AHSANULLAH UNIVERSITY OF SCIENCE AND TECHNOLOGY (AUST)

ME-3105: FLUID MECHANICS-II (LEC-4: NON-DIMENSIONAL CONSTANT)

BY

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Non-dimensional Constant

In most of the fluid phenomena the following variables may important,

- ✓ Length (L)
- ✓ Acceleration due to gravity (g)
- ✓ Density (ρ)
- ✓ Velocity (V)
- ✓ Pressure (P)
- ✓ Viscosity (μ)
- ✓ Surface Tension (σ)
- ✓ Velocity of Sound (C)

Non-dimensional Constant (Diff. Forces)

The following forces can be formed with these variables:

$$\text{Inertia force} = \text{mass} \times \text{acceleration} = \rho \times L^3 \times \frac{V}{t} = \rho L^2 V^2$$

$$\text{Viscous} = \text{Area} \times \text{Shear Stress} = L^2 \times \mu \times \frac{V}{L} = \mu V L$$

$$\text{Gravity} = \text{mass} \times g = \rho \times L^3 \times g = \rho L^3 g$$

$$\text{Pressure force} = \text{Pressure} \times \text{area} = P L^2$$

$$\text{Elastic Force} = \text{Bulk Modulus of Elasticity} \times \text{area} = E L^2 = \rho C^2 L^2$$
$(E = \rho c^2)$

$$\text{Surface Tension} = \sigma \cdot L$$

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Non-dimensional Group/No.

Reynolds No. (N_{Re}):

- ✓ It is the ratio of inertia force and viscous force.
- ✓ $N_{Re} = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho \cdot V^2 \cdot L^2}{\mu \cdot V \cdot L} = \frac{\rho \cdot V \cdot L}{\mu}$
- ✓ It is used when viscous force is predominant, such as flow in pipe, submerged flow, flow through venturi meter, etc.

Froude No. (F_r):

- ✓ It is the ratio of inertia force and gravity force.
- ✓ $F_r = \frac{\text{Inertia force}}{\text{Gravity force}} = \frac{\rho \cdot V^2 \cdot L^2}{\rho \cdot L^3 \cdot g} = \frac{V^2}{L \cdot g}$
- ✓ It is used when gravitational force is predominant, such as open channel flow, wave motion in ocean, forces on bridge pier and offshore structures, etc.

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Non-dimensional Group/No.

Euler Number (E):

- ✓ It is the ratio of pressure force and inertia force.
- ✓ $\mathrm{E} = \frac{\text{Pressure force}}{\text{Inertia force}} = \frac{\mathrm{P} \cdot \mathrm{L}^2}{\rho \cdot L^2 \cdot V^2} = \frac{\mathrm{P}}{\rho \cdot V^2} = \frac{F}{\rho \cdot V^2 \cdot L^2}$
- ✓ It is used when pressure force is predominant such as flow in pipe, submerged flow, flow through venturi meter, etc.

Mach Number (M):

- ✓ It is square root of the ratio of inertia force to the elastic force.
- ✓ $\mathrm{M} = \left(\frac{\text{Inertia force}}{\text{Elastic force}}\right)^{\frac{1}{2}} = \left(\frac{\rho \cdot \mathrm{V}^2 \cdot \mathrm{L}^2}{\rho \cdot C^2 L^2}\right)^{\frac{1}{2}} = \frac{V}{\mathrm{C}}$
- ✓ The ratio V^2/C^2 is known as Cauchy's number. Mach no. is important in compressive fluid flow at high velocity, such high velocity flow in pipe, projectiles, and missiles, etc.

Non-dimensional Group/No.

Weber Number (W):

- ✓ It is the ratio of inertia force and surface tension force.
- ✓ $\mathrm{W} = \frac{\text{Inertia force}}{\text{Surface tension force}} = \frac{\rho \cdot V^2 \cdot \mathrm{L}^2}{\sigma \cdot L} = \frac{\rho \cdot \mathrm{L} \cdot V^2}{\sigma}$
- ✓ It is used when surface force is predominant, such as capillary tube flow, droplet formation, human blood flow, etc.

Non-dimensional Group or No.

Example: Application of non-dimensional constant

$$F = f(D, V, \rho, \mu) \longrightarrow \frac{F}{\rho V^2 D^2} = f_1\left(\frac{\rho V D}{\mu}\right)$$

The prototype and the model must have the same phenomenon.

$$\frac{F_m}{\rho_m V_m{}^2 D_m{}^2} = f_1\left(\frac{\rho_m V_m D_m}{\mu_m}\right) \quad \left(\frac{F}{\rho V^2 D^2}\right)_{prototype} = f_1\left(\frac{\rho V D}{\mu}\right)_{prototype}$$

Design conditions. $\left(\frac{\rho V D}{\mu}\right)_{model} = \left(\frac{\rho V D}{\mu}\right)_{prototype}$

Then … $\left(\frac{F}{\rho V^2 D^2}\right)_{model} = \left(\frac{F}{\rho V^2 D^2}\right)_{prototype}$

Non-dimensional Group or No.

Example: Application of non-dimensional constant

$$D = f(w, h, \mu, \rho, V) \longrightarrow \frac{D}{w^2 \rho V^2} = \Phi\left(\frac{w}{h}, \frac{\rho V w}{\mu}\right)$$

The prototype and the model must have the same phenomenon.

$$\frac{D_m}{w_m{}^2 \rho_m V_m{}^2} = \Phi\left(\frac{w_m}{h_m}, \frac{\rho_m V_m w_m}{\mu_m}\right) \qquad \left(\frac{D}{w^2 \rho V^2}\right)_{prototype} = \Phi\left(\frac{w}{h}, \frac{\rho V w}{\mu}\right)_{prototype}$$

Design conditions. $\frac{w_m}{h_m} = \frac{w_p}{h_p} \qquad \frac{\rho_m V_m w_m}{\mu_m} = \frac{\rho_p V_p w_p}{\mu_p}$

Then … $\frac{D_p}{w_p^2 \rho_p V_p^2} = \frac{D_m}{w_m{}^2 \rho_m V_m{}^2}$

Non-dimensional Group or No.: Problem-1

A model is used to predict the pressure drop in a large air duct. The pressure drop in the prototype is 2 kN/m^2 and in model 162 kN/m^2 . Water is used in the model test, which is 900 times denser and 90 times more viscous than air. Find the scale ratio.

(SEE HAND ANALYSIS)

Non-dimensional Group or No.: Problem-2

A 1:45 model of airplane is tested in water, which is 50 times more viscous and 750 times denser than air. If the model experiences a drag force of 0.98 N. Find the corresponding drag force on the prototype.

(SEE HAND ANALYSIS)

Non-dimensional Group or No.: Problem-3

A 1:60 model of a boat has a wave resistance of 0.36N when operating at a velocity of 1.25 m/s . Find the corresponding prototype wave resistance. Also find the power requirement for the prototype and the model.

SEE HAND ANALYSIS