Digital Communication II

Adaptive Predistortion

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Summary

Non-linear distortion caused by power amplifiers is one of the impairments that limit the performance of digital transmission systems.

In this talk, the power amplifier is modelled as a memoryless non linear system, hence its distortion can be compensated by putting before the amplifier a pre-distorter that, in this case, is a memoryless non linear system that inverts the non liner transformation made by the amplifier. Of course, for inversion to be possible, the input signal must be in the range where the transformation made by the amplifier is invertible.

Adaptivity of the pre-distorter will be discussed. Adaptivity is commonly adopted in the practice because it allows to track slow time variations of amplifier's characteristic.

Outline of the talk

- Amplifier's characterization
- Ideal amplification: the role of predistortion
- Output back off and crest factor
- Characterization of the input signal
- System model and design consideration
- Predistortion with non-parametric amplifier's model
- Adaptive predistortion
- Results
- Project guidelines

Amplifier's Characterization

Consider baseband equivalent and let y be the complex input to the power amplifier. The output of a memoryless power amplifier can be written as

$$z = yG(|y|) = y|G(|y|)| \cdot e^{j\phi(|y|)}, \tag{1}$$

where $|y|G(|y|)|$ is the AM-AM input-output characteristic, $|G(|y|)|$ is the AM-AM gain and $\phi(|y|) = \angle G(|y|)$ is the AM-PM gain of the power amplifier. The modulus of the gain is often expressed in decibel:

$$|G(|y|)||_{dB} = 20\log_{10}(|G(|y|)|).$$

The most important figure of merit of the power amplifier is the saturation output power P_{sat}:

$$P_{sat} = \max_{0\leq|y|\leq\infty} |z(|y|)|^2 = \max_{0\leq|y|\leq\infty} |y|^2|G(|y|)|^2, \tag{2}$$

that is the maximum output power of the amplifier. By (1) we see that the peak power of the output signal can be lower than P_{sat} (this typically happens when the peak power of the actual input signal is low), but, whatever is the input signal, it can never be greater than P_{sat}.

Example: the Rapp Model

The Rapp model of the power amplifier implemented in Simulink has only AM-AM distortion, not AM-PM distortion:

$$z = \frac{K_a y}{\left(1 + \left(\frac{K_a |y|}{\sqrt{P_{sat}}}\right)^{2s}\right)^{\frac{1}{2s}}}, \tag{3}$$

where y is the input to the amplifier, K_a is the amplifier's gain at low amplitude, and s is the smoothing parameter. When $s \longrightarrow \infty$ the amplifier input-output amplitude characteristic tends to the ideal soft limiter with slope K_a till the saturation at $|z|^2 = P_{sat}$. If desired, AM-PM distortion can be introduced in the form

$$z = \frac{K_a e^{j\phi(|y|)} y}{\left(1 + \left(\frac{K_a |y|}{\sqrt{P_{sat}}}\right)^{2s}\right)^{\frac{1}{2s}}}, \tag{4}$$

where $\phi(|y|)$ is any function that one wants to use as phase distortion.

Example: the Rapp Model

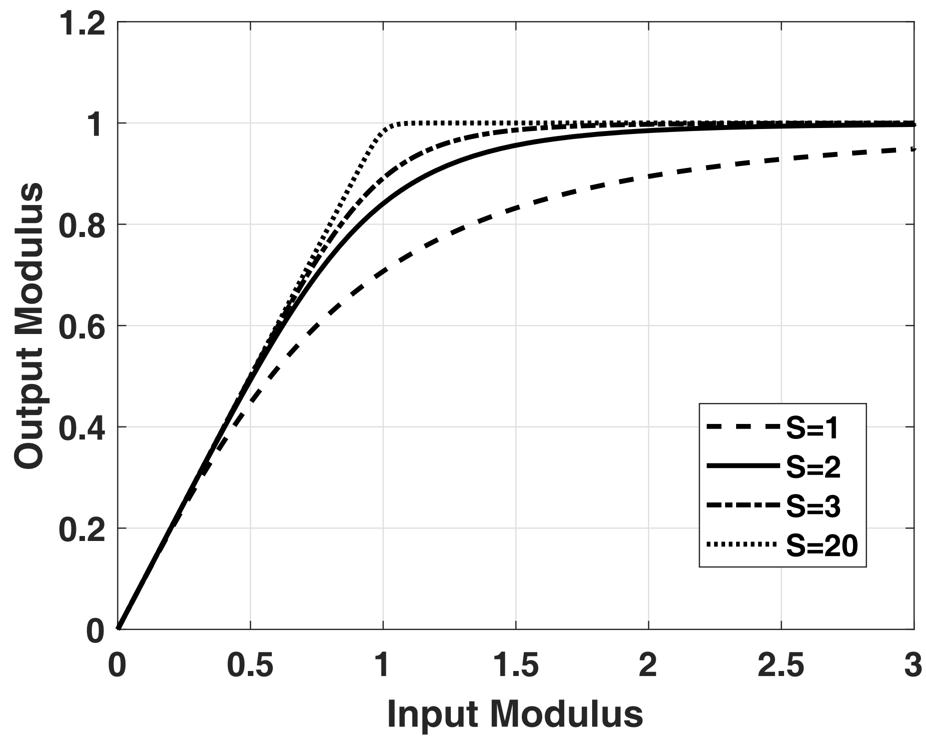

Fig. 1: AM-AM Input-output characteristic of the Rapp model for different values of the smoothing parameter. The gain K_a and the saturation power P_{sat} are set to one. For smoothing parameter $s = 20$ the model is virtually that of the soft limiter.

Example: the Rapp Model

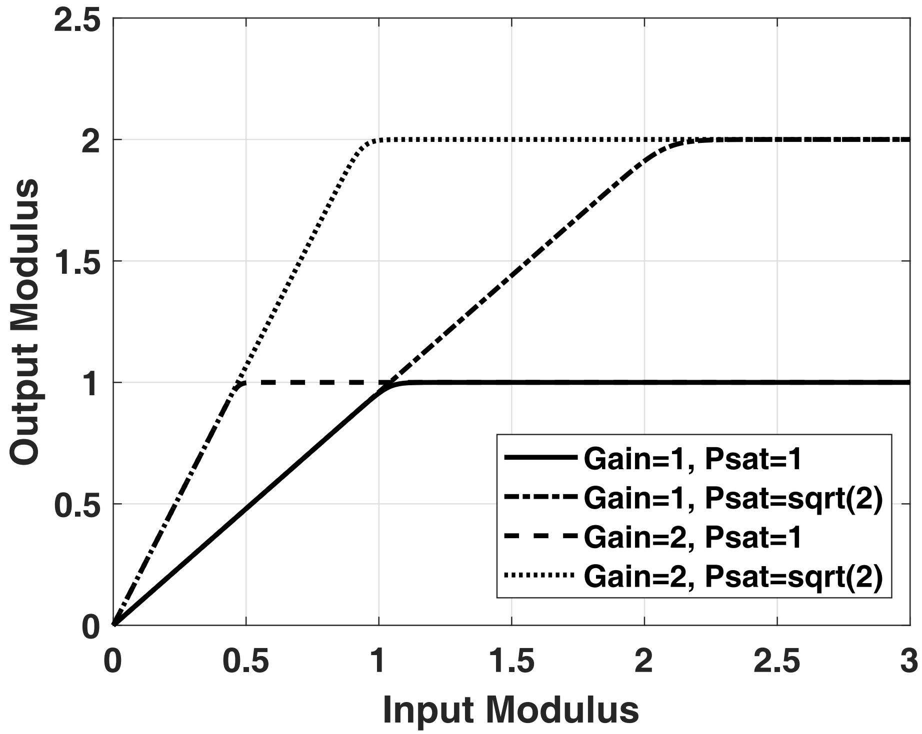

Fig. 2: Role of gain K_a and saturation power P_{sat} in the AM-AM Input-output characteristic of the Rapp model. The smoothing parameter is $s = 20$.

Output and Input Back Off

The definition of *Output Back Off* (OBO) of the power amplifier is

$$\text{OBO} = \frac{P_{sat}}{E\{|z|^2\}}. \tag{5}$$

With real-world amplifiers it is difficult to measure the saturation power, hence one often takes the saturation power equal to the output power at the so-called 1 dB compression point, which is the point where the actual AM-AM of the amplifier is 1 dB below the ideal linear-gain AM-AM. Here we will use synthetic models, where the saturation power is known, thus we will take the OBO from saturation. The *Input Back Off* (IBO) is referred to the mean power of the input signal and to the power that the input signal would have at the saturation output power if the amplifier were ideally linear till saturation (virtually S=20 in the Rapp model):

$$\text{IBO} = \frac{\alpha P_{sat}}{K_a^2 E\{|y|^2\}}, \tag{6}$$

where $\alpha = 1.26$ (1 dB) when the 1 dB compression point is considered for saturation, while in our synthetic case $\alpha = 1$.

Spectral Broadening Due to Non-Ideal Gain

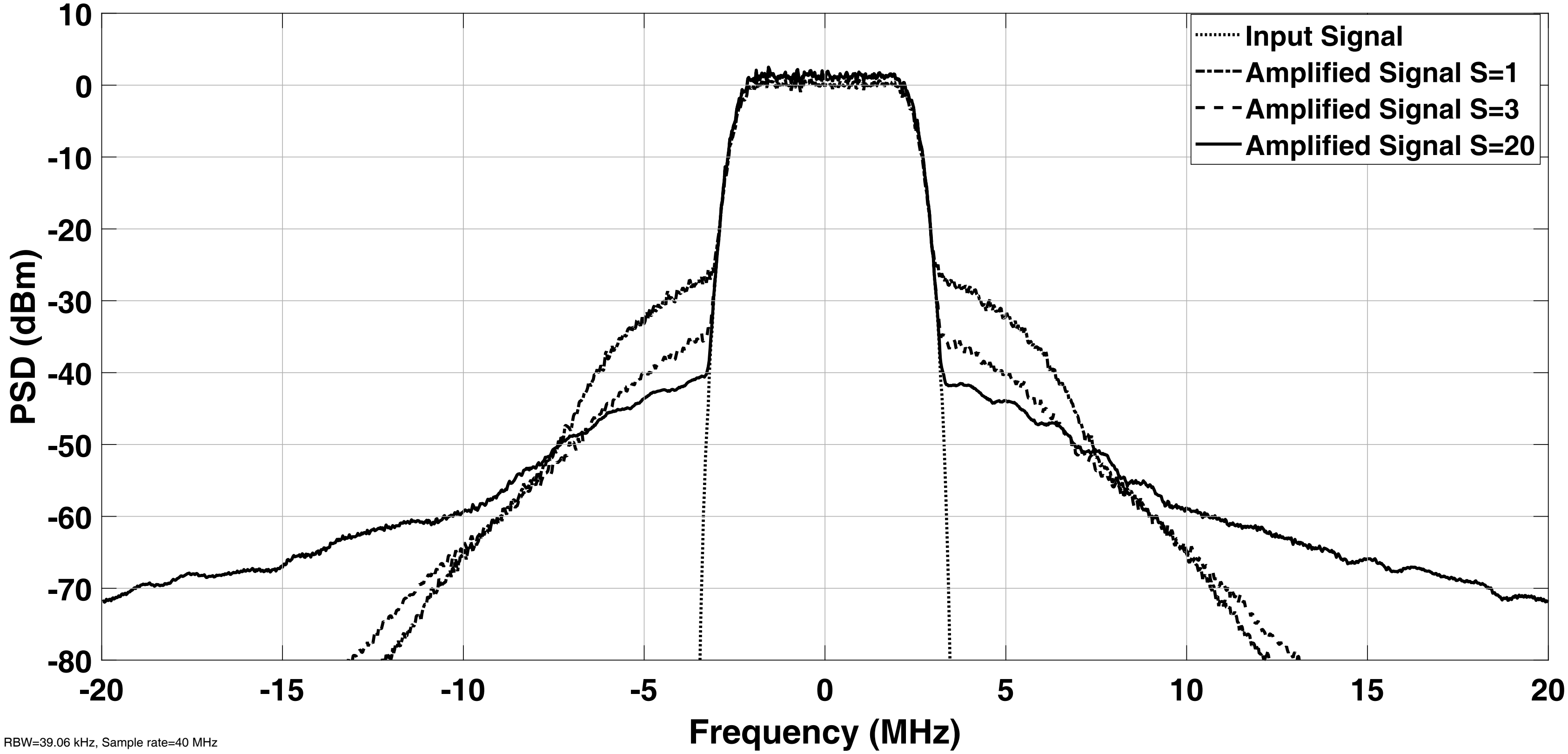

Fig. 3: Spectral broadening for smoothing parameter of the Rapp model $s = 1, 3, 20$. IBO=7.87 dB in all the three cases. OBO= 8.98 dB (s=1), 7.98 dB (s=3), 7.87 dB (s=20).

Ideal Amplification: the Role of Predistortion

We can try to compensate the non-constant gain of the actual amplifier by putting a predistorter with gain $P(|x|)$, that is a non-linear transformation

$$y = xP(|x|),$$

between the signal x that we want to amplify and the amplifier. Ideal amplification means $G(|x|) = K$, hence we want to force the condition

$$z = Kx, \quad 0 \leq |x| \leq |x|_{max}, \tag{7}$$

where $|x|^2_{max}$ is the peak power of x, therefore $P(|x|)$ must be such that

$$z = yG(|y|) = xP(|x|)G(|xP(|x|)|) = Kx. \tag{8}$$

$P(|x|)$ exists if the peak power of the output signal is lower than P_{sat}:

$$K^2|x|^2_{max} = |z|^2_{max} < P_{sat}. \tag{9}$$

Note that $|z|^2_{max}$ depends on $|x|^2_{max}$ and on K^2, while P_{sat} depends on the amplifier.

Example: Ideal Predistortion of the Rapp Model

When the condition

$$K^2|x|^2_{max} < P_{sat}$$

is met, it can be checked by substitution in (8) that the condition of ideal amplification

$$z = Kx,$$

can be imposed on the system made by the cascade of predistorter and Rapp amplifier by

$$y = \frac{Kx}{K_a \left(1 - \left(\frac{K|x|}{\sqrt{P_{sat}}}\right)^{2s}\right)^{\frac{1}{2s}}}.$$

Note that the amplifier's output never arrives at saturation if the condition $|z|^2_{max} = K^2|x|^2_{max} < P_{sat}$ is met. Also note that, as expected, predistorter's gain diverges when $K^2|x|^2 = P_{sat}$.

Limits of Predistortion - AM-AM

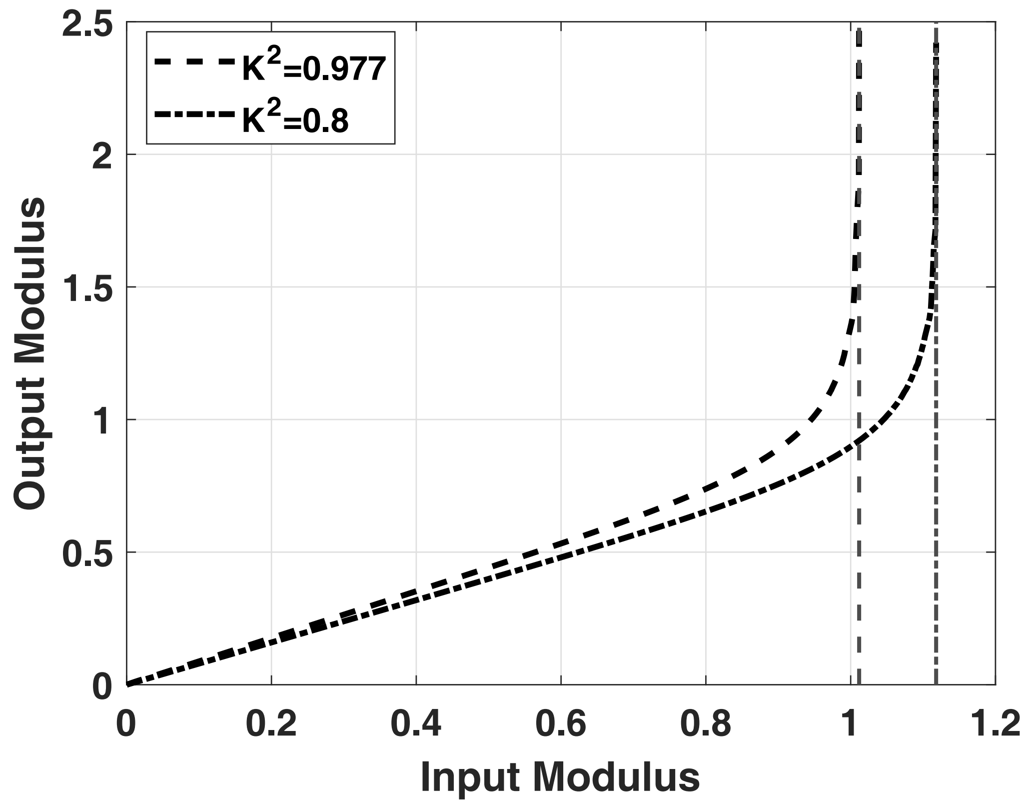

Fig. 4: Predistorter's AM-AM input-output characteristic for the Rapp model and two values of the gain K of the cascade of predistorter and amplifier. The saturation power is $P_{sat} = 1$, smoothing parameter $s = 3$. The figure shows that the system must meet the condition $K^2|x|^2_{max} \leq P_{sat}$.

Limits of Predistortion - Gain

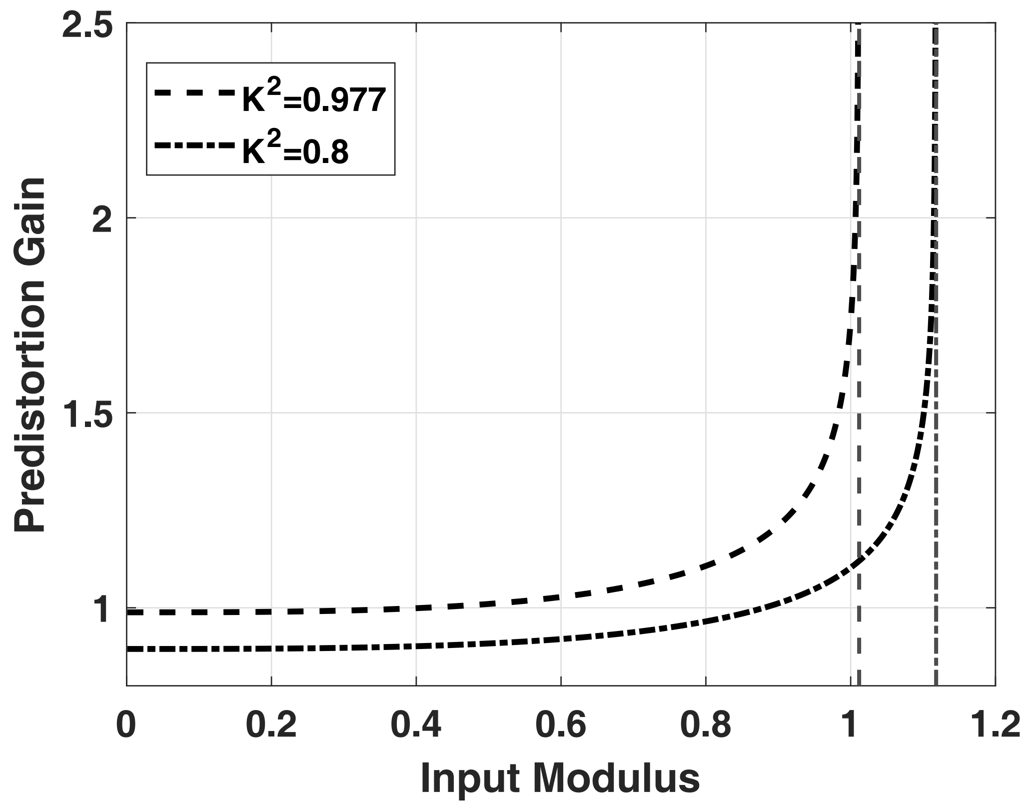

Fig. 5: Predistorter's gain for the Rapp model and two values of the gain K of the cascade of predistorter and amplifier. The saturation power is $P_{sat} = 1$, smoothing parameter $s = 3$. The figure shows that the system must meet the condition $K^2|x|^2_{max} \leq P_{sat}$.

Ideal Predistortion with AM-PM

Ideal predistortion is always possible when the AM-AM gain of the amplifier can be compensated, also if AM-PM is present. In presence of AM-PM distortion, the optimal predistortion function is

$$y = xP_A(|x|)e^{-j\phi(|y|)} = xP_A(|x|)e^{-j\phi(|xP_A(|x|)|)},$$

where $\phi(|y|)$ is the AM-PM complex gain of the amplifier and $P_A(|x|)$ is the optimal predistorter's gain when $\phi(|y|) = 0$. For instance, with (4) one has

$$y = \frac{Kxe^{-j\phi(|y|)}}{K_a\left(1-\left(\frac{K|x|}{\sqrt{P_{sat}}}\right)^{2s}\right)^{\frac{1}{2s}}} = \frac{Kxe^{-j\phi\left(\left|\frac{Kx}{K_a\left(1-\left(\frac{K|x|}{\sqrt{P_{sat}}}\right)^{2s}\right)^{\frac{1}{2s}}}\right|\right)}}{K_a\left(1-\left(\frac{K|x|}{\sqrt{P_{sat}}}\right)^{2s}\right)^{\frac{1}{2s}}}. \tag{10}$$

Ideal Predistortion with AM-PM

Note that the Rapp model does not introduce AM-PM distortion, but in real world amplifiers this distortion can be large. Our analysis simply guarantees that, even if, in doing a more realistic amplifier than the pure Rapp model, we include AM-PM to the Rapp model, then, when it is possible to compensate for the AM-AM distortion, it will be possible also to compensate for the AM-PM distortion by taking (10) for the predistortion function.

Characterization of the Input Signal

The distribution of the squared modulus of Gaussian-like complex signals as OFDM is χ^2 with two degrees of freedom. Normalizing the χ^2 random variable in such a way that the mean value of the squared modulus is $E\{|x|^2\} = 2$, the cumulative distribution results

$$F(\alpha) = 1 - P(|x|^2 > \alpha) = 1 - e^{-\alpha/2}.$$

For instance, for the probability that the squared modulus of the complex signal is 15 dB ($10^{1.5} = 31.6$ in linear scale), that is the probability that the squared modulus is 12 dB above the mean power, we have

$$P(|x|^2 > 31.6) = 1.37 \cdot 10^{-7}.$$

The probability that the squared modulus is 7.9 dB above the mean power (hence the squared modulus is 10.9 dB, corresponding to $10^{1.09} = 12.3$ in linear scale) is

$$P(|x|^2 > 12.3) = 2.13 \cdot 10^{-3}.$$

Characterization of the Input Signal

With real Gaussian input signal with zero mean and unit variance, the cumulative distribution results

$$P(|x|^2 > \alpha) = 2Q\left(\sqrt{\alpha}\right) \approx \frac{2e^{-\alpha/2}}{\sqrt{2\pi\alpha}},$$

To compute the probability of a signal with squared modulus 12 dB above the mean power we put $\alpha = 10^{1.2} = 15.8$, getting

$$P(|x|^2 > 15.8) \approx 7.44 \cdot 10^{-5}.$$

The probability of a signal with squared modulus 7.9 dB above the mean power is obtained with $\alpha = 10^{0.79} = 6.16$, leading to

$$P(|x|^2 > 6.16) \approx 1.48 \cdot 10^{-2}.$$

Hence the real signal is more spiky than the complex signal.

Cumulative Distribution of the Squared Modulus

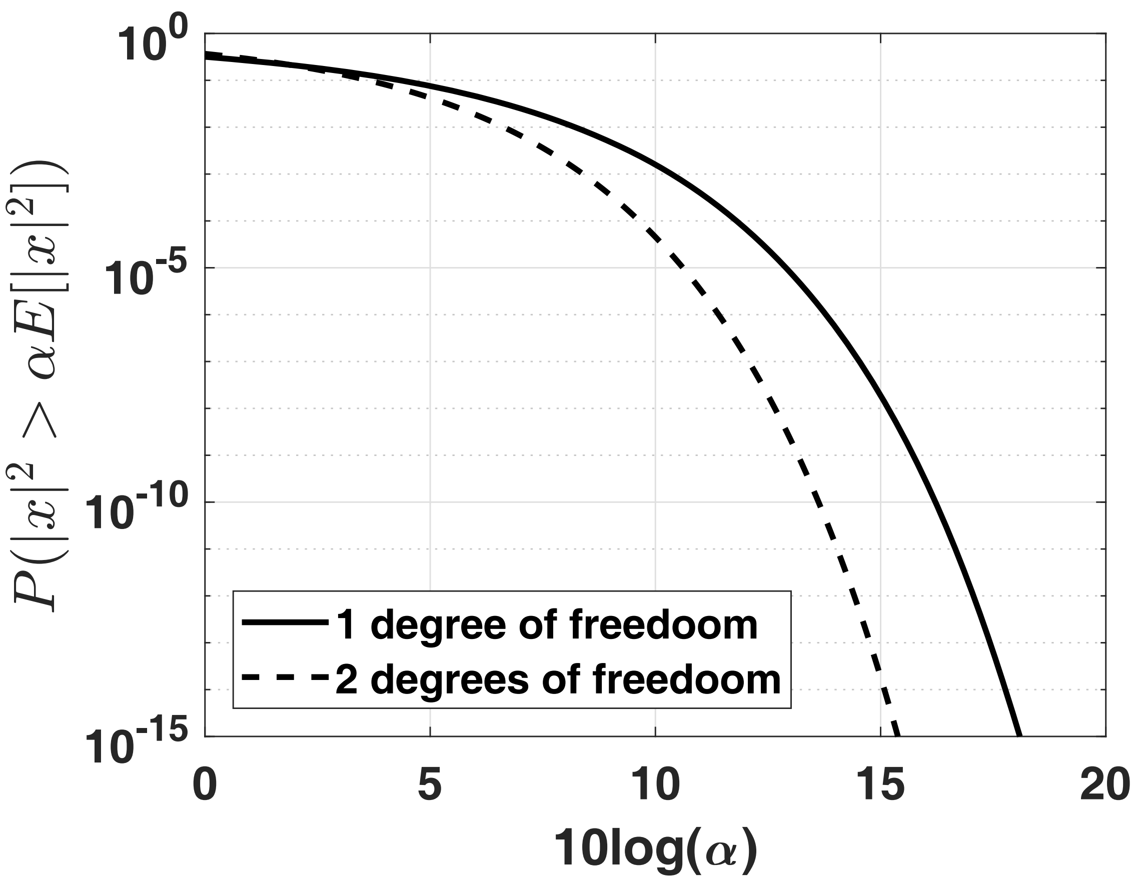

Fig. 6: Probability that the squared modulus of the Gaussian signal exceeds the abscissa.

Crest Factor Reduction

With Gaussian-like signals, the only way to meet the condition $K^2|x|^2_{max} < P_{sat}$ is to intentionally reduce the peak power of the Gaussian source signal. This is done by passing the Gaussian signal through the ideal soft limiter (see for instance the case $s = 20$ of Fig. 1) and by taking the peak-limited signal as input to the predistorter. In the technical parlance, this operation is called *crest factor reduction*, where the *Crest Factor* (CF), which is often called also *Peak to Average Power Ratio* (PAPR), is

$$\mathrm{CF}_x = \frac{|x|^2_{max}}{E\{|x|^2\}}. \quad (11)$$

Design of the Crest Factor

After that distortion has been intentionally introduced before predistortion, the condition for ideal amplification ca be met and the cascade of predistortion and amplification can ideally reproduce the intentionally distorted signal. Compared to the approach where a non-crest-limited signal is distorted by amplifier, the advantage here is that the distortion is intentional and therefore it is known and under control.

Crest reduction is subject to the following compromise. Low crest factor is desired to keep high output mean power (low OBO), hence high system gain. However, low crest factor means high non-linear distortion, hence large spectral broadening and substantial Bit Error Rate (BER) degradation at the receive side due to the non-linear intermodulation noise. On the other hand, with high crest factor there is no problem of spectral broadening and non-linear distortion, but the mean output power will be low (high OBO). In practice, the crest factor is kept the smaller possible allowed by the spectral mask by the BER floor imposed by the regulations.

Spectra of Crest-reduced Signals

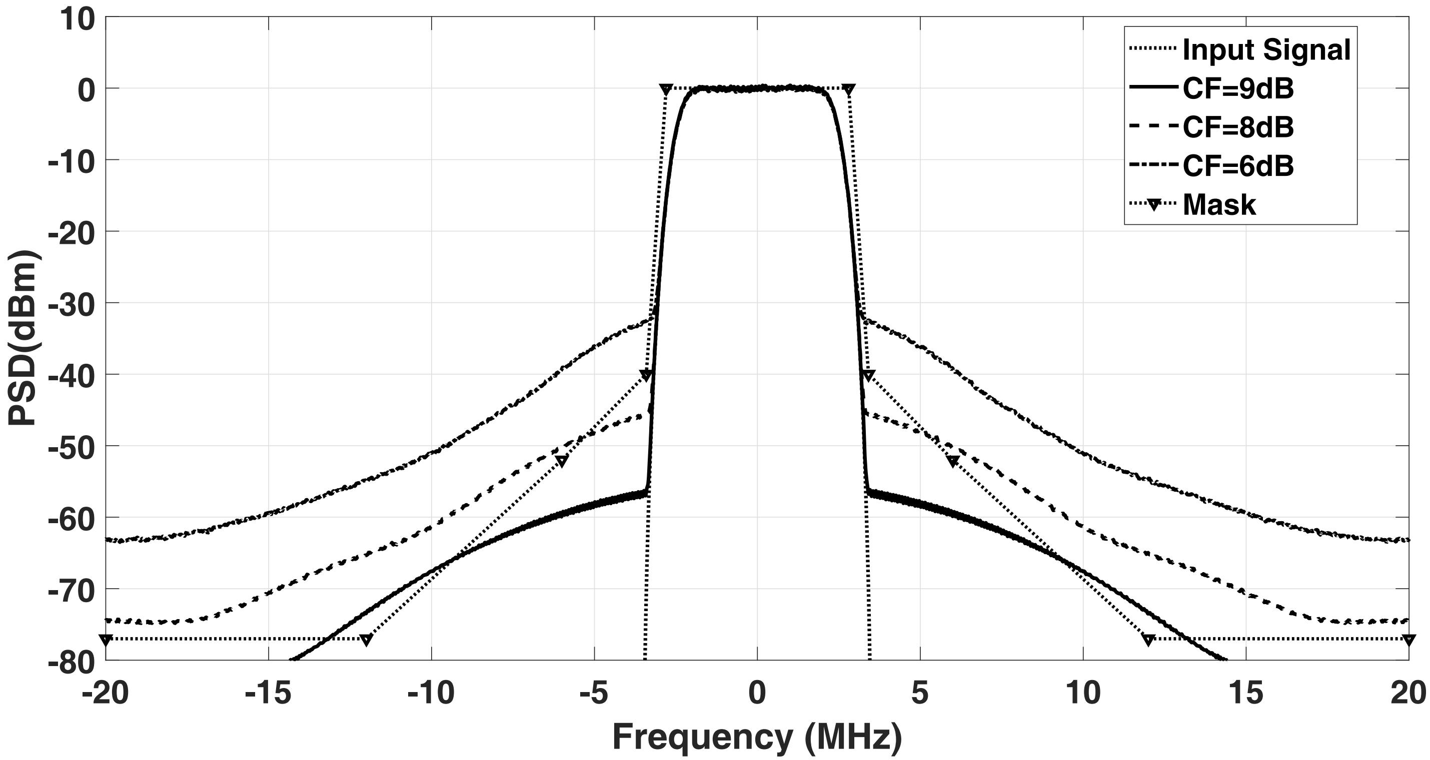

Fig. 7: Spectral broadening due to crest reduction. The spectral mask is that of DVB-T2. Far from the main spectral lobe a RF post-amplifier filter will bring the spectrum below the mask, but close to the main lobe the RF filter cannot be effective in controlling the spectrum.

Automatic Gain Control

Given that the crest factor has been designed to match system's constraint, that it is fixed and that we are happy with it, the system gain should be made as large as possible compatibly with the constraint of ideal amplification, that is

$$K^2 = \frac{P_{sat}}{|x|^2_{max}}.$$

However, P_{sat} can depend on the specific unit, that is, on the specific device, on the temperature, on aging, hence it is not perfectly known and under control.

When this happens, K can be adaptively (automatically) controlled by a feedback mechanism that keeps the maximum predistorter's gain to a desired value. If the maximum predistorter's gain is below the desired value, then K is increased, elsewhere it is decreased to prevent predistorter's divergence. Note that this is much different from the conventional automatic gain control that fixes the mean output power to a desired level. Here we cannot fix the mean output power to a desired level, because, if this level is too high, then the gain becomes too large leading to predistorter's divergence.

System Model

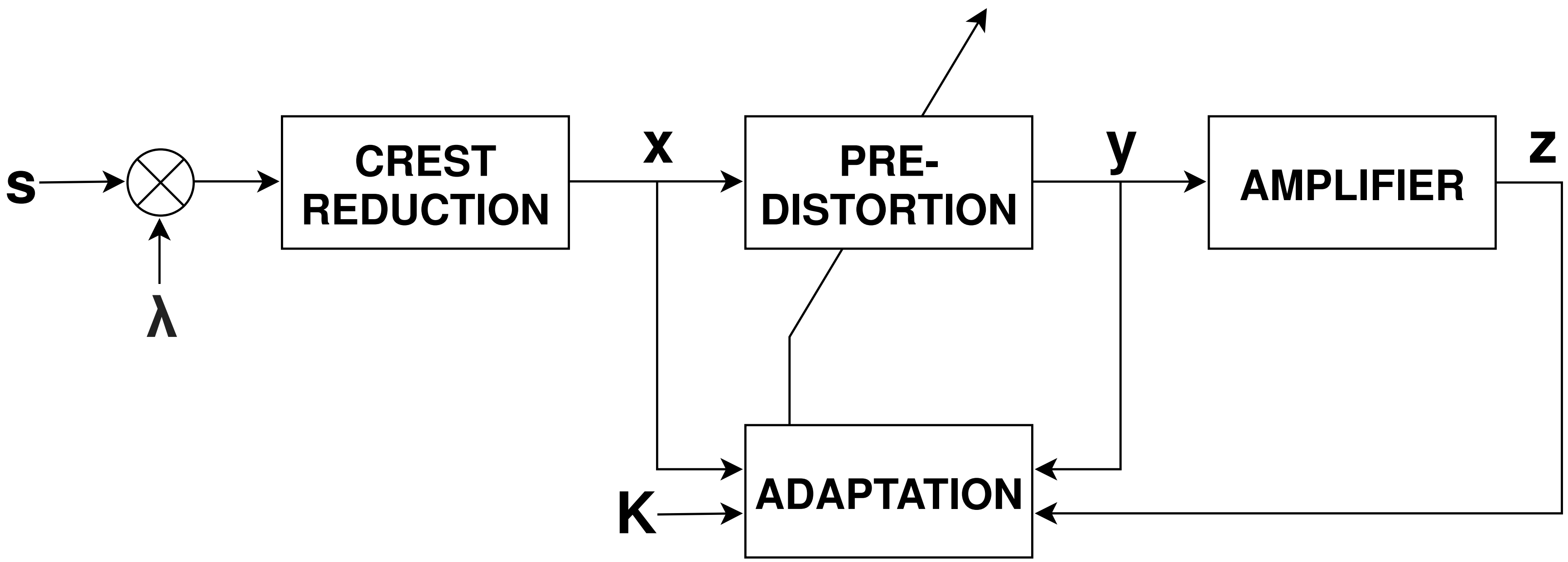

Fig. 8: Block diagram of the system.

The crest reduction block is a soft limiter. The goal of predistortion is to impose $z = Kx$, where K is the desired gain. For Gaussian-like signal s, the crest factor of x is, in systems with 5 dB $\leq$ CF$_x$ ≤ 10 dB, approximately proportional to λ^{-2}.

Predistortion with Non-Parametric Amplifier Model

In the practice one doesn't have a parametric model (as, for instance, the Rapp model) for the power amplifier and has only the results of measurements of the AM-AM and AM-PM characteristic. In this case the predistorter cannot be found in closed form the optimal predistorter must be found with some numerical method. Let the predistorter's gain $P(|x|)$ be a function of the $N+1$ parameters $\{a_0, a_1, \cdots, a_N\}$. Ideal amplification can be imposed by numerically searching for the parameters that minimize $|z - Kx|^2$:

$$\frac{\partial |z - Kx|^2}{\partial a_n} = 2(z - Kx)\left(\frac{\partial z}{\partial a_n}\right)^* = 0, \ \ \forall x : |x| \leq |x|_{max}, \ \ n = 0, 1, \cdots N. \quad (12)$$

The partial derivative can be written as

$$\frac{\partial z}{\partial a_n} = \frac{\partial z}{\partial y}\frac{\partial y}{\partial a_n} = \frac{\partial z}{\partial y} x \frac{\partial P(|x|)}{\partial a_n}. \quad (13)$$

Derivative of a Complex Function of Complex Variable

Let us consider the first factor in the right hand side of (13), which is independent of the specific implementation of the predistortion function, and write it as the familiar limit

$$\left[\frac{\partial z}{\partial y}\right]_{y=y_0} = \lim_{\Delta y \to 0} \frac{z(y_0 + \Delta y) - z(y_0)}{\Delta y}, \tag{14}$$

where Δy is complex. The basic concept of derivation with respect to a complex variable is that, the continuity of the derivative, hence its uniqueness, requires that the derivative is independent of the phase of Δy or, in other words, it is independent of the specific point in the circle of radius $|\Delta y|$ around y_0 we consider to compute the limit. Equivalently, we can say that the derivative is unique when the result of (14) is independent of direction that we take in the complex plane spanned by y to reach y_0 starting from $y_0 + \Delta y$. This extends to the complex variable the concept of continuity from the left and from the right of the derivative w.r.t. the real variable. Note that the derivative can exist even when it is not unique, exactly as it happens in the case of different values of the derivative w.r.t. the real variable, where we can have two different values of the derivative, one from the left, the other from the right.

Derivative of a Complex Function of Complex Variable

For instance, uniqueness of the derivative requires that

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial \Re\{y\}} = \frac{\partial z}{j\partial \Im\{y\}},$$

with

$$\frac{\partial z}{\partial y} = \lim_{\Delta y \to 0} \frac{z(y+\Delta y) - z(y)}{\Delta y} = \lim_{\Delta \Re\{y\} \to 0} \frac{z(y+\Delta \Re\{y\}) - z(y)}{\Delta \Re\{y\}}, \tag{15}$$

$$\frac{\partial z}{\partial y} = \lim_{\Delta y \to 0} \frac{z(y+\Delta y) - z(y)}{\Delta y} = \lim_{j\Delta \Im\{y\} \to 0} \frac{z(y+j\Delta \Im\{y\}) - z(y)}{j\Delta \Im\{y\}}. \tag{16}$$

Derivative of a Complex Function of Complex Variable

In our case, the derivative is

$$\frac{\partial z}{\partial y} = \frac{\partial y G(|y|)}{\partial y} = G(|y|) + y\frac{\partial G(|y|)}{\partial y}. \tag{17}$$

Let us check uniqueness of the derivative. To do this, we write ∂y in polar coordinates and take the derivative w.r.t. the phase, thus setting

$$\partial y = |y|\partial e^{j\angle y} = |y|je^{j\angle y}\partial\angle y = jy\partial\angle y,$$

that, for the second term in (17), leads to

$$y\frac{\partial G(|y|)}{\partial y} = y\frac{\partial G(|y|)}{jy\partial\angle y} = 0,$$

because $G(|y|)$ depends only on the modulus, it is independent of the phase.

Derivative of a Complex Function of Complex Variable

Taking the derivative w.r.t. the modulus we set $\partial y = e^{j\angle y}\partial|y|$, which, since the gain depends on the modulus, leads to

$$y\frac{\partial G(|y|)}{e^{j\angle y}\partial|y|} = |y|\frac{\partial G(|y|)}{\partial|y|} \neq 0.$$

Hence we conclude that the derivative

$$\frac{\partial z}{\partial y}$$

is not unique. However, as discussed before, the derivative can exist with different values that depend on the specific Δy that we take. In this situation, the designer can decide which among the infinitely many Δy to take and then must check if the result fits his scope.

Derivative of a Complex Function of Complex Variable

For reasons that go beyond the scope of this lecture and that are related to the AM-PM distortion, in the practice it is convenient to take the derivative with respect to the phase, hence to use in (17)

$$\partial y = jy\partial\angle y,$$

leading to

$$\frac{\partial z}{\partial y} = \frac{\partial y G(|y|)}{\partial y} = G(|y|) + y\frac{\partial G(|y|)}{\partial y} = G(|y|) + y\frac{\partial G(|y|)}{jy\partial\angle y} = G(|y|), \qquad (18)$$

where, as already discussed, the derivative of the gain w.r.t. the phase of y is zero because the gain depends only on the modulus of y. Since

$$G(|y|) = \frac{z}{y},$$

in the adaptive implementation presented in the following we will set

$$\frac{\partial z}{\partial y} = \frac{z}{y}. \qquad (19)$$

Predistortion Based on Piecewise Linear Interpolation

For instance, let the predistortion function be based on uniform quantization of the range $[0, |x|_{max})$ into $2N$ intervals $\{I_0, I_1, \cdots, I_{2N-1}\}$ with interval size $\Delta = \frac{|x|_{max}}{2N}$ (don't quantize signal $|x|$, quantize only the range where it takes its values). Moreover, let I_{2N} be the interval $|x| \geq |x|_{max}$. For

$$|x| \in I_{2n} \cup I_{2n+1},$$

predistortion is obtained by piecewise linear interpolation as

$$P(|x|) = \frac{(|x| - (2n+1)\Delta)(a_{n+1} - a_n)}{2\Delta} + \frac{a_{n+1} + a_n}{2}, \tag{20}$$

where conventionally we put $a_{N+1} = a_N$. Note that a_n is the result of the interpolation when $|x|$ falls on the border between I_{2n} and I_{2n-1}. In general, a closed form solution of (12) is not available. When this happens, one resorts to the gradient descent method.

Predistortion Based on Piecewise Linear Interpolation

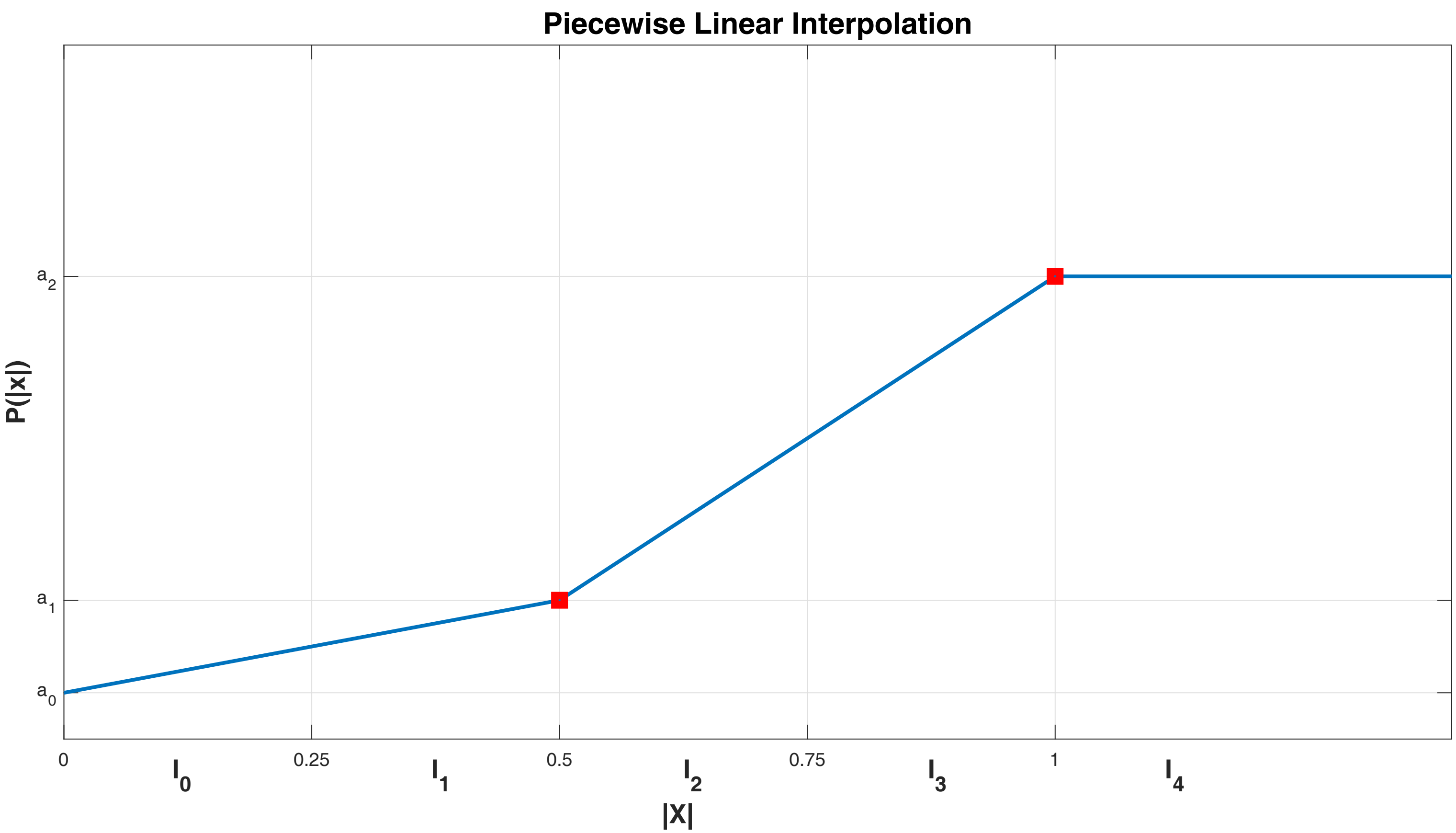

Fig. 9: Piecewise linear predistorter's gain with $N = 2$, 3 parameters $\{a_0, a_1, a_2\}$, and 4 intervals $\{I_0, I_1, I_2, I_3\}$ plus the interval I_4 that is visited by the input signal only when $|x| = |x|_{max} = 1$.

Adaptive Predistortion Based on Piecewise Linear Interpolation

The $N+1$ points $\{a_n\}$ are found by stochastic gradient search with objective

$$a_n = \arg\min E\{|z - Kx|^2\}, \quad n = 0, 1, \cdots, N. \tag{21}$$

For

$$|x(l)| \in I_{2n-1} \cup I_{2n}, \tag{22}$$

where l is the time index, the adaptive algorithm is

$$a_n(l+1) = a_n(l) - \gamma(z(l) - Kx(l))\frac{z^*(l)}{y^*(l)}x^*(l), \tag{23}$$

$$a_i(l+1) = a_i(l), \quad i \neq n. \tag{24}$$

Note that, at the l-th time instant, $x(l)$ is one number and only one of the $N+1$ points $\{a_i\}$ is updated at that time instant. The specific subscript n of the point updated at time l is found using (22): it depends on which quantization interval $x(l)$ belongs to.

Adaptive Predistortion Based on Piecewise Linear Interpolation

The two equations (23) and (24) are derived as follows. When

$$|x| \in I_{2n} \cup I_{2n+1},$$

for the derivatives of y w.r.t. a_{n+1} and a_n one has

$$\frac{\partial y}{\partial a_n} = x\frac{\Delta - (|x| - (2n+1)\Delta)}{2\Delta}, \tag{25}$$

$$\frac{\partial y}{\partial a_{n+1}} = x\frac{\Delta + (|x| - (2n+1)\Delta)}{2\Delta}. \tag{26}$$

Adaptive Predistortion Based on Piecewise Linear Interpolation

Note that

$$2\Delta \geq \Delta \pm (|x| - (2n+1)\Delta) \geq 0, \quad |x| \in I_{2n} \cup I_{2n+1}. \tag{27}$$

This observation suggests hard quantization of the term $\Delta \pm (|x| - (2n+1)\Delta)$ in the two values 2Δ and 0, leading to

$$\frac{\partial y}{\partial a_n} = x, \quad |x| \in I_{2n}, \tag{28}$$

$$\frac{\partial y}{\partial a_{n+1}} = x, \quad |x| \in I_{2n+1}, \tag{29}$$

Adaptive Predistortion Based on Piecewise Linear Interpolation

In the end, when

$$|x(l)| \in I_{2n-1} \cup I_{2n}, \quad n = 0, 1, \cdots, N, \quad I_{-1} \equiv \{\oslash\},$$

the adaptive algorithm is

$$\begin{aligned} a_n(l+1) &= a_n(l) - \gamma(z(l) - Kx(l))\frac{z^*(l)}{y^*(l)}x^*(l), \\ a_i(l+1) &= a_i(l), \quad i \neq n. \end{aligned} \tag{30}$$

Since the modulus of the factor $z^* x^* y^{-*}$ is virtually constant inside the quantization region, in the implementation it can be absorbed it into the step size and only the complex exponential of the phase of that term as a factor of the error can be used in the first equation of (30).

FPGA Implementation

We have implemented the predistorter on Xilinx Kintex xc7k160tfbg484-2. In the implementation we used piecewise linear interpolation with $K = 64$ intervals. The sample frequency of the baseband I/Q signal is 40 MHz, which allows for an over-sampling factor equal to 4 signals having maximum single-sided bandwidth of the I and Q signals of 5 MHz (10 MHz bandwidth at RF), a case that occurs, for instance, in digital video broadcasting. The resources used by our design/the total resources on the FPGA are

- LUTs: 13341/101400
- Registers: 12165/202800
- RAM blocks: 18/325
- DSP blocks: 115/600

The probing signal is approximately Gaussian with peak to average power ratio of 12 dB and 3dB single-sided bandwidth of 2.5 MHz and is represented on 18 bits.

Experimental Results

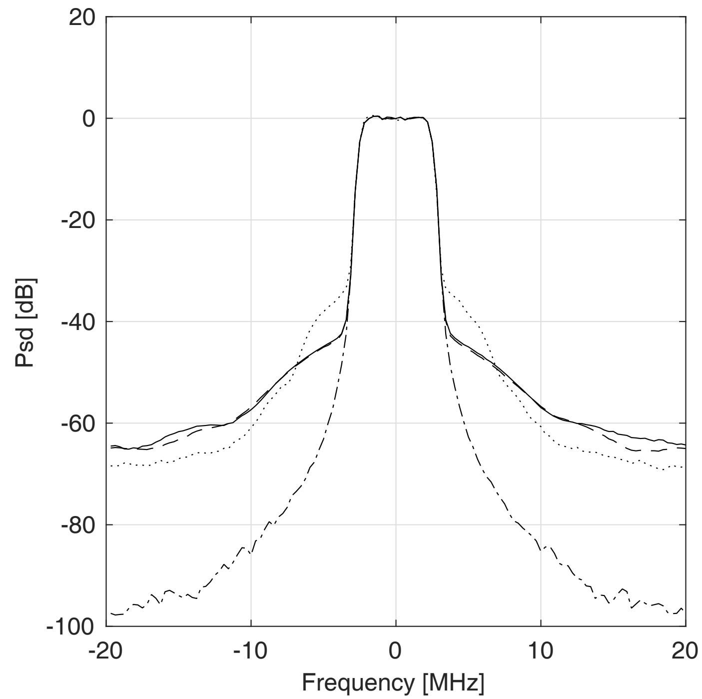

Fig. 10: Power spectral densities of various signals. Dash-dotted line: source OFDM signal. Dashed line: after crest factor reduction to 7.9 dB. Solid line: after the amplifier with crest factor reduction to 7.9 dB, predistortion and $K^2 = 0.977$ (OBO=8 dB). Dotted line: after the amplifier with no predistortion, OBO= 8 dB.

Experimental Results

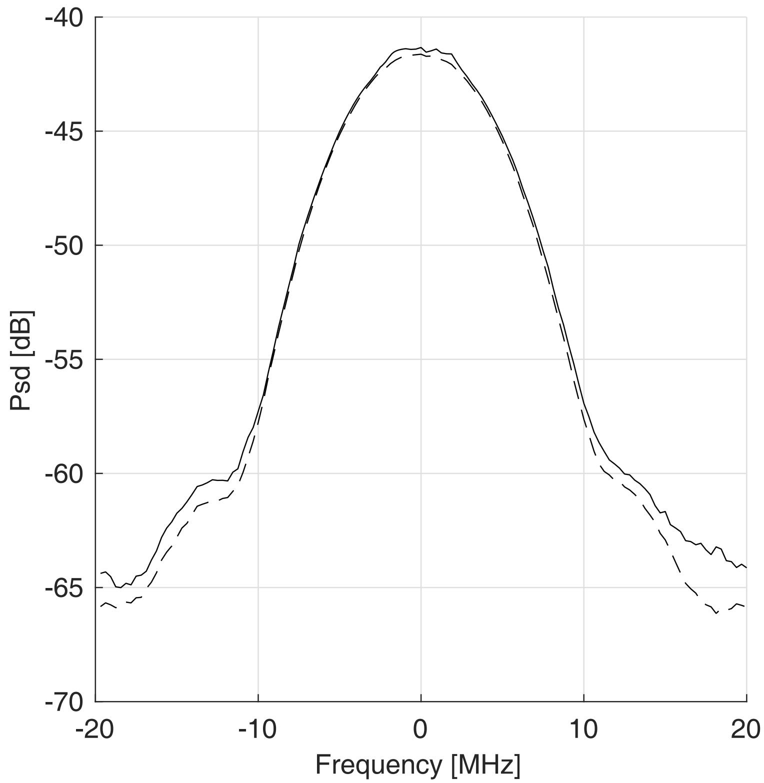

Fig. 11: Power Spectral Density (PSD) of the non-linear distortion with CF$_x$=7.9 dB and OBO= 8 dB ($K^2 = 0.977$). Dashed line: PSD of $\lambda s - x$. Solid line: PSD of $\lambda s - K^{-1} z$.

Experimental Results

A meaningful figure of merit is the Signal-to-Distortion Ratio, defined as

$$\text{SDR} = \frac{\lambda^2 E\{|s|^2\}}{E\{|\lambda s - u|^2\}}, \tag{31}$$

where $\lambda^2 E\{|s|^2\}$ is the power of the distortion free signal before the crest reduction factor. When $u = x$ the distortion $E\{|\lambda s - x|^2\}$ is the integral of the error spectrum of the dashed line of Fig. 11 and it includes only the effect of the crest factor reduction. When $u = K^{-1}z$ the distortion $E\{|\lambda s - K^{-1}z|^2\}$ is the integral of the error spectrum of the solid line of Fig. 11 and it includes the effect of the crest factor reduction and the possibly non ideal cascade of predistortion and amplifier. Also, you can drop the predistortion and feed the amplifier with the output of the crest reduction block. In this case, $u = z$. The results obtained with these three different definitions of u are reported in Fig. 12

Experimental Results

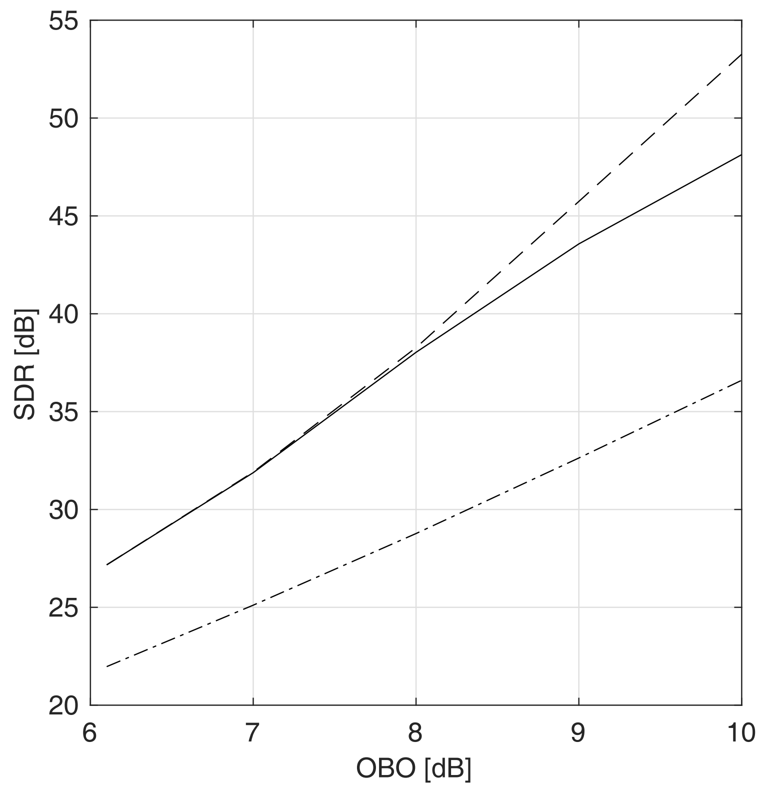

Fig. 12: SDR vs OBO. Dash-dotted line: amplifier alone with OFDM input. Solid line: cascade of crest factor reduction with CF_x= OBO -0.1 dB, predistorter, amplifier. Dashed line: ideal soft limiter.

Simulink/Matlab Project

The system block diagram is that of Fig. 8. Encode two systems, one with real Gaussian input, the other with complex Gaussian input (two Gaussian generators with different seeds or a random source complex Gaussian). The random generators are followed by a square root Nyquist filter with oversampling factor 8. Keep to 1 the saturation and the gain of the crest reduction block and the saturation of the amplifier exactly as in Fig. 1. Produce the results of all the Figures reported in these slides, duplicating them for the real and the complex case, and make a comparison between the real and the complex case. For Fig. 5 fix the CF and increase the desired gain K till you observe divergence. For each K plot $P(|x|)$ and measure the OBO (that is, the output power, the saturation power is fixed to 1). For the other figures set $0.95 < K < 1$. For instance, set $K^2 = 0.977$ to get CF$_x$ $=$ OBO $- \, 0.1$dB independently of λ. Then manually tune λ to tune CF$_x$ and, with it, OBO. While tuning λ, keep under control the spectrum of the crest-reduced signal. In general, put spectrum analyzers and power meters here and there to help the debug.

Thank You!