

primery Matrix mechanics

汤浩戈, 2018190506040

Abstrct

This is an eassay focused on mind of Matrix mechanics and relationship between Matrix multiplication and its application in quantum mechanics.

Keywords

Matrix mechanics Heisenberg energy level model analogy

1.Introduction

Matrix mechanics is a very important part in quantum mechanics. It is an extremely difficult part in physics but is also related to our linear algebra,expecially the Matrix calculatation.

2.History and background of Matrix mechanics's development

2.1 main physicist

During the early 1920s many famous physicists, including Einstein, Bohr, Born, and Heisenberg, worked to resolve several difficulties with the existing quantum theory.

2.2 invention of inital Matrix mechanics

Finally, in June of 1925, seeking relief from a bout of hay fever, Heisenberg went to the pollen-free island of Helgoland and invented what came to be called matrix mechanics,which was the first version of modern quantum mechanics. Heisenberg's reasoning as he progressed from classical mechanics to "the fundamental equations of the new quantum mechanics" was not always perfectly clear. For some of the key steps he made use of the phrase :

"it seems that the simplest and most natural assumption would be...".
and then wrote down the quantum mechanical expression.

2.3 recommend of other physicist

Regarding this paper the great modern physicist Steven Weinberg wrote:

I have tried several times to read the paper that Heisenberg wrote on returning from Helgoland, and, although I think I understand quantum mechanics, I have never understood Heisenberg's motivations for the mathematical steps in his paper.

According to Weinberg's account, Heisenberg in this paper was acting as a "magician-physicist", i.e., one who "does not seem to be reasoning at all but who jumps over all intermediate steps to a new insight about nature." Of course, Heisenberg himself acknowledged that his reasoning was not to be regarded as an axiomatic derivation. As he later wrote:

It should be distinctly understood that this cannot be a deduction in the mathematical sense of the word, since the equations to be obtained form themselves the postulates of the theory. Although they are made highly plausible by the following considerations, their ultimate justification lies in the agreement of their predictions with experiment.

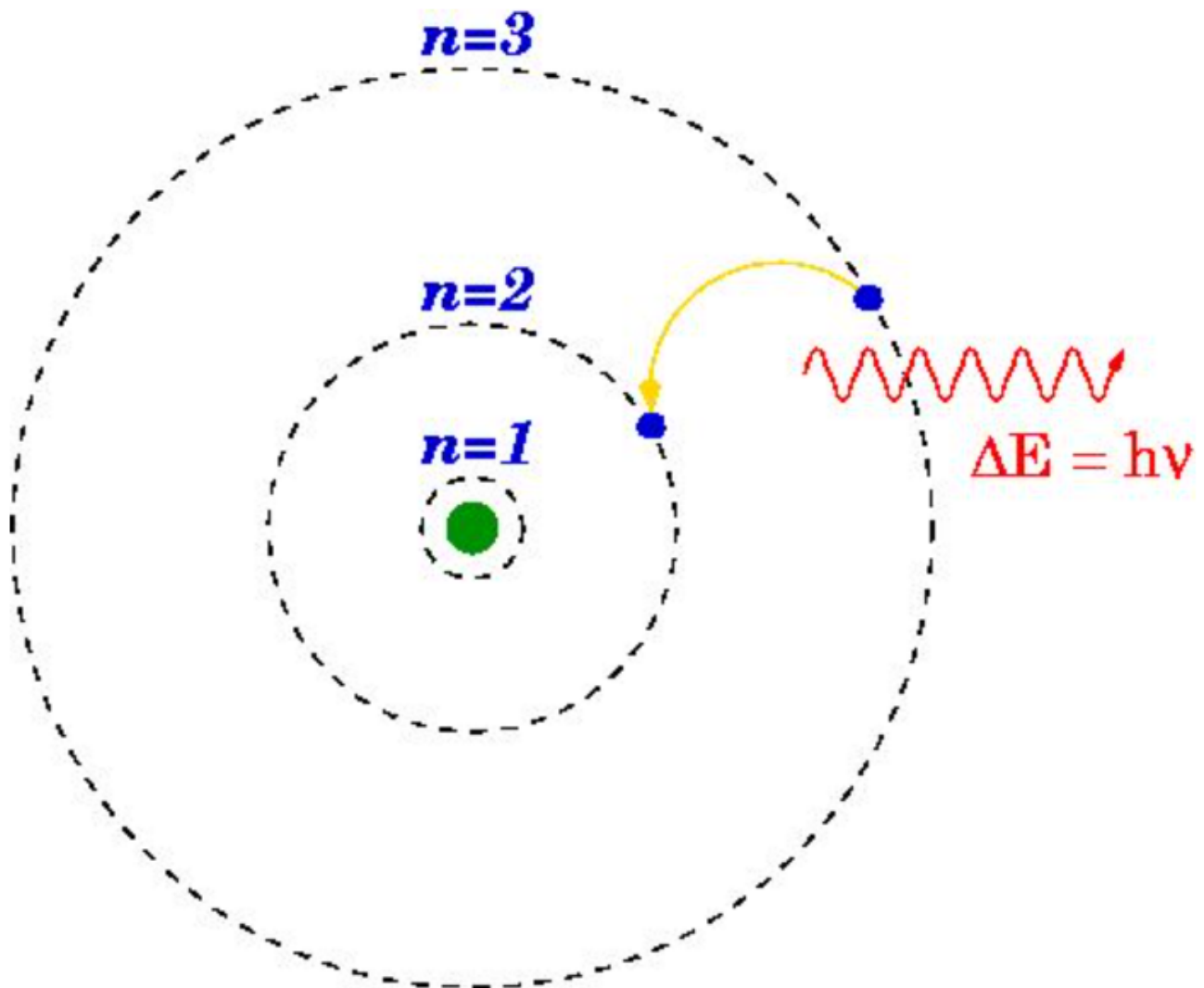
2.4 summary

Nevertheless, it's interesting to examine his considerations, to see just how they can serve as conceptual steps leading to the fundamental postulates of quantum mechanics.

3. Classical Bohr's energy level model

3.1. Model

The electrons in atoms are moving in a specific pathway with a specific frequency and the electrons warp to other pathway from time to time. Every electron's pathway are symbolized a energy level. Therefore when electrons warping between pathways, they absorb energy which is equal to the difference between two energy levels.



3.2. confusion of energy pathway

However, what is exactly is the energy pathway? We can not observe it. Can any experiment prove this pathway's existence? Can any method calculate the distance between two energy levels? We are not sure.

3.3. example of bus station to analogize energy pathway

Here we suppose a bus starts from station A, passing through station B, station C, station D, and eventually approaches station E. We make a charge standard. AB we charge 1 dollar, BC we charge 0.5 dollar, CD we charge 1 dollar, DE we charge 2 dollars. Then we analogize two methods to explain.

- Bohr's explanation:

Bohr considers that every station has its absolute coordinates, so this charge problem can be translated into a distance problem. This means if we make sure of the distance between stations, we will calculate the charge fees directly.

We call this charge level:

A	0
B	1
C	1.5
D	2.5
E	4.5

- Heisenberg's explanation: Heisenberg disagrees with Bohr's absolute coordinates view; he thinks passengers can never know an absolute coordinate; they can only know the fact that: if I transport from C to D, I need to spend 1 dollar, so he uses another two-dimensional table to present his solution:

	A	B	C	D	E
A	0	1	1.5	2.5	4.5
B	1	0	0.5	1.5	3.5
C	1.5	0.5	0	1	3
D	2.5	1.5	1	0	2
E	4.5	3.5	3	2	0

Why does Heisenberg choose to make the thing so difficult? Why don't we just choose to make this problem easy to solve, just like Bohr?

4. relation to Matrix

4.1. another bus station model

Let us suppose two charge standards below:

method1		
	A	B
A	1	2
B	3	1

method2		
	A	B
A	1	3
B	4	1

so this time Heisenberg geniusly start to consider what does multiply means. How can we multiply this 2 tables? What does this multiply mean?

4.2 the meaning of multiplication

let us consider a kind of meaning. Method1 means from A to B, I need to spend 2 dollars, but from B to A, I need to spend 3 dollars. The game rule has changed, so we can not use original ways to solve this problem.

Then let us to explain the meaning of Method1 * Method 2. since method1 is 2x2 table, method2 is 2x2 table, their multiplication can also be 2x2. The first column and first row means all the possible chance of A to A, including 2 ways (A → A + A → A and A → B + B → A)

$$A \rightarrow A + A \rightarrow A = 1 + 1 = 2$$

$$A \rightarrow B + B \rightarrow A = 2 + 4 = 6$$

Finally we get the whole charge is 9

However what if we change the order of method1 and method2? what will happen if we change the turns? we calculate again:

$$A \rightarrow A + A \rightarrow A = 1 + 1 = 2$$

$$A \rightarrow B + B \rightarrow A = 3 + 3 = 6$$

you can suprisingly find if we change the order the value is different.

With the help of Heisenberg's teacher Born, they finally find this algorithm is the same as Matrix, this is the beginning of the Matrix mechanics.

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method1 =
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$$\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$$

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>> method2=[1 3;4 1]
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method2 =
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$$\begin{pmatrix} 1 & 3 \\ 4 & 1 \end{pmatrix}$$

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>> method1*method2
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ans =
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$$\begin{pmatrix} 9 & 5 \\ 7 & 10 \end{pmatrix}$$

5. Conclusion

I have researched on the history of Matrix mechanics, and do most of the analogy to explain the Bohr's energy level model in Heisenberg's way to simply explain the relationship between Matrix calculation and quantum mechanics.

Reference

[1]<https://mathpages.com/home/kmath698/kmath698.htm>

[2]量子力学史话 曹天元著