Homework 2: SAR Interferometry (InSAR)

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Part 1: Mono-dimensional SAR data simulation

1)The topography is modeled as a Gaussian distribution with the largest value equal to 20 m and the span of 1 km. The mean of the Gaussian is computed as

$$c = h * \tan(\theta_i) \qquad (1)$$

Considering the constraints on the spacing between the pixels of $\rho_{rg}/4$, the scenario has total 801 pixels.

2)In order to define the slant range *r,* we can estimate all the distances from the master to the topography, from the slave to the topography, and take the largest and the smallest value them all. In order to avoid some errors, the values are taken slightly larger in absolute value. The discrization is equal to the ρ_{rg}.

3+4)The acquisitions are formed as a coherent sum of all single scatterers in the scene

$$I^n(r) = \sum_{i=1}^{N_p} t_i \sin c\left[\frac{r - R^n(p_i)}{\rho_{rg}}\right] e^{-j\frac{4\pi}{\lambda}R^n(p_i)} \qquad (2)$$

Since images are acquired in the slant-range and not in the ground range, we would expect them to not be aligned as in the figure 1, since slant range is a radar-target difference, and the satellites have a non-zero baseline.

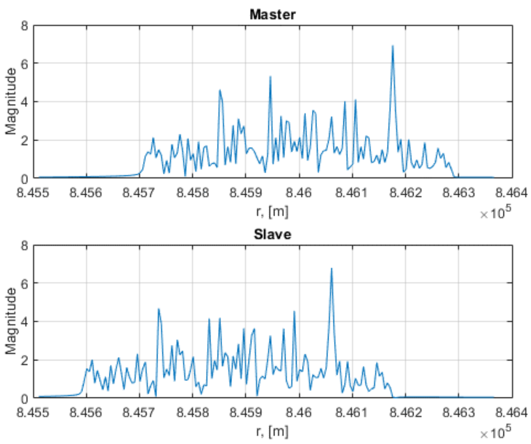

Figure 1 SAR images (the master and the slave)

Part 2: Image coregistration

In order to coregister the images, coregistration by geocoding was used. The idea behind the coregistration is to use the known reference elevation model, which is a flat surface for this specific case. Then you project the images on the ground range coordinates through the interpolation. The final result (aligned images) can be seen in the figure 2.

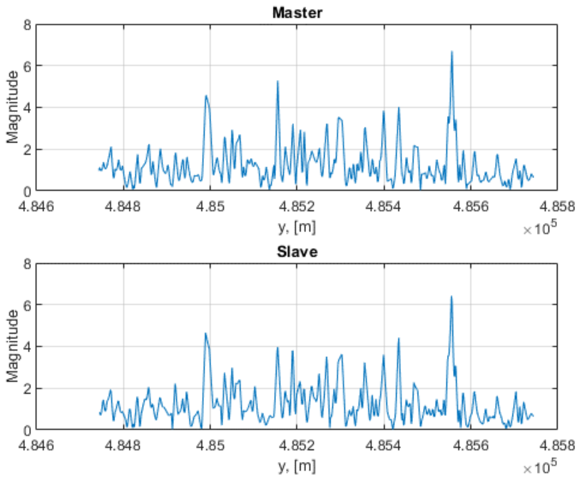

Figure 2 Coregistered by geocoding images

Part 3: The interferometric phase

1)In order to create an interferogram, with the reference to (2) we can write the signals as

$$X_M(p) = A_M(p)e^{j\phi_M(p)};\ X_S(p) = A_S(p)e^{j\phi_S(p)} \tag{3, 4}$$

for each point *P*. In order to get an interferogram, we need to multiply $X_M(P)$ by the complex conjugate of $X_S(P)$. The phase in this case will be

$$\psi(p) = \frac{4\pi}{\lambda}(R_M - R_S) = \frac{4\pi}{\lambda}B_{\|} = \frac{4\pi}{\lambda}B\sin(\theta_p)\,. \tag{5}$$

This term is present even in case of no elevation and will have to be taken into account later. In case of the topography (fig. 4) we can apply Tailor expansion to estimate the interferogram at point P′

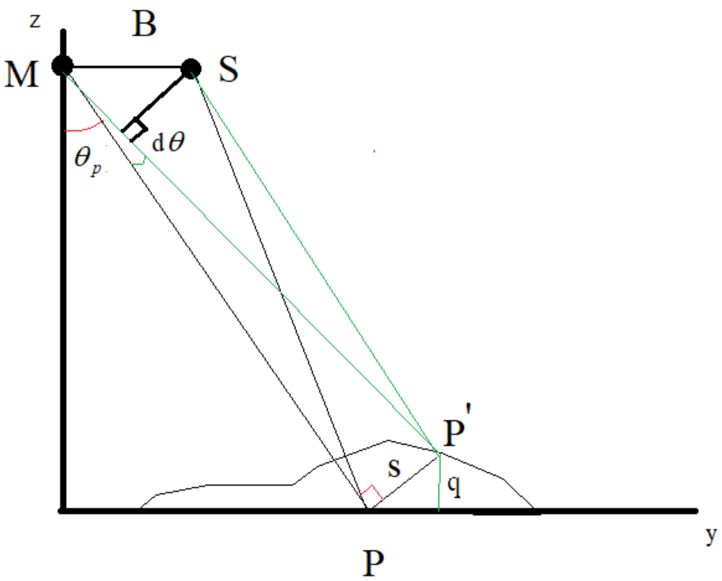

Figure 4 Scenario

$$\psi(p^{'}) = \psi(p + d\theta) = \psi(p) + \frac{\partial \psi(p)}{\partial \theta}\Delta\theta = \left\{ \Delta\theta = \frac{q}{R_M(p)\sin(\theta_p)} \right\} = \frac{4\pi}{\lambda}B\sin(\theta_p) + \frac{4\pi}{\lambda}B\cos(\theta_p) \qquad (6)$$

The phase after doing the multiplication between the M and the S can be seen in the fig 5

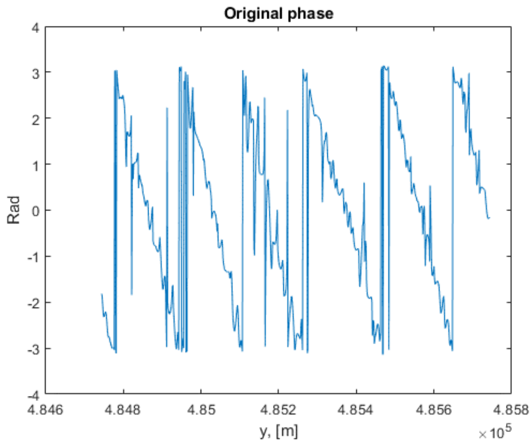

Figure 5 Original interferogram

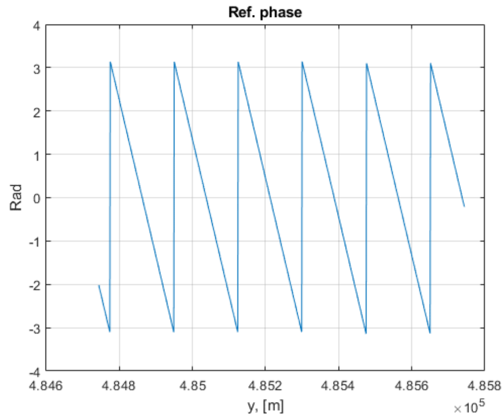

Figure 6 Flat term

We can see from fig 5 the presence of the fast varying term (first element in the sum in (6)), which is represented in fig 6. In order to flatten the interferogram, we can get this term from the reference model and multiply the interferogram with its complex conjugate. The result of this step and the smoothing with an average filter with a window 80 can be seen in fig 7 and 9.

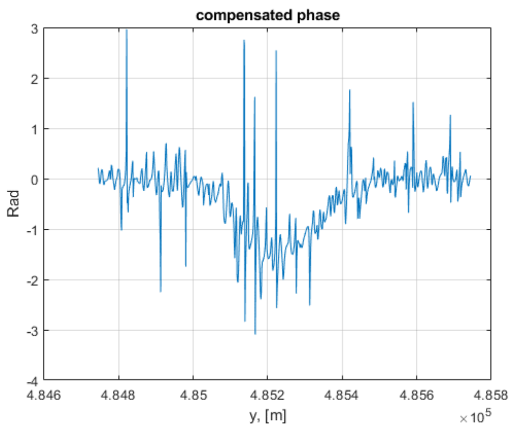

Figure 7 Flattened phase interferogram

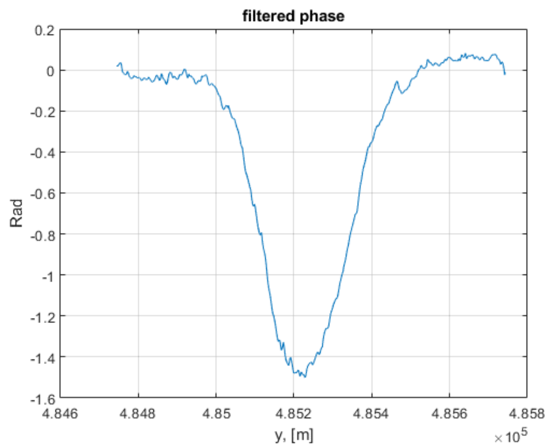

Figure 8 Filtered interferogram

Now since due to the phase ambiguity of 2pi, we need to do the unwrapping. Luckily, for this case it is not needed, but in general case it can be done with the following code

```
phase_unwr = phase_comp_filt;

    for i=2:length(phase_comp_filt)% make a decision
        difference = phase_comp_filt(i)-phase_comp_filt(i-1);
        if difference > pi
            phase_unwr(i:end) = phase_unwr(i:end) - 2*pi;
        elseif difference < -pi
            phase_unwr(i:end) = phase_unwr(i:end) + 2*pi;
        end
    end
```

, whcich simply makes a decision of adding/substractiong a 2pi depending on the jump size.

Finally, since we know that now the phase contains only the elevation, we can reconstruct the topography (fig 9)

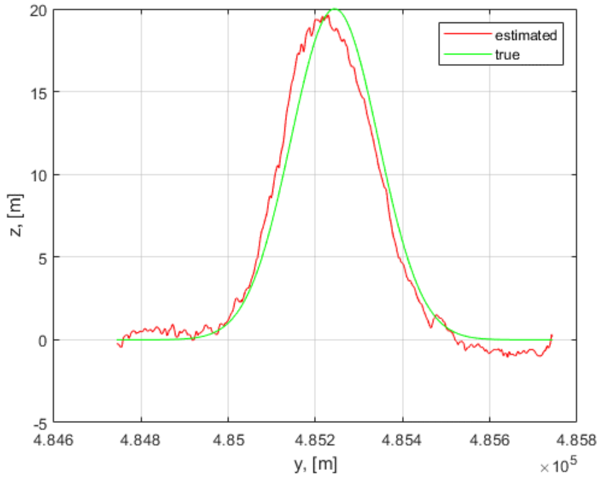

Figure 9 True vs estimated topography

We can see that the simulation is done properly since the truth coincides with the simulation results. For further smoothing, further filtering can be applied if needed.

Part 4 (optional): Decorrelation, subsidence and multi-baseline interferometry

1)Decorrelation

Previously we assumed complete correlation between the complex reflectivities for M and S acquisitions. In this scenario we assume partial and complete decorrelation between the images.

To do so, a correlation matrix 2x2 was generated, which was used with th Cholevski decomposition was used to correlate the acquisitions. Since the further steps are exactly the same as discussed in the previous parts, we can skip them and see the results

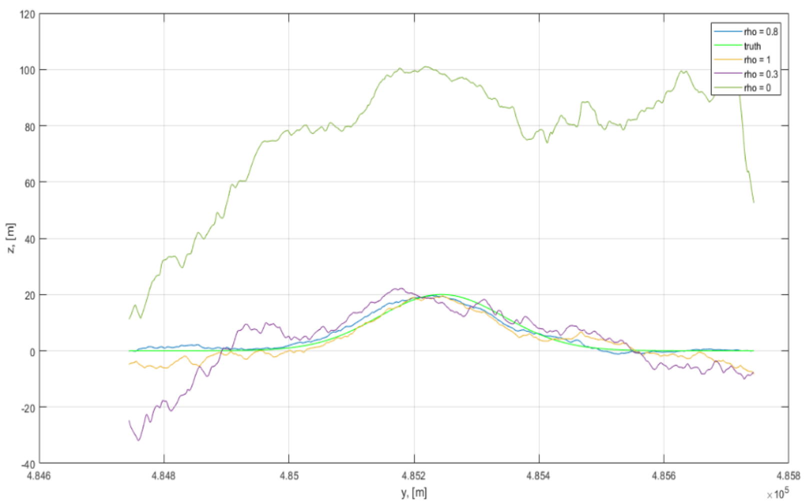

Figure 9 Decorrelated results

As we can see from the results, the larger is the correlation, the less accurate our estimation is. In case of full correlation is before we get nearly perfect estimation, whilst when the correlation is zero, the result is nearly random.

2) Subsidence

In this case we assume to have no topography, but the terrain sinks between the 2 acquisitions. In this scenario the terrain sinks by 3 cm between two acquisitions (fig 10).

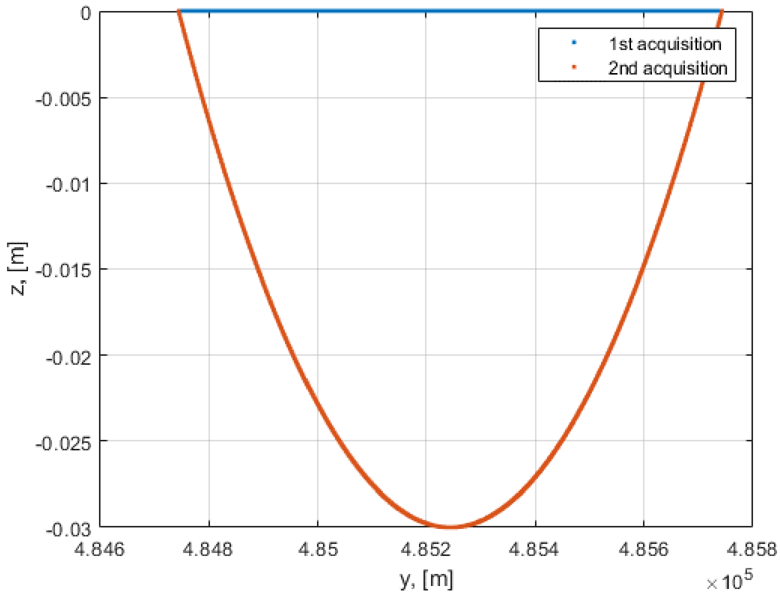

Figure 10 Subsidence

Although we have no topography, the contribution in the phase is due to the deformation, in other words

$$\psi(p') = \frac{4\pi}{\lambda}(B\sin(\theta_p) + \Delta R_{LOS}) \tag{7}$$

From where we can estimate how much the terrain has sunk down as

$$h \approx \frac{\Delta r_{LOS}}{\cos(\theta_p)} \tag{8}$$

The result after all the steps (including the unwrapping) are the following (fig. 11)

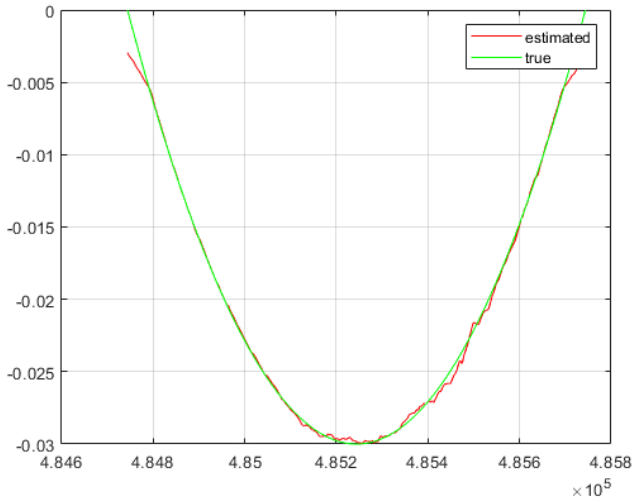

Figure 11 Estimated subsidence

Although the subsidence is very small, the estimated range difference in the LOS projected on the height coincides with the ground truth.

3)Multi-baseline interferometry

With unknown topography and subsidence, the task is to estimate the topography and subsidence rate using several SAR images.

N = 20 - #images

dz = 5mm – subsidence rate

Topography is the same as in the previous parts (Gaussian).

We can represent the phase as

$$\phi(t) = \psi_P - \frac{4\pi}{\lambda} \cdot v_P \cdot t - \frac{2\pi}{q_a} \cdot q_P \qquad (9)$$

By parametrizing and discretizing the phase, we can write in matrix notation

$$\boldsymbol{\phi} = \mathbf{A\Theta} \qquad (10)$$

, where

$$\mathbf{A\Theta} = \begin{bmatrix} 1 & k_v B_t(1) & k_q B_n(1) \\ 1 & k_v B_t(2) & k_q B_n(2) \\ \dots & \dots & \dots \\ 1 & k_v B_t(N) & k_q B_n(N) \end{bmatrix}; \Theta = \begin{bmatrix} \psi_P \\ v_P \\ q_P \end{bmatrix}$$

$$k_v = -\frac{4\pi}{\lambda};\ k_q = \frac{4\pi}{\lambda}\frac{1}{R\sin\theta}$$

for every pixel.

Thus, we can estimate the parameters using the Least Square approach

$$\hat{\mathbf{\Theta}} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\boldsymbol{\phi} = \mathbf{A}^\dagger\boldsymbol{\phi} \qquad (11)$$

To simplify the model, only the 2^{nd} and 3^{rd} columns of matrix A were used, therefore what we estimate is subsidence rate and the subsidence.

However, a problem when there is a need to invert the matrix occurs.

If we take a look at the condition number for inversion of matrix A, we will see an extremely large value

cond(A) = 2.4727e+19. Since the condition number of A is much larger than 1, the matrix is sensitive to the inverse calculation. That means since we use random values for the simulation will provide us inconsistent results due to high sensitivity.

Vp=0.0063 m/acquisition

q_p =   2.1374e-06 m.

We can see that resulting value of the velocity estimated in the same order of magnitude as the true value, while the subsidence has a significant difference in values. One of the possible workarounds is to adjust the units of the parameters inside the matrix A as to make the values of the same magnitude, which will result in better results after the inversion and lower value of conditional number of A.