Lec-1 alone

01798455193

Fluid properties

Fazlar Rahman

$\Sigma F_x = 0$

$\Sigma F_y = 0$ | $v = 0$

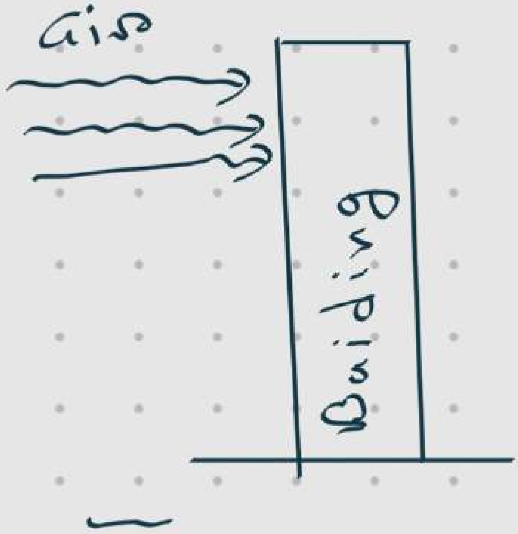
$\Sigma F_z = 0$

also

$\Sigma M_x = 0$

$\Sigma M_y = 0$

$\Sigma M_z = 0$



Static fluid   Moving fluid

(I) Fluid Mechanics   Frank M White

(II) Intro to fluid Mechanics
Robert Fox McDonald

(III) Fluid - Yunus Cengel.

(IV) Khurmi

Fluid Properties

- Fluid in Rest or in motion

- Duct - Rectangular - low pressure
- Pipe - Circular - High pressure fluid.

□ What is the diff between Duct, pipe, tube.

liquid - Non-Compresible

Gas - Compressible

$$\rho = \frac{m}{V}$$

$$v = 1/\rho$$

$$\gamma = \rho g = \frac{mg}{V}$$

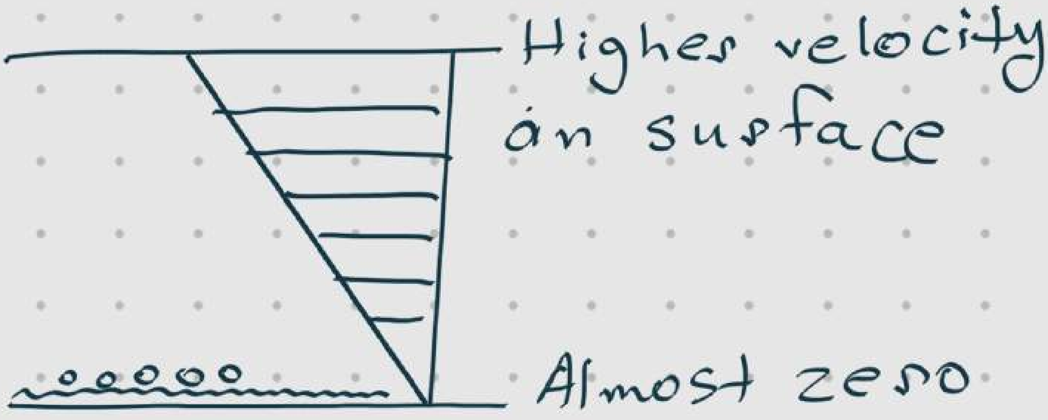
$$SG = \gamma/\gamma_{water} = \frac{\rho}{\rho_{water}}$$

Standard reference density is water

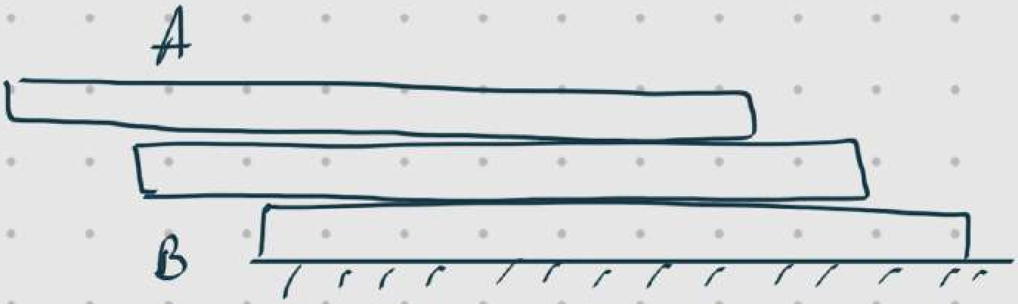
Viscosity: $\frac{Ns}{m^2}$ - Dynamic viscosity

Kinematic viscosity $= \frac{\mu}{\rho}$ m^2/s

Fluid moves in layer



Upper fluid will flow more



- Compressibility of fluid

Bulk Modulus of Elasticity (K)

ratio of infinitesimal increase of pressure to the resulting relative decrease in volume

change of pressure

$$\boxed{K = -\,dP \Big/ \left(\frac{dv}{v}\right)}$$

relative ration of volume change

$v = 1/\rho$

$v\rho = 1$

$dv \cdot \rho + d\rho \cdot v = 0$

$$\frac{dv}{v} = -\frac{d\rho}{\rho}$$

$$K = -\frac{dP}{\frac{dv}{v}}$$

$$= \frac{dP}{\frac{d\rho}{\rho}} \quad N/m^2$$

Coefficient of Compressibility

$$\beta = \frac{1}{K}$$

Surface tension { Cohesion (water, honey); Adhesion (murcury) }

10/12/24

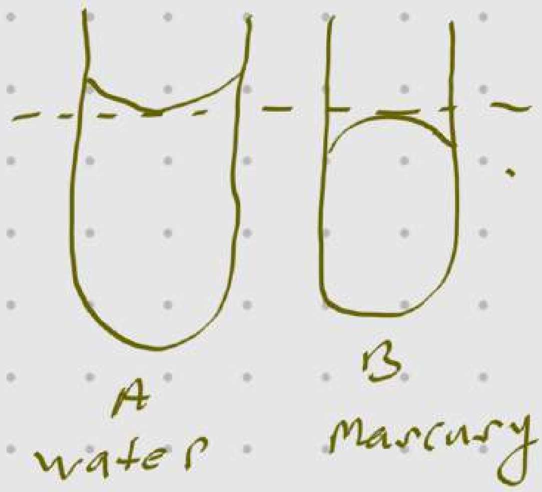
Cohesion - resist to tensile stress

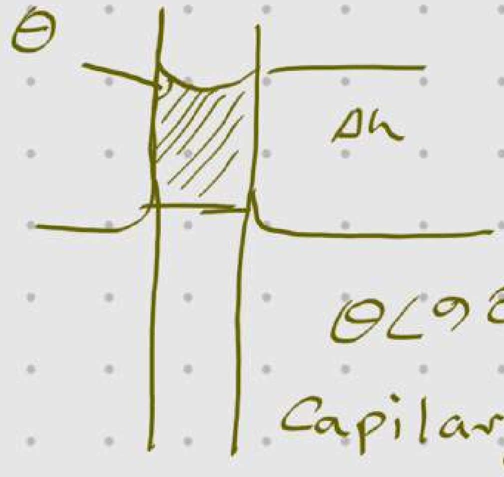
adhesion - enables fluid to adhere another body.

Surface Tension

$$\sigma = F/L \quad (N/m)$$

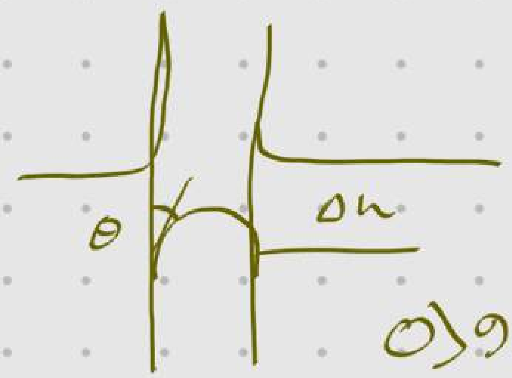
Capilarity of fluid:





$\theta < 90$

Capilary rise



$\theta > 90$

What is Meniscus

Capilary depression

Depressio

$$A = \frac{\pi d^2}{4}$$

$$Vol = A \Delta h$$

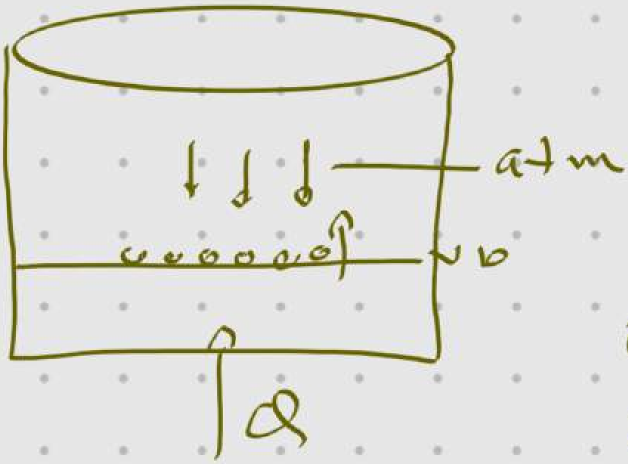
Vapor Pressure: pressure exerted by the vapor at free surface

14.7 Psi

101.3 kPa

100°C

80°C



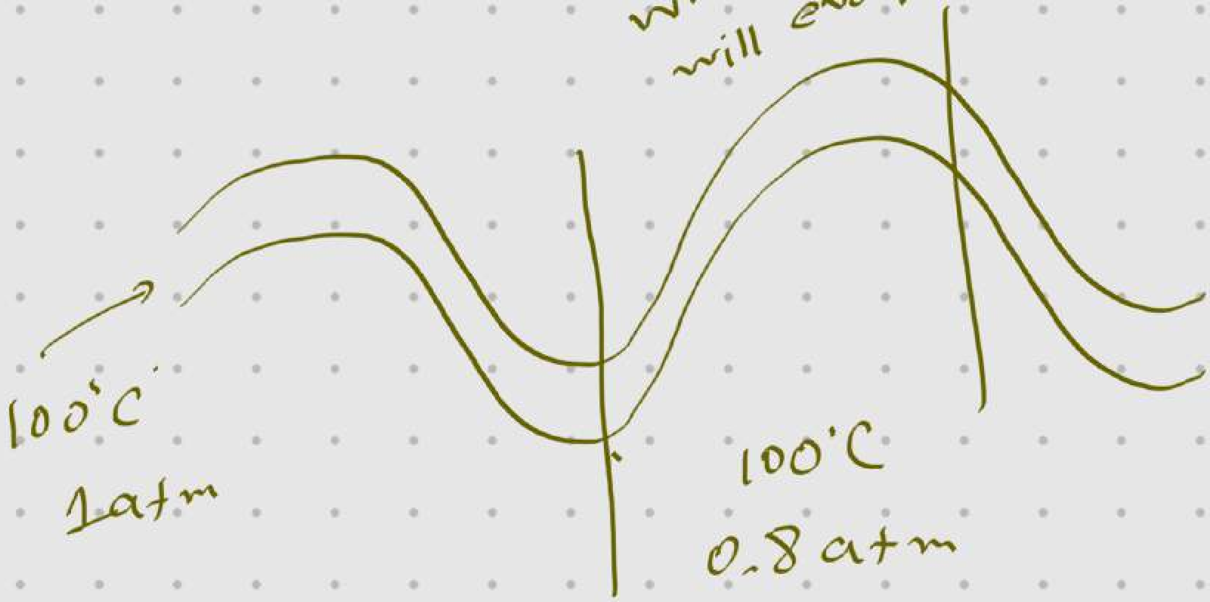
$v_p > P_{atm}$

On mountain it will boil faster because atm will be less

$T_{boil} = 100°C$

$P = 1\ atm$

water here evaporate



Cavitation One to callapsing bouble

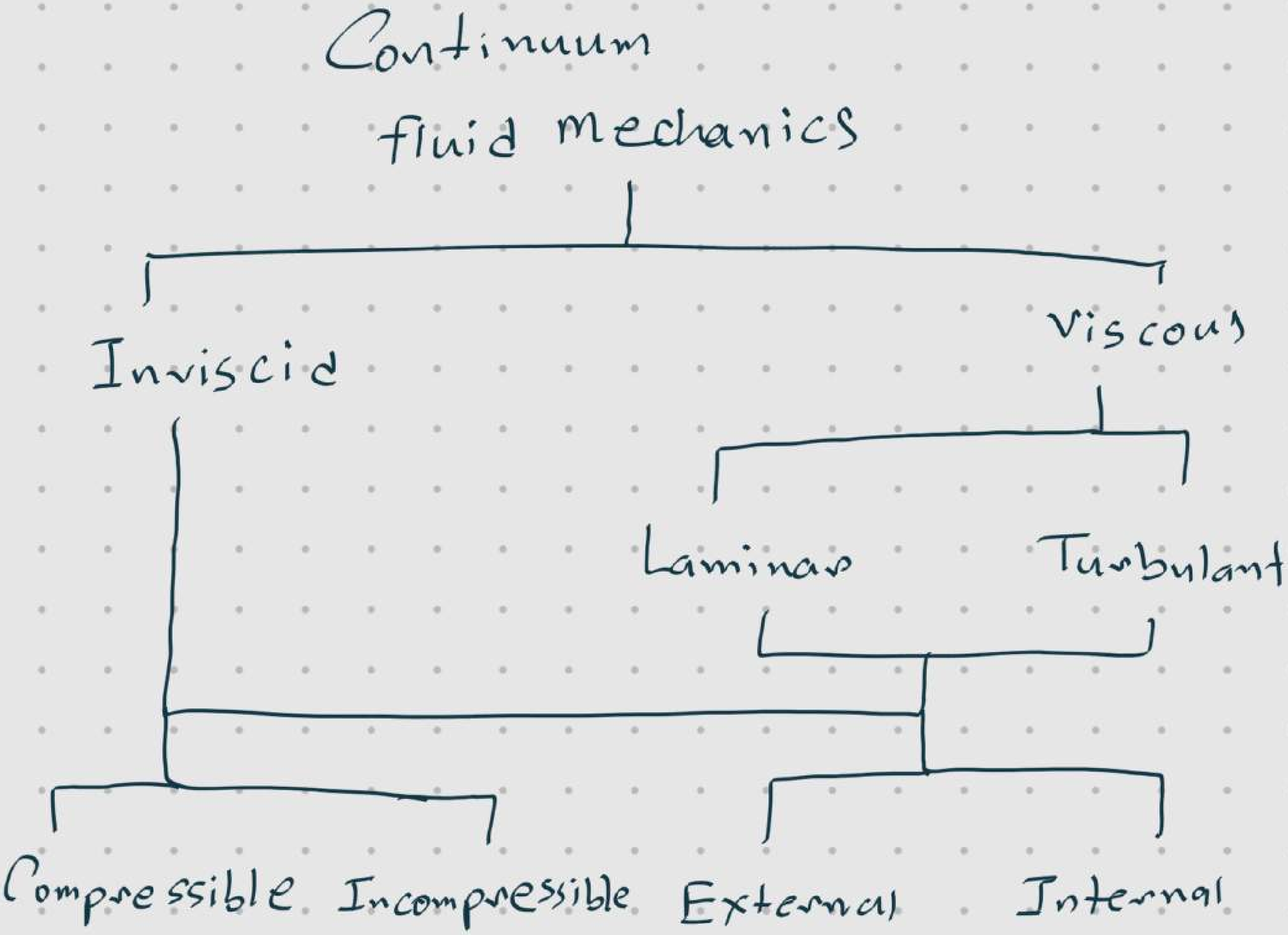
□ Fluid Pressure:

$$P = \frac{F}{A}$$

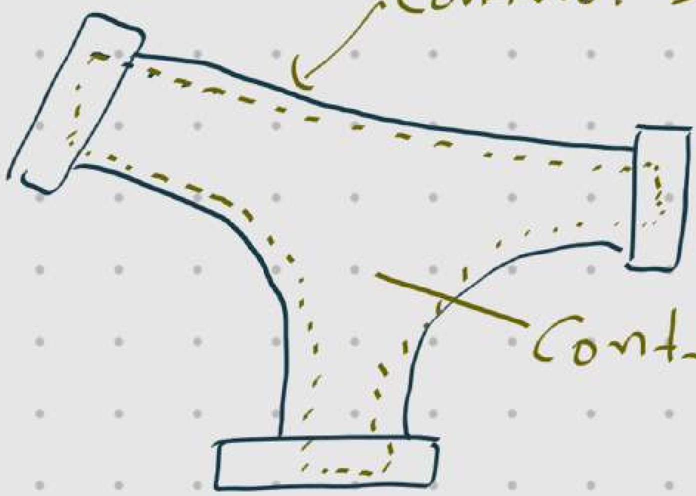
Continum Concept of Fluid

mathematical idealization of fluid as a continious distribution of matter

$$\rho = f(x, y, z, t)$$



System: fixed ammount of matter with a boundary



$dA = dx \cdot dy$

$dQ = v\,dA$

$Q = \int v\,dA$

Fluid Particle:

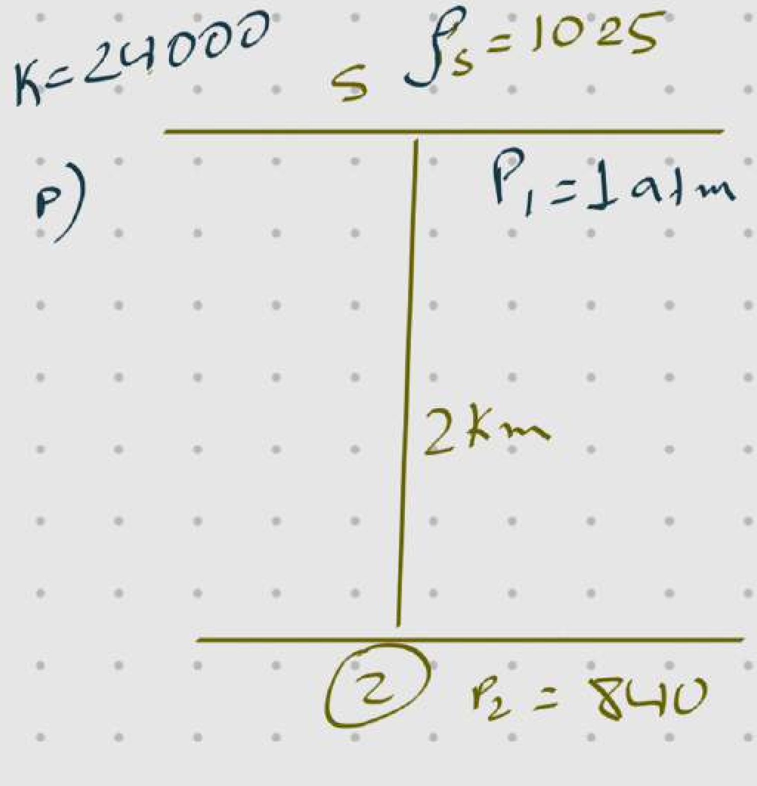
$\delta v \quad \delta m \quad \rho$

$\rho = \delta m / \delta v$

P1 at depth of 2 km in ocean the gauge pressure is 840 kg/cm² density and bulk modulus of elasticity of water at the surface $1025\ kgf/m^2$ and $24000\ kgf/cm^2$

a| Density of water at the depth 2Km

b| Specific volume and change of volume at that depth.



$P_2 = 840\ \frac{kg.f}{cm^2}$ (Gage P)

$\rho_s = 1025\ \frac{kg}{m2}$

$K = 24000\ \frac{kgf}{cm2}$

a| $\rho_2 = ?$

$\vartheta = \frac{1}{\rho}$ $\quad P_2 = 840 + P_{atm}$

$K = \frac{dP}{dv} = \frac{dP}{\frac{d\rho}{\rho}}$

$dP = P_2 - P_1$

$= 840 - P_1$

$$\Rightarrow 24000 \frac{kg.f}{cm^2} = \frac{840}{\frac{d\rho}{\rho_1}}$$

$= 840$

$\rho = \rho_1$

$d\rho = \rho_2 - \rho_1$

$=$

$$\frac{d\rho}{\rho_1} = 0.035$$

$d\rho = 0.035 \rho_1$

$\rho_2 - \rho_1 = 0.035 \rho_1$

$\rho_2 = 1.035 \rho_1$

$= 1.035 \times 1025$

$= 1060.80 \frac{kg}{m^2}$

Specific volume

$v_2 = \frac{1}{\rho_2} = 9.43 \times 10^{-4} \; m^3/kg$

$d\rho = 0.035 \rho_1$

$= 35.875 \; kg/m^2$

$$dv = \frac{1}{d\rho} = 0.0278 \; kg/m^2 \text{ (decreased)}$$

My name is Fluffy

Basic concept of Mathematics in F.M

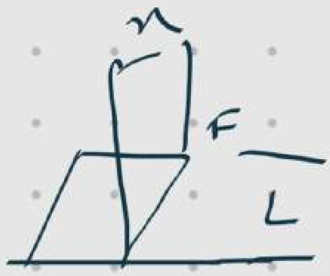
Normal stress $\sigma = P/A$

Shear Stress $\tau = F_s/A$

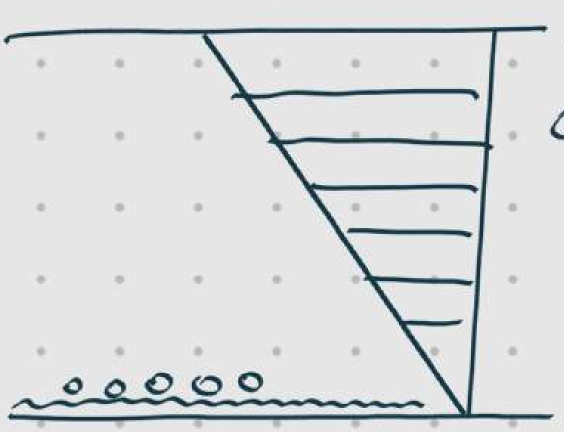
Deformation Rate "Change in length
per unit length
per unit time"

Deformation Rate $= \dfrac{dL}{dT}$

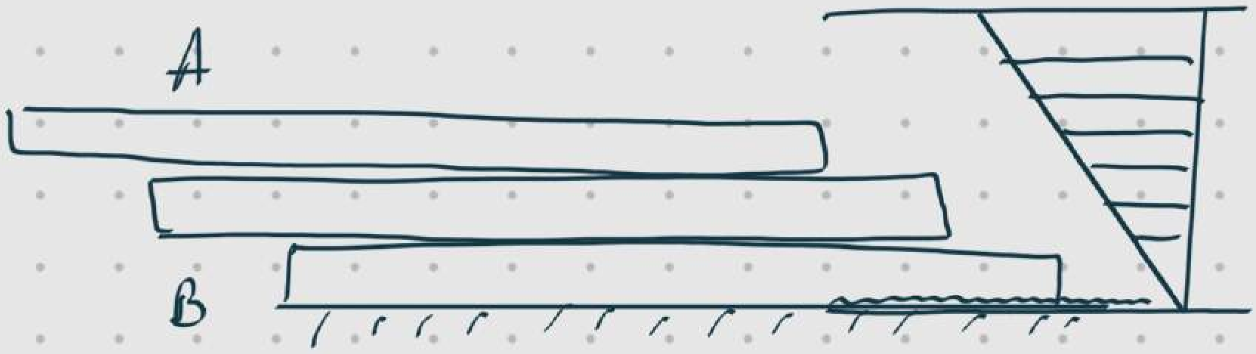
Strain Rate $= \dfrac{dL}{L} \times dT$



$\gamma = n/L$

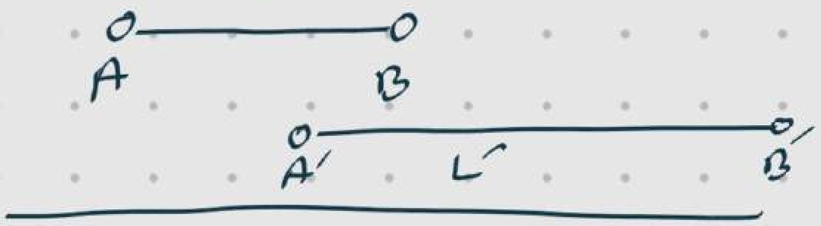


Upper fluid will flow more



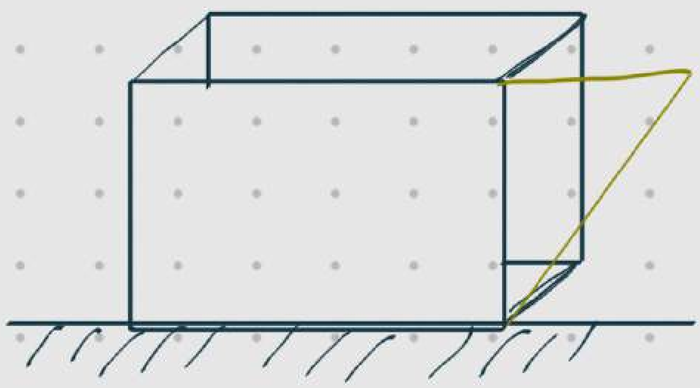
μ, ρ

$\Delta L = L' - L$

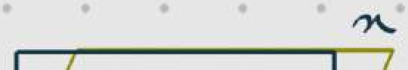


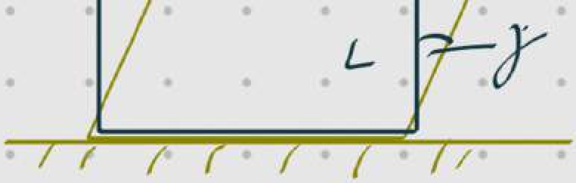
$$\varepsilon = \frac{\Delta L}{L}$$

$$\dot{\varepsilon} = \frac{\Delta L}{L} \Big/ \Delta t$$



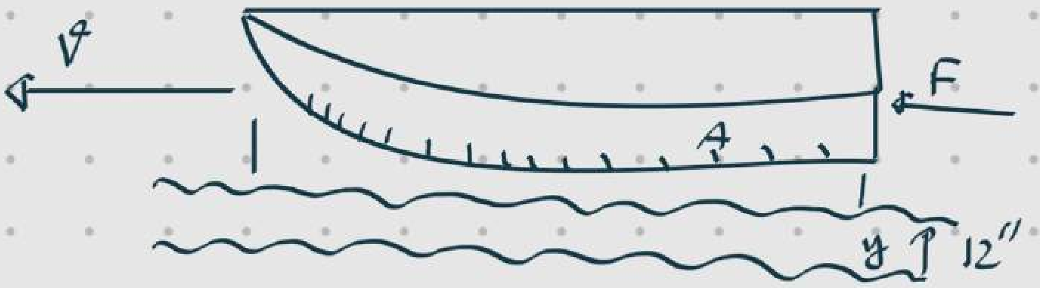
$$\tan \gamma = \frac{x}{L}$$





$$\gamma = \frac{x}{L}$$

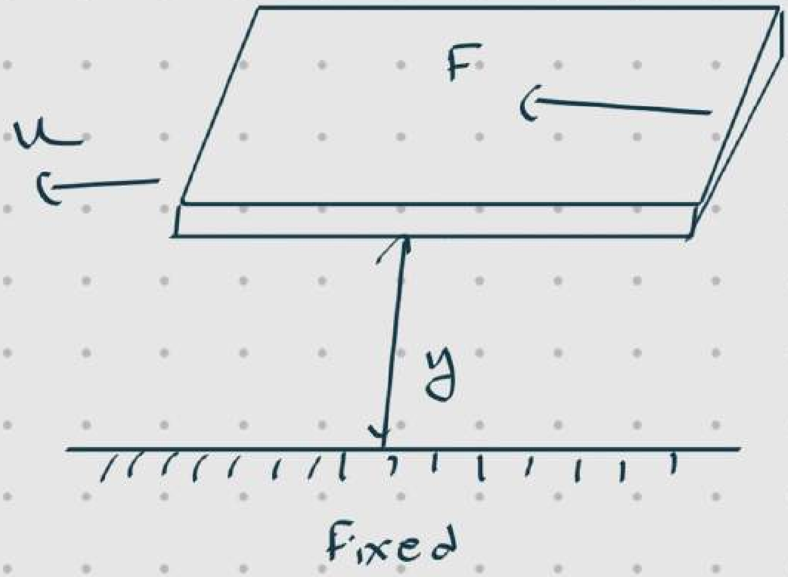
Why Resistance decreases when Depth decreases.



$F \propto \frac{1}{y}$

$F \propto A$

$F \propto V$



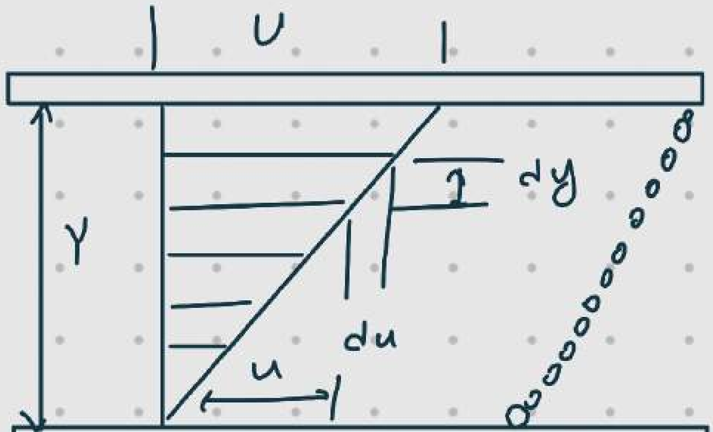
$F \propto \frac{Av}{y}$

$\frac{F}{A} \propto \frac{v}{y}$ Shear stress

$\tau \propto \frac{u}{y}$

$$\boxed{\tau = \mu \frac{u}{y} = \mu \frac{du}{dy}}$$

Newtons law of Viscosity



$$\frac{F}{A} \propto \frac{v}{y} \approx \frac{du}{dy}$$ $\frac{du}{dy}$ - Rate of strain

$$\frac{F}{A} \propto \frac{du}{dy} \quad \Big| \quad \tau = \mu \frac{du}{dy}$$

$$\tau \propto \frac{du}{dy}$$

$$\mu = \frac{\tau\, dy}{du} = \frac{\frac{N}{m^2} \times m}{m/s} = \frac{N.S}{m^2}$$

Problem 2: A cylindrical shaft of 95 mm Diameter rotates at a speed of 50 rpm inside a Cylinder of 96.4 mm diameter. Both shaft and Cylinder are 0.50 m long. If torque is 1 N-m to rotate shaft, find viscosity of oil.

$d_s = 95$ mm $= 0.095$ mm

$N_s = 50$ rpm

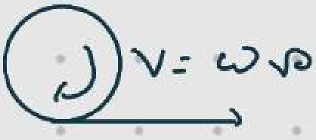
$$\omega_s = \frac{2\pi N_s}{60} = 5.24 \text{ rad/s}$$

Radial Distance bet^n shaft and cylinder

$$\Delta r = \frac{d_c}{2} - \frac{d_s}{2} = \frac{96.4}{2} - \frac{95}{2} = 0.7\text{mm}$$

Length of the shaft $L_s = 0.5$m

$T = 1\ N\cdot m$



$\mu_{oil} = ?$

$$\tau = \mu \frac{V}{Y} \Rightarrow \mu = \frac{\tau \cdot Y}{V} = \frac{\frac{F}{A} \times Y}{V}$$

A = Cylindrical Surface Area

$$= 2\pi r_s \cdot L = 2\pi \frac{d_s}{2} \cdot 0.5$$

$$= 0.15\ m^2$$

$Y = 0.7\ mm$

$= 0.7 \times 10^{-3}\ m$

$$U = \omega_s \times r_s = 5.24 \times \frac{d_s}{2} = 0.25\ m/s$$

$$T = F \cdot r_s \Rightarrow F = \frac{T}{r_s} = 21.05\ N$$

So, $$\mu = \frac{F/A \cdot Y}{U} = \frac{\frac{21.05}{0.15} \times 0.7 \times 10^{-3}}{0.25}\ \frac{Ns}{m^2}$$

$$= 0.393\ \frac{Ns}{m^2}$$

$$= 3.93\ \text{Poise}$$

☐ Newtonian and Non newtonian fluid

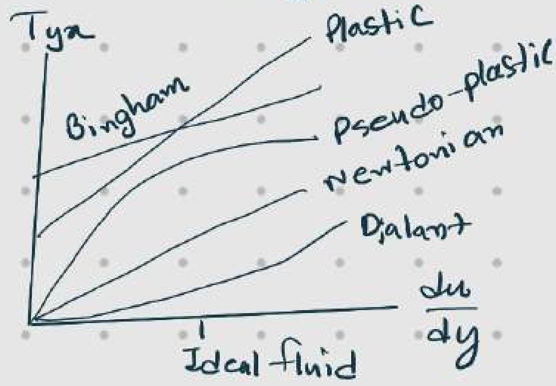
Ideal fluid - No viscosity

$$\tau = \mu \boxed{\frac{du}{dy}} - x$$

$$y = mx$$

Non newtonian

$$\tau = A + B\left(\frac{\delta u}{\delta y}\right)$$



If you know value of a function in one point you can know the value of the function of neighbouring point.

└ Application of Taylor

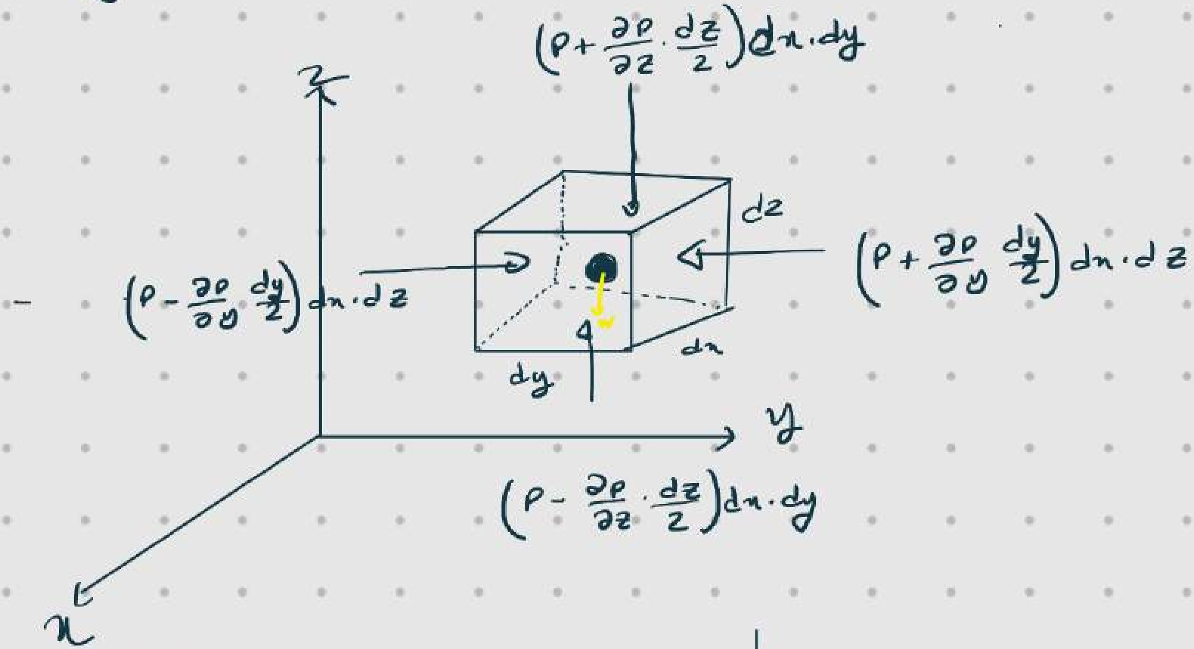
$y = x^3 + 3x^2 - 4x + 1$

$y(0) = 1$

$y(1) = 5$

$y(1.10) = ?$

Hydrostatic Pressure Apply



Apply

$$f(x+h) = f(x) + f'(x)\frac{h}{2!} \cdots$$

$$\xrightarrow{+} \sum F_y = 0$$

$$\Rightarrow \left(p - \frac{\partial p}{\partial y}\cdot\frac{dy}{2}\right) - \left(p + \frac{\partial p}{\partial y}\cdot\frac{dy}{2}\right) = 0$$

$f(x) = P \qquad P = ?$ $\qquad dx \cdot dz \qquad dx \cdot dz$

$y + \frac{dy}{2}$

$f(y+h) = f(y) + f'(y)\frac{h}{2!} + \cdots = \cancel{P dx dz} - \frac{\partial P}{\partial y} \cdot \frac{dy}{2} \cdot dx \cdot dz$

$- \cancel{P dx dz} - \frac{\partial P}{\partial y} \frac{dy}{2} \cdot dx dz = 0$

$$P\Big|_{y + \frac{dy}{2}} = P + \frac{\partial P}{\partial y} \cdot \frac{dy}{2}$$

$$\Rightarrow -\frac{1}{2}\frac{\partial P}{\partial y} dx\, dy\, dz - \frac{1}{2}\frac{\partial P}{\partial y} \cdot dx \cdot dy \cdot dz = 0$$

$$\Rightarrow \frac{\partial P}{\partial y} \cdot dx \cdot dy\, dz = 0$$

$$\frac{\partial P}{\partial y} \Delta V = 0 \qquad \Delta V = dx \cdot dy \cdot dz$$

$$\frac{\partial P}{\partial y} = 0 \, ; \qquad \Delta V \neq 0$$

$$\int \partial P = 0 \Rightarrow P = \text{Constant}$$

If we swim under 10 m water we will not feel any change in pressure.

$\overset{+}{\rightarrow} \sum F_x = 0$

$$\Rightarrow -\left(P + \frac{\partial P}{\partial x} \cdot \frac{dy}{2}\right) dy\, dz + \left(P - \frac{\partial P}{\partial x} \cdot \frac{dx}{2}\right) dy\, dz = 0$$

$$\Rightarrow \frac{\partial P}{\partial x} \Delta V = 0$$

$$\frac{\partial P}{\partial x} = 0 \Rightarrow P = \text{constant},$$

$+\uparrow \sum F_z = 0$

$$\left(P - \frac{\partial P}{\partial z} \cdot \frac{\partial z}{2}\right) dx\, dy - \left(P + \frac{\partial P}{\partial z} \cdot \frac{\partial z}{2}\right) dx\, dy$$

$= \left(P - \frac{\partial P}{\partial z}\cdot\frac{z}{2}\right) dx\,dy - \left(P + \frac{\partial P}{\partial z}\cdot\frac{z}{2}\right) dx\,dy$

$- \rho g\, dx\,dy\,dz = 0$

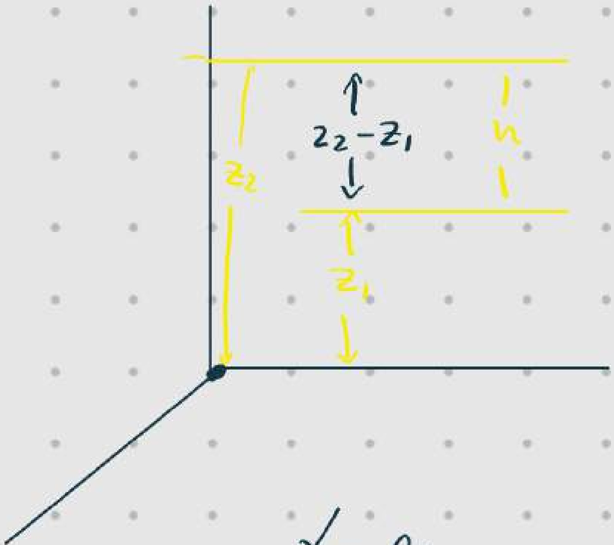
$\Rightarrow -\frac{1}{2}\frac{\partial P}{\partial z}\cdot\frac{\partial z}{2}\, dx\,dy\,dz - \frac{1}{2}\frac{\partial P}{\partial z} dx\,dy\,dz - \rho g\, dx\,dy\,dz = 0$

$\Rightarrow \left(-\frac{\partial P}{\partial z} - \rho g\right)\Delta V = 0 \Rightarrow \left(\frac{\partial P}{\partial z} + \rho g\right)\Delta V = 0$

$\Delta V \neq 0$ So,

$\frac{\partial P}{\partial z} + \rho g = 0 \Rightarrow \frac{\partial P}{\partial z} = -\rho g \Rightarrow \partial P = -\rho g\, dz$

$\int \partial P = -\int \rho g\, dz = \boxed{P = -\rho g z}$



$= \int_1^2 dP = -\rho g \int_1^2 dz$

$P_2 - P_1 = -\rho g (z_2 - z_1)$

$P_{atm} - P_1 = -\rho g h$

$P_1 = P_{atm} + \rho g h$ (absolute)

$\gamma = \rho g$

$\frac{N}{m^3}$

$\rho g h \rightarrow$ (gage Pressure)

$\boxed{P = \gamma \cdot h = \rho g h}$

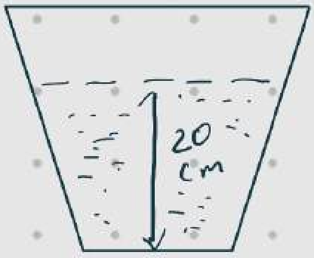
$= P$

$h = P/\gamma = P/\rho g$

$P = \rho_{oil} g h \Rightarrow 300 \frac{N}{m^2} = 2000 \frac{kg}{m^3} \times 9.81\ m/s^2 \times h_{oil}$

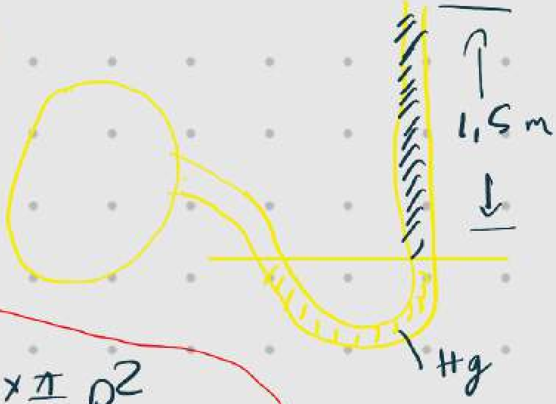
$\Rightarrow h_{oil} = 15\ m$

$P = h\rho g$



$1 - D - 1$

$P = \frac{F}{A} \Rightarrow F = PA$
$= \rho g h \times \frac{\pi}{4} D^2$



What is the force on the Bottom of the bucket.

D Hydrostatic Pressure of Compressible fluid

$$\frac{dP}{dz} = -\rho g = -\gamma \qquad P = \rho R T \qquad \rho = \frac{P}{RT}$$

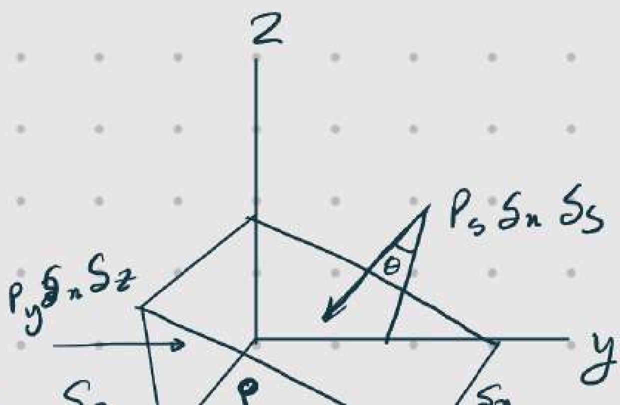
$$\frac{dP}{dz} = -\frac{P}{RT} g$$

For isothermal Condition

$$\int_1^2 \frac{dP}{dz} = -\frac{g}{RT} \int_{z_1}^{z_2} dz$$

$$\ln \frac{P_2}{P_1} = -\frac{g}{RT} \int_{z_1}^{z_2} \frac{dz}{T}$$

Fluid Pressure Same in all Direction – Pascal Law



$\overset{+}{\rightarrow} \sum F_y = 0$

$\Rightarrow \partial z \partial x \cdot P_y - \delta_x \delta_s \sin\theta \, P_s = 0$

$\sin\theta = \frac{\partial z}{\partial s}$

$\Rightarrow P_y \delta_x \delta_s \sin\theta - P_s \delta_x \delta_s \sin\theta = 0$

$\Rightarrow \quad P_y = P_s$

$\gamma \dfrac{\delta x\,\delta y\,\delta z}{2} \to 0$ small dimension

$$\sum F_z = P_z\,\delta x\,\delta y - P_s\,\delta x\,\delta s\cos\theta - \gamma\frac{\delta x\,\delta y\,\cancel{\delta z}}{\cancel{2}} = 0$$

$$\Rightarrow P_z\,\partial x\,\partial y - P_s\,\frac{\cos\theta\,\partial s\,\partial x}{\partial y} = 0$$

$$\Rightarrow P_z\,\partial x\,\cancel{\partial y} - P_s\,\cancel{\partial y}\,\cancel{\partial x} = 0$$

$$\Rightarrow \quad P_z = P_s$$

$\sigma\cos\theta(2\pi r) - w = 0$

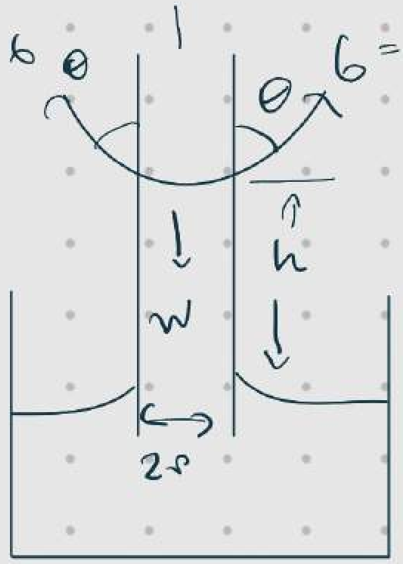
□ Capilarity Rise of the fluid

$A = \pi r^2 \qquad \gamma = \rho g$

$v = \pi r^2 h$

$w = \gamma \pi r^2 h$

$2\pi h \times \sigma\cos\theta$



$\sigma = \dfrac{F}{L} = \dfrac{N}{m}$

$\uparrow \sum F_y = -\gamma\pi r^2 h + \sigma\cos\theta(2\pi h) = 0$

$$h = \frac{2\sigma\cos\theta}{\gamma r}$$

$\theta = 0$

$r = 0.3\,mm \simeq 3\times10^{-4}\,m$

$\sigma = 0.073$

$$h = \frac{2\times\sigma\cos\theta}{\gamma r} = 2\times0.073$$

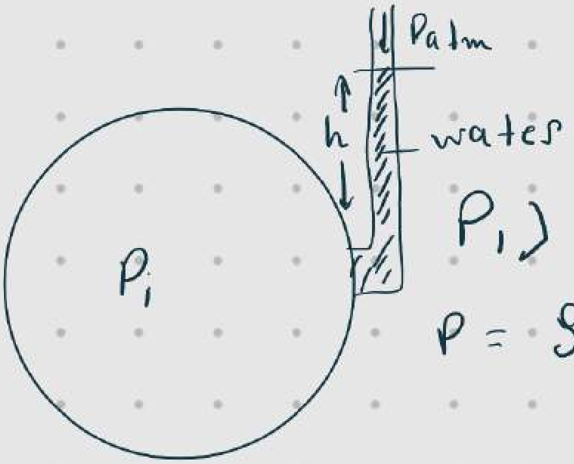
$$= \frac{2\times0.073\times\cos 0}{1000\times9.8\times3\times10^{-4}}$$

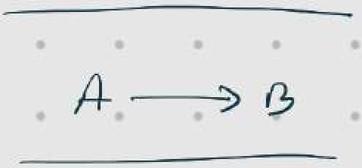
$=$

I love you ♡

Name Date

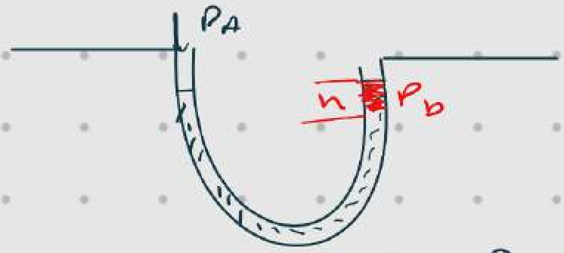
Shanto Ezba





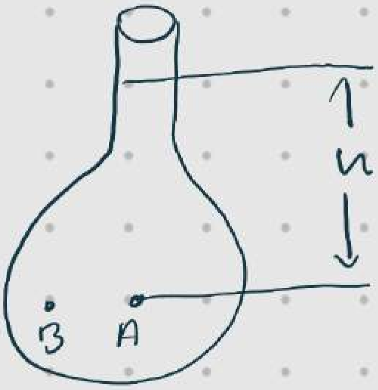
$P_1 > P_{atm}$

$P = \rho g h$

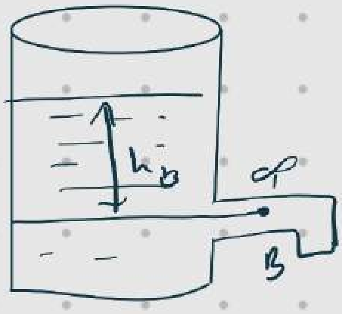


$P_A = P_B$

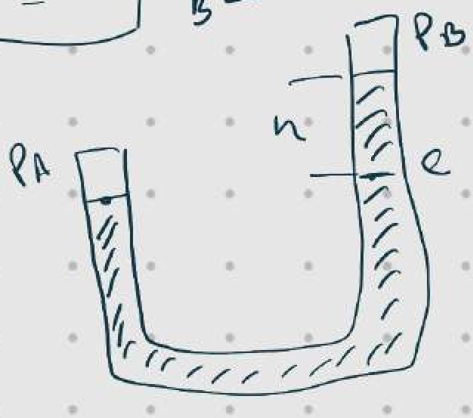
$P_A - P_B = \rho g h$



$P_A = P_B = \rho g h$

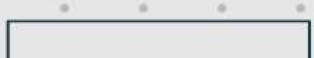


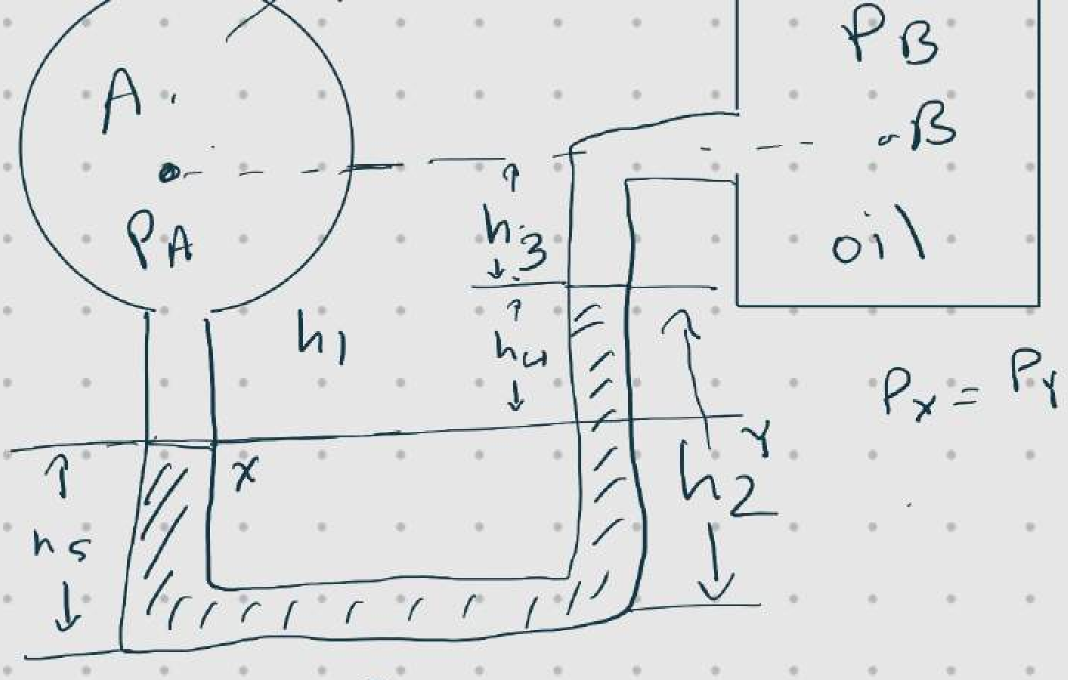
$P_B = \rho g h_B$



$P_B + h\rho g = P_A = P_C$

(Taylor)

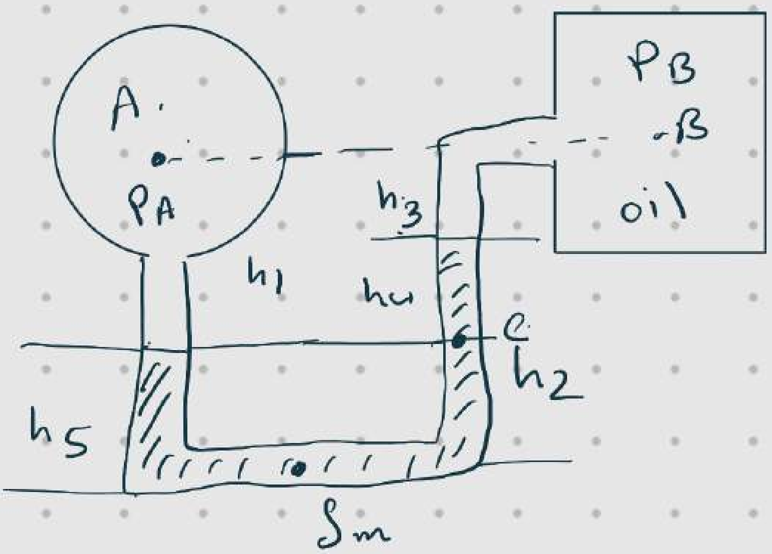




$P_x = P_Y$

$$P_A + h_1 \rho_w g = P_B + h_3 \rho_{oil} g + h_4 \rho_m g$$

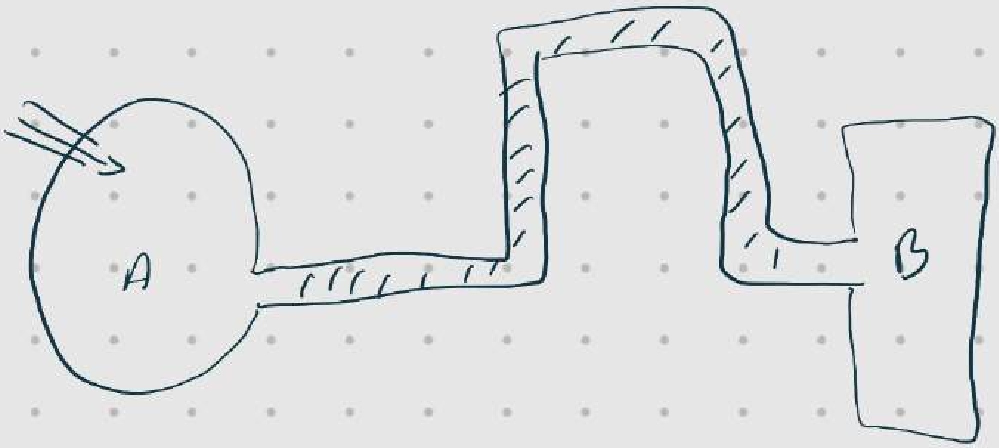
$$P_A = P_B + h_3 \rho_{oil} g + h_4 \rho_m g - h_1 \rho_w g$$



$$P_A + h_1 \rho_w g + h_3 \rho_m g - h_5 \rho_m g - h_4 \rho_m g - h_3 \rho_{oil} g = P_B$$

33600

$$P_A + h_1 \rho_w g - h_4 \rho_m g - h_3 \rho_{oil} g = P_B$$

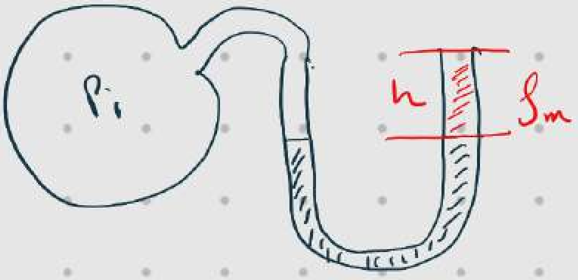


$$\frac{P}{\gamma} + \frac{v^2}{2g} + z_1 = C$$

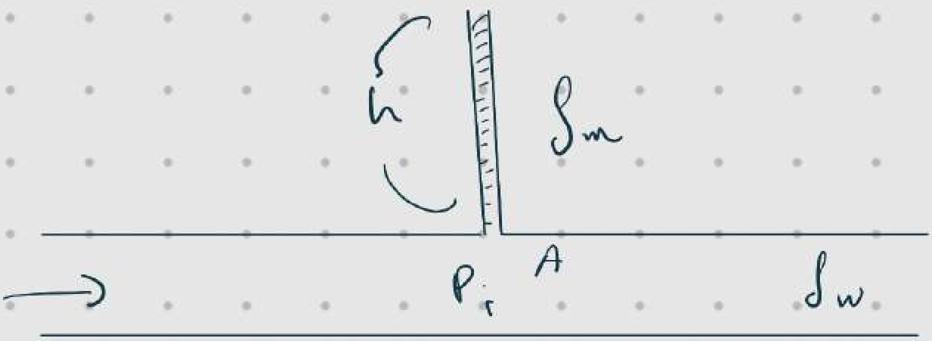
FM-3105

24-12-24

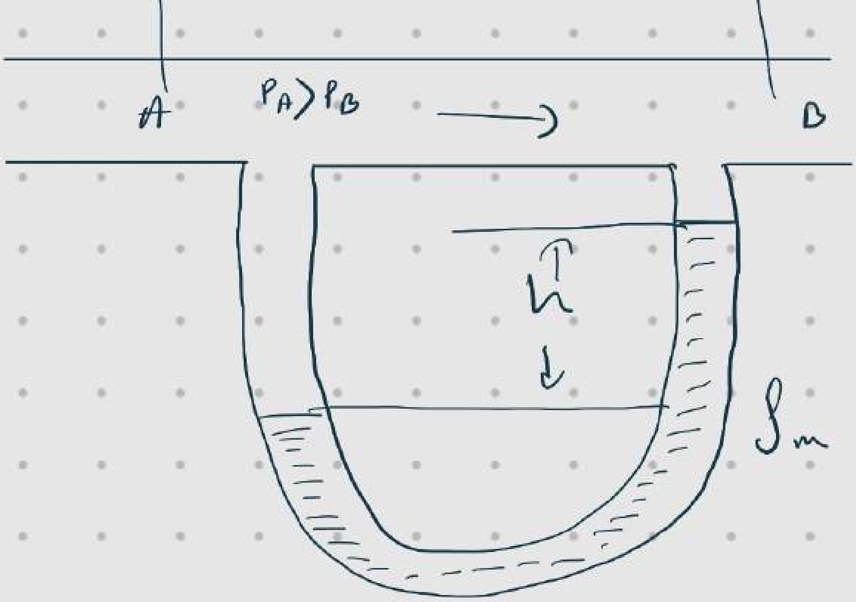
Measurement of Preasure

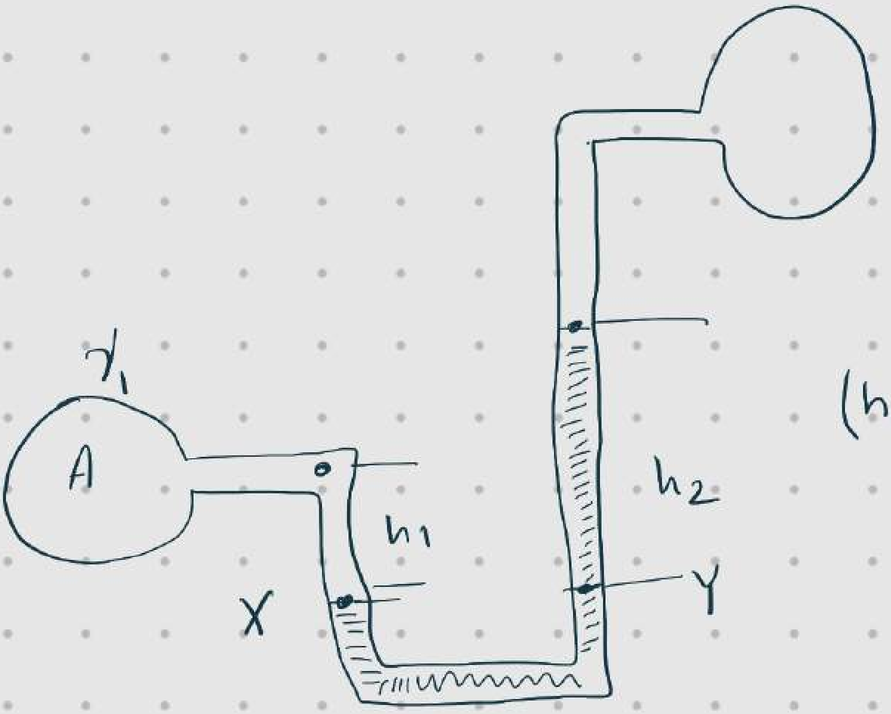


$p = h\rho_m g$



$$\rho_m > \rho_w$$





$$(h_2 - h_1)\rho_2 g = \Delta P_{AB}$$

$$P_X = P_Y$$

$$P_X = P_A + h_1 \rho_1 g \quad \Big| \quad P_Y = P_B + h_3 \rho_3 g + h_2 \rho_2 g$$

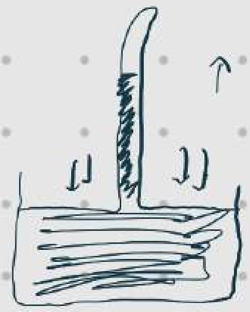
$$P_A + h_1 \rho_1 g = P_B + h_3 \rho_3 g + h_2 \rho_2 g$$

$$P_A - P_B = h_3 \rho_3 g + h_2 \rho_2 g - h_1 \rho_1 g$$

again

$$P_A + h_1 \rho g - h_2 \rho_2 g - h_3 \rho_2 g = P_B$$

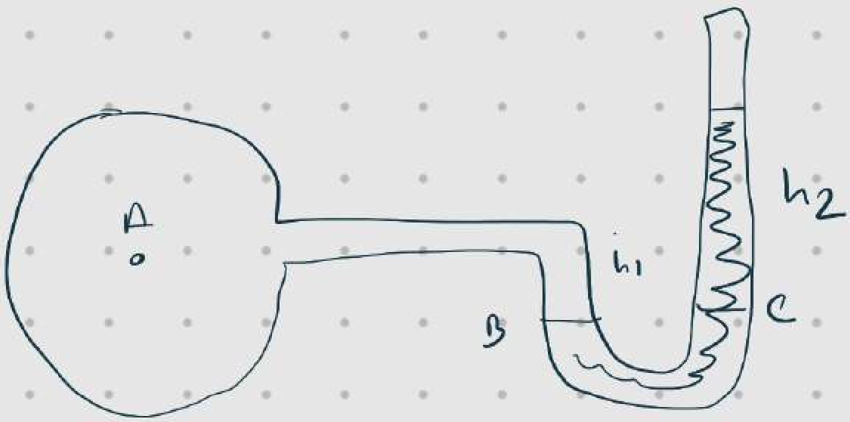
□ Measurement of Atmospheric preasure



$\underline{101 \, kPa} = N/m^2$

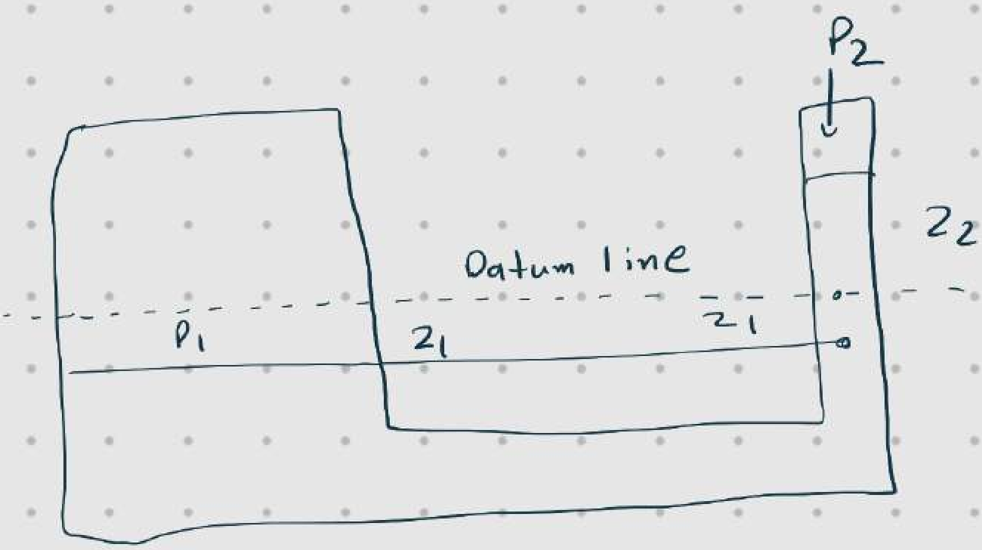
$P_{atm} = h \rho_w g$

$h_m = 20 cm$



$P_B = P_C$

$P_A + h_1 \rho g = P_{atm} + h_2 \rho_m g$



$\frac{\pi}{4} D^2 z_1 = \frac{\pi}{4} d^2 z_2$

$$z_1 = \left(\frac{d}{D}\right)^2 z_2$$

$$P_1 - z_1 \rho g - z_2 \rho g = P_2$$

$$P_1 - P_2 = \rho g \left[z_1 + z_2\right]$$

$$P_1 - P_2 = \rho g \left[\left(\frac{d}{D}\right)^2 z_2 + z_2\right]$$

$$= \rho g z_2 \left[\left(\frac{d}{D}\right)^2 + 1\right]$$

$$= \rho g z_2$$

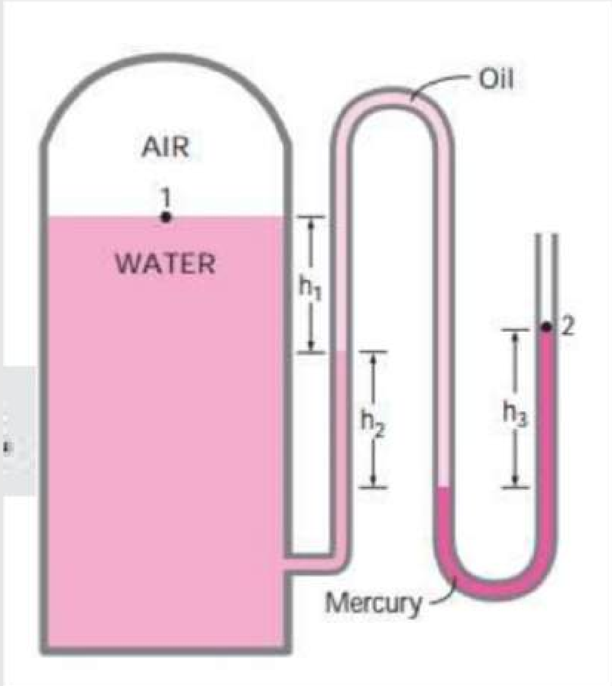
when $d <<<<<< D$

$$\frac{d}{D} <<<< 1$$

$$\frac{d}{D} \approx 0$$

The water in a tank is pressurized by air, and the pressure is measured by a multifluid manometer as shown in Fig . The tank is located on a mountain at an altitude of 1400 m where the atmospheric pressure is 85.6 kPa. Determine the air pressure in the tank if $h_1 = 0.1$ m, $h_2 = 0.2$ m, and $h_3 = 0.35$ m. Take the densities of water, oil, and mercury to be 1000 kg/m³, 850 kg/m³, and 13,600 kg/m³, respectively.

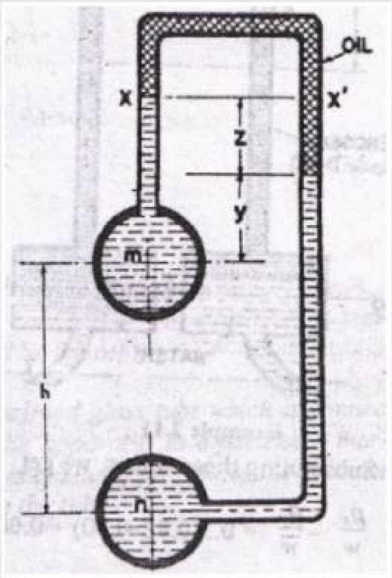
$\rho_{oil} = 850$



$$P_1 + h_1 \rho_w g + h_2 \rho_{oil} g - h_3 \rho_m g = P_{atm}$$

$$P_1 = P_{atm} - h_1 \rho_w g - h_2 \rho_{oil} g - h_3 \rho_m g$$

Water fills the vessels shown in the figure below. Specific gravity of manometric liquid is 0.9 ; (a). Find the difference in pressure intensity at m and n when h = 1.25 m and z = 0.3 m; (b). Instead of water mercury in the vessel and manometric liquid has specific gravity of 1.6; find in the pressure intensity at m and n if h = 0.6 m and z = 1.0 m.

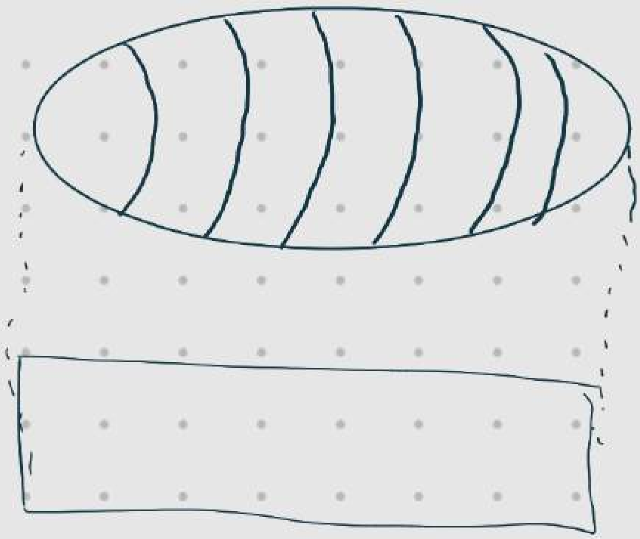


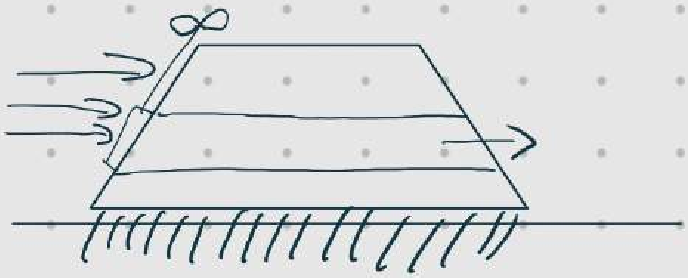
$$P_A + 0.12\,\rho_w g - 1.35\,\rho_w g = P_B$$

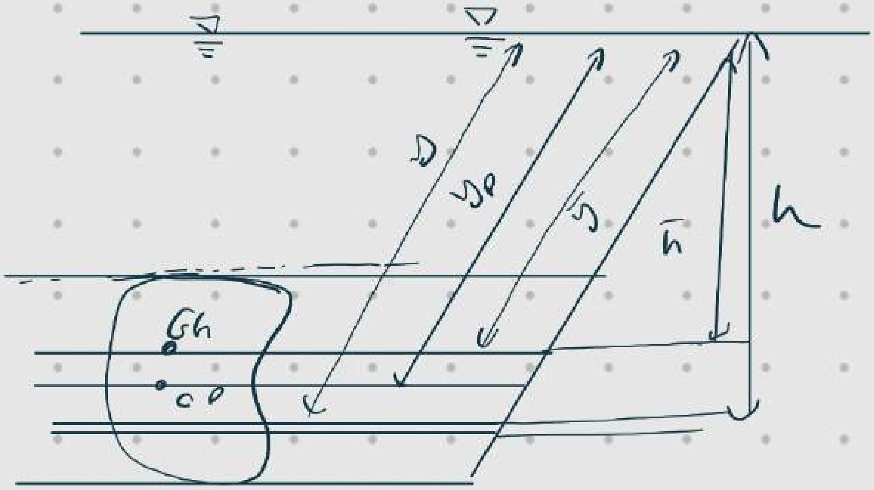
Nei Negga Nei Nei
Negga Negga Nei, Nei
Nei Negga Nei Nei
Negga Negga Nei, Nei

☐ Hydrostatic Forces on Surface

$P_{atm} = 101325\ Pa$







$$\sin\theta = \frac{\bar{h}}{\bar{y}} \qquad \sin\theta = \frac{h_p}{y_p} \qquad \sin\theta = \frac{h}{y}$$

$$F = PA$$

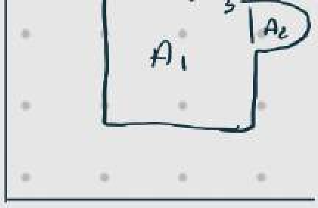
$$dF = P\,dA$$

$$F = \int_A P\,dA = \int_A \rho g h\,dA = \int_A \rho g y \sin\theta\,dA$$

$$F = \rho g \sin\theta \int y\,dA$$

$$= \rho g \sin\theta\, A\bar{y}$$

$= \gamma A h$



$$\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3}{A_1 + A_2 + A_3}$$

$$F y_p = \int_A y \, dF$$

$$dF = P \cdot dA$$
$$= \rho g h \cdot dA \qquad \Big| \sin\theta = \frac{h}{y}$$
$$= \rho g \sin\theta \, y \, dA$$

$$\bar{y} = \frac{\int y \, dA}{A}$$

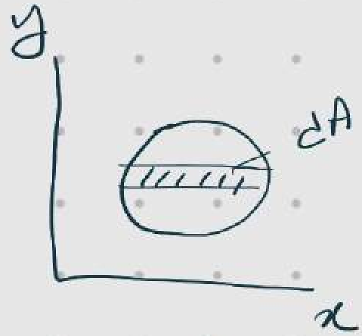
$$\int y \, dA = A\bar{y}$$

$$F = \rho g \bar{h} A$$
$$= \rho g \sin\theta \, \bar{y} A$$

$$\int_A y \, dF = \int_A y \rho g \sin\theta \, y \, dA = \rho g \sin\theta \int y^2 dA$$

$$\cancel{\rho g \sin\theta} \, \bar{y} A y_p = \cancel{\rho g \sin\theta} \int y^2 dA$$

$$y_p = \frac{\int_A y^2 dA}{A\bar{y}} = \frac{I_o + A\bar{y}^2}{A\bar{y}}$$

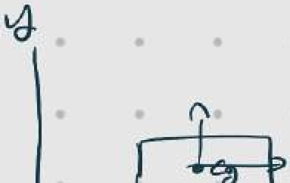


$$\boxed{I_x = \int_A y^2 dA = I_o + A\bar{y}^2}$$

$$(y \cdot dA) \, y$$

$$I = \int y^2 dA$$

$$y_p = \bar{y} + \frac{I_o}{A\bar{y}}$$



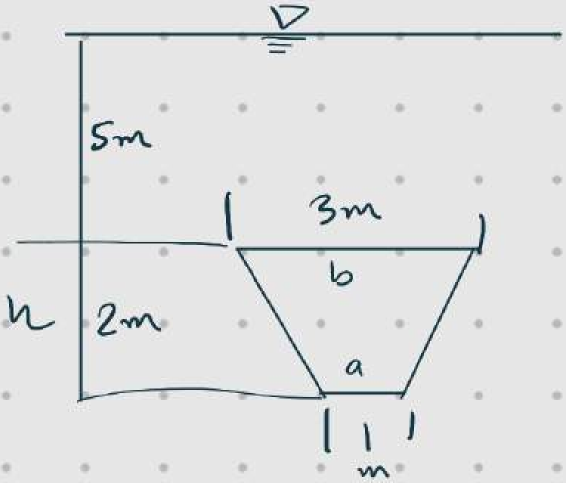
$I_{xcg} = \frac{1}{2} bh^3 = I_o$

$I_{ycg} = \frac{1}{12} hb^3$

$I_x = I_o + Ad^2$

Problem-1

$I_x = I_o + A\bar{y}^2 \qquad y_p = \frac{I_o + A\bar{y}^2}{A\bar{y}}$



$I_o = (y_p - \bar{y}) A\bar{y} \qquad = \frac{I_o}{A\bar{y}} + \bar{y}$

$F = \gamma \sin\theta A\bar{y} = \gamma \bar{h} A$

$\theta = 90^\circ \quad \rho = 1000 \text{ kg/m3} \qquad A = ?$

C.g (location) = ?

$(\bar{y}, \bar{h}) = ?$

$A = \frac{1}{2}(a+b) \times h$

$= 0.5 \times (1+3) \times 2 = 4 \text{ m}^2$

$y_{cg} = \frac{h(2a+b)}{3(a+b)} = \frac{2(2\times 1+3)}{3 \cdot (1+3)} = 0.833$

$I_o =$

$P_2 = \rho g z_2 = \gamma z_2$

$P_1 = \rho g z_1 = \gamma z_1$

$P_1 = -\rho g z_1 = -\gamma z_1$

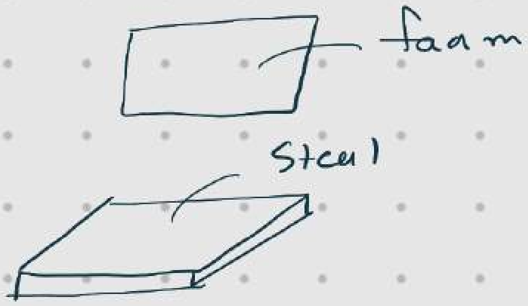
$$F_B = \int_{body} (P_2 - P_1)\, dA_H$$

$$= -\gamma \int (z_2 - z_1)\, dA_H = -\gamma \cdot V = \text{weight of the fluid Displaced by the object.}$$

□ Condition of Floating Body

$\rho_b < \rho_f \rightarrow$ float

$\rho_b > \rho_f \rightarrow$ sink



$W_b = \rho_b V_T g$

$F_B = \rho_f g V_{sub}$

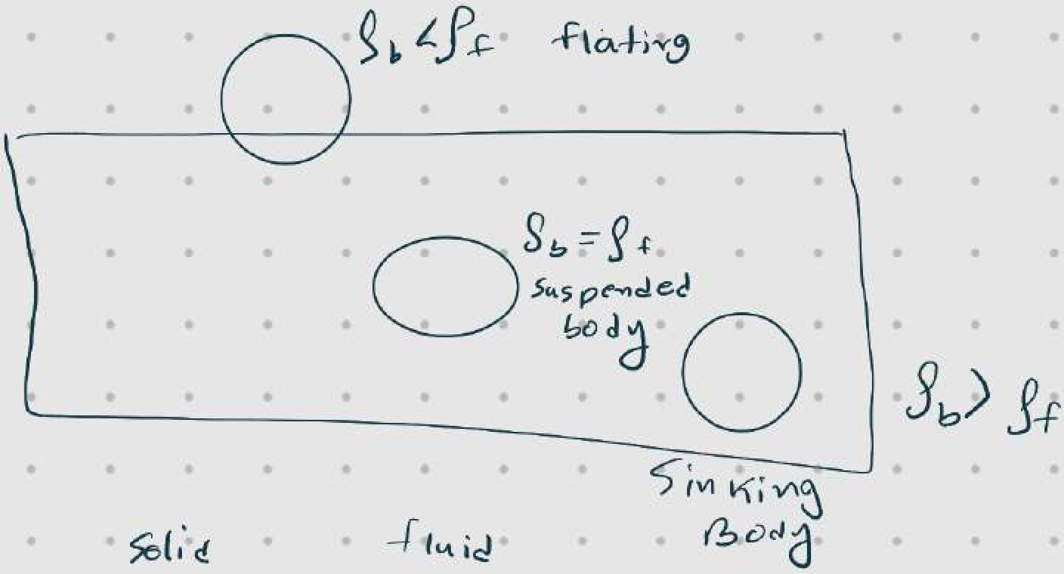
If floats

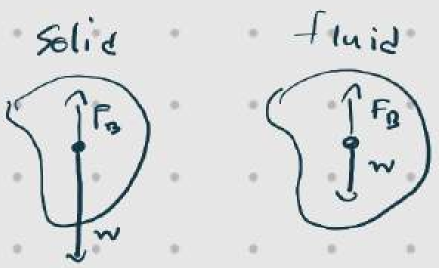
$\rho_b V_T g = \rho_f g V_{sub}$

$$\frac{\rho_b}{\rho_f} = \frac{V_{sub}}{V_T} \quad \Big| \quad \rho_b < \rho_f$$

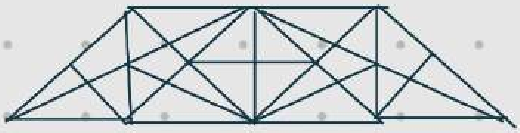
$$\frac{\rho_b}{\rho_f} < 1$$

ρ_f





C_g and F_B maybe of may not work in same point based on size of object

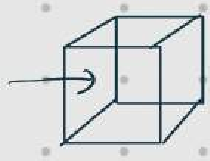


$\boxed{\text{Shanto}}$

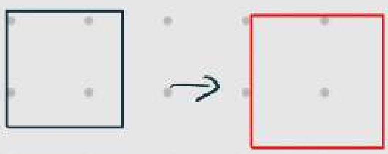
Fluid Kinematics

☐ Fundamentals of Fluid Flow or Fluid Kinematics

Front Side view of Fluid element

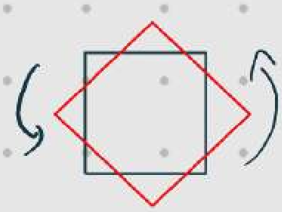


a)

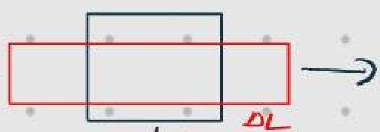


- Translation
- Rotation
- Linear strain
- Shear Strain

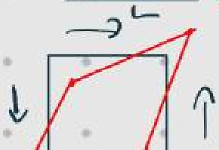
b)



c)



d)



Velocity → rate of translation

angular

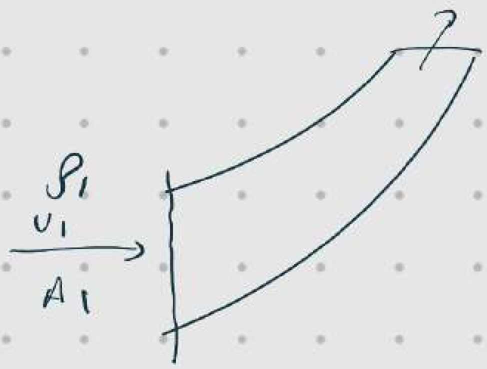
velocity → rate of rotation

$\dot{\varepsilon} = \frac{\Delta L}{L} / t$

linear strain → rate of linear strain

Shear strain → rate of shear strain

Continuity Equation For one Dimensional Flow



$Q = AV \quad m^3/s$

$\dot{m} = Q \cdot \rho = \frac{m^3}{s} \frac{kg}{m^3}$

$= kg/s$

$\dot{m}_1 = \dot{m}_2$

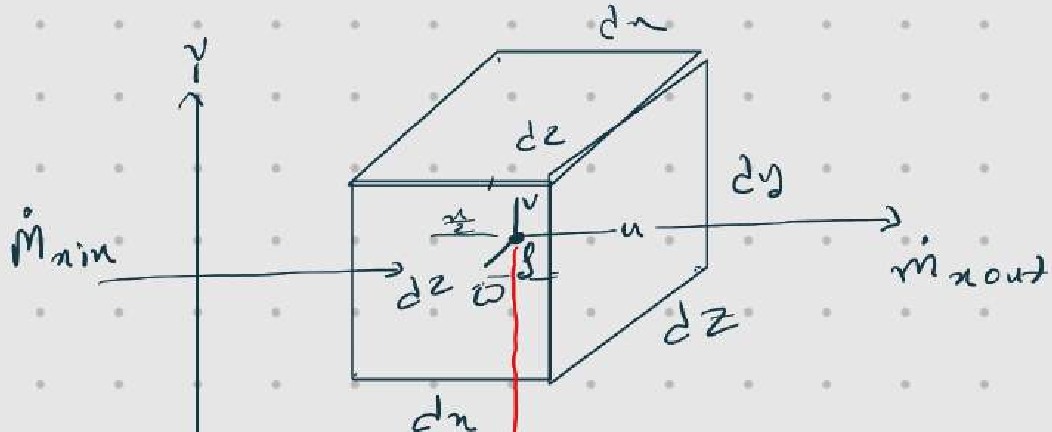
Continuity Equation $\quad \rho_1 v_1 A_1 = \rho_2 v_2 A_2$ | If $\rho_1 = \rho_2$

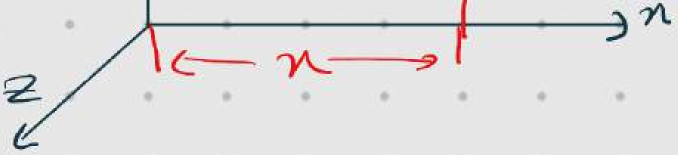
for incompressible Flow $\quad Q = v_1 A_1 = v_2 A_2$

□ Continuity Equation For Three Dimensional Compressible Flow

Consider infinitesimal control volume dx, dy, dz

Final Exam





$$\dot{m} = \rho v A = \rho_{in} v_{in}\, dy\, dz = \dot{m}_{x\,in}$$

$$P = \rho g h$$

$$f(x+h) = f(x) + f'(x)\frac{h}{1!} + \cdots$$

$$\rho_{x-\frac{dx}{2}} = \rho_x + \frac{\partial \rho}{\partial x}\left(-\frac{dx}{2}\right)$$

$\dot{m}_{x\,in}$

$$U_{x-\frac{dx}{2}} = U_x + \frac{\partial u}{\partial x}\left(-\frac{dx}{2}\right)$$

$$\dot{m}_{x\,in} = \rho_{x-\frac{dx}{2}}\, U_{x-\frac{dx}{2}}\, dy\, dz$$

$$= \left(\rho_x - \frac{\partial \rho}{\partial x}\frac{dx}{2}\right) \times \left(U_x - \frac{\partial u}{\partial x}\frac{dx}{2}\right) dy\, dz$$

$$= \left[\rho U - \rho\frac{\partial U}{\partial x}\frac{dx}{2} - U\frac{\partial \rho}{\partial x}\frac{dx}{2} + \cancelto{0}{\frac{\partial \rho}{\partial x}\frac{\partial u}{\partial x}\left(\frac{dx}{2}\right)^2}\right] \times dy\, dz$$

$$= \left[\rho U - \rho\frac{\partial U}{\partial x}\frac{dx}{2} - U\frac{\partial \rho}{\partial x}\frac{dx}{2}\right] dy\, dz$$

$$\dot{m}_{x\,out} = \rho_{x+\frac{dx}{2}}\, U_{x+\frac{dx}{2}}\, dy \cdot dz$$

$$= \left(\rho_x + \frac{\partial \rho}{\partial x}\frac{dx}{2}\right) \times \left(U + \frac{\partial u}{\partial x}\frac{dx}{2}\right) \times dy\, dz$$

$$= \left[\rho U + \rho\frac{\partial u}{\partial x}\frac{dx}{2} + U\frac{\partial \rho}{\partial x}\frac{dx}{2}\right] dy\, dz$$

$$\dot{m}_{x\,net} = \dot{m}_{x\,in} - \dot{m}_{x\,out}$$

$$= \left[\rho u - \rho \frac{\partial u}{\partial x}\frac{dx}{2} - u\frac{\partial \rho}{\partial x}\frac{dx}{2}\right] dy\,dz - \left[\rho u + \rho \frac{\partial u}{\partial x}\frac{dx}{2} + u\frac{\partial \rho}{\partial x}\cdot\frac{dx}{2}\right] dy\,dz$$

$$\dot{m}_{x\,net} = -\left[\rho \frac{\partial u}{\partial x} dx\,dy\,dz + u\frac{\partial \rho}{\partial x} dx\,dy\,dz\right]$$

$$= -\left[\rho \frac{\partial u}{\partial x} + u\frac{\partial \rho}{\partial x}\right] dx\,dy\,dz$$

$$= -\frac{\partial}{\partial x}(\rho u)\, dx\,dy\,dz$$

In y direction

$$\dot{m}_{y\,net} = -\frac{\partial}{\partial y}(\rho v)\,dx\,dy\,dz$$

$$\dot{m}_{z\,net} = -\frac{\partial}{\partial z}(\rho v)\,dx\,dy\,dz$$

mass changes inside the Control Volume

$$\dot{m}_{change} = \frac{\partial}{\partial t}(\rho\, dx\,dy\,dz) = \frac{\partial \rho}{\partial t}\cdot dx\,dy\,dz$$

So,

$$\dot{m}_{x\,net} + \dot{m}_{y\,net} + \dot{m}_{z\,net} = \dot{m}_{change}$$

$$-dx\,dy\,dz\left(\frac{\partial}{\partial x}\rho u + \frac{\partial}{\partial y}\rho v + \frac{\partial}{\partial z}\rho w\right) = \frac{\partial \rho}{\partial t}\,dx\,dy\,dz$$

$$\boxed{\frac{\partial}{\partial x}\rho u + \frac{\partial}{\partial y}\rho v + \frac{\partial}{\partial z}\rho w + \frac{\partial \rho}{t} = 0}$$

- Continuity Equation for Compressible Three Dimensional Unsteady flow

(i) For steady flow

$$\frac{\partial \rho}{\partial t} = 0$$

$$\boxed{\frac{\partial}{\partial x}\rho u + \frac{\partial}{\partial y}\rho v + \frac{\partial}{\partial z}\rho w = 0}$$

(ii) For incompressible flow $\rho = \text{constant}$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$U = u(x, y, z, t)$$
$$V = v(x, y, z, t)$$
$$W = w(x, y, z, t)$$

Lec-6

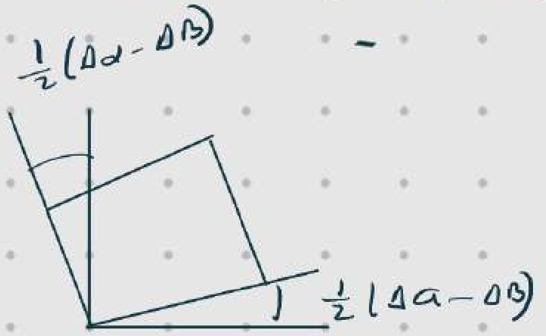
☐ Movement or kinematics of fluid Particle

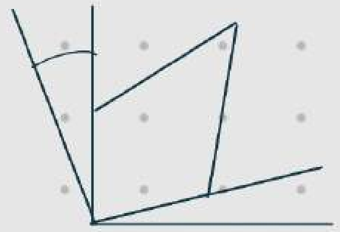
Fluid element in a flow and undergo with one of the three type of motion

→ Pure o Irrotational Translation

→ Pure rotation or Rotational Translation

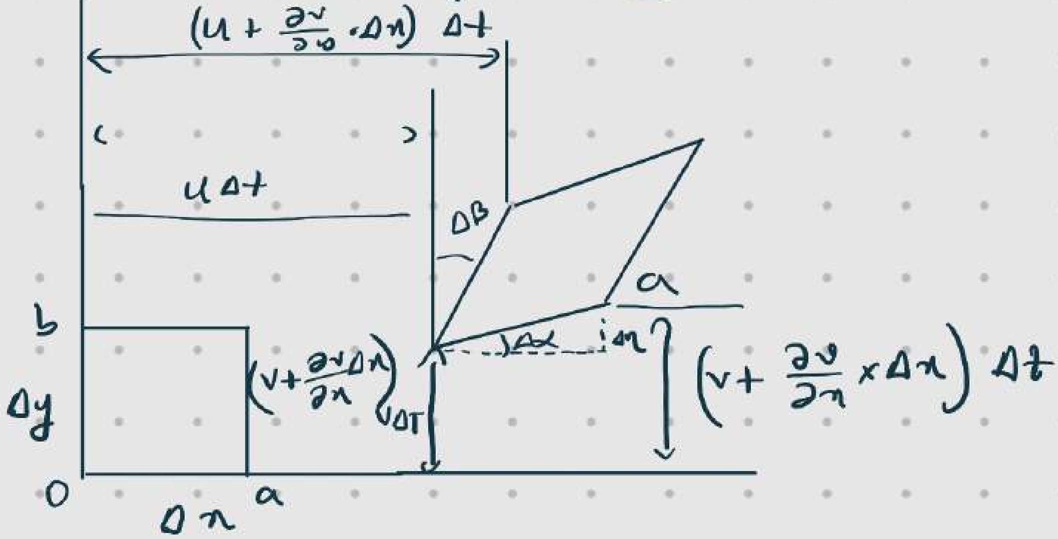
→ Pure Distortion or Deformation





Rotational OR Angular Deformation Rate of Fluid

$$\vec{\omega} = \hat{i}\,\omega_x + \hat{j}\,\omega_y + \hat{k}\,\omega_z$$



$$\Delta\eta = \left(v + \frac{\partial v}{\partial x}\Delta x\right)\Delta T - v\Delta T$$

$$= \frac{\partial v}{\partial x}\Delta x\,\Delta T$$

$$\Rightarrow \frac{\Delta\eta}{\Delta x} = \frac{\partial v}{\partial x}\cdot\Delta T$$

$$\Rightarrow \tan(\Delta\alpha) = \frac{\partial v}{\partial x}\Delta T$$

$$\Delta\alpha = \frac{\partial v}{\partial x}\Delta T$$

$$\omega_{OA} = \lim_{\Delta t \to 0}\left[\frac{\frac{\partial v}{\partial x}\cdot\Delta T}{\Delta T}\right] = \frac{\partial v}{\partial x}$$

$$\Delta \varepsilon = \left(u + \frac{\partial u}{\partial y}\cdot\Delta y\right)\Delta t - v\,\Delta t$$

$$= \frac{\partial u}{\partial y}\cdot\Delta Y\cdot\Delta t$$

$$\frac{\Delta \varepsilon}{\Delta Y} = \frac{\partial u}{\partial y} \Rightarrow \tan\Delta\beta = \frac{\partial u}{\partial y}\Delta t$$

$$\Delta\beta = \frac{\partial u}{\partial y}\Delta t$$

$$\omega_{OB} = \lim_{\Delta t \to 0}\left[\frac{\frac{\partial u}{\partial y}\cdot\Delta t}{\Delta t}\right] = \frac{\partial u}{\partial y}$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

$$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

$$\omega = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} + \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\vec{j}$$

$$+ \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{K}$$

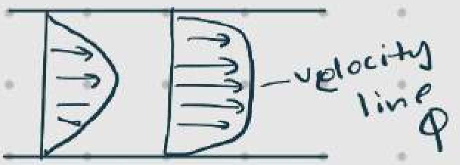
$$\vec{V} = u\,\vec{i} + v\,\vec{j} + w\,\vec{K}$$

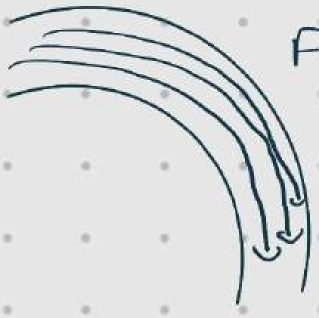
$$\vec{\nabla} = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\hat{k}$$

$$\vec{\nabla} \times \vec{v} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

$$\omega = \frac{1}{2}\left(\vec{\nabla} \times \vec{v}\right) = \frac{1}{2}\left(\mathrm{Curl}\, \vec{v}\right)$$

Vorticity $\xi = 2\vec{\omega} = \vec{\nabla} \times \vec{v}$





Flow rate

Stream line

ψ

$$-\frac{\partial \phi}{\partial z} = w$$

$$-\frac{\partial \phi}{\partial x} = u$$

$$-\frac{\partial \phi}{\partial y} = v$$

Continuity for incompressible

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial y}\right) + \frac{\partial}{\partial z}\left(-\frac{\partial \phi}{\partial z}\right) = 0$$

$$\frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial y}\right) + \frac{\partial}{\partial z}\left(-\frac{\partial \phi}{\partial z}\right) = 0$$

laplace equation

$$\boxed{\Rightarrow \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0}$$

Irrotational flow

Potential Flow

Stream Function

$\psi = f(x,y) \rightarrow$ Steady flow

$\psi = f(x,y,z,t) \rightarrow$ unstead "

$$-\frac{\partial \psi}{\partial x} = v$$

$$\frac{\partial \psi}{\partial y} = u$$

Continuity for 2D incompressible

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\Rightarrow \frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial x}\right) = 0$$

$$0 = 0$$

Relationship between ψ and ϕ

$$\psi = \psi(x,y)$$

$$d\psi = \frac{\partial \psi}{\partial x} \cdot dx + \frac{\partial \psi}{\partial y} \cdot dy$$

$$0 = -v\,dx + u \cdot dy$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{\psi = cons} = \frac{v}{u}$$

$$\phi = \phi(x,y)$$

$$d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy$$

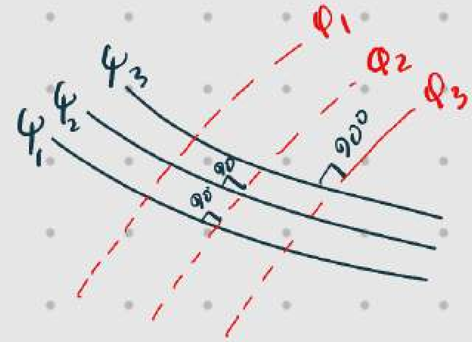
$$0 = -u\,dx - v\,dy$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{\phi = cons} = -\frac{u}{v}$$

$$\left.\frac{dy}{dx}\right|_{\phi = cons} \times \left.\frac{dy}{dx}\right|_{\psi = const} = -1$$

$$m_1 \times m_2 = -1$$

Flow net :



The velocity component

$$u = \frac{y^3}{3} + 2x - x^2 y$$

$$v = xy^2 - 2y - \frac{x^3}{3}$$

Show the flow is

a) Incompressible and irrotational

b) Obtain expression of φ

c) " " of ϕ

a) $u, v = f(x, y, z, t)$

Continuity equation

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) + \cancelto{0}{\frac{\partial \rho}{\partial t}} = 0$$

Incompressible

$$\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(\frac{y^3}{3} + 2x - x^2 y\right) = 2 - 2xy$$

$$\frac{\partial v}{\partial y} = \frac{\partial}{\partial y}\left(xy^2 - 2y - \frac{x^3}{3}\right) = y^2 - x^2$$

$$\frac{\partial v}{\partial x} = y^2 - x^2$$

$$\frac{\partial u}{\partial y} = y^2 - x^2$$

$$\Rightarrow 2 - 2xy + 2xy - 2 = 0 \Rightarrow 0 = 0$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0 \quad \text{irrotational}$$

b) $\psi \rightarrow$

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x}$$

$$y^3/3 + 2x - x^2 y = \frac{\partial \psi}{\partial y}$$

$$\Rightarrow \partial \psi = \left(\frac{1}{3} y^3 + 2x - x^2 y\right)\partial y$$

$$\psi = \frac{1}{12}y^4 + 2xy - \frac{1}{2}x^2y^2 + f(x)$$

$$-\frac{\partial \psi}{\partial x} = v$$

$$-(\cancel{2}y - \cancel{x}y^2 - f'(x)) = \cancel{x}y^2 - \cancel{2}y - \frac{x^3}{3}$$

$$f'(x) = \frac{x^3}{3} \Rightarrow f(x) = \frac{1}{12}x^4 + C$$

$$\psi = \frac{1}{12}y^4 + \frac{1}{12}x^4 + 2xy - \frac{1}{2}x^2y^2 + C$$

c) $u = -\frac{\partial \phi}{\partial x}$ $\quad v = -\frac{\partial \phi}{\partial y}$

$$\partial\phi = -u\,dx = (-y^3/3 - 2x + x^2y\ \partial x$$

$$\phi = -\frac{1}{3}y^3 x - x^2 + \frac{1}{3}x^3y$$

$$\frac{\partial \phi}{\partial y} = -xy^2 + \frac{1}{3}x^3 + f'(y)$$

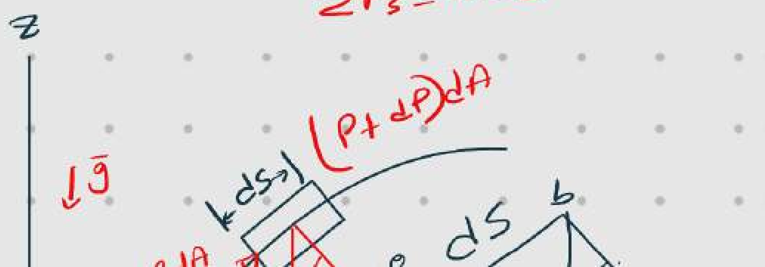
$\boxed{}$ Bernoullis equation

$\sin\theta = \frac{dz}{ds}$

Derivation

$m = ds\cdot dA\cdot\rho$

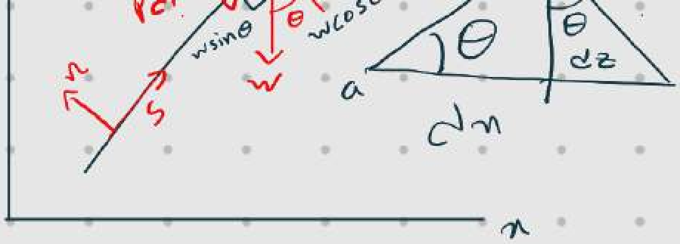
$\Sigma F_s = ma_s$



$$PdA - (P+dP)dA - w\sin\theta = ma$$

$$\Rightarrow -dP\cdot\cancel{dA} - \rho g\,ds\,\cancel{dA}\sin\theta$$

$$= \rho\, ds\, \cancel{dA} \cdot a$$



$$\Rightarrow -dp - \rho g\, ds \cdot \frac{dz}{ds} = \rho\, ds\, a$$

s = distance along streamline

$v = f(s,t)$

$$dv = \frac{\partial v}{\partial s} \cdot ds + \frac{\partial v}{\partial t} \cdot dt$$

$$-dp - \rho g\, dz = \rho v\, dv$$

$$\frac{dv}{dt} = \frac{\partial v}{\partial s} \cdot \frac{ds}{dt} + \cancel{\frac{\partial v}{\partial t}}^{\,0}$$

$$\Rightarrow \frac{dP}{\rho g} + dz + \frac{1}{g} v\, dv = 0$$

$$\Rightarrow \frac{dP}{\gamma} + \frac{1}{g} v\, dv + dz = 0$$

$$a = v \frac{\partial v}{\partial s} = v \frac{dv}{ds}$$

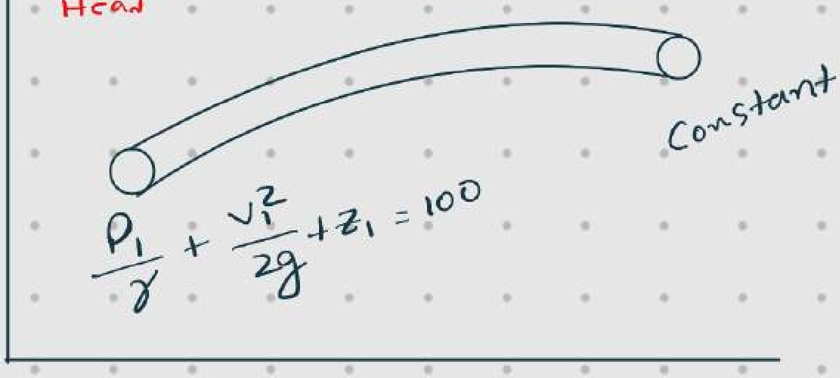
$$= \int \frac{dP}{\gamma} + \frac{1}{g} \int v\, dv + \int dz = \text{const}$$

$$\Rightarrow \boxed{\frac{P}{\gamma}} + \boxed{\frac{v^2}{2g}} + \boxed{z} = \text{constant}$$

Pressure head

Velocity Head

Potential head



$$P + \rho \frac{v^2}{2} + \rho g z = \text{Const}$$

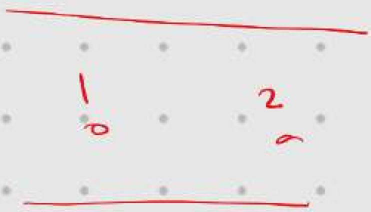
Pressure + Hydrostatic pressure + Dynamic pressure

$$\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \cancel{\frac{v_2^2}{2g}}^{\,0} + z_2$$

Hydraulic Grade line

Hydraulic Grade line:

EGL

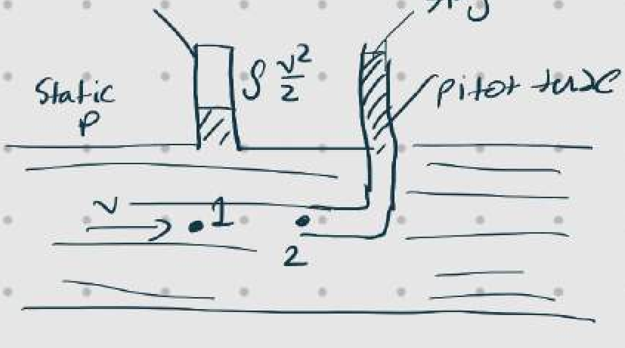


$$\frac{P_1}{\gamma}+\frac{v_1^2}{2g}+\cancel{z_1}=\frac{P_2}{\gamma}+\cancel{\frac{v_2^2}{2g}}^{\,0}+\cancel{z_2}$$

$$\underbrace{P_2=P_s=P+\rho\frac{v^2}{2}}_{\text{Stagnation Pressure}}$$

☐ Static Dynamic and Stagnation Pressure

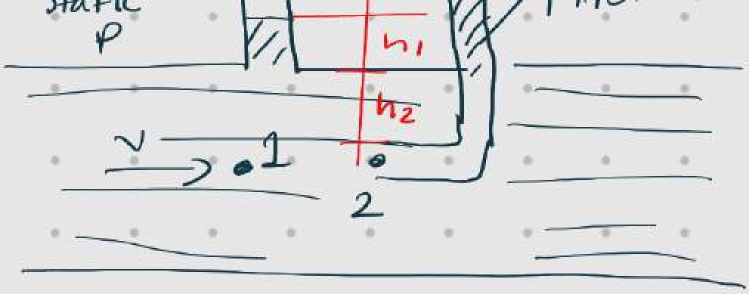
$$P_{stag}=P+\rho\frac{v^2}{2}$$



$$\frac{P_1}{\gamma}+\frac{v_1^2}{2g}+z_1=\frac{P_2}{\gamma}+\frac{v_2}{2g}+z_2$$

$$\frac{P_1}{\gamma}+\frac{v_1^2}{2g}=\frac{P_s}{\gamma}$$

$$\frac{v^2}{2g}=\frac{P_s-P_1}{\gamma}=\frac{P_s-P_1}{\rho g}\Rightarrow v_1=\sqrt{\frac{2(P_s-P_1)}{\rho}}$$



$$P_s = P_{atm} + h\rho g + h_1 \rho_f g + h_2 \rho_f g$$

$$P_1 = P_{atm} + h\rho g + h_1 \rho g + h_2 \rho g$$

$$P_s - P_1 = h\rho g$$

$$v_1 = \sqrt{\frac{2(P_s - P_1)}{\rho}} = \sqrt{2gh}$$

Problem-1

Is bernoullis Equation 100% Correct for Real fluid

$$\frac{P_A}{\gamma} + \cancel{\frac{v_A^2}{2g}} + \cancel{z_A} = \frac{P_B}{\gamma} + \cancel{\frac{v_B^2}{2g}} + \cancel{z_B} + h_{Lf}$$

$z_A = z_B$

$Q = AV$

$A_1 v_1 = A_2 v_2$

$$h_{Lf} = \frac{P_A - P_B}{\gamma} = \frac{h\rho g}{\rho g} = h$$

Gross frictional loss | Minor frictional loss

How to calculate the Frictional Head loss in the pipe

Hegen Poisellie's Equation $Q = \dfrac{\Delta P \pi D^4}{128 \mu L}$

Q = discharge $\frac{m^3}{s}$

Only suitable for laminar flow incompressible Newtonial fluid.

Darcy weisbach $h_f = \dfrac{f L V_{avg}^2}{2g D}$ f = Darcy friction factor

valid for both laminar and turbulent $V_{avg} = \dfrac{Q}{A}$

Laminar $\boxed{f = \dfrac{64}{Re}}$ ~ Raynolds number

Raynolds Number:-

If flow is Turbulent

$$\boxed{\frac{1}{\sqrt{F}} = -2 \log_{10} \left(\frac{\frac{\varepsilon}{D}}{3.7} + \frac{2.51}{Re \sqrt{f}} \right)}$$

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}} \qquad \tau = \mu \frac{u}{Y} = \frac{\partial u}{\partial Y}$$

$$= \frac{ma}{\tau A} = \frac{\rho \cdot L^3 \frac{v}{t}}{\mu \cdot \frac{v}{l} \times l^2} = \frac{\rho L^2 v^2}{\mu v L} = \frac{\rho \cdot vL}{\mu}$$

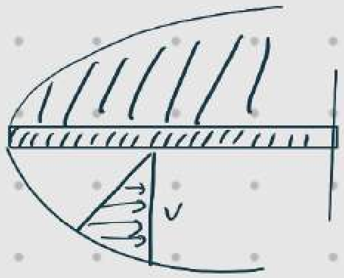
$$= \frac{\rho L^2 \frac{L}{t} v}{\mu v L}$$

$$Re = \frac{\rho v L}{\mu}$$ | L = Characteristics length

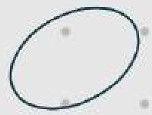
$$= \frac{\rho L^2 v v}{\mu v L}$$

$$= \frac{\rho v D_H}{\mu}$$ | D_H = Hydraulic Diameter

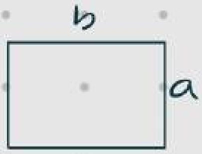
$$= \frac{4 \times \text{Cross sectional area}}{\text{wetted Perimeter}}$$



$$= \frac{4 \times \frac{\pi}{4} D^2}{\pi D} = D$$

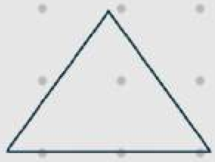


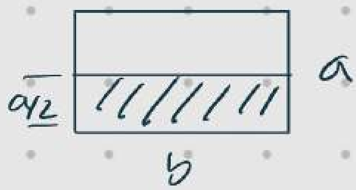
$$D_H = \frac{4 \times A}{P} = \frac{4 \times ab}{2(a+b)}$$



$$= \frac{2ab}{(a+b)}$$

$$= \frac{4 \frac{\pi}{4} D^2}{\frac{\pi D}{2}}$$



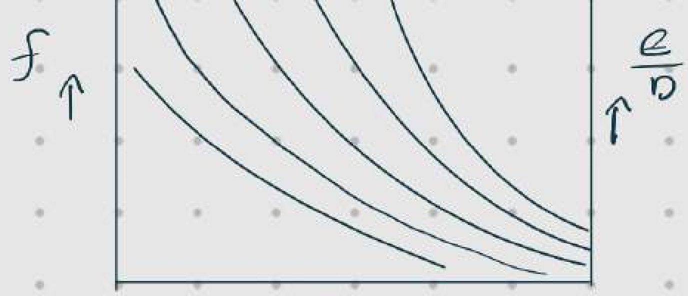


$$D_H = \frac{4ab}{\frac{a}{2} + b + \frac{a}{2}} = \frac{4ab}{a+b}$$

$Re \leq 2300$ laminar flow

$2300 \leq Re \leq 4000$ transitional

$Re \geq 4000$ turbulent flow



How in the flow we now h_{lf} / Loss

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + Z_B + h_{Lf}$$

$+ h_{pump}$

$$P_{pump} = \rho g Q h_{pump}$$

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A + h_{pump} = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + Z_B + h_{Lf}$$
$$+ h_{turbine} + e_{mech\ loss}$$

Powes of T+P

$$\text{Fluid Powes} = \rho g Q h \begin{cases} P_{pump} = \rho g Q h_{pump} \\ P_{turbine} = \rho g Q h_{turbine} \end{cases}$$

$$\frac{J}{s} = \text{watt}$$

$P_1 = 200\ N/m^2$ $\quad P_2 = ?$

$h_{Lf} = \frac{\Delta P}{} = P_1 - P_2$

$$P_2 = P_1 - h_{lf}\gamma$$

Efficiency $\eta_{pump} = \dfrac{\rho g Q h_{pump}}{P_{in}}$

" $\eta_{turbine} = \dfrac{P_{out}}{\rho g Q h_{turbine}}$

$V_{avg} = \dfrac{Q}{A}$

Frictional head loss $h_{lf} = \dfrac{f L V_{avg}^2}{2gD}$

Pressure drop $\Delta P = \rho g h_{lf}$

Total Head loss in the Pipe

$$h_{L\,total} = \underset{\substack{\downarrow \\ \text{(Pipe length)}}}{h_{L\,major}} + \underset{\text{fitting}}{h_{L\,minor}} = \sum_i h_{L\,major} + \sum h_{L\,minor}$$

$$= \sum_i f_i \frac{L_i}{D_i} \frac{V_i^2}{2g} + \sum K_{Lj} \frac{V_j^2}{2g}$$

h_{lf} (major) $= \dfrac{f_1 L_1 V_{avg_1}^2}{2gD_1} = \dfrac{f_2 L_2 V_{avg_2}^2}{2gD_2}$

h_{lf} (minor) $= K_L \cdot \dfrac{V^2}{2g}$ [K_L minor loss coefficient]

Problem-2

$P_1 \to atm$ $V_1^2 \to 0$ $P_2 \to atm$ $V_2^2 \to 0$ $4m$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + \cancel{z_2} + h_{Lf}$$

$$h_{Lf} = h_{Lf\text{-minor}} + h_{Lf\text{ major}}$$

$Q = 6\ L/s$
$= 6000\ cm^3/s$
$= \frac{6000}{100^3}$
$= 6 \times 10^{-3}\ m^3/s$

$$= \Sigma K_L \left(\frac{V^2}{2g}\right) + \frac{f L V_{avg}^2}{2 g D}$$

$P_1 = P_2 = atm$

$D = 5cm$
$= 0.05m$

$$V_{avg} = \frac{Q}{A} = \frac{6 \times 10^{-3}}{\pi/4 \times 0.5^2} = 3.05\ m/s$$

$V_2 = 0$ | Static

$$Z_1 = 4 + h_{lf} = 4 + 1.18 + 27.03 = 32.21$$

$$h_{lf\text{ minor}} = \Sigma K_L \frac{V^2}{2g}$$

$$= (0.5 + 0.3 + 0.2) \frac{3.05^2}{2 \times 9.8}$$

$0.3 + 1.06$

$$= 2.36 \frac{3.05^2}{2 \times 9.81}$$

$$= 1.118$$

$L = 89$
$D = 0.05m$

$$h_{lf\text{ major}} = \frac{f L V_{avg}^2}{2 g D} = \frac{0.032 \times 89 \times 3.05^2}{2 \times 9.8 \times 0.05} = 27.03$$

$$R_e = \frac{\rho V D}{\mu} = \frac{1000 \times 3.05 \times 0.05}{1.307 \times 10^{-3}}$$

$$= 116672.4185$$

$$= 1.17 \times 10^5$$

Re higher than 4000
So turbulent

$f = 0.032$ [Chart]

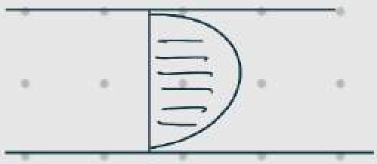
$\frac{\varepsilon}{D} = \frac{0.26}{50} = 5.2 \times 10^{-3}$

$= 0.005$

$$\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_{Lf}$$

$Re = \frac{\rho v_1 D_H}{\mu}$ $\quad$ $Re = \frac{\rho v_2 D_H}{\mu}$ $\quad$ (minor + major) (head loss)

Kinetic energy Correction factor



$\alpha = \frac{K_E \text{ actual}}{K_E \text{ avg}}$

$\alpha = 2 \rightarrow$ laminar

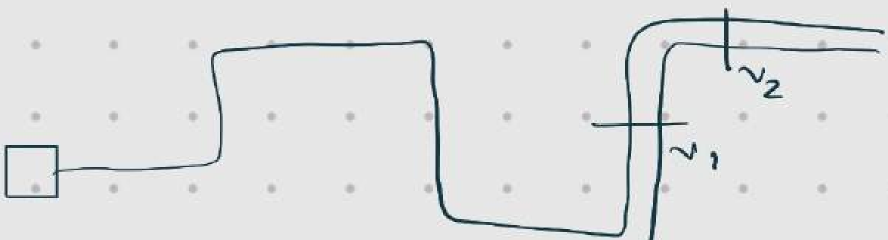
$= 1.5 \rightarrow$ Turbulent

Impulse moment Equation

$$F = ma = m\left(\frac{\Delta V}{\Delta t}\right) = m \cdot \frac{V_2 - V_1}{\Delta t} = \frac{mV_2 - mV_1}{\Delta t}$$

Rate of change of momentum

$$\frac{m \cdot V_2 - m \cdot V_1}{\Delta t} = F \quad \text{[For Solid]}$$



Impulse $I = F \cdot \Delta t = m\,\Delta v$

$$F = \frac{m}{\Delta t}(v_2 - v_1)$$
$$= \dot{m}\,(v_2 - v_1)$$
$$\dot{m} = \rho A v = \rho Q \qquad = \rho A v\,(v_2 - v_1)$$
$$= \rho Q\,(v_2 - v_1)$$

If the moment equation is applied to a control Volume

$$F = \rho_1 Q_1 v_1 - \rho_2 v_2 Q_2$$

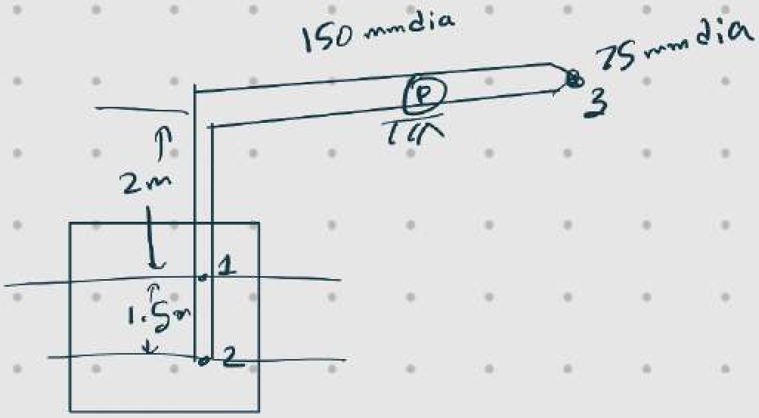
Problem-1

$$F = \rho Q (v_2 - v_1)$$
$$\frac{P}{\gamma} + \frac{v^2}{2g} + z = \text{Cont}$$
$$h_{lf} = \frac{f l v^2}{2 g D}$$
$$Re = \frac{\rho v D_H}{\mu}$$

$$D_H = \frac{4A}{P}$$
$$V = \frac{Q}{A}$$

Power $P = \rho Q g h$

Head of Pump/ Turbine



$$F = \rho Q\,(v_2 - v_1)$$
$$P_{pump} = \rho g Q\, h_{pump}$$
$$Q = A * V$$

$P_{u\,pump} = 8\,kW$

$$\cancel{\frac{P_1}{\gamma}}^{0} + \cancel{\frac{v_1^2}{2g}}^{0} + \cancel{z_1} + h_{pump} = \cancel{\frac{P_3}{\gamma}}^{0} + \frac{v_3^2}{2g} + z_3 + h_{Lf}$$

$P_1 = atm$ | $z_1 = 0$ | Assumed $h_{lf} = 0$

$P_2 = atm$ | $z_3 = 2.m$

$$\frac{v_1^2}{2g} + h_{pump} = \frac{v_3^2}{2g} + 2$$

$$\Rightarrow 3621.85 = v_3^3 + 39.24 v_3$$

$Q = A_1 v_1 = A_3 v_3$

$v_1 = \left(\frac{A_3}{A_1}\right) v_3$

$= \frac{\pi/4\ 75^2}{\pi/5\ 150^2} v_3$

$v_1 = \frac{v_3}{4}$

$Pump = \rho g Q h_{pump}$

$8 \times 10^3 = \cancel{1000} \times 9.81 \times Q h$

$\frac{8}{9.81 Q} = h$

$=$

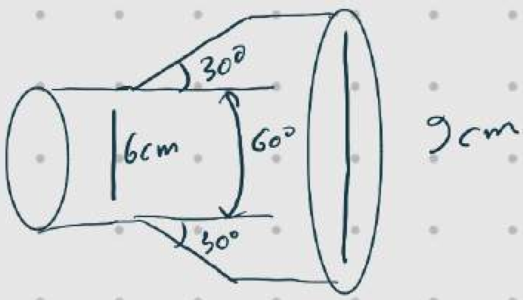
h

Problem - 3

$z_1 = z_2$

$v_1 = 7\,m\,s^{-1}$

$P_1 = 150 \times 1000$



$D_2 = 0.09\,m$

$D_1 = 0.06m$ D_2

$\rho = 1000 \text{ kg/m}^3$ $\mu = 1.307 \times 10^{-3} \frac{Ns}{m^2}$ $h_L = ?$ $P_2 = ?$

$v_2 = 3.11$

$$v_2 = \frac{Q}{A_2}$$

$$Q = A_1 v_1 = A_2 v_2$$

$$v_2 = \left(\frac{A_1}{A_2}\right) v_1 = \left(\frac{6^2}{9^2}\right) v_1$$

$$v_2 = 3.11 \text{ ms}^{-1}$$

$$\frac{P_1}{\gamma} + \left(\frac{v_1^2}{2g}\right)\alpha_1 + \cancel{z_1}^{0} = \frac{P_2}{\gamma} + \left(\frac{v_2^2}{2g}\right)\alpha_2 + \cancel{z_2}^{0} + h_{Lf}$$

major + minor loss

$\alpha = 2$ Laminar

$\alpha = 1.05$ Turbulent

$$h_{Lf-major} = \frac{2fLv^2}{2gD} = 0 \text{ (neglected)}$$

$$h_{Lf\,minor} = K_L \left(\frac{v^2}{2g}\right)$$

$$\boxed{\text{For } \theta = 60^\circ \quad K_L = 0.07}$$

$$h_{lf} = K_1 \frac{v_1^2}{2g} = 0.175 \text{ m}$$

$$\frac{P_1}{\gamma} + \left(\frac{v_1^2}{2g}\right)\alpha_1 + \cancel{z_1}^{0} = \frac{P_2}{\gamma} + \left(\frac{v_2^2}{2g}\right)\alpha_2 + \cancel{z_2}^{0} + h_{Lf}$$

$$Re_1 = \frac{\rho v_1 D_{1H}}{\mu}$$

$$= 411.76 \times 10^3$$

$\rho = 1000$
$\mu = \checkmark$
$v_1 = \checkmark$
$D = \checkmark\checkmark$

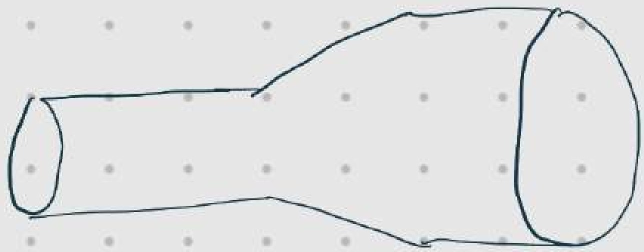
$Re_1 > 4000$ so Turbulant $\alpha_1 = 1.05$

$Re_2 = \square > 4000$ $\alpha_2 = 1.05$

$$\frac{P_1}{\gamma} + \left(\frac{v_1^2}{2g}\right)^{\alpha_1} + \cancel{z_1}^{\,0} = \frac{P_2}{\gamma} + \left(\frac{v_2^2}{2g}\right)^{\alpha_2} + \cancel{z_2}^{\,0} + h_{Lf}$$

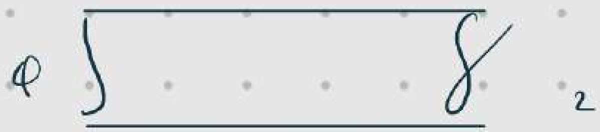
$$\frac{P_1}{\gamma} + \left(\frac{v_1^2}{2g}\right) \times 1.05 = \frac{P_2}{\gamma} + \left(\frac{v_2^2}{2g}\right) 1.05 + 0.175$$

$$P_2 = 18.93 \text{ kPa}$$



$$\frac{P_1}{\gamma} + \left(\frac{v_1^2}{2g}\right) + z_1 = \frac{P_2}{\gamma} + \left(\frac{v_2^2}{2g}\right) + z_2$$
$$\times \alpha_1 \qquad \times \alpha_2 \qquad + h_L$$

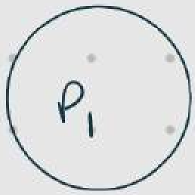
$$h_L = \frac{-P_2 + P_1}{\gamma}$$



$$\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_L$$

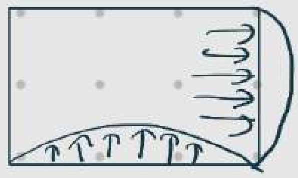
$$h_L = \frac{P_1 - P_2}{\gamma} = \frac{\Delta P}{\gamma}$$

$$\frac{P_1}{\gamma} + \frac{v_1^2}{2g}\alpha_1$$



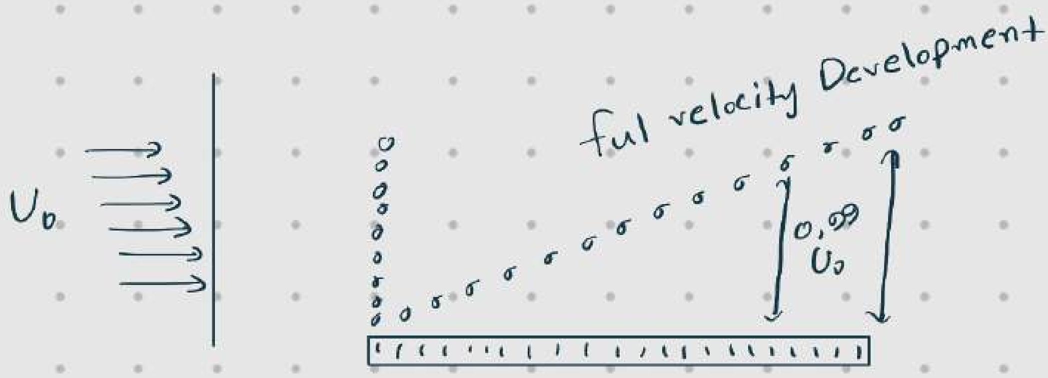
$P_0 = P_{atm}$

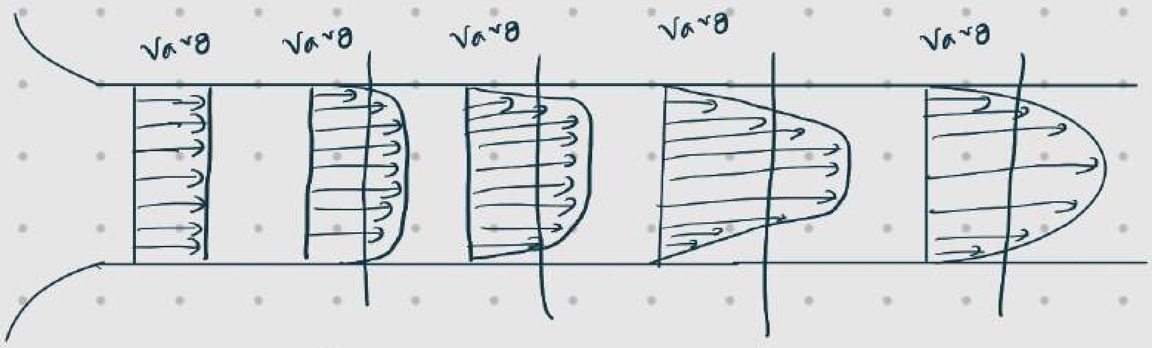
$P_1 - P_0 = \uparrow$ circular



$\Delta P = P_1 - P_0 \downarrow$

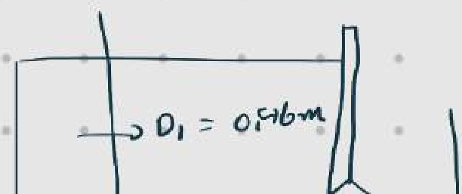
Development of Boundary Layer in pipe

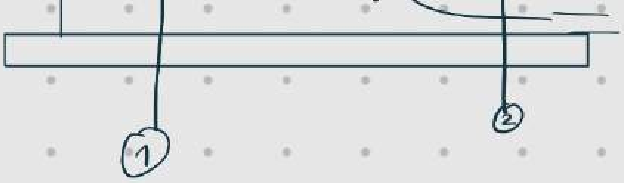




Hydrodynamic Developed Region

Problem 04





$$\frac{P_1}{\gamma} + \left(\frac{v_1^2}{2g}\right)_{\alpha_1} + z_1 = \frac{P_2}{\gamma} + \left(\frac{v_2^2}{2g}\right)_{\alpha_2} + z_2 + h_{Lf} \approx 0$$

$$\alpha_1 = \alpha_2 = 1$$

$v_1 = 0$ $\quad z_1 = 0.46\ m$ $\quad z_2 = 0.051$

$P_1 = atm$ $\quad P_2 = atm$

$$\frac{P_{atm}}{\gamma} + 0 + 0.46 = \frac{P_{atm}}{\gamma} + \frac{v_2^2}{2g} \times 1 + 0.051 + 0$$

$$v_2 = \boxed{}\ m/s$$

$$Q = AV$$
$$= D_2 \cdot w \cdot v$$

$$\frac{Q}{w} = D_2 \cdot v_2$$
$$= \boxed{}\ m^3/s/m$$

☐ Dimensional Analysis

$$\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = C$$

$$P + \frac{v^2}{2}\rho + z_1 \rho h$$

$$1 + \frac{v_1^2}{2\rho P} + \frac{z_1 \rho g}{P} = C$$

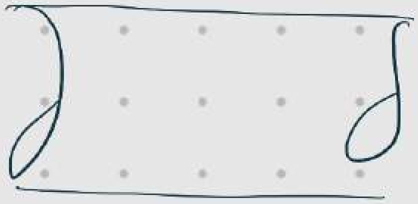
$$\frac{\rho^2 D^2 v^2}{\mu^2} = Re^2$$

L)

W
H } 1:200

$m = 1$ $S = 1:200$

$F_m = 1000\,N$



ε h_{cen}

L

v

D

$\mu = \frac{N.s}{m^2}$

T ρ

g

K

$\frac{N.S}{m^2}$

$\frac{MLT^{-2}\,T}{L^2}$

$= ML^{-1}T^{-1}$

$\Delta P = f(\varepsilon, L, \rho, v, \mu, D)$

$\Delta P = f(\rho, v, D, \mu)$

$\Delta P = K \rho^a v^b D^c \mu^d$

$ML^{-1}T^{-2} = K(ML^{-3})^a (LT^{-1})^b (L)^c (ML^{-1}T^{-1})^d$

Problem-1

$$R = \rho D^2 v^2 f\left(\frac{\mu}{\rho v D}\right)$$

$R =$ Resistance (force) $= N = MLT^{-2}$

$D =$ Diameter of Pipe $=$ (Length) $= L$

$V =$ velocity $= LT^{-2}$

$\mu =$ Viscocity of fluid $= ML^{-1}T^{-1}$

$\rho =$ Density $= ML^{-3}$

$$R = f(D, v, \rho, \mu)$$

$$= K\, D^a v^b \rho^c \mu^d$$

$K =$ non-dimensional Constant

$$\Rightarrow MLT^{-2} = K\, L^a (LT^{-1})^b (ML^{-3})^c (ML^{-1}T^{-1})^d$$

$$\Rightarrow MLT^{-2} = K\left[M^{c+d}\, L^{a+b-3c-d},\, T^{-b-d}\right]$$

Eqate the power

$1 = c+d$

$1 = a+b-3c-d$

$-2 = -b-d$

$2 = b+d$

$c = 1-d$

$b = 2-d$

$1 = a + (2-d) - 3(1-d) - d$

$1 = a + 2 \cancel{-d} - 3 \cancel{+d} - d$

$a = 2-d$

$a = b$

$$R = f(D, v, \rho, \mu)$$

$$= K\, D^a v^b \rho^c \mu^d$$

$$= K\, D^{2-d} v^{2-d} \rho^{1-d} \mu^d$$

$$= K \frac{D^2}{D^d} \frac{v^2}{v^d} \frac{\rho}{\rho^d} \mu^d$$

$$R = K\, D^2 v^2 \rho \left(\frac{\mu}{Dv\rho}\right)^d$$

$$R = D^2 v^2 \rho\, f\left(\frac{\mu}{Dv\rho}\right) = D^2 v^2 \rho\, f\left(\frac{1}{Re}\right)$$

Exp-3

$$P = F \cdot V$$
$$= N m s^{-1}$$
$$= kg\, m s^{-2} m s^{-1}$$
$$= M L^2 T^{-3}$$

Body wt = 70 kg
Climbing Top floor of a 6 story building in 8 min. How much calory will be burned?

$$P = f(\rho\, N\, D\, H\, g) = K\left[\rho^a N^b D^c H^d g^e\right]$$

$$\boxed{\frac{J}{s} = \frac{N\cdot m}{s} = \frac{kg\, m}{s^2} = M L^2 T^{-3} = P}$$

$\rho = M L^{-3}$

$D = L$

$H = L$

$g = L T^{-2}$

$N = T^{-1}$

$$\Rightarrow M L^2 T^{-3} = K\left[(M L^{-3})^a (T^{-1})^b \cdot L^c \cdot L^d \cdot (L T^{-2})^e\right]$$

$a = 1$

$2 = -3a + c + d + e$

$5 = c + d + e$

$-3 = -b - 2e \Rightarrow b = 3 - 2e$

$c = 5 - d - e$

$$P = K\left[\rho\, N^{3-2e} \cdot D^{5-d-e}\, H^d\, g^e\right]$$
$$= K\left[\rho\, N^3 D^5 \cdot N^{-2e}, D^{-e-d} \cdot H^d g^e\right]$$

$$\left(\frac{g}{N^2}\right)^e H^d \frac{1}{D^{e+d}}$$

$$= K\left[\rho\, N^3 D^5 \left(\frac{g}{N^2 D}\right)^e \left(\frac{H}{D}\right)^d\right]$$

$\boxed{e = 1 \quad d = 1}$

$e + d = 2$

$$= \rho N^3 D^5 f_1\left(\frac{gH}{D^2 N^2}\right)$$

$$= \rho N^3 D^5 f\left(\frac{D^2 N^2}{gH}\right)$$

$$\frac{P}{\rho N^3 D^5} = f\left(\frac{N^2 D^2}{gH}\right)$$

Buckingham's π Theorem

$K = 6$

$r = 3$

$K - r = 6 - 3 = 3$

$\square = f_1(\ )$

$\square = f_2(\ )$

$\square = f_3(\ )$

P, ρ, g, N, H, D

$\pi_1 = P \ f(D, \rho, H)$

$\pi_2 = g \ f(D, \rho, H)$

$\pi_1 = \phi(\pi_2, \pi_3 \ldots \pi_{K-r})$

$\pi_1 = f(\pi_2, \pi_3)$

Problem 3

$$Q = \nu D^2 f\left[\frac{\sqrt{gD}}{\nu}, \frac{H}{D}\right]$$

$K - r = 3$

$Q = f(v, D, H, g)$ $\quad K = 5 \quad r = 2$

$\Rightarrow f(Q, v, D, H, g) = 0$

$Q = L^3 T^{-1}$

$v = L T^{-1}$

$D = L$

$H = L$

$g = L T^{-2}$

Geometry, Fluid Property

D.

$\pi_1 = Q v^a D^b \mid \pi_2 = g v^a D^b$

$M^0 L^0 T^0 = L^3 T^{-1} (L T^{-1})^a (L^b)$

$0 = 3 + a + b$

$0 = -1 - a$

$a = -1$

$b = -2$

$\pi_1 = Q v^{-1} D^{-2}$

$\pi_2 = g v^a D^b$

$\pi_2 = \frac{\sqrt{gD}}{v}$

$\pi_3 = H v^{a_2} D^{b_2}$

$= \frac{H}{D}$

$\pi_1 = f_1(\pi_2, \pi_3)$

$\frac{Q}{vD^2} = f_1\left(\frac{\sqrt{gD}}{v}, \frac{H}{D}\right)$

$\frac{Q}{vD^2} = f\left(\frac{\sqrt{gD}}{2}, \frac{D}{H}\right)$

$O = L^3 T^{-1}$

$V = L T^{-1}$

$D = L$

The geometry is very important for this Chapter

Lec-2 Part-B

☐ Undistorted model analysis

or

Similarity and similitude

Lec-4

Non dimentional Constant

Problem-2

Given,

$$\frac{\rho_m}{\rho_p} = 750 \quad \Big| \quad \frac{\mu_m}{\mu_p} = 50$$

$S = 45$

$F_m = 0.98\ N$

$F_p = ?$

Euler Number:

$$\left(\frac{F}{\rho v^2 L^2}\right)_m = \left(\frac{F}{\rho v^2 L^2}\right)_p$$

$$\frac{F_m}{\rho_m v_m^2 L_m^2} = \frac{F_p}{\rho_p v_p^2 L_p^2} \qquad \frac{L_m}{L_p} = \frac{1}{5} = \frac{1}{45}$$

$$\frac{F_p}{F_m} = \frac{\rho_p}{\rho_m}\left(\frac{L_p}{L_m}\right)^2\left(\frac{v_p}{v_m}\right)^2 \qquad \frac{L_p}{L_m} = 5 = 45$$

$$\frac{F_p}{0.98} = \frac{1}{750} \times (45)^2 \times \left(\frac{v_p}{v_m}\right)^2 \quad \text{— (1)}$$

$$F_p = \frac{0.98}{750} \times (45)^2 \times \left(\frac{v_p}{v_m}\right)^2$$

$$\frac{\rho V L}{\mu}$$

$$\frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p}$$

$$\Rightarrow \frac{v_p}{v_m} = \frac{\rho_m}{\rho_p}\frac{L_m}{L_p}\frac{\mu_p}{\mu_m}$$

$$= \cancel{750}^{15}\ \frac{1}{\cancel{45}_{3}} \times \frac{1}{\cancel{50}} = \frac{1}{3}$$

$$\frac{v_p}{v_m} = \frac{1}{3} = 0.33$$

$$F_p = \frac{0.98}{750} \times (45)^2 \times \left(\frac{v_p}{v_m}\right)^2$$

$$= \frac{0.98}{750} \times (45)^2 \times \left(\frac{1}{3}\right)^2$$

$$= 0.29\ N$$

Exp-3

$S = 60$ $F_m = 0.36\,N$ $V_m = 1.25\ m/s$

$F_p = ?$ $P_p = ?$ $P_m = ?$

Euler Numbers

$$\frac{F_m}{\cancel{\rho_m} V_m^2 L_m^2} = \frac{F_p}{\cancel{\rho_p} V_p^2 L_p^2}$$

$\frac{L_p}{L_m} = 60 = S$

$\rho_m = \rho_p$

$V_m = 1.25\ m/s$

$F_m = 0.36\,N$

$$\frac{F_p}{F_m} = \left(\frac{V_p}{V_m}\right)^2 \times \left(\frac{L_p}{L_n}\right)^2 \frac{\rho_p}{\rho_m}$$

$$\frac{F_p}{F_m} = \left(\frac{V_p}{1.25}\right)^2 (60)^2$$

$$\frac{F_p}{0.36} = \left(\frac{V_p}{1.25}\right)^2 (60)^2$$

Froude Number

$$\frac{V_p^2}{L_p g} = \frac{V_m^2}{L_m g}$$

$$\left(\frac{V_p}{1.25}\right)^2 = \frac{L_p}{L_m} = 60$$

$V_p = 9.68\ m/s$

$$\frac{F_p}{0.36} = \left(\frac{V_p}{1.25}\right)^2 (60)^2$$

$(9.68)^2$ 60×0.36

$F_P = \left(\frac{}{1.25}\right) 60 \times 0.5$

$= 77.7\ KN$

Power $= F \times V$

$P_m = F_m V_m = 0.36 \times 1.25$

$P_p = F_p \times V_p = 77.7 \times 10^3 \times 9.68$

Lec 4- Compressible Fluid Flow

$PV = RT$

$P\vartheta = RT$

$\frac{P}{\rho} = RT$

$Pv^k = cons$

$Pu^k = Const$

$\frac{P}{\rho} = constant$

$\frac{P}{\rho} = \frac{P_1}{\rho_1} = \frac{P_2}{\rho_2}$

$\frac{P}{\rho^K} = \frac{P_1}{\rho_1^K} = \frac{P_2}{\rho_2^K}$

$x^m = y$

$x = y^{\frac{1}{m}}$

$\sqrt[a]{x} = x^{\frac{1}{a}}$

$\frac{1}{x^{1/K}} = x^{-\frac{1}{K}}$

Derivation of Energy Equation for Compressive Fluid Considering Both isothermal and Adiabatic Process.

$P_1\ \rho_1\ T_1$

$P_2\ \rho_2\ T_2$

$\frac{P}{\gamma} + \frac{v^2}{2g} + z = const$

$\frac{P}{\rho} + \frac{v^2}{2} + gz = const$

$\frac{dP}{\rho}\quad vdv + 0 = 0$

$$\frac{dp}{\rho} + v\,dv = 0$$

$$\Rightarrow \frac{P_1}{\rho_1}\int_1^2 \frac{dP}{P} + \int_1^2 v\,dv = 0$$

$$\Rightarrow RT_1 \ln\frac{P_2}{P_1} + \frac{1}{2}\left(v_2^2 - v_1^2\right) = 0$$

Energy Eqn for isothermal Condition

For adiabatic

$$\frac{P}{\rho^K} = \text{constant}$$

$$\frac{P}{\rho^K} = \frac{P_1}{\rho_1^K} = \frac{P_2}{\rho_2^K}$$

$$\frac{1}{\rho^K} = \frac{P_1}{\rho_1^K} \cdot \frac{1}{P}$$

$$\frac{1}{\rho} = \frac{P^{\frac{1}{K}}}{\rho} \cdot \left(\frac{1}{P}\right)^{1/K}$$

Now

$$\frac{dP}{\rho} + v\,dv = 0$$

$$\Rightarrow \frac{P_1^{1/K}}{\rho_1}\int_1^2 \frac{dP}{P^{1/K}} + \int_1^2 v\,dv = 0$$

$$\Rightarrow \frac{P_1^{\frac{1}{K}}}{\rho}\left[\frac{P^{-1/K+1}}{\ldots}\right] + \frac{1}{2}\left(v_2^2 - v_1^2\right) = 0$$

$$\rho_1 \left[\quad -\tfrac{1}{K}+1 \right] \quad (\quad)$$

$$\Rightarrow \frac{K}{K-1} \frac{P_1^{1/K}}{\rho_1} \left[P_2^{\frac{K-1}{K}} - P_1^{\frac{K-1}{K}} \right] + \frac{1}{2}(v_2^2 - v_1^2) = 0$$

$$\frac{1}{2}(v_2^2 - v_1^2) = \frac{K}{K-1} \frac{P_1^{1/K}}{\rho_1} \left[P_1^{\frac{K-1}{K}} - P_2^{\frac{K-1}{K}} \right]$$

$$= \frac{K}{K-1} \left[\frac{1}{\rho_1} P_1^{\frac{K-1}{K}+\frac{1}{K}} - \frac{P_2^{1/K}}{\rho_2} P_2^{\frac{K-1}{K}} \right]$$

$$\frac{P_1}{\rho_1^K} = \frac{P_2}{\rho_2^K}$$

$$= \frac{K}{K-1} \left[\frac{P_1}{\rho_1} - \frac{P_2}{\rho_2} \right]$$

$$\frac{P_1^{1/K}}{\rho_1} = \frac{P_2^{1/K}}{\rho_2}$$

$$= \frac{K}{K-1} \frac{P_1}{\rho_1} \left[1 - \frac{\rho_1}{P_1} \cdot \frac{P_2}{\rho_2} \right]$$

$$\frac{\rho_1^K}{P_1} = \frac{\rho_2^K}{P_2}$$

$$\left(\frac{\rho_1}{\rho_2}\right)^K = \frac{P_1}{P_2}$$

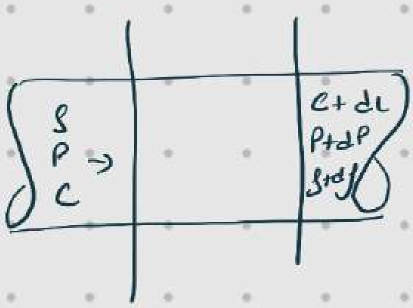
$$\frac{\rho_1}{\rho_2} = \left(\frac{P_1}{P_2}\right)^{1/K} = \left(\frac{P_2}{P_1}\right)^{-1/K}$$

$$\frac{1}{2}(v_2^2 - v_1^2) = \frac{K}{K-1} \frac{P_1}{\rho_1} \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{1}{K}} \cdot \frac{P_2}{P_1} \right]$$

$$= \frac{K}{K-1} \frac{P_1}{\rho_1} \left[1 - \left(\frac{P_2}{\rho_2}\right)^{\frac{K-1}{K}} \right]$$

$$\frac{P_1}{\rho_1^K} = RT$$

Derive expression for the area velocity relationship for one dimensional compressible flow. Show the effects of variation of area on subsonic, sonic and supersonic flow.



dc dρ → small

$$A c\rho = A(c+dc)(\rho+d\rho)$$

$$\cancel{Ac\rho} = \cancel{Ac\rho} + c\,d\rho\,A + \cancel{A\,dc\,d\rho}^{\,0} + A\,dc\,\rho$$

$$\Rightarrow c\,d\rho\,\cancel{A} = -\cancel{A}\,dc\,\rho$$

$$\Rightarrow dc = -\frac{d\rho}{\rho}\cdot c$$

$$F = ma$$
$$= m\frac{dv}{dt}$$
$$= \frac{m}{dt}dv$$
$$= \dot{m}(v_2 - v_1)$$

$$\dot{m} = A\rho v$$

$$F = ma$$

$$P.A - (P+dP).A$$

$$= \rho c A\left[\cancel{c} + dc - \cancel{c}\right]$$

$$\cancel{P.A} - \cancel{P.A} - dP\cancel{A} = \rho c \cancel{A}\,dc$$

$$dc = -\frac{dP}{\rho c}$$

$$-\frac{d\rho}{\rho}\cdot c = -\frac{dP}{\rho c}$$

Bulk Modulus Elasticity of

$$c^2 = \frac{dP}{d\rho}$$

$$C = \sqrt{\frac{dP}{d\rho}} = \sqrt{\frac{E}{\rho}}$$

fluid

$$E = -\frac{dP}{\frac{dv}{v}} \approx \frac{dP}{\frac{d\rho}{\rho}}$$

$$E = \frac{dP}{d\rho}\rho$$

$$\frac{dP}{d\rho} = \frac{E}{\rho}$$

Isothermal Condition

$$PV = \text{cosnt}$$

$$E = -\frac{dP}{\frac{dv}{v}} = \frac{dP}{\frac{d\rho}{d\rho}}$$

$$dP \cdot v + p\,dv = 0$$

$$P = -\frac{dP}{\frac{dv}{v}} = E$$

$$C = \sqrt{\frac{P}{\rho}} = \sqrt{RT}$$

$$Pv^k = \text{constant}$$

$$K v^{K-1} dv\, P + v^K dP = 0$$

$$K v^{-1} dv\, P + dp = 0$$

$$K \frac{dv}{v} \cdot P + dP = 0$$

$$KP = -\frac{dP}{\frac{dv}{v}} = \frac{dP}{\frac{d\rho}{\rho}}$$

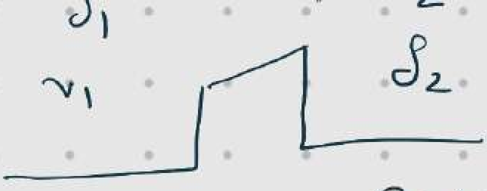
$$KP = E$$

For adiabatic

$$C = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{k \cdot P}{\rho}}$$

$$C = \sqrt{KRT}$$

Reservoir



Recievers

$$\frac{1}{2}(v_2^2 - v_1^2) = \frac{k}{k-1}\frac{P_1}{\rho_1}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right] = \frac{k}{k-1}\left[\frac{P_1}{\rho_1} - \frac{P_2}{\rho_2}\right]$$

$$= \frac{kR}{k-1}\left[T_1 - T_2\right]$$

$$v_2 = \sqrt{\frac{2k}{k-1}\frac{P_1}{\rho_1}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]}$$

$$\dot m = A\rho v = A_c \rho_2 v_2 \qquad \frac{P_1}{\rho_1^k} = \frac{P_2}{\rho_2^k}$$

$$= A_2 \rho_1 \left(\frac{P_2}{P_1}\right)^{\frac{1}{k}} \qquad \rho_2^k = \frac{P_2}{P_1}\rho_1^k$$

$$\times \sqrt{\frac{2k}{k-1}\frac{P_1}{\rho_1}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]} \qquad \rho_2 = \left(\frac{P_2}{P_1}\right)^{\frac{1}{k}}\rho_1$$

$$= A_2 \rho_1 \sqrt{\frac{2k}{k-1}\frac{P_1}{\rho_1} \times \frac{P_2}{P_1}^{\frac{2}{k}}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]}$$

$$\dot m = A_2 \rho_1 \sqrt{\frac{2k}{k-1}\frac{P_1}{\rho_1}}\sqrt{\left(\frac{P_2}{P_1}\right)^{2/k} - \left(\frac{P_2}{P_1}\right)^{\frac{k+1}{k}}}$$

$$\frac{P_2}{P_1} = ? \quad \text{For } \dot m(max) \qquad \frac{d\dot m}{d\underbrace{\left(\frac{P_2}{P_1}\right)}_{x}} = \frac{d\dot m}{dx} \approx 0$$

$$\dot m = A_c \rho_1 \sqrt{\frac{2k}{k-1}\frac{P_1}{\rho_1}} \times \sqrt{x^{\frac{2}{k}} - x^{\frac{k+1}{k}}}$$

$$\frac{d\dot m}{\ } \quad \sqrt{\frac{2k}{\ }\frac{P_1}{\ }} \; \frac{1}{\ }\left(x^{\frac{2}{k}} - x^{\frac{k+1}{k}}\right)^{-\frac{1}{2}}$$

$$\frac{d\dot{m}}{dx} = 0 \Rightarrow A_2 \rho_1 \sqrt{\frac{2k}{k-1} \cdot \frac{p_1}{\rho_1}} \cdot \frac{1}{2}\left(\ldots\right) \cdot \left(\frac{2}{k} x^{\frac{2}{k}-1} - \frac{k+1}{k} x^{\frac{k+1}{k}-1}\right) = 0$$

$$\Rightarrow \frac{2}{k} x^{\frac{2}{k}-1} - \frac{k+1}{k} x^{\frac{k+1}{k}-1} = 0$$

$$\Rightarrow \frac{2}{\cancel{k}} x^{\frac{2-k}{k}} = \frac{k+1}{\cancel{k}} x^{\frac{1}{k}}$$

$$\Rightarrow \frac{x^{\frac{2-k}{k}}}{x^{1/k}} = \frac{k+1}{2}$$

$$\Rightarrow x^{\frac{2-k}{k} - \frac{1}{k}} = \frac{k+1}{2}$$

$$\Rightarrow x^{\frac{1-k}{k}} = \frac{k+1}{2} \Rightarrow x = \left(\frac{k+1}{2}\right)^{\frac{k}{1-k}}$$

$$k = 1.4 = \frac{c_p}{c_v} \qquad \Rightarrow \frac{P_2}{P_1} = \left(\frac{2}{k+1}\right)^{\frac{k}{1-k}} = 0.528$$

Critical Pressure Ratio for max $\dot{m}$ in adiabatic condition

$$\dot{m}_{max} = A_2 \rho_1 \sqrt{\frac{2k}{k-1} \cdot \frac{p_1}{\rho_1}} \sqrt{x^{\frac{2}{k}} - x^{\frac{k+1}{k}}}$$

$$= A_2 \rho_1 \sqrt{\frac{2 \times 1.4}{1.4-1} \times \frac{P_1}{\rho_1}} \sqrt{(0.528)^{\frac{2}{1.4}} - (0.528)^{\frac{1.4+1}{1.4}}}$$

$$= A_2 \sqrt{P_1 \rho_1} \cdot \sqrt{\frac{2k}{k-1}} \left[\left(\frac{2}{k+1}\right)^{\frac{2}{k-1}} - \left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\right]^{1/2}$$

$$= A_2 \sqrt{P_1 \rho_1} \sqrt{\frac{2k}{k-1}} \left(\frac{2}{k+1}\right)^{\frac{2}{k-1} \times \frac{1}{2}} \left[1 - \left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1} - \frac{2}{k-1}}\right]^{1/2}$$

$$= \ldots \sqrt{\frac{2k}{\ldots}} \left(\frac{2}{\ldots}\right)^{\frac{1}{k-1}} \left[1 - \left(\frac{2}{\ldots}\right)^{\frac{k+1-2}{k-1}}\right]^{1/2}$$

$$= A_2 \sqrt{P_1 \rho_1} \sqrt{\frac{\quad}{K-1}} \left(\frac{\quad}{K+1}\right) \left[1 - \left(\frac{\quad}{K+1}\right)\right]$$

$$= \underline{\qquad\qquad\qquad\qquad} \left[1 - \left(\frac{2}{K+1}\right)^{\frac{K-1}{K-1}}\right]^{1/2}$$

$$= \underline{\qquad\qquad\qquad\qquad} \left[1 - \frac{2}{K+1}\right]^{1/2}$$

$$= A_2 \sqrt{P_1 \rho_1} \cdot \sqrt{\frac{2K}{K-1} \left(\frac{2}{K+1}\right)^{\frac{1}{K-1}}} \cdot \frac{\sqrt{K-1}}{\sqrt{K-1}}$$

$$= A_2 \sqrt{P_1 \rho_1} \sqrt{\frac{2K}{K+1}} \left(\frac{2}{K+1}\right)^{\frac{1}{K-1}}$$

$$= A_2 \sqrt{P_1 \rho_1 K} \left(\frac{2}{K+1}\right)^{1/2} \cdot \left(\frac{2}{K+1}\right)^{\frac{1}{K-1}}$$

$$= A_2 \sqrt{P_1 \rho_1 K} \left(\frac{2}{K+1}\right)^{\frac{1}{2} + \frac{1}{K-1}}$$

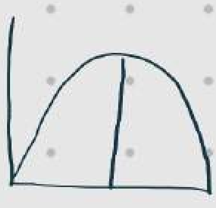
$$= A_2 \sqrt{P_1 \rho_1 K} \left(\frac{2}{K+1}\right)^{\frac{K+1}{2(K-1)}}$$

$$\frac{P_2}{P_1} = \left(\frac{2}{K+1}\right)^{\frac{K}{K-1}}$$

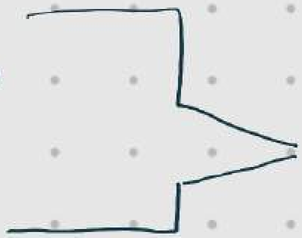
$K = 1.4$

$= 0.528$

$$\dot{m}_{max} = A_2 \sqrt{K P_1 \rho_1} \left(\frac{2}{K+1}\right)^{\frac{K+1}{2K-1}}$$



P_1
v_1
ρ_1
T_1



P_2
ρ_2
T_2
v_2

$$\frac{P_2}{P_1} = \left(\frac{2}{K+1}\right)^{\frac{K}{K-1}}$$

$$\frac{1}{2}\left(v_2^2 - v_1^2\right) = \frac{KR}{K-1}\left(T_1 - T_2\right)$$

$v_1 = 0$

$$\frac{1}{2} v_2^2 = \frac{K R}{K-1} T_2 \left(\frac{T_1}{T_2} - 1 \right)$$

$$\frac{P_1}{\rho_1^K} = \frac{P_2}{\rho_2^K}$$

$$\frac{\rho_2}{\rho_1} = \left(\frac{P_2}{P_1} \right)^{\frac{1}{K}}$$

$$P_1 = \rho_1 R T_1$$

$$P_2 = \rho_2 R T_1$$

$$\frac{T_1}{T_2} = \frac{P_1}{P_2} \times \frac{\rho_2}{\rho_1} = \left(\frac{P_2}{P_1} \right)^{-1} \cdot \left(\frac{P_2}{P_1} \right)^{\frac{1}{K}} = \left(\frac{P_2}{P_1} \right)^{\frac{1-K}{K}}$$

$$\Rightarrow \frac{1}{2} v_2^2 = \frac{K R T_2}{K-1} \left[\left(\frac{2}{K+1} \right)^{\frac{1-K}{\cancel{K}} \cdot \frac{\cancel{K}}{K-1}} - 1 \right]$$

$$\Rightarrow \frac{1}{2} v_2^2 = \frac{K R T_2}{K-1} \left[\frac{K+1}{2} - 1 \right]$$

$$\frac{1}{2} v_2^2 = \frac{K R T_2}{K-1} \left(\frac{K-1}{2} \right)$$

$$v_{2\,max} = \sqrt{K R T_2} = c$$

$$M = \frac{v}{c} = \frac{c}{c} = 1$$

Derive expression for the area velocity relationship for one dimensional compressible flow. Show effect of variation of area on subsonic, sonic, supersonic flows.

$$M = \frac{v}{c}$$

$$\ldots \; v^2 + z = \text{Const}$$

$$\frac{dp}{\gamma} + \frac{v^2}{2g} + z = \text{const}$$

$$\frac{p}{\rho^k} = \text{const}$$

$$\Rightarrow \frac{dp}{\rho} + \frac{v^2}{2} + zg = \text{const}$$

$$\Rightarrow \frac{dp}{\rho} + v\,dv = \text{const} \qquad (z = \text{const})$$

$$c = \sqrt{\frac{kp}{\rho}} \Rightarrow c^2 = \frac{kp}{\rho}$$

$$\Rightarrow kp \cdot \frac{d\rho}{\rho^2} + v\,dv = 0$$

$$E = kp \quad \Rightarrow kp = \rho c^2$$

$$= c^2 \rho \, \frac{d\rho}{\rho^2} + v\,dv = 0$$

$$\frac{dp}{\frac{d\rho}{\rho}} = k.p. \frac{d\rho}{\rho}$$

$$\Rightarrow \frac{c^2}{v^2} \cdot \frac{d\rho}{\rho} + \frac{dv}{v} = 0 \qquad \left[\frac{1}{v^2} \times\right]$$

$$\Rightarrow \frac{1}{M^2} \cdot \frac{d\rho}{\rho} + \frac{dv}{v} = 0$$

$$\Rightarrow \frac{d\rho}{\rho} = -M^2 \cdot \frac{dv}{v}$$

$$\dot{m} = A v \rho$$

$$\log(\dot{m}) = \log A + \log v + \log \rho$$

$$\Rightarrow 0 = \frac{dA}{A} + \frac{dv}{v} + \frac{d\rho}{\rho}$$

$$\Rightarrow 0 = \frac{dA}{A} + \frac{dv}{v} - M^2 \frac{dv}{v} = 0$$

$$\Rightarrow \frac{dA}{A} = (M^2 - 1) \frac{dv}{v}$$

$$\Rightarrow \frac{dA}{dv} = \frac{A}{v} (M^2 - 1)$$

(1) Subsonic flow $\quad M < 1 \quad \frac{dA}{dv} = (+ve)$

$dA = +ve$
$dv = -ve$

$dA = -ve$
$dv = +ve$

$v_2 < v_1$ $A_2 < A_1$ $v_2 > v_1$

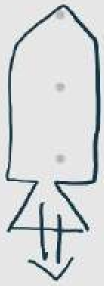
$A_1 < A_2$

(II) Supersonic flow, $M > 1$

$$\frac{dA}{dv} = +ve$$

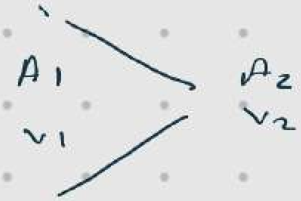
$dA = +ve$
$dv = +ve$

$dA = -ve$
$dv = -ve$



compressible

$v_2 > v_1$



$v_2 < v_1$

(III) when $M = 1$

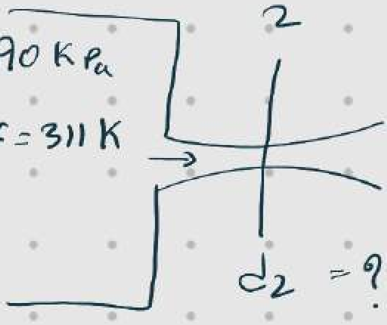
$$\frac{dA}{dv} = 0 \qquad dA = 0$$

Problem-3

$P_1 = 690\ KPa$

$T_1 = 38^\circ C = 311\ K$

$v_1 = 0$



2

$d_2 = ?$

$P_3 = P_{atm} = 101.4\ KPa$

$d_3 = 50mm = 0.05\ m$

$\dot{m}_{max} = ?$

For $m_{max} = \frac{P_2}{P_1} = 0.528$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}}$$

$$= (0.528)^{\frac{1.4-1}{1.4}}$$

$v_2 = V_{max} = \sqrt{KRT_2}$

$= 322.67\ m/s$

$= 259.13^\circ K$

$T_2 = 0.833\, T_1 = 2\ldots$

$P_1 = \rho_1 R T_1$

$P_2 = \rho_2 R T_2$

$\frac{P_1}{\rho_1^{k}} = \frac{P_2}{\rho_2^{k}}$

$\frac{T_3}{T_1} = \left(\frac{P_3}{P_1}\right)^{\frac{k-1}{k}}$

$T_3 = 179.8\ ^\circ K$

$\frac{1}{2}\left(V_3^2 - V_2^2\right) = \frac{kR}{k-1}\left(T_2 - T_3\right)$ | $R = 287\ J/kgK$

$V_3 = 513.29\ m/s$

$P = \rho R T$

$m_{max} = A_3 V_3 \rho_3$

$= \frac{\pi}{4} d_3^2 V_3 \rho_3$

$\rho_3 = \frac{P_3}{R T_3} = 1.965\ kg/m^3$

$= 1.98\ kg/s$,

$\rho_2 = \frac{P_2}{R T_2}$ $\frac{P_2}{P_1} = 0.528$

$P_2 = 364.32\ KPa$

$1.98 = A_2 V_2 \rho_2 =$

$\rho_2 = 4.9\ kg/m3$

$= \frac{\pi}{4} d_2^2 \times 322.67 \times 4.9\ kg/m^3$

$1.98 = \frac{\pi}{4} d_2^2 \times 322.67 \times 4.9$

$d_2 = 40\ mm$

Problem-2

$v_1 = 350$ m/s

$P_1 = 80$ KPa

$T_1 = 40°C = 313$ K

$V_2 = ?$

$T_2 = ?$

$P_2 = 120$ KPa

$$M_1 = \frac{v_1}{c_1} = \frac{v_1}{\sqrt{KRT_1}} \qquad M_2 = \frac{v_2}{c} = \frac{v_2}{\sqrt{KRT_2}}$$

$R = 287$ J/kg $\quad K = 1.4$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}} \Rightarrow T_2 = 351.44°K$$

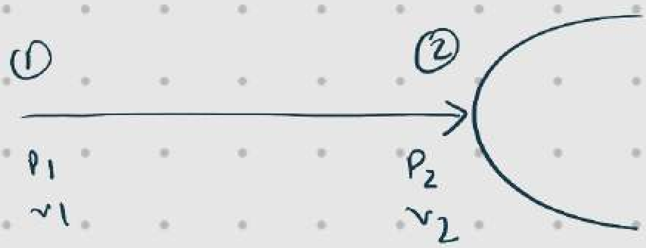
$$M_1 = \frac{350}{\sqrt{1.4 \times 287 \times 313}} = 0.99$$

$$\frac{1}{2}(v_2^2 - v_1^2) = \frac{KR}{K-1}\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}}\right]$$

$V_2 = 212.76$ m/s

$$M_2 = \frac{212.76}{\sqrt{KR\,351.46}} =$$

Incompressive



$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \cancel{\frac{v_2^2}{2g}}^{\,0}$$

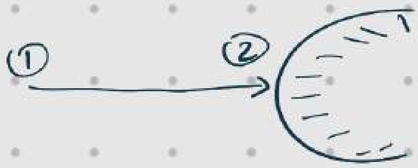
$$\Rightarrow \frac{P_1}{\rho} + \frac{v_1^2}{2} = \frac{P_s}{\rho}$$

$$P_s = P_1 + \frac{1}{2}\rho v_1^2$$

$$\Rightarrow P_s = P_1 + \frac{1}{2}\rho v_1^2$$

$$\Rightarrow P_s = P_1 + \frac{1}{2}\rho v_1^2 (1 + m^2)$$

Compressive



$$h = C_p T$$

7

$$\frac{v_0^2}{2} + h_0 = \frac{v_s^2}{2} + h_s$$

$$\Rightarrow \frac{v_0^2}{2} + C_p T_0 = 0 + C_p T_s$$

$$\Rightarrow T_s = T_0 + \frac{v_0^2}{2C_p}$$

$$C_p - C_v = R$$

$$C_p\left(1 - \frac{C_v}{C_p}\right) = R$$

$$C_p\left(1 - \frac{1}{K}\right) = R$$

$$C_p = \frac{KR}{K-1}$$

$$\Rightarrow \frac{T_s}{T_0} = 1 + \frac{v_0^2}{2T_0}\left(\frac{K-1}{KR}\right)$$

$$= 1 + \frac{v_0^2}{2c^2}(K-1)$$

$$\Rightarrow \frac{T_s}{T_1} = 1 + \frac{m^2}{2}(K-1)$$

$$\Rightarrow \left(\frac{P_s}{P_0}\right)^{\frac{K-1}{K}} = 1 + \frac{m^2}{2}(K-1)$$

$$\Rightarrow \frac{P_s}{P_0} = \left[1 + \frac{m^2}{2}(K-1)\right]^{\frac{K}{K-1}}$$

$$c = \sqrt{KRT_0} \qquad M = \frac{v}{c}$$

$$c^2 = KRT_0$$

$$m = \frac{v_0}{c}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} \ldots$$

$$\Rightarrow \frac{P_s}{P_0} = 1 + \left(\frac{K}{K-1}\right)\frac{m^2}{2}(K-1) + \left(\frac{K}{K-1}\right)\left(\frac{K}{K-1} - 1\right)\frac{1}{2}\frac{m^4}{4}(K-1)^2 + \ldots$$

$$= 1 + \frac{m^2}{2}K + \left(\frac{K}{\ldots}\right)\left(\frac{1}{\ldots}\right)\cdot\frac{m^4}{\ldots}(K-1)^2$$

$= 1 + \frac{}{2} k + \left(\frac{}{k-1}\right)\left(\frac{}{k-1}\right) \quad 8 \left(\ \right)$

$$\Rightarrow \frac{P_s}{P_0} = 1 + \frac{M^2}{2} k + \frac{k}{8} M^4 = 1 + \frac{kM^2}{2}\left[1 + \left(\frac{M^2}{2}\right)^2\right]$$

$$\Rightarrow P_s = P_0 + \frac{1}{2} k M^2 P_0 \left[1 + \left(\frac{M^2}{2}\right)^2\right]$$

$$= P_0 + \frac{1}{2} \rho_0 C_0^2 \cdot \frac{v_0^2}{C_0^2}\left[1 + \frac{M^4}{4}\right]$$

$$= P_0 + \frac{1}{2} \rho_0 v_0^2 \left(1 + \frac{M^4}{4}\right)$$

$C = \sqrt{\frac{Kp}{\rho}}$

$C^2 = \frac{Kp}{\rho}$

$Kp = \rho_0 C_0^2$

$$P_s = P_1 + \frac{1}{2} \rho v_1^2$$

Derivation

(I) Bernoulli

(II) Newton law of viscosity

(III) $P = h\rho g$

(IV) Force is Hydrostatic Plate in water

$PA\rho \sin\theta$

(V) ψ line an ϕ line always perpendicular

(VI) Continuity Equation
- Compressible
- Incompressible

Math

Bernoulli

$h_{Lm} = \frac{fLV^2}{2gD}$

head losses

Kinetic energy
connection
velocity correction

$$Re = \frac{\rho v D_H}{\mu} — \frac{4A}{P_w}.$$

$$h_{fL} = K \frac{v^2}{2g}$$

$\alpha \Rightarrow T = 1 \qquad L = 1.05$

Part-B — 2.5 question