AHSANULLAH UNIVERSITY OF SCIENCE AND TECHNOLOGY (AUST)

# ME-3105: FLUID MECHANICS

(LC-9: Application of Bernoulli's Equation)

BY

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**Study List of Previous Lecture !!!**

1) Derive Bernoulli's equations with assumption and necessary sketches.
2) Is it possible to apply Bernoulli's equation directly in the case of real fluid flow in the pipe? Explain briefly and write the equation in the case of real fluid flow.
3) Write Bernoulli's equation and explain the each terms and notation, briefly.
4) State the Bernoulli's equation in one sentence.
5) What is Stagnation pressure? How it measures in the fluid flow. What is the application of stagnation pressure?
6) What is Pitot tube? Explain it's application briefly.
7) Define Hydraulic Grade Line (HGL) and Energy Grade Line (EGL)? Show the ideal and real HGL, EGL and also Dynamic head line in a graph.
8) Write Bernoulli's equation between two points of a fluid system with a pump and turbine, explain each term of the equation.

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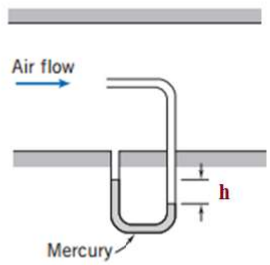
## Assignment

A Pitot tube is inserted in an air flow (at STP) to measure the flow speed. The tube is inserted so that it points upstream into the flow and the pressure sensed by the tube is the stagnation pressure. The static pressure is measured at the same location in the flow, using a wall pressure tap. Show that velocity of the air,

$$V_{air} = \sqrt{2 \cdot g \cdot h \cdot \left(\frac{\rho_m}{\rho_{air}} - 1\right)}$$

Where $\rho_m$ is the density of the mercury and h is the deflection of manometric fluid.




Also find the velocity of air if difference of height of manometric fluid is equal to the last two digits of your ID.
h = X (cm) for single digit; and XX (mm) for double digit.

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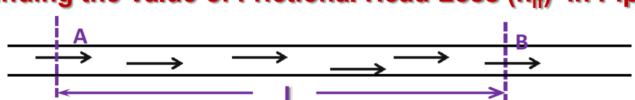
## Finding the Value of Frictional Head Loss ($h_{lf}$) in Pipe




1) Hagen-Poisellie's Equation, $Q = \frac{\Delta P \cdot \pi \cdot D^4}{128 \cdot \mu \cdot L}$, $h_{lf} = \frac{P_A - P_B}{\gamma} = \frac{\Delta P}{\gamma}$

where Q = discharge ( $\frac{m^3}{s}$ )

$\Delta P$ = Pressure difference between two points ( Pa or $\frac{N}{m^2}$ )

$\mu$ = Viscosity of fluid also called dynamic or absolute viscosity ( Poise or $\frac{N \cdot s}{m^2}$ ) and

L = Length of pipe between two points (m)

* Applicable for only Laminar flow, incompressible and Newtonian fluid. This is the limitation of Hagen-Poisellie's equation.

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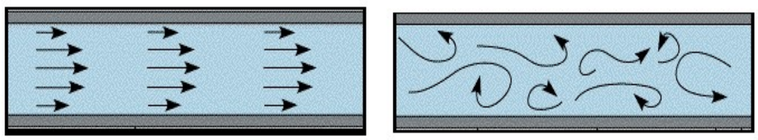
### TYPES OF FLUID FLOW (Refreshing Your Mind)

***Laminar Flow:*** A flow is said to be laminar when the fluid particles are move in a layers (or laminate) with *one layer of fluid sliding smoothly over adjacent layer and there is no momentum transfer in between the layers*. Viscosity of fluid plays important role in development of Laminar Flow.

***Turbulent Flow:*** A fluid in motion is said to be turbulent when the fluid particles move in an entirely haphazard or disorderly manner, that results in *a rapid and continuous mixing of the fluid leading to momentum transfer as flow occurs*.



Laminar Flow    Turbulent Flow

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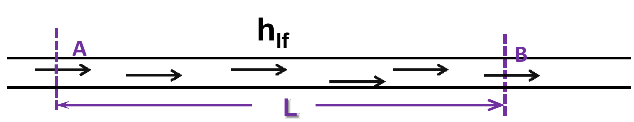
### Finding the Value of Frictional Head Loss ($h_{lf}$) in Pipe




2) Darcy-Weisbach , $h_{lf} = \dfrac{f \cdot L \cdot V_{avg}^{2}}{2 \cdot g \cdot D}$ ;

where $h_{lf}$ = Frictional head loss in pipe (m)

$L$ = Length of pipe between two points (m)

$V_{avg} = \dfrac{Q}{A}$ = Average velocity of fluid in the pipe ( $\dfrac{m}{s}$ )

$D$ = Diameter of the pipe (m)

$f$ = Darcy friction factor

This equation is valid for both laminar and turbulent flow. However, how you will find the value of the Darcy friction factor ' **f** ' (see next slide).

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## Finding the Value of Darcy Friction Factor ' f '

Darcy-Weisbach , $h_{lf} = \frac{f \cdot L \cdot V_{avg}^{2}}{2 \cdot g \cdot D}$

For Laminar flow, $f = \frac{64}{R_e}$ for laminar flow (not valid for turbulen flow)

For turbulent flow, value of friction factor ' f ' need to find from the following equation or Moody diagram.

$$\frac{1}{\sqrt{f}} = -2 \cdot \log_{10}\left(\frac{\frac{\varepsilon}{D}}{3.7} + \frac{2.51}{R_e \cdot \sqrt{f}}\right)$$

Where $\varepsilon$ = Roughness factor, D = Diameter of pipe, $R_e$ = Reynolds number

what is Reynolds number ' $R_e$ ' (see next slide).

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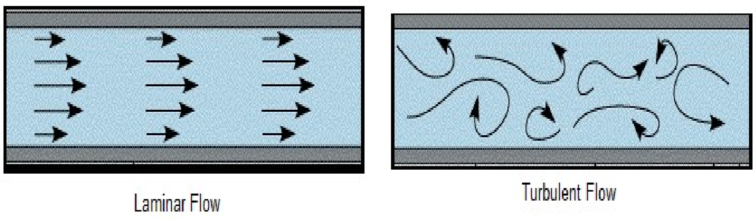
## Refresh Your Mind: Turbulent and Laminar Flow

**Laminar Flow:** One layer of fluid sliding smoothly over adjacent layer and there is no momentum transfer in between the layers.

**Turbulent Flow:** Fluid flows in haphazard or disorderly manner, a rapid and continuous mixing of the fluid leading to momentum transfer as flow occurs.

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## Reynolds's Number, $R_e$

- A flow to be Laminar or turbulent depends on the geometry, surface roughness, flow velocity, surface temperature, type of fluid and some other factors. In 1880's Reynolds discovered that the flow regime (laminar or turbulent) depends mainly on the ratio of inertial forces to viscous forces in the fluid.
- Reynolds Number: The ratio of inertial force to viscous force in the fluid flow is called the Reynolds number, and it is expressed for internal flow in a circular pipe as ' $R_e$ '

$$R_e = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{m \cdot a}{\tau \cdot A} = \frac{\rho \cdot L^3 \cdot \frac{V}{t}}{\mu \cdot \frac{V}{L} \cdot L^2} = \frac{\rho \cdot L^2 \cdot V^2}{\mu \cdot V \cdot L} = \frac{\rho \cdot V \cdot L}{\mu}$$

$$\tau = \mu \cdot \frac{dv}{dy}$$

For pipe flow, $R_e = \frac{\rho \cdot V \cdot D_h}{\mu}$ (general form for internal flow)

where $D_h$ is known as hydraulic diameter and

$$D_h = \frac{4 \times \text{Cross Sectional Area}}{\text{Wetted Perimeter}} = \frac{4 \times A}{P}$$

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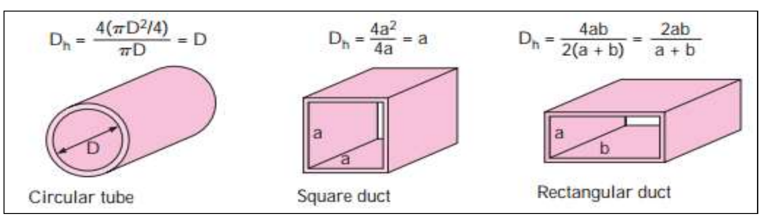
## Reynolds's Number, $R_e$ (Continue)

- The Reynolds number at which fluid becomes Laminar to turbulent is called critical Reynolds number.
- For internal flow through pipe, the critical Reynolds number is 2300.
- If Reynolds number is higher than 4000, flow is turbulent.
- If Reynolds number is less than 2300, flow is laminar.

| | |
|---|---|
| $Re \lesssim 2300$ | laminar flow |
| $2300 \lesssim Re \lesssim 4000$ | transitional flow |
| $Re \gtrsim 4000$ | turbulent flow |

$$D_h = \frac{4 \times \text{Cross Sectional Area}}{\text{Wetted Perimeter}} = \frac{4 \times A}{P}$$

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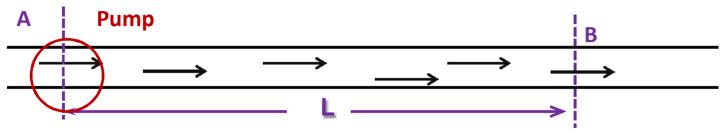
## Pump and Turbine in Fluid System

**Pumps** add energy to the fluid, which is equal to the head of pump, $h_{pump}$ .




- Apply Bernoulli's equation between point 'A' and 'B'.

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2\cdot g} + Z_A + h_{pump} = \frac{P_B}{\gamma} + \frac{V_B^2}{2\cdot g} + Z_B + h_{Lf}$$

Energy of fluid at point 'A'

Energy added to fluid by Pump

Energy remaining in the fluid.

Frictional Loss of Energy between Points A and B.

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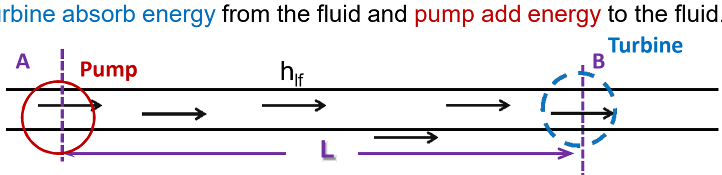
## Pump and Turbine in Fluid System

Turbine absorb energy from the fluid and pump add energy to the fluid.




$$\frac{P_A}{\gamma} + \frac{V_A^2}{2\cdot g} + Z_A + h_{pump} = \frac{P_B}{\gamma} + \frac{V_B^2}{2\cdot g} + Z_B + h_{Lf} + h_{turbine} + e_{mech\_loss}$$

Energy remains in the fluid when leaving point B.

Energy absorb by Turbine

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## Power and Efficiency of Pump and Turbine

Fluid Power, $P = \rho \cdot g \cdot Q \cdot h$; Power of pump, $P_{pump} = \rho \cdot g \cdot Q \cdot h_{pump}$ and Power of Turbine, $P_{turbine} = \rho \cdot g \cdot Q \cdot h_{turbine}$;

Checking Unit: $\frac{kg}{m^3} \cdot \frac{m}{s^2} \cdot \frac{m^3}{s} \cdot m = \frac{kg \cdot m}{s^2} \cdot \frac{m}{s} = \frac{N \cdot m}{s} = \frac{J}{s} = \text{Watts}$

Efficiency, $\eta_{pump} = \frac{\rho \cdot g \cdot Q \cdot h_{pump}}{P_{input}}$ and $\eta_{turbine} = \frac{P_{output}}{\rho \cdot g \cdot Q \cdot h_{turbine}}$

Frictional head loss in the pipe, $h_{lf} = \frac{f \cdot L \cdot V_{avg}^2}{2 \cdot g \cdot D}$

Pressure drop in the pipe due to friction, $\Delta P = \rho \cdot g \cdot h_{lf}$

$\frac{P}{\gamma} + \frac{V^2}{2 \cdot g} + Z = \text{Const}$

$Q = A \cdot V = \text{Const}$

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## PROBLEM-1

Water is flowing at the rate of 0.15 m³/s through the piping system as shown in figure below. Find power requirement by the pump. Assume frictional head loss in the pipe is 3.22 m.

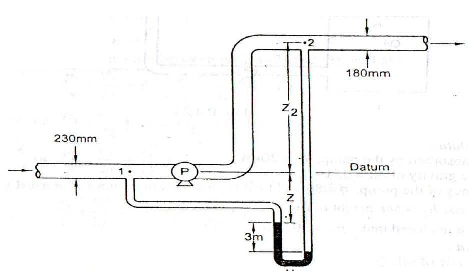
$Q = 0.15 \cdot \frac{m^3}{s}$ $\quad Q = A_1 \cdot V_1 = A_2 \cdot V_2 = \text{Const}$

$h_{pump} = ??$ $\quad P_1 = ???$ $\quad P_2 = ???$;

$D_1 = 230 \cdot mm = 0.23 \cdot m$

$D_2 = 180 \cdot mm = 0.18 \cdot m$

$h_{lf} = 3.22 \cdot m$




$P_{pump} = \rho \cdot g \cdot Q \cdot h_{pump}$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2 \cdot g} + Z_1 + h_{pump} = h_{lf} + \frac{P_2}{\gamma} + \frac{V_2^2}{2 \cdot g} + Z_2$$

Let's solve it !!!

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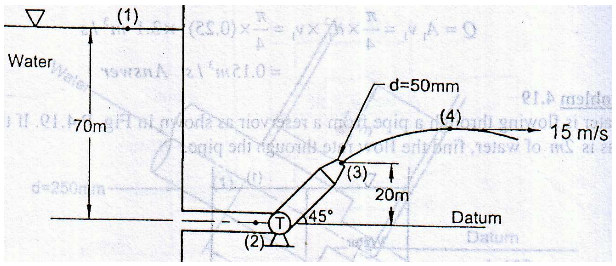
**PROBLEM-2**

Water is flowing from a turbine as shown in the figure below. Find the power developed by the turbine if efficiency of turbine is 80% and frictional head loss in the pipe is 5 m.




$P_{turbine} = \rho \cdot g \cdot Q \cdot h_{turbine}$  $\eta_{turbine} = \dfrac{P_{output}}{\rho \cdot g \cdot Q \cdot h_{turbine}}$

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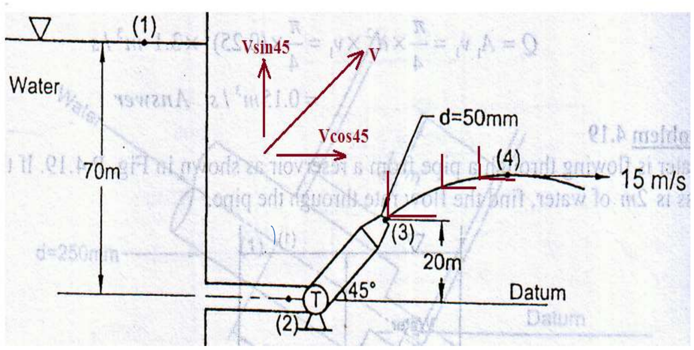
**PROBLEM-2 (Continue)**




Horizontal component, $V_h = V*Cos(45)$ is always constant up to point 4 and hitting the ground.

i.e. $V_h = V_3 *Cos\ (45) = 15$ m/s

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## Study List of this Lecture !!!

1) How do you measure the frictional head loss in the pipe flow? Write the corresponding two equations with their name and limitation.
2) What is Darcy friction factor? How do you find the value of Darcy friction factor in a pipe flow? Explain briefly.
3) What is Moody Diagram? Explain it's application briefly. What type of data/parameter needed to find the useful data from the Moody diagram?
4) Is it possible to find Darcy friction factor without using Moody diagram? Explain briefly.
5) Define Reynolds' number. Explain it's application.
6) What are the ranges of the Reynolds number for laminar, transition and turbulent flow?
7) Define the term 'Hydraulic Diameter'. Prove that hydraulic diameter of a rectangular cross section, Dh = 2ab/(a+b)
8) Write the power and efficiency equation of the pump and turbine.

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