{"Fluid Mechanics"}

Problem on Bulk Modulus of Elasticity: (Lec-1)

Problem-1 (Lec-1).

(a) Soln: Given that,

$\rho_1 = \rho_w = 1025 \text{ kg/m}^3$

$\rho_w = \rho_1$ $\quad K = 24000 \text{ kg(f)/cm}^2$.

$P_2 = 840 \text{ kg(f)/cm}^2$

$\rho_2 = ?$ We know,

$$K = \frac{dP}{\frac{d\rho}{\rho}}$$

$$\Rightarrow 24000 = \frac{840}{\frac{d\rho}{\rho_1}}$$

$$\Rightarrow \frac{d\rho}{\rho_1} = \frac{840}{24000}$$

$$\Rightarrow \frac{d\rho}{\rho_1} = 0.035.$$

$d\rho = \rho_2 - \rho_1 = 0.035\,\rho_1$

$$\rho_2 - \rho_1 = 0.035\rho_1$$

$$\Rightarrow \rho_2 = \rho_1 + 0.035\rho_1$$

$$\Rightarrow \rho_2 = \rho_1(1+0.035)$$

$$\Rightarrow \rho_2 = 1.035\,\rho_1$$

$$\therefore \rho_2 = 1.035 \times 1025$$

$$\therefore \rho_2 = 1060.87 \text{ kg/m}^3.$$

$$d\rho = 0.035\rho_1 = 0.035 \times 1060.87$$

$$= 37.13 \text{ kg/m}^3.$$

$$V_2 = \frac{1}{\rho_2} = \cancel{9.43\times10^{-4}\ \frac{m^3}{kg}}$$

$$= \frac{1}{1060.87} = 9.42\times10^{-4} \text{ m}^3/\text{kg}$$

Change in vol $dV = \dfrac{1}{d\rho} = \dfrac{1}{37.13} = 6.0278 \text{ kg/m}^3$ (Ans).

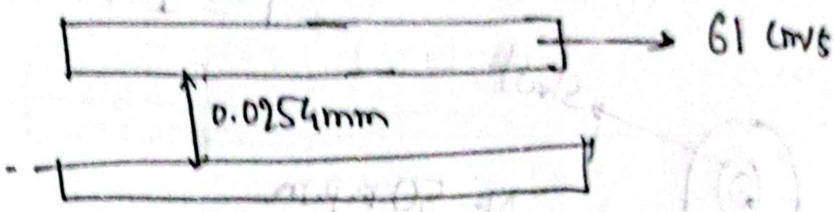
~~1mm~~

$\cancel{1m} = 100cm$
$= 1cm$
$= 10mm$

Lec-2 [Math Problems]

Newton Law of viscosity.

1. Problem: 1



$Y = 0.0254mm$
$= 2.54\times10^{-5} m$

$U = 61\ cm/s$
$= 0.61\ m/s.$

$\frac{F}{A} = 0.2\ kgf/m^2$

$T = (0.2 \times 9.81)$
$= 1.962 N/m^2$
$= \tau$

$\cancel{\tau = F}$

$$\tau = \mu \frac{du}{dy}$$

$$\mu = \frac{\tau}{\frac{du}{dy}}$$

$$= \frac{\tau}{\frac{U}{Y}}$$

$$= \frac{1.962}{24015} = 8.1\times10^{-5}\ Ns/m^2$$

(Ans.)

$$\frac{du}{dy} = \frac{U}{Y}$$

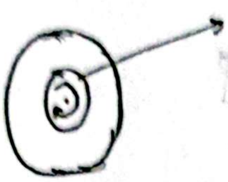
$$= \frac{0.61}{2.54\times10^{-5}}$$

$$\frac{U}{Y} = 24015.74$$

Problem-2: [Journal bearing prob.]

$$\tau = \mu \frac{du}{dy} \qquad \tau = \boxed{\frac{F}{A} = \mu \frac{U}{Y}}$$

$$T = 1 \text{ N-m}.$$



$N = 50$ r.p.m.

$$\omega = \frac{2\pi N}{60} = \frac{2 \times 3.1416 \times 50}{60}$$

$$= 5.235 \text{ rad/s}.$$

$$\boxed{v = \omega r}$$

$L_s = 0.50$ m

$d_s = 95$ mm

$d_c = 96.4$ mm

$r_s = \frac{95}{2} = 47.5$ mm

$= 0.0475$ m

$r_c = \frac{96.4}{2} = 48.2$ mm

$= 0.0482$ m.

$\therefore Y = r = r_c - r_s$ ~~1.2~~

$= 48.2 - 47.5 = \cancel{0.7 \text{ mm} = 7 \times 10^{-3} \text{ m}}$

$\cancel{= 0.7 \times 10^{-3} \text{ m}}$ $0.7 \text{ mm} = 0.7 \times 10^{-3} \text{ m}.$

$U = \omega r_s$

$= 5.236 \times 0.0475$

$= 0.248$ m/s.

$Y = r_c - r_s$

$= 0.7 \times 10^{-3}$ m

$\cancel{\tau = \frac{F}{A} =}$

$T = F \cdot r_s$

$\Rightarrow F = \frac{T}{r_s}$

$= \frac{1}{0.0475}$

$= 21.05$ N.

$A = 2\pi r \times l$

$= 2\pi r_s \times l$

$= 2 \times 3.1416 \times 0.0475 \times 0.5$

$= 0.149$ m^2

$\tau = \frac{F}{A} = \frac{21.05 \text{ N}}{0.149 \text{ m}^2}$

$= 141.06$ N/m^2.

$\mu = \frac{\tau}{\frac{u}{y}} = \frac{141.06}{\frac{0.248}{0.7 \times 10^{-3}}} = 0.398$ Ns/m^2 Ans

Lec-2 # Math: 3 [Capillary rise of fluid].

Soln:

Here,

$d_t = 0.60\,mm$

$r = \frac{d_t}{2}$

$= 0.3\,mm$

$r = 3\times10^{-4}\,m$

$\rho = 1000\,kg/m^3$

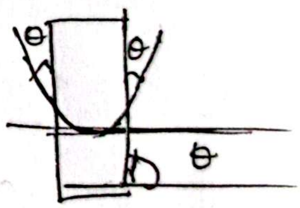
$g = 9.81\,m/s$

$\sigma = 0.073$ from chart

$\theta = 0 = \text{hman}$

$\cancel{\theta = 90^\circ = 0.}$

$\theta = 0^\circ$



$$h = \frac{2\sigma\cos\theta}{\gamma r}$$

$$= \frac{2\sigma\cos\theta}{\rho g r}$$

$$= \frac{2\times0.073\times\cos0^\circ}{1000\times9.81\times0.3\times10^{-4}}$$

$$= \frac{2\times0.073}{1000\times9.8\times3\times10^{-4}}$$

$$= \cancel{0.4965\,m}$$

$$= 0.0496598\,m$$

$$= 49.65\,mm$$

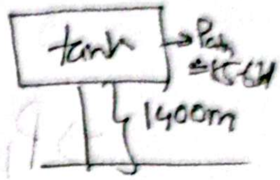
(Ans)

$\gamma = \rho g$

Lec-3 [Math problems] $(\downarrow +)(\uparrow -)$

1. Problem-1:

Here Given



$P_{atm} = 85.6\ kPa$

$h_1 = 0.1m$, $h_2 = 0.2m$, $h_3 = 0.35\ m.$

$\rho_w = 1000\ kg/m^3$, $\rho_{oil} = 850\ kg/m^3$,

$\rho_{hg} = 13{,}600\ kg/m^3.$

$$\cancel{P_p = P_{atm} + \rho_{hg} h_3 g + \rho_{oil} h_2 g + \rho_w h_2 g}$$

$P_A = P_1 + \rho_w g h_1$

$P_B = P_{atm} + \rho_{hg} g h_3 - \rho_{oil} g h_2$

$P_A = P_B$

$\Rightarrow P_1 + \rho_w g h_1 = P_{atm} + \rho_{hg} g h_3 - \rho_{oil} g h_2$

$\Rightarrow P_1 = P_{atm} + \rho_{hg} g h_3 - \rho_{oil} g h_2 - \rho_w g h_1$

$$P_1 + \rho_w g h_1 + \rho_{oil} g h_2 - \rho_{Hg} g h_3 = P_{atm}$$

$$\Rightarrow P_1 = P_{atm} - \rho_w g h_1 - \rho_{oil} g h_2 + \rho_{Hg} g h_3$$

$$= P_{atm} + g(\rho_{Hg} h_3 - \rho_w h_1 - \rho_{oil} h_2)$$

$$= 130 kPa.$$

~~Ans~~

Ans

$$\boxed{\begin{array}{l} 1 kPa = 1000 N/m^2. \\ 1N = 1 kg \cdot m/s^2. \end{array}}$$

Problem-2

Soln:

$$S.G. = \frac{\rho_f}{\rho_w}$$

$$\Rightarrow 0.9 = \frac{\rho_f}{1000}$$

$$\Rightarrow \rho_f = 1000 \times 0.9$$

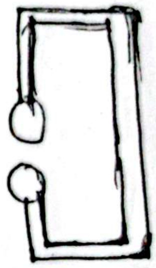
$$= 900\ kg/m^3$$

$h = 1.25\ m$

$z = 0.3\ m.$

$\rho_w = 1000\ kg/m^3$

$\Delta P_{mn} = ?$



$$P_1 = P_m - \rho_w g(z+y)$$

$$P_2 = P_n - \rho_w g(h+y) - \rho_m g z.$$

Now,

$$P_1 = P_2$$

$$P_m - P_n = \rho_w g z + \cancel{\rho_w g y} - \rho_w g h - \cancel{\rho_w g y} - \rho_m g z$$

$$= \rho_w g z - \rho_w g h - \rho_m g z.$$

$$= 12\ kN/m^2$$

(Ans).

Hydrostatic force. [Lec-4] "Math Problems"

Formulas:

$$F = \gamma \sin\theta A\bar{y}$$

$$= \gamma \bar{h} A \qquad (\bar{h} = \bar{y}\sin\theta).$$

$$I_x = I_o + A\bar{y}^2 \qquad y_p = \frac{I_x}{A\bar{y}} = \frac{I_o + A\bar{y}^2}{A\bar{y}}$$

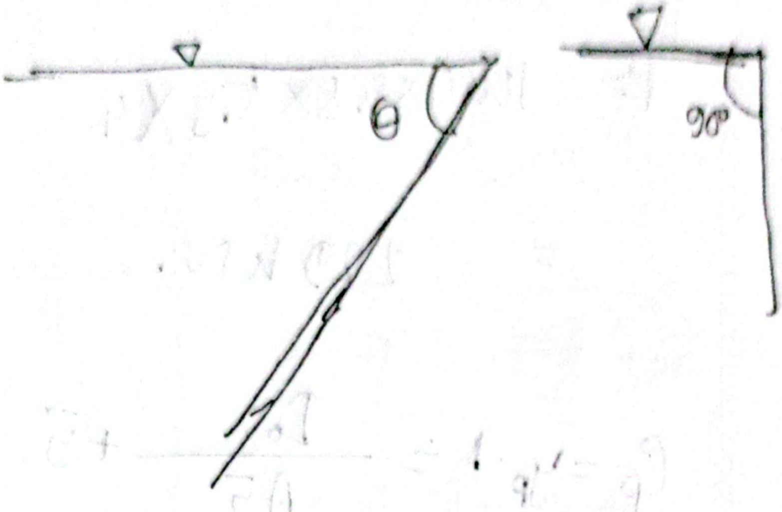
$$= \frac{I_o}{A\bar{y}} + \bar{y}$$

$$I_o = ? \quad \bar{y} = ?$$

Formula chart

Attached

Problem-1



Here $\theta = 90°$ $\therefore \bar{h} = \bar{y}$

$I_x = I_o + A\bar{y}^2$

$F = \gamma \sin\theta A\bar{y}$

$= \gamma \bar{h} A$

$=$

$a = 1$ m

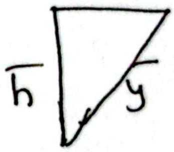
$b = 3$ m.

$h_{depth} = 5$m.

$h = 2$m.

$y_p = \frac{I_o}{A\bar{y}} + \bar{y}$

$\rho = 1000$ kg/m^3



$\bar{h} = 2$ m.

$I_o = \frac{h^3(\cancel{a^2} + 4ab + b^2)}{3(a+b)}$

$A = \frac{h(a+b)}{2}$

$= \frac{2(1+3)}{2} = 4.$

$I_o = 1.22$ m^4.

$\bar{y} = 5 + 0.83$

$= 5.83$ m.

$y_{cg} = \frac{h(2a+b)}{3(a+b)}$

$= 0.83$m.

$P = 1000 \times 9.8 \times 5.8 \times 4$

$= 229 \text{ kN}.$

$$C_p = y_p \cdot d = \frac{I_o}{A\bar{y}} + \bar{y}$$

$$= \frac{1.22}{4 \times 5.83} + 5.83$$

$= 5.88 \text{ m}$ (Ans).

Problem-2

Here,

$\theta = 45°$

$h = 6m$ $\quad F = ?$ $\quad y_p = ?$

$r = 4m$

here

$A = \frac{1}{2}\pi r^2$

$y_{cg} = \frac{4r}{3\pi}$

$I_o = \frac{(9\pi^2 - 64) r^4}{72\pi}$

$\cancel{r = 4m}$

$\bar{h} = 6 + y_1$

$\cancel{\bar{y} = \bar{h}/\sin\theta}$

$\boxed{\bar{h} = 5 \sin\theta}$

We know

$F = \gamma \bar{h} A \quad [\bar{h} = \bar{y} \sin\theta]$

$y_p = \frac{I_o}{A\bar{y}} + \bar{y}$

$A = \frac{1}{2}\pi (4)^2$

$= 25.13 m^2$.

$y_{cg} = \frac{4r}{3\pi} = \frac{4 \times 4}{3\pi}$

$= 1.70 m$.

$$I_o = \left(\frac{9\pi^2-64}{72\pi}\right) r^4 \qquad = \left(\frac{9\pi^2-64}{72\pi}\right)(4)^4$$

$$= 28.10$$

$\bar{h} = 6 + 1.70 = 7.70\,m.$

$\cancel{\bar{y} = \bar{h}\sin\theta}$

$$\bar{y} = \frac{\bar{h}}{\sin\theta} = \frac{7.70}{\sin 45^\circ}$$

$\therefore\ \bar{h} = \bar{y}\sin\theta$

$$= 10.88\,m$$

$$\boxed{P = 1000 \times 9.8 \times 7.70 \times 25.13 = 1898\ kN.}$$

$$y_P = \frac{I_o}{A\bar{y}} + \bar{y}$$

$$= \frac{28.1}{25.13 \times 10.88} + 10.88$$

$$= 10.98\ m$$

Ans

$\boxed{\text{Lec-7}}$ Math

Math Problems:

$u =$

$v =$

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$$

$\phi = ?\ \phi \to ?$

$$-\frac{\partial \phi}{\partial x} = u$$

$$-\frac{\partial \phi}{\partial y} = v$$

$$\partial \phi = -u \, \partial x$$

$$\partial \phi = -v \, \partial y$$

Pblm-Q.1

Soln:

$u = x - 4y$

$$\frac{\partial u}{\partial x} = 1$$

$$\frac{\partial u}{\partial y} = -4$$

$v = -y - 4x$

$$\frac{\partial v}{\partial y} = -1$$

$$\frac{\partial v}{\partial x} = -4.$$

(a) **Continuity eqn:**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 1 - 1 = 0$$ [Satisfies continuity eqn]

(b) Stream function.

$u = x - 4y$

$v = -y - 4x$

$u = \frac{\delta \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$

$\Rightarrow \delta\psi = (x-4y)\,\delta y$ $\quad u = x-4y$

$\Rightarrow \psi = xy - \frac{4y^2}{2} + f(x)$ — (1) [Integrating both sides].

$\Rightarrow$

$$\frac{\partial \psi}{\partial x} = y - 0 + \frac{\delta f(x)}{\partial x}.$$

$$-v = y + \frac{\delta f(x)}{\delta x}.$$

$$-(-y-4x) = y + \frac{\delta f(x)}{\delta x}$$

$$\Rightarrow -(-y-4x) = y + \frac{\delta f(x)}{\delta x}$$

$$\frac{\delta f(x)}{\delta x} = 4x.$$

$$\frac{\delta \psi}{\delta x} = y - 0 + \frac{\delta f}{\delta x}$$

$$\frac{\delta \psi}{\delta y} = x -$$

$$v = -y - 4x$$

$$\cancel{\frac{\delta\psi}{\delta y} = x - 4y + 0.}$$

$\cancel{f(x) \Rightarrow}$

$$\frac{\delta f(x)}{\delta x} = 4x$$

$$\Rightarrow f(x) = 4\frac{x^2}{2} + C_1 \quad \text{[Integration on both]}.$$

$$\Rightarrow f(x) = 2x^2 + C_1 \quad \ldots$$

$$\psi = x.y - 4\left(\frac{y^2}{2}\right) + 4\left(\frac{x^2}{2}\right) + C_1$$

$$\Rightarrow \psi = xy - 2y^2 + 2x^2 + C_1$$

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(c) Potential function:

$$u = x - 4y$$

$$v = -y - 4x$$

$$u = -\frac{\delta\phi}{\delta x}$$

$$\delta\phi = -(x-4y)\,\delta x$$

$$\Rightarrow \phi = -\frac{1}{2}x^2 + 4xy + f(y) \text{ ——— } (4) \text{ eqn no } (2)$$

$$\Rightarrow \frac{\delta\phi}{\delta y} = 4x + \frac{\delta f(y)}{\delta y}.$$

$$\Rightarrow -(-y-4x) = 4x + \frac{\delta f(y)}{\delta y}$$

$$\frac{\delta f(y)}{\delta y} = y$$

$$f(y) = \frac{y^2}{2} + c_2 \text{ ——— } (3)$$

from ② & ③ we get

$$\phi = -\frac{1}{2}x^2 + 4xy + \frac{y^2}{2} + C_2.$$

From Laplace eqn

$$\frac{\delta^2\phi}{\partial x^2} + \frac{\delta^2\phi}{\delta y^2} + \frac{\delta^2\phi}{\delta z^2} = 0.$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

Problem

$u = x - 4y - 4y \quad v = -y - 4x$

Problem-2

$u = \frac{y^3}{3} + 2x - x^2 y$

(a) Soln:

$v = xy^2 - 2y - \frac{x^3}{3}$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\therefore \frac{\partial u}{\partial x} = 2 - 2xy$$

$$\frac{\partial v}{\partial y} = 2xy - 2$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \cancel{2} - \cancel{2xy} + \cancel{2xy} - \cancel{2}$$

$$= 0.$$

$\therefore$ Fluid is incompressible

(Ans)

(b)

$$\frac{\partial \psi}{\partial x} = v \qquad \frac{\partial \psi}{\partial y} = u.$$

$$\psi = v\, \partial x$$

$$\Rightarrow \psi = \int v\, dx$$

$$= \int \left(xy^2 - 2y - \frac{x^3}{3}\right) dx$$

$$= \frac{x^2y^2}{2} - 2xy - \frac{x^4}{12} + F(y) + C \quad \text{--- (i)}$$

Similarly

$$\frac{\partial \psi}{\partial y} = -u$$

$$\psi = \int -u\, dy$$

$$\psi = \int -\left(\frac{y^3}{3} + 2x - x^2y\right) dy$$

$$\Rightarrow \psi = -\frac{y^4}{12} - 2xy + \frac{x^2y^2}{2} + f(x) + C \quad \text{--- (ii)}$$

From (i) + (ii) ~~করে পাই~~

$$\psi = \frac{x^2y^2}{2} - 2xy - \frac{x^4}{12} - \frac{y^4}{12} + C.$$

(Ans)

(c) Velocity Potential (ϕ).

$$\frac{\partial \phi}{\partial x} = -u.$$

$$\Rightarrow \phi = -\int u.dx$$

$$\Rightarrow \phi = -\int \left(\frac{y^3}{3} + 2x - x^2y\right) dx$$

$$= -\left[\frac{y^3}{3} + \frac{\cancel{2}x^2}{\cancel{2}} - \frac{x^3y}{3}\right] + f(y) + C \text{ — (i)}$$

$$= -\frac{y^3}{3} + x^2 - \frac{x^3y}{3} + f(y) + C \text{ —— (i)}$$

$$\frac{d\phi}{\partial y} = -v$$

$$\phi = -v\,dy$$

$$= -\int \left(xy^2 - 2y - \frac{x^3}{3}\right) dy$$

$$= -\left[\frac{xy^3}{3} - y^2 - \frac{x^3 y}{3}\right] + f(x) + c \quad \text{(ii)}$$

From (i) ~~and~~ (ii)

$$\phi = \frac{x^3 y}{3} - \frac{xy^3}{3} - x^2 + y^2 + c = 0$$

Ans

Pblm-4 Solution:

To show the incompressibility of the vector the velocity vector must be zero.

Now

$$\frac{\partial}{\partial x}(x^3y) + \frac{\partial}{\partial y}(y^2z) + \frac{\partial}{\partial z}\left(-(3x^2yz + yz^2)\right)$$

$$= 3x^2 + 2yz - (3x^2y + 2yz) = 0.$$

[incompressible]

Hen

$$\frac{\partial \vec{v}}{\partial t} = 0.$$ [Steady].

Now substitute $x=1$, $y=2$, $z=3$ into the velocity vector

It becomes:

$$\vec{V} = (1^3 \cdot 2)\hat{i} + (2^2 \cdot 3)\hat{j} - (3 \cdot 1^2 \cdot 2 \cdot 3 + 2 \cdot 3^2)\hat{k}.$$

$$= 2\hat{i} + 12\hat{j} - (18 + 18)\hat{k}.$$

$$= 2\hat{i} + 12\hat{j} - 36\hat{k}$$

Since As it is a steady flow.

$$\frac{\partial \vec{v}}{\partial t} = 0.$$

$$\vec{a} = (2\hat{i} + 12\hat{j} - 36\hat{k}).\nabla\,(x^3y\hat{i} + y^2z\hat{j} - (3x^2yz + yz^2)\hat{k})$$

$$\therefore \quad \vec{a} = 0.$$

$$\vec{a} = (2\hat{i} + 12\hat{j} - 36\hat{k}) \cdot (3x^2y\hat{i} + 2yz\hat{j} - (3x^2y + 2yz)\hat{k}).$$

$(1, 2, 3) \Rightarrow$ give $\vec{a} = 0$ Ans

Pblm-3

[L-8] {Math Problems}

1. Solution:

$$v = \sqrt{2gh\frac{\rho_m}{\rho_f}}$$

$$= \sqrt{2 \times 9.8 \times 0.03 \times \frac{13000}{1.29}}$$

$$= \sqrt{}$$

$$= 76.97 \text{ m/s} \quad \text{Ans}$$

$$v = \cancel{\sqrt{2gh\frac{\rho_m}{\rho_f}}}$$

$$= \cancel{\sqrt{2 \times 9.8 \times 0.03 \times \frac{13000}{1000}}}$$

$$= \cancel{\sqrt{2 \times 9}}$$

$$= \cancel{4.76681} \text{ m/s} \quad (\text{Ans})$$

0

2. Solution:

From Continuity eqn

$$Q = A_1V_1 = A_2V_2$$

$$\Rightarrow V_1 = \left(\frac{A_2}{A_1}\right) V_2$$

$$\Rightarrow V_1 = \left(\frac{0.02}{0.1}\right) \times 50$$

$$\Rightarrow V_1 = 10 \text{ m/s}$$

Applying Bernaullis eqn

$\rho_{air} = 1.29\, kg/m^3$.

$\gamma = \rho g \Rightarrow \gamma = 1.29 \times 9.8$

$= 12.64$

$= 11$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{Z_1} = \frac{\cancel{P_2}^{\,0}}{\gamma} + \frac{V_2^2}{2g} + \cancel{Z_2}$$

$$\Rightarrow \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{V_2^2}{2g}$$

$$\Rightarrow \frac{P_1}{\gamma} + \frac{(10)^2}{2g} = \frac{(50)^2}{2g}$$

$$\Rightarrow \frac{P_1}{\gamma} = \frac{50^2}{2g} - \frac{10^2}{2g}$$

$$\Rightarrow P_1 = \gamma\left(\frac{50^2 - 10^2}{2g}\right)$$

$$\cancel{\Rightarrow P_1 = \rho g \frac{2400}{2g}}$$

$$\cancel{\Rightarrow P_1 = \rho \times 1200}$$

$$\cancel{\Rightarrow P_1 = 12.64 \times 1200 =}$$

$\gamma = \rho g \Rightarrow \gamma = 9.8 \times 1.2$

$= 11.86 \ N/m^3$

$$P_1 = \gamma \left(\frac{\cancel{P_2}^{\,0}}{\gamma} + \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \right)$$

$\Rightarrow P_1 = 11.8 \times 122.44$

$= 1.44 \times 10^3 \ Pa$

$\therefore P_1 = 1.44 \ kPa$ (Ans)

0

Pblm: 3 Soln:

Applying Bernoulli's equation:-

$$\frac{P_2}{\gamma} + 0 + Z_2 = \frac{P_1}{\gamma} + 0 + 0$$

$$\Rightarrow \frac{\cancel{P_2}^{\to 0}}{\gamma} + Z_2 = \frac{P_1}{\gamma}$$

$$\Rightarrow Z_2 = \frac{400\text{ kPa}}{\gamma}$$

$$\Rightarrow Z_2 = \frac{400 \times 1000}{\rho \times g}$$

$$= \frac{400 \times \cancel{1000}}{\cancel{1000} \times 9.8}$$

$$Z_2 = h = 40.81 \text{ m}$$

$$\therefore Z_2 = h = 40.81$$

(Ans)...

Problem-4 : Sol^n:

Given that,

$D_1 = 0.46m = z_1$

$D_2 = 0.051m = z_2$ $\quad$ $Q = A_2 \times V_2$

$V_1 = 0$ $\quad$ $Q = V_2 \times W \times Z$

$V_2 = ?$ $\quad$ $\frac{Q}{W} = ?$

Applying Bernoulli's eqn -

$P_1 = P_{atm} = P_2$

$$\cancel{\frac{P_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}}^{\to 0} + z_1 = \cancel{\frac{P_2}{\gamma}} + \frac{V_2^2}{2g} + z_2$$

$$\Rightarrow z_1 = \frac{V_2^2}{2g} + z_2$$

$$\Rightarrow V_2^2 = 2g(z_1 - z_2)$$

$$\Rightarrow v_2 = \sqrt{2g(0.46 - 0.051)}$$

$$\Rightarrow v_2 = 2.831 \text{ m/s}.$$

$$\boxed{Q = A_2 V_2} \Rightarrow Q = W.h \times V_2$$

$$\frac{Q}{W} = h \times V_2$$

$$\Rightarrow \frac{Q}{W} = 0.051 \times 2.831$$

$$\frac{Q}{W} = 0.14438$$

$$\frac{Q}{W} = 0.144 \ \frac{m^3/s}{m/1}$$

$$= 0.144 \ \frac{m^3}{s} \times \frac{1}{m}$$

$$= 0.144 \ m^2/s$$

Answer

[Lec-9] Math Problem

Problem 1: Solution:

Given that, $Q = 0.15\ m^3/s$

$h_{Lf} = 3.22\ m$

$Q = A_1V_1 = A_2V_2 = \text{Const.} \Rightarrow V = \frac{Q}{A}$

$h_{pump} = ?$ $P_1 = ?$ $P_2 = ?$

$D_1 = 230\ mm = 0.23\ m$

$D_2 = 180\ mm = 0.18\ m$

$V_1 = \frac{Q}{A}$

$= \frac{0.15}{\frac{\pi}{4}(0.23)^2}$

$= 3.61\ m/s$

$V_2 = \frac{0.15}{\frac{\pi}{4}(0.18)^2}$

$= 5.89\ m/s$

$P_{pum} = \rho g Q h_{pump}$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 + h_{pump} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_{Lf} \longrightarrow (1)$$

2)

$\rho_w = 1000\ kg/m^3$

~~$\rho_m = 13.5$~~ $\rho_m = 13.56 \times 1000$

$= 1356 \times 10^4\ kg/m^3$.

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From eqn ①. $z_1 = 0$ [red line] datum

$$h_{pump} = \frac{P_2}{\gamma_w} - \frac{P_1}{\gamma_w} + \frac{V_2^2}{2g} - \frac{V_1^2}{2g} + z_2 + h_{L(f)}$$

$$P_1 + z\rho_w g + h\rho_m g = P_2 + (z_2 + z + h)\rho_w g.$$

$$\Rightarrow P_2 - P_1 = z\gamma_w + h\gamma_m - z_2\gamma_w - z\gamma_w - h\gamma_w$$

$$\Rightarrow \frac{P_2 - P_1}{\gamma_w} = \frac{h\gamma_m}{\gamma_w} - z_2 - h.$$

$$\Rightarrow \frac{P_2 - P_1}{\gamma_w} = h\left(\frac{\gamma_m}{\gamma_w} - 1\right) - z_2 \quad \text{--- ②.}$$

$$\therefore h_{pump} = h\left(\frac{\gamma_m}{\gamma_w} - 1\right) \cancel{-z_2} + \cancel{z_2} + \frac{V_2^2}{2g} - \frac{V_1^2}{2g} + h_f$$

Now

$$h_{pump} = h.\left(\frac{\gamma_m}{\gamma_w} - 1\right) + \frac{V_2^2 - V_1^2}{2g} + h_{lf}$$

$$= 3\left(\frac{13600}{1000} - 1\right) + \frac{(5.895)^2 - (3.61)^2}{2\times 9.8} + 3.22$$

$$= 37.68 + 3.22 + 1.108$$

$$\therefore h_{pump} = 42.008 \text{ m.}$$

$$\simeq 42 \text{m.}$$

$$P_{pump} = \rho_w \times h_{pump} \times g \times Q$$

$$= 1000 \times 42 \times 9.8 \times 0.15$$

$$= 6.174 \times 10^4 \text{ W.}$$

$$= 61 \text{ kW. (Ans)}$$

Problem-2 Solution:

Given that,

$z_1 = 70m$ $\quad h_{L} = 5m$ $\quad \eta_t = 80\%$

$z_3 = 20m$ $\quad D_n = 50mm$ $\quad = 0.80$

$\rightarrow 0.05m$

$V_4 = 15 m/s$ $\quad \theta_n = \cancel{450}\ 45^\circ$

$$Q = AV$$

$$\Rightarrow Q = \frac{\pi}{4} D_n^2 \times V_3$$

$$=$$

$$P_{turbine} = \rho g Q \cdot h_{turbine} = \text{Input}$$

$$\eta_t = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{\rho \cdot g \cdot Q\, h_{turbine}}$$

$P_{out} = ?$

$$Q = AV$$

$$\Rightarrow Q = \frac{\pi}{4} D_n^2 \times V_3$$

$$V_3 (\cos 45^\circ) = V_4$$

$$\rightarrow V_3 = \frac{V_4}{\cos 45^\circ}$$

$$\therefore V_3 = \frac{15}{\cos 45^\circ} = 21.21 m/s$$

$$Q = A_3 V_3$$

$$= \frac{\pi}{4}(D_3)^2 \times V_3$$

$$= \frac{\pi}{4} \times (0.05)^2 \times 21.21$$

$$= 0.0416$$

Applying Bernoulli's eqn between 1 and 3 -

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = h_{turbine} + h_f + \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + z_3$$

$$\rightarrow \cancel{\frac{P_{atm}}{\gamma}} + \cancel{\frac{0}{2g}} + 70 = h_{turbine} + 5 + 20 + \cancel{\frac{P_{atm}}{\gamma}} + \frac{V_3^2}{2g}$$

$$\rightarrow h_{turbine} = 70 - 25 - \frac{V_3^2}{2 \times 9.8}$$

$$= 45 - \frac{(21.21)^2}{19.6}$$

$$= 22.05$$

$h_{turbine} = 22.08$ m

$$\eta_t = \frac{P_{out}}{\rho g Q h_{turbine}}$$

$$\Rightarrow 0.80 = \frac{P_{out}}{1000 \times 9.8 \times 0.0416 \times 22.08}$$

$$\Rightarrow P_{out} = 0.80 \times 1000 \times 9.8 \times 0.0416 \times 22.08$$

$$\Rightarrow P_{out} = 7194.7386$$

$$= 7.194 \text{ kW}$$

$$\therefore P_{out} = 9.64 \text{ hp}$$

(Ans)

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~~3 hours there~~

musk melon

Math of Lec-10

1. Soln: $F = \rho . Q . (v_2 - v_1)$; $P_{pump} = \rho g \boxed{Q . h_{pump}} \to ?$

$F = \rho Q (v_3 - v_1)$ $\quad Q = AV$ $\quad P_{pump} = 8\ kW$.

$V_1 = 0,\ z_1 = 0$ $\quad P_1 = P_{atm} = P_3$ $\quad z_3 = 2\ m$ $\quad V_3 \to ?$ $\quad h_f = 0$. Assume

$$\cancel{\frac{P_1}{\gamma_w}} + \frac{V_1^2}{2g} + \cancel{z_1} + h_{pump} = \cancel{\frac{P_3}{\gamma_w}} + \frac{V_3^2}{2g} + \cancel{z_3}^{\,2} + \cancel{h_f}$$

$$\Rightarrow h_{pump} = \frac{V_3^2}{2g} + 2 \quad \cdots\cdots (1)$$

$P_{pump} = \rho g Q . h_{pump}$ $\quad$ and $\quad Q = A_3 V_3$.

$\therefore Q = 0.064\ m^3/s$ $\quad Q = \frac{\pi}{4} d_3^2 V_3$ (14.50)

$$\Rightarrow 8000 = 1000 \times 9.8 \times \frac{\pi}{4} (0.075)^2 \cdot V_3 \times h_{pump}.$$

$$\Rightarrow \cancel{8000}\, h_{pump} = \frac{184.60}{V_3} \quad \text{———} (2)$$

~~hpump~~ Now,

$$\frac{184.60}{V_3} = \frac{V_3^2}{2g} + 2$$

$$\Rightarrow 3621.85 = V_3^3 + 39.24 V_3$$

$\therefore V_3 = 14.50\ m/s$ by calculator.

$\therefore F = \rho_w \cdot Q \cdot (v_3 - v_1)$ $\quad v_1 = 0, \quad v_3 = 14.5$

$= 1000 \times 0.064 \times 14.5$

$= 928.856 \, N$ (Ans)