Exact analytical solution

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$$y' = \sin^2 x + y \cot x$$

Make a substitution:

$$y = y_1 * u$$

Solve the complementary equation:

$$y_1' - y_1 \cot x = 0$$

$$\frac{\mathrm{d}y_1}{\mathrm{d}x} = y_1 \cot x$$

$$\int \frac{\mathrm{d}y_1}{y_1} = \int \cot x \mathrm{d}x$$

$$\ln y_1 = \ln \sin x$$

$$y_1 = \sin x$$

If:

$$y = y_1 * u$$

Then:

$$\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{\sin^2 x}{\sin x}$$

$$\int \mathrm{d}u = \int \sin x \mathrm{d}x$$

$$u = -\cos x + C$$

$$y = -\cos x \sin x + C \sin x$$

Solve Initial Value Problem for:

$$x_0 = 1, y_0 = 1$$

$$y_0 = -\cos x_0 \sin x_0 + C \sin x_0$$

$$1 = 0 + C$$

$$C = 1$$

The solution for IVP is:

$$y = -\cos x \sin x + \sin x$$