Modeling Without Conditional Independence: Gaussian Process Regression for Time-Series Cross-Sectional Analyses

David Carlson

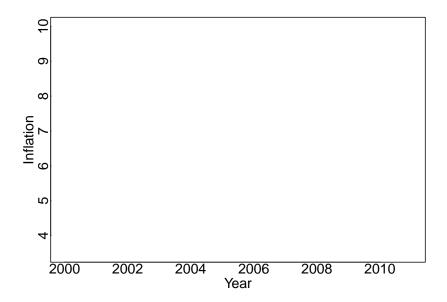
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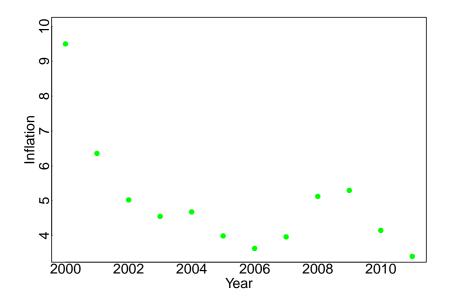
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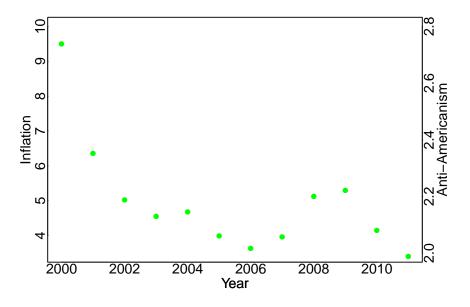
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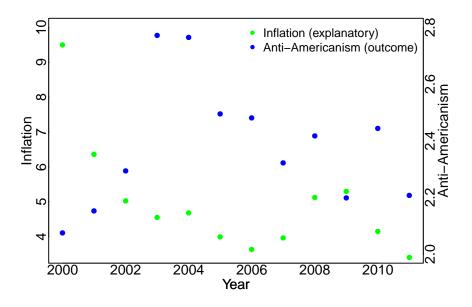
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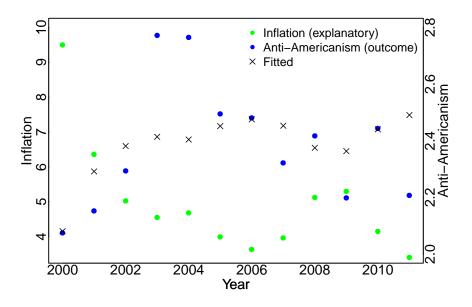
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- Variables in political science violate modeling assumptions
- Observations not conditionally independent
- Example: How does inflation explain anti-Americanism?

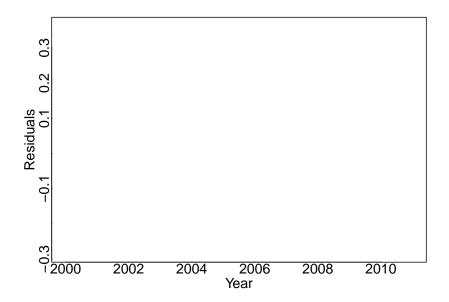


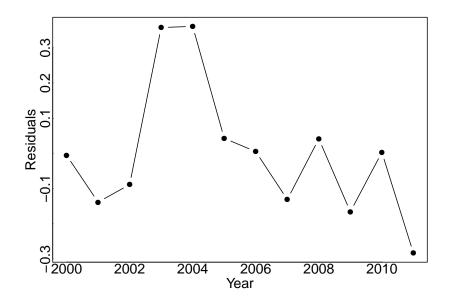












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- Machine learning algorithm models outcomes jointly as a process

Current practices

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- Further applications

TSCS 3-year review: APSR, AJPS, JOP, CPS (320 articles)

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- Random effects (RE) $\approx 15\%$ (n = 48)

Issues with Current Practice

Esarey and Menger 2017

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- Estimate variance-covariance in a learning kernel
- Very flexible, still interpretable

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- ullet Data in mean and Ω do not have to be the same

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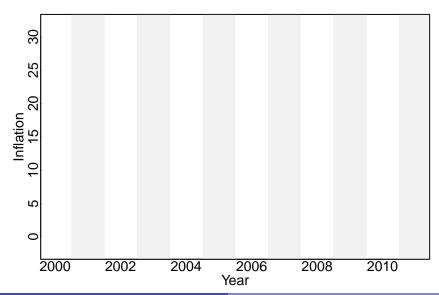
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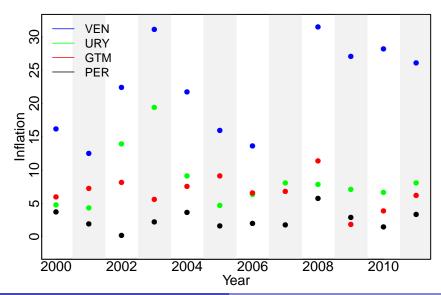
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Inflation in Latin America

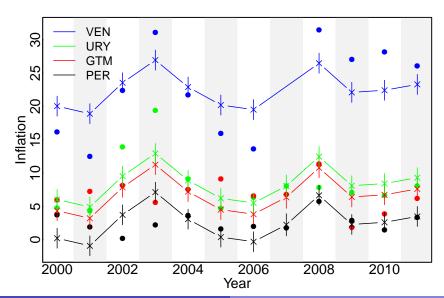
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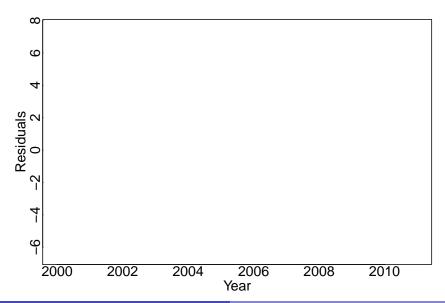
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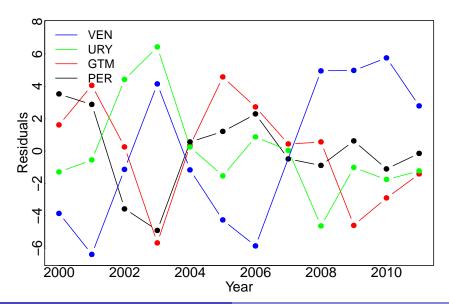
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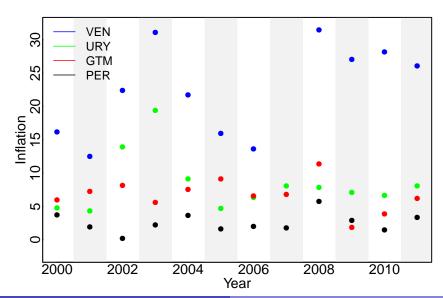
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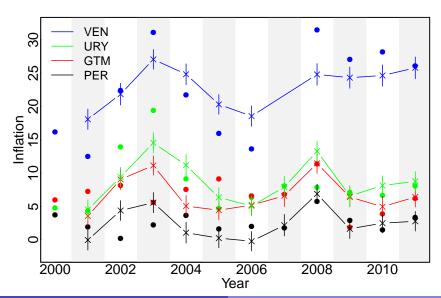
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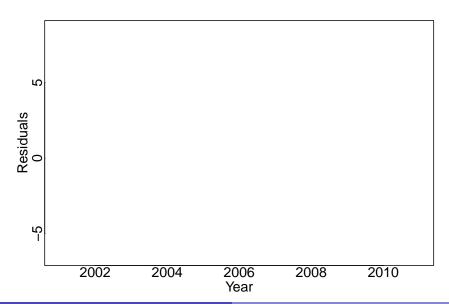
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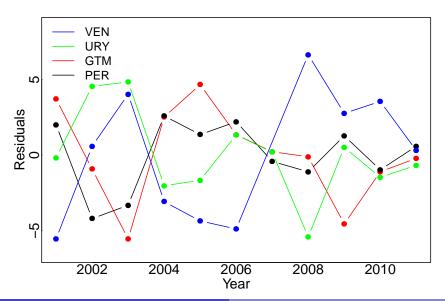
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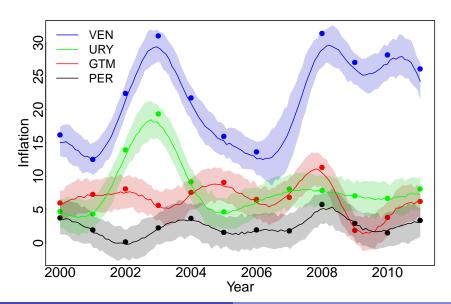
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Residuals in LDV model show same trends



GPR allows correlated but different time trends



Overview

- Current practices
- GPR discussion
- Specification
- Simulations
- Application: Inflation and anti-Americanism in Latin America
- Future work

Simulations

Serial correlation

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- Small number of units

Serial correlation within units (errors and explanatory)

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- 15 units, 15 observations (time) per unit

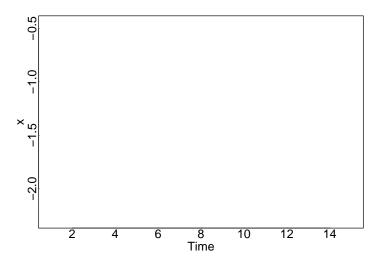
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- GPR best on coverage probability and mean squared error

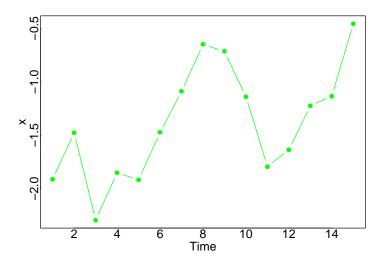
Serial Correlation Example

An example when $\rho = 0.9$



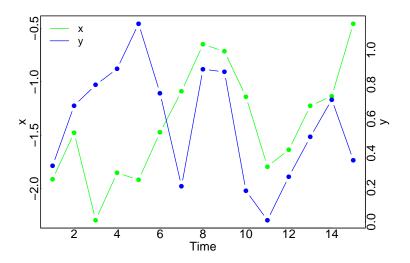
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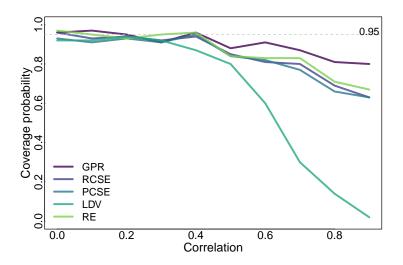
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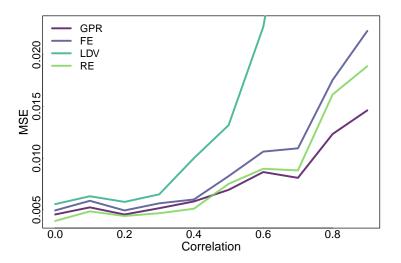
Serial Correlation Results

Coverage probability



Serial Correlation Results

Mean squared error of estimates



David Carlson

Correlation between unrelated variable (z) and unit intercept

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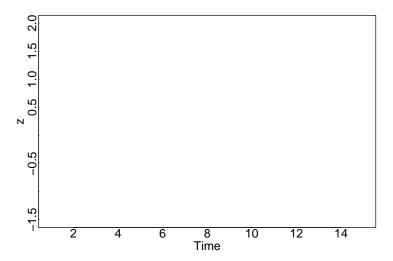
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- RE very bad at false positives

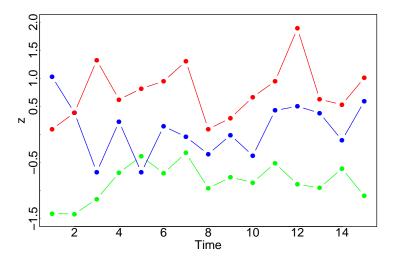
Intercept Correlation Example

Example when $\rho = 0.9$, $\gamma_{1:3} = [-1, 0, 1]$



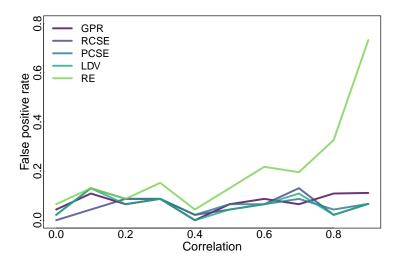
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Intercept Unrelated Correlate Results

False positive rates



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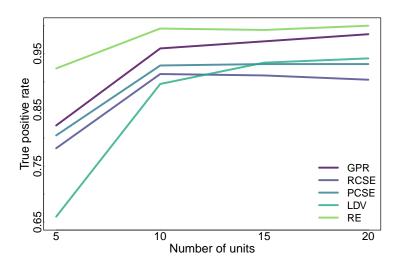
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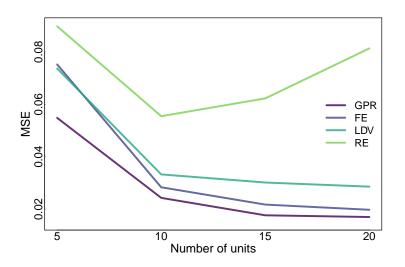
Varying N Results

True positive rates as units increase



Varying N Results

Mean squared of estimates as units increase



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- ullet Good economy o focus on foreign policy o decreased sentiment

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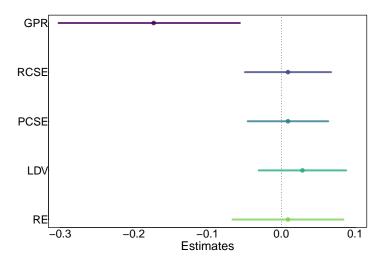
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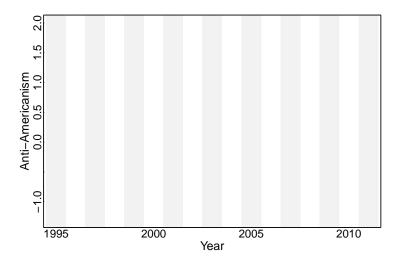
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• 18 countries, 1995–2011, 256 observations (unbalanced)

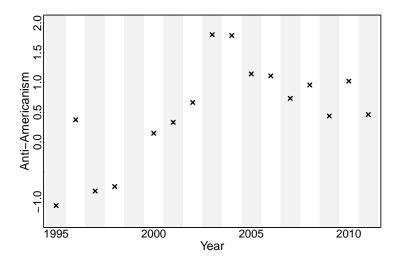
Effect of inflation on anti-Americanism



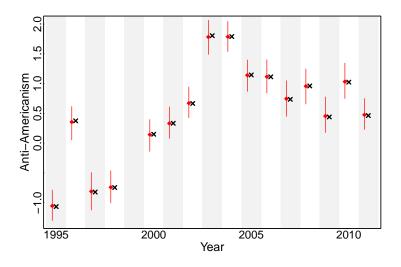
Model fit in Mexico



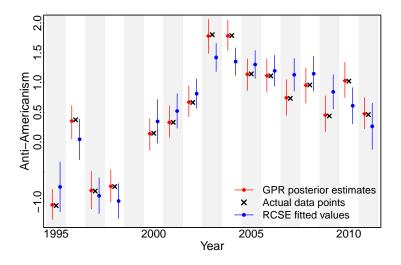
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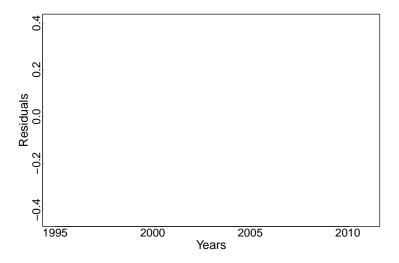
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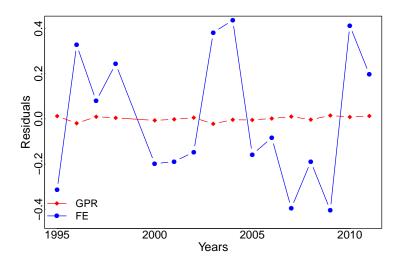
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Residuals in Mexico



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- Code available: https://github.com/carlson9/GPRPanel

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- Text analysis
- Causal GPR (Huang, Zhang, Schölkopf 2015)

How being within range of Palestian rockets impacts the right bloc vote

• Outcome: Right bloc vote

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Explanatory: Within range

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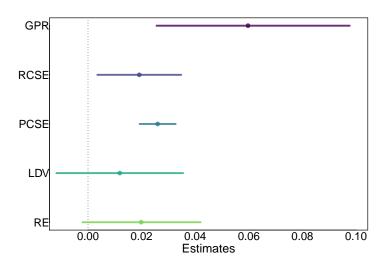
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 - ▶ Local fatalities due to suicide attacks three months prior

Replication: Results

Effect of rocket range on right vote



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- Including inflation in example significantly outperforms null with both measures

Priors

$$\mathbf{y} \sim \mathcal{MVN}(\tilde{\mathbf{X}}eta, \sigma^2\mathbf{\Omega}),$$
 $\mathbf{\Omega}^*(\mathbf{x}_j^*, \mathbf{x}_i^*|oldsymbol{\zeta}) = \exp\left\{-\sum_{p=1}^m rac{|x_{pj}^* - x_{pi}^*|^2}{\zeta_p}
ight\},$
 $\mathbf{\Omega}(\mathbf{x}_j^*, \mathbf{x}_i^*) = \mathbf{\Omega}^*(\mathbf{x}_j^*, \mathbf{x}_i^*) + \delta g_{j,i},$
 $g_{j,i} = \begin{cases} 1 & \text{if } j = i, \\ 0 & \text{otherwise.} \end{cases}$
 $\delta \sim Exp(1)$
 $\zeta_p = rac{\zeta_{p1} + \zeta_{p2}}{2}$
 $\zeta_{p1} \sim G(1, 20)$
 $\zeta_{p2} \sim G(10, 10)$
 $\sigma^2 \sim IG(1, 1)$
 $eta \sim \mathcal{N}(0, 3)$

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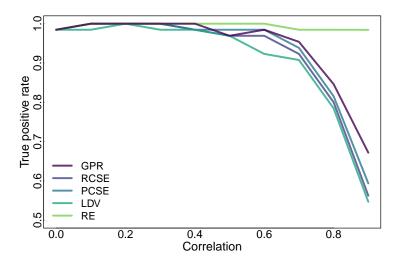
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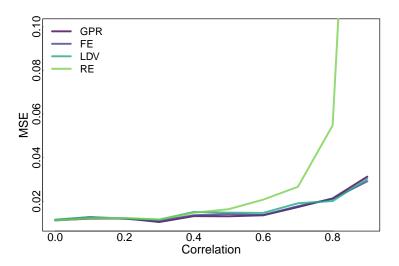
Intercept Correlation Results

True positive rates

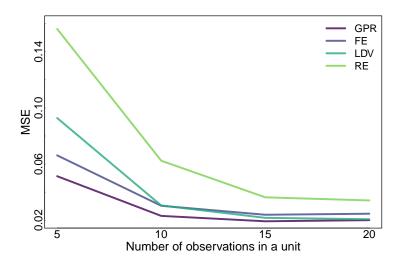


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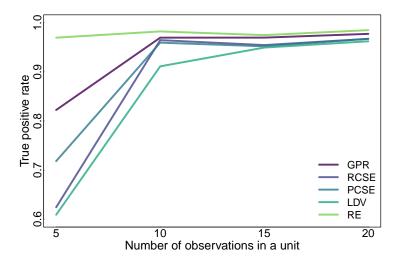
Mean squared error of estimates



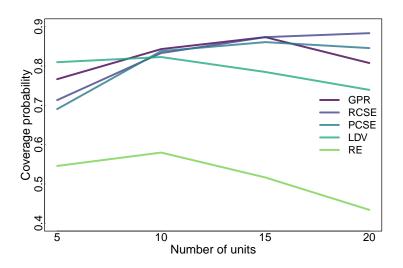
Mean squared of estimates as observations increase



True positive rates as observations increase



Coverage probabilities as units increase



Coverage probabilities as observations increase

