Homework 1: Multipath in automotive radar imaging

Prof. Andrea Virgilio Monti Guarnieri

Dr. Marco Manzoni

Student: Usevalad Milasheuski

ID: 10816982

Politechnico di Milano 2022

Part 1: The monostatic array

1)The signal model of the demodulated and range can be written as

$$S_{DD}^{n}(r) = e^{j\pi k t_n^2} T_p e^{-j\frac{4\pi}{\lambda} r_n} \mathrm{sinc}\left(\frac{r - r_n}{\rho_{rg}}\right) \tag{1}$$

Ignoring the first two constants in this equation, we can write

$$S_{DD}^{n}(r) = \mathrm{sinc}\left(\frac{r - r_n}{\rho_{rg}}\right) e^{-j\frac{4\pi}{\lambda} r_n} \tag{2}$$

By reading the signal along the channels (antennas), we can observe that the signal is a rectangle, since we sample the x-axis with an antenna array, and a sinc along the range.

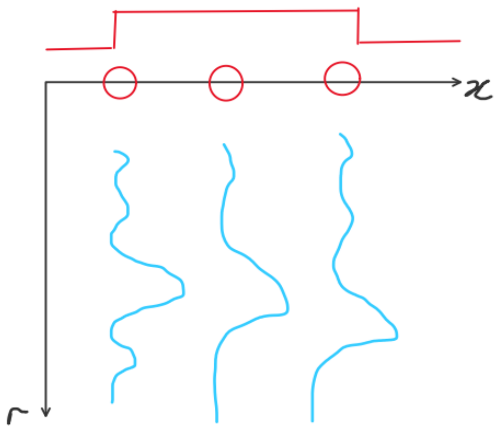


Figure 1 Signal representation along x and r axis

The range of can be extracted as the *r,* which corresponds to a peak of the signal along the channel. However, in order to extract DOA, we need to take a closer look at the phase of our signal.

First, since we are in a far-field scenario (wavefront is assumed to be a plane, not spherical), we can expand the distance in Taylor series around zero

$$r_n = r_0 + \left.\frac{\partial r_n}{\partial x_n}\right|_{x_n = x_0} = r_0 + \frac{2(x_0 - x_t)}{2\sqrt{(x_0 - x_t)^2 + y_t^2}}(ndx) = r_0 - \sin\theta_T ndx \tag{3}$$

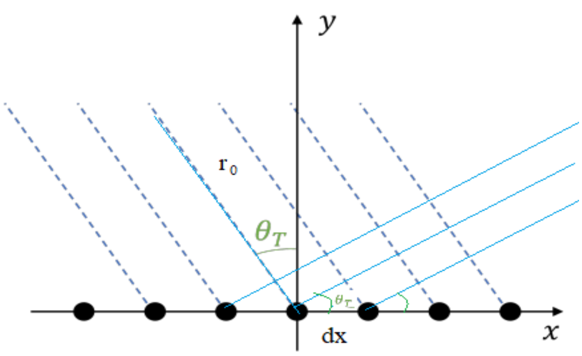


Figure 2 Far field approximation

By putting in inside the phase of equation (2), we get

$$e^{-j\frac{4\pi}{\lambda}r_n} = e^{-j\frac{4\pi}{\lambda}r_0} e^{j2\pi\frac{2}{\lambda}\sin\theta_T ndx} \tag{4}$$

The second multiplier is none other than a sampled complex sinusoid. By taking a Fourier transform, we can extract its frequency and, therefore, extract the angle.

2) Since we have mentioned the formulation of the problem as the peak extraction on the frequency domain, we can now write the Nyquist condition for the problem

$$f_{spat} = \frac{1}{d} = 2\max\{\frac{2}{\lambda}\sin\theta_T\} -> d \le \frac{\lambda}{4} = 9.7403\mathrm{e-04\ m} \text{ -antialiasing condition} \tag{5}$$

As said before, the signal is rectangular along the channels (x axis), therefore we can write

$$e^{-j\frac{4\pi}{\lambda}r_n} = ...e^{j2\pi\frac{2}{\lambda}\sin\theta_T x} rect(x/L) \quad \text{-FFT->} \quad \sin c(L(f - f_{spat}))$$

The first zero of the acquired sinc is in the position +-1/L, which means, we can resolve two sinusoids if the peak of one of them corresponds to zero of the other, therefore the frequency resolution is $\rho_f = 1/L$

To get the angular resolutions (considering equation (5)), we can write

$$\frac{\Delta f}{\Delta\theta} = \frac{\partial f}{\partial\theta}; \qquad \Delta f = \frac{2}{\lambda}\cos\theta_T\Delta\theta \quad \text{or} \quad \rho_f = \frac{2}{\lambda}\cos\theta_T * \rho_\theta \text{, therefore}$$

$$\rho_\theta = \frac{\lambda}{2L\cos\theta_T} \tag{6}$$

Considering the requirements, L =0.031 m, Ntx = 33.

3. The target is located in the position (-1, 16), r_0=18.8680 m

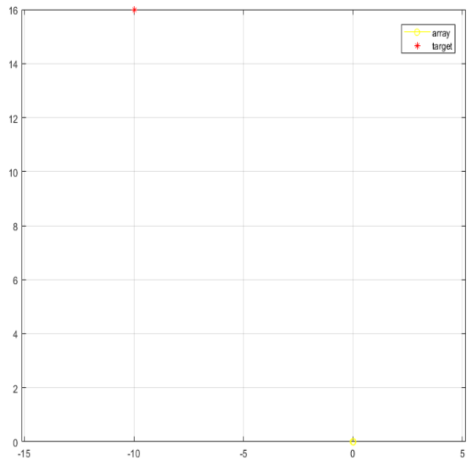


Figure 3 Scenario

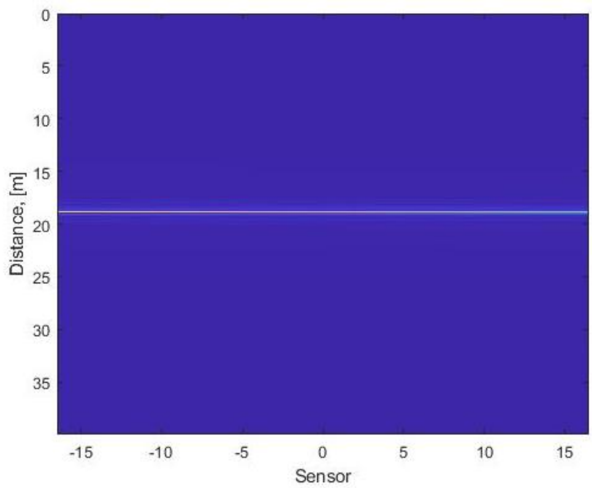


Figure 4 Range-compress matrix

After taking the FFT of the signal, we have (fig 5):

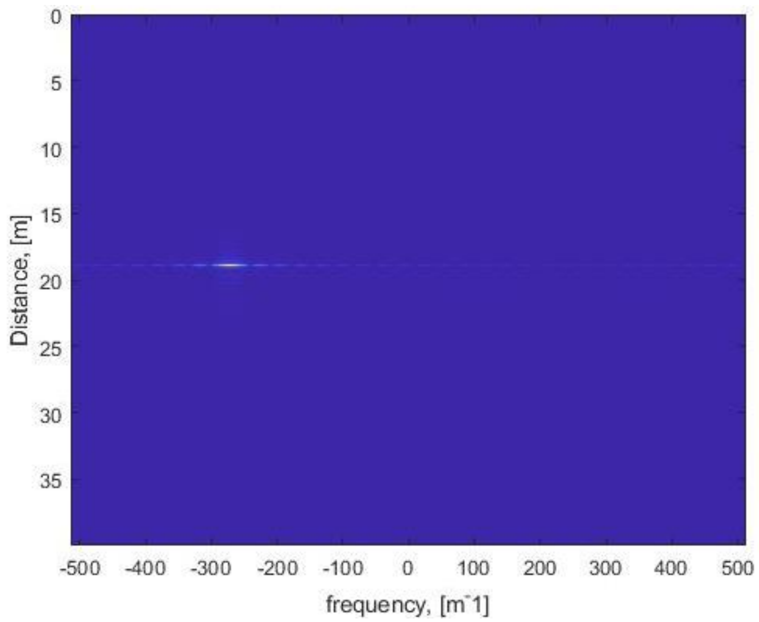


Figure 5 FFT(1024) of the range compressed matrix along the channels

The signal we observe now is a bidirectional sinc both along ranges and along frequencies

mean(R_n_est) = 18.8680 m

theta_est = -31.9580 deg

theta_true = -32.0054 deg

4) The coordinates of the image wrt. the target can be written as (-[X_t-2D], Y_t). Following similar derivation steps as before, the signal is

$$S_{DD}^{n}(r) = \mathrm{sinc}\left(\frac{r - 0.5[r_n(p_T) + r_n(p_I)]}{\rho_{rg}}\right) e^{-j\frac{2\pi}{\lambda} r[r_n(p_T)+r_n(p_I)]} \tag{7}$$

Applying formula (3) lo linearize both distances, for the phase we get (ignoring constant terms):

$$e^{j\frac{2\pi}{\lambda}[\sin\theta_T + \sin\theta_I]ndx} = e^{j2\pi\frac{2}{\lambda}\sin\theta_\beta ndx}\ ;\ \sin\theta_\beta = \frac{\sin\theta_T + \sin\theta_I}{2} \tag{8}$$

Therefore, by following the same procedure as before, we should find neither the angle wrt. the target, nor the angle wrt. the image, but rather an angle somewhere in between (ghost target).

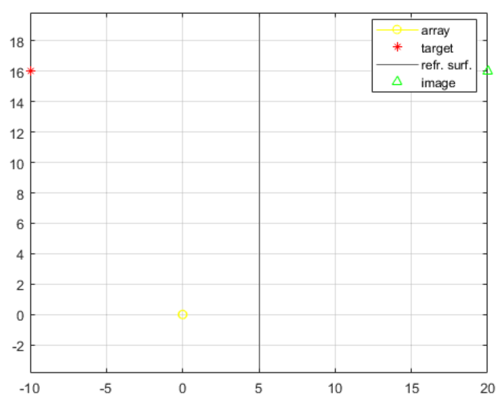


Figure 6 Multipath scenario

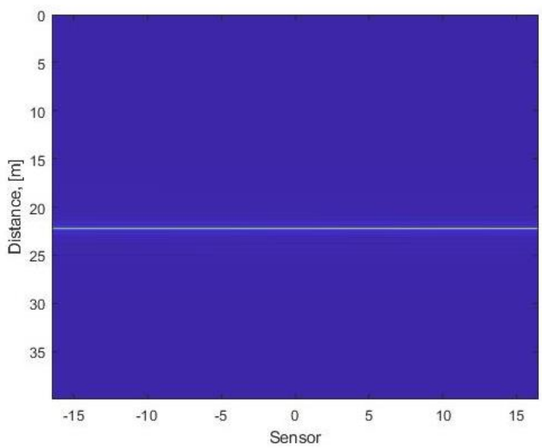
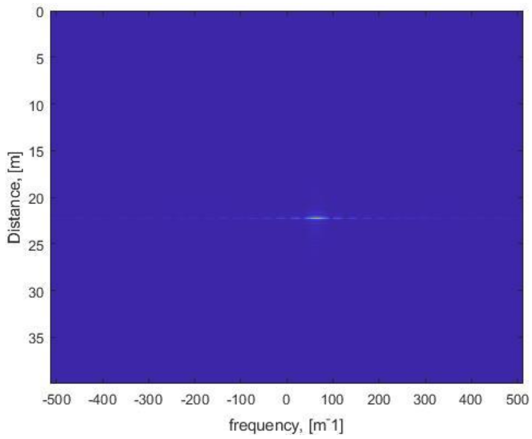


Figure 7 Range-compress matrix for multipath



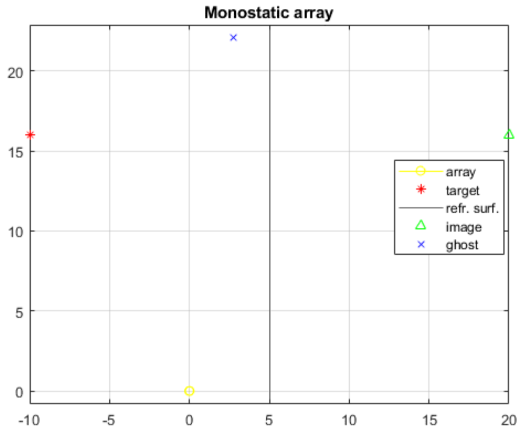


Figure 8 FFT(1024) of the range compressed matrix along the channels for the multipath

Figure 9 Full scenario and focused target

mean(R_n_est_m) = 22.2402 m

theta_est = 7.1808 deg

theta_true_ghost = 7.2059 deg

theta_true_image = 51.3402

The predicted angle is not an angle of the image, but it is what equation (8) claims to be. The distance is also different wrt. the target, since we take an average distance between the target and the image.

We can summarize everything done above with the following figures

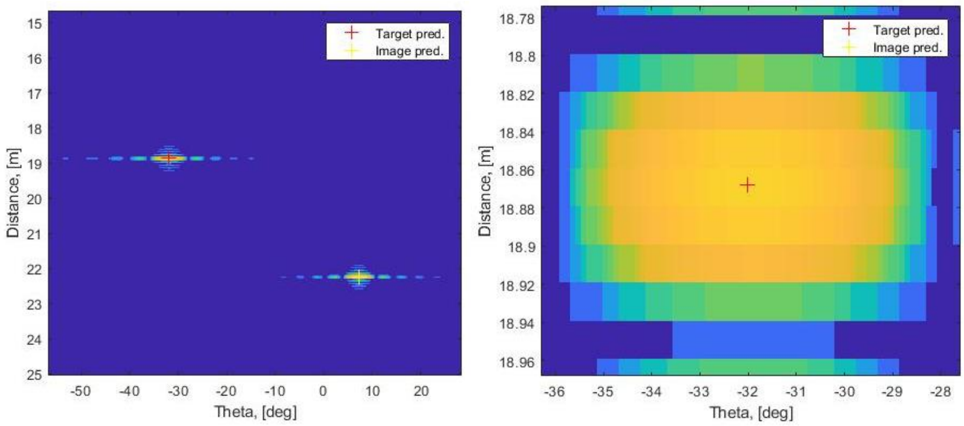


Figure Range-angle representation of the simulation (monostatic)

The main problem was in transition between figures 5 and 8 is since the DoA and the frequency are connected not linearly (formula 4 has sin), to imagesc function uniformly spaces the bins, which is not what we need. The 'uimage' function does what is needed, however it takes up to 5-15 min to get the output.

From these plots we can see the true target focused in the beginning (red cross, upper left corner) as well as the focused target in case of multipath (lower right corner) In addition, we can see by the main lobe of the bidimentional sinc that the resolutions, both, angular and range, are met. In addition, the crosses in the plot correspond to the theoretically predicted values where the target is to be estimated, so we see that theory meets the simulation.

Part 2: The bistatic array

1)Ignoring the constant terms, the signal for the nth antenna is

$$S_{DD}^{n}(r) = \mathrm{sinc}\left(\frac{r - 0.5[r_0 + r_n]}{\rho_{rg}}\right) e^{-j\frac{2\pi}{\lambda} r_n} \qquad (9)$$

This modification of (2) is due to the path that a signal takes: it has a constant term, which is the propagation from the central antenna, and the term, which depends on the receiving antenna.

2) Linearizing using (3), the phase is

$$e^{-j\frac{2\pi}{\lambda} r_n} = ...e^{j2\pi\frac{1}{\lambda}\sin\theta_T n dx} \qquad (10)$$

$$f_{spat} = \frac{1}{d} = 2\frac{1}{\lambda}\sin\theta_T -> d \le \frac{\lambda}{2} = 0.0019 \text{ m} \text{ -antialiasing condition} \qquad (11)$$

With angular resolution $\rho_\theta = \dfrac{\lambda}{L\cos\theta_T}$.

Considering the requirements, L =0.062 m, Ntx = 33.

Therefore, we need the same number of antennas as before, but the spacing can be two times larger.

If we keep the same length of the array as before, then we will be able to work on 2 times larger frequency.

3) Scenario is the same as in figure (3), except for the part that the middle antenna is the only TX.

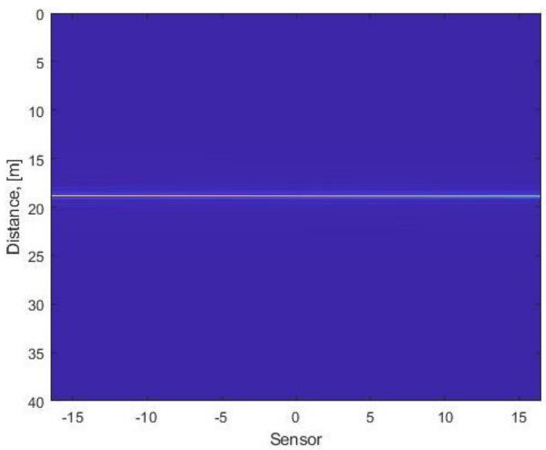


Figure 10 Range-compressed matrix (bist.)

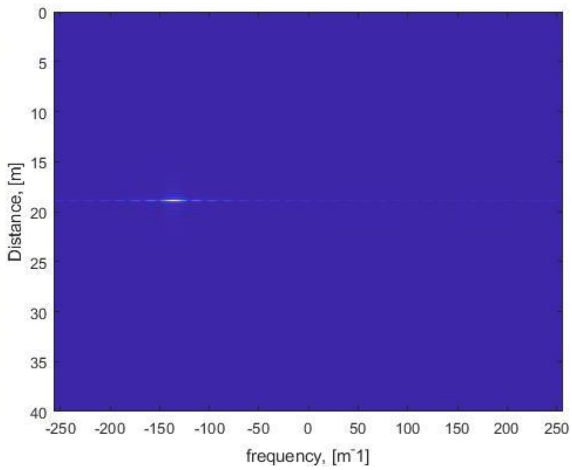


Figure 11 FFT(1024) of the range compressed matrix along the channels (bist.)

The vertical axis of figures 10 and 11 are not exactly distances. Since, as we have said, the spacing is larger, the corresponding values are bins and should be multiplied by a factor of 2.

```
mean(R_n_est) = 18.8679 m
theta_est =  -31.9580 deg

theta_true = -32.0054 deg
```

4) The coordinates of the image wrt. the target, as before, are (-[X_t-2D], Y_t). The signal for multipath is (ignoring constant terms)

$$S_{DD}^{n}(r) = \mathrm{sinc}\left(\frac{r - 0.5[r_0(p_T) + r_n(p_I)]}{\rho_{rg}}\right) e^{-j\frac{2\pi}{\lambda} r_n(p_I)} \qquad (12)$$

Applying formula (3) to linearize both distances, for the phase we get (ignoring constant terms):

$$e^{j\frac{2\pi}{\lambda}\sin\theta_I ndx} \qquad (8)$$

which, as before, corresponds to a complex sinusoid. By extracting its frequency what we will get is the angle wtr. the image target. Scenario geometrically is the same as in figure 6.

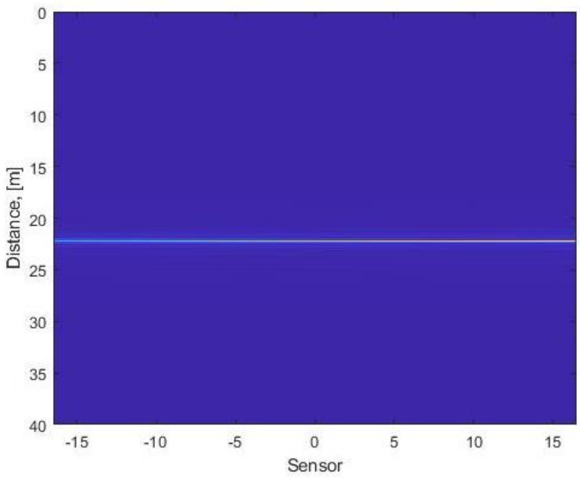


Figure12 Range-compressed matrix for multipath(bist.)

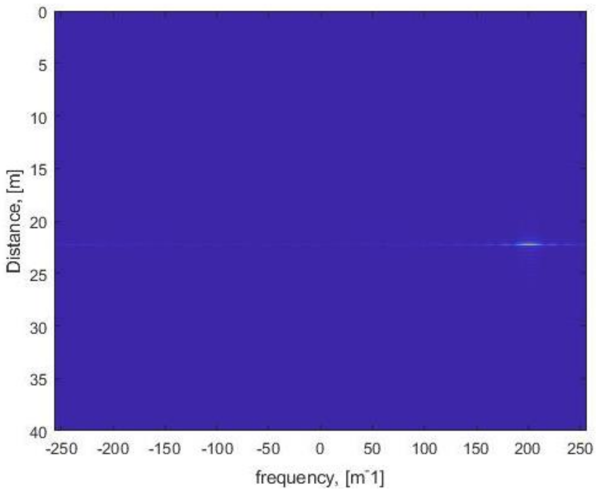


Figure13 FFT(1024) of the range comp. (bist.)

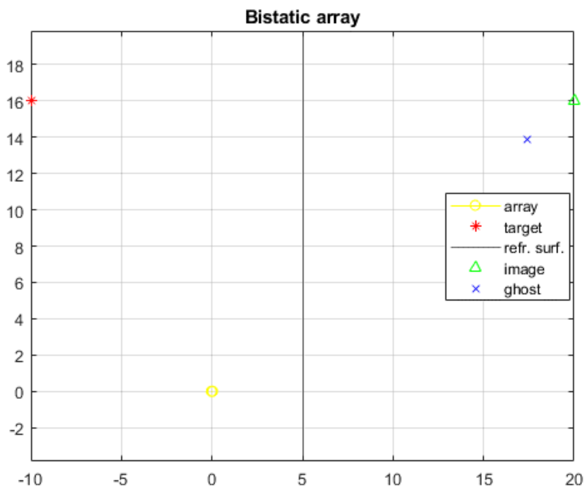


Figure 14 Full scenario and focused target

mean(R_n_m_est)= 22.2403 m

theta_est = 51.3752 deg

theta_true_ghost = 51.3402 deg

theta_true_image = 51.3402 deg

The multipath is reconstructed in a completely different position wrt. monostatic array. It is hard to tell which of the two cases is more dangerous, since neither of the two are perfectly safe. Since we are way better at estimating the image target, in case we know the position of the wall, we can "flip" the estimated ghost target along y axis and have a better estimation of the position of the true target (from the estimated range and angle and the coordinates position (-[X_t-2D], Y_t)). In the monostatic case there is nothing we can do to extract one of the two required angles.

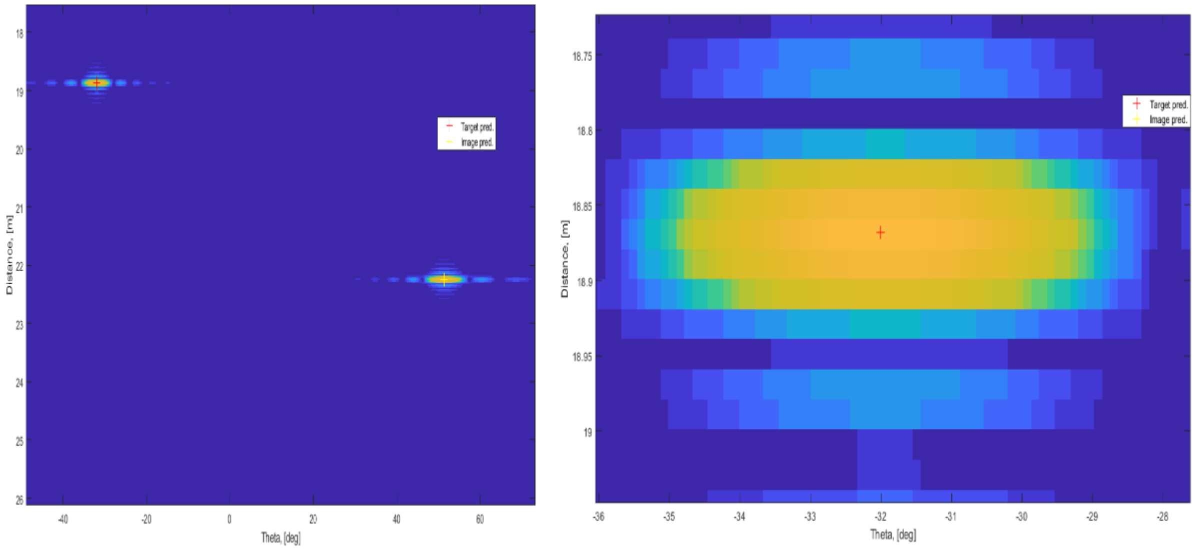


Figure Range-angle representation of the simulation (bistatic)

Similar to before, we clearly see the effect of multipath on the target's location. Considering new requirements on the array, we meet the given requirements on the angular resol, since the distance from the peak to the first zero along x axis is 3.6 deg. Having used the formulas written before, the theory meets the simulation results.

Part 3: The MIMO array

0)Proof of virtual phase channel's position

Let us look at the problem from the geometrical point of view. In a bistatic case, the curve of equal distances is an ellipse, where Tx and Rx are located in the focuses. In the monostatic case this figure is a circle with an antenna in its center. In order to see if we can equivalently replace a bistatic case with a monostatic, we need to make sure the that the total distance to the target is the same. Therefore, referring to the fig. 15 we write

$$r_1 + r_2 = 2r_3$$

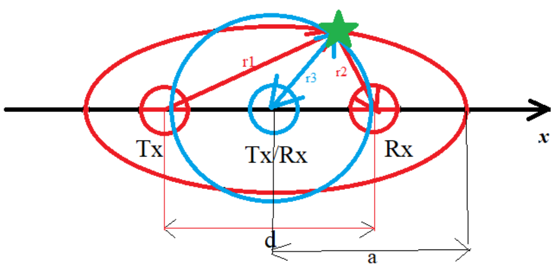


Figure 15 Bistatic and an equivalent monostatic case

From an ellipse's properties, we know that

$$r_1 + r_2 = 2a\,,$$

therefore, $a = r_3$. The prop. of an ellipse is that is a=b, than it converges to a circle, or equivalently, as d->0, which corresponds to the central point where Tx-Rx will overlap.

1)Ignoring the constant terms, the signal when mth antenna transmits and nth antenna receives is

$$S_{DD}^{n,m}(r) = \mathrm{sinc}(\frac{r - 0.5[r_m + r_n]}{\rho_{rg}})e^{-j\frac{2\pi}{\lambda}(r_m + r_n)} \quad (9)$$

If linearization is applied to both ranges, we can see that the phase is the product of two sinusoids sampled with different frequencies. However, adopting the virtual phase channel principle we can treat the system as if it is an MxN monostatic case.

2) To get the spacings for Tx and for Rx antennas, we can try the 'brute force' method by looking for the positions such that the spacing for virtual monostatic channels is preserved as in part 1.

a) By incrementing number of Rx antennas, and making sure spacing and position is correct, we can claim that $dx_{Rx} = \frac{\lambda}{2}$

b) As we increment the number of Tx antennas, we need to make sure the spacing between the last channel of one antenna and the first of the other is still $\frac{\lambda}{4}$. The spacing of Tx depends on the number of Rx and equals to $dx_{Rx} = N_{Rx}\frac{\lambda}{2}$.

Therefore, for case 4x8(TX x RX), we have

dx_{RX} =0.0019 m, dx_{TX} = 0.0156 m.

3) Scenario is the same as before, but the array has the following structure

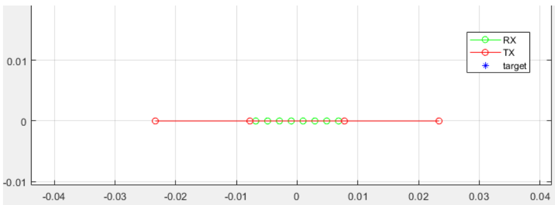
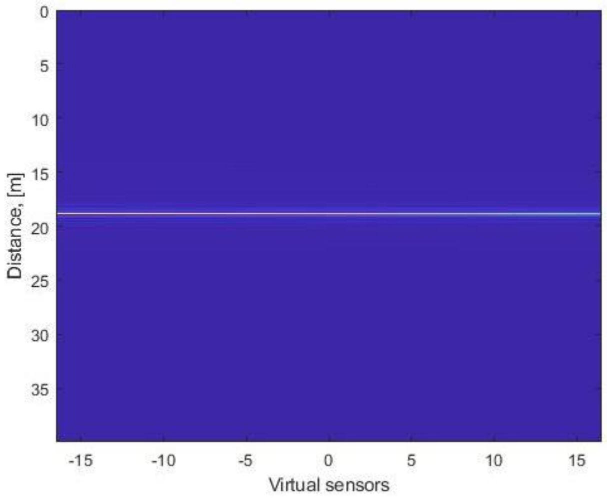


Figure 16 MIMO 4x8 Array

The range compression matrix has dimensions 20534 x 8 x 4. Since we know that each Tx antenna generates 8 virtual phase channels consequentially, we can convert the matrix as if it is acquired from a monostatic case.



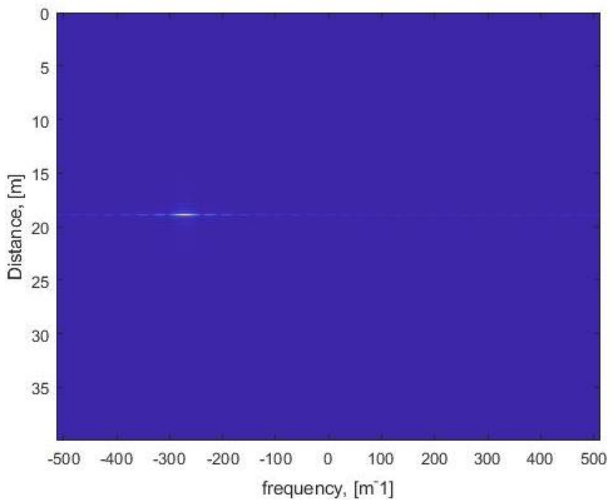


Figure 17 Range-compressed matrix (virt.)

Figure 18 FFT(1024) of the range compressed matrix along the channels (virt.)

mean(R_n_m_est) = 18.8680 m

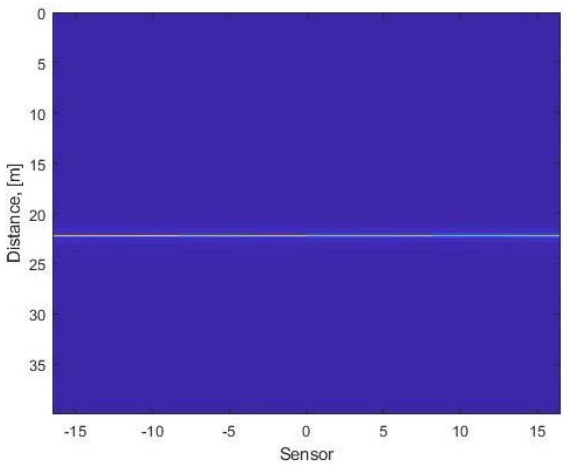
theta_est = -31.9580 deg

theta_true = -32.0054 deg

4The coordinates of the image wrt. the target, as before, are (-[X_t-2D], Y_t. Scenario and target's position is the same as before. The signal for multipath is (ignoring constant terms)

$$S_{DD}^{n,m}(r) = \mathrm{sinc}\left(\frac{r - 0.5[r_m(p_T) + r_n(p_I)]}{\rho_{rg}}\right) e^{-j\frac{2\pi}{\lambda}(r_m(p_T)+r_n(p_I))} \tag{12}$$

By leveraging the virtual phase centers (as in the previous step), we can estimate the frequency of the signal (as in part 1)



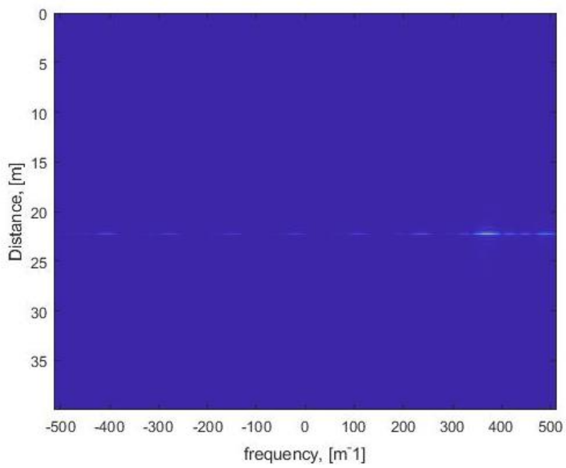


Figure17 Range-compressed matrix for multipath(virt.)

Figure18 FFT(1024) of the range comp. (virt.)

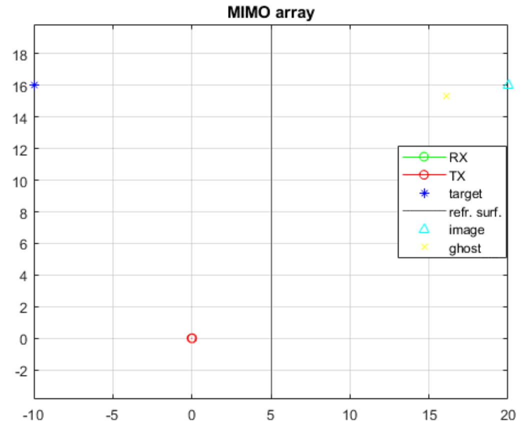


Figure 19 Full scenario and focused target

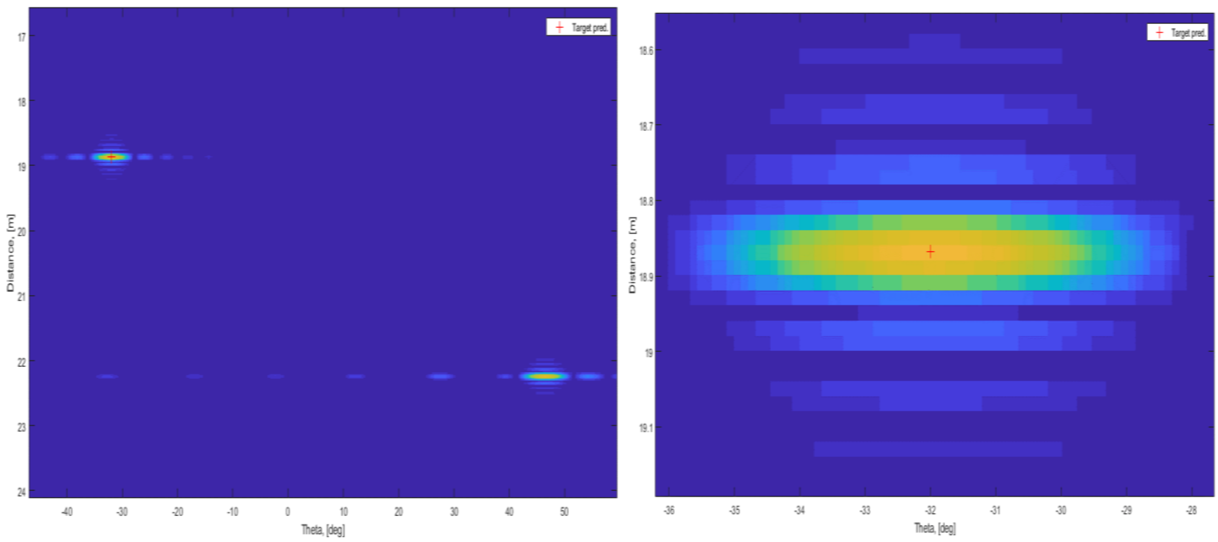


Figure Range-angle representation of the simulation (MIMO)

In this example, it is very complicated to provide a theoretical estimation of the target's position in a closed form solution. Although we have used virtual phase channels (monostatic), the result is similar to a bistatic case. In addition, by the first zeros positions we can see that we also satisfy the range and angular resolution constraints.

Conclusion

Now we can compare the acquired results from application point of view:

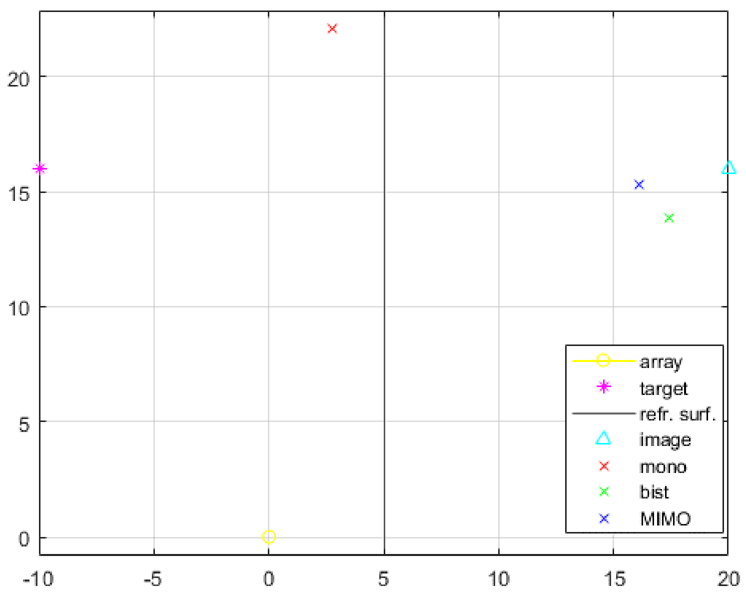
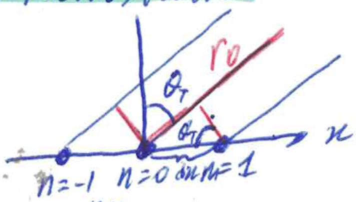


Figure 20 Focusing for different arrays

From the results we can see that MIMO estimates an image target better wrt. monostatic array, although the virtual phase centers were used for DOA extraction, but a little worse than bistatic one. The main advantage of MIMO is the ability to have smaller number of antennas required wrt. the other constellations, which should make it cheaper wrt. the monostatic one. In addition, although the performance of the MIMO is almost the same as bistatic array, the length of the MIMO is smaller than the bistatic case, which might be useful for some applications (0.062 m(bis); 0.0468 m (mimo).

I) Monostatic



$$\boxed{r_n = r_0 - \sin\theta_T n dx}$$

$t = \frac{2 r_n}{c}$

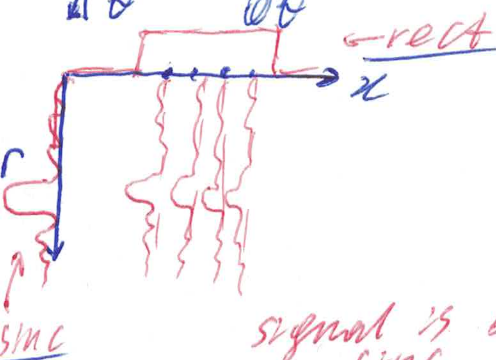
$$S_{Rx}^{nn}(r) = \mathrm{sinc}\left(\frac{r - r_n(P_T)}{\rho_r}\right) e^{-j\frac{4\pi}{\lambda} r_n(P_T)}$$

$$e^{-j\frac{4\pi}{\lambda} r_n(P_T)} = e^{-j\frac{4\pi}{\lambda}(r_0 - \sin\theta_T d x)} = \mathrm{const}\cdot \underbrace{e^{j2\pi\cdot\left(\frac{2}{\lambda}\sin\theta_T\right) dx}}_{\text{compl. sin}}$$ frequency

$$f_{sampl.} = \frac{1}{d} = 2\cdot\max\{\tfrac{2}{\lambda}\overset{1}{\sin\theta_T}\} = \frac{4}{\lambda} \Rightarrow \boxed{d = \lambda/4}$$ - spacing

Resolution:

$$\frac{\Delta f}{\Delta\theta} = \frac{\partial f}{\partial\theta};\ \Delta f = \rho_f = \frac{\partial f}{\partial\theta} = \frac{\partial}{\partial\theta}\left(\frac{2}{\lambda}\sin\theta_T\right) = \frac{2}{\lambda}\cos(\theta_T)\underbrace{\Delta\theta}_{\rho_\theta}$$



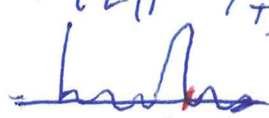
$$\exp\left(-j\frac{4\pi}{\lambda}r_n\right) = \exp\left(-j\frac{4\pi}{\lambda}r_0\right)\exp\left(j2\pi\cdot\frac{2}{\lambda}\sin(\theta_T)\underbrace{n dx}_{x}\right) = \dots \underbrace{\exp\left(j2\pi\frac{2}{\lambda}\sin\theta_T x\right)}_{\sin}\underbrace{\mathrm{rect}\left(\frac{x}{L}\right)}_{\mathrm{rect}}$$

signal is bidiment. sinc →

FT

$\mathrm{sinc}[L(f - f_T)]$ ← shifted

First zero in $1/L$



$\rho_f = 1/L$ ← freq. resol.

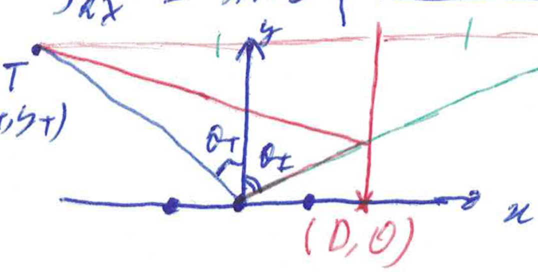
(max of one peak should correspond to the first zero of the other)

$$\rho_\theta = \frac{\lambda}{\underbrace{2L\cos(\theta)}_{\text{depends on direct.}}} \simeq \frac{\lambda}{2L} \rightarrow \underbrace{L = \frac{\lambda}{2\rho_\theta}}_{\text{array length}};\ \overset{\#\text{antennas}}{N} = \left\lceil\frac{L}{d} + 1\right\rceil$$

Multipath

$t = \frac{r_n^T + r_n^I}{2}$

$$S_{Rx}^{nn} = \mathrm{sinc}\left(\frac{r - 0.5[r^n(P_T) + r^n(P_I)]}{\rho_r}\right) e^{-j\frac{2\pi}{\lambda}[r^n(P_T) + r^n(P_I)]}$$



$(-x_T + 2D,\ y_T)$ - coordinate of the image target

(dist → shift → flip)

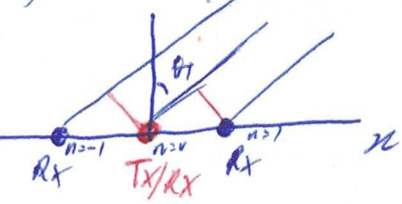
$r^{T}(p_{T}) = r_0 - n dx \sin\theta_T$; $r^{I}(p_I) = r_0^{I} - n dx \sin\theta_I$

$$\underbrace{\exp\left(-\frac{2\pi}{\lambda}\left[r_0^{T} + r_0^{I}\right]\right)}_{const} \exp\left(j 2\pi \cdot \frac{\sin(\theta_T + \sin\theta_I)}{\lambda}\, n dx\right)$$

$= \frac{2\sin(\beta)}{\lambda}$; $\sin\beta = \frac{1}{2}(\sin\theta_T + \sin\theta_I)$

Your ~~est~~ estimated angle is expected to be between the true target and the image target.

II) Bistatic



$r_n = r_0 - \sin\theta_T\, n dx$ $\quad t = \frac{r_0 + r_n}{2}$

$$S_{Rx}^{nn}(r) = \operatorname{sinc}\left(\frac{r - 0.5[r_0 + r_n]}{\rho_r}\right) e^{-j\frac{2\pi}{\lambda} r_n} = const \cdot e^{j2\pi \left(\frac{1}{\lambda}\sin\theta\right) n dx}$$

freq.

$f_{samp} = \frac{1}{d} = 2\max\left\{\frac{1}{\lambda}\sin\theta_T\right\} = \frac{2}{\lambda} \rightarrow \boxed{d = \lambda/2}$

Resolution: $\rho_f = 1/L = \frac{1}{\lambda}\cos\theta_T\, \Delta\theta$; $\Delta\theta = \rho_\theta = \frac{\lambda}{L\cos\theta_T}$

$\rho_\theta = \frac{\lambda}{L} \rightarrow L = \lambda/\rho_\theta$; $N = \left\lfloor \frac{L}{d} + 1 \right\rfloor$

Multipath

$t = \frac{r_0^T + r_n^I}{2}$

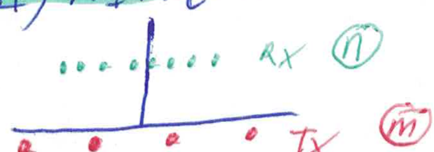
$$S_{Rx}^{nn}(r) = \operatorname{sinc}\left(\frac{r - 0.5[r_0^T + r_n^I]}{\rho_r}\right) e^{-\frac{2\pi}{\lambda}[r_0^T + r_n^I]}$$

$$e^{-\frac{2\pi}{\lambda}[r_0^T + r_n^I]} = e^{-\frac{2\pi}{\lambda}[r_0^T + r_0^I - \sin\theta_I\, n dx]} = const \cdot e^{2\pi \frac{\sin\theta_I}{\lambda} n dx}$$

freq

Your estimated angle should correspond to the angle of the image target

III) MIMO



Each of N_{Rx} receives an echo from each N_{Tx}

$S_{Tx}^{m}(t) = e^{j\pi k t^2} e^{j2\pi f_0 t}\operatorname{rect}(t/T_p)$; $B = k \cdot T$

n,m ← rec. n from trans m

$S_{Rx}^{n,m}(t) = S_{Tx}^{m}(t - t_{m,n}) = \{t_{m,n}$ - delay from tx "m" to rx "n"$\}$ =

$$= e^{j\pi k (t - t_{m,n})^2} e^{j2\pi f_0 (t - t_{m,n})} \operatorname{rect}\left(\frac{t - t_{m,n}}{T_p}\right) = e^{j\pi k t^2} e^{j\pi t_{m,n}^2} e^{-j2\pi k t_{m,n} t} \cdot e^{j2\pi f_0 t} \cdot e^{j2\pi f_0 t_{m,n}} \operatorname{rect}(t/T_p)$$

$\approx \operatorname{rect}(t/T_p)$

$S_{tx}^{m}(t)$

Demodul. and deramping

$$S_{DD}^{n,m}(t) = S_{Rx}^{n,m}(t) = S_{Rx}^{n,m}(t) \cdot [S_{tx}^{m}(t)]^{*} = e^{j\pi k t_{mn}^2} \underbrace{e^{j2\pi(-k t_{m,n}) t}}_{compl.\ sin.} \cdot e^{-j2\pi f_0 t_{m,n}} \operatorname{rect}(t/T_p)$$

$$S_{DD}^{n,m}(f) = e^{j\pi k t_{m,n}^2} e^{-j2\pi f_0 t_{m,n}}\, T_p \operatorname{sinc}(T_p(f + k t_{m,n})) = \{f = -kt\}$$

$$S_{DD}^{n,m}(t) = \ldots T_p \operatorname{sinc}(T_p(-kt + k t_{mn})) = \ldots T_p \operatorname{sinc}(T_p k (t - t_{mn})) = \left\{t = \frac{r_m + r_n}{2}\right\}$$

$$S_{DD}^{n,m}(r) = \ldots T_p \operatorname{sinc}\left(B\left(\frac{2r}{c} - \frac{r_m + r_n}{c}\right)\right) = e^{j\pi t_{m,n}^2} e^{-j2\pi(r_m + r_n)} T_p \cdot \operatorname{sinc}\left(\frac{r - 0.5(r_m + r_n)}{\rho_r}\right) \rightarrow$$

$$S_{DD}^{n,m}(r) = \operatorname{sinc}\left(\frac{r - 0.5[r_m + r_n]}{\rho_r}\right) e^{-j2\pi(r_m + r_n)}$$

DIFFICULT TO FIND A CLOSED-FORM SOL. FOR R ⇒ VIRTUAL CHANNEL

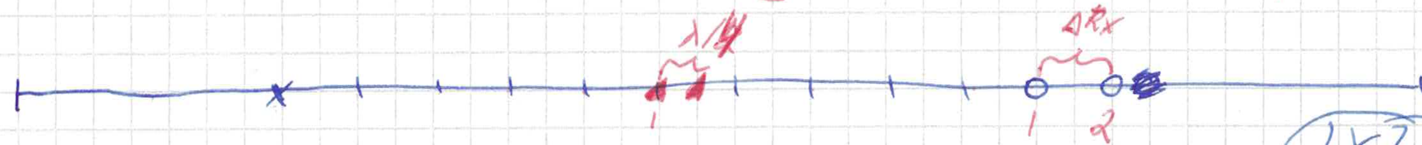
Antennas positioning

$m^{Tx} \times n^{Rx}$

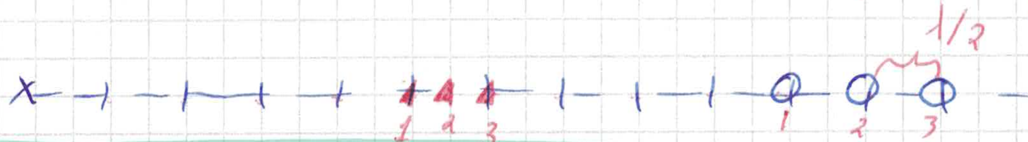
X – Tx
O – Rx
Δ – virt

a) (1x2)

← monostatic → $\Delta R_x = 2 \cdot \lambda/4$



b) (1x3)

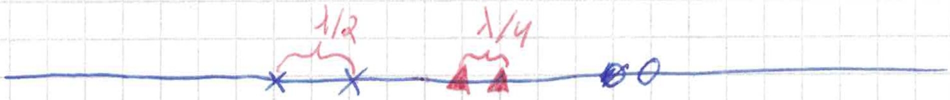


Spacing between R_x is $\lambda/2$! (Always)

equivalently

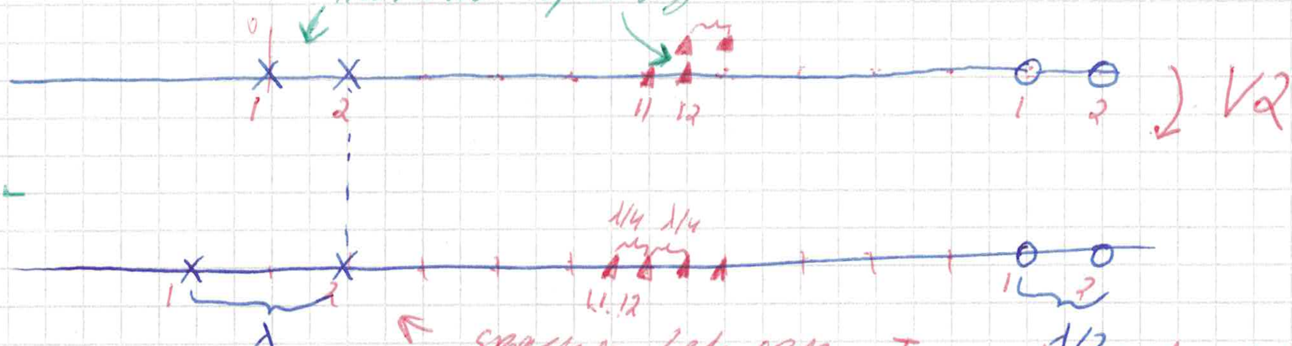
c) (2x1)

Increment # Rx



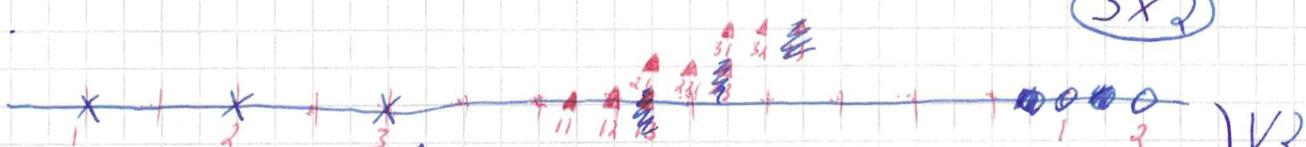
d) (2x2)

increase spacing

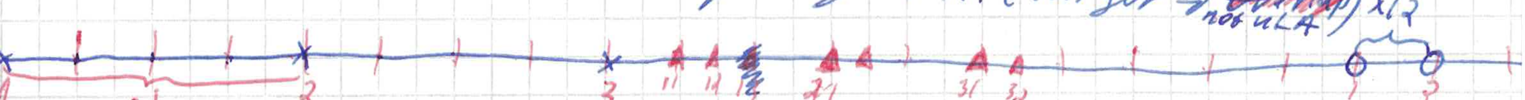


spacing between Tx is λ ← λ/2 ← for 2 antennas

e) (3x2)

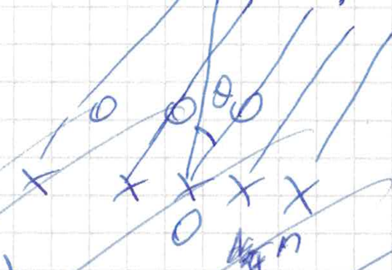


As before, spacing is λ. (larger → not ULA)



$$R_m = R_0^{Tx} - m\,dx_{tx}\sin\theta$$

$$R_n = R_0^{Rx} - n\,dx_{Rx}\sin\theta$$



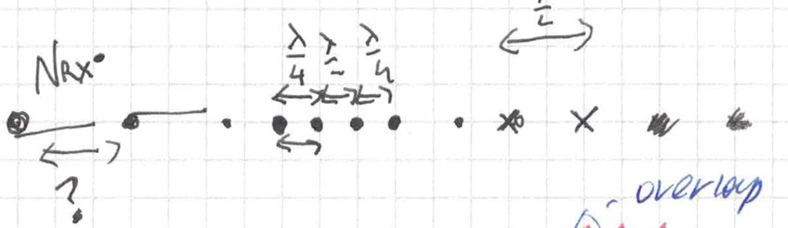
$$e^{-j\frac{2\pi}{\lambda}(R_m+R_n)} = e^{-j\frac{2\pi}{\lambda}\left(R_0^{Tx} - n\,dx_{tx}\sin\theta + R_0^{Rx} - m\,dx_{Rx}\sin\theta\right)} =$$

$$= e^{-j\frac{2\pi}{\lambda}(R_0^{Tx}+R_0^{Rx})}\; \underbrace{e^{j2\pi\frac{\sin\theta}{\lambda}n\,dx_{tx}}}_{free}\; e^{j2\pi\frac{\sin\theta}{\lambda}m\,dx_{Rx}}$$

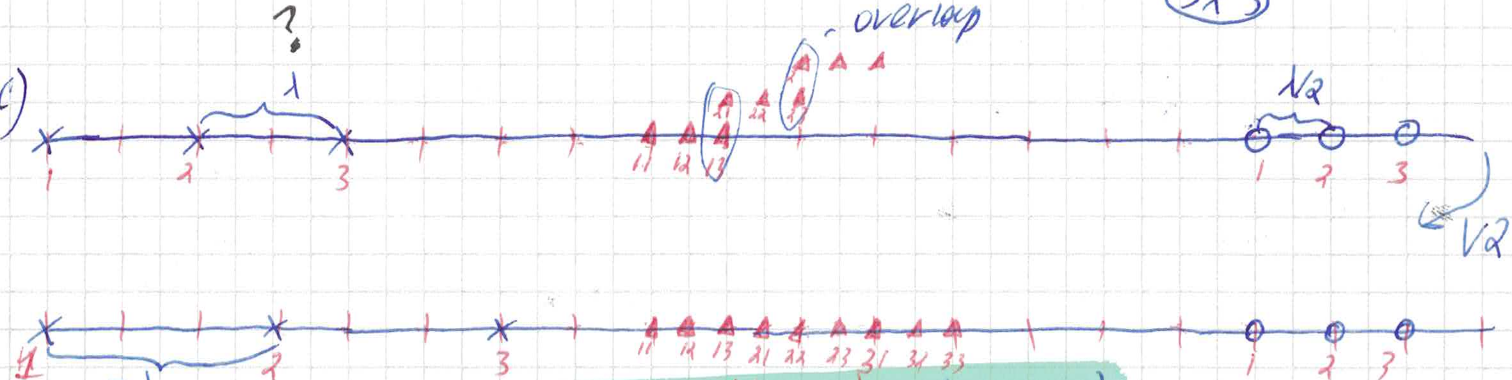
N_{Rx} $\quad dx_{Rx} \le \lambda/2$

m_sends_to_ N_{Rx} receivers

4×8



3×3

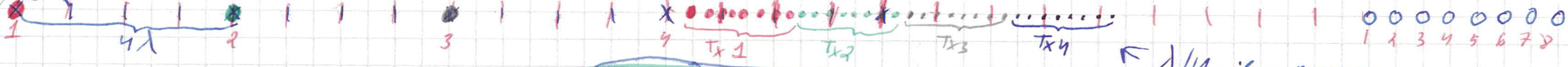


← spacing is $N_{Rx}\cdot\frac{\lambda}{2}$ (of Tx)

case for $\overset{Tx}{4}\times 8^{-Rx}$; $d_{Tx} = 8\cdot\frac{\lambda}{2} = 4\lambda$; $d_{Rx} = \lambda/2$ $\left(d_{virt} = \frac{\lambda}{4}\right)$

(λ – 2 squares)

$e^{j2\pi\frac{\sin\theta}{\lambda}}$



4×8 scenario

← λ/4 is preserved

Multipath

$$S_{nn}^{n,m}(r) = \mathrm{sinc}\left(\frac{r - 0.5[r_m(P_T) + r_n(P_I)]}{\rho_r}\right) e^{-j\pi(r_m(P_T)+r_n(P_I)}$$