AHSANULLAH UNIVERSITY OF SCIENCE AND TECHNOLOGY (AUST)

ME-3105: FLUID MECHANICS-II
(LEC-3: DISTORTED MODEL ANALYSIS
OR
SIMILARITY AND SIMILITUDE)

BY

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Distorted Model Analysis

Definition:

- ✓ The model does not have complete geometrical similarity with prototype is called Distorted Model.
- ✓ Example rivers, harbors, reservoirs, etc. have large horizontal dimension compared to vertical dimension.
- ✓ If model of such prototype have full/complete geometric similarity then vertical depth of water of the model will be too small and can not be measured accurately.
- ✓ To overcome this difficulty the model of such prototype or structures are made with two different scales: horizontal and vertical scales (i.e. greater vertical scales than the horizontal ones).

Distorted Model Analysis

Definition:

- ✓ A model having complete geometrical similarity with prototype but working under different head of water and behaves as a distorted model.
- ✓ The prediction of a distorted model is relatively difficult and the result of the models being distorted can not be easily transferred to the prototype, though basic conditions of geometric similarity is not satisfied.

Advantages and Disadvantages of Distorted Model

Advantages:

- ✓ Model size can be reduced sufficiently (by its distortion) as a result cost of model is considerably decrease, and its operation is simplified.
- ✓ Vertical exaggeration results in steeper slope of water surface, which can be easily and accurately measured.
- ✓ Reynold's number of a model is considerably increased and surface resistance is decreased due to exaggerated water slopes. It helps in simulation of the flow conditions in the model and prototype.

Advantages and Disadvantages of Distorted Model

Disadvantages:

1) An unfavorable psychological effect on the observer.
2) The Behavior of flow of a model differs from that of the prototype.
3) Magnitude and direction of pressure is not correctly reproduced.
4) Velocities are not correctly reproduced, as the vertical exaggeration causes distortion of lateral velocity and kinetic energy.

(Note: despite of disadvantages of distorted model, it is sometimes preferred to use. By exercising an utmost care, the results of the model may be transferred to the prototype.)

Prediction of Parameter in Distorted Model

Following parameters can be predicted from distorted model:

- ✓ Velocity of water in the prototype for the given velocity at the corresponding point of model.
- ✓ Discharge of the prototype for the given discharge of the model.
- ✓ Speed of the prototype for the given speed of the model
- ✓ Power developed by the prototype for the given power of the model.
- ✓ Time of emptying a prototype for the given time of emptying a model.

Prediction of Velocity

Consider a distorted model of prototype like a river weir, dam or spillway, etc.

Let h_m = Head of water over the model

v_m = Velocity of water at some point of the model

h_p, v_p = Corresponding value for the prototype

$\frac{1}{s_h}$ = Horizontal scale ratio of the model to prototype

$\frac{1}{s_v}$ = Vertical scale ratio of the model to prototype

Velocity of water in the model,

$v_m = c_{vm}\sqrt{2gh_m}$

velocity of water in the prototype,

$v_p = c_{vp}\sqrt{2gh_p}$

$$\frac{v_m}{v_p} = \frac{c_{vm}\sqrt{2gh_m}}{c_{vp}\sqrt{2gh_p}}$$

$$\frac{v_m}{v_p} = \sqrt{\frac{h_m}{h_p}}; \quad \frac{h_m}{h_p} = \frac{1}{s_v} \quad ; \quad \frac{v_m}{v_p} = \sqrt{\frac{1}{s_v}} \quad ; \quad v_p = v_m \times \sqrt{s_v}$$

Prediction of Discharge

Consider a distorted model of a prototype like a river, weir, notch, spillway, etc.

Let L_m = Length of the model

h_m = Height of the model through which discharge is taking place

v_m = Actual velocity of water in the model

L_p, h_p, v_p = Corresponding value for the prototype

$\frac{1}{s_h}$ = Horizontal scale ratio of the model to prototype

$\frac{1}{s_v}$ = Vertical scale ratio of the model to prototype

Discharge of the model and prototype,

Q_m = Area $\times$ velocity

$= L_m \times h_m \times v_m$

$Q_p = L_p \times h_p \times v_p$

$$\frac{Q_m}{Q_p} = \left(\frac{L_m}{L_p}\right)\cdot\left(\frac{h_m}{h_p}\right)\cdot\left(\frac{v_m}{v_p}\right) \; ; \quad \frac{h_m}{h_p} = \frac{1}{s_v} \; ; \; \frac{L_m}{L_p} = \frac{1}{s_h} \quad \text{and} \quad \frac{v_m}{v_p} = \frac{1}{\sqrt{s_v}}$$

$$\frac{Q_m}{Q_p} = \left(\frac{1}{s_h}\right)\cdot\left(\frac{1}{s_v}\right)\cdot\left(\frac{1}{\sqrt{s_v}}\right) = \left(\frac{1}{s_h}\right)\cdot\left(\frac{1}{s_v^{1.5}}\right) \; ; \quad Q_p = Q_m \times s_h \times s_v^{1.5}$$

Prediction of Speed (Rotational)

Consider a distorted model of a prototype like a centrifugal pump or turbine.

Let D_m = Diameter of the model impeller
v_m = Tangential velocity of the model impeller
N_m = Speed of the model runner in rpm
D_p , v_p , N_p = Corresponding value of the prototype
$\frac{1}{s_h}$ = Horizontal scale ratio of the model to the prototype
$\frac{1}{s_v}$ = Vertical scale ratio of the model to the prototype

Tangential velocity of the model impeller,

$$v_m = \frac{\pi D_m N_m}{60} \; ; \; N_m = \frac{60\, v_m}{\pi D_m}$$

Similarly speed of prototype,

$$N_p = \frac{60\, v_p}{\pi D_p}$$

$$\frac{N_m}{N_p} = \left(\frac{60\, v_m}{60\, v_p}\right)\cdot\left(\frac{\pi D_p}{\pi D_m}\right) \; ; \; \frac{N_m}{N_p} = \left(\frac{v_m}{v_p}\right)\cdot\left(\frac{D_p}{D_m}\right)$$

$$\frac{v_m}{v_p} = \frac{1}{\sqrt{s_v}} \; ; \; \frac{D_m}{D_p} = \frac{1}{s_h} ; \; \frac{N_m}{N_p} = \frac{1}{\sqrt{s_v}} \times s_h \; ; \; N_p = \frac{N_m\sqrt{s_v}}{s_h}$$

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Prediction of Power

Consider a distorted model of a prototype like a turbine.

Let Q_m = Discharge of the model in m^3/s,
h_m = Head of water over the model in meters,
P_m = Power developed by the model,
Q_p , h_p , P_p = Corresponding value of the prototype
$\frac{1}{s_h}$ = Horizontal scale ratio of the model to the prototype
$\frac{1}{s_v}$ = Vertical scale ratio of the model to the prototype

Power developed by the model,

$$P_m = \rho_m g Q_m h_m$$

Power developed by the prototype,

$$P_p = \rho_p g Q_p h_p$$

$$\frac{P_m}{P_p} = \left(\frac{\rho_m}{\rho_p}\right)\cdot\left(\frac{Q_m}{Q_p}\right)\cdot\left(\frac{h_m}{h_p}\right)$$

$$\frac{Q_m}{Q_p} = \left(\frac{1}{s_h}\right)\cdot\left(\frac{1}{s_v^{\,1.5}}\right) \text{ and } \frac{h_m}{h_p} = \frac{1}{s_v}$$

$$\frac{P_m}{P_p} = \left(\frac{\rho_m}{\rho_p}\right)\cdot\left(\frac{1}{s_h}\right)\cdot\left(\frac{1}{s_v^{\,1.5}}\right)\cdot\left(\frac{1}{s_v}\right) \; ; \; P_p = P_m\cdot\left(\frac{\rho_p}{\rho_m}\right)\times s_h \times s_v^{\,2.5}$$

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Prediction of Emptying Time

Consider a distorted model of a prototype like a tank or reservoir.

Let V_m = Total volume of water or a reservoir,

Q_m = Rate of discharge of water in the model,

V_p, Q_p, = Corresponding value of the prototype

$\frac{1}{s_h}$ = Horizontal scale ratio of the model to the prototype

$\frac{1}{s_v}$ = Vertical scale ratio of the model to the prototype

Time of emptying the model and prototype,

$$t_m = \frac{V_m}{Q_m} \text{ and } t_p = \frac{V_p}{Q_p}$$

$$\frac{t_m}{t_p} = \left(\frac{V_m}{V_p}\right)\cdot\left(\frac{Q_p}{Q_m}\right);$$

$$\frac{V_m}{V_p} = \frac{A_m \cdot h_m}{A_p \cdot h_p} = \frac{(L_m \cdot b_m)\cdot h_m}{(L_p \cdot b_p)\cdot h_p} = \left(\frac{L_m}{L_p}\right)\cdot\left(\frac{b_m}{b_p}\right)\cdot\left(\frac{h_m}{h_p}\right); \quad \frac{V_m}{V_p} = \left(\frac{1}{s_h}\right)^2\cdot\left(\frac{1}{s_v}\right) \text{ and } \frac{Q_m}{Q_p} = \left(\frac{1}{s_h}\right)\cdot\left(\frac{1}{s_v^{1.5}}\right)$$

$$\frac{t_m}{t_p} = \left(\frac{1}{s_h}\right)^2\cdot\left(\frac{1}{s_v}\right)\cdot(s_h)\cdot\left(s_v^{1.5}\right) = \left(\frac{1}{s_h}\right)\times s_v^{0.5}; \quad t_p = \frac{t_m \times s_h}{\sqrt{s_v}}$$

Summery of Formula for Distorted Model

All formula:

$\frac{1}{s_h}$ = Horizontal scale ratio of the model to prototype

$\frac{1}{s_v}$ = Vertical scale ratio of the model to prototype

$$\frac{h_m}{h_p} = \frac{1}{s_v}; \quad \frac{v_m}{v_p} = \sqrt{\frac{1}{s_v}}; \quad \frac{N_m}{N_p} = \frac{1}{\sqrt{s_v}}\times s_h;$$

$$\frac{Q_m}{Q_p} = \left(\frac{1}{s_h}\right)\cdot\left(\frac{1}{s_v^{1.5}}\right); \quad \frac{P_m}{P_p} = \left(\frac{\rho_m}{\rho_p}\right)\cdot\left(\frac{1}{s_h}\right)\cdot\left(\frac{1}{s_v^{2.5}}\right);$$

$$\frac{t_m}{t_p} = \left(\frac{1}{s_h}\right)\times s_v^{0.5};$$

Problem-1 (Distorted Model)

A diversion weir 240 m long has discharge capacity of 250 m^3/s under a head of 1.2 m. A model of this weir has to be constructed in a laboratory where the available channel is 3.0 m long and 500 m deep. Design the suitable model for the weir if the water available in the laboratory is 25 liters/s.

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Problem-2 (Distorted Model)

A turbine model of 1:8 is tested under a head of 6.0 m of water. The full-scale turbine is required to work under head of 30 m of water and run at 428 rpm. At what speed the model must be run?

If the model develops 5 kw and uses 110 liters of water per second, at this speed what power will be obtained from the full-scale turbine, assuming that its efficiency is 3% better than that of the model?

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