

# Exact analytical solution

$$y' = \sin^2 x + y \cot x$$

Solve the complementary equation:

$$y'_1 - y_1 \cot x = 0$$

$$\frac{dy_1}{y_1} = \cot x \, dx$$

$$\int \frac{dy_1}{y_1} = \int \cot x \, dx$$

$$\ln y_1 = \ln \sin x$$

$$y_1 = \sin x$$

Make a substitution:

$$y = y_1 u$$

Then:

$$\frac{du}{u} = \frac{\sin^2 x}{\sin x} \, dx$$

$$\int \frac{du}{u} = \int \sin x \, dx$$

$$u = -\cos x + C$$

$$y = -\cos x \sin x + C \sin x$$

Solve Initial Value Problem for:

$$x_0 = 1, y_0 = 1$$

$$y_0 = -\cos x_0 \sin x_0 + C \sin x_0$$

$$1 = 0 + C$$

$$C = 1$$

The solution for IVP is:

$$y = -\cos x \sin x + \sin x$$