ME 3101: Mechanics of Machinery

Gear & Gear Trains

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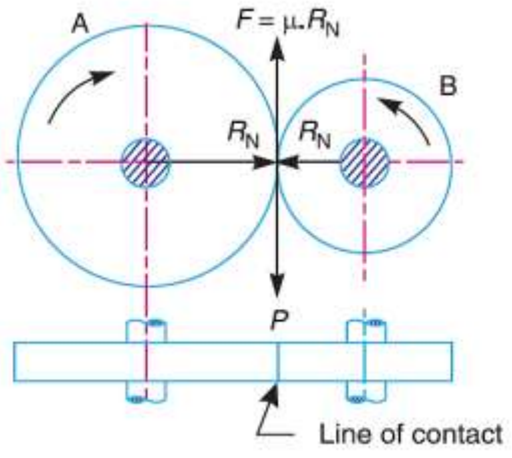
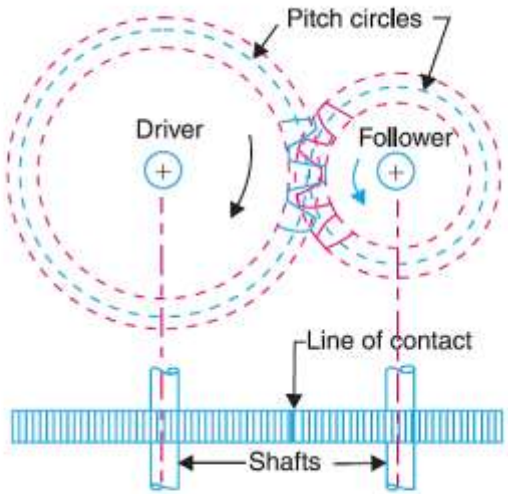
FRICTION WHEELS

- The motion and power transmitted by gears is kinematically equivalent to that transmitted by friction wheels or discs.
- Consider two plain circular wheels *A* and *B* mounted on shafts, having sufficient rough surfaces and pressing against each other.
- Let the wheel *A* be keyed to the rotating shaft and the wheel *B* to the shaft, to be rotated.
- When the wheel *A* is rotated by a rotating shaft, it will rotate the wheel *B* in the opposite direction.
- The wheel *B* will be rotated (by the wheel *A*) so long as the tangential force exerted by the wheel *A* does not exceed the maximum frictional resistance between the two wheels. But when the tangential force (*P*) exceeds the *frictional resistance (*F*), slipping will take place between the two wheels.
- In order to avoid the slipping, a number of projections (called teeth) as shown in (*b*), are provided on the periphery of the wheel *A*, which will fit into the corresponding recesses on the periphery of the wheel *B*. A friction wheel with the teeth cut on it is known as ***toothed wheel* or *gear*.** The usual connection to show the toothed wheels is by their **pitch circles.



(*a*) Friction wheels.



(*b*) Toothed wheels.

CLASSIFICATION OF GEARS

1. According to the position of axes of the shafts.

The axes of the two shafts between which the motion is to be transmitted, may be

(*a*) Parallel

- The two parallel and co-planar shafts connected by the gears. These gears are called ***spur gears*** and the arrangement is known as ***spur gearing***. These gears have teeth parallel to the axis of the wheel as shown in Fig.

- Another name given to the spur gearing is ***helical gearing***, in which the teeth are inclined to the axis. The single and double helical gears connecting parallel shafts are shown in Fig. and respectively. The double helical gears are known as ***herringbone gears***. A pair of spur gears are kinematically equivalent to a pair of cylindrical discs, keyed to parallel shafts and having a line contact.

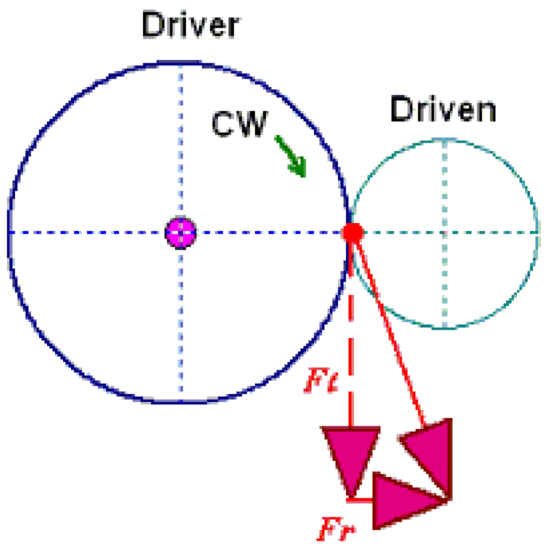
spur gears

helical gears

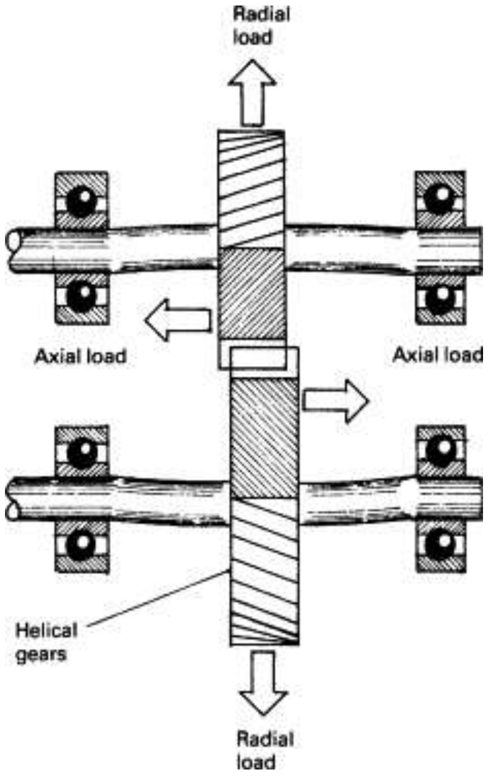
herringbone gears

FORCES ON GEARS

Gear Type	Radial Thrust (Force)	Axial Thrust (Force)
Spur Gear	Present	Absent
Helical Gear	Present	Present
Double Helical Gear	Present	Canceled Out



Forces in spur gears



Forces in helical gears

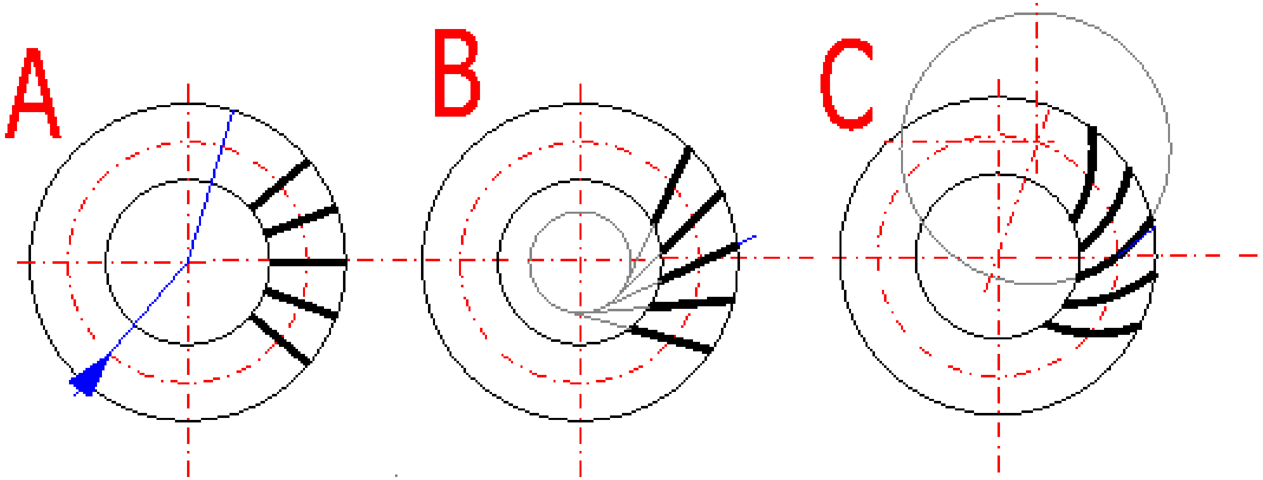
Axial thrust is cancelled out in double helical gears

CLASSIFICATION OF GEARS

(*b*) Intersecting

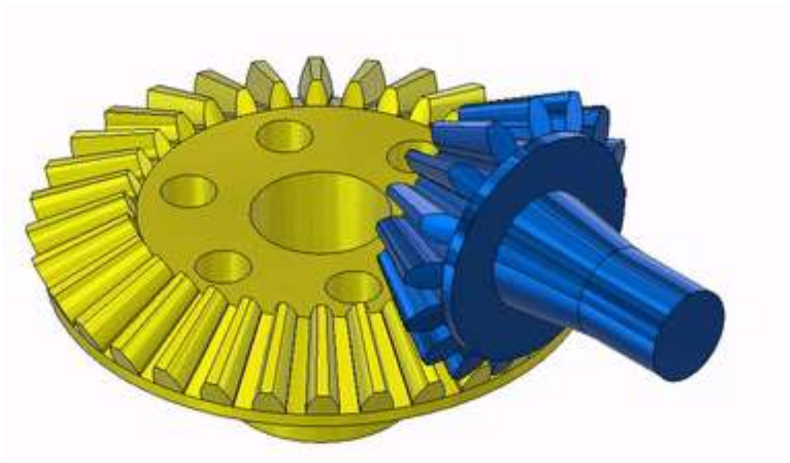
- The two non-parallel or intersecting, but coplanar shafts connected by gears is shown in Fig. These gears are called ***bevel gears*** and the arrangement is known as ***bevel gearing***.
- The bevel gears, like spur gears, may also have their teeth inclined to the face of the bevel, in which case they are known as ***helical bevel gears***.
- If the teeth are curved and gradually engage, then the gears are called ***spiral gears***.
- When equal bevel gears (having equal teeth) connect two shafts whose axes are mutually perpendicular, then the bevel gears are known as ***mitres (1:1 Ratio Bevel Gear)***.

bevel gears ***spiral gears***

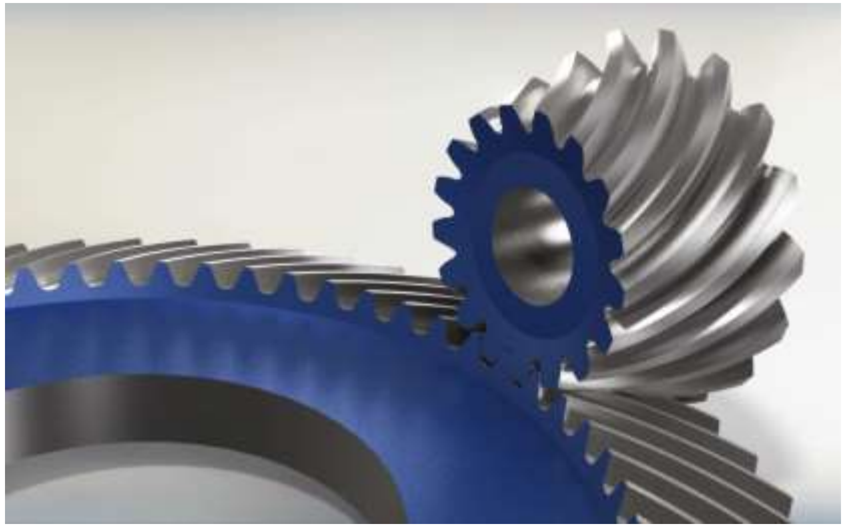


straight bevel gears ***helical bevel gears*** ***spiral gears***

CLASSIFICATION OF GEARS



bevel gears



spiral gears

helical bevel gears

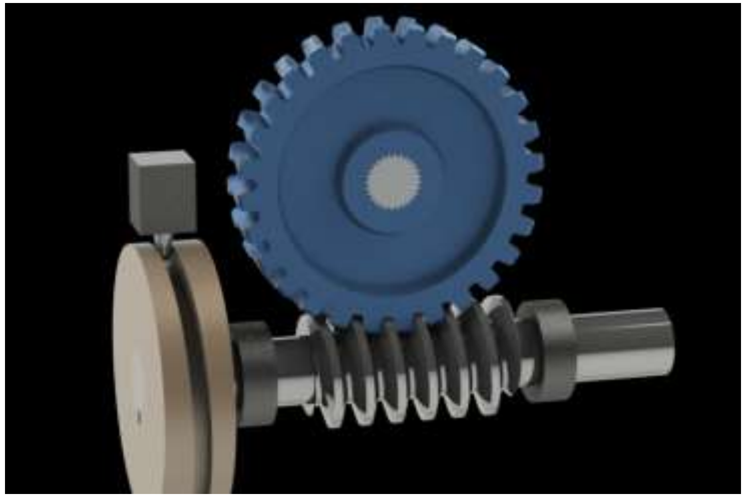
mitres

CLASSIFICATION OF GEARS

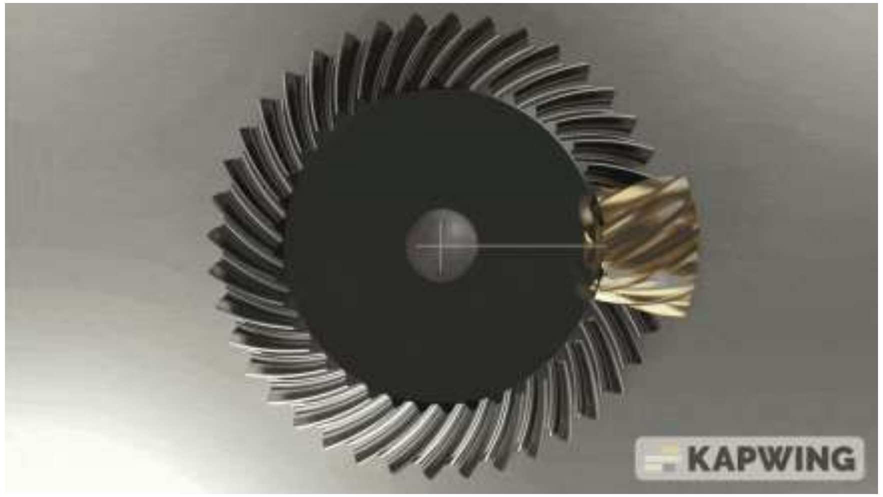
(c) Non-intersecting and non-parallel.

- The two non-intersecting and non-parallel *i.e.* non-coplanar shaft connected by gears is shown in Fig.
- ***Worm Gear***
- Consists of a worm (screw-like gear) and a worm wheel.
- ***Hypoid Gear (Special Type of Bevel Gear)***
- Similar to spiral bevel gears but with offset shafts (not truly intersecting).
- ***Hyperboloid/ Crossed Helical Gear/ Screw Gear***
- Two helical gears with non-parallel, non-intersecting shafts.

Hypoid Gear



worm gears



Hypoid Gear

Screw Gear

CLASSIFICATION OF GEARS

CLASSIFICATION OF GEARS

2. *According to the peripheral velocity of the gears.* The gears, according to the peripheral velocity of the gears may be classified as :

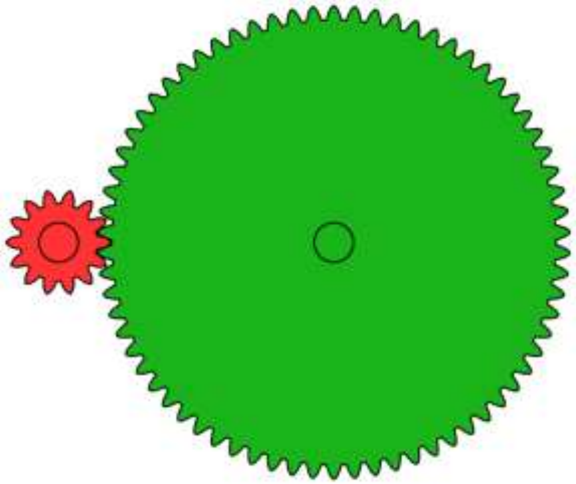
(*a*) Low velocity, **(*b*)** Medium velocity, and **(*c*)** High velocity.

The gears having velocity less than 3 m/s are termed as ***low velocity*** gears and gears having velocity between 3 and 15 m/s are known as ***medium velocity gears***. If the velocity of gears is more than 15 m/s, then these are called ***high speed gears***.

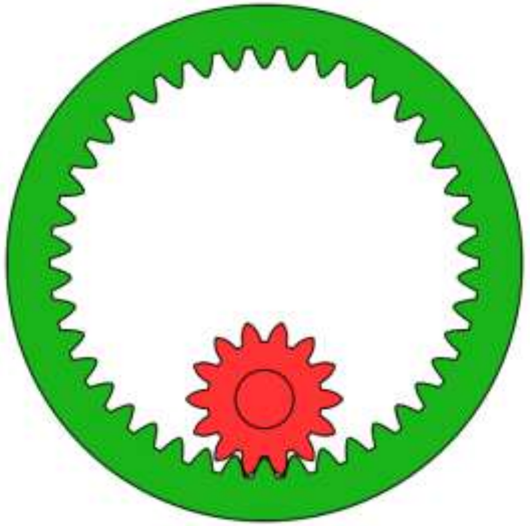
3. *According to the type of gearing*. The gears, according to the type of gearing may be classified as :

(*a*) External gearing, **(*b*)** Internal gearing, and **(*c*)** Rack and pinion.

- In ***external gearing***, the gears of the two shafts mesh externally with each other as shown in Fig. The larger of these two wheels is called ***spur wheel or gear*** and the smaller wheel is called **pinion.** In an external gearing, the motion of the two wheels is always ***unlike***, as shown in Fig.
- In ***internal gearing***, the gears of the two shafts mesh ***internally*** with each other as shown in (*b*). The larger of these two wheels is called ***annular wheel*** and the smaller wheel is called ***pinion***. In an internal gearing, the motion of the two wheels is always ***like***, as shown in Fig.



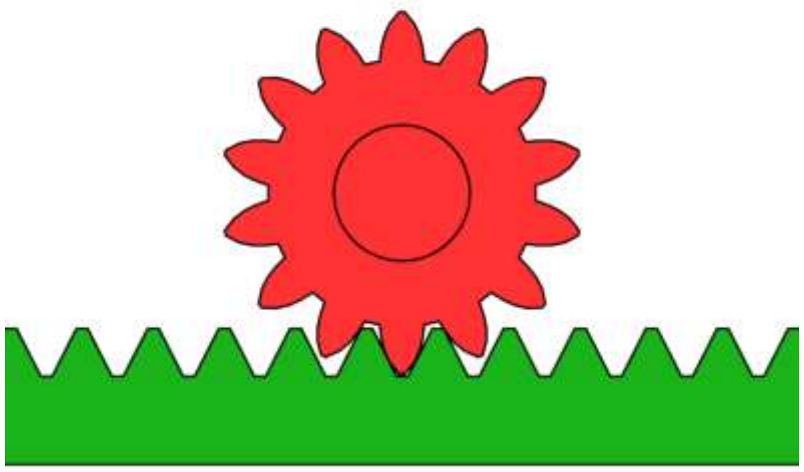
external gearing

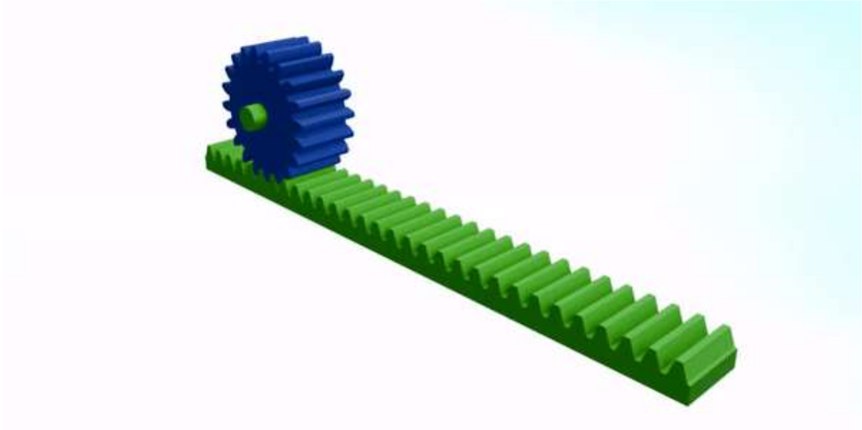


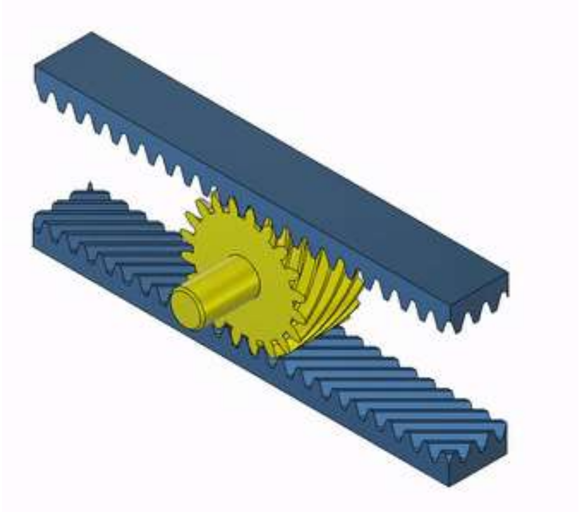
internal gearing

- Sometimes, the gear of a shaft meshes externally and internally with the gears in a *straight line, as shown in Fig. Such type of gear is called ***rack and pinion***. The straight-line gear is called rack and the circular wheel is called pinion. A little consideration will show that with the help of a rack and pinion, we can convert linear motion into rotary motion and ***vice-versa*** as shown in Fig.

4. *According to position of teeth on the gear surface*. The teeth on the gear surface may be
(*a*) straight, **(*b*)** inclined, and **(*c*)** curved.
We have discussed earlier that the spur gears have straight teeth where as helical gears have their teeth inclined to the wheel rim. In case of spiral gears, the teeth are curved over the rim surface.







rack and pinion

APPLICATIONS OF GEARS

Gear Type	Unique Application
Spur Gear	Gear trains in mechanical clocks
Helical Gear	Automotive transmissions
Double Helical Gear	Marine propulsion systems
Bevel Gear	Hand drills (to change rotation direction)
Helical Bevel Gear	Industrial conveyors
Spiral Bevel Gear	Automotive differentials
Miter Gear	Robotics (changing shaft direction without speed change)
Worm Gear	Elevators and conveyor systems
Rack and Pinion	Steering mechanism in automobiles

GEAR TERMINOLOGY

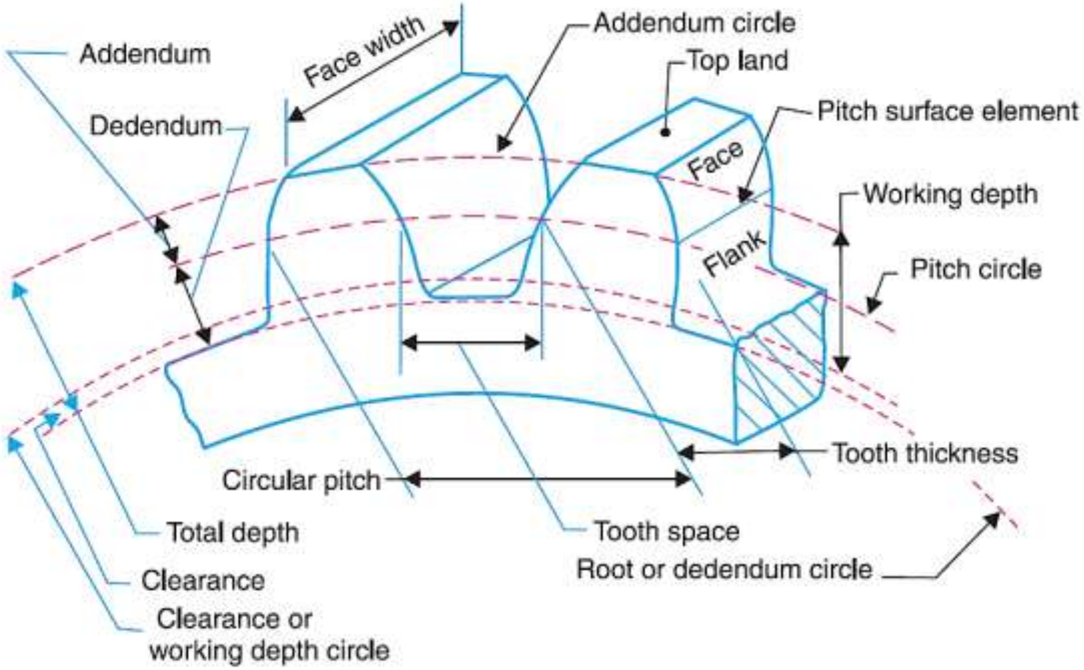
1. *Pitch circle*. It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

* A straight line may also be defined as a wheel of infinite radius.

2. *Pitch circle diameter*. It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also known as ***pitch diameter***.

3. *Pitch point*. It is a common point of contact between two pitch circles.

4. *Pressure angle or angle of obliquity*. It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by φ. The standard pressure angles are 14.5 ° and 20°.



5. *Addendum*. It is the radial distance of a tooth from the pitch circle to the top of the tooth.

6. *Dedendum*. It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

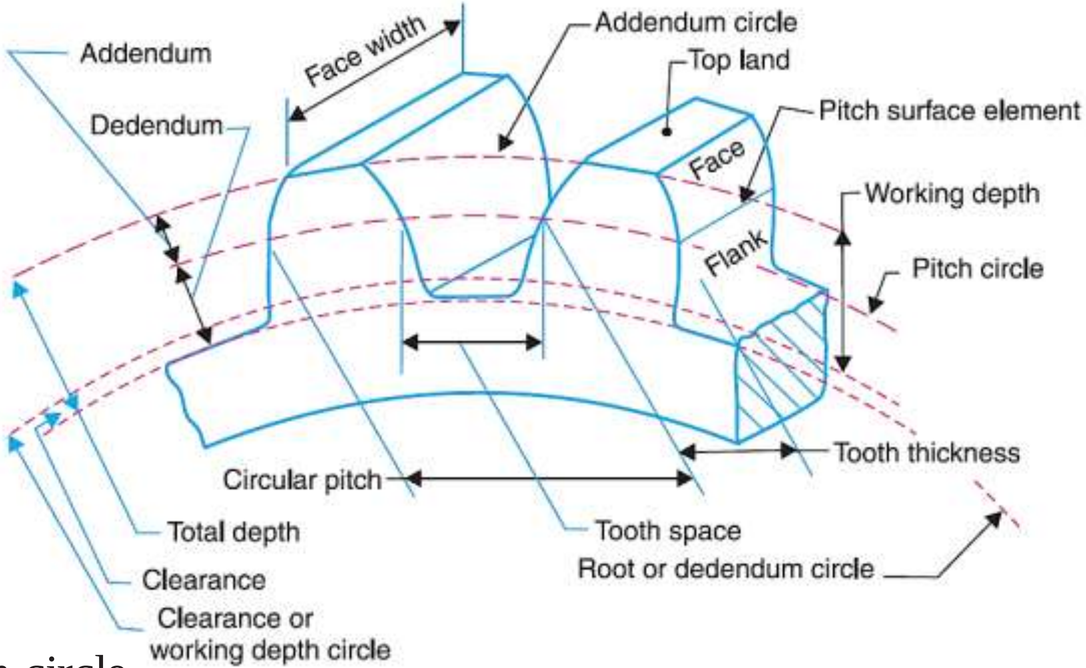
7. *Addendum circle*. It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

8. *Dedendum circle*. It is the circle drawn through the bottom of the teeth. It is also called root circle.

Note : Root circle diameter = Pitch circle diameter × cos φ, where φ is the pressure angle.

9. *Circular pitch*. It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by p_c.

GEAR TERMINOLOGY



Mathematically, Circular pitch, $p_c = \pi D/T$
where D = Diameter of the pitch circle, and T = Number of teeth

10. *Module.* It is the ratio of the pitch circle diameter in millimeters to the number of teeth. It is usually denoted by *m*. Mathematically, Module, $m = D/T$

11. *Total depth*. It is the radial distance between the addendum and the dedendum circles of a gear. It is equal to the sum of the addendum and dedendum.

12. *Working depth*. It is the radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.

13. *Tooth thickness*. It is the width of the tooth measured along the pitch circle.

14. *Tooth space* . It is the width of space between the two adjacent teeth measured along the pitch circle.

15. *Backlash*. It is the difference between the tooth space and the tooth thickness, as measured along the pitch circle. Theoretically, the backlash should be zero, but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion.

16. *Face of tooth*. It is the surface of the gear tooth above the pitch surface element.

17. *Flank of tooth*. It is the surface of the gear tooth below the pitch surface element.

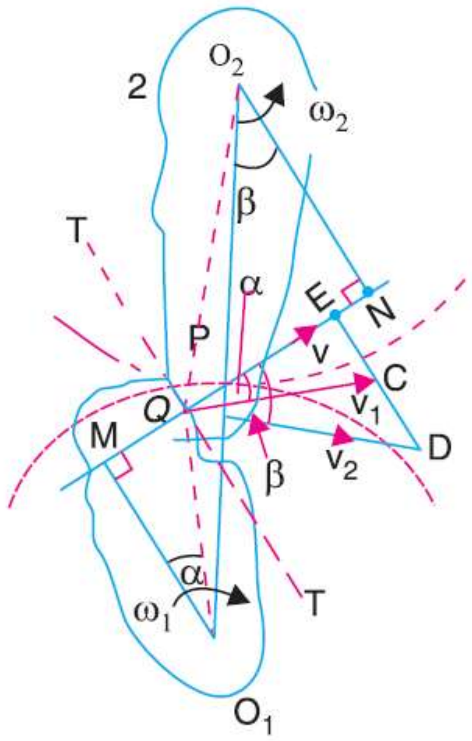
18. *Top land*. It is the surface of the top of the tooth.

19. *Face width*. It is the width of the gear tooth measured parallel to its axis.

20. *Path of contact*. It is the path traced by the point of contact of two teeth from the beginning to the end of engagement.

LAW OF GEARING

- **Condition for Constant Velocity Ratio of Toothed Wheels**
- Consider the portions of the two teeth, one on the wheel 1 (or pinion) and the other on the wheel 2, as shown by thick line curves in Fig. Let the two teeth come in contact at point *Q*, and the wheels rotate in the directions as shown in the figure.
- Let *TT* be the common tangent and *MN* be the common normal to the curves at the point of contact *Q*. From the centres O_1 and O_2 , draw O_1M and O_2N perpendicular to *MN*. A little consideration will show that the point *Q* moves in the direction *QC*, when considered as a point on wheel 1, and in the direction *QD* when considered as a point on wheel 2.
- Let v_1 and v_2 be the velocities of the point *Q* on the wheels 1 and 2 respectively. If the teeth are to remain in contact, then the components of these velocities along the common normal *MN* must be equal.



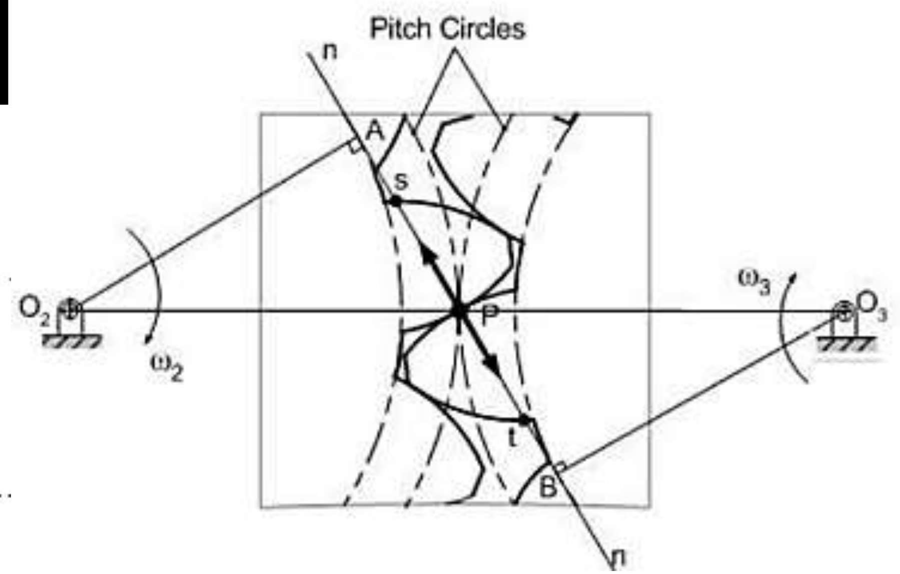
$$\therefore \quad v_1 \cos\alpha = v_2 \cos\beta$$

$$\text{or} \quad (\omega_1 \times O_1Q)\cos\alpha = (\omega_2 \times O_2Q)\cos\beta$$

$$(\omega_1 \times O_1Q)\frac{O_1M}{O_1Q} = (\omega_2 \times O_2Q)\frac{O_2N}{O_2Q} \quad \text{or} \quad \omega_1 \times O_1M = \omega_2 \times O_2N$$

$$\therefore \quad \boxed{\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M}} \qquad \ldots(i)$$

LAW OF GEARING



Also from similar triangles O_1MP and O_2NP,

$$\frac{O_2N}{O_1M} = \frac{O_2P}{O_1P}$$

Combining equations (*i*) and (*ii*), we have

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P}$$

From above, we see that the angular velocity ratio is inversely proportional to the ratio of the distances of the point P from the centres O_1 and O_2, or the common normal to the two surfaces at the point of contact Q intersects the line of centres at point P which divides the centre distance inversely as the ratio of angular velocities. Therefore, in order to have a constant angular velocity ratio for all positions of the wheels, the point P must be the fixed point (called pitch point) for the two wheels. In other words, ***the common normal at the point of contact between a pair of teeth must always pass through the pitch point.*** This is the fundamental condition which must be satisfied while designing the profiles for the teeth of gear wheels. It is also known as ***law of gearing.***

Notes :

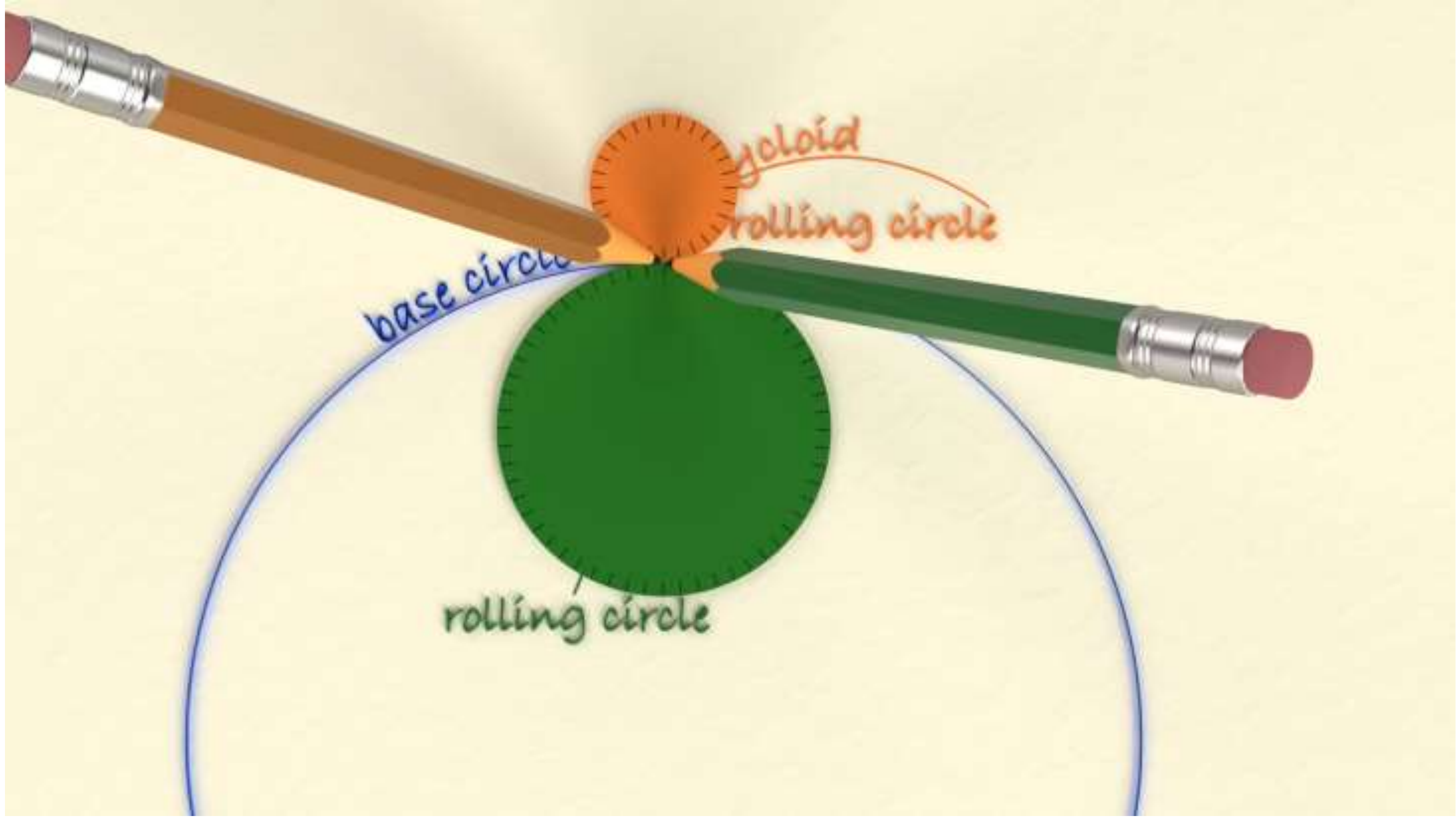
1. The above condition is fulfilled by teeth of involute form, provided that the root circles from which the profiles are generated are tangential to the common normal.

2. If D_1 and D_2 are pitch circle diameters of wheels 1 and 2 having teeth T_1 and T_2 respectively, then velocity ratio,

$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{D_2}{D_1} = \frac{T_2}{T_1}$$

FORMS OF GEAR TEETH (CYCLOIDAL)

- **Cycloidal Teeth**
- A ***cycloid*** is the curve traced by a point on the circumference of a circle which rolls without slipping on a fixed straight line. When a circle rolls without slipping on the outside of a fixed circle, the curve traced by a point on the circumference of a circle is known as ***epi-cycloid***. On the other hand, if a circle rolls without slipping on the inside of a fixed circle, then the curve traced by a point on the circumference of a circle is called ***hypo-cycloid***.

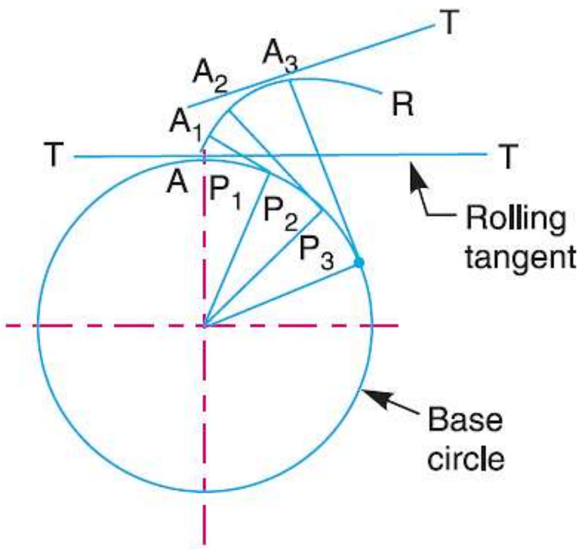


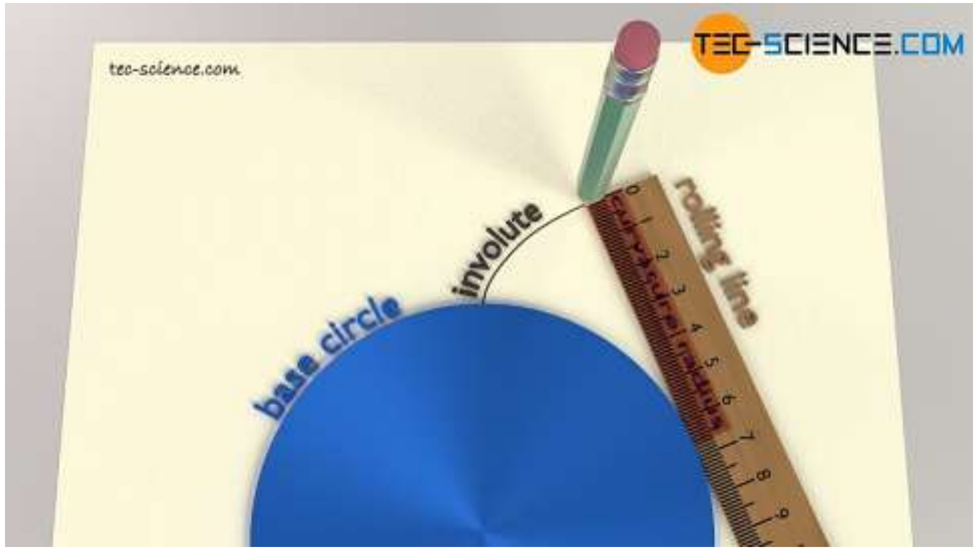
[Cycloidal Teeth]

FORMS OF GEAR TEETH (INVOLUTE)

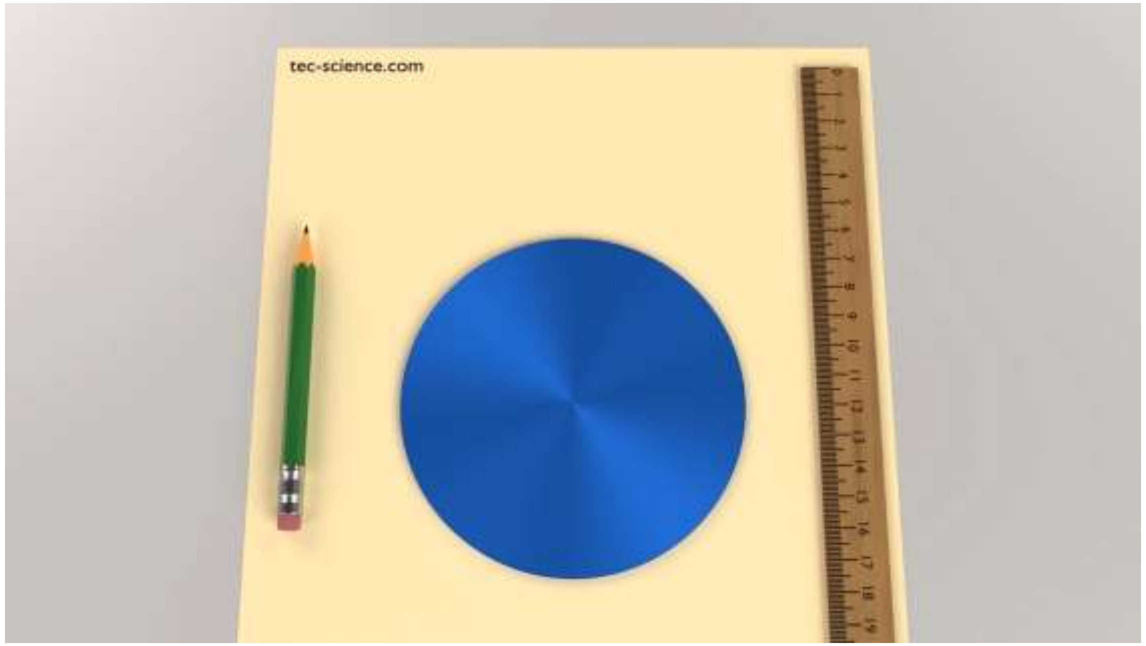
- **Involute Teeth**
- An involute of a circle is a plane curve generated by a point on a tangent, which rolls on the circle without slipping or by a point on a taut string which is unwrapped from a reel. The **involute profile** ensures that the **velocity ratio between gears remains constant**
- In connection with toothed wheels, the circle is known as base circle. The involute is traced as follows :
- Let A be the starting point of the involute. The base circle is divided into equal number of parts *e.g.* AP_1, P_1P_2, P_2P_3 etc. The tangents at P_1, P_2, P_3 etc. are drawn and the length P_1A_1, P_2A_2, P_3A_3 equal to the arcs AP_1, AP_2 and AP_3 are set off. Joining the points A, A_1, A_2, A_3 etc. we obtain the involute curve AR. A little consideration will show that at any instant A_3, the tangent A_3T to the involute is perpendicular to P_3A_3 and P_3A_3 is the normal to the involute. In other words, ***normal at any point of an involute is a tangent to the circle.***

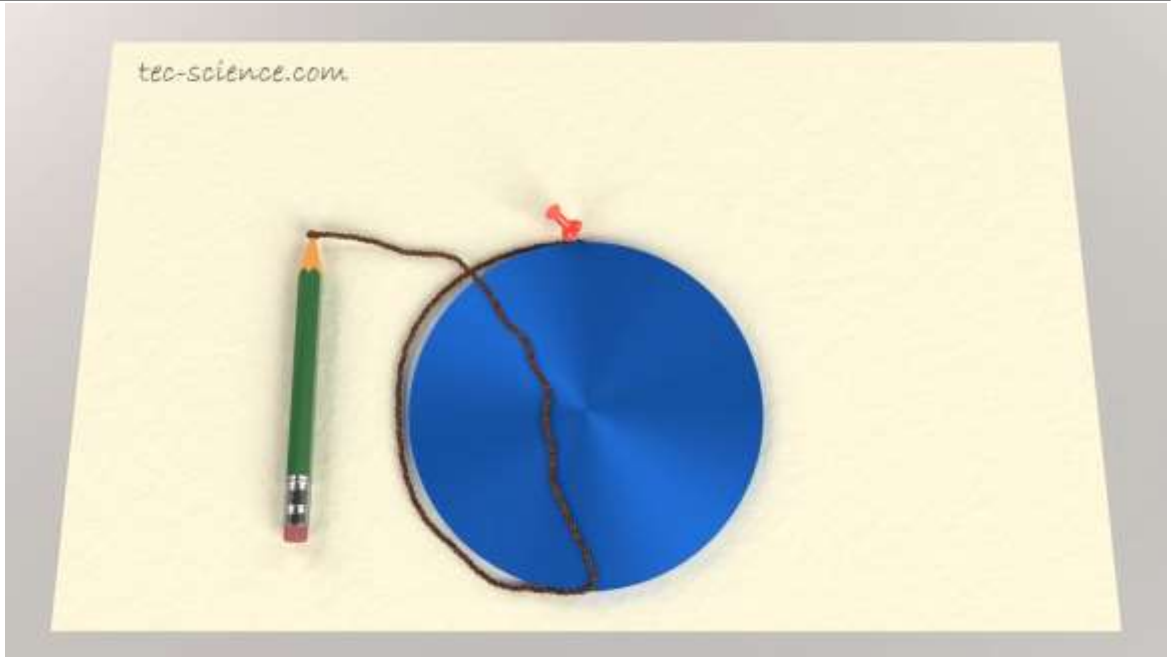
[Involute Teeth]()

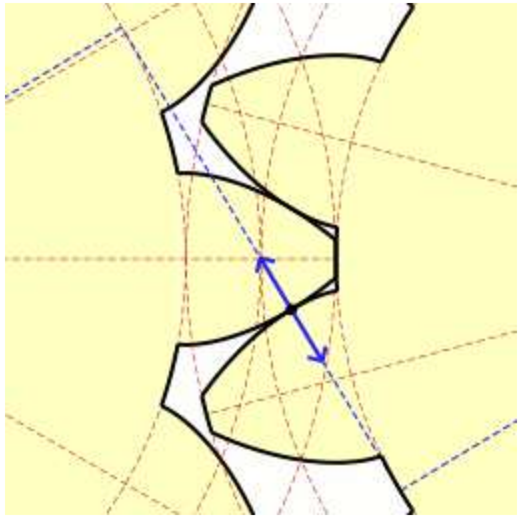




FORMS OF GEAR TEETH (INVOLUTE)



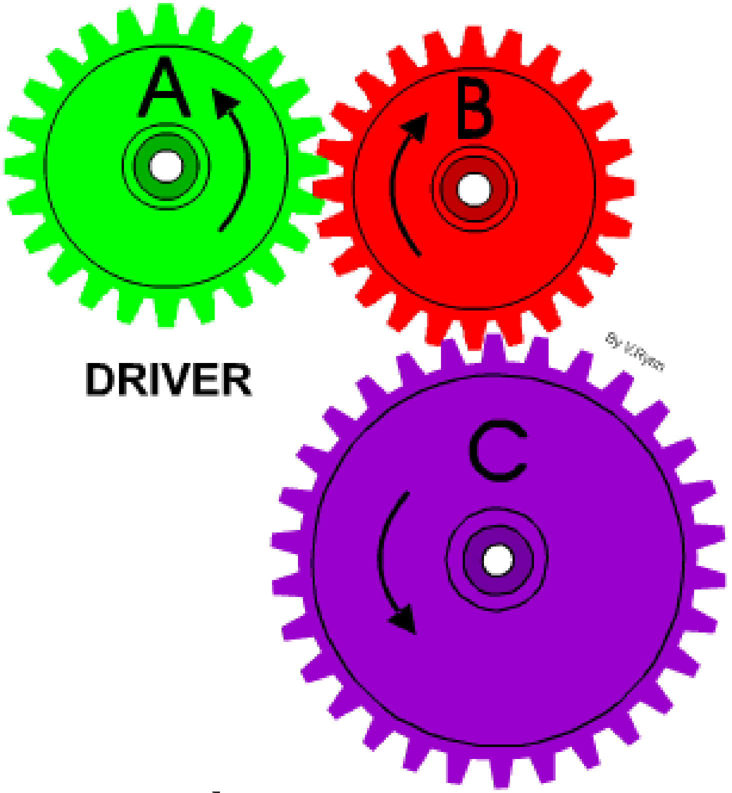




[Gear Types, Design Basics, Applications and More - Basics of Gears]

GEAR TRAIN

- Sometimes, two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called ***gear train*** or ***train of toothed wheels.***
- The nature of the train used depends upon the **velocity ratio required** and the relative position of the axes of shafts. A gear train may consist of spur, bevel or spiral gears.



Types of Gear Trains

Following are the different types of gear trains, depending upon the arrangement of wheels :

1. Simple gear train,
2. Compound gear train,
3. Reverted gear train, and
4. Epicyclic gear train

- In the first three types of gear trains, the axes of the shafts over which the gears are mounted are **fixed relative to each other**
- But in case of epicyclic gear trains, the axes of the shafts on which the gears are mounted **may move relative to a fixed axis**

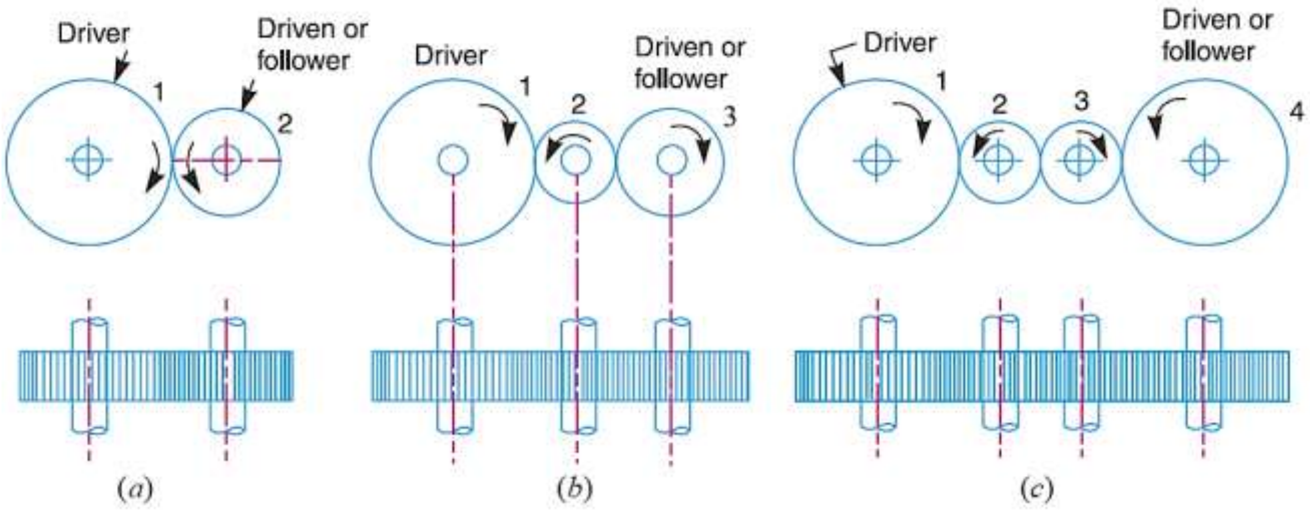
SIMPLE GEAR TRAIN

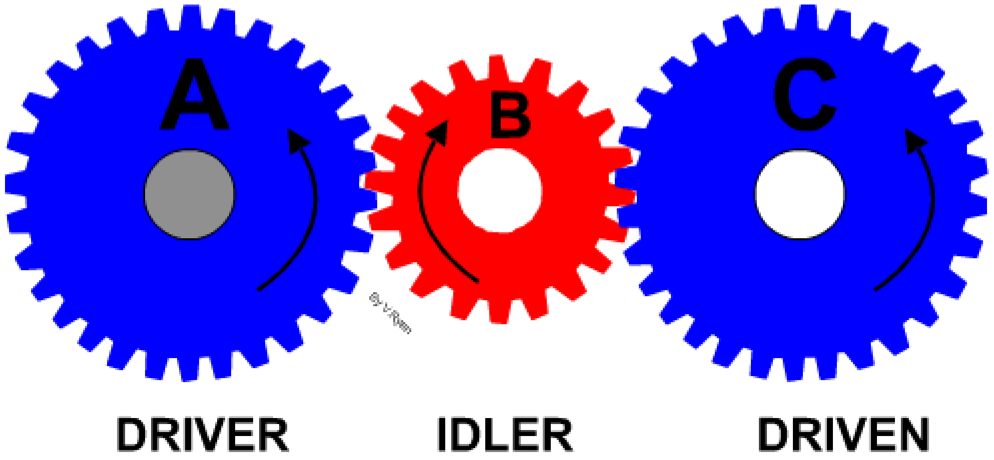
- When there is only one gear on each shaft, as shown in Fig., it is known as ***simple gear train.*** The gears are represented by their pitch circles
- When the distance between the two shafts is small, the two gears 1 and 2 are made to mesh with each other to transmit motion from one shaft to the other, as shown in Fig. (*a*). Gear 1 is called the ***driver*** and the gear 2 is called the ***driven*** or ***follower.*** It may be noted that the motion of the driven gear is opposite to the motion of driving gear.
- The speed ratio in a simple train of gears, is independent of the size and number of intermediate gears. These intermediate gears are called ***idle gears,*** as they do not affect the speed ratio or train value of the system. The idle gears are used for the following two purposes:

1. To connect gears where a large centre distance is required, and

2. To obtain the desired direction of motion of the driven gear (*i.e.* clockwise or anticlockwise)

- When the number of intermediate gears are ***odd,*** the motion of both the gears (*i.e.* driver and driven or follower) is ***like*** as shown in Fig (*b*). But if the number of intermediate gears are ***even,*** the motion of the driven or follower will be in the opposite direction of the driver as shown in Fig (*c*).



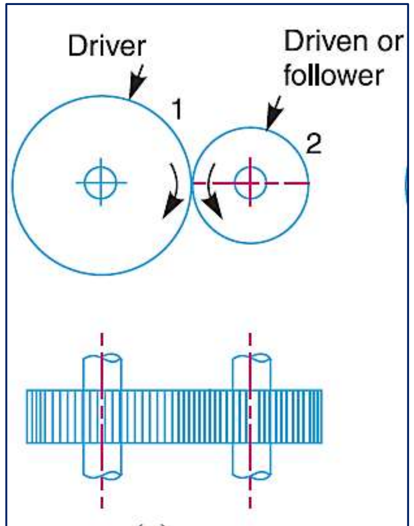


VELOCITY RATIO (SPEED RATIO) & TRAIN VALUE

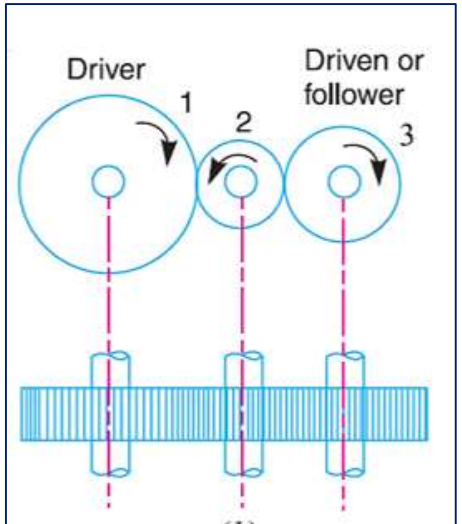
Let N_1 = Speed of gear 1(or driver) in r.p.m.,
N_2 = Speed of gear 2 (or driven or follower) in r.p.m.,
T_1 = Number of teeth on gear 1, and
T_2 = Number of teeth on gear 2.

Since the speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth, therefore

$$\text{Speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$



(a)



(b)

It may be noted that ratio of the speed of the driven or follower to the speed of the driver is known as ***train value*** of the gear train. Mathematically,

$$\text{Train value} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

The speed ratio of the gear train as shown in Fig. (b):

$$\frac{N_1}{N_2} \times \frac{N_2}{N_3} = \frac{T_2}{T_1} \times \frac{T_3}{T_2} \quad \text{or} \quad \frac{N_1}{N_3} = \frac{T_3}{T_1}$$

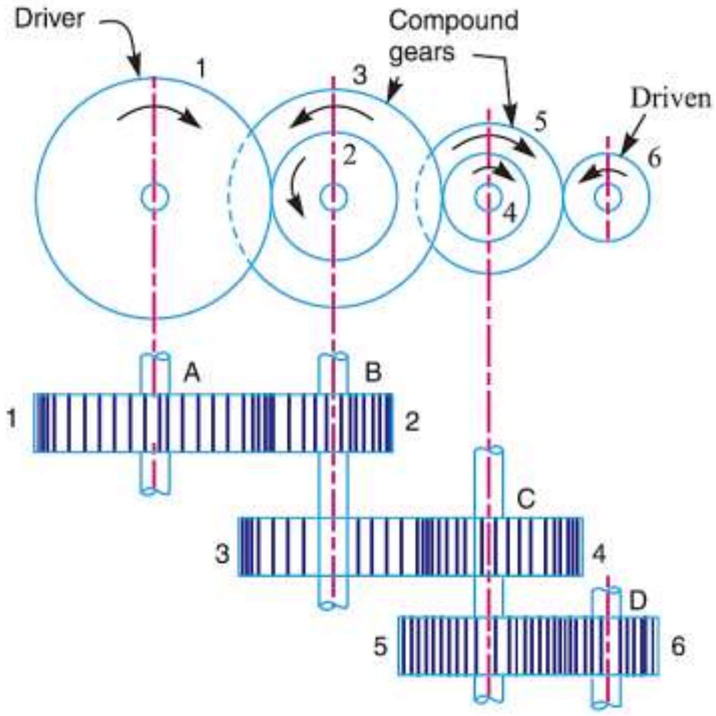
In general,

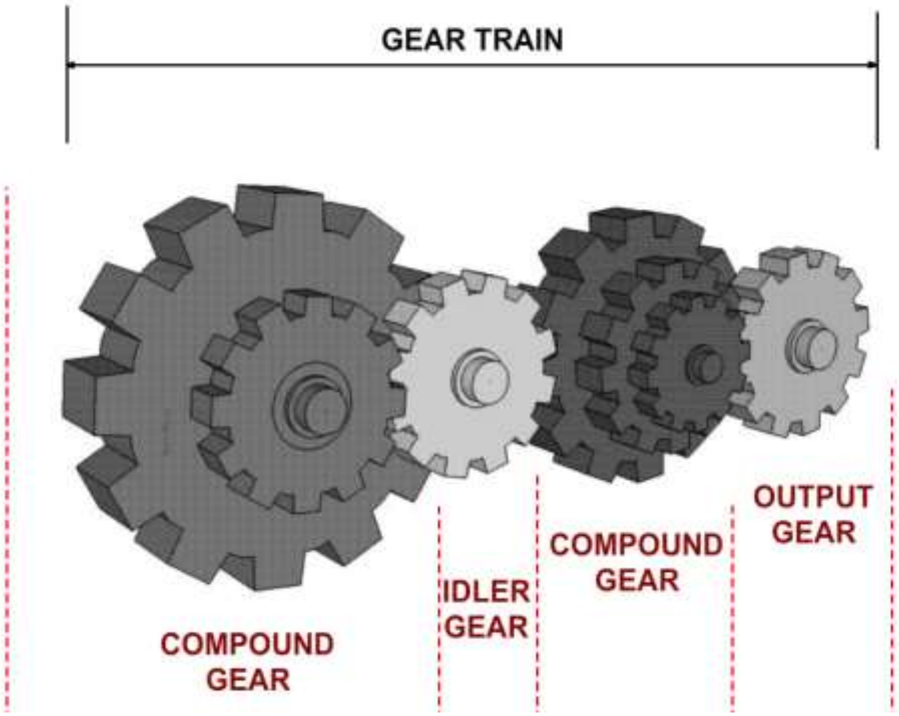
$$\text{Speed ratio} = \frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of teeth on driven}}{\text{No. of teeth on driver}}$$

$$\text{Train value} = \frac{\text{Speed of driven}}{\text{Speed of driver}} = \frac{\text{No. of teeth on driver}}{\text{No. of teeth on driven}}$$

COMPOUND GEAR TRAIN

- When there are more than one gear on a shaft, as shown in Fig., it is called a ***compound train of gear***
- The idle gears, in a simple train of gears do not affect the speed ratio of the system. But these gears are useful in bridging over the space between the driver and the driven.
- But whenever the distance between the driver and the driven or follower has to be bridged over by intermediate gears and at the same time a great (or much less) speed ratio is required, then the advantage of intermediate gears is intensified by providing compound gears on intermediate shafts.
- In this case, each intermediate shaft has two gears rigidly fixed to it so that they may have the same speed. One of these two gears meshes with the driver and the other with the driven or follower attached to the next shaft as shown in Fig.





COMPOUND GEAR TRAIN

Let $N_1, N_2, N_3 \ldots$ = Speed of respective gears in r.p.m.
$T_1, T_2, T_3 \ldots$ = Number of teeth on respective gears.

The speed ratio of compound gear train is obtained by:

$$\frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5} \quad \text{or} \quad * \frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

Since gears 2 and 3 are mounted on one shaft *B*, therefore $N_2 = N_3$. Similarly gears 4 and 5 are mounted on shaft *C*, therefore $N_4 = N_5$.

$$\text{Speed ratio} = \frac{\text{Speed of the first driver}}{\text{Speed of the last driven or follower}}$$

$$= \frac{\text{Product of the number of teeth on the drivens}}{\text{Product of the number of teeth on the drivers}}$$

$$\text{Train value} = \frac{\text{Speed of the last driven or follower}}{\text{Speed of the first driver}}$$

$$= \frac{\text{Product of the number of teeth on the drivers}}{\text{Product of the number of teeth on the drivens}}$$

The advantage of a compound train over a simple gear train is that a much larger speed reduction from the first shaft to the last shaft can be obtained with small gears. If a simple gear train is used to give a large speed reduction, the last gear has to be very large. Usually for a speed reduction in excess of 7 to 1, a simple train is not used and a compound train or worm gearing is employed.

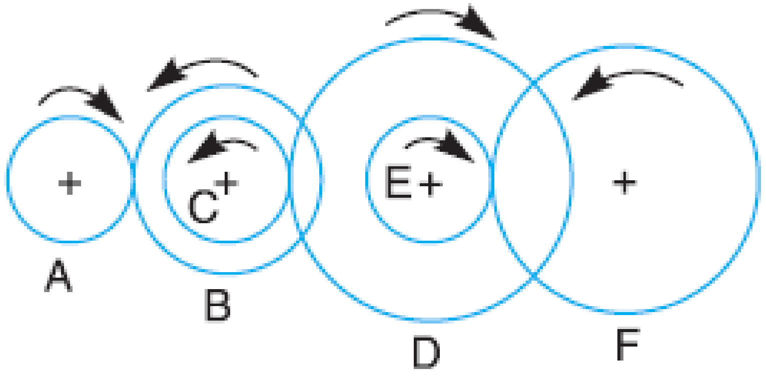
Note: The gears which mesh must have the same circular pitch or module. Thus gears 1 and 2 must have the same module as they mesh together. Similarly gears 3 and 4, and gears 5 and 6 must have the same module.

MATH

The gearing of a machine tool is shown in Fig. The motor shaft is connected to gear A and rotates at 975 r.p.m. The gear wheels B, C, D and E are fixed to parallel shafts rotating together. The final gear F is fixed on the output shaft. What is the speed of gear F ? The number of teeth on each gear are as given below :

Gear A B C D E F

No. of teeth 20 50 25 75 26 65



$$\boxed{N_F = 52 \text{ r. p. m.} = \textbf{Ans.}}$$

SOLUTION

Solution. Given : $N_A = 975$ r.p.m. ; $T_A = 20$; $T_B = 50$; $T_C = 25$; $T_D = 75$; $T_E = 26$; $T_F = 65$

From Fig. we see that gears A, C and E are drivers while the gears B, D and F are driven or followers. Let the gear A rotates in clockwise direction. Since the gears B and C are mounted on the same shaft, therefore it is a compound gear and the direction or rotation of both these gears is same (*i.e.* anticlockwise). Similarly, the gears D and E are mounted on the same shaft, therefore it is also a compound gear and the direction of rotation of both these gears is same (*i.e.* clockwise). The gear F will rotate in anticlockwise direction.

Let N_F = Speed of gear F, *i.e.* last driven or follower.

We know that

$$\frac{\text{Speed of the first driver}}{\text{Speed of the last driven}} = \frac{\text{Product of no. of teeth on drivens}}{\text{Product of no. of teeth on drivers}}$$

or

$$\frac{N_A}{N_F} = \frac{T_B \times T_D \times T_F}{T_A \times T_C \times T_E} = \frac{50 \times 75 \times 65}{20 \times 25 \times 26} = 18.75$$

$$\therefore \quad N_F = \frac{N_A}{18.75} = \frac{975}{18.75} = 52 \text{ r. p. m.} \quad \textbf{Ans.}$$

DESIGN OF SPUR GEARS

Sometimes, the spur gears (*i.e.* driver and driven) are to be designed for the given velocity ratio and distance between the centres of their shafts.

Let x = Distance between the centres of two shafts,

N_1 = Speed of the driver,

T_1 = Number of teeth on the driver,

d_1 = Pitch circle diameter of the driver,

N_2 , T_2 and d_2 = Corresponding values for the driven or follower, and

p_c = Circular pitch.

We know that the distance between the centres of two shafts,

$$x = \frac{d_1 + d_2}{2} \qquad \text{...}(i)$$

and speed ratio or velocity ratio,

$$\frac{N_1}{N_2} = \frac{d_2}{d_1} = \frac{T_2}{T_1} \qquad \text{...}(ii)$$

From the above equations, we can conveniently find out the values of d_1 and d_2 (or T_1 and T_2) and the circular pitch (p_c). The values of T_1 and T_2, as obtained above, may or may not be whole numbers. But in a gear since the number of its teeth is always a whole number, therefore a slight alterations must be made in the values of x, d_1 and d_2, so that the number of teeth in the two gears may be a complete number.

MATH

Two parallel shafts, about 600 mm apart are to be connected by spur gears. One shaft is to run at 360 r.p.m. and the other at 120 r.p.m. Design the gears, if the circular pitch is to be 25 mm.

The number of teeth on the first and second gear must be 38 and 114 and their pitch circle diameters must be 302.36 mm and 907.1 mm respectively. The exact distance between the two shafts must be 604.73 mm.

SOLUTION

Solution. Given : $x = 600$ mm ; $N_1 = 360$ r.p.m. ; $N_2 = 120$ r.p.m. ; $p_c = 25$ mm

Let d_1 = Pitch circle diameter of the first gear, and

d_2 = Pitch circle diameter of the second gear.

We know that speed ratio,

$$\frac{N_1}{N_2} = \frac{d_2}{d_1} = \frac{360}{120} = 3 \quad \text{or} \quad d_2 = 3d_1 \qquad \ldots(i)$$

and centre distance between the shafts (x),

$$600 = \frac{1}{2}(d_1 + d_2) \quad \text{or} \quad d_1 + d_2 = 1200 \qquad \ldots(ii)$$

From equations (*i*) and (*ii*), we find that

$d_1 = 300$ mm, and $d_2 = 900$ mm

∴ Number of teeth on the first gear,

$$T_1 = \frac{\pi d_2}{p_c} = \frac{\pi \times 300}{25} = 37.7$$

SOLUTION

and number of teeth on the second gear,

$$T_2 = \frac{\pi d_2}{p_c} = \frac{\pi \times 900}{25} = 113.1$$

Since the number of teeth on both the gears are to be in complete numbers, therefore let us make the number of teeth on the first gear as 38. Therefore for a speed ratio of 3, the number of teeth on the second gear should be 38 × 3 = 114.

Now the exact pitch circle diameter of the first gear,

$$d_1' = \frac{T_1 \times p_c}{\pi} = \frac{38 \times 25}{\pi} = 302.36 \text{ mm}$$

and the exact pitch circle diameter of the second gear,

$$d_2' = \frac{T_2 \times p_c}{\pi} = \frac{114 \times 25}{\pi} = 907.1 \text{ mm}$$

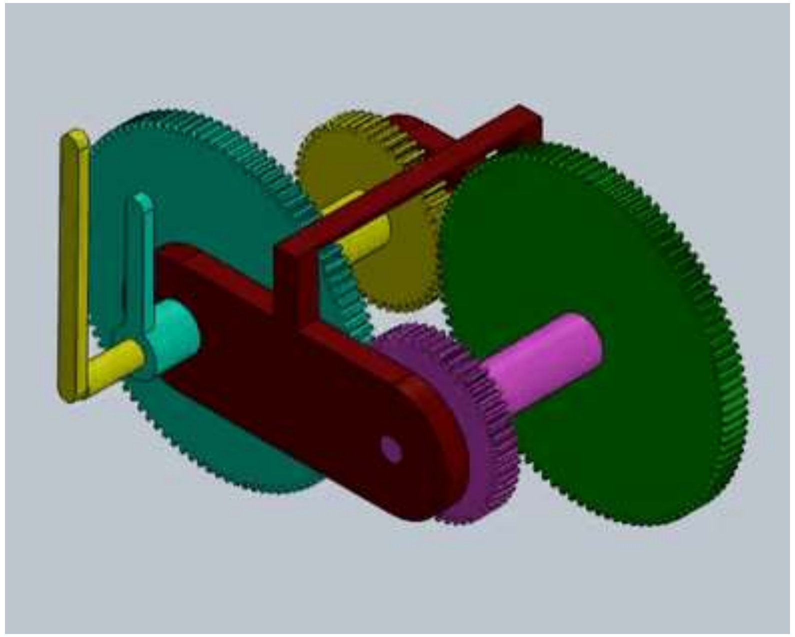
∴ Exact distance between the two shafts,

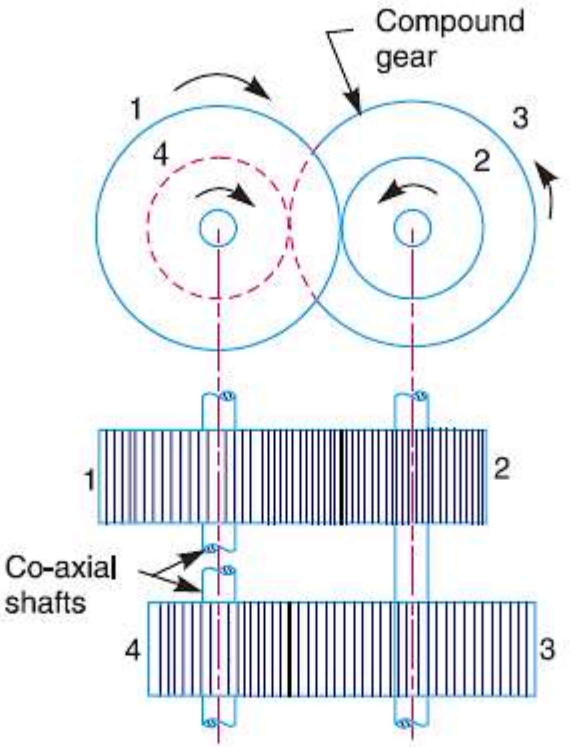
$$x' = \frac{d_1' + d_2'}{2} = \frac{302.36 + 907.1}{2} = 604.73 \text{ mm}$$

Hence the number of teeth on the first and second gear must be 38 and 114 and their pitch circle diameters must be 302.36 mm and 907.1 mm respectively. The exact distance between the two shafts must be 604.73 mm. **Ans.**

REVERTED GEAR TRAIN

- When the axes of the first gear (*i.e.* first driver) and the last gear (*i.e.* last driven or follower) are co-axial, then the gear train is known as ***reverted gear* train** as shown in Fig.
- We see that gear 1 (*i.e.* first driver) drives the gear 2 (*i.e.* first driven or follower) in the opposite direction. Since the gears 2 and 3 are mounted on the same shaft, therefore they form a **compound gear** and the gear 3 will rotate in the same direction as that of gear 2. The gear 3 (which is now the second driver) drives the gear 4 (*i.e.* the last driven or follower) in the same direction as that of gear 1. Thus, we see that in a reverted gear train, the motion of the first gear and the last gear is ***like.***
- Let T_1 = Number of teeth on gear 1,
- r_1 = Pitch circle radius of gear 1, and
- N_1 = Speed of gear 1 in r.p.m.
- Similarly,
- T_2, T_3, T_4 = Number of teeth on respective gears,
- r_2, r_3, r_4 = Pitch circle radii of respective gears, and
- N_2, N_3, N_4 = Speed of respective gears in r.p.m.





REVERTED GEAR TRAIN

Since the distance between the centres of the shafts of gears 1 and 2 as well as gears 3 and 4 is same, therefore

$$r_1 + r_2 = r_3 + r_4 \quad \text{...(i)}$$

Also, the circular pitch or module of all the gears is assumed to be same, therefore number of teeth on each gear is directly proportional to its circumference or radius.

$$\therefore \; *T_1 + T_2 = T_3 + T_4 \quad \text{...(ii)}$$

and

$$\text{Speed ratio} = \frac{\text{Product of number of teeth on drivens}}{\text{Product of number of teeth on drivers}}$$

or

$$\frac{N_1}{N_4} = \frac{T_2 \times T_4}{T_1 \times T_3} \quad \text{... (iii)}$$

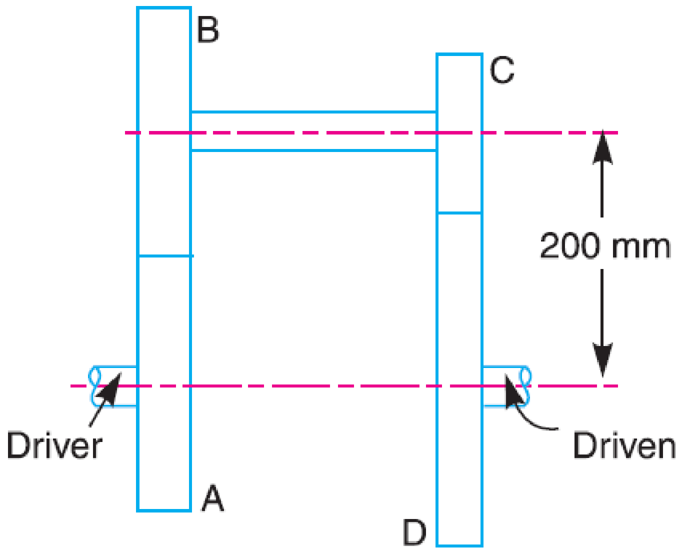
From equations (*i*), (*ii*) and (*iii*), we can determine the number of teeth on each gear for the given centre distance, speed ratio and module only when the number of teeth on one gear is chosen arbitrarily.

The reverted gear trains are used in automotive transmissions, lathe back gears, industrial speed reducers, and in clocks (where the minute and hour hand shafts are co-axial).

MATH

The speed ratio of the reverted gear train, as shown in Fig., is to be 12 and the ratio is same between gear A and B, and between C and D. The module pitch of gears A and B is 3.125 mm and of gears C and D is 2.5 mm.

- *Calculate the suitable numbers of teeth for the gears. No gear is to have less than 24 teeth.*



$T_A = 28$
$T_B = 100$
$T_C = 36$
$T_D = 124$

SOLUTION

Solution. Given : Speed ratio, $N_A/N_D = 12$;
$m_A = m_B = 3.125$ mm ; $m_C = m_D = 2.5$ mm

Let N_A = Speed of gear A,

T_A = Number of teeth on gear A,

r_A = Pitch circle radius of gear A,

N_B, N_C, N_D = Speed of respective gears,

T_B, T_C, T_D = Number of teeth on respective gears, and

r_B, r_C, r_D = Pitch circle radii of respective gears.

---

* We know that circular pitch,

$$p_c = \frac{2\pi r}{T} = \pi m \qquad \text{or} \qquad r = \frac{m.T}{2}, \text{ where } m \text{ is the module.}$$

$$\therefore \qquad r_1 = \frac{m.T_1}{2} \; ; \; r_2 = \frac{m.T_2}{2} \; ; \; r_3 = \frac{m.T_3}{2} \; ; \; r_4 = \frac{m.T_4}{2}$$

Now from equation (*i*),

$$\frac{m.T_1}{2} + \frac{m.T_2}{2} = \frac{m.T_3}{2} + \frac{m.T_4}{2}$$

$$T_1 + T_2 = T_3 + T_4$$

SOLUTION

Since the speed ratio between the gears A and B and between the gears C and D are to be same, therefore

$$* \frac{N_A}{N_B} = \frac{N_C}{N_D} = \sqrt{12} = 3.464$$

Also the speed ratio of any pair of gears in mesh is the inverse of their number of teeth, therefore

$$\frac{T_B}{T_A} = \frac{T_D}{T_C} = 3.464 \qquad \text{...}(i)$$

We know that the distance between the shafts

$$x = r_A + r_B = r_C + r_D = 200 \text{ mm}$$

or
$$\frac{m_A . T_A}{2} + \frac{m_B . T_B}{2} = \frac{m_C . T_C}{2} + \frac{m_D . T_D}{2} = 200 \qquad \text{...}\left(\because r = \frac{m.T}{2}\right)$$

$$3.125\,(T_A + T_B) = 2.5\,(T_C + T_D) = 400 \qquad \text{...}(\because m_A = m_B, \text{ and } m_C = m_D)$$

$\therefore$
$$T_A + T_B = 400 / 3.125 = 128 \qquad \text{...}(ii)$$

and
$$T_C + T_D = 400 / 2.5 = 160 \qquad \text{...}(iii)$$

From equation (*i*), $T_B = 3.464\,T_A$. Substituting this value of T_B in equation (*ii*),

$$T_A + 3.464\,T_A = 128 \quad \text{or} \quad T_A = 128 / 4.464 = 28.67 \text{ say } 28 \quad \textbf{Ans.}$$

and
$$T_B = 128 - 28 = 100 \ \textbf{Ans.}$$

Again from equation (*i*), $T_D = 3.464\,T_C$. Substituting this value of T_D in equation (*iii*),

$$T_C + 3.464\,T_C = 160 \quad \text{or} \quad T_C = 160 / 4.464 = 35.84 \text{ say } 36 \quad \textbf{Ans.}$$

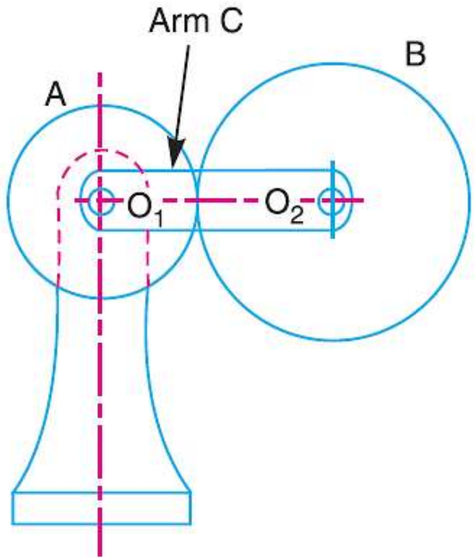
and
$$T_D = 160 - 36 = 124 \ \textbf{Ans.}$$

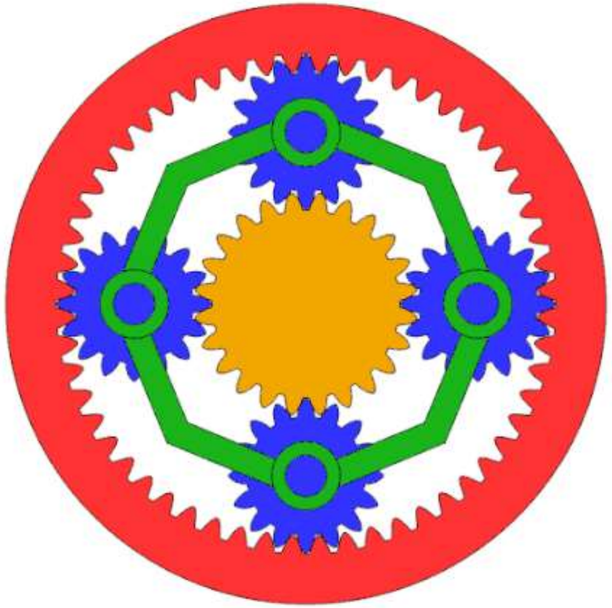
Note : The speed ratio of the reverted gear train with the calculated values of number of teeth on each gear is

$$\frac{N_A}{N_D} = \frac{T_B \times T_D}{T_A \times T_C} = \frac{100 \times 124}{28 \times 36} = 12.3$$

EPICYCLIC GEAR TRAIN

- A simple epicyclic gear train is shown in Fig., where a gear *A* and the arm *C* have a common axis at O_1 about which they can rotate. The gear *B* meshes with gear *A* and has its axis on the arm at O_2, about which the gear *B* can rotate. If the arm is fixed, the gear train is simple and gear *A* can drive gear *B* or ***vice- versa***, but if gear *A* is fixed and the arm is rotated about the axis of gear *A* (*i.e.* O_1), then the gear *B* is forced to rotate ***upon*** and ***around*** gear *A*. Such a motion is called **epicyclic** and the gear trains arranged in such a manner that one or more of their members move upon and around another member are known as ***epicyclic gear trains*** (***epi.*** means upon and ***cyclic*** means around). The epicyclic gear trains may be ***simple*** or ***compound.***
- The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space. The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.





SIMPLE EPICYCLIC GEAR TRAIN

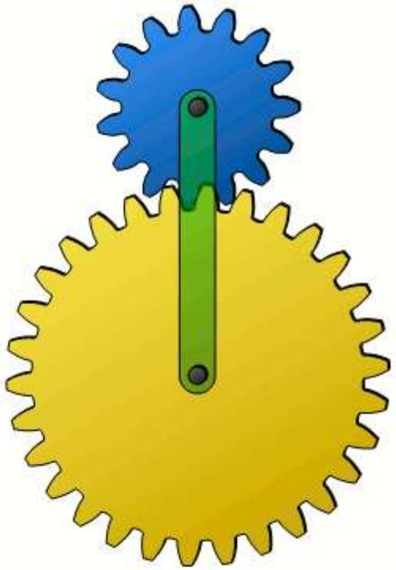
- *When there is only one gear on each shaft in any epicyclic gear trains, then they are called as* ***simple epicyclic gear trains.***

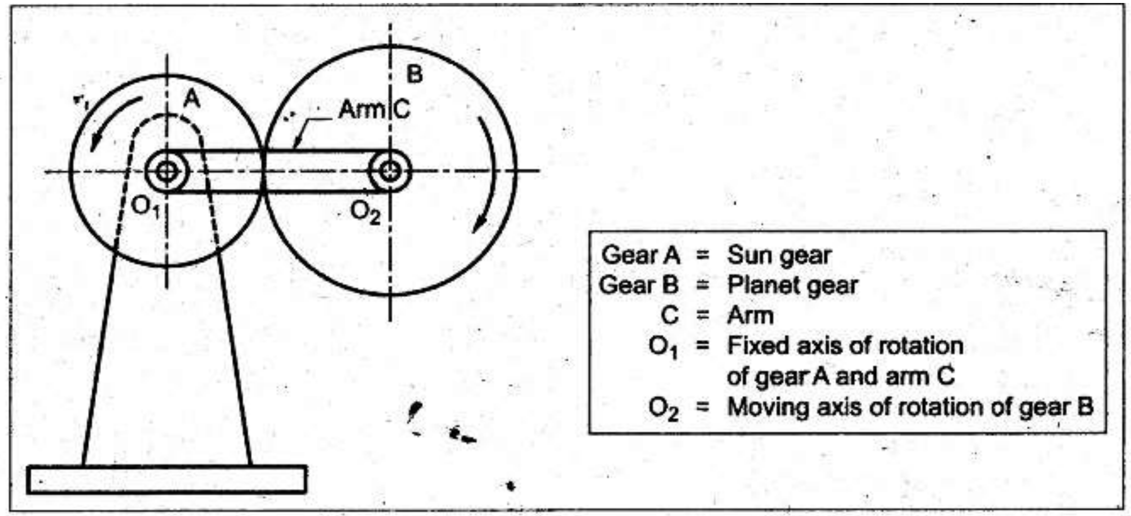
Three ***main elements of the any epicyclic gear train*** are:

(i) ***Sun gear:*** The gear at the centre, whose axis is fixed, is called the sun gear.

(ii) ***Planet gear:*** The gear whose axis moves (which is free to revolve around sun gear) is called planet gear.

(iii) ***Arm:*** The arm carries the planet gear and is free to revolve about the fixed axis.





EPICYCLIC GEAR TRAIN

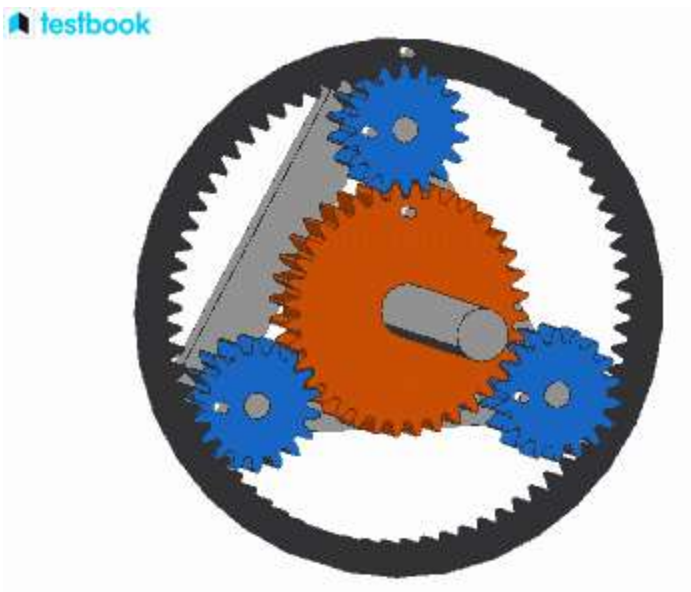
Configuration 1 – Sun Gear Fixed

- In this configuration, the sun gear remains fixed while the ring gear, which is responsible for driving the planet carrier, is supplied with power. This arrangement of the gear train results in a relatively lower gear ratio than other possible configurations.

Here,

The planet gear rotates in an anti clockwise direction

The ring gear rotates in a clockwise direction.



Configuration 1

Configuration 2 – Ring Gear Fixed

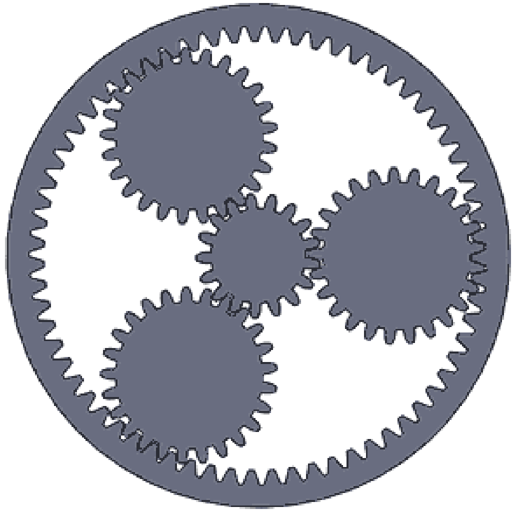
- In this configuration, the ring gear remains fixed while the sun gear behaves as the driver gear. The planet carrier acts as the driven gear. The epicyclic gear train assembled in this configuration offers the highest gear ratio possible.

- As a result, higher torque requirements are fulfilled while maintaining a lower speed.

Here,

There is a clockwise rotation of sun gear

The planet gear rotates in an anti-clockwise direction.



Configuration 2

EPICYCLIC GEAR TRAIN

Configuration 3 – Planet Carrier Fixed

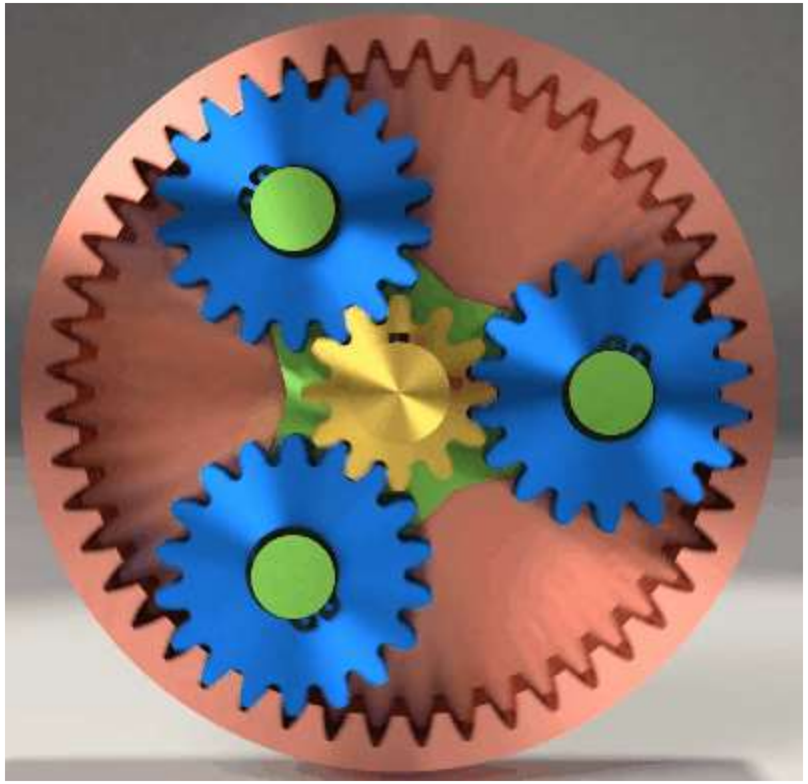
- In this configuration, the planet carrier is fixed making the sun and ring gear rotate in the opposite or reverse directions. Because of this, the configuration is used in the vehicle to travel in reverse after applying the reverse gear.

Here,
There is again a clockwise rotation of sun gear
The planet and ring gear rotate in an anti-clockwise direction.

Configuration 4 – Direct Drive

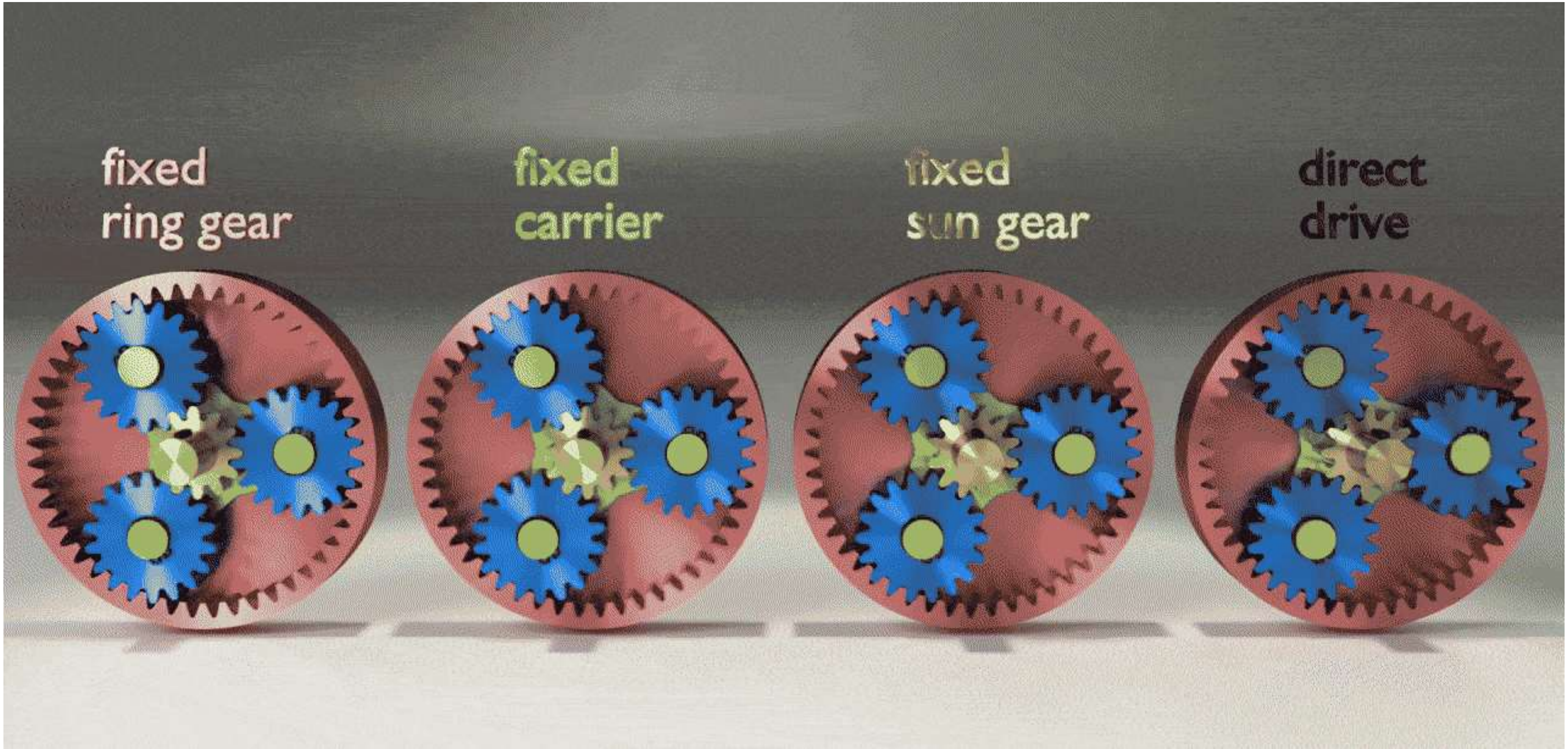
✓ A planetary gear can also be used as a so-called direct drive. The carrier and the sun gear are firmly fixed to the ring gear.

✓ In this case, the rotary motion is transmitted directly from the input shaft to the output shaft (transmission ratio 1:1).



Configuration 3

EPICYCLIC GEAR TRAIN



EPICYCLIC GEAR TRAIN

Here, Z = No. of Teeth; Transmission Ratio = Overall Gear Ratio

Fixed	Ring Gear	Planet Carrier	Sun Gear	None (Direct Drive)
Input	Sun Gear	Sun Gear	Ring Gear	Sun/Ring/Carrier
Output	Planet Carrier	Ring Gear	Planet Carrier	Other two components except input
Transmission Ratio	$i_r = 1 + (z_r/z_s)$ $2 < i_r < \infty$	$i_c = -(z_r/z_s)$ $-\infty < i_c < -1$	$i_s = 1 + (z_s/z_r)$ $1 < i_s < 2$	1:1
Reverse Input	Planet Carrier	Ring Gear	Planet Carrier	Sun/Ring/Carrier
Reverse Output	Sun Gear	Sun Gear	Ring Gear	Other two components except input
Inverse Transmission Ratio	$i^*_r = 1/i_r$ $0 < i^*_r < 0.5$	$i^*_c = -1/i_c$ $-1 < i_c < 0$	$i^*_s = 1/i_s$ $0.5 < i^*_s < 1$	1:1
Direction of Output Shaft/ Revolution of Planet Gears	Same as input shaft	Opposite of the input shaft	Same as input shaft	Total system move in same direction together
Direction of Spin of Planet Gears	Opposite of Sun Gear	Opposite of Sun Gear	Opposite of Ring Gear	No Spin

EPICYCLIC GEAR TRAIN

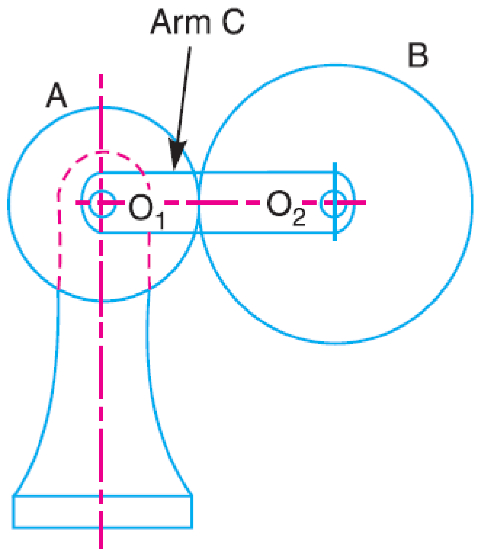
Higher Gear (Reduction) Ratio > Higher Torque > Lower Output Speed
Lower Gear (Reduction) Ratio > Lower Torque > Higher Output Speed

Stationery	Ring Gear	Carrier	Sun Gear
Input	Sun Gear	Sun Gear	Ring Gear
Output	Carrier	Ring Gear	Carrier
Transmission Ratio	$i = 1 + \frac{Z_{ring}}{Z_{sun}}$	$i = -\frac{Z_{ring}}{Z_{sun}}$	$i = 1 + \frac{Z_{sun}}{Z_{ring}}$
Example: Sun Gear - 12 teeth Ring Gear - 48 teeth	i = 5 (5:1)	i = -4 (-4:1)	i = 1.25 (5:4)

Higher Torque & Lower Output Speed	*Higher Torque & Lower Output Speed & Reverse Drive*	*Lower Torque & Higher Output Speed*

VELOCITY RATIOS OF EPICYCLIC GEAR TRAIN

1. *Tabular method.* Consider an epicyclic gear train as shown in Fig.



Let T_A = Number of teeth on gear *A*, and T_B = Number of teeth on gear *B*.

- First of all, let us suppose that the arm is fixed. Therefore, the axes of both the gears are also fixed relative to each other. When the gear *A* makes one revolution anticlockwise, the gear *B* will make *T_A / T_B revolutions, clockwise.
- Assuming the anticlockwise rotation as positive and clockwise as negative, we may say that when gear *A* makes + 1 revolution, then the gear *B* will make ($-T_A / T_B$) revolutions. This statement of relative motion is entered in the first row of the table (see Table).
- Secondly, if the gear *A* makes + *x* revolutions, then the gear *B* will make $-x \times T_A / T_B$ revolutions. This statement is entered in the second row of the table. In other words, multiply each motion (entered in the first row) by *x*.
- Thirdly, each element of an epicyclic train is given + *y* revolutions and entered in the third row. Finally, the motion of each element of the gear train is added up and entered in the fourth row.
- A little consideration will show that when two conditions about the motion of rotation of any two elements are known, then the unknown speed of the third element may be obtained by substituting the given data in the third column of the fourth row.

Table of motions

Step No.	Conditions of motion	*Revolutions of elements*		
		Arm C	*Gear A*	*Gear B*
1.	Arm fixed-gear *A* rotates through + 1 revolution *i.e.* 1 rev. anticlockwise	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear *A* rotates through + *x* revolutions	0	$+x$	$-x \times \frac{T_A}{T_B}$
3.	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_A}{T_B}$

VELOCITY RATIOS OF EPICYCLIC GEAR TRAIN

2. Algebraic method.

In this method, the motion of each element of the epicyclic train relative to the arm is set down in the form of equations. The number of equations depends upon the number of elements in the gear train. But the two conditions are, usually, supplied in any epicyclic train *viz*. some element is fixed and the other has specified motion. These two conditions are sufficient to solve all the equations ; and hence to determine the motion of any element in the epicyclic gear train.

Let the arm *C* be fixed in an epicyclic gear train as shown in Fig. Therefore, speed of the gear *A* relative to the arm *C*

$$= N_A - N_C$$

and speed of the gear *B* relative to the arm *C*,

$$= N_B - N_C$$

Since the gears *A* and *B* are meshing directly, therefore they will revolve in ***opposite*** directions.

$$\therefore \quad \frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B}$$

Since the arm *C* is fixed, therefore its speed, $N_C = 0$.

$$\therefore \quad \frac{N_B}{N_A} = -\frac{T_A}{T_B}$$

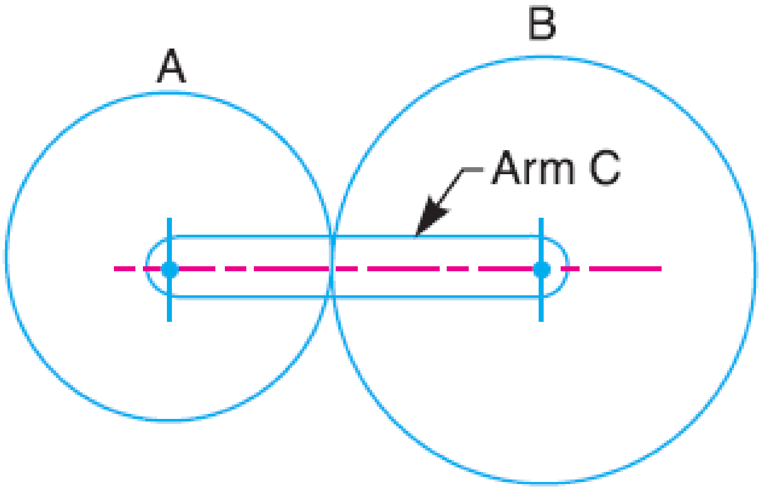
If the gear *A* is fixed, then $N_A = 0$.

$$\frac{N_B - N_C}{0 - N_C} = -\frac{T_A}{T_B} \quad \text{or} \quad \frac{N_B}{N_C} = 1 + \frac{T_A}{T_B}$$

Note : The tabular method is easier and hence mostly used in solving problems on epicyclic gear train.

MATH

In an epicyclic gear train, an arm carries two gears A and B having 36 and 45 teeth respectively. If the arm rotates at 150 r.p.m. in the anticlockwise direction about the centre of the gear A which is fixed, determine the speed of gear B. If the gear A instead of being fixed, makes 300 r.p.m. in the clockwise direction, what will be the speed of gear B ?



N_B (A fixed)= 270 r.p.m. (anticlockwise)
N_B (A makes 300 r.p.m. clockwise) =510 r.p.m. (anticlockwise)

SOLUTION

1. *Tabular method*

First of all prepare the table of motions as given below :

Table of motions.

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear A rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear A rotates through + x revolutions	0	$+x$	$-x \times \frac{T_A}{T_B}$
3.	Add + y revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_A}{T_B}$

Speed of gear B when gear A is fixed

Since the speed of arm is 150 r.p.m. anticlockwise, therefore from the fourth row of the table,

$$y = +150 \text{ r.p.m.}$$

Also the gear A is fixed, therefore

$$x + y = 0 \quad \text{or} \quad x = -y = -150 \text{ r.p.m.}$$

$$\therefore \text{ Speed of gear } B, \quad N_B = y - x \times \frac{T_A}{T_B} = 150 + 150 \times \frac{36}{45} = +270 \text{ r.p.m.}$$

$$= 270 \text{ r.p.m. (anticlockwise)} \quad \textbf{Ans.}$$

SOLUTION

Speed of gear B when gear A makes 300 r.p.m. clockwise

Since the gear A makes 300 r.p.m.clockwise, therefore from the fourth row of the table,

$$x + y = -300 \qquad \text{or} \qquad x = -300 - y = -300 - 150 = -450 \text{ r.p.m.}$$

$\therefore$ Speed of gear B,

$$N_B = y - x \times \frac{T_A}{T_B} = 150 + 450 \times \frac{36}{45} = +510 \text{ r.p.m.}$$

$$= 510 \text{ r.p.m. (anticlockwise)}$$ **Ans.**

2. *Algebraic method*

Let N_A = Speed of gear A.

N_B = Speed of gear B, and

N_C = Speed of arm C.

Assuming the arm C to be fixed, speed of gear A relative to arm C

$$= N_A - N_C$$

and speed of gear B relative to arm $C = N_B - N_C$

SOLUTION

Since the gears A and B revolve in *opposite* directions, therefore

$$\frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B} \qquad \ldots(i)$$

Speed of gear B when gear A is fixed

When gear A is fixed, the arm rotates at 150 r.p.m. in the anticlockwise direction, *i.e.*

$$N_A = 0, \quad \text{and} \quad N_C = +150 \text{ r.p.m.}$$

$$\therefore \quad \frac{N_B - 150}{0 - 150} = -\frac{36}{45} = -0.8 \qquad \ldots\text{[From equation } (i)\text{]}$$

or $$N_B = -150 \times -0.8 + 150 = 120 + 150 = 270 \text{ r.p.m.}$$ **Ans.**

Speed of gear B when gear A makes 300 r.p.m. clockwise

Since the gear A makes 300 r.p.m. clockwise, therefore

$$N_A = -300 \text{ r.p.m.}$$

$$\therefore \quad \frac{N_B - 150}{-300 - 150} = -\frac{36}{45} = -0.8$$

or $$N_B = -450 \times -0.8 + 150 = 360 + 150 = 510 \text{ r.p.m.}$$ **Ans.**

COMPOUND EPICYCLIC GEAR TRAIN

- *When there are more than one gear on a shaft in any epicyclic gear trains, then they are called as* ***compound epicyclic gear trains.***

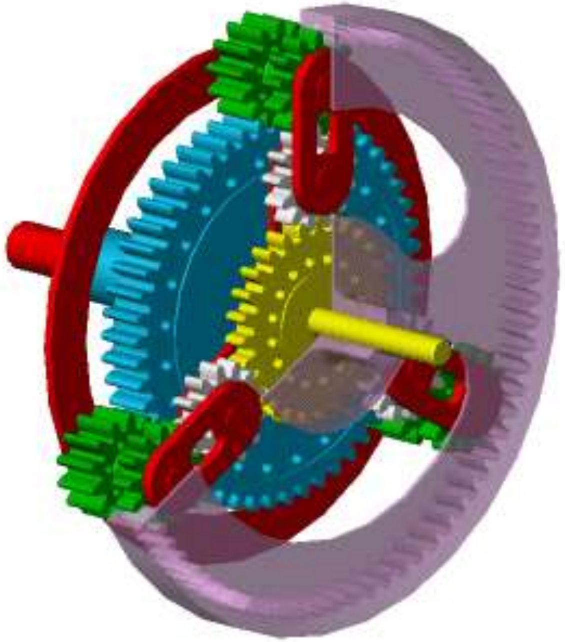
Three ***main elements of the any epicyclic gear train*** are:

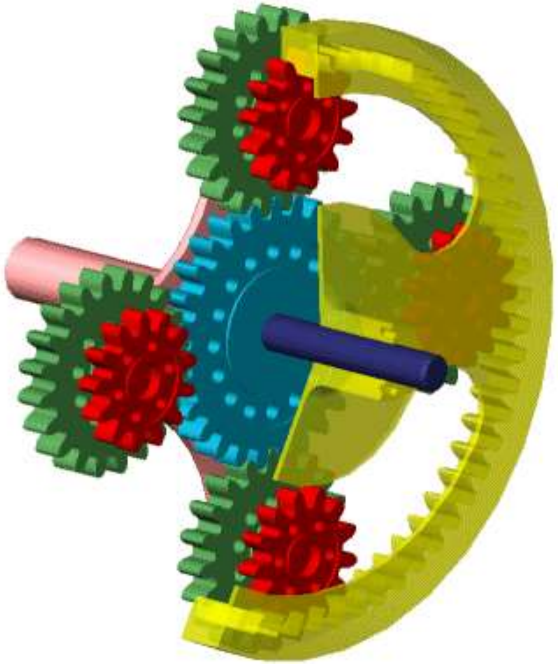
(i) ***Sun gear:*** The gear at the centre, whose axis is fixed, is called the sun gear.

(ii) ***Planet gear:*** The gear whose axis moves (which is free to revolve around sun gear) is called planet gear.

(iii) ***Arm:*** The arm carries the planet gear and is free to revolve about the fixed axis.

(iv) ***Annulus gear:*** Annulus gear is nothing but the internal gear.

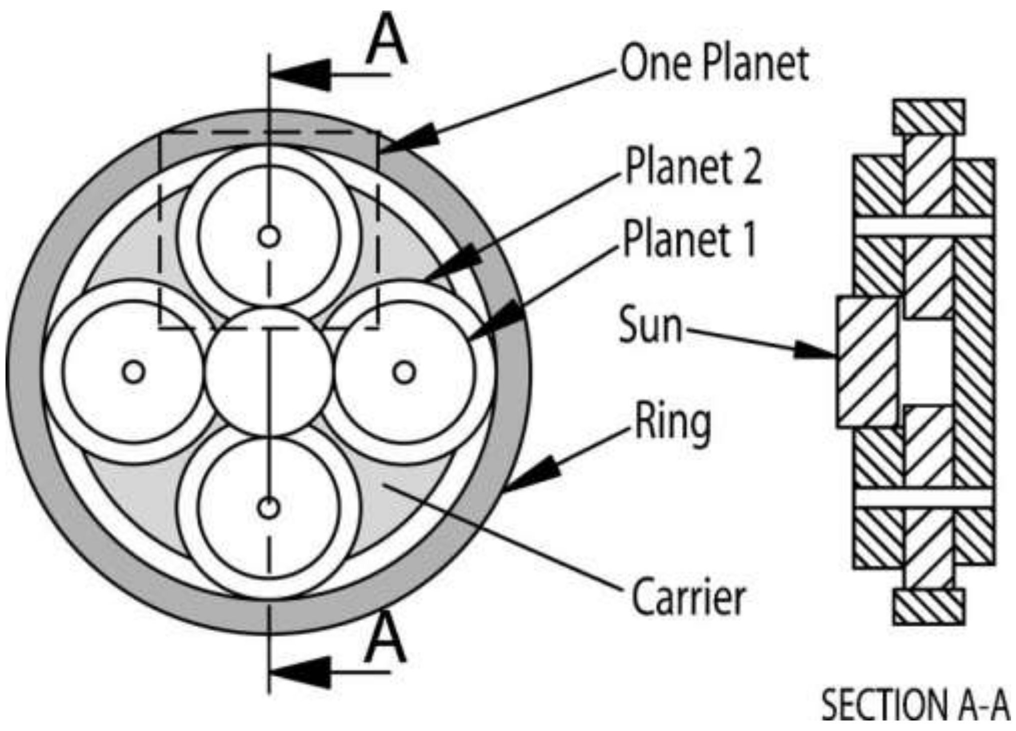


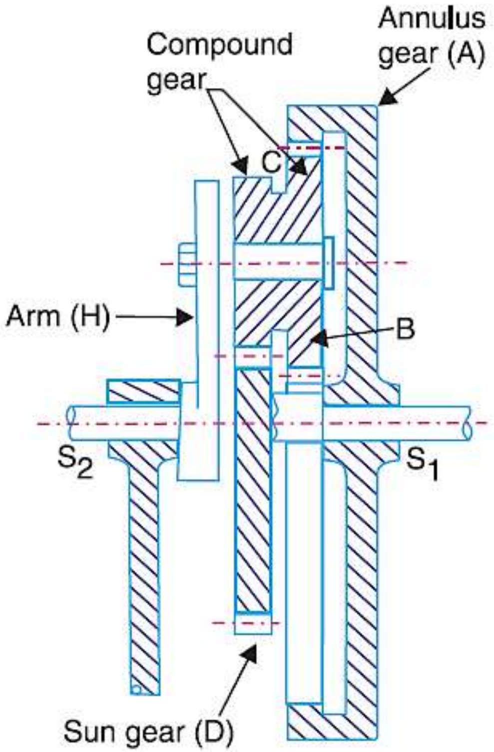


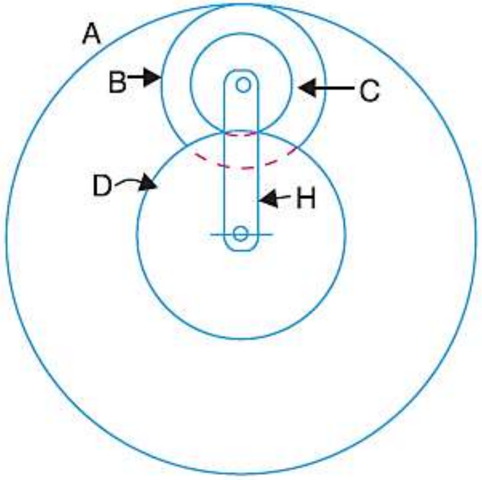
COMPOUND EPICYCLIC GEAR TRAIN — SUN AND PLANET GEAR

A compound epicyclic gear train is shown in Fig. It consists of two co-axial shafts S_1 and S_2, an annulus gear *A* which is fixed, the compound gear (or planet gear) *B*-*C*, the sun gear *D* and the arm *H*. The annulus gear has internal teeth and the compound gear is carried by the arm and revolves freely on a pin of the arm *H*. The sun gear is co-axial with the annulus gear and the arm but independent of them. The annulus gear *A* meshes with the gear *B* and the sun gear *D* meshes with the gear *C*. It may be noted that when the annulus gear is fixed, the sun gear provides the drive and when the sun gear is fixed, the annulus gear provides the drive. In both cases, the arm acts as a follower.

Note : The gear at the centre is called the ***sun gear*** and the gears whose axes move are called ***planet gears.***





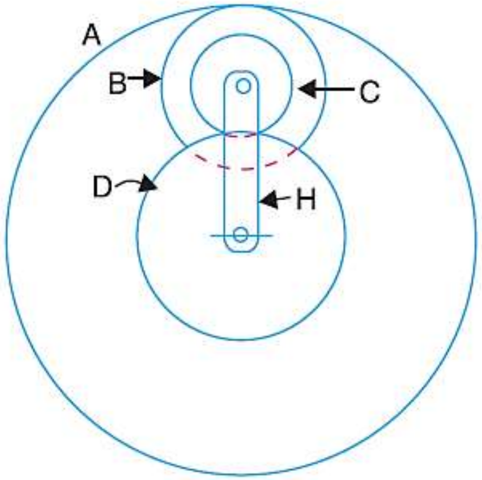


COMPOUND EPICYCLIC GEAR TRAIN — SUN AND PLANET GEAR

- Let T_A , T_B , T_C , and T_D be the teeth and N_A, N_B, N_C and N_D be the speeds for the gears *A*, *B*, *C* and *D* respectively. A little consideration will show that when the arm is fixed and the sun gear *D* is turned anticlockwise, then the compound gear *B*-*C* and the annulus gear A will rotate in the clockwise direction. The motion of rotations of the various elements are shown in the table below.

Table of motions.

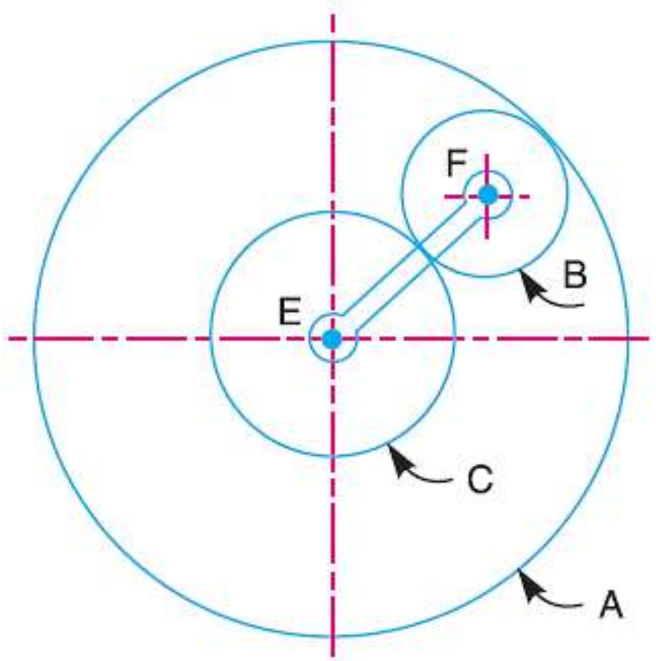
Step No.	*Conditions of motion*	*Revolutions of elements*			
		Arm	*Gear D*	*Compound gear B-C*	*Gear A*
1.	Arm fixed-gear *D* rotates through + 1 revolution	0	+ 1	$-\frac{T_D}{T_C}$	$-\frac{T_D}{T_C} \times \frac{T_B}{T_A}$
2.	Arm fixed-gear *D* rotates through + *x* revolutions	0	$+x$	$-x \times \frac{T_D}{T_C}$	$-x \times \frac{T_D}{T_C} \times \frac{T_B}{T_A}$
3.	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_D}{T_C}$	$y - x \times \frac{T_D}{T_C} \times \frac{T_B}{T_A}$



Note : If the annulus gear *A* is rotated through one revolution anticlockwise with the arm fixed, then the compound gear rotates through T_A / T_B revolutions in the same sense and the sun gear *D* rotates through $T_A / T_B \times T_C / T_D$ revolutions in clockwise direction

MATH

An epicyclic gear consists of three gears A, B and C as shown in Fig. The gear A has 72 internal teeth and gear C has 32 external teeth. The gear B meshes with both A and C and is carried on an arm EF which rotates about the centre of A at 18 r.p.m.. If the gear A is fixed, determine the speed of gears B and C.



Speed of Gear C = 58.5 r.p.m. in the direction of arm.
Speed of Gear B = 46.8 r.p.m. in the opposite direction of arm.

SOLUTION

Solution. Given : $T_A = 72$; $T_C = 32$; Speed of arm $EF = 18$ r.p.m.

Considering the relative motion of rotation as shown in Table 13.5.

Table of motions.

Step No.	Conditions of motion	Revolutions of elements			
		Arm EF	Gear C	Gear B	Gear A
1.	Arm fixed-gear C rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_C}{T_B}$	$-\frac{T_C}{T_B} \times \frac{T_B}{T_A} = -\frac{T_C}{T_A}$
2.	Arm fixed-gear C rotates through $+x$ revolutions	0	$+x$	$-x \times \frac{T_C}{T_B}$	$-x \times \frac{T_C}{T_A}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_C}{T_B}$	$y - x \times \frac{T_C}{T_A}$

SOLUTION

Speed of gear C

We know that the speed of the arm is 18 r.p.m. therefore,

$$y = 18 \text{ r.p.m.}$$

and the gear A is fixed, therefore

$$y - x \times \frac{T_C}{T_A} = 0 \quad \text{or} \quad 18 - x \times \frac{32}{72} = 0$$

$\therefore$ $x = 18 \times 72 / 32 = 40.5$

$\therefore$ Speed of gear C $= x + y = 40.5 + 18$

$= +\,58.5$ r.p.m.

$= 58.5$ r.p.m. in the direction of arm. **Ans.**

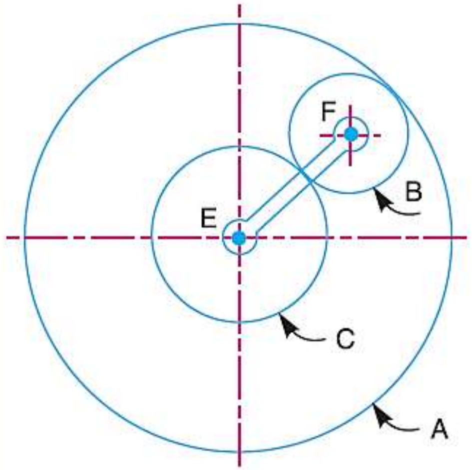


Fig. 13.10

Speed of gear B

Let d_A, d_B and d_C be the pitch circle diameters of gears A, B and C respectively. Therefore, from the geometry of Fig. 13.10,

$$d_B + \frac{d_C}{2} = \frac{d_A}{2} \qquad \text{or} \qquad 2\,d_B + d_C = d_A$$

Since the number of teeth are proportional to their pitch circle diameters, therefore

$$2\,T_B + T_C = T_A \qquad \text{or} \qquad 2\,T_B + 32 = 72 \qquad \text{or} \qquad T_B = 20$$

$\therefore$ Speed of gear B $= y - x \times \frac{T_C}{T_B} = 18 - 40.5 \times \frac{32}{20} = -\,46.8$ r.p.m.

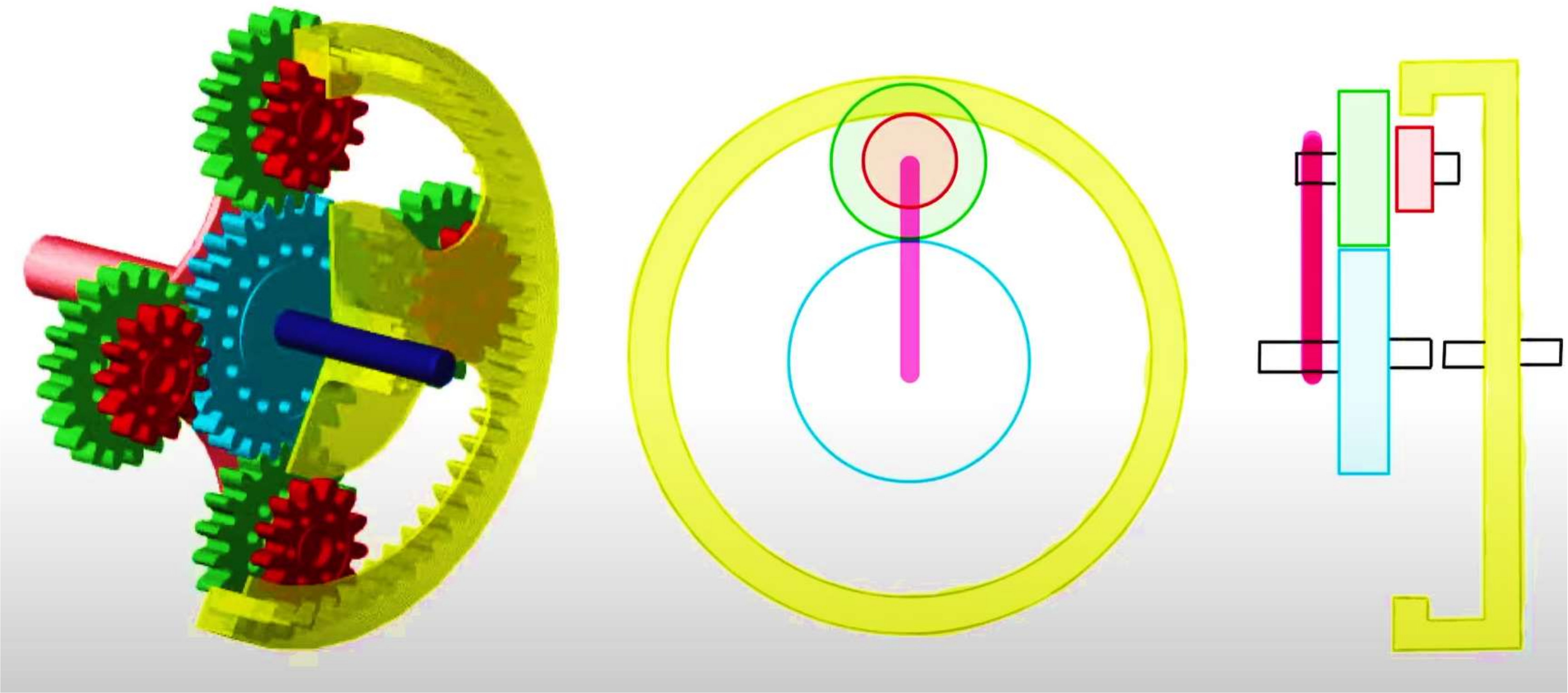
$= 46.8$ r.p.m. in the opposite direction of arm. **Ans.**

MATH

Two shafts A and B are co-axial. A gear C (50 teeth) is rigidly mounted on shaft A. A compound gear D-E gears with C and an internal gear G. D has 20 teeth and gears with C and E has 35 teeth and gears with an internal gear G. The gear G is fixed and is concentric with the shaft axis. The compound gear D-E is mounted on a pin which projects from an arm keyed to the shaft B. Sketch the arrangement and find the number of teeth on internal gear G assuming that all gears have the same module. If the shaft A rotates at 110 r.p.m., find the speed of shaft B.

T_G = 105 Ans.
Speed of shaft B = 50 r.p.m. anticlockwise

SOLUTION



Solution. Given : $T_C = 50$; $T_D = 20$; $T_E = 35$; $N_A = 110$ r.p.m.

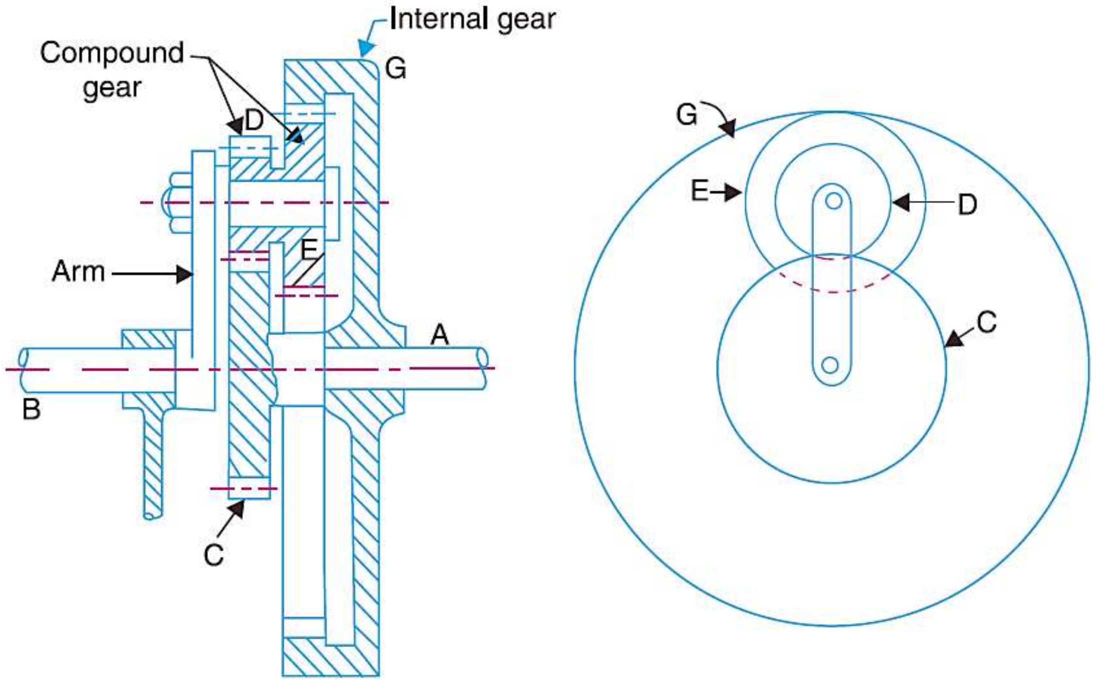
The arrangement is shown in Fig.

Number of teeth on internal gear G

Let d_C , d_D , d_E and d_G be the pitch circle diameters of gears *C*, *D*, *E* and *G* respectively. From the geometry of the figure,

$$\frac{d_G}{2} = \frac{d_C}{2} + \frac{d_D}{2} + \frac{d_E}{2}$$

or $$d_G = d_C + d_D + d_E$$



Let T_C , T_D , T_E and T_G be the number of teeth on gears *C*, *D*, *E* and *G* respectively. Since all the gears have the same module, therefore number of teeth are proportional to their pitch circle diameters.

$\therefore$ $$T_G = T_C + T_D + T_E = 50 + 20 + 35 = 105 \textbf{ Ans.}$$

SOLUTION

Speed of shaft B

The table of motions is given below :

Table of motions.

Step No.	Conditions of motion	Revolutions of elements			
		Arm	*Gear C (or shaft A)*	*Compound gear D-E*	*Gear G*
1.	Arm fixed - gear C rotates through + 1 revolution	0	$+1$	$-\frac{T_C}{T_D}$	$-\frac{T_C}{T_D} \times \frac{T_E}{T_G}$
2.	Arm fixed - gear C rotates through $+x$ revolutions	0	$+x$	$-x \times \frac{T_C}{T_D}$	$-x \times \frac{T_C}{T_D} \times \frac{T_E}{T_G}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_C}{T_D}$	$y - x \times \frac{T_C}{T_D} \times \frac{T_E}{T_G}$

SOLUTION

Since the gear G is fixed, therefore from the fourth row of the table,

$$y - x \times \frac{T_C}{T_D} \times \frac{T_E}{T_G} = 0 \qquad \text{or} \qquad y - x \times \frac{50}{20} \times \frac{35}{105} = 0$$

$$\therefore \qquad y - \frac{5}{6}x = 0 \qquad \ldots(i)$$

Since the gear C is rigidly mounted on shaft A, therefore speed of gear C and shaft A is same. We know that speed of shaft A is 110 r.p.m., therefore from the fourth row of the table,

$$x + y = 100 \qquad \ldots(ii)$$

From equations (*i*) and (*ii*), $x = 60$, and $y = 50$

$\therefore$ Speed of shaft B = Speed of arm = $+y$ = 50 r.p.m. anticlockwise **Ans.**

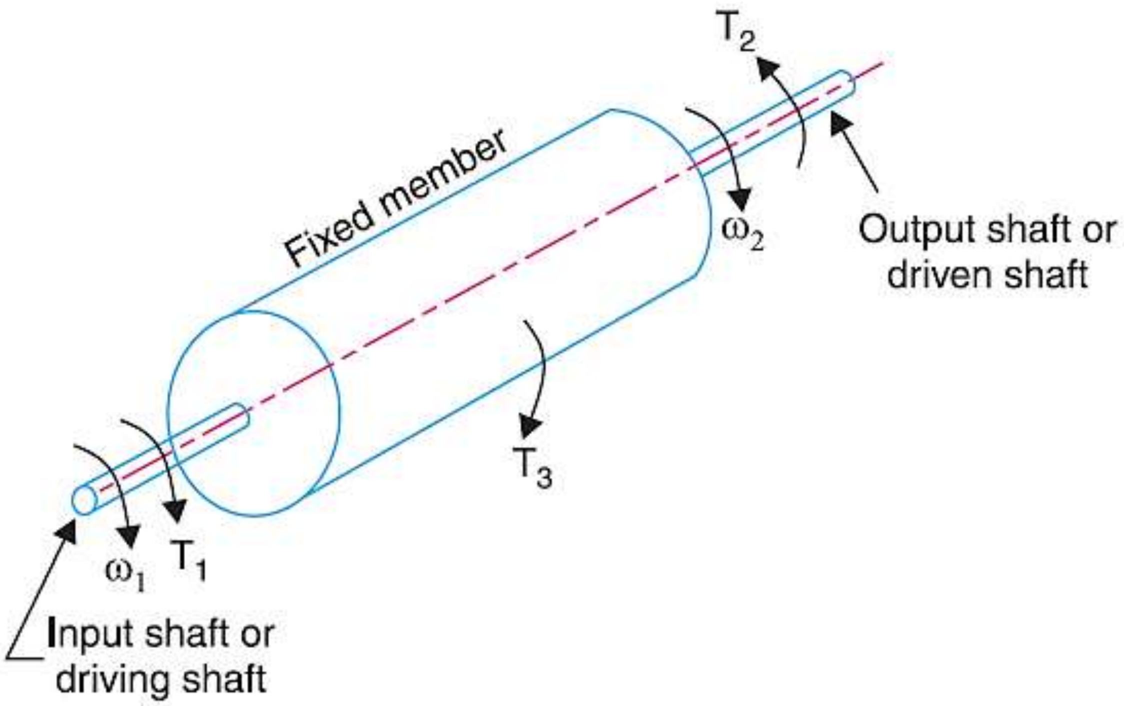
TORQUES IN EPICYCLIC GEAR TRAINS

When the rotating parts of an epicyclic gear train, as shown in Fig, have no angular acceleration, the gear train is kept in equilibrium by the three externally applied torques, *viz*.

1. Input torque on the driving member (T_1),

2. Output torque or resisting or load torque on the driven member (T_2),

3. Holding or braking or fixing torque on the fixed member (T_3).



TORQUES IN EPICYCLIC GEAR TRAINS

The net torque applied to the gear train must be zero. In other words,

$$T_1 + T_2 + T_3 = 0 \quad \text{...}(i)$$

$$\therefore F_1.r_1 + F_2.r_2 + F_3.r_3 = 0 \quad \text{...}(ii)$$

where F_1, F_2 and F_3 are the corresponding externally applied forces at radii r_1, r_2 and r_3. Further, if ω_1, ω_2 and ω_3 are the angular speeds of the driving, driven and fixed members respectively, and the friction be neglected, then the net kinetic energy dissipated by the gear train must be zero, *i.e.*

$$T_1.\omega_1 + T_2.\omega_2 + T_3.\omega_3 = 0 \quad \text{...}(iii)$$

But, for a fixed member, $\omega_3 = 0$

$$\therefore T_1.\omega_1 + T_2.\omega_2 = 0 \quad \text{...}(iv)$$

Notes : 1. From equations **(*i*)** and **(*iv*),** the holding or braking torque T_3 may be obtained as follows :

$$T_2 = -T_1 \times \frac{\omega_1}{\omega_2} \quad \text{...[From equation } (iv)]$$

and

$$T_3 = -(T_1 + T_2) \quad \text{...[From equation } (i)]$$

$$= T_1\left(\frac{\omega_1}{\omega_2} - 1\right) = T_1\left(\frac{N_1}{N_2} - 1\right)$$

2. When input shaft (or driving shaft) and output shaft (or driven shaft) rotate in the same direction, then the input and output torques will be in opposite directions. Similarly, when the input and output shafts rotate in opposite directions, then the input and output torques will be in the same direction.

Solve by Yourself

Book: Mechanics of Machines: Advanced Theory and Examples

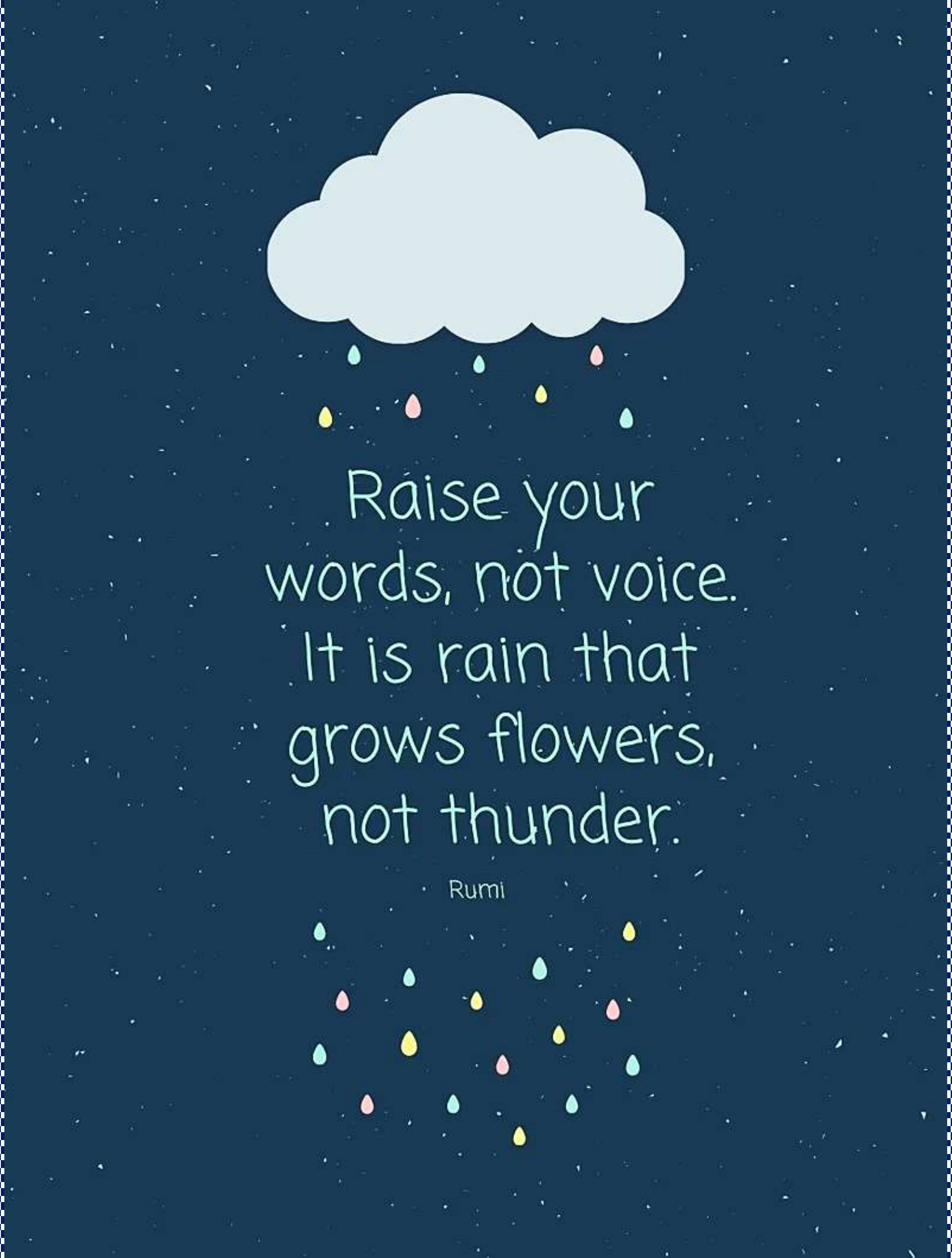
Chapter 12

Exercise: 16,19,20,21

Book: Theory of Machines by RS Khurmi

Chapter 13

Example: 13.7, 13.8,13.9,13.10,13.11,13.15,13.19,13.22
Exercises: 13


Raise your
words, not voice.
It is rain that
grows flowers,
not thunder.
Rumi