

# Exact analytic solution

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$$y' = \sin^2 x + y \cot x$$

Make a substitution:

$$y = y_1 * u$$

Solve the complementary equation:

$$y_1' - y_1 \cot x = 0$$

$$\frac{dy_1}{dx} = y_1 \cot x$$

$$\int \frac{dy_1}{y_1} = \int \cot x dx$$

$$\ln y_1 = \ln \sin x$$

$$y_1 = \sin x$$

If:

$$y = y_1 * u$$

Then:

$$\frac{du}{dx} = \frac{\sin^2 x}{\sin x}$$

$$\int du = \int \sin x dx$$

$$u = -\cos x + C$$

$$y = -\cos x \sin x + C \sin x$$

Solve Initial Value Problem for:

$$x_0 = 1, \quad y_0 = 1$$

$$y_0 = -\cos x_0 \sin x_0 + C \sin x_0$$

$$1 = 0 + C$$

$$C = 1$$

The solution for IVP is:

$$y = -\cos x \sin x + \sin x$$