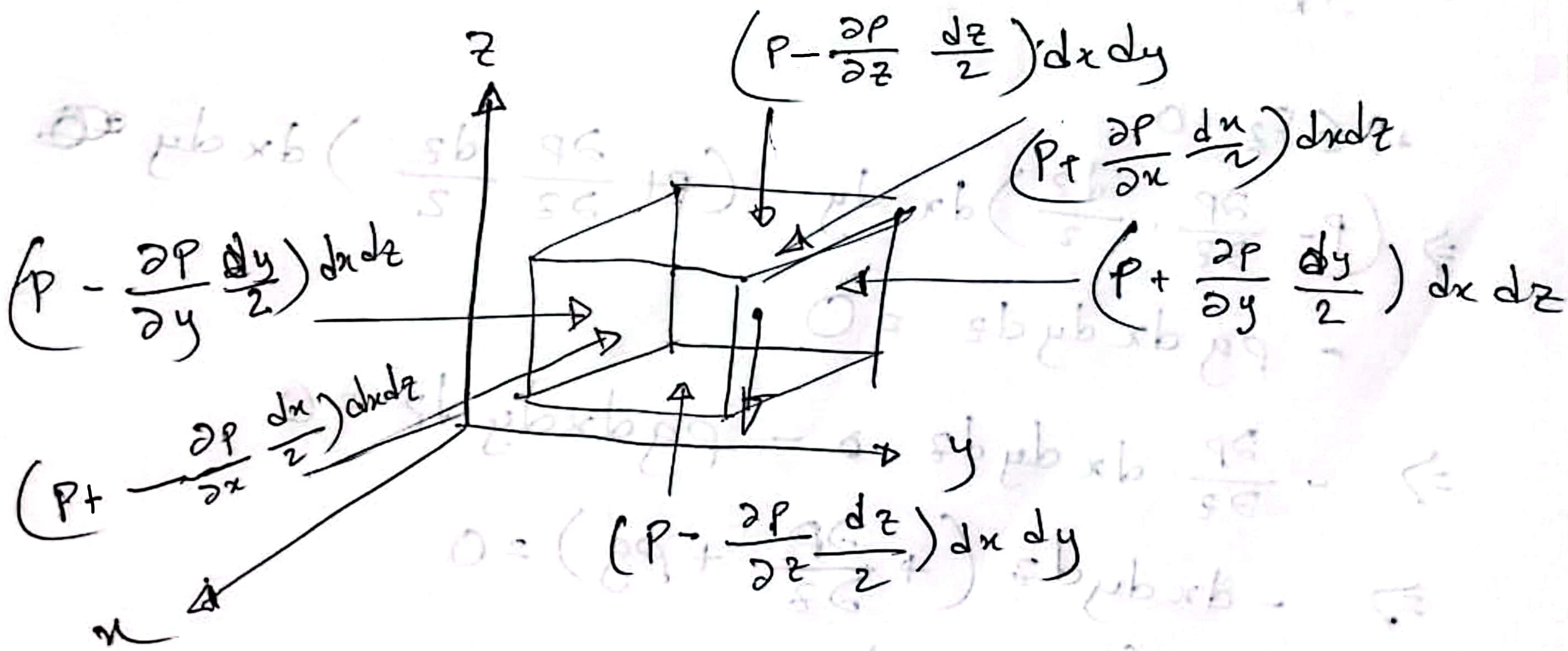
$\frac{du}{dy}$

$$\tau = A + B\left(\frac{\delta u}{\delta y}\right)^n$$

For newtonian fluids, $A = 0$, $B = \mu$ and $n = 1$

$$\tau = \mu \frac{\delta u}{\delta y}$$

* <u>Hydrostatic pressure of fluid</u> →



$$f(x+h) = f(x) + \frac{\partial f}{\partial x}\frac{h}{1!} + \frac{\partial^2 f}{\partial x^2}\frac{h^2}{2!}$$

$$P_{y+\frac{dy}{2}} = P + \left(\frac{\partial P}{\partial x}\right)\frac{dx}{2}$$

$$\xrightarrow{+} \sum F_y = 0$$

$$\Rightarrow \left(p - \frac{\partial p}{\partial y}\frac{dy}{2}\right) dx.dz - \left(p + \frac{\partial p}{\partial y}\frac{\partial y}{2}\right) dx\,dz = 0$$

$$\Rightarrow -\frac{1}{2}\frac{\partial p}{\partial y} dx\,dy\,dz - \frac{1}{2}\frac{\partial p}{\partial y} dx\,dy\,dz = 0$$

$$\Rightarrow \frac{\partial p}{\partial y}\Delta V = 0$$

$$\overset{+}{\swarrow} \sum F_x = 0$$

$$\Rightarrow \left(p - \frac{\partial p}{\partial x}\frac{dx}{2}\right) dy.dz - \left(p + \frac{\partial p}{\partial x}\frac{dx}{2}\right) dy.dz = 0$$

$$\Rightarrow -\frac{1}{2}\frac{\partial p}{\partial x}\Delta V - \frac{1}{2}\frac{\partial p}{\partial x}\Delta V = 0$$

$$\Rightarrow \frac{\partial p}{\partial x}\Delta V = 0$$

$$+\uparrow \sum F_z = 0$$

$$\Rightarrow \left(p - \frac{\partial p}{\partial z}\cdot\frac{dz}{2}\right) dx.dy - \left(p + \frac{\partial p}{\partial z}\frac{dz}{2}\right) dx\,dy$$

$$- \rho g\,dx\,dy\,dz = 0$$

$$\Rightarrow -\frac{\partial p}{\partial z} dx\,dy\,dz - \rho g\,dx\,dy\,dz = 0$$

$$\Rightarrow -dx\,dy\,dz\left(+\frac{\partial p}{\partial z} + \rho g\right) = 0$$

$$\Rightarrow \Delta V\left(\frac{\partial p}{\partial z} + \rho g\right) = 0$$

$\Delta V \neq 0$

$\therefore \quad \frac{\partial p}{\partial z} + \rho g = 0$

$\Rightarrow \quad \partial p = -\rho g \, \partial z$

$\Rightarrow \quad \int_1^2 dp = -\rho g \int_1^2 dz$

$\Rightarrow \quad P_2 - P_1 = -\rho g (z_2 - z_1) = -\rho g h$

$\Rightarrow \quad P_{atm} - P_1 = -\rho g h$

$\Rightarrow \quad P_1 = \rho g h + P_{atm}$ (absolute)

$\therefore \quad P = \rho g h$ (gage)

Hydrostatic pressure of compressible fluid

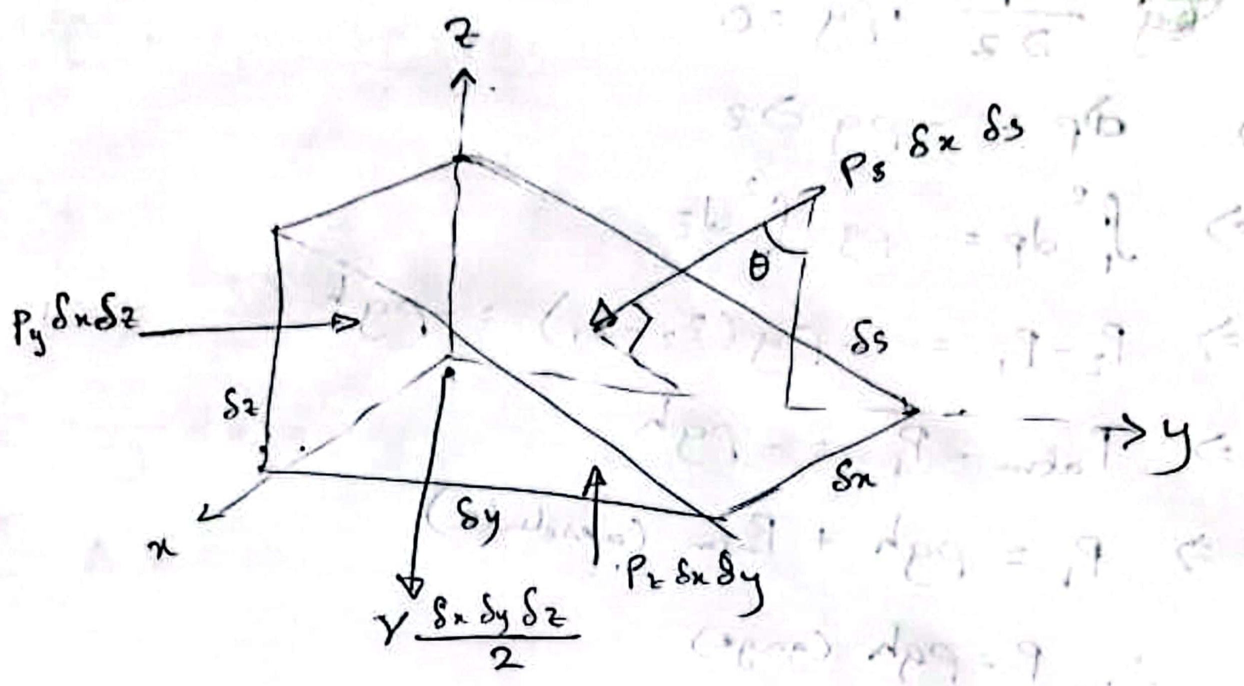
$$\frac{dp}{dz} = -\rho g$$

$$P = \rho RT \quad \Rightarrow \rho = \frac{P}{RT}$$

$$\frac{dp}{dz} = -\frac{gP}{RT} \quad \Rightarrow \frac{dp}{P} = -\frac{g\,dz}{RT}$$

$$\Rightarrow \int_{P_1}^{P_2} \frac{dp}{P} = \ln \frac{P_2}{P_1} = -\frac{g}{R} \int_{z_1}^{z_2} \frac{dz}{T}$$

Fluid pressure same in all direction (Pascal's law)



$$\Sigma F_y = P_y \delta x \delta z - P_s \delta x \delta s \sin\theta = 0$$

$$\Rightarrow P_y \delta x \delta s \sin\theta - P_s \delta x \delta s \sin\theta = 0$$

$$P_y = P_s$$

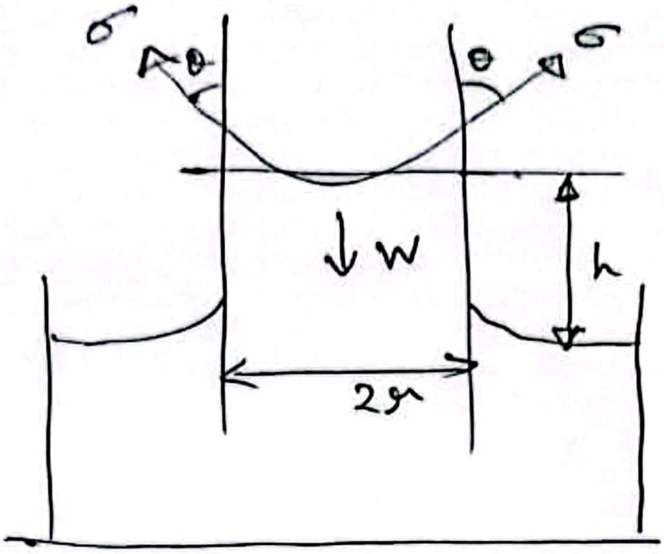
$$\Sigma F_z = P_z \delta x \delta y - P_s \delta x \delta s \cos\theta - \gamma \frac{\delta x \delta y \delta z}{2} = 0$$

$$= P_z \delta x \delta y - P_s \delta x \delta y = 0$$ [$\delta x \delta y \delta z$ is negligible]

$$P_z = P_s$$

$$P_y = P_z = P_s$$

Capillary rise of fluid



$+\uparrow \sum F_y = 0$

$\Rightarrow \sigma \cos\theta (2\pi r) - W = 0$

$\Rightarrow \sigma \cos\theta (2\pi r) = \gamma \pi r^2 h$

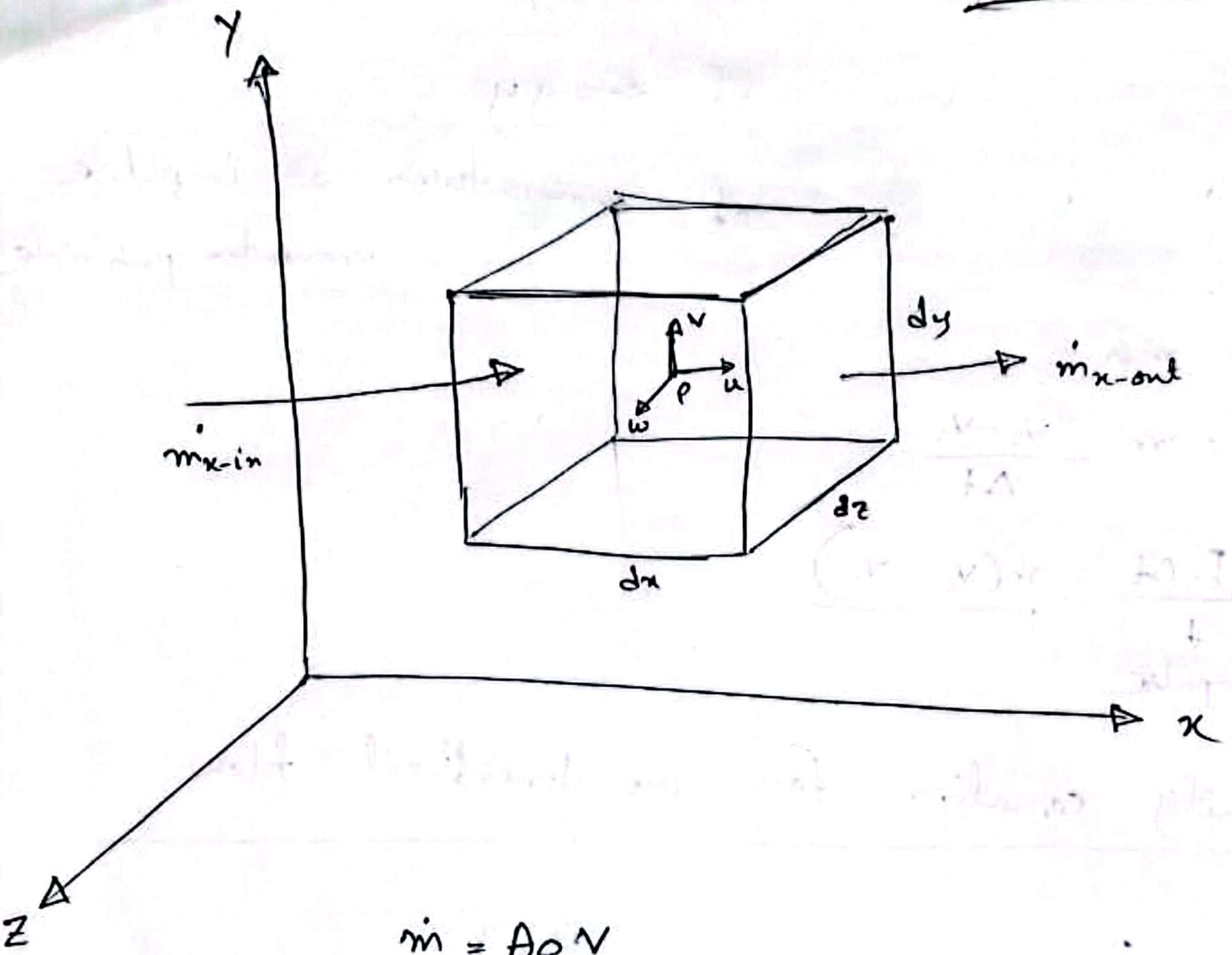
$W = \gamma \pi r^2 h$

$$h = \frac{\sigma \cos\theta \, 2\pi r}{\gamma \pi r^2 h} = \frac{2\sigma \cos\theta}{\gamma r}$$

Continuity equation for three dimensional flow →

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) + \frac{\partial \rho}{\partial t} = 0$$

$\boxed{\text{Important for exam}}$ $\boxed{\text{final}}$



$$\dot{m} = A\rho V$$

$$f(x+h) = f(x) + f'(x)\frac{h}{\lfloor 1} + f''(x)\frac{h^2}{\lfloor 2} + \ldots\ldots$$

$$\rho\Big|_{x-\frac{dx}{2}} = \rho + \frac{\partial \rho}{\partial x}\left(-\frac{dx}{2}\right) + \ldots\ldots$$

$$u_{x-\frac{dx}{2}} = u + \frac{\partial u}{\partial x}\left(-\frac{dx}{2}\right)$$

$$\rho_{x+\frac{dx}{2}} = \rho + \frac{\partial \rho}{\partial x} \cdot \frac{dx}{2}$$

$$u_{x+\frac{dx}{2}} = u + \frac{\partial u}{\partial x} \cdot \frac{dx}{2}$$

$$\dot{m}_{in-x} = \left(\rho - \frac{\partial \rho}{\partial x}\frac{dx}{2}\right)\left(u - \frac{\partial u}{\partial x} \cdot \frac{dx}{2}\right) dy\, dz$$

$$= \left[\rho u - \rho \frac{\partial u}{\partial x} \cdot \frac{\partial x}{2} - u \frac{\partial \rho}{\partial x} \cdot \frac{dx}{2}\right] dy.dz$$

$$\dot{m}_{x-out} = \left(\rho + \frac{\partial \rho}{\partial x}\frac{dx}{2}\right)\left(u + \frac{\partial u}{\partial x} \cdot \frac{dx}{2}\right) dy\, dz$$

$$= \left[\rho u + \rho \frac{\partial u}{\partial x}\frac{dx}{2} + u \frac{\partial \rho}{\partial x} \cdot \frac{dx}{2}\right] dy\, dz$$

$$\dot{m}_{x\,net} = \dot{m}_{x-in} - \dot{m}_{x-out}$$

$$= \left[-\rho \frac{\partial u}{\partial x}\frac{dx}{2} - u \frac{\partial \rho}{\partial x} \cdot \frac{dx}{2}\right] dy\, dz$$

$$- \left[\rho \frac{\partial u}{\partial x} \cdot \frac{dx}{2} + u \frac{\partial \rho}{\partial x} \cdot \frac{dx}{2}\right] dy\, dz$$

$$= \left(-\rho \frac{\partial u}{\partial x} dx - u \frac{\partial \rho}{\partial x} dx\right) dy.dz$$

$$= -\left(\rho \frac{\partial u}{\partial x} + u \frac{\partial \rho}{\partial x}\right) dx\, dy\, dz$$

$$\therefore \dot{m}_{x-net} = -\frac{\partial}{\partial x}(\rho u) \cdot dx\, dy\, dz$$

$$\therefore \dot{m}_{y-net} = -\frac{\partial}{\partial y}(\rho v)\, dx\, dy\, dz$$

$$\therefore \dot{m}_{z\text{-net}} = -\frac{\partial}{\partial z}(\rho w)\, dx\, dy\, dz$$

$$\dot{m}_{change} = \frac{\partial \rho}{\partial t}\, dx.dy.dz$$

$$m = \rho V$$

$$\frac{d}{dt}(m) = \frac{d}{dt}(\rho V)$$

$$\dot{m}_{chang} = \frac{\partial \rho}{\partial t}\cdot V$$

$$\Rightarrow \dot{m}_{net\text{-}x} + \dot{m}_{net\text{-}y} + \dot{m}_{net\text{-}z} = \frac{\partial \rho}{\partial t}\, dx\, dy\, dz$$

$$\Rightarrow -\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right]\cdot dx\, dy\, dz$$

$$= \frac{\partial \rho}{\partial t}\, dx\, dy\, dz$$

$$\Rightarrow \left\{\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) + \frac{\partial \rho}{\partial t} = 0\right.$$

$$\Rightarrow \vec{\nabla}\cdot\rho\vec{V} + \frac{\partial \rho}{\partial t} = 0$$

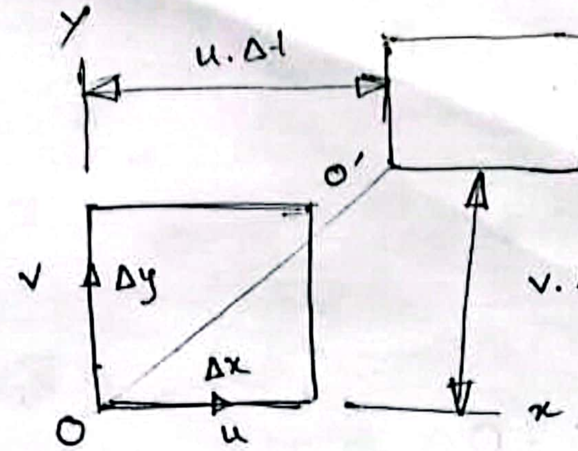
$$\nabla\cdot\rho\vec{V} = \frac{\partial}{\partial x}\rho u + \frac{\partial}{\partial y}\rho v + \frac{\partial}{\partial z}\rho w$$

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

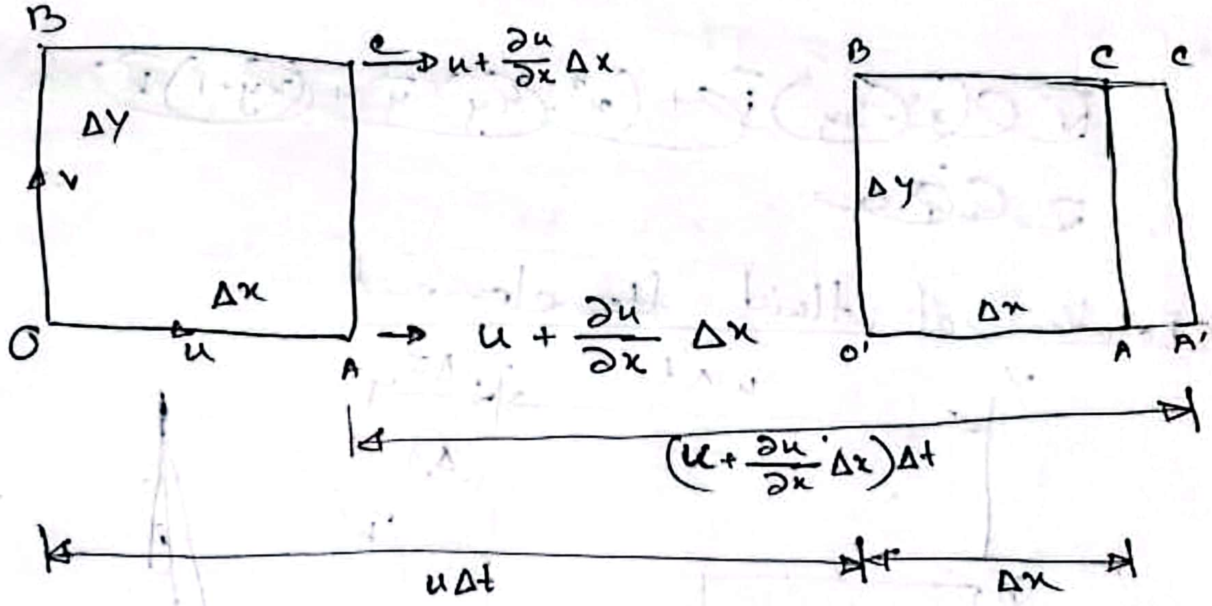
[compressible and steady flow]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \rightarrow \text{if flow is incompressible}$$

two dimension



$\Delta t\ ,\ O \rightarrow O'$



$$AA' = OA' + OA$$

$$AA' = \left(u + \frac{\partial u}{\partial x}\cdot\Delta x\right)\Delta t + \Delta x - (u.\Delta t + \Delta x)$$

$$= u.\Delta t + \frac{\partial u}{\partial x}\Delta x\Delta t + \Delta x - u\Delta t - \Delta x$$

$$= \frac{\partial u}{\partial x}\Delta x\,\Delta t$$

$$\Rightarrow \frac{AA'}{\Delta x} = \frac{\partial u}{\partial x}\Delta t$$

$$\Rightarrow \epsilon_x = \frac{\partial u}{\partial x} \Rightarrow \dot{\epsilon}_x = \frac{\partial u}{\partial x}$$

$$\epsilon_{xx} = \frac{\partial u}{\partial x}$$

$$\epsilon_{yy} = \frac{\partial v}{\partial y}$$

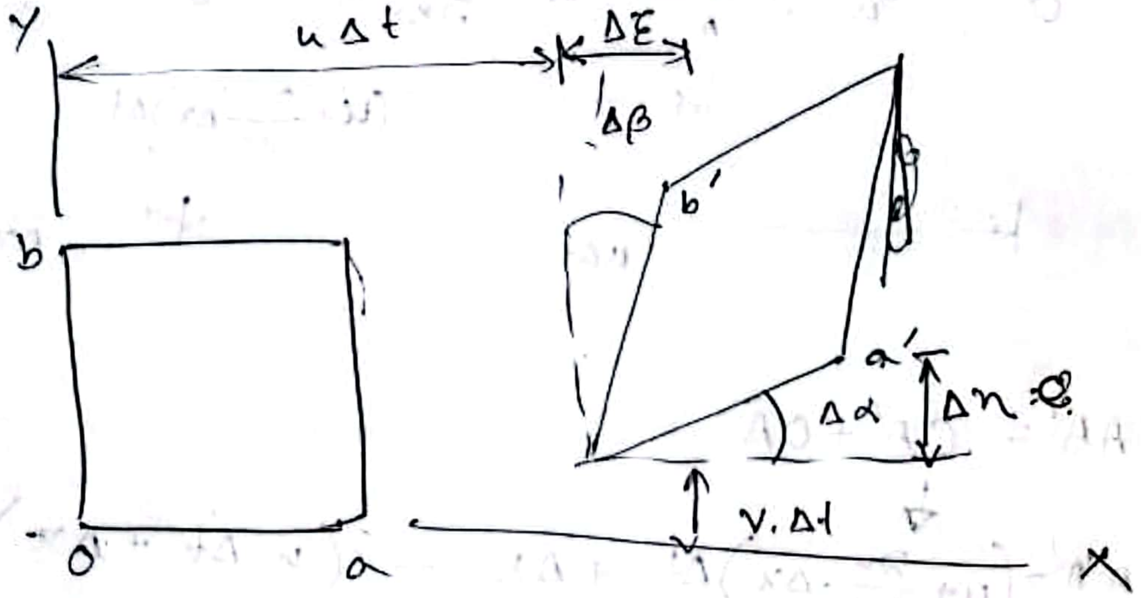
$$\epsilon_{zz} = \frac{\partial w}{\partial z}$$

$$\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} = 0$$

$$\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\cancel{\vec{V} = (x^2 - 3y)\vec{i} + (x^3 - 4y)\vec{j} + (2xy - 1)\vec{k}}$$

* Rotation of fluid ~~flow~~ element



bb'

$$\boxed{\tan \Delta\beta = \frac{\Delta \mathcal{E}}{\Delta y}}$$

$$\Delta \mathcal{E} = \left(u + \frac{\partial u}{\partial y} \cdot \Delta y\right)\Delta t - u\Delta t$$

$$= \frac{\partial u}{\partial y} \Delta y \, \Delta t \Rightarrow \frac{\Delta \mathcal{E}}{\Delta y} = \frac{\partial u}{\partial y} \Delta t$$

$$\Rightarrow \tan \Delta\beta = \frac{\partial u}{\partial y} \cdot \Delta t \Rightarrow \Delta\beta = \frac{\partial u}{\partial y} \Delta t$$

$$\Delta \eta = \left(v + \frac{\partial v}{\partial x} \Delta x \right) \Delta t - v \Delta t$$

$$= \frac{\partial v}{\partial x} \Delta x \, \Delta t$$

$$\cancel{\tan \Delta\beta = \frac{\Delta \epsilon}{\Delta y}}$$

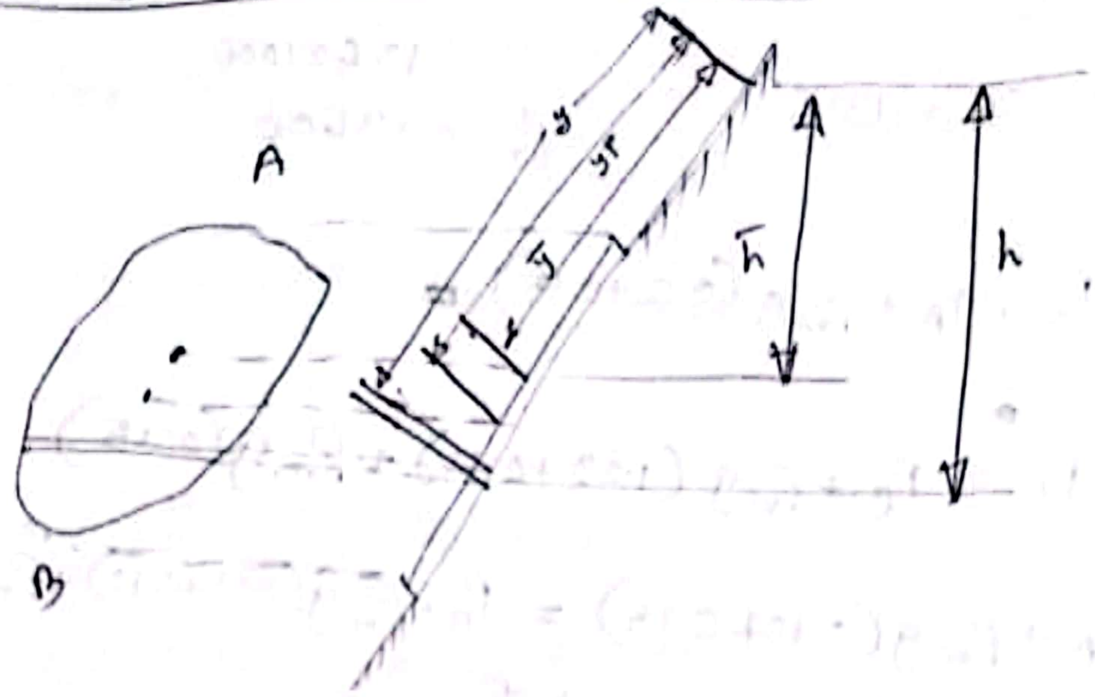
$$\Rightarrow \frac{\Delta \eta}{\Delta x} = \frac{\partial v}{\partial x} \Delta t$$

$$\Rightarrow \Delta \alpha = \frac{\partial v}{\partial x} \Delta t$$

$$w_z = \lim_{\Delta t \to 0} \left(\frac{\Delta \beta - \Delta \alpha}{\Delta t} \right)$$

$$= \frac{1}{2} \left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right.$$

Hydrostatic Forces on Flat surfaces



Total pressure force on the strip.

$$dF = P\,dA$$

$P = \gamma h$
$= \gamma y \sin\theta$

$\Rightarrow \cancel{\int dF = \int_A \rho g h\, dA}$

$\cancel{= \rho g \int_A h\, dA}$

$\cancel{= \theta}$

$$\Rightarrow F = \int dF = \int_A \gamma y \sin\theta\, dA = \gamma \sin\theta \int_A y\, dA$$

$$= \gamma \sin\theta\, A\bar{y}$$

or,

$$= \rho g \int_A h\, dA = \rho g \int_A \sin\theta . y\, dA$$

$$F = \gamma \bar{h} A$$

$$\boxed{\left[\begin{array}{l} \bar{y} = \dfrac{\int_A y\, dA}{A} \\ \bar{h} = \bar{y}\sin\theta \end{array} \right]}$$

$$\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3}{A}$$

$$F \cdot y_P = \int_A y \, dF$$

$$\Rightarrow y_P = \frac{\int_A y \, dF}{F} = \frac{\int_A y \cdot y \sin\theta \, dP}{\gamma \sin\theta \, A \bar{y}}$$

$$\left[\because dF = y \sin\theta \, dP \atop F = \gamma \sin\theta \, A\bar{y} \right]$$

$$= \frac{\gamma \sin\theta \int_A y^2 \, dF}{\gamma \sin\theta \, \bar{y} A}$$

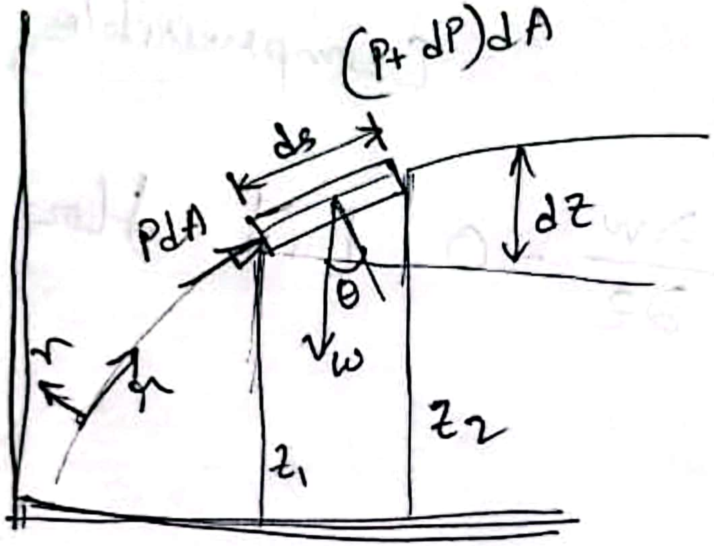
$$= \frac{\int_A y^2 \, dA}{A\bar{y}} = \frac{I_x}{A\bar{y}}$$

$$\left[I_x = I_0 + Ay^2 \atop = I_{cg} + Ay^2 \right]$$

$$= \frac{I_{cg} + A\bar{y}^2}{\bar{y} A}$$

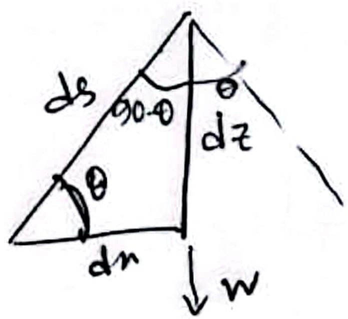
$$y_P = \frac{I_0}{A\bar{y}} + \bar{y}$$

Bernoulli's equation



$$W = ds.dA.\rho.g$$

newton's 2nd law of motion, $F = ma \Rightarrow F = PA$
$= PdA$



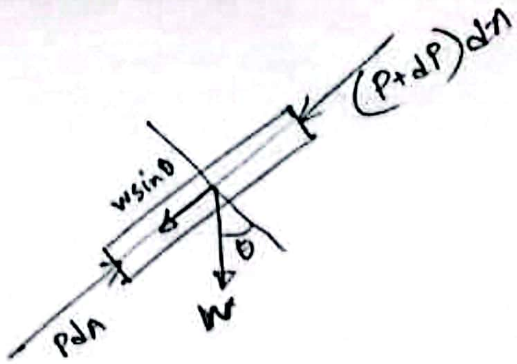
$$\sin\theta = \frac{dz}{ds}$$

$$\cos\theta = \frac{dx}{ds}$$

$$x+90-\theta+90=0$$

$$\Rightarrow x-\theta=0$$

$$\Rightarrow x=\theta$$



$$W = ds \cdot dA \cdot \rho g$$

$$\Rightarrow P dA - (P+dP) \cdot dA - W \sin\theta = ds\, dA\, \rho\, a$$

$$\Rightarrow -dp\, dA - ds \cdot dA \rho g \sin\theta = ds\, dA\, \rho \cdot a$$

$$\Rightarrow -dp - \rho g\, dz = ds\, \rho \cdot a$$

$$\Rightarrow -dp - \rho g\, dz = ds\, \rho \frac{dv}{dt}$$

$$\Rightarrow -dp - \rho g dz = ds \cdot v \cdot \frac{dv}{ds} \rho$$

$$\Rightarrow -dp - \rho g dz = \cancel{ds} \cdot v \frac{dv}{\cancel{ds}} \rho$$

$$P = p(x, y)$$

$$\Rightarrow dp = \frac{\partial P}{\partial x} \cdot dx + \frac{\partial P}{\partial y} dy$$

$$\Rightarrow dv = \frac{\partial v}{\partial s} ds + \frac{\partial v}{\partial t} \cdot dt$$

$$\Rightarrow \frac{dv}{dt} = \frac{\partial v}{\partial s} \frac{ds}{dt} + \cancel{\frac{\partial v}{\partial t}}^{0}$$

$$\Rightarrow \frac{dv}{dt} = v \frac{\partial v}{\partial s}$$

$$\Rightarrow \frac{dv}{dt} = v \frac{dv}{ds}$$

$$\Rightarrow dp + \rho g dz + \rho v\, dv = 0$$

$$\Rightarrow \frac{dp}{\rho g} + \frac{1}{g} v\, dv + dz = 0$$

$$\Rightarrow \frac{dp}{\rho g} + \frac{1}{g} v\, dv + dz = 0$$

$$\Rightarrow \frac{dP}{\gamma} + \frac{v\, dv}{g} + dz = 0$$

$$\Rightarrow \frac{1}{\gamma} \int dp + \frac{1}{g} \int v\, dv + \int dz = \text{const.}$$

$$\Rightarrow \frac{P}{\gamma} + \frac{V^2}{2g} + z = \text{const.}$$