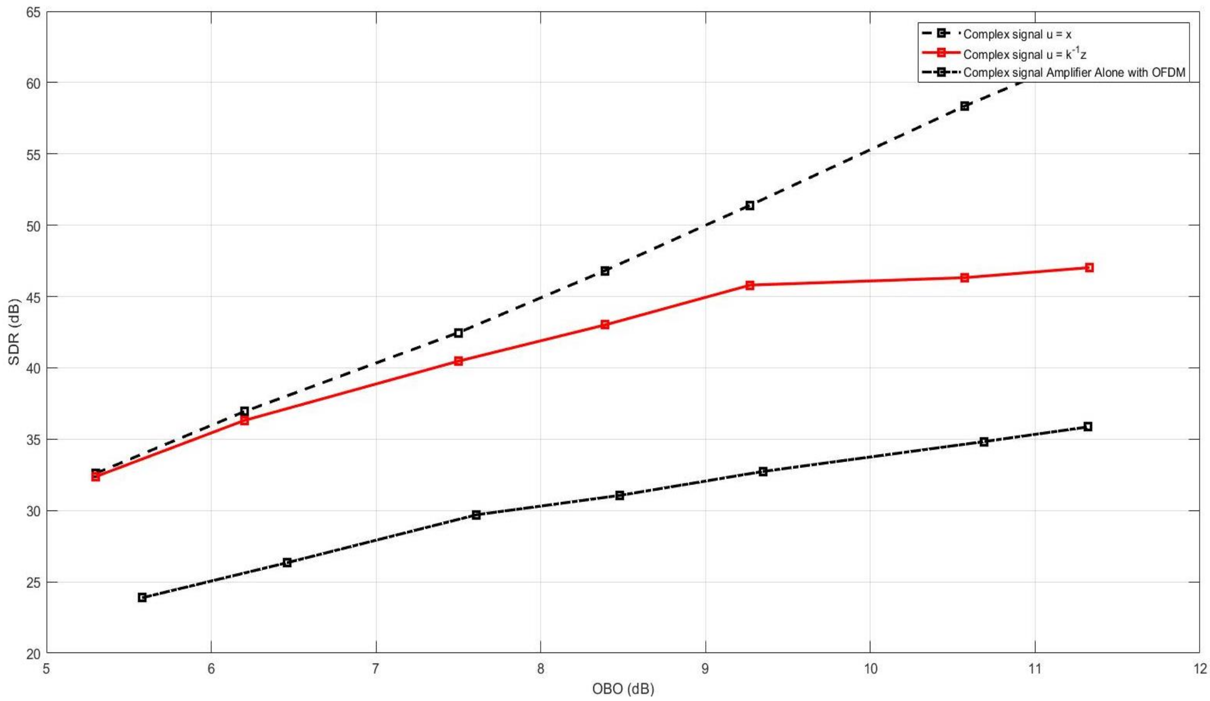

Figure 13. **SDR vs OBO for the complex signal (Figure 12 *in th.*)**

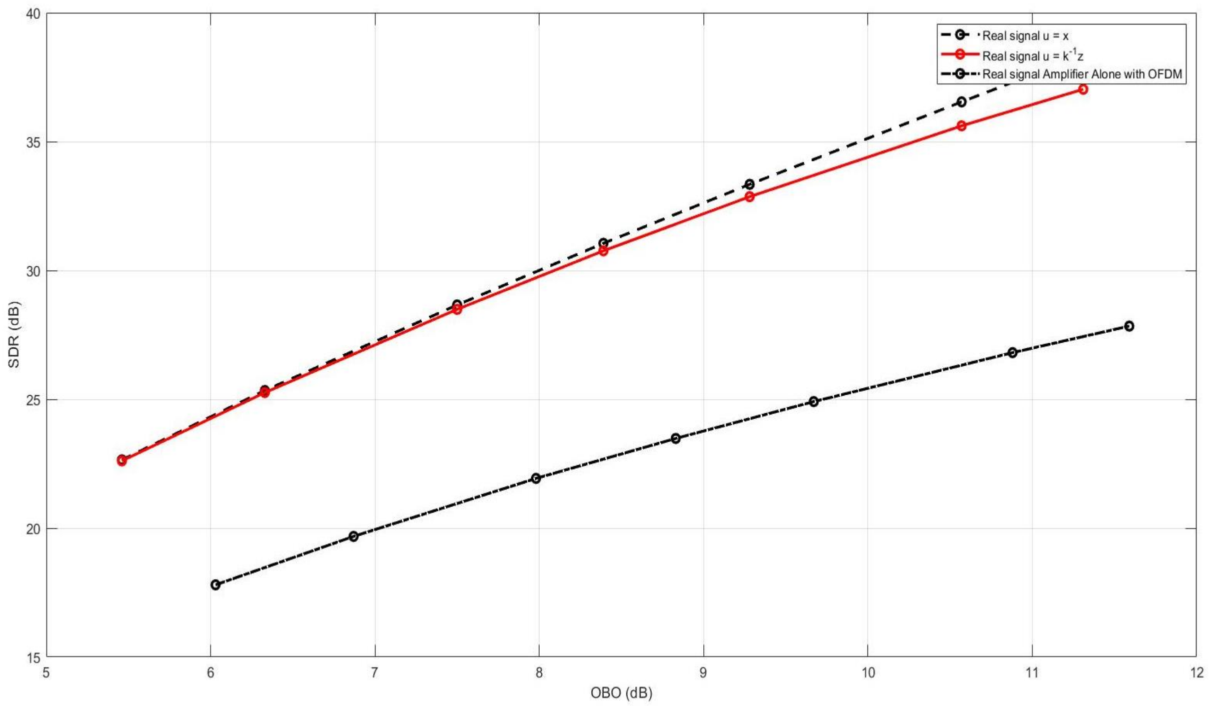

Figure 14. **SDR vs OBO for the real signal (*Figure 12* in th.)**

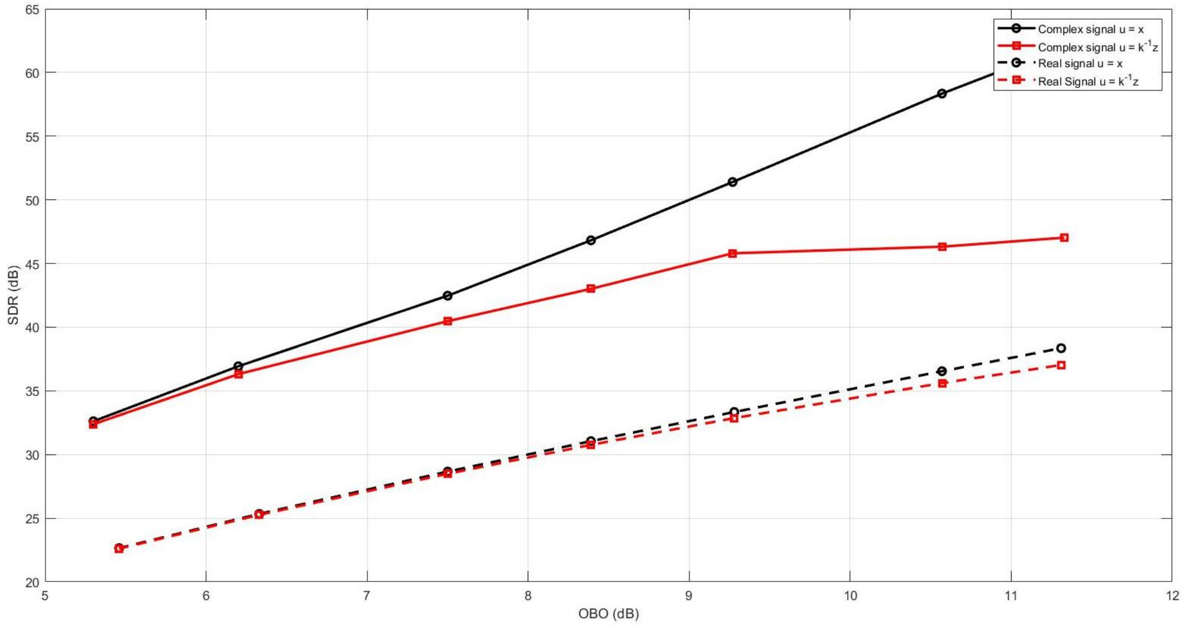

Figure 14. **SDR vs OBO for both signals (*Figure 12* in th.)**

λ	**OBO [dB]**	**SDR (x) [dB]**	**SDR(K^{-1}z) [dB]**	**SDR (No PD)**	**OBO (No PD**
1.25	5.3	32.61	32.38	23.88	5.58
1.15	6.2	36.93	36.31	26.34	6.46
1.05	7.5	42.47	40.47	29.70	7.61
0.99	8.39	46.82	43.01	31.06	8.48
0.94	9.27	51.41	45.8	32.74	9.35
0.88	10.57	58.34	46.32	34.83	10.59
0.85	11.33	62.05	47.03	35.86	11.32

Complex

λ	**OBO [dB]**	**SDR (x) [dB]**	**SDR(K^{-1}z) [dB]**	**SDR (No PD)**	**OBO (No PD**
1.25	5.46	22.65	22.61	17.8	6.03
1.15	6.33	25.34	25.26	19.68	6.87
1.05	7.5	28.66	28.49	21.93	7.98
0.99	8.39	31.05	30.76	23.48	8.83
0.94	9.28	33.34	32.86	24.91	9.67
0.88	10.57	36.54	35.61	26.81	10.88
0.85	11.31	38.35	37.03	27.84	11.59

Real

Comparing SDR and OBO values for the complex case increasing SDR corresponds to lower distortion. SDR for $u = x$ is the ideal soft limiter, represented as the dashed line. The red solid line is SDR for $u = K^{-1}z$ and that is the SDR value after predistortion. It follows the ideal soft limiter case with slowly lower values. As the OBO increases the difference between the ideal case increases. SDR values for the complex signal without predistortion is lower than our predistortion block proving that our predistortion block reduces nonlinear distortion significantly.

For the real case, all the SDR values are lower than the complex signal, meaning there is more nonlinear distortion present for the real signal, this was also the case for the PSD of nonlinearity as the real signal had higher values. Even though the overall distortion is more on the real signal compared to complex one, it is more possible to stay close to the ideal soft limiter case for the real signal. Again, checking the SDR values for no predistortion case, we can see that our predistortion block increases the performance of the system and decreases the nonlinear distortion significantly.