AHSANULLAH UNIVERSITY OF SCIENCE AND TECHNOLOGY (AUST)

ME-3105: FLUID MECHANICS

(LC-5: Fluid Kinematics)

BY

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FUNDAMENTALS OF FLUID FLOW OR FLUID KINEMATICS

Kinematics: Fluid kinematics is the *study of how fluids flow and how to describe* **fluid motion** *without considering the forces and moments that cause the fluid in motion.*

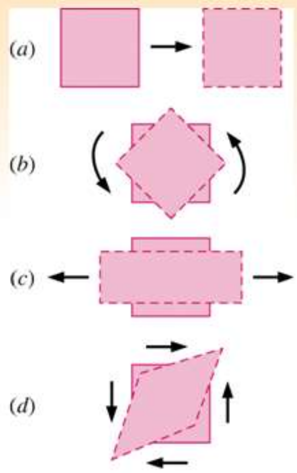
Kinetics: *The science which deals with the action of the forces in producing or changing motion of the fluid is known as hydrokinetics or simply kinetics.*

- The study of fluids in motion involves the consideration of both the Kinetics and Kinematics.

Kinematics: *It involves position, velocity, and acceleration but not force.*

- Fluid kinemetics describes how a fluid particle translates, distorts, and rotates, and how to visualize flow fields.

FUNDAMENTALS OF FLUID FLOW OR FLUID KINEMATICS

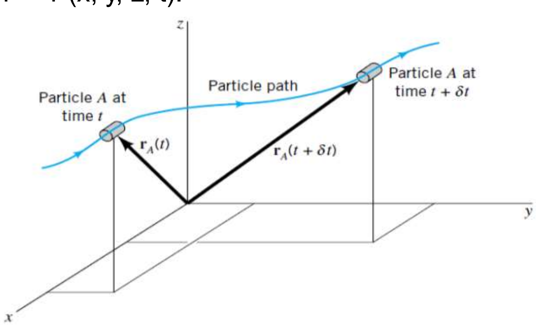


- In fluid mechanics, an element may undergo four fundamental types of motion.
  - Translation
  - Rotation
  - Linear strain
  - Shear strain
- Because fluids are in constant motion, motion and deformation are described in terms of rates
  - velocity: rate of translation
  - angular velocity: rate of rotation
  - linear strain : rate of linear strain
  - shear strain : rate of shear strain

FLOW FIELD REPRESENTATION

The representation of fluid parameters as functions of the space and time coordinates is termed a Field Representation of the flow.

Particle locations in terms of its position vector is shown in the figure below. *Velocity and Position vector* of fluid particle, V = V(x, y, z, t) and P = P(x, y, z, t).



EULERIAN AND LAGRANGIAN METHOD

In the study of fluid flow, *it is necessary to observe the motion of fluid particles at various points in space and at successive instants of time.* There are two methods by which the motion of a fluid me be described are *Lagrangian Method* and *Eulerian Method*.

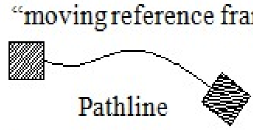
Eulerian Method: *The flow quantities such as velocity, density, temperature and pressure are described as a function of space and time at a particular point without referring to any individual identity of the fluid particle.* Eulerian method is most commonly adopted in fluid mechanics. In this method, u = u(x, y, z, t); v = v(x, y, z, t) and w = w(x, y, z, t).

Lagrangian Method: *The flow quantities are described based on behavior of each individually identifiable fluid particle while moving through flow field of interest*.

EULERIAN AND LAGRANGIAN METHOD

Lagrangian

"moving reference frame"

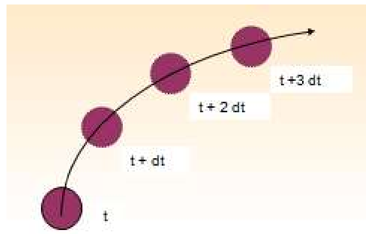
- 

- Focus on behavior of particular particles as they move with the flow

Eulerian

- "stationary reference frame"
- Focus on behavior of group of particles at a particular point

MATERIAL DERIVATIVE, TOTAL DERIVATIVE AND DEL OPERATOR



The total derivative operator d/dt is given a special name material derivative D/Dt, which is formed by following a fluid particle as it moves through the flow field.

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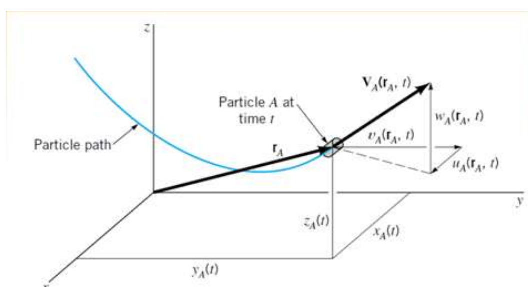
MATERIAL DERIVATIVE, TOTAL DERIVATIVE AND DEL OPERATOR (note)



$$\mathbf{V}_A = \mathbf{V}_A(\mathbf{r}_A, t) = \mathbf{V}_A[x_A(t), y_A(t), z_A(t), t]$$

$$\mathbf{a}_A(t) = \frac{d\mathbf{V}_A}{dt} = \frac{\partial \mathbf{V}_A}{\partial t} + \frac{\partial \mathbf{V}_A}{\partial x}\frac{dx_A}{dt} + \frac{\partial \mathbf{V}_A}{\partial y}\frac{dy_A}{dt} + \frac{\partial \mathbf{V}_A}{\partial z}\frac{dz_A}{dt}$$

$$\mathbf{a}_A = \frac{\partial \mathbf{V}_A}{\partial t} + u_A\frac{\partial \mathbf{V}_A}{\partial x} + v_A\frac{\partial \mathbf{V}_A}{\partial y} + w_A\frac{\partial \mathbf{V}_A}{\partial z} \qquad \mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z}$$

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MATERIAL DERIVATIVE, TOTAL DERIVATIVE AND DEL OPERATOR (Note)

$$\mathbf{a} = \frac{D\mathbf{V}}{Dt} \qquad \frac{D(\ )}{Dt} \equiv \frac{\partial(\ )}{\partial t} + u\frac{\partial(\ )}{\partial x} + v\frac{\partial(\ )}{\partial y} + w\frac{\partial(\ )}{\partial z}$$

Gradient or del operator: $\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$

$$\mathbf{V} = u\,\mathrm{i} + v\,\mathrm{j} + w\,\mathrm{k}$$

$$\mathbf{V}\cdot\nabla = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

the rate of change of temperature as

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \frac{\partial T}{\partial t} + \mathbf{V}\cdot\nabla T$$

$$\frac{D(\ )}{Dt} \equiv \frac{\partial(\ )}{\partial t} + u\frac{\partial(\ )}{\partial x} + v\frac{\partial(\ )}{\partial y} + w\frac{\partial(\ )}{\partial z}$$

$$\frac{D(\ )}{Dt} = \frac{\partial(\ )}{\partial t} + (\mathbf{V}\cdot\nabla)(\ )$$

termed the material derivative or substantial derivative

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TYPES OF FLUID FLOW

Fluid flows may be classified as below:

- Steady flow and Unsteady flow
- Uniform flow and Non-uniform flow
- One dimensional, Two dimensional and Three dimensional flow
- Rotational flow and Irrotational flow
- Laminar flow and Turbulent flow

Steady Flow: Fluid flow is said to be steady if at any point in the flowing fluid various characteristics such as velocity, pressure, density, temperature etc., *which describe the behavior of the fluid in motion do not change with time.*

Unsteady Flow: Fluid flow is said to be unsteady if at any point in the flowing fluid any one or *all the characteristics which describe the behavior of the fluid in motion change with time*.

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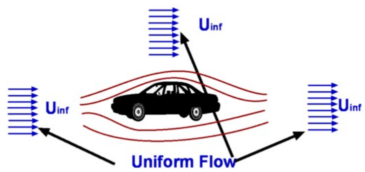
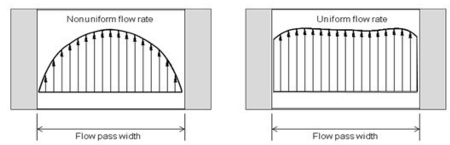
TYPES OF FLUID FLOW

Uniform Flow: *When velocity of flow of fluid does not change both in magnitude and direction, from point to point in the flowing fluid for any given instant of time, the flow is said to be uniform.*

Non-Uniform Flow: *If the velocity of flow of fluid changes from point to point in the flowing fluid at any given instant of time, the flow is said to be non-uniform flow.*





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TYPES OF FLUID FLOW

All these types flows can exist independent of each other. So that any of the four types of combination of flow is possible:

(a) Steady uniform flow

(b) Steady-non-uniform flow

(c) Unsteady uniform flow and

(d) Unsteady non-uniform flow

Example: constant flow rate in a constant diameter pipe (Steady uniform flow); *either decreasing or increasing flow rate in a constant diameter pipe (Unsteady uniform flow);* constant flow rate flow in a taper pipe (Steady non-uniform flow); and ***either increasing or decreasing flow rate in a taper pipe (Unsteady non-uniform flow).***

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TYPES OF FLUID FLOW

One-dimensional, Two-dimensional and Three-dimensional Flow: The various characteristics of flowing fluid such *as velocity, pressure, density, temperature etc are in general function of space (x, y, z) and time (t). Such a flow is known as Three-dimensional flow.*

If fluid properties (velocity, density, temperature etc) vary in two direction then the flow is called Two-dimensional flow; and if fluid properties vary in one-directions then the flow is called One-dimensional flow.

Types of flow	Unsteady	Steady
Three-dimensional	V = f(x, y, z, t)	V = f (x, y, z)
Two-dimensional	V = f(x, y, t)	V = f(x, y)
One-dimensional	V = f(x, t)	V = f(x)

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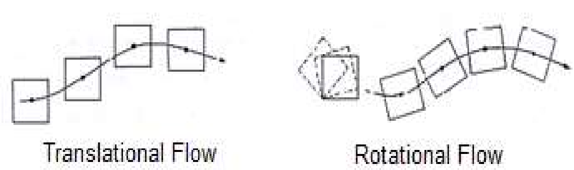
TYPES OF FLUID FLOW



Rotational Flow: A flow is said to be rotational if fluid particles are rotate about their mass centers while moving in the direction of flow.

Irrotational Flow: A flow is said to be irrotational or translation if fluid particles are not rotate about their mass centers while moving in the direction of flow.

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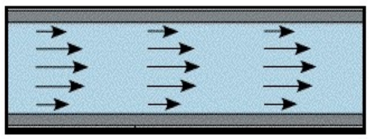
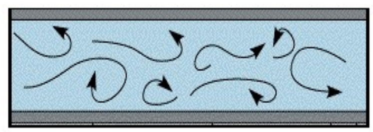
TYPES OF FLUID FLOW

Laminar Flow: A flow is said to be laminar when the fluid particles are move in a layers (or laminate) with *one layer of fluid sliding smoothly over adjacent layer and there is no momentum transfer in between the layers.* Viscosity of fluid plays important role in development of Laminar Flow.

Turbulent Flow: A fluid in motion is said to be turbulent when the fluid particles move in an entirely haphazard or disorderly manner, that results in *a rapid and continuous mixing of the fluid leading to momentum transfer as flow occurs*.



Laminar Flow



Turbulent Flow

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FUNDAMENTALS OF FLOW VISUALIZATION

Flow visualization is the visual examination of flow-field features or characteristics. It is important for both physical experiments and numerical (CFD) solutions. While *quantitative study of fluid dynamics requires advanced mathematics*, however, much can be learned by flow visualization.

There are many types of flow patterns, which can be visualized, both physically or experimentally, and also through CFD or computational analysis. The flow pattern may be described by means of streamlines, stream-tubes, path lines and streak-lines.

Spinning baseball

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DESCRIPTION OF FLOW PATTERN

Streamline: A streamline is an *imaginary curve drawn through a flowing fluid* in such a way that the *tangent to it at any point gives the direction of the velocity of flow at that point*. So, the pattern of flow of fluid may be represented by a series of stream-lines in such a way that velocity vector at any point is tangent to the curves.

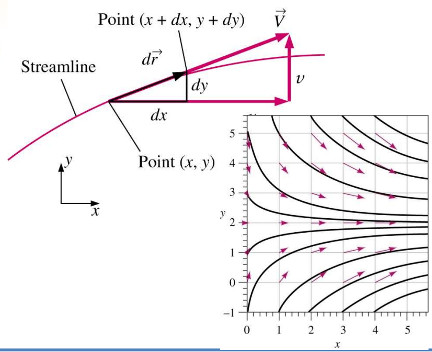
$$d\vec{r} = dx\vec{i} + dy\vec{j} + dz\vec{k}$$

$d\vec{r}$ must be parallel to the local velocity vector

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

Geometric arguments results in the equation for a streamline

$$\frac{dr}{V} = \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$



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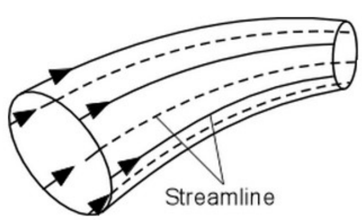
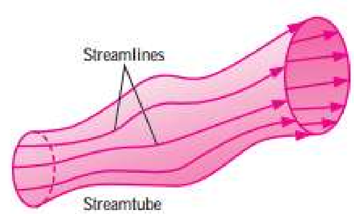
DESCRIPTION OF FLOW PATTERN

Stream-Tube: A stream-tube is an *imaginary tube, which is* formed by a group of streamlines passing through a small closed curve, which may or may not be circular. *There is no flow across the bounding surface of the stream-tube and velocity of fluid has no component normal to the streamline.*





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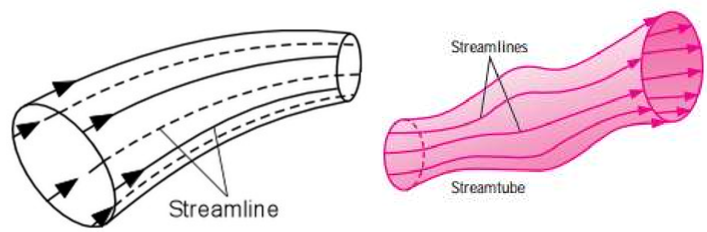
DESCRIPTION OF FLOW PATTERN



The mass flow rate passing through any cross-sectional slice of a given stream tube must remain the same. Shape of the stream tube changes from one instant to another because of change in position of streamlines. Examples- pipes, nozzle and diffuser.

Two stream lines can never intersect each other, as the instantaneous velocity vector at any given point is unique.

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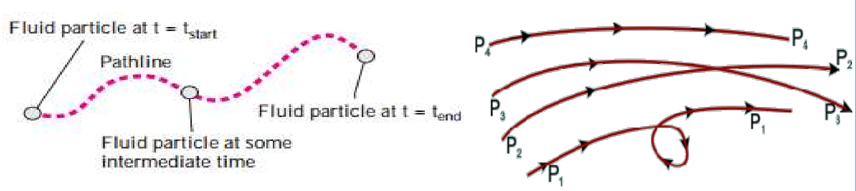
DESCRIPTION OF FLOW PATTERN

Path-line: A path-line may be defined as the *actual path traveled by an individual fluid particle over some time of period.*



Two path lines can intersect each other or a single path line can form a loop as different particles or even same particle can arrive at the same point at different instants of time.

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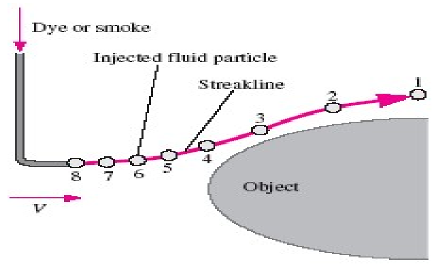
DESCRIPTION OF FLOW PATTERN

Streak-line: *A streak line is the locus of the temporary locations of all particles that have passed though a fixed point in the flow field at any instant of time.* It is a line traced by a fluid particle passing through a fixed point in a flow field.

Easy to generate in experiments: dye in a water flow, or smoke in an airflow.



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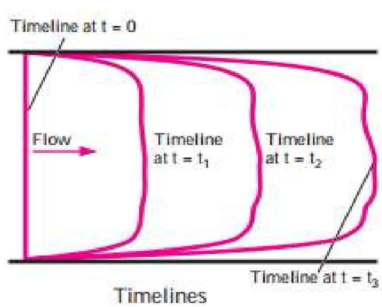
DESCRIPTION OF FLOW PATTERN

Timeline: A timeline is a set of adjacent fluid particles that were marked at the same (earlier) instant in time. Timelines are particularly useful in situations where the uniformity of a flow (or lack thereof) is to be examined.



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DESCRIPTION OF FLOW PATTERN (COMPARISONS)

For steady flow, streamlines, path-lines, and streak-lines are identical but for unsteady flow, they can be very different.

- *Streamlines* are instantaneous pictures of the flow field.
- Path-lines and Streak-lines are flow patterns that have a time history associated with them.
- *Streak-line:* instantaneous snapshot of a time-integrated flow pattern.
- *Path-line:* time-exposed flow path of an individual particle.

BASIC PRINCIPLES OF FLUID FLOW

Alike solid mechanics there are three basic principles used in the analysis of problems of fluid in motion:

- *Principle of conservation of mass:* Mass neither be created nor destroyed. On the basis of this principle the continuity equation is derived.
- *Principle of conservation of energy:* Energy neither be created nor destroyed. On the basis of this principle the energy equation is derived.
- *Principle of conservation of momentum or impulse momentum principle:* the impulse of the resultant force, or the product of force and time increment during which it acts, is equal to the change in the momentum of the body. On the basis of this principle the momentum equation is derived.

DERIVATION OF CONTINUITY EQUATION (Note)

The Taylor series of a real or complex-valued function $f(x)$ that is infinitely differentiable at a real or complex number a is the power series

$$f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f^{(3)}(a)}{3!}(x-a)^3+\cdots.$$

which can be written in the more compact sigma notation as

$$\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$$

Example:

For a function $f(x)$

$$f(x+h)=f(x)+f'(x)h+\frac{f''(x)}{2!}h^2+\frac{f'''(x)}{3!}h^3+\cdots$$

$$\rho)_{x+dx/2}=\rho+\left(\frac{\partial\rho}{\partial x}\right)\frac{dx}{2}+\left(\frac{\partial^2\rho}{\partial x^2}\right)\frac{1}{2!}\left(\frac{dx}{2}\right)^2+\cdots$$

$$\rho)_{x+dx/2}=\rho+\left(\frac{\partial\rho}{\partial x}\right)\frac{dx}{2}\quad\left(\text{Neglecting higher-order terms}\right)$$

DERIVATION OF CONTINUITY EQUATION (Note)

Given, $f(x)=\sin(x)$ find the value of $\sin(2)$?

$\sin\left(\frac{\pi}{2}\right)=1 \quad x=\frac{\pi}{2} \quad x+h=2 \quad h=2-x=2-\frac{\pi}{2}=0.42920$

$$f(x+h)=f(x)+f'(x)h+f''(x)\frac{h^2}{2!}+f'''(x)\frac{h^3}{3!}+f''''(x)\frac{h^4}{4!}+\cdots$$

$x=\frac{\pi}{2} \quad h=0.42920$

$f(x)=\sin(x),\ f\left(\frac{\pi}{2}\right)=\sin\left(\frac{\pi}{2}\right)=1 \quad f'(x)=\cos(x),\ f'\left(\frac{\pi}{2}\right)=0 \quad f''(x)=-\sin(x),\ f''\left(\frac{\pi}{2}\right)=-1$

$f'''(x)=-\cos(x),\ f'''\left(\frac{\pi}{2}\right)=0 \quad f''''(x)=\sin(x),\ f''''\left(\frac{\pi}{2}\right)=1$

Hence

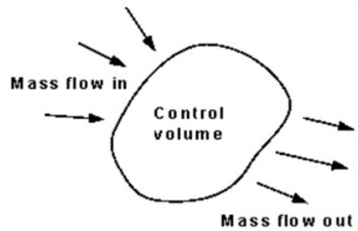
$$f\left(\frac{\pi}{2}+h\right)=f\left(\frac{\pi}{2}\right)+f'\left(\frac{\pi}{2}\right)h+f''\left(\frac{\pi}{2}\right)\frac{h^2}{2!}+f'''\left(\frac{\pi}{2}\right)\frac{h^3}{3!}+f''''\left(\frac{\pi}{2}\right)\frac{h^4}{4!}+\cdots$$

$$f\left(\frac{\pi}{2}+0.42920\right)=1+0(0.42920)-1\frac{(0.42920)^2}{2!}+0\frac{(0.42920)^3}{3!}+1\frac{(0.42920)^4}{4!}+\cdots$$

$$=1+0-0.092106+0+0.00141393+\cdots\cong 0.90931$$

CONTINUITY EQUATION

Matter cannot be created or destroyed - (it is simply changed in to a different form of matter). This principle is know as the *conservation of mass* and we use it in the analysis of flowing fluids.
The principle is applied to fixed volumes, known as control volumes (or surfaces), like that in the figure below:



For any control volume the principle of *conservation of mass* says

Mass entering per unit time − Mass leaving per unit time = Increase of mass in the control volume per unit time

For **steady** flow there is no increase in the mass within the control volume, so

Mass entering per unit time = Mass leaving per unit time

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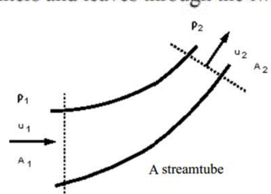
CONTINUITY EQUATION FOR ONE DIMENSIONAL FLOW

A streamtube shown below such as that no fluid flows across the boundary made by the streamlines so mass only enters and leaves through the two ends of this streamtube section.



We can then write

mass entering per unit time at end 1 = mass leaving per unit time at end 2

$$\rho_1 \delta A_1 u_1 = \rho_2 \delta A_2 u_2$$

Or for steady flow, $\rho_1 \delta A_1 u_1 = \rho_2 \delta A_2 u_2 = \text{Constant} = \dot{m}$ This is the equation of continuity.

$$\rho_1 A_1 u_1 = \rho_2 A_2 u_2 = \text{Constant} = \dot{m}$$

When the fluid can be considered incompressible, i.e. the density does not change, $\rho_1 = \rho_2 = \rho$

$$A_1 u_1 = A_2 u_2 = Q$$

This is the form of the continuity equation most often used.

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CONTINUITY EQUATION FOR THREEEE DIMENSIONAL COMPRESSIBLE FLOW

(1). Continuity equation in Rectangular Coordinates or Cartesian Coordinate System.

Consider an infinitesimal control volume of dimension dx, dy and dz. The density of the fluid at the center of the control volume is ρ and velocity of fluid is $\vec{V}$, where $\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$

Ideal fluid with no viscosity
Fluid is compressible.

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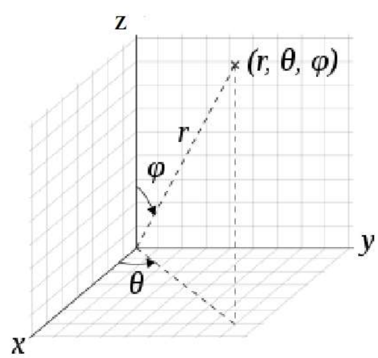
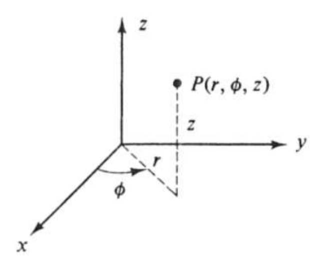
CONTINUITY EQUATION FOR THREEEE DIMENSIONAL FLOW



Spherical Coordinate System



Cylindrical Coordinate System

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CONTINUITY EQUATION FOR THREEEE DIMENSIONAL FLOW

(1). *Continuity equation in Rectangular Coordinates or Cartesian Coordinate System.*

(NOTE :DERIVATION: SEEE HAND ANALYSIS)

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} + \frac{\partial \rho}{\partial t} = 0 \qquad \nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = \nabla \cdot \rho \vec{V} \qquad \nabla \cdot \rho \vec{V} + \frac{\partial \rho}{\partial t} = 0$$

(2). Continuity equation in Cylindrical Coordinates Coordinate System.

$$\frac{1}{r}\frac{\partial (r\rho V_r)}{\partial r} + \frac{1}{r}\frac{\partial (\rho V_\theta)}{\partial \theta} + \frac{\partial (\rho V_z)}{\partial z} + \frac{\partial \rho}{\partial t} = 0$$

$$\nabla = \hat{e}_r \frac{\partial}{\partial r} + \hat{e}_\theta \frac{1}{r}\frac{\partial}{\partial \theta} + \hat{k}\frac{\partial}{\partial z}$$

$$\nabla \cdot \rho \vec{V} + \frac{\partial \rho}{\partial t} = 0$$

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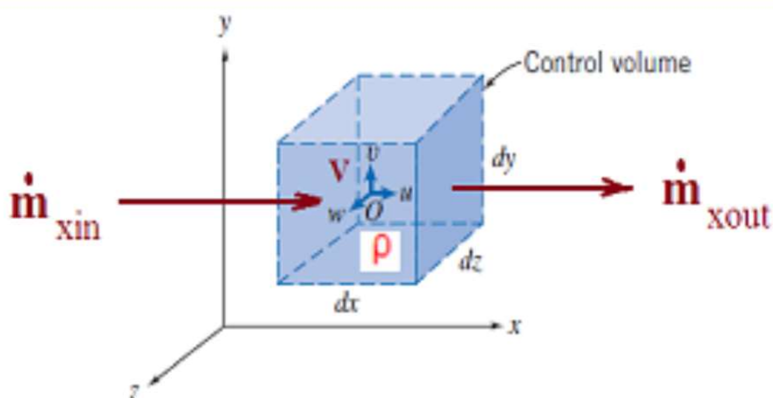

$$\rho_{x+\frac{dx}{2}} = \rho + \left(\frac{\delta \rho}{\delta x}\right)\cdot\frac{dx}{2} \qquad \rho_{x-\frac{dx}{2}} = \rho - \left(\frac{\delta \rho}{\delta x}\right)\cdot\frac{dx}{2}$$

$$u_{x+\frac{dx}{2}} = u + \left(\frac{\delta u}{\delta x}\right)\cdot\frac{dx}{2} \qquad u_{x-\frac{dx}{2}} = u - \left(\frac{\delta u}{\delta x}\right)\cdot\frac{dx}{2}$$

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