ME 3101: Mechanics of Machinery

Cams & Followers

Prepared by
Muhammad Ifaz Shahriar Chowdhury
Lecturer, MPE Dept

Importance

One of the simplest mechanisms

One of the most important mechanisms found in modern machinery today

Motion

Cam's Motion

Rotation at Uniform Speed by a shaft

Follower's Motion

Predetermined according to the shape of the cam

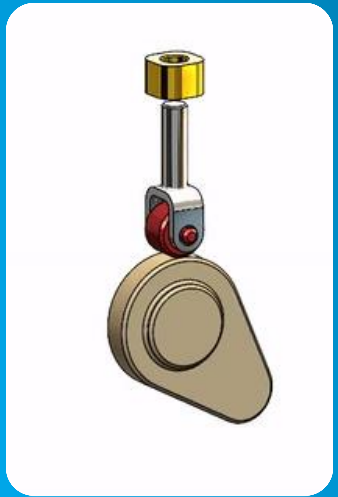
Pair Type

The cam and the follower have a line contact

*Constitute a **higher pair***

CAM

Rotating Machine element that gives



Reciprocating Motion to element called

FOLLOWER

Feed Mechanism of Automatic Lathes

Spinning and Weaving Textile Machineries

Paper Cutting Machines

Automatic Attachment of Machineries

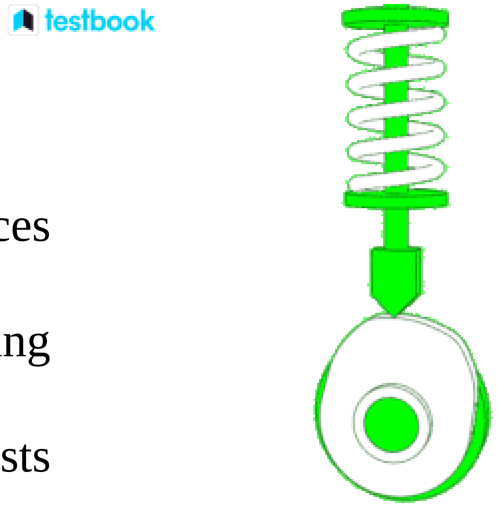
Inlet and Outlet Valve of IC Engine

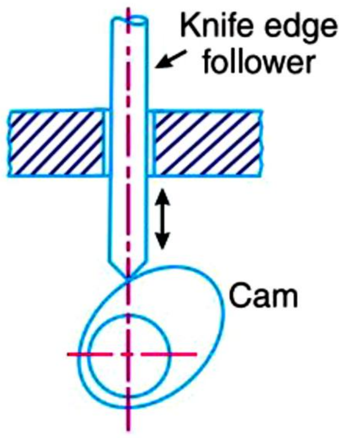
Application

1. *According to the surface in contact*

a. Knife edge follower

- Contacting end of the follower has **a sharp knife edge**
- Sliding motion takes place between the contacting surfaces (i.e. the knife edge and the cam surface).
- Seldom used in practice because the small area of contacting surface results in excessive wear.
- In knife edge followers, a considerable side thrust exists between the follower and the guide.

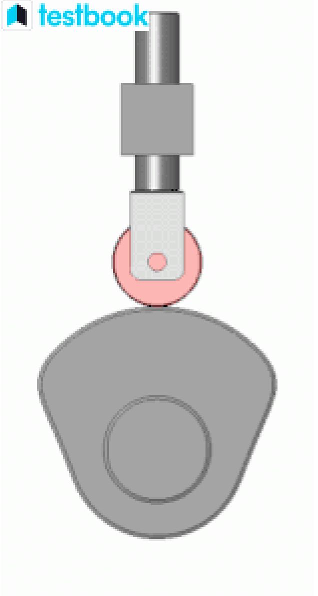


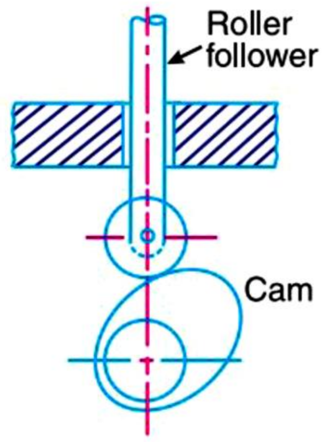


(*a*) Cam with knife edge follower.

b. Roller follower

- ✓ Contacting end of the follower is **a roller**
- ✓ Rolling motion takes place between the contacting surfaces (*i.e.* the roller and the cam), therefore the rate of **wear is greatly reduced**
- ✓ Side thrust exists between the follower and the guide
- ✓ Extensively used where more space is available such as in **stationary gas** and **oil engines** and **aircraft engines**

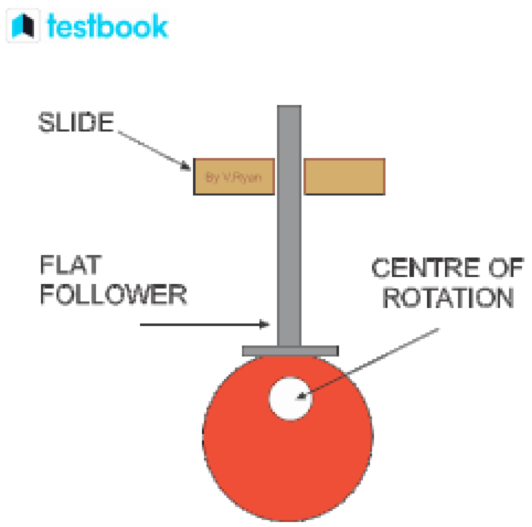


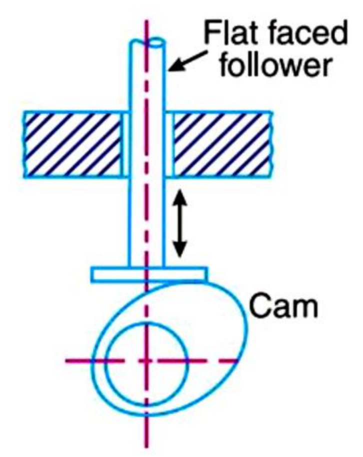


(*b*) Cam with roller follower.

c. Flat faced or mushroom follower

- Contacting end of the follower is a **perfectly flat face**
- Side thrust between the follower and the guide is much reduced. Only side thrust is due to friction between the contact surfaces of the follower and the cam
- The relative motion between these surfaces is largely of sliding nature but wear may be reduced by off setting the axis of follower, so that when cam rotates, the follower also rotates about its own axis.
- The flat faced followers are generally used where space is limited such as in cams which operate the **valves of automobile engines**
- When the flat faced follower is **circular**, it is then called a **mushroom follower**

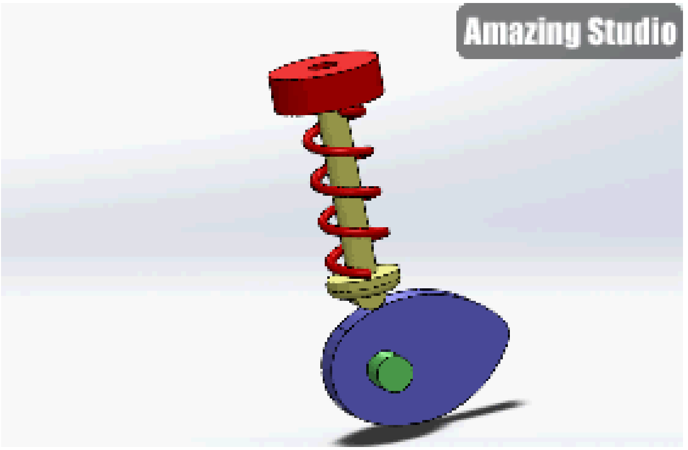


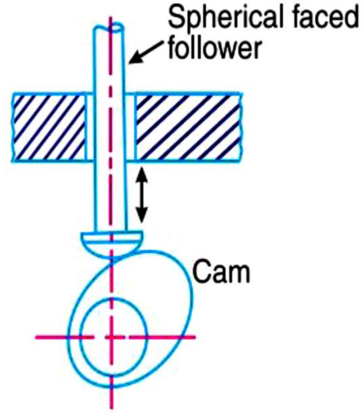


(*c*) Cam with flat faced follower.

d. Spherical faced follower

- Contacting end of the follower is of spherical shape
- When used in automobile engines, high surface stresses are produced. In order to minimize these stresses, the flat end of the follower is **machined to a spherical shape**





(*d*) Cam with spherical faced follower.

Classification of Followers

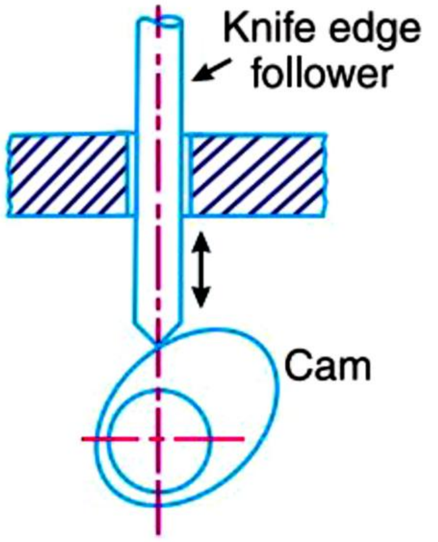
2. *According to the motion of the follower*

a. Reciprocating or translating follower

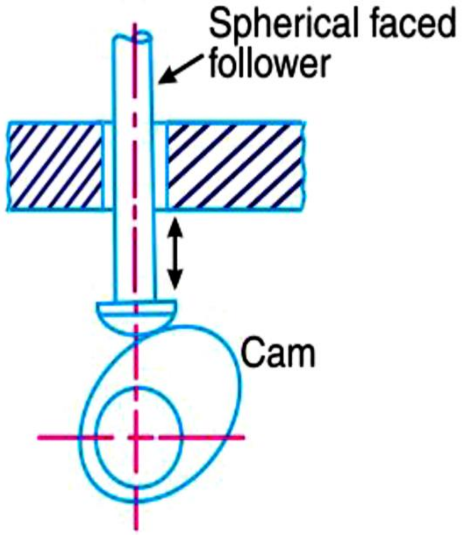
- When the follower reciprocates in guides as the cam rotates uniformly, it is known as reciprocating or translating follower

b. Oscillating or rotating follower

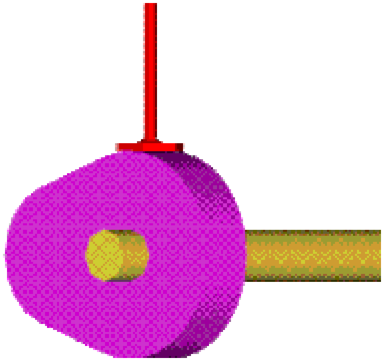
- ✓ When the uniform rotary motion of the cam is converted into predetermined oscillatory motion of the follower, it is called oscillating or rotating follower

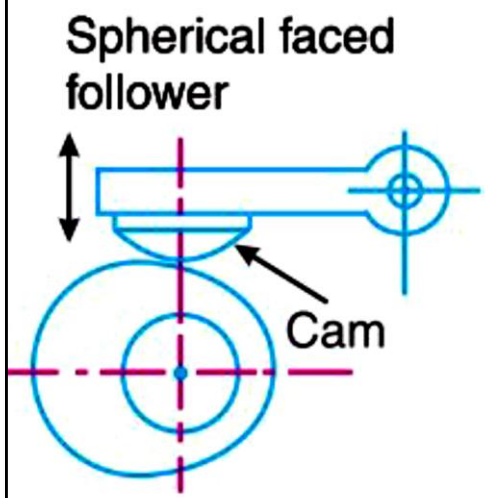


(*a*) Cam with knife edge follower.

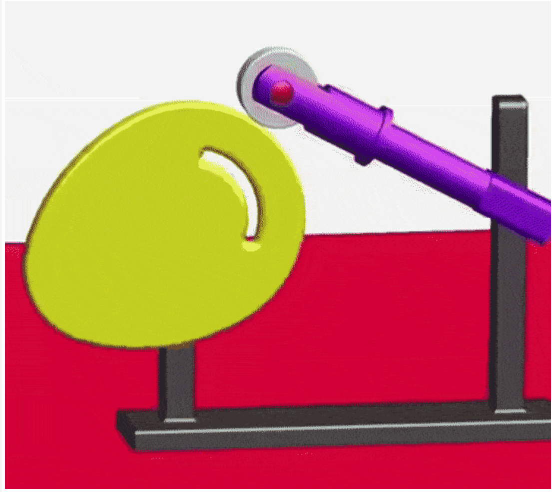


(*d*) Cam with spherical faced follower.





(*e*) Cam with spherical

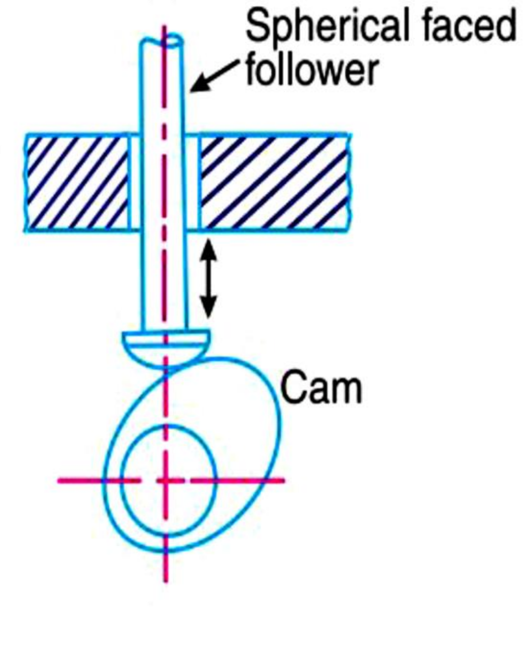


Classification of Followers

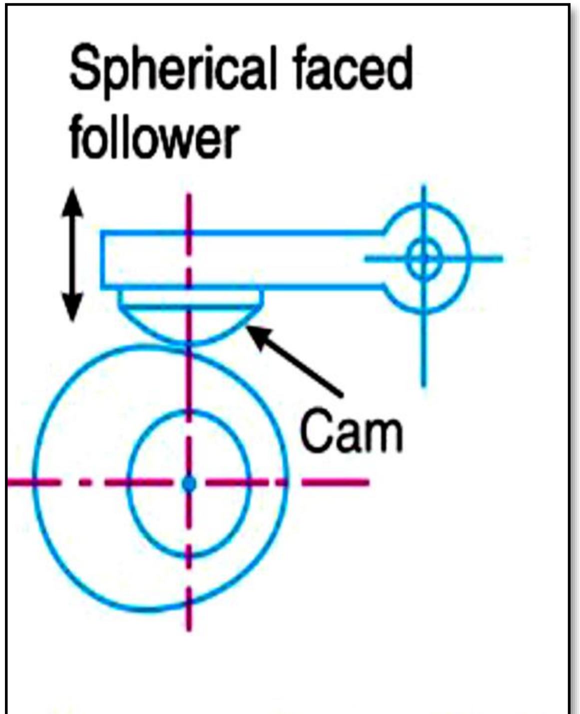
3. According to the path of motion of the follower

a. Radial follower

- When the motion of the follower is along an axis passing through the center of the cam, it is known as radial follower



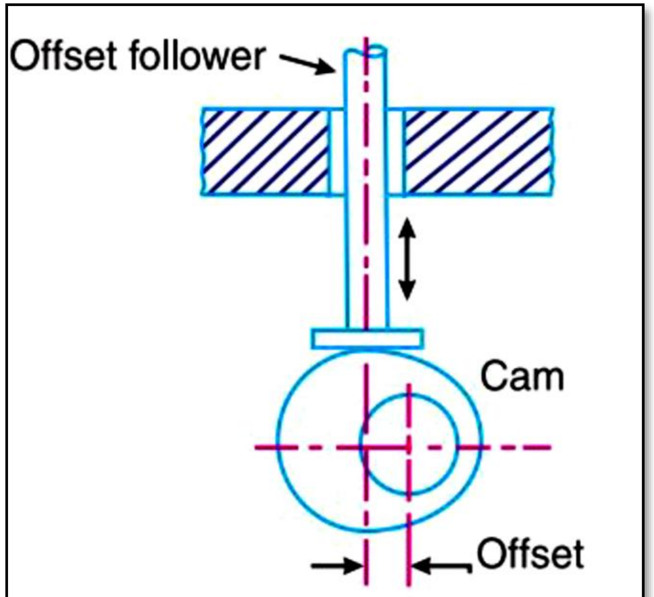
(d) Cam with spherical faced follower.



(e) Cam with spherical

b. Off-set follower

- ✓ When the motion of the follower is along an axis away from the axis of the cam center, it is called off-set follower
- ✓ The follower must be constrained to follow the cam. This may be done by springs, gravity or hydraulic means. In some types of cams, the follower may ride in a groove.



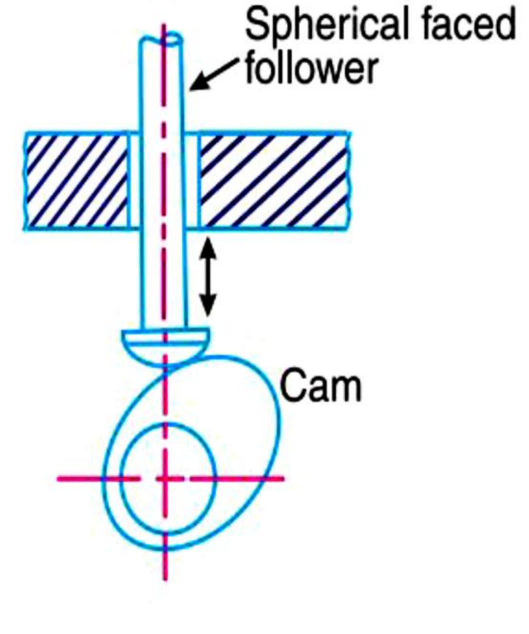
(f) Cam with offset

Classification of Cams

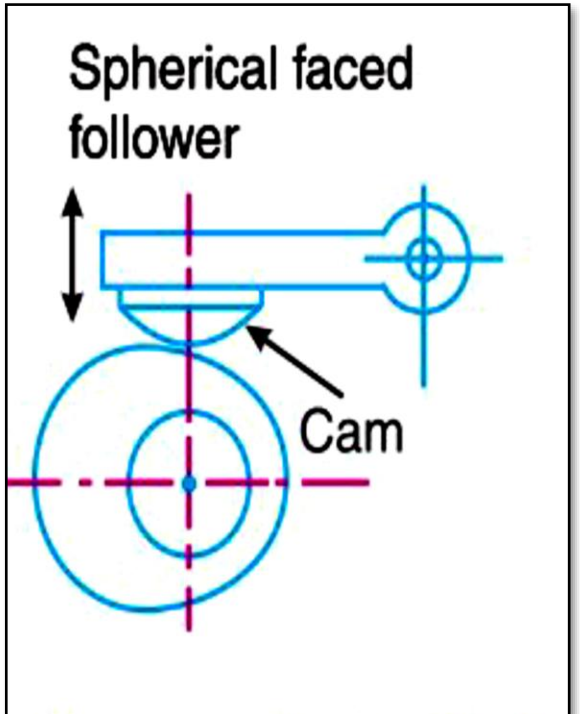
Though the cams may be classified in many ways, yet the following two types are important from the subject point of view :

1. *Radial or disc cam*

In radial cams, the follower reciprocates or oscillates in a direction perpendicular to the cam axis.



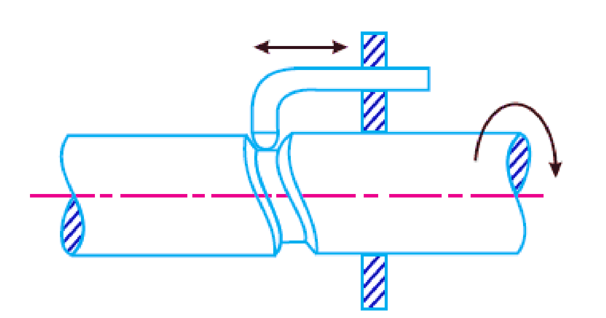
(*d*) Cam with spherical faced follower.



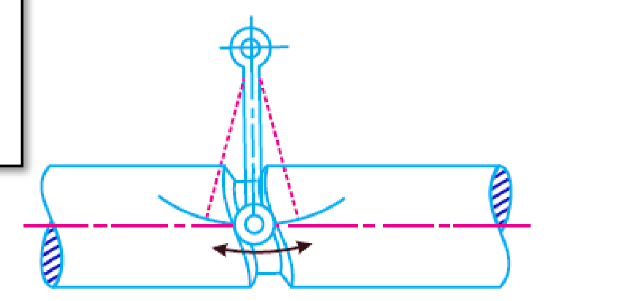
(*e*) Cam with spherical

2. *Cylindrical cam*

Follower reciprocates or oscillates in a direction parallel to the cam axis. The follower rides in a groove at its cylindrical surface. A cylindrical grooved cam with a reciprocating and an oscillating follower is shown respectively.



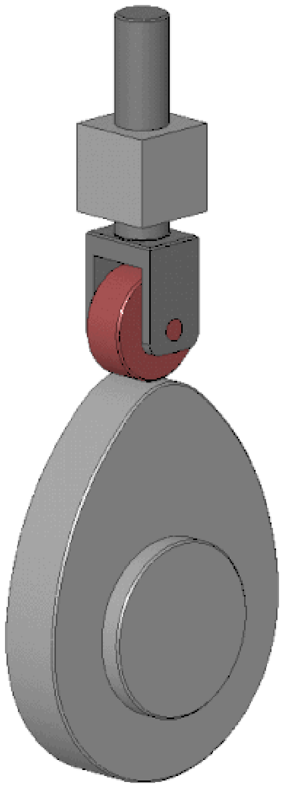
(*a*) Cylindrical cam with reciprocating follower.



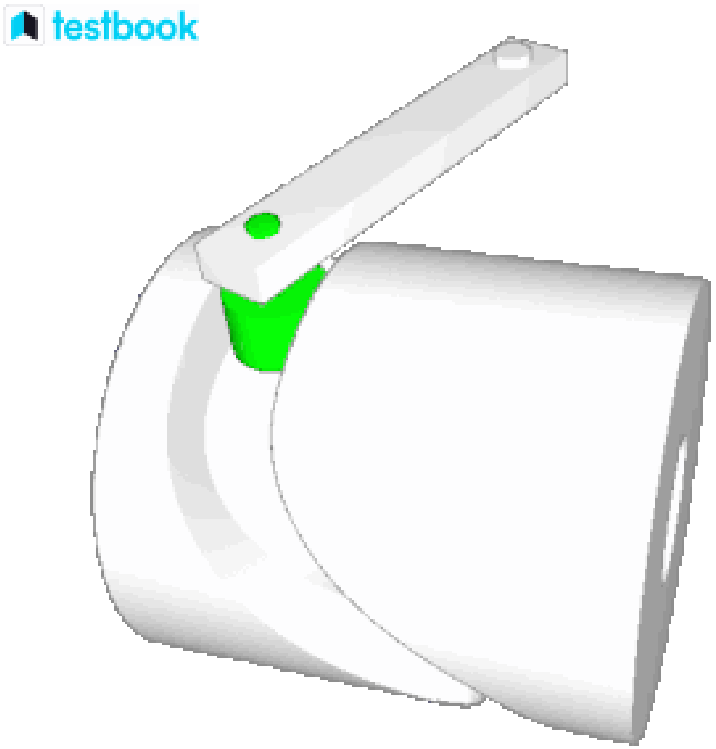
(*b*) Cylindrical cam with oscillating follower.

Classification of Cams

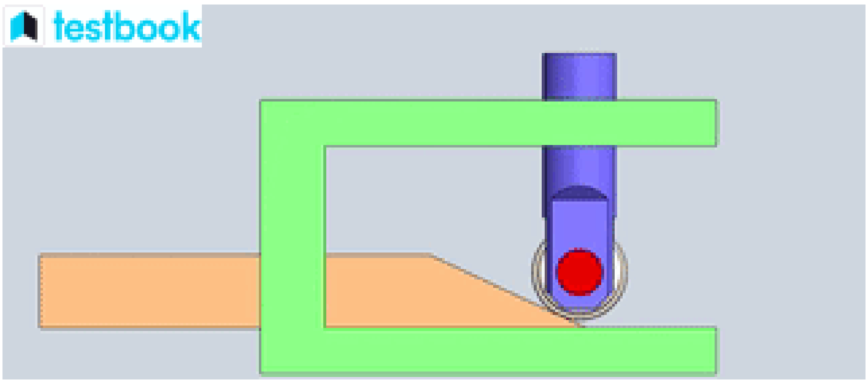
1. *Radial or disc cam*



2. *Cylindrical cam*

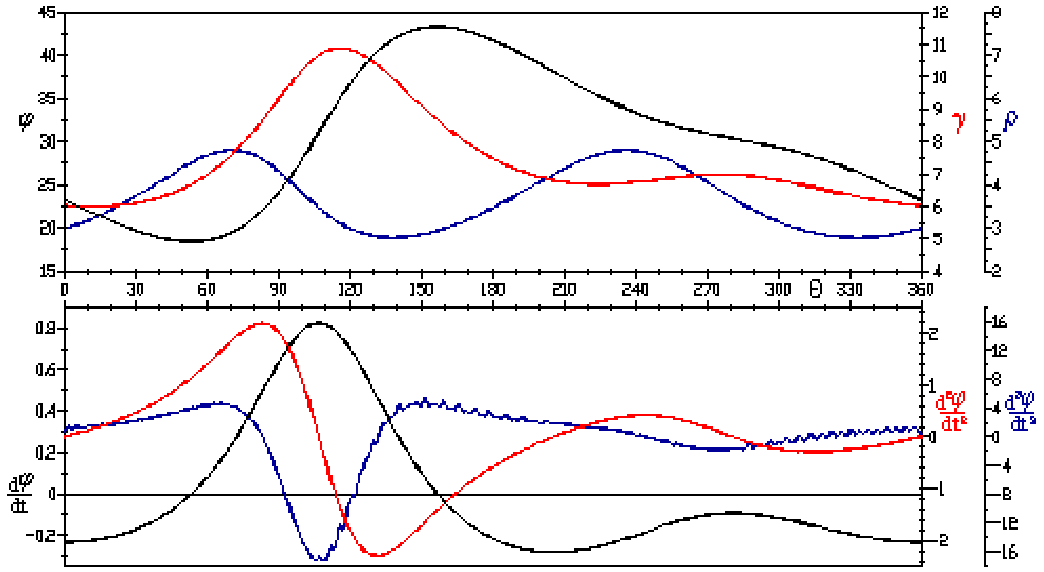


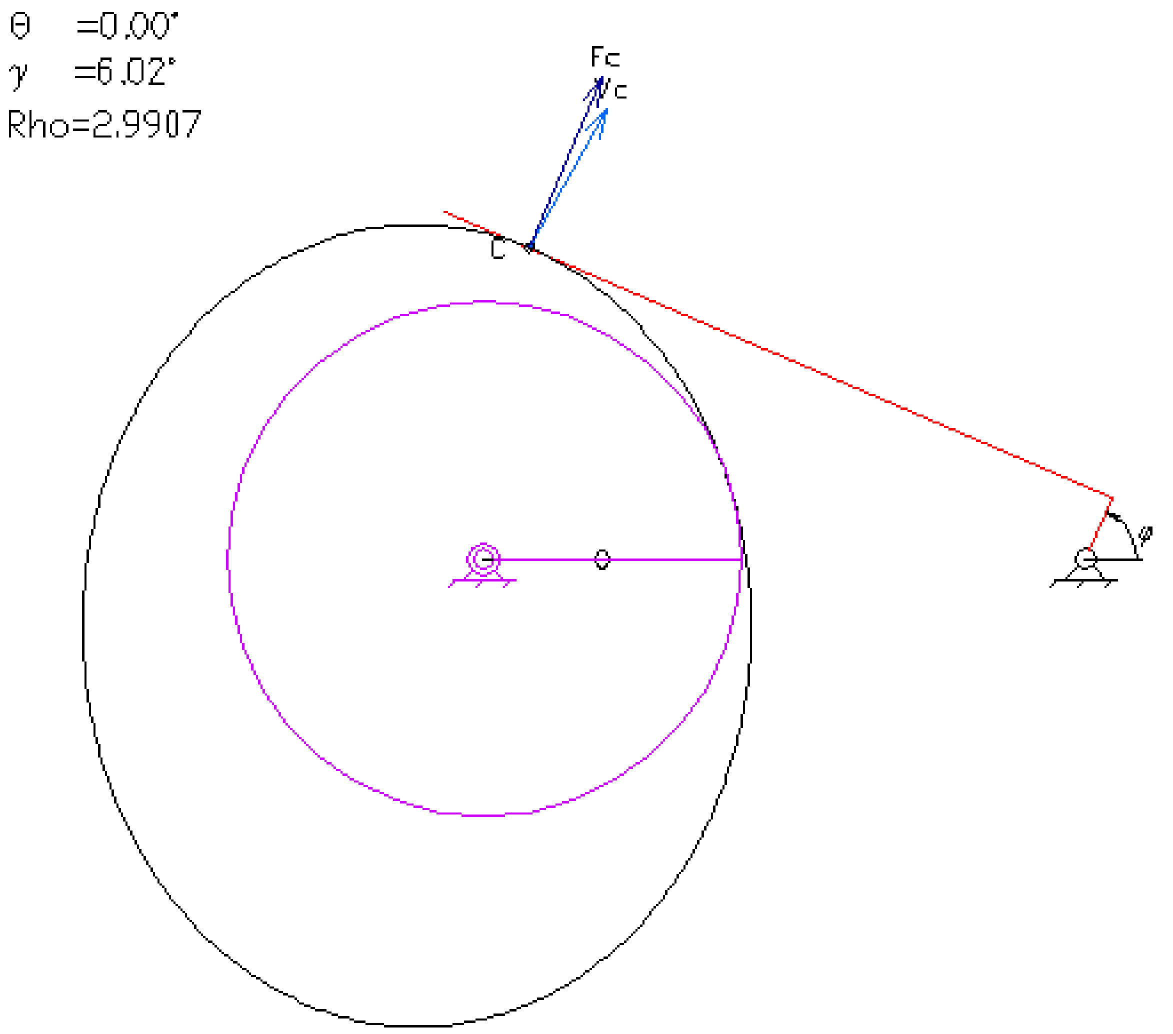
3. *Wedge cam*



Cam Profile

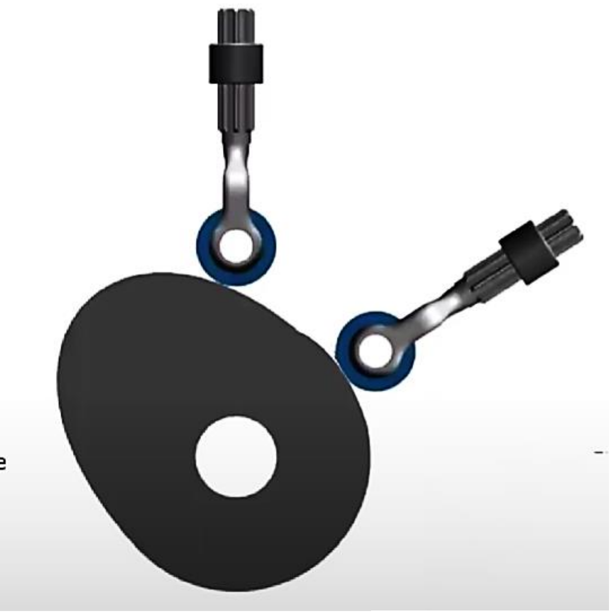
- A **cam profile** refers to the **shape** or **contour** of a cam, which **determines the motion of the follower** in a cam-follower mechanism. The cam is a rotating or sliding piece that imparts motion to the follower, which translates or oscillates depending on the cam's design. The **cam profile** is crucial because it defines the exact path and movement of the follower, which can be a linear, oscillating, or complex motion.
- The cam profile is designed to achieve specific motion characteristics, such as smooth acceleration or deceleration, depending on the mechanical requirements of the system

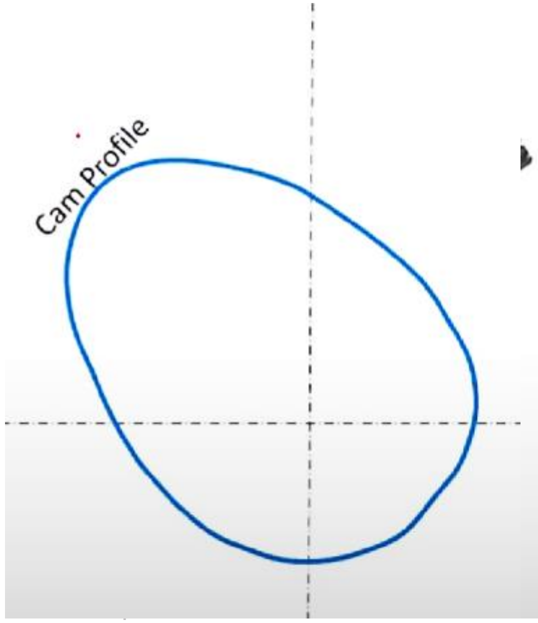


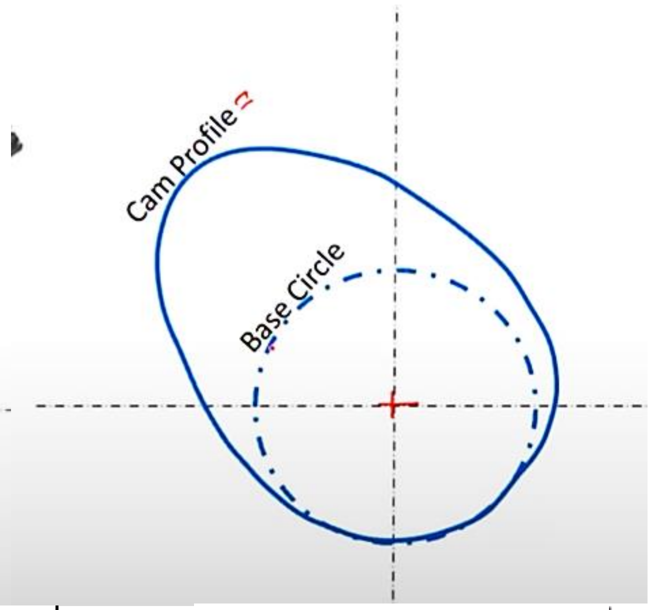


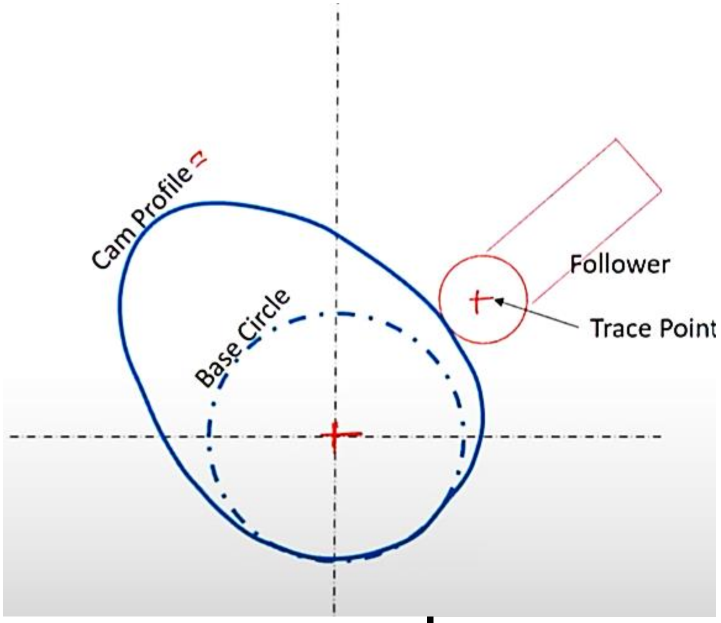
Cam (Mechanism)

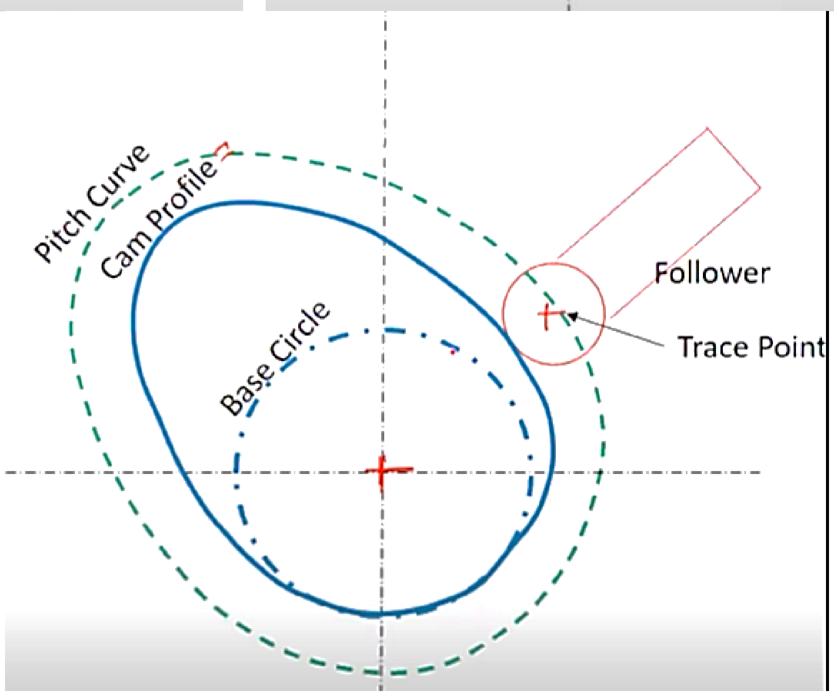
Terminology in Radial Cams

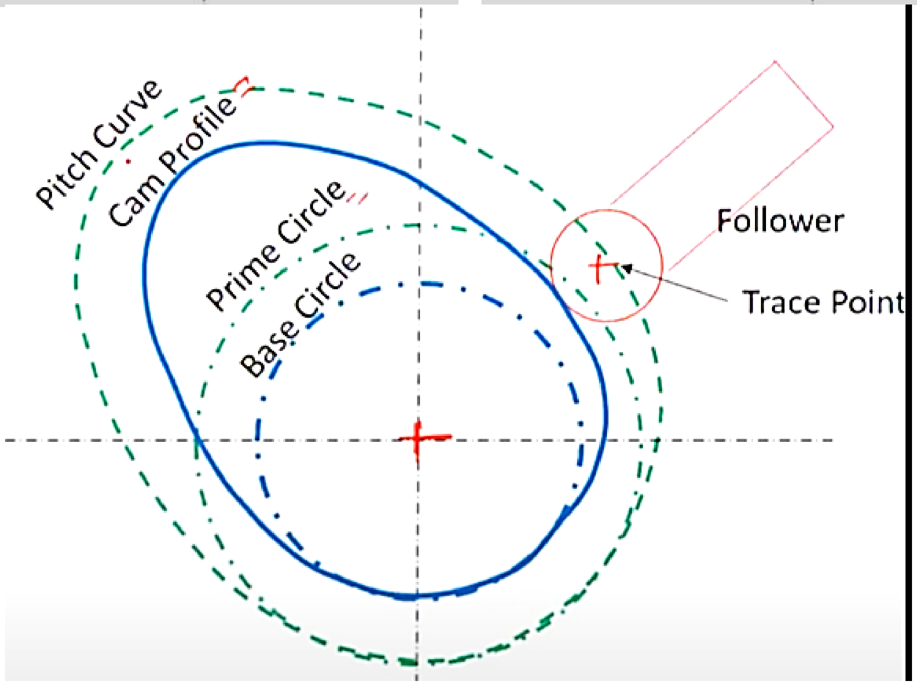




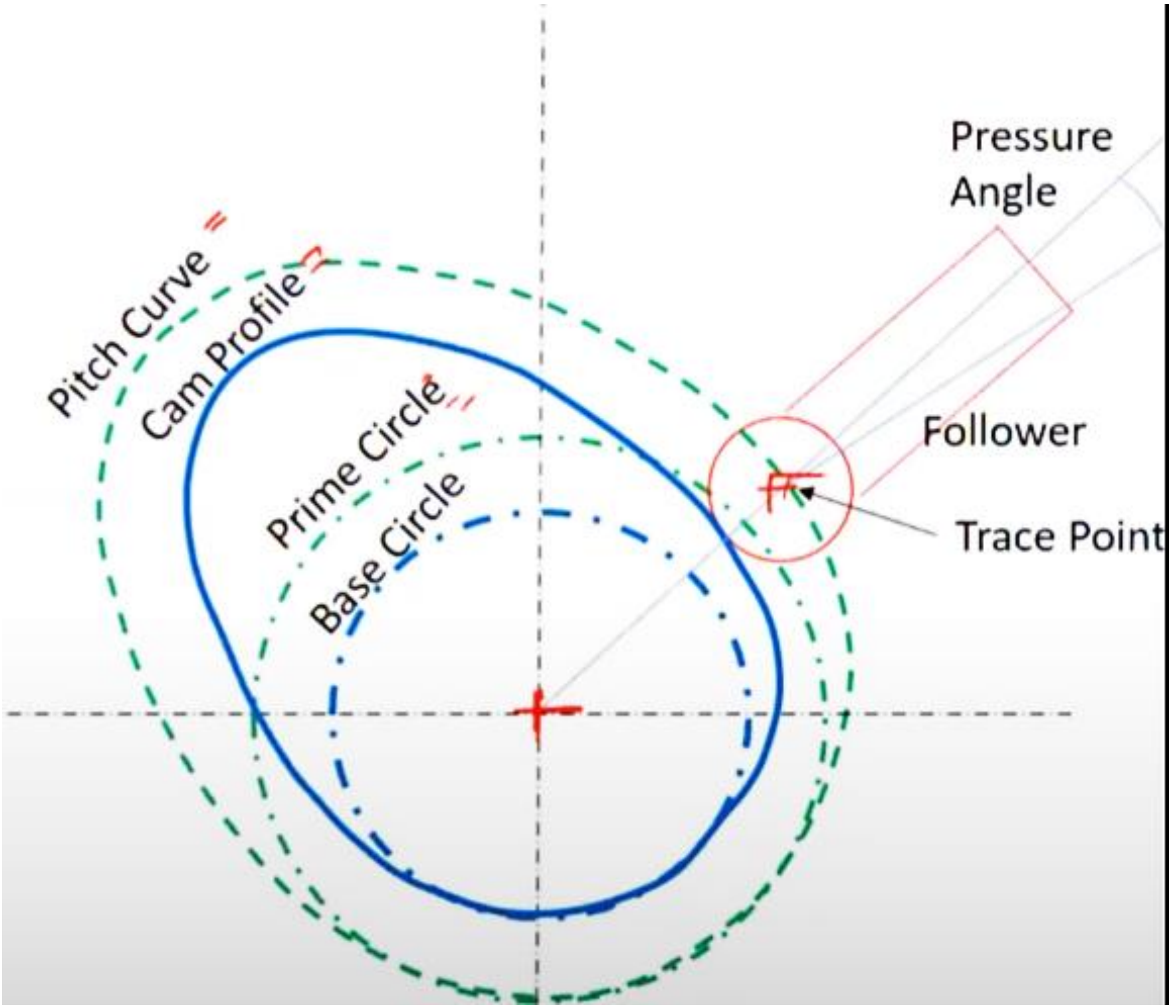


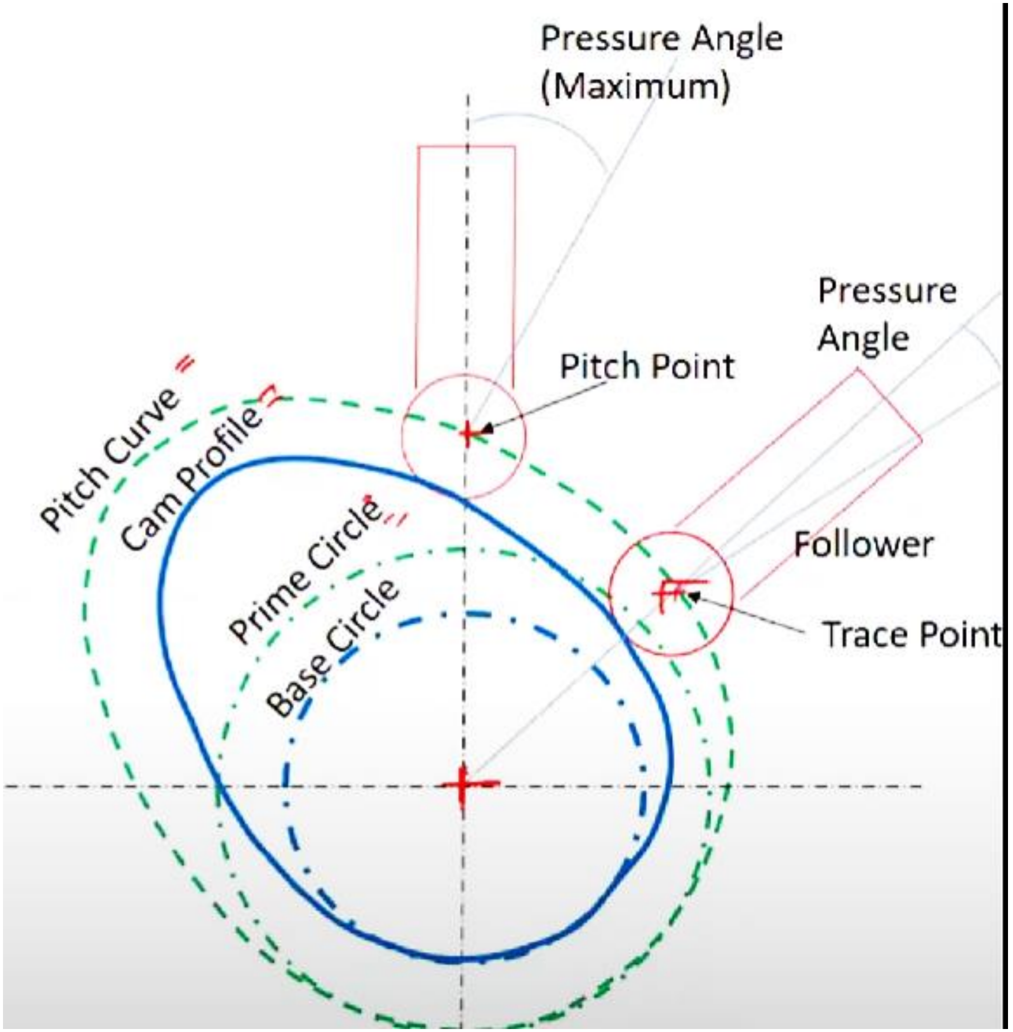




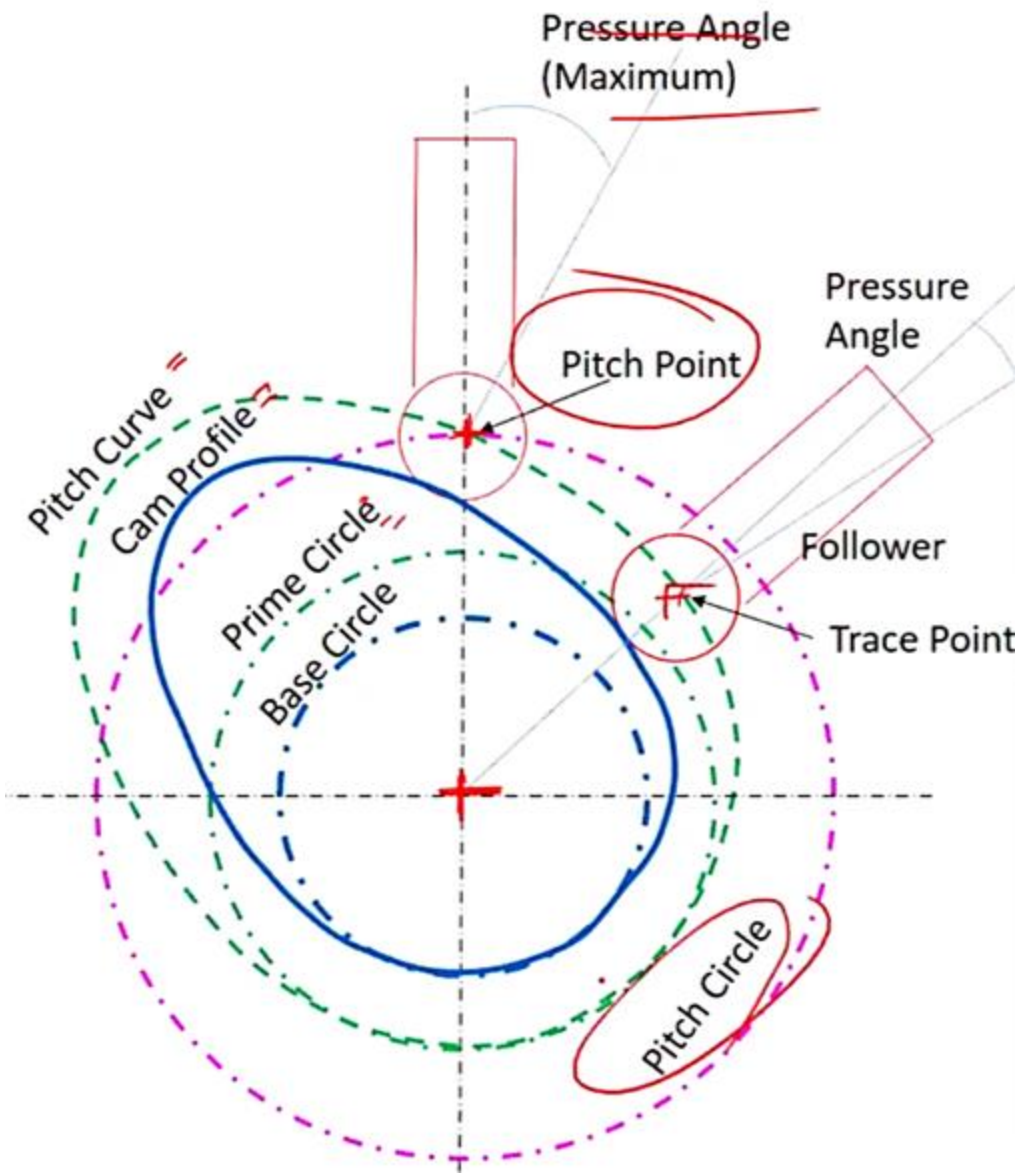


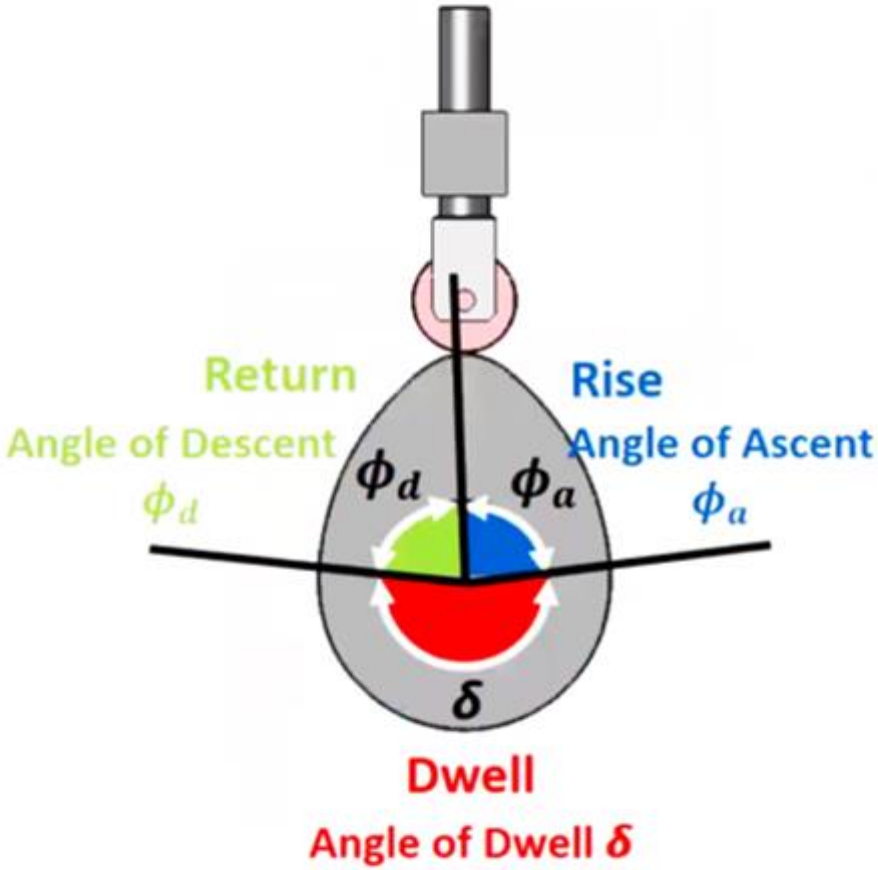
Terminology in Radial Cams





Terminology in Radial Cams





Terminology in Radial Cams

1. *Base circle.* It is the smallest circle that can be drawn to the cam profile.

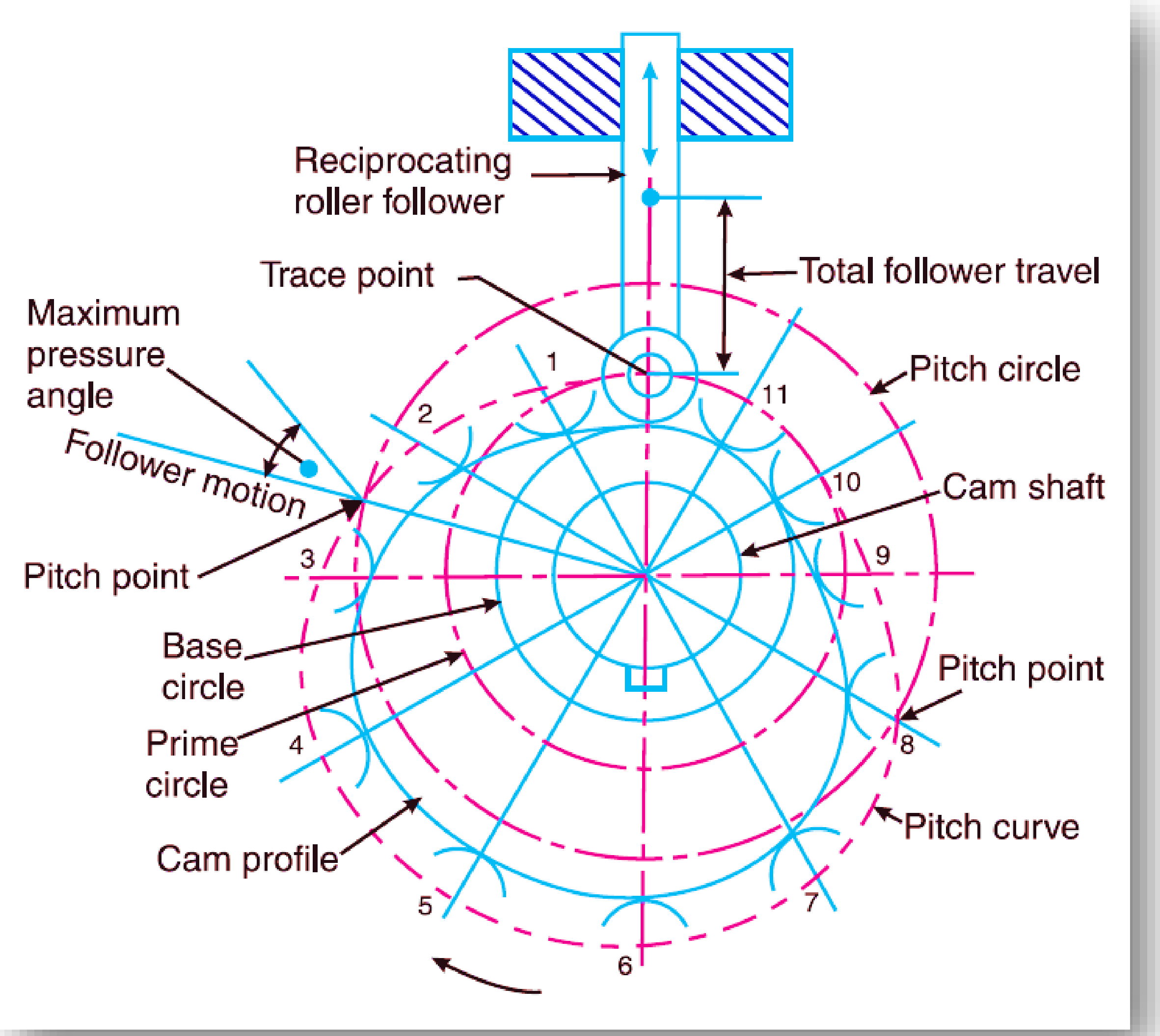
2. *Trace point.* It is a reference point on the follower and is used to generate the *pitch curve*. In case of knife edge follower, the knife edge represents the trace point and the pitch curve corresponds to the cam profile. In a roller follower, the center of the roller represents the trace point.

3. *Pressure angle.* It is the angle between the direction of the follower motion and a normal to the pitch curve. This angle is very important in designing a cam profile. If the pressure angle is too large, a reciprocating follower will jam in its bearings.

4. *Pitch point.* It is a point on the pitch curve having the maximum pressure angle.

5. *Pitch circle.* It is a circle drawn from the centre of the cam through the pitch points.

6. *Pitch curve.* It is the curve generated by the trace point as the follower moves relative to the cam. For a knife edge follower, the pitch curve and the cam profile are same whereas for a roller follower, they are separated by the radius of the roller.



7. *Prime circle.*

It is the smallest circle that can be drawn from the centre of the cam and tangent to the pitch curve. For a knife edge and a flat face follower, the prime circle and the base circle are identical. For a roller follower, the prime circle is larger than the base circle by the radius of the roller.

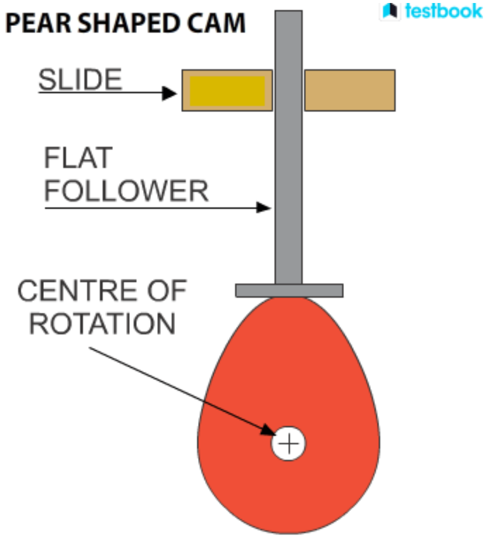
8. *Lift or stroke.*

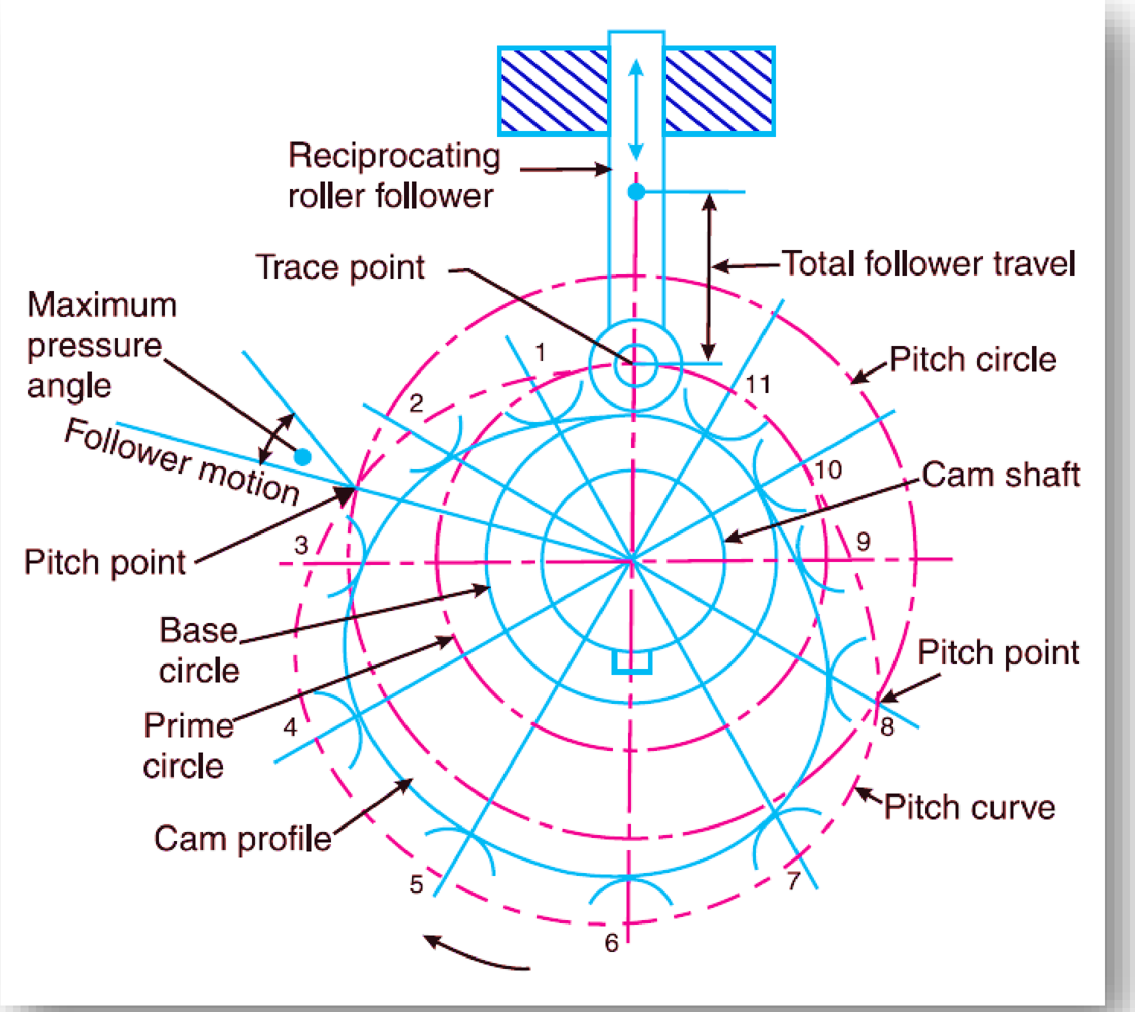
It is the maximum travel of the follower from its lowest position to the topmost position.

Motion of the Follower

The follower, during its travel, may have one of the following motions.

1. Uniform velocity
2. Simple harmonic motion
3. Uniform acceleration and retardation
4. Cycloidal motion

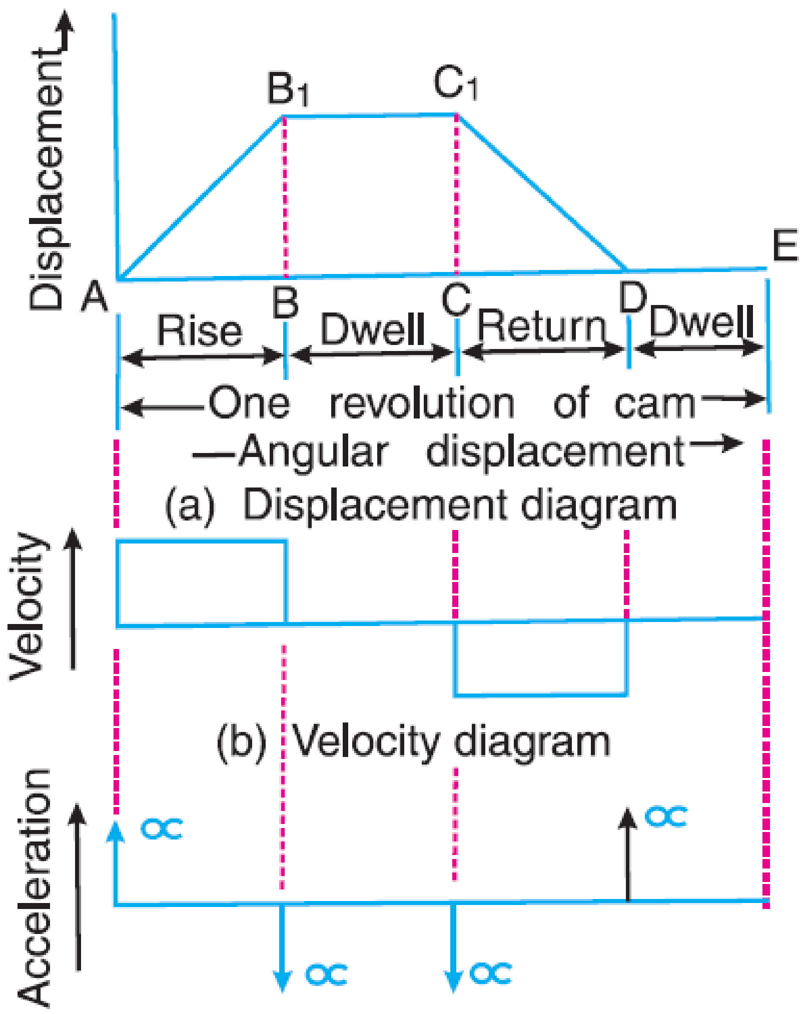




Follower with Uniform Velocity

Displacement, Velocity and Acceleration Diagrams

1. The follower moves with **uniform velocity** during **its rise** and **return stroke**, therefore the slope of the displacement curves must be **constant**. In other words, AB_1 and C_1D must be straight lines.
2. A little consideration will show that the follower remains at rest during part of the cam rotation. The periods during which the follower remains at rest are known as ***dwell periods,*** as shown by lines B_1C_1 and DE in Fig.(*a*).
3. From Fig.(*c*) we see that the **acceleration** or **retardation** of the follower at the beginning and at the end of each stroke is **infinite**. This is due to the fact that the follower is required to start from rest and has to gain a velocity within no time. This is only possible if the acceleration or retardation at the beginning and at the end of each stroke is infinite. These conditions are however, impracticable.

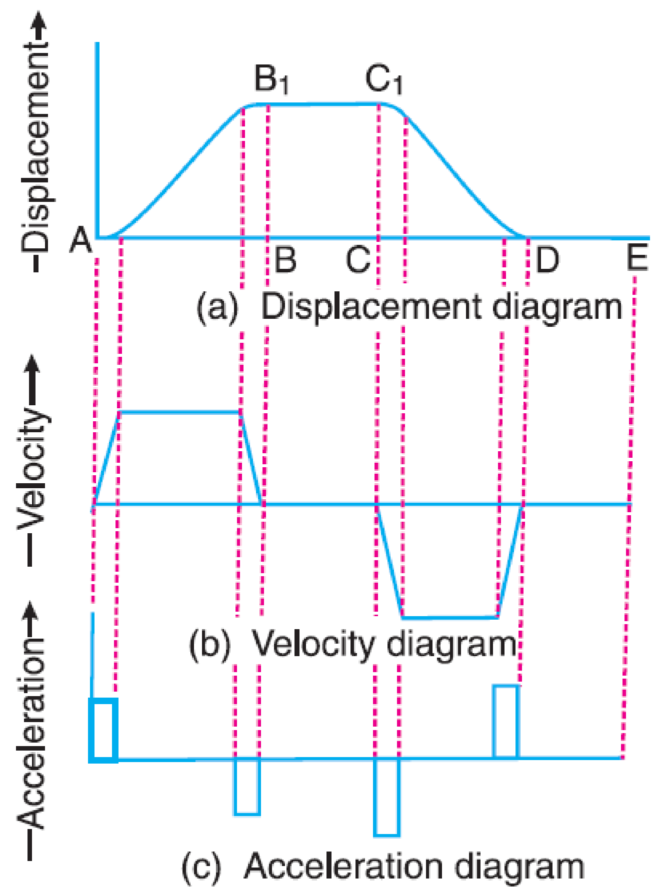


(a) Displacement diagram
(b) Velocity diagram
(c) Acceleration diagram

Follower with Uniform Velocity

Modified Displacement, Velocity and Acceleration Diagrams

1. In order to have the acceleration and retardation within the finite limits, it is necessary to modify the conditions which govern the motion of the follower. This may be done by **rounding off** the sharp corners of the displacement diagram at the beginning and at the end of each stroke.
2. By doing so, the velocity of the follower increases gradually to its maximum value at the beginning of each stroke and decreases gradually to zero at the end of each stroke. The round corners of the displacement diagram are usually **parabolic curves** because the **parabolic motion** results in a very low acceleration of the follower for a given stroke and cam speed.



Modified displacement, velocity and acceleration diagrams when the follower moves with uniform velocity.

Follower with Simple Harmonic Motion

Displacement, Velocity and Acceleration Diagrams

Displacement Diagram

a. Draw a semi-circle on the follower stroke as diameter

b. Divide the semi-circle into any number of even equal parts (say eight)

c. Divide the angular displacements of the cam during out stroke and return stroke into the same number of equal parts. The displacement diagram is obtained by projecting the points as shown in **Fig. (a)**

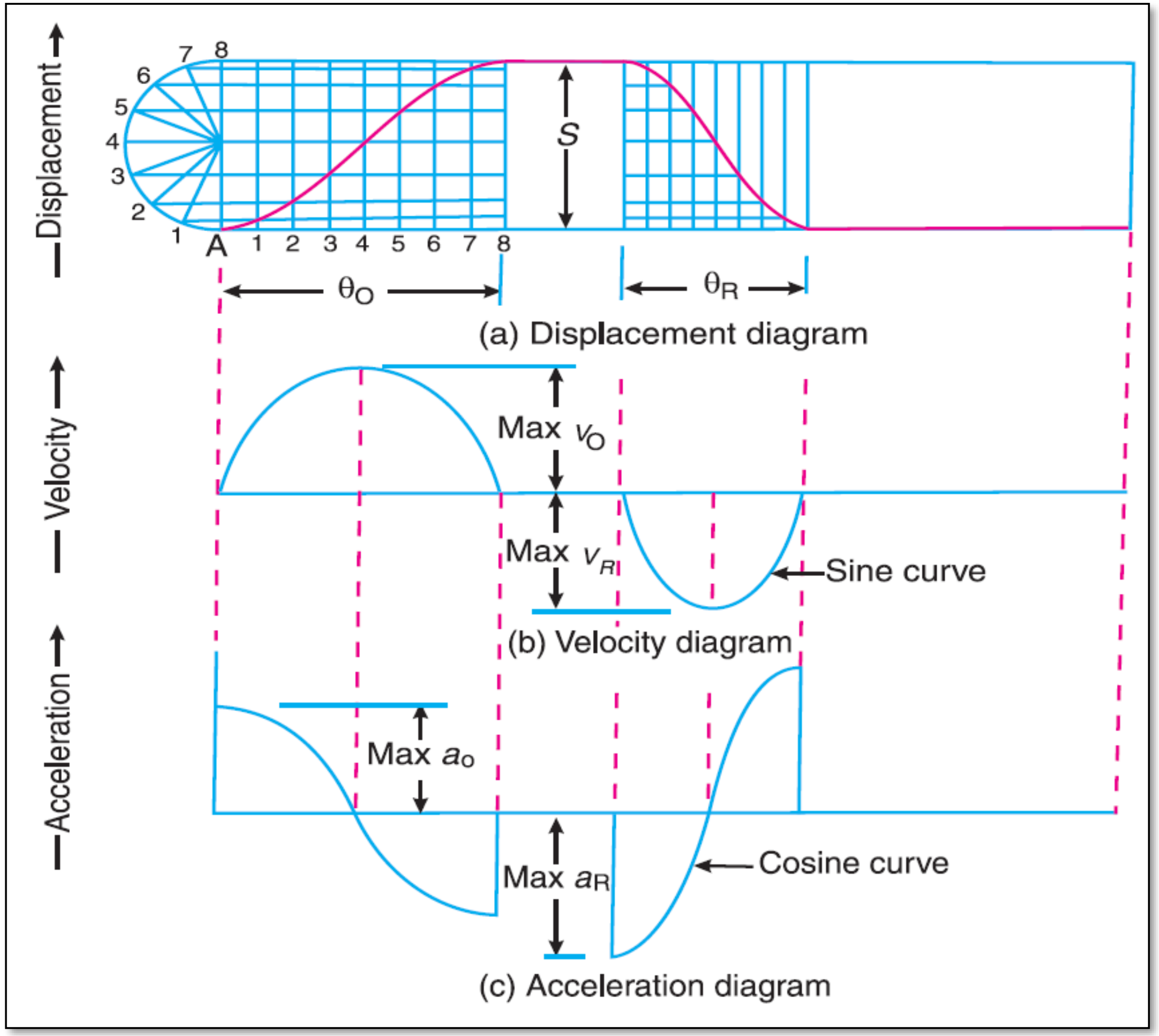
Velocity Diagram

a. The follower moves with a simple harmonic motion. Therefore, velocity diagram consists of a sine curve and the acceleration diagram is a cosine curve

b. We see from **Fig. (*b*)** that the velocity of the follower is zero at the beginning and at the end of its stroke and increases gradually to a maximum at mid-stroke

Acceleration Diagram

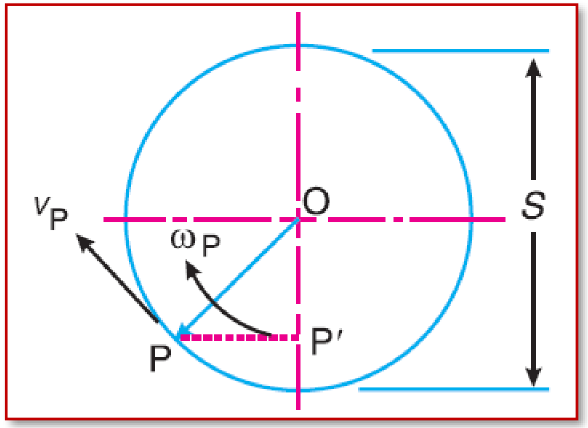
a. Acceleration of the follower is maximum at the beginning and at the ends of the stroke and diminishes to zero at mid-stroke



Follower with Simple Harmonic Motion

Let S = Stroke of the follower,
θ_O and θ_R = Angular displacement of the cam during out stroke and return stroke of the follower respectively, in radians, and
ω = Angular velocity of the cam in rad/s

Time required for the out stroke of the follower in seconds, $t_O = \theta_O / \omega$



Consider a point P moving at a uniform speed ω_P (rad/s) round the circumference of a circle with the stroke S as diameter, as shown in Fig. The point P' (which is the projection of a point P on the diameter) executes a simple harmonic motion as the point P rotates. The motion of the follower is similar to that of point P'.

$\therefore$ Peripheral speed of the point P',

$$v_P = \frac{\pi S}{2} \times \frac{1}{t_O} = \frac{\pi S}{2} \times \frac{\omega}{\theta_O}$$

maximum velocity of the follower on the outstroke,

$$v_O = v_P = \frac{\pi S}{2} \times \frac{\omega}{\theta_O} = \frac{\pi \omega . S}{2\theta_O}$$

We know that the centripetal acceleration of the point P,

$$a_P = \frac{(v_P)^2}{OP} = \left(\frac{\pi \omega . S}{2\theta_O}\right)^2 \times \frac{2}{S} = \frac{\pi^2 \omega^2 . S}{2(\theta_O)^2}$$

$\therefore$ Maximum acceleration of the follower on the outstroke,

$$a_O = a_P = \frac{\pi^2 \omega^2 . S}{2(\theta_O)^2}$$

Similarly, maximum velocity of the follower on the return stroke,

$$v_R = \frac{\pi \omega . S}{2\theta_R}$$

maximum acceleration of the follower on the return stroke,

$$a_R = \frac{\pi^2 \omega^2 . S}{2(\theta_R)^2}$$

Follower with Uniform Acceleration & Retardation

Displacement, Velocity and Acceleration Diagrams

Displacement Diagram

a. Divide the angular displacement of the cam during outstroke (θ_O) into any even number of equal parts (say eight) and draw vertical lines through these points as shown in Fig.(*a*).

b. Divide the stroke of the follower (*S*) into the same number of equal even parts.

c. Join *Aa* to intersect the vertical line through point 1 at *B*. Similarly, obtain the other points *C*, *D* etc. as shown in Fig. (*a*). Now join these points to obtain the parabolic curve for the out stroke of the follower.

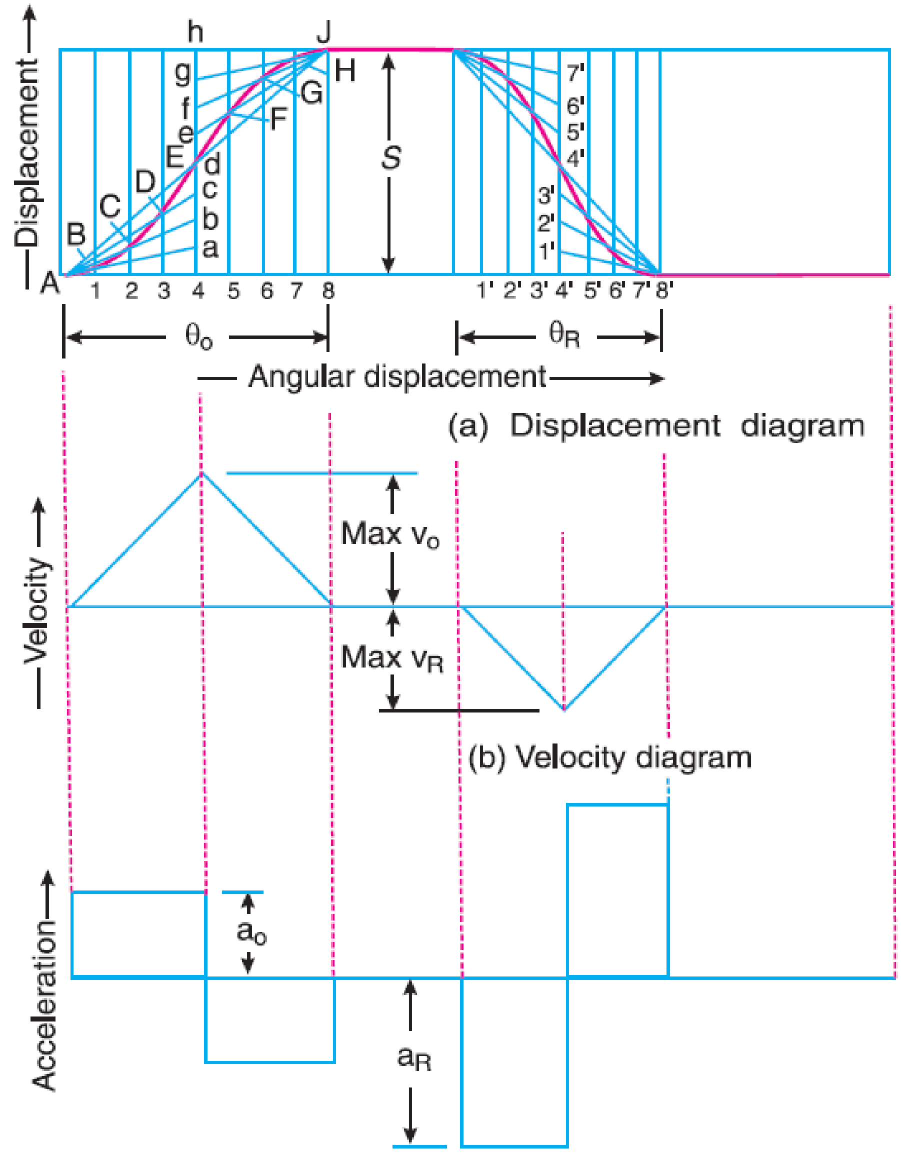
d. In the similar way as discussed above, the displacement diagram for the follower during return stroke may be drawn.

Velocity Diagram

a. Since the acceleration and retardation are uniform, therefore the velocity varies directly with the time. The velocity diagram is shown in Fig. (*b*).

Acceleration Diagram

a. Shown in Fig. (c) for uniform acceleration and retardation



(a) Displacement diagram

(b) Velocity diagram

(c) Acceleration diagram

Follower with Uniform Acceleration & Retardation

Let S = Stroke of the follower,
θ_O and θ_R = Angular displacement of the cam during out stroke and return stroke of the follower respectively, in radians, and
ω = Angular velocity of the cam in rad/s

Time required for the follower during outstroke, $t_O = \theta_O/\omega$

Time required for the follower during return stroke, $t_R = \theta_R/\omega$

Mean velocity of the follower during outstroke = S/t_O

Mean velocity of the follower during return stroke= S/t_R

Maximum velocity of follower is equal to twice the mean velocity, therefore maximum velocity of the follower during outstroke,

$$v_O = \frac{2S}{t_O} = \frac{2\omega S}{\theta_O}$$

Maximum velocity of the follower during return stroke,

$$v_R = \frac{2\omega S}{\theta_R}$$

From the acceleration diagram, in Fig. (*c*), that during first half of the outstroke there is uniform acceleration and during the second half of the out stroke there is uniform retardation. Thus, the maximum velocity of the follower is reached after the time $t_O/2$ (during out stroke) and $t_R/2$ (during return stroke).

Maximum acceleration of the follower during outstroke

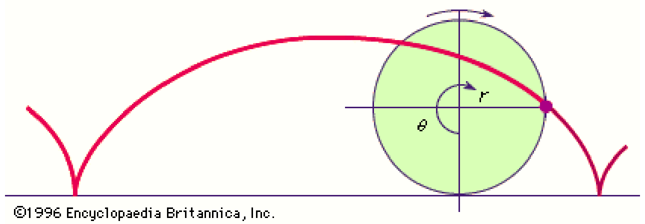
$$a_O = \frac{v_O}{t_O/2} = \frac{2 \times 2\omega.S}{t_O.\theta_O} = \frac{4\omega^2.S}{(\theta_O)^2}$$

Maximum acceleration of the follower during return stroke

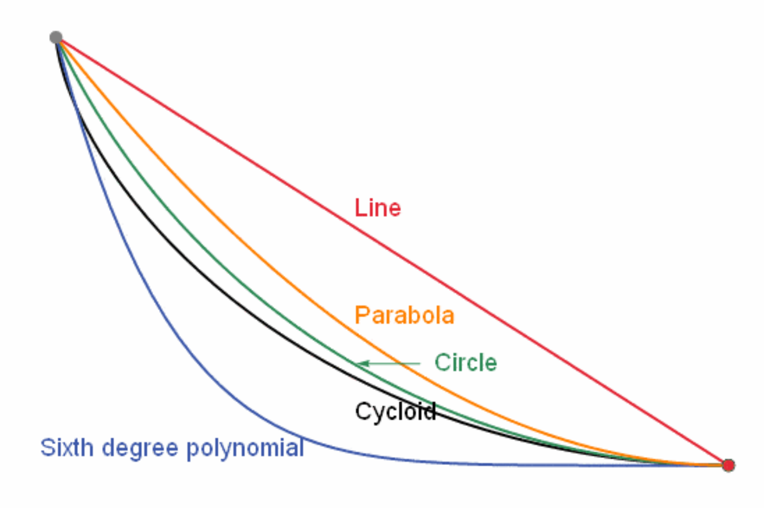
$$a_R = \frac{4\omega^2.S}{(\theta_R)^2}$$

Follower with Cycloidal Motion

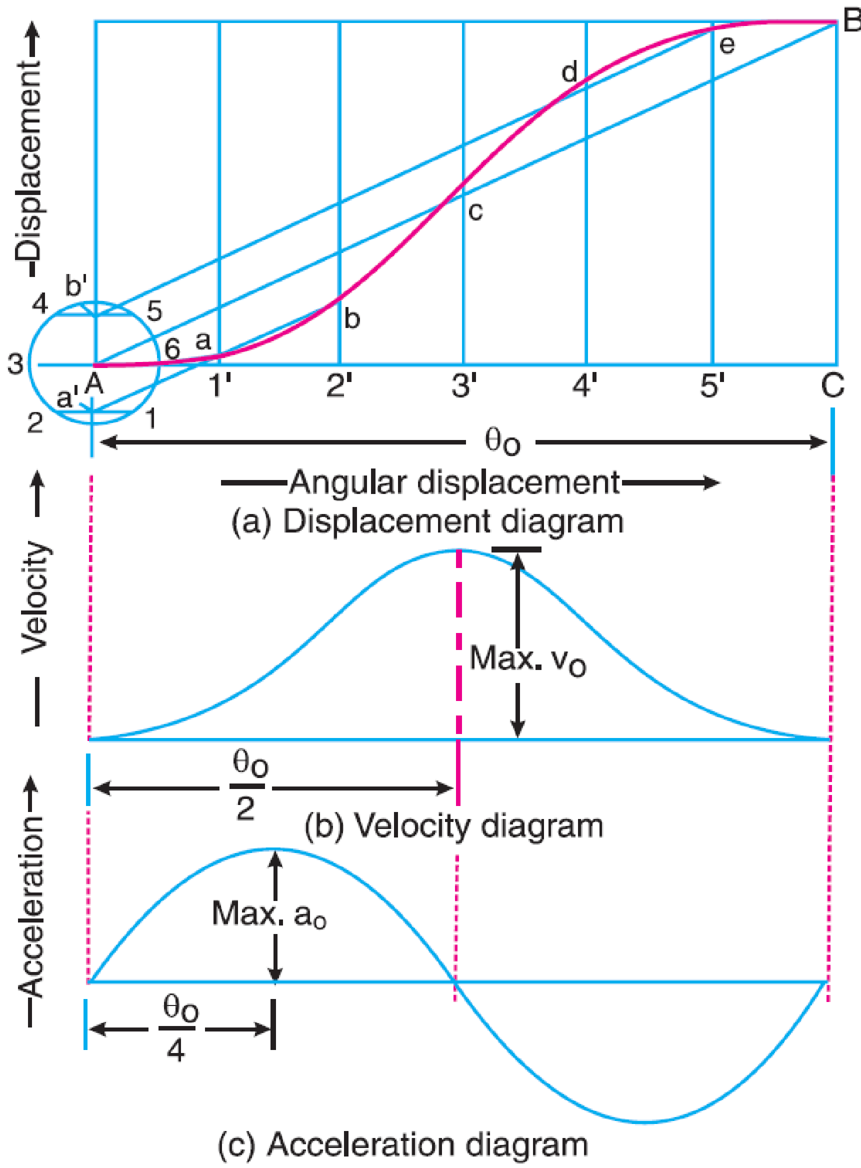
- Cycloidal motion refers to the movement traced by a point on the circumference of a circle as it rolls along a straight line without slipping
- The resulting curve is called a **cycloid**



- Among all possible paths connecting two points (such as a straight line, parabola, etc.), the cycloid is the path that allows a ball or object to reach the destination in the shortest possible time. Even though a straight line seems shorter in distance, the cycloidal curve allows the object to gain speed more quickly because the initial steep section of the curve provides a faster acceleration. This is called Brachistochrone Property



Brachistochrone Curve



Follower with Cycloidal Motion

The displacement, velocity and acceleration diagrams when the follower moves with cycloidal motion are shown in Fig. (*a*), (*b*) and (*c*) respectively. We know that cycloid is a curve traced by a point on a circle when the circle rolls without slipping on a straight line. In case of cams, this straight line is a stroke of the follower which is translating and the circumference of the rolling circle is equal to the stroke (S) of the follower. Therefore, the radius of the rolling circle is $S / 2\pi$.

Displacement Diagram

1. Draw a circle of radius $S / 2\pi$ with A as centre.

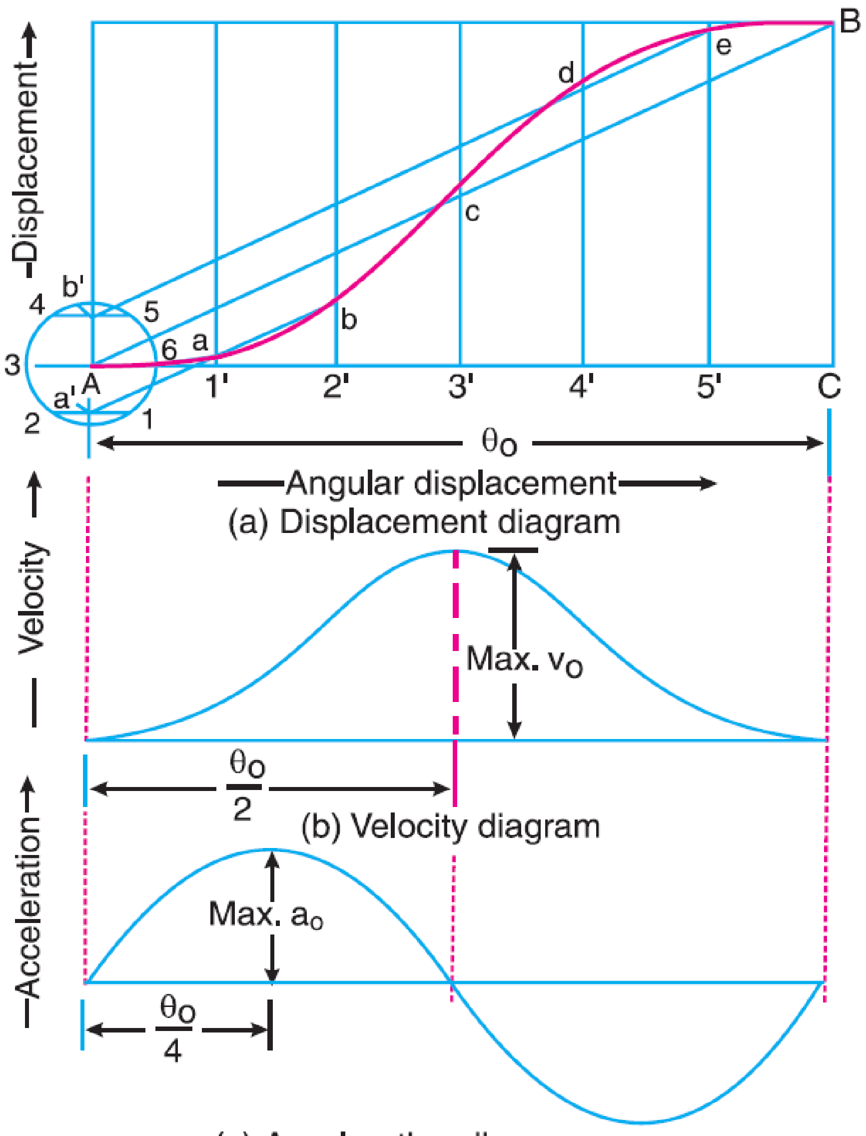
2. Divide the circle into any number of equal even parts (say six). Project these points horizontally on the vertical centre line of the circle. These points are shown by *a'* and *b'* in Fig. (*a*).

3. Divide the angular displacement of the cam during outstroke into the same number of equal even parts as the circle is divided. Draw vertical lines through these points.

4. Join *AB* which intersects the vertical line through *3'* at *c*. From *a'* draw a line parallel to *AB* intersecting the vertical lines through *1'* and *2'* at *a* and *b* respectively.

5. Similarly, from *b'* draw a line parallel to *AB* intersecting the vertical lines through *4'* and *5'* at *d* and *e* respectively.

6. Join the points *A a b c d e B* by a smooth curve. This is the required cycloidal curve for the follower during outstroke.



(a) Displacement diagram

(b) Velocity diagram

(c) Acceleration diagram

Follower with Cycloidal Motion

Let S = Stroke of the follower,
θ = Angle through which the cam rotates in time t seconds
ω = Angular velocity of the cam in rad/s

We know that displacement of the follower after time t seconds,

$$x = S\left[\frac{\theta}{\theta_O} - \frac{1}{2\pi}\sin\left(\frac{2\pi\theta}{\theta_O}\right)\right] \qquad \ldots (i)$$

$\therefore$ Velocity of the follower after time t seconds,

$$\frac{dx}{dt} = S\left[\frac{1}{\theta_O}\times\frac{d\theta}{dt} - \frac{2\pi}{2\pi\theta_O}\cos\left(\frac{2\pi\theta}{\theta_O}\right)\frac{d\theta}{dt}\right]$$

. . . [Differentiating equation (*i*)]

$$= \frac{S}{\theta_O}\times\frac{d\theta}{dt}\left[1-\cos\left(\frac{2\pi\theta}{\theta_O}\right)\right] = \frac{\omega.S}{\theta_O}\left[1-\cos\left(\frac{2\pi\theta}{\theta_O}\right)\right] \qquad \ldots (ii)$$

The velocity is maximum, when

$$\cos\left(\frac{2\pi\theta}{\theta_O}\right) = -1 \quad \text{or} \quad \frac{2\pi\theta}{\theta_O} = \pi \quad \text{or} \quad \theta = \theta_O / 2$$

Substituting $\theta = \theta_O / 2$ in equation (*ii*), we have maximum velocity of the follower during outstroke,

$$v_O = \frac{\omega.S}{\theta_O}(1+1) = \frac{2\,\omega.S}{\theta_O}$$

Follower with Cycloidal Motion

Similarly, maximum velocity of the follower during return stroke,

$$v_R = \frac{2\,\omega.S}{\theta_R}$$

Now, acceleration of the follower after time t sec,

$$\frac{d^2x}{dt^2} = \frac{\omega.S}{\theta_O}\left[\frac{2\pi}{\theta_O}\sin\left(\frac{2\pi\theta}{\theta_O}\right)\frac{d\theta}{dt}\right] \qquad \text{. . . [Differentiating equation } (ii)]$$

$$= \frac{2\,\pi\,\omega^2.S}{(\theta_O)^2}\sin\left(\frac{2\pi\theta}{\theta_O}\right) \qquad \text{. . . } \left(\because \frac{d\theta}{dt} = \omega\right) \qquad \text{. . . } (iii)$$

The acceleration is maximum, when

$$\sin\left(\frac{2\pi\theta}{\theta_O}\right) = 1 \quad \text{or} \quad \frac{2\pi\theta}{\theta_O} = \frac{\pi}{2} \quad \text{or} \quad \theta = \theta_O / 4$$

Substituting $\theta = \theta_O / 4$ in equation (iii), we have maximum acceleration of the follower during outstroke,

$$a_O = \frac{2\pi\,\omega^2.S}{(\theta_O)^2}$$

Similarly, maximum acceleration of the follower during return stroke,

$$a_R = \frac{2\pi\,\omega^2.S}{(\theta_R)^2}$$

The ***velocity*** and ***acceleration diagrams*** are shown in Fig. (b) and (c) respectively

Cam Profile for Radial Cam

- In order to draw the cam profile for a radial cam, first of all the displacement diagram for the given motion of the follower is drawn. Then by constructing the follower in its proper position at each angular position, the profile of the working surface of the cam is drawn.
- In constructing the cam profile, the principle of kinematic inversion is used, *i.e.* the cam is imagined to be stationary and the follower is allowed to rotate in the *opposite direction* to the *cam rotation.*

How to Draw: Displacement Diagram & Cam Profile

1. PROBLEM

A cam is to give the following motion to a knife-edged follower :

1. *Outstroke during 60° of cam rotation*
2. *Dwell for the next 30° of cam rotation*
3. *Return stroke during next 60° of cam rotation*
4. *Dwell for the remaining 210° of cam rotation*

The stroke of the follower is 40 mm and the minimum radius of the cam is 50 mm. The follower moves with uniform velocity during both the outstroke and return strokes.

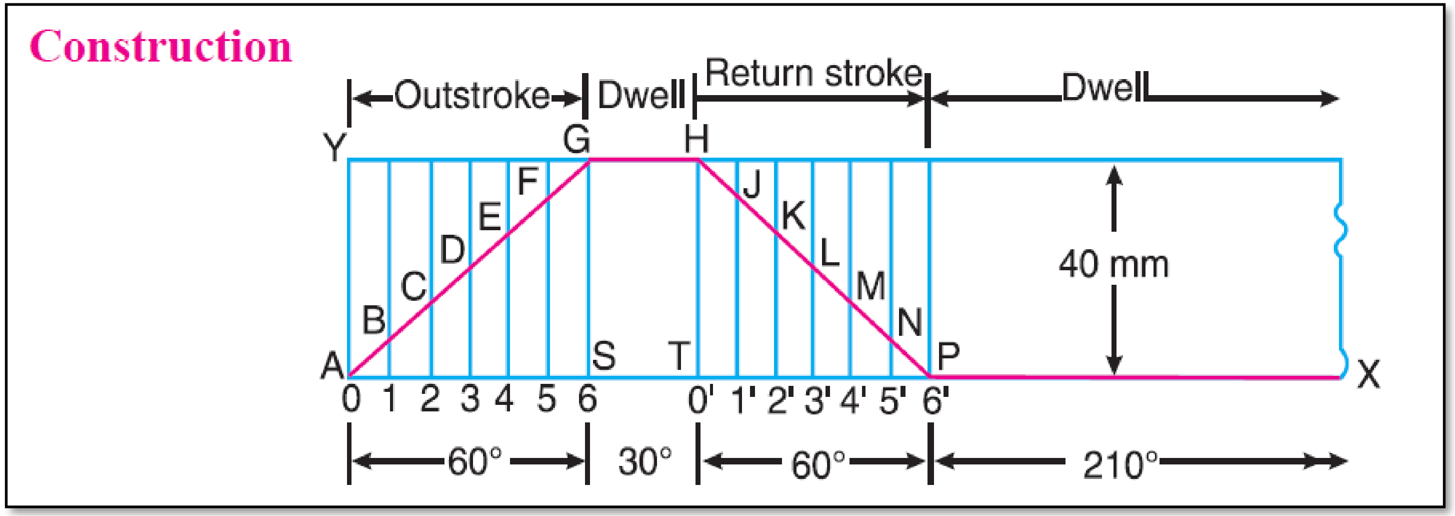
Draw the profile of the cam when

(a) the axis of the follower passes through the axis of the cam shaft, and

(b) the axis of the follower is offset by 20 mm from the axis of the cam shaft.

Solution

Displacement diagram



First of all, the displacement diagram, as shown in Fig. is drawn as discussed in the following steps :

1. Draw a horizontal line *AX* = 360° to some suitable scale. On this line, mark *AS* = 60° to represent outstroke of the follower, *ST* = 30° to represent dwell, *TP* = 60° to represent return stroke and *PX* = 210° to represent dwell.

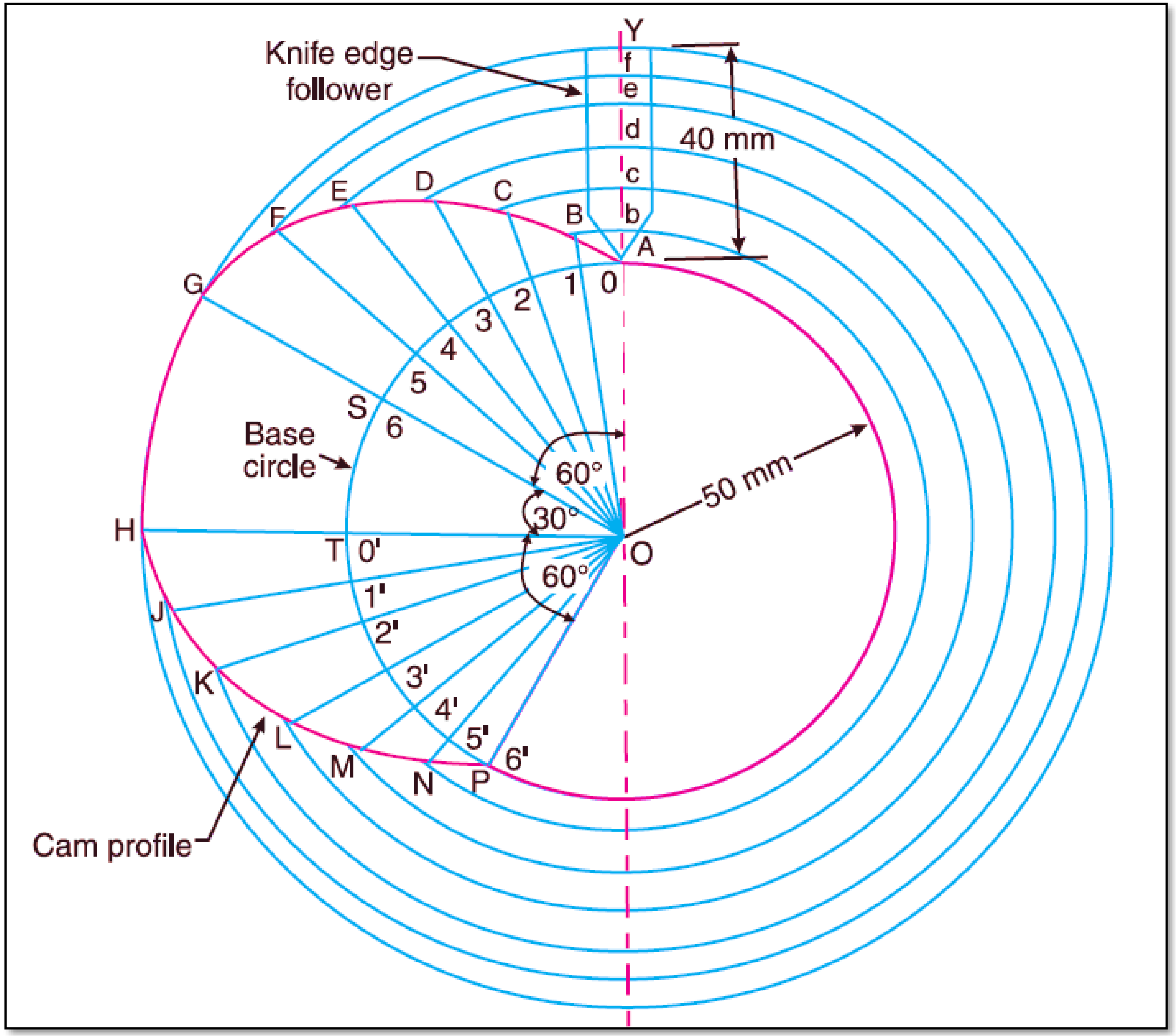
2. Draw vertical line *AY* equal to the stroke of the follower (*i.e.* 40 mm) and complete the rectangle as shown in Fig.

3. Divide the angular displacement during outstroke and return stroke into any equal number of even parts (say six) and draw vertical lines through each point.

4. Since the follower moves with uniform velocity during outstroke and return stroke, therefore the displacement diagram consists of straight lines. Join *AG* and *HP*.

5. The complete displacement diagram is shown by *AGHPX* in Fig.

Solution



(a) Profile of the cam when the axis of follower passes through the axis of cam shaft

The profile of the cam when the axis of the follower passes through the axis of the cam shaft, as shown in Fig., is drawn as discussed in the following steps:

1. Draw a base circle with radius equal to the minimum radius of the cam (*i.e.* 50 mm) with *O* as centre.

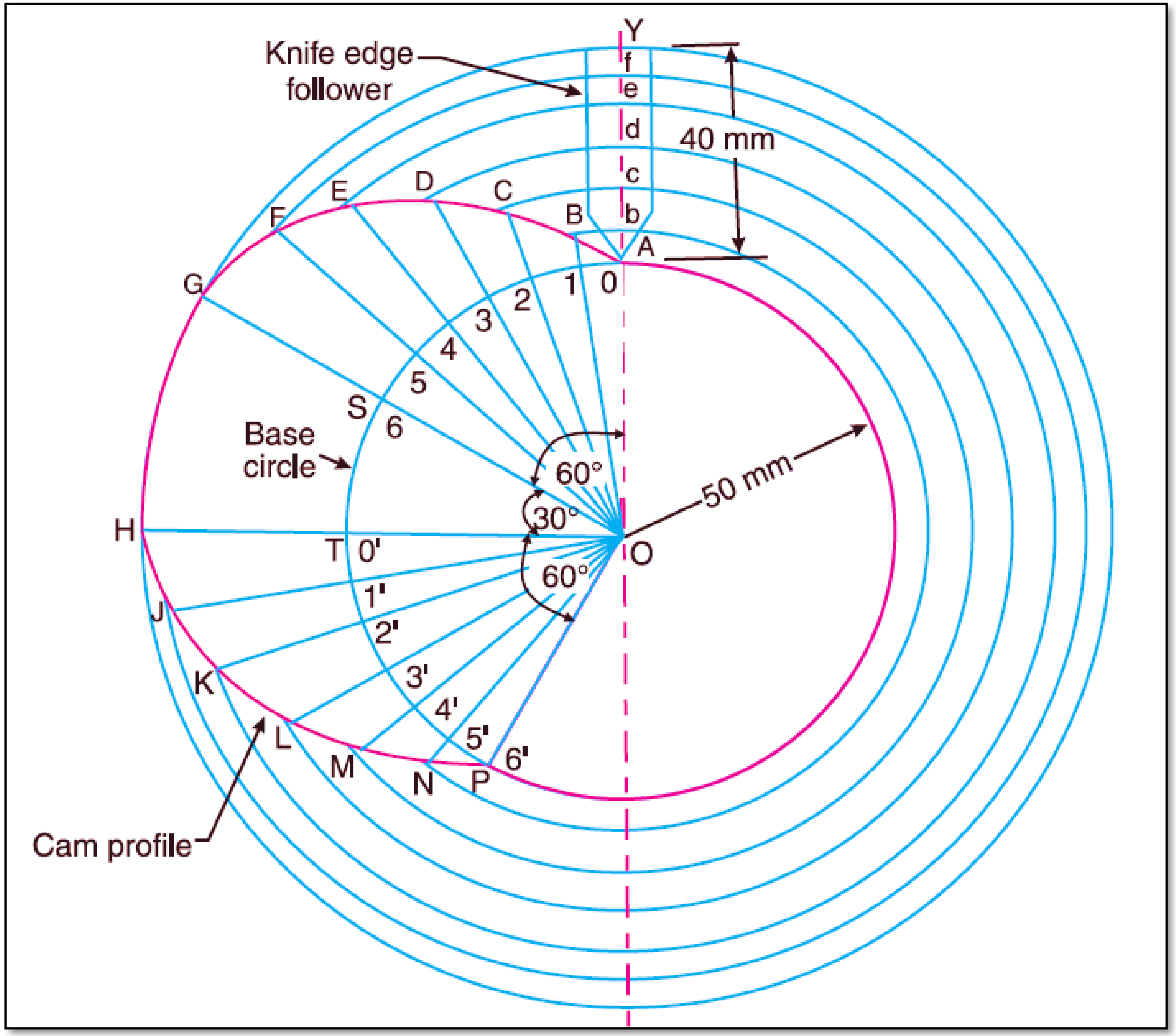
2. Since the axis of the follower passes through the axis of the cam shaft, therefore mark trace point *A*, as shown in Fig.

3. From *OA*, mark angle *AOS* = 60° to represent outstroke, angle *SOT* = 30° to represent dwell and angle *TOP* = 60° to represent return stroke.

4. Divide the angular displacements during outstroke and return stroke (*i.e.* angle *AOS* and angle *TOP*) into the same number of equal even parts as in displacement diagram.

5. Join the points 1, 2, 3 ...etc. and *0′* ,*1′* , *2′* , *3′* , ... etc. with centre *O* and produce beyond the base circle as shown in Fig.

Solution



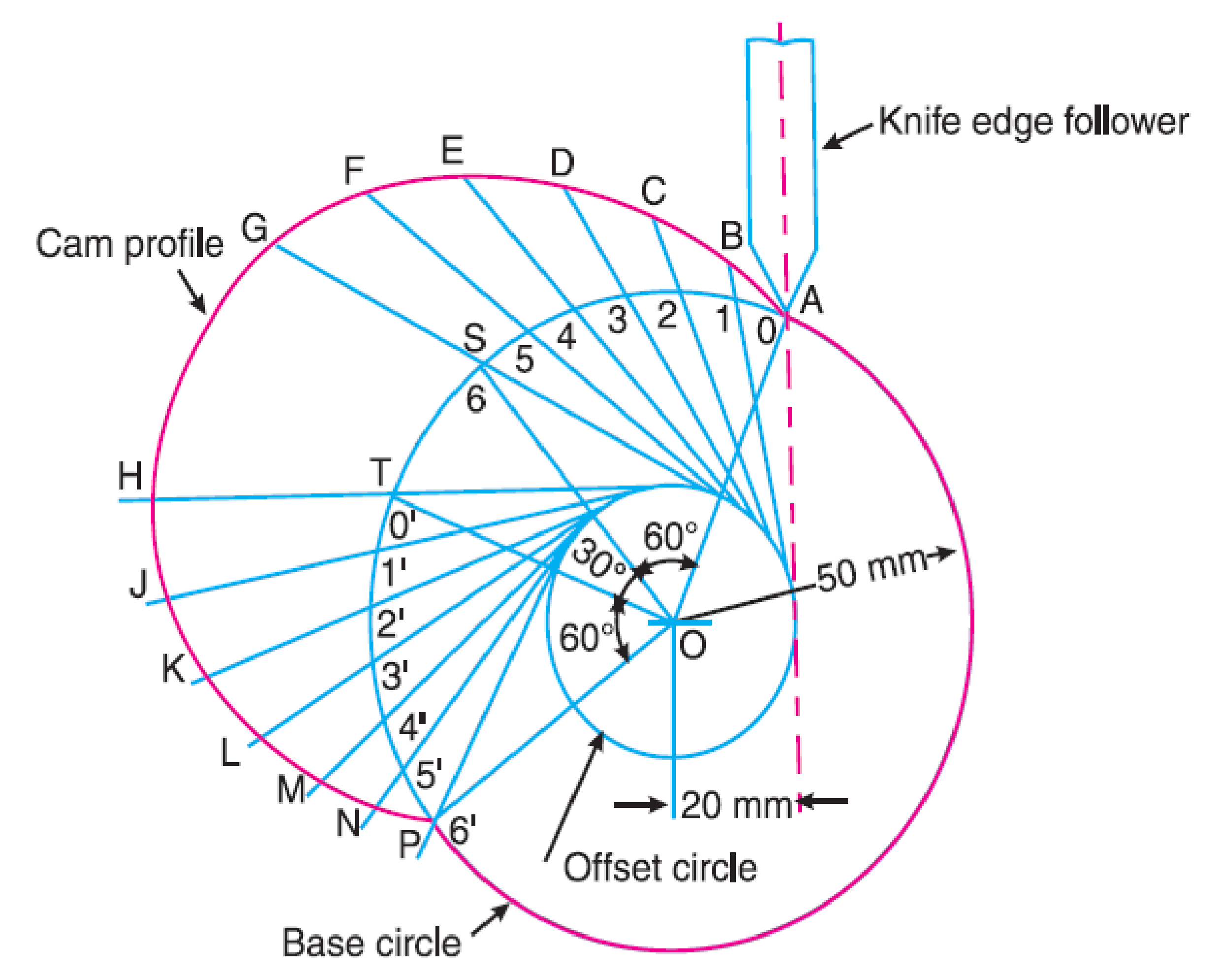
6. Now set off 1*B*, 2*C*, 3*D* ... etc. and 0′ *H*,1′ *J* ... etc. from the displacement diagram.

7. Join the points *A*, *B*, *C*,... *M*, *N*, *P* with a smooth curve. The curve *AGHPA* is the complete profile of the cam.

Notes : The points *B*, *C*, *D* .... *L*, *M*, *N* may also be obtained as follows :

1. Mark *AY* = 40 mm on the axis of the follower, and set of *Ab*, *Ac*, *Ad*... etc. equal to the distances 1*B*, 2*C*, 3*D*... etc. as in displacement diagram.

2. From the centre of the cam *O*, draw arcs with radii *Ob*, *Oc*, *Od* etc. The arcs intersect the produced lines *O*1, *O*2... etc. at *B*, *C*, *D* ... *L*, *M*, *N*.



(b) Profile of the cam when the axis of the follower is offset by 20 mm from the axis of the cam shaft

The profile of the cam when the axis of the follower is offset from the axis of the cam shaft, is drawn as discussed in the following steps :

1. Draw a base circle with radius equal to the minimum radius of the cam (*i.e.* 50 mm) with *O* as centre.

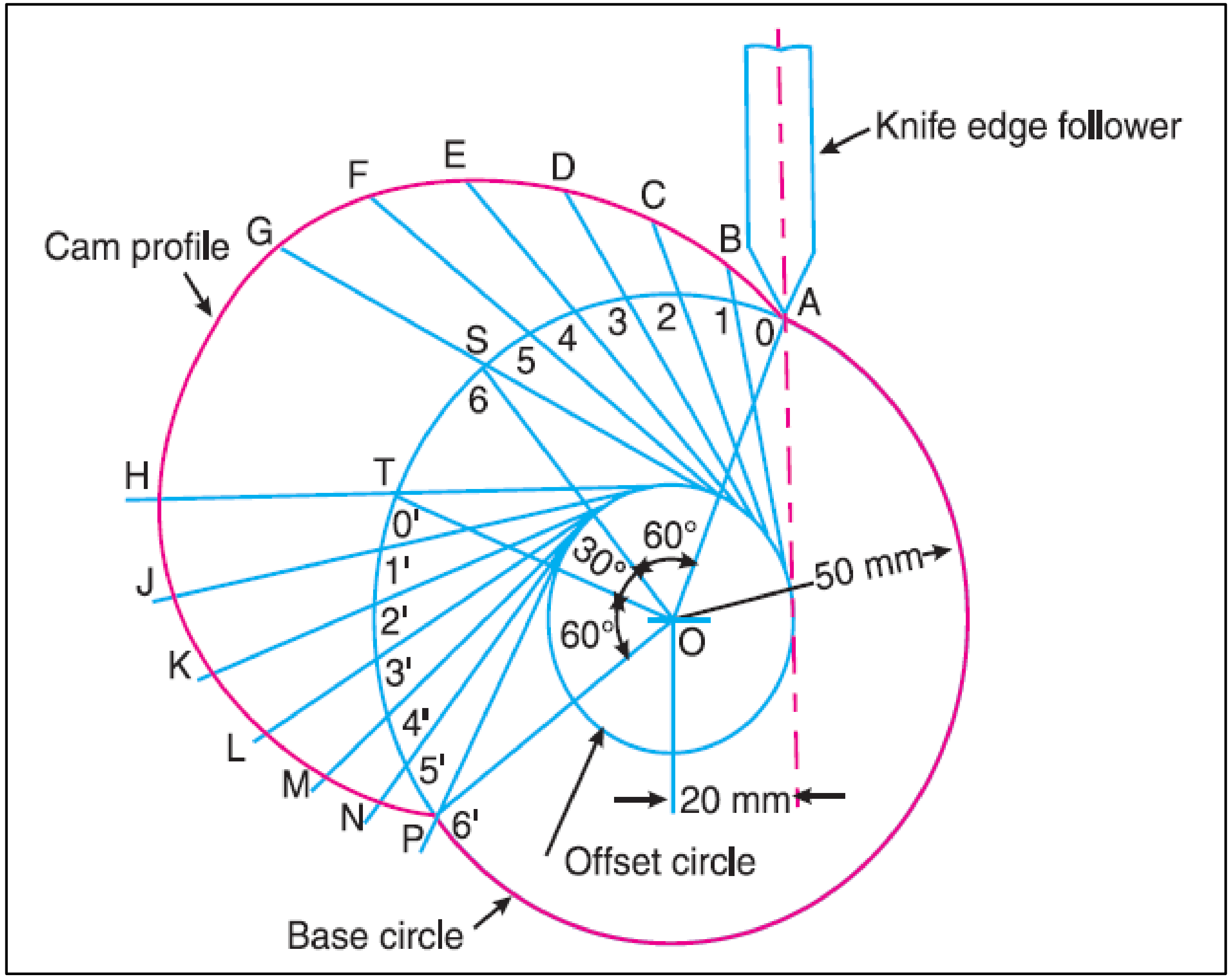
2. Draw the axis of the follower at a distance of 20 mm from the axis of the cam, which intersects the base circle at *A*.

3. Join *AO* and draw an offset circle of radius 20 mm with centre *O*.

4. From *OA*, mark angle *AOS* = 60° to represent outstroke, angle *SOT* = 30° to represent dwell and angle *TOP* = 60° to represent return stroke.

5. Divide the angular displacement during outstroke and return stroke (*i.e.* angle *AOS* and angle *TOP*) into the same number of equal even parts as in displacement diagram.

Solution



6. Now from the points 1, 2, 3 ... etc. and 0′,1′, 2′,3′ ... etc. on the base circle, draw tangents to the offset circle and produce these tangents beyond the base circle as shown in Fig.

7. Now set off 1*B*, 2*C*, 3*D* ... etc. and 0′ *H*,1′ *J* ... etc. from the displacement diagram.

8. Join the points *A*, *B*, *C* ...*M*, *N*, *P* with a smooth curve. The curve *AGHPA* is the complete profile of the cam.

2. PROBLEM

A cam is to be designed for a knife edge follower with the following data :

1. *Cam lift = 40 mm during 90° of cam rotation with simple harmonic motion.*
2. *Dwell for the next 30°.*
3. *During the next 60° of cam rotation, the follower returns to its original position with simple harmonic motion.*
4. *Dwell during the remaining 180°.*

Draw the profile of the cam when

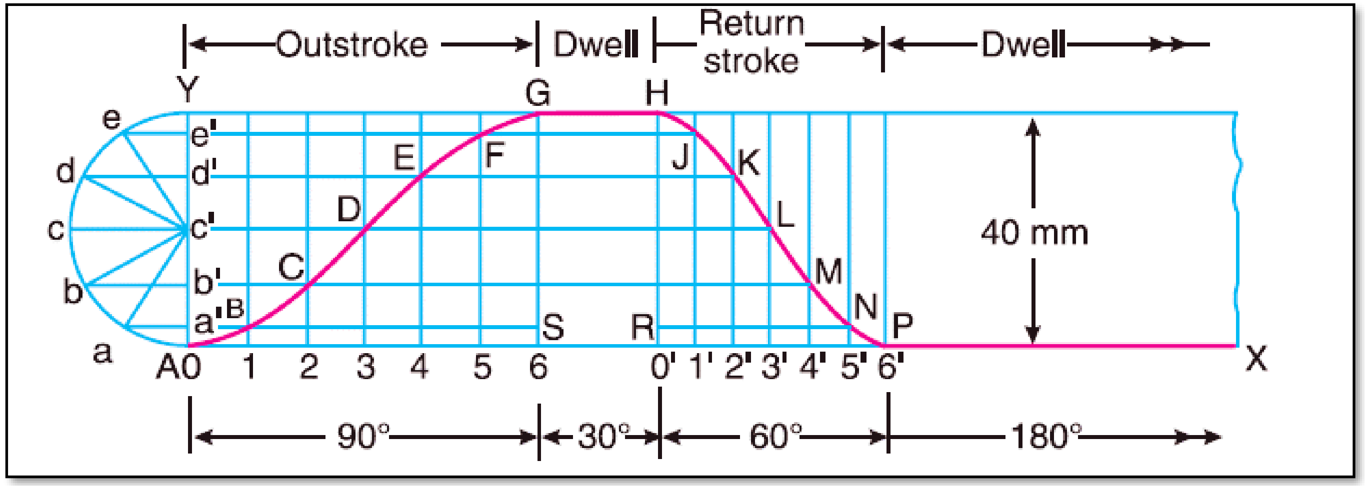
(a) the line of stroke of the follower passes through the axis of the cam shaft, and

(b) the line of stroke is offset 20 mm from the axis of the cam shaft.

The radius of the base circle of the cam is 40 mm. Determine the maximum velocity and acceleration of the follower during its ascent and descent, if the cam rotates at 240 r.p.m.

Solution

Given : S = 40 mm = 0.04 m
$\theta_O = 90° = \pi/2$ rad = 1.571 rad
$\theta_R = 60° = \pi/3$ rad = 1.047 rad
N = 240 r.p.m.



Displacement diagram

First of all, the displacement diagram is drawn as discussed in the following steps :

1. Draw horizontal line AX = 360° to some suitable scale. On this line, mark AS = 90° to represent out stroke ; SR = 30° to represent dwell ; RP = 60° to represent return stroke and PX = 180° to represent dwell.

2. Draw vertical line AY = 40 mm to represent the cam lift or stroke of the follower and complete the rectangle.

3. Divide the angular displacement during out stroke and return stroke into any equal number of even parts (say six) and draw vertical lines through each point.

4. Since the follower moves with simple harmonic motion, therefore draw a semicircle with AY as diameter and divide into six equal parts.

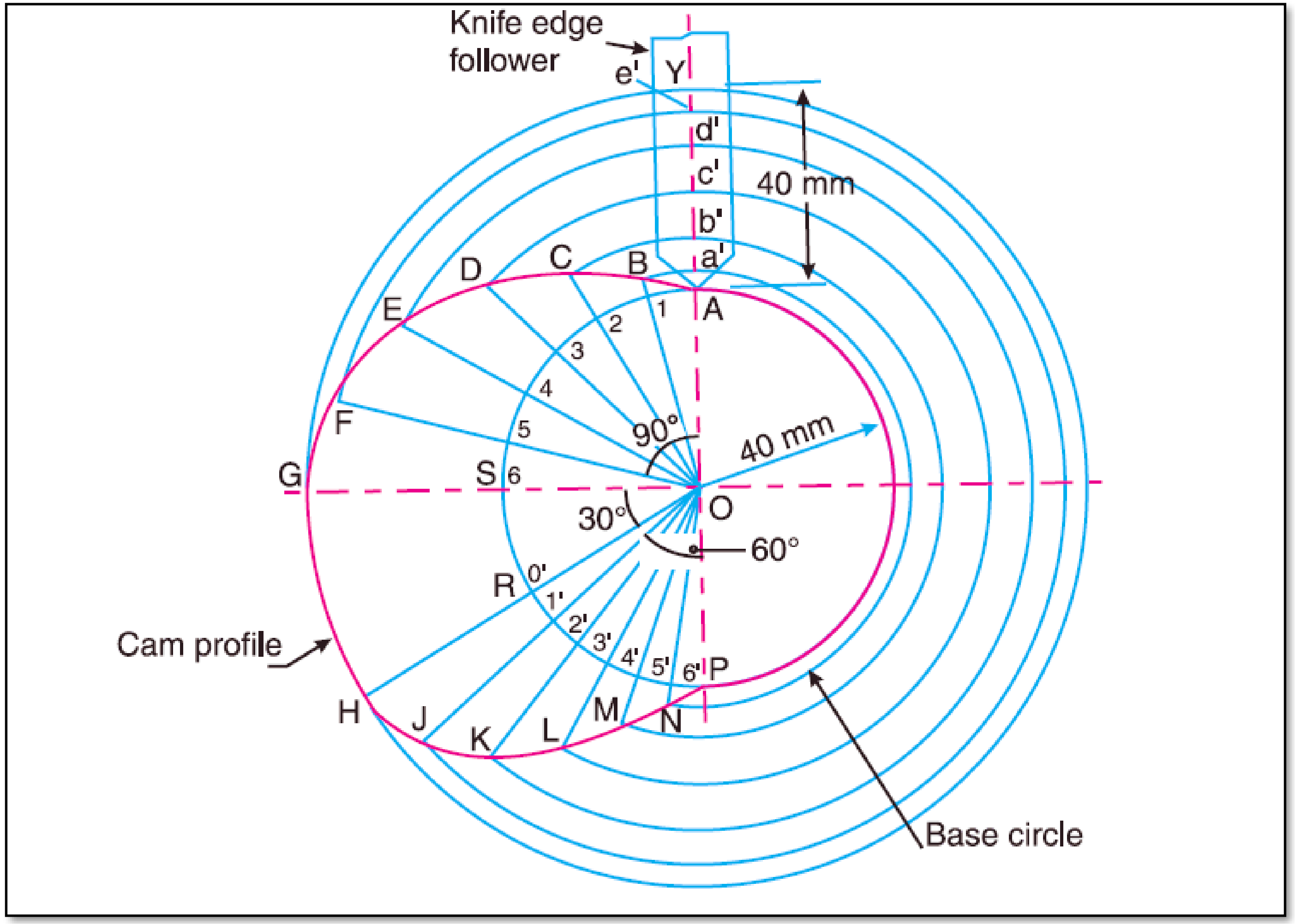
5. From points a, b, c ... etc. draw horizontal lines intersecting the vertical lines drawn through 1, 2, 3 ... etc. and 0′ ,1′ , 2′ ...etc. at B, C, D ... M, N, P.

6. Join the points A, B, C ... etc. with a smooth curve as shown in Fig. This is the required displacement diagram.

Solution

(a) Profile of the cam when the line of stroke of the follower passes through the axis of the cam shaft

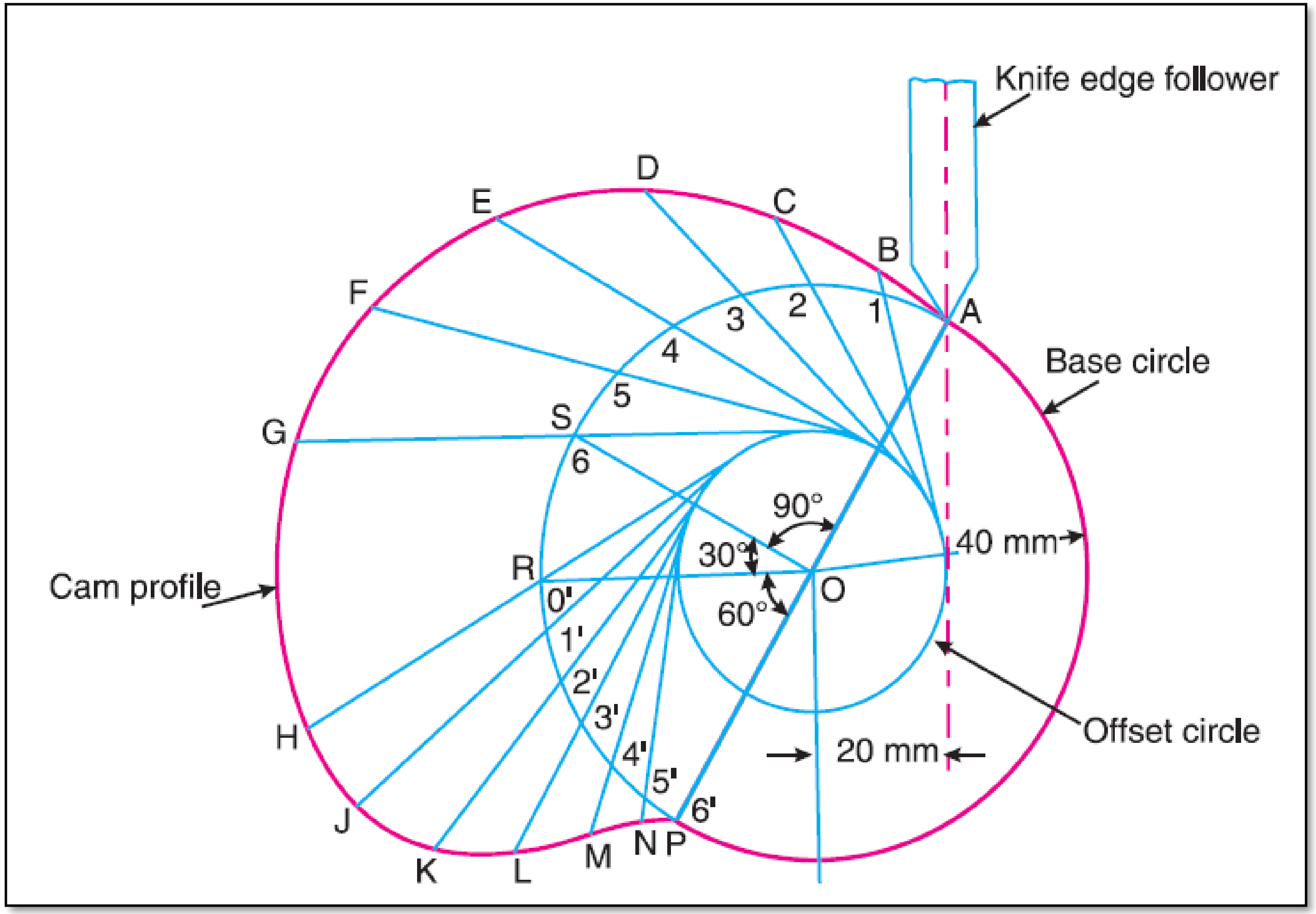
The profile of the cam when the line of stroke of the follower passes through the axis of the cam shaft, as shown in Fig., is drawn in the similar way as is discussed in problem 1.



Solution

(b) Profile of the cam when the line of stroke of the follower is offset 20 mm from the axis of the cam shaft

The profile of the cam when the line of stroke of the follower is offset 20 mm from the axis of the cam shaft, as shown in Fig, is drawn in the similar way as discussed in problem 1.



Solution

Maximum velocity of the follower during its ascent and descent

We know that angular velocity of the cam,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60} = 25.14 \text{ rad/s}$$

We also know that the maximum velocity of the follower during its ascent,

$$v_O = \frac{\pi \omega . S}{2\theta_O} = \frac{\pi \times 25.14 \times 0.04}{2 \times 1.571} = 1 \text{ m/s} \quad \textbf{Ans.}$$

and maximum velocity of the follower during its descent,

$$v_R = \frac{\pi \omega . S}{2\theta_R} = \frac{\pi \times 25.14 \times 0.04}{2 \times 1.047} = 1.51 \text{ m/s} \quad \textbf{Ans.}$$

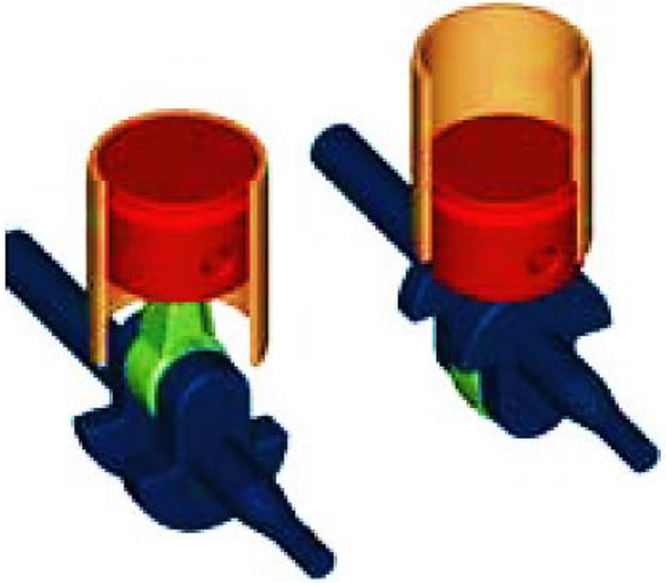
Maximum acceleration of the follower during its ascent and descent

We know that the maximum acceleration of the follower during its ascent,

$$a_O = \frac{\pi^2 \omega^2 . S}{2(\theta_O)^2} = \frac{\pi^2 (25.14)^2 0.04}{2(1.571)^2} = 50.6 \text{ m/s}^2 \quad \textbf{Ans.}$$

and maximum acceleration of the follower during its descent,

$$a_R = \frac{\pi^2 \omega^2 . S}{2(\theta_R)^2} = \frac{\pi^2 (25.14)^2 0.04}{2(1.047)^2} = 113.8 \text{ m/s}^2 \quad \textbf{Ans.}$$



Role of cams in piston movement.

3. PROBLEM

A cam, with a minimum radius of 50 mm, rotating clockwise at a uniform speed, is required to give a knife edge follower the motion as described below :

1. *To move outwards through 40 mm during 100° rotation of the cam ;*
2. *To dwell for next 80°;*
3. *To return to its starting position during next 90°, and*
4. *To dwell for the rest period of a revolution i.e. 90°.*

Draw the profile of the cam

(i) when the line of stroke of the follower passes through the centre of the cam shaft, and

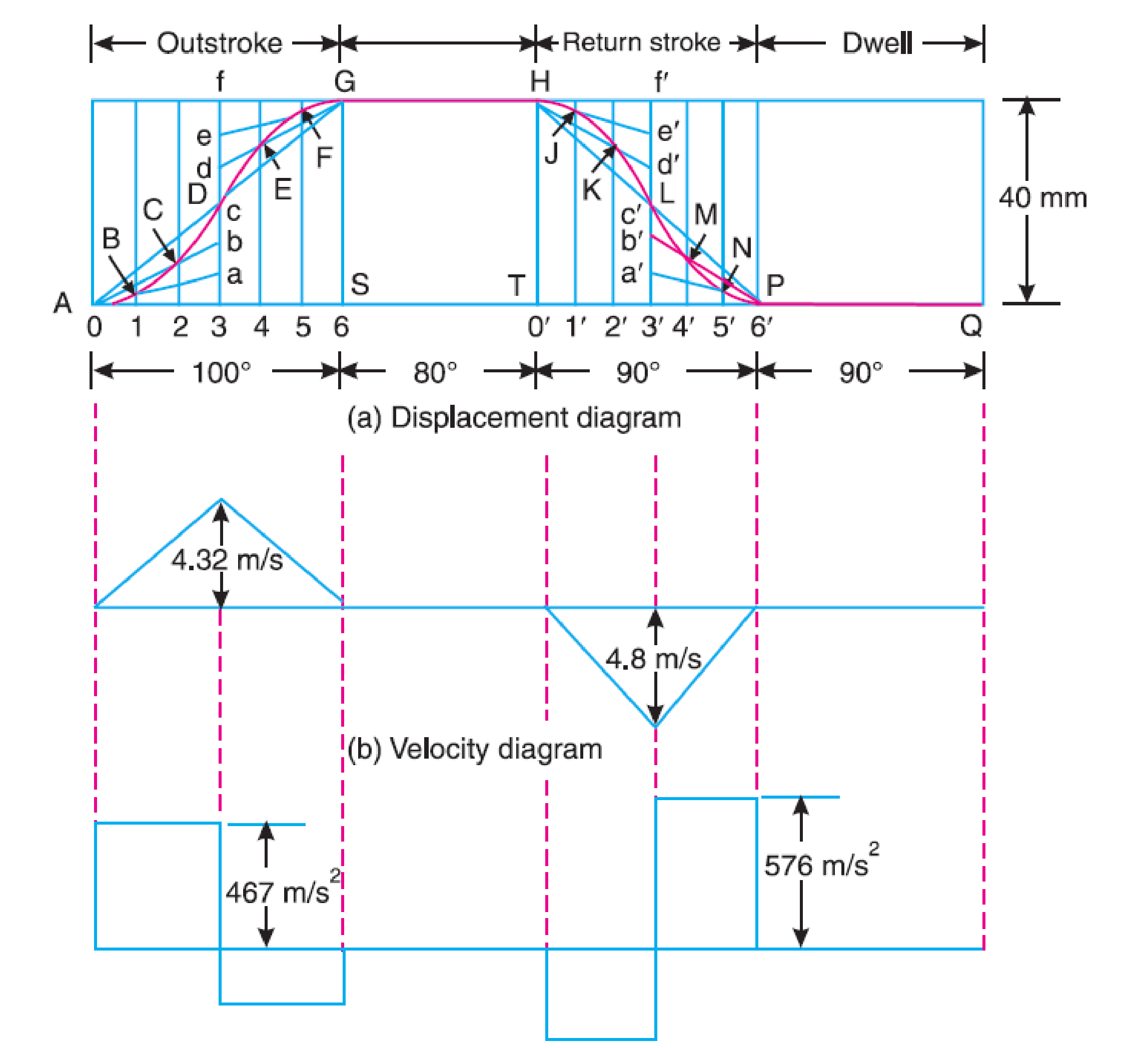
(ii) when the line of stroke of the follower is off-set by 15 mm.

The displacement of the follower is to take place with uniform acceleration and uniform retardation.

Draw the displacement, velocity and acceleration diagrams for one complete revolution of the cam.

Solution

Given : S = 40 mm = 0.04 m; θ_O =100° = 100 × π /180 = 1.745 rad ; θ_R = 90° = π /2 = 1.571 rad ; N = 900 r.p.m.



(a) Displacement diagram

(b) Velocity diagram

(c) Acceleration diagram

1. Draw a horizontal line *ASTPQ* such that *AS* represents the angular displacement of the cam during outward stroke (*i.e.* 100°) to some suitable scale. The line *ST* represents the dwell period of 80° after outward stroke. The line *TP* represents the angular displacement of the cam during return stroke (*i.e.* 90°) and the line *PQ* represents the dwell period of 90° after return stroke.

2. Divide *AS* and *TP* into any number of equal even parts (say six).

3. Draw vertical lines through points 0, 1, 2, 3 etc. and equal to the lift of the valve *i.e.* 40 mm.

4. Divide the vertical lines 3-*f* and 3′- *f* ′ into six equal parts as shown by points *a*, *b*, *c* . . . and *a*′ , *b*′ , *c*′ . . . in Fig. (*a*).

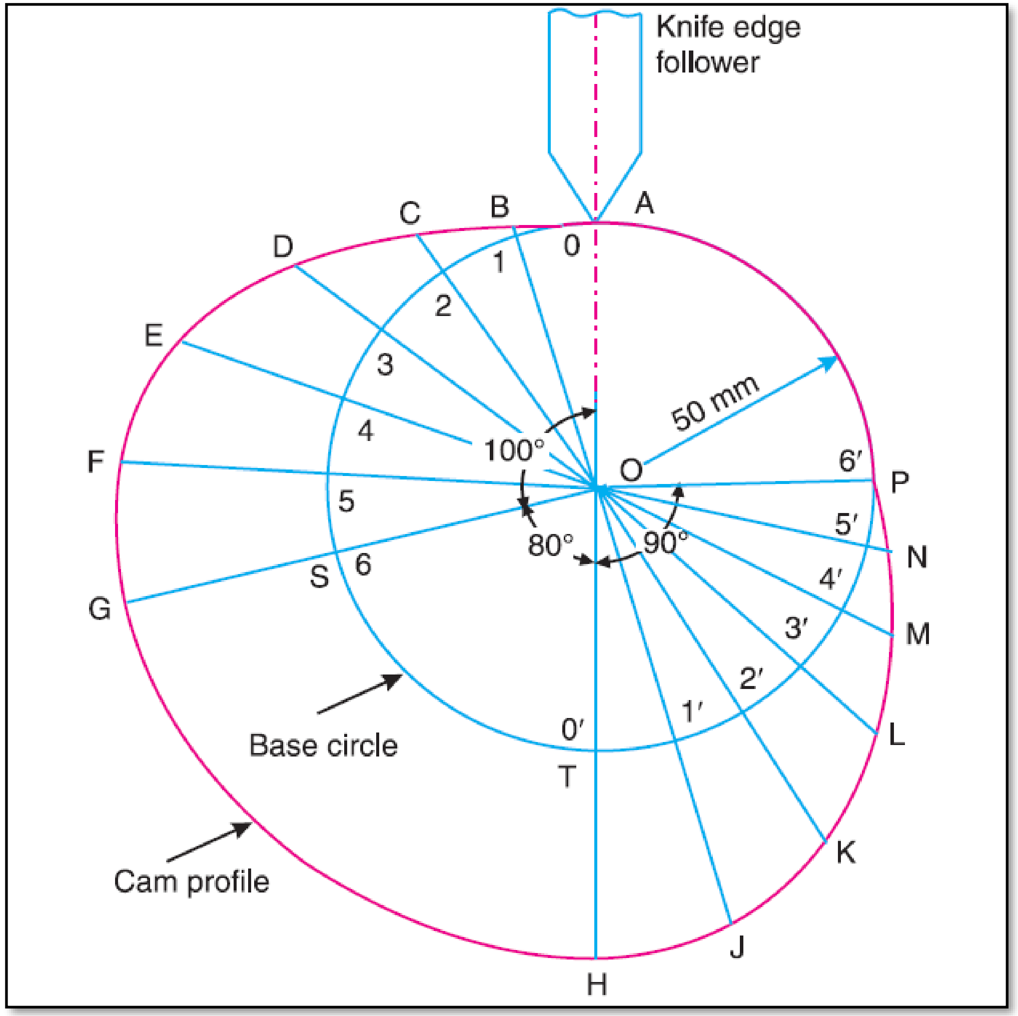
5. Since the follower moves with equal uniform acceleration and uniform retardation, therefore the displacement diagram of the outward and return stroke consists of a double parabola.

6. Join *Aa*, *Ab* and *Ac* intersecting the vertical lines through 1, 2 and 3 at *B*, *C* and *D* respectively.

7. Join the points *B*, *C* and *D* with a smooth curve. This is the required parabola for the half outstroke of the valve. Similarly the other curves may be drawn as shown in Fig.

8. The curve *A B C . . N P Q* is required displacement diagram

Solution



(i) Profile of the cam when the line of stroke of the follower passes through the centre of the cam shaft

The profile of the cam when the line of stroke of the follower passes through the centre of cam shaft, as shown in, may be drawn as discussed in the following steps :

1. Draw a base circle with centre *O* and radius 50 mm (equal to minimum radius of the cam).

2. Divide the base circle such that angle *AOS* = 100° ; angle *SOT* = 80° and angle *TOP* = 90°.

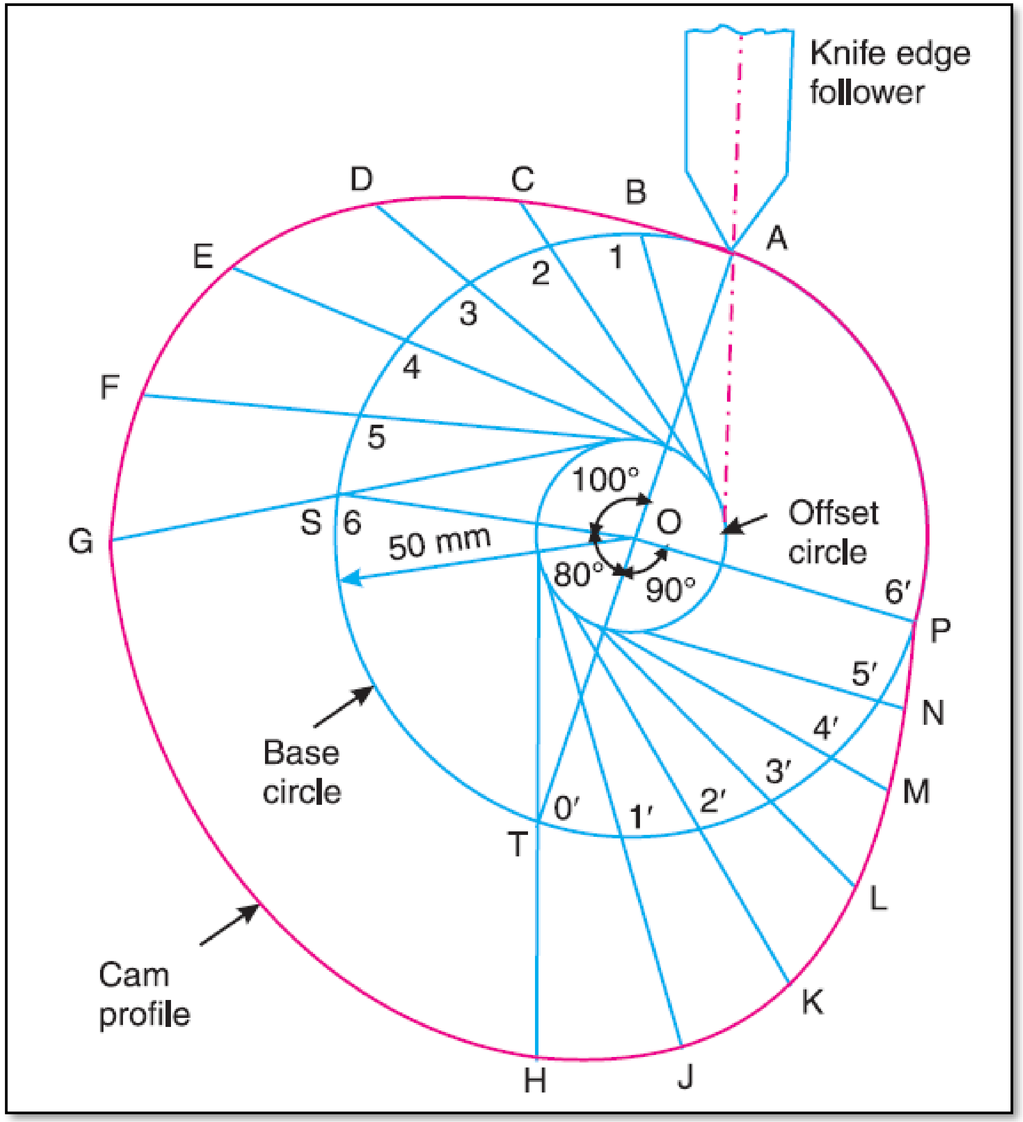
3. Divide angles *AOS* and *TOP* into the same number of equal even parts as in displacement diagram (*i.e.* six parts).

4. Join the points 1, 2, 3 . . . and 1′ , 2′ , 3′ , . . . with centre *O* and produce these lines beyond the base circle.

5. From points 1, 2, 3 . . . and 1′ , 2′ , 3′ , . . . mark the displacements 1*B*, 2*C*, 3*D* . . . etc. as measured from the displacement diagram.

6. Join the points *A*, *B*, *C* . . . *M*, *N*, *P* with a smooth curve as shown in Fig. This is the required profile of the cam.

Solution



(ii) Profile of the cam when the line of stroke of the follower is offset by 15 mm

The profile of the cam when the line of stroke of the follower is offset may be drawn as discussed in Problem 2. The profile of cam is shown in Fig.

Maximum velocity of the follower during out stroke and return stroke

We know that angular velocity of the cam shaft,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 900}{60} = 94.26 \text{ rad/s}$$

Solution

We also know that the maximum velocity of the follower during out stroke,

$$v_O = \frac{2\omega.S}{\theta_O} = \frac{2\times94.26\times0.04}{1.745} = 4.32 \text{ m/s}$$ **Ans.**

and maximum velocity of the follower during return stroke,

$$v_R = \frac{2\omega.S}{\theta_R} = \frac{2\times94.26\times0.04}{1.571} = 4.8 \text{ m/s}$$ **Ans.**

The velocity diagram is shown in Fig. (*b*).

A type of roller follower.

Maximum acceleration of the follower during out stroke and return stroke

We know that the maximum acceleration of the follower during out stroke,

$$a_O = \frac{4\omega^2.S}{(\theta_O)^2} = \frac{4(94.26)^2 0.04}{(1.745)^2} = 467 \text{ m/s}^2$$ **Ans.**

and maximum acceleration of the follower during return stroke,

$$a_R = \frac{4\omega^2.S}{(\theta_R)^2} = \frac{4(94.26)^2 0.04}{(1.571)^2} = 576 \text{ m/s}^2$$ **Ans.**

The acceleration diagram is shown in Fig. (*c*).

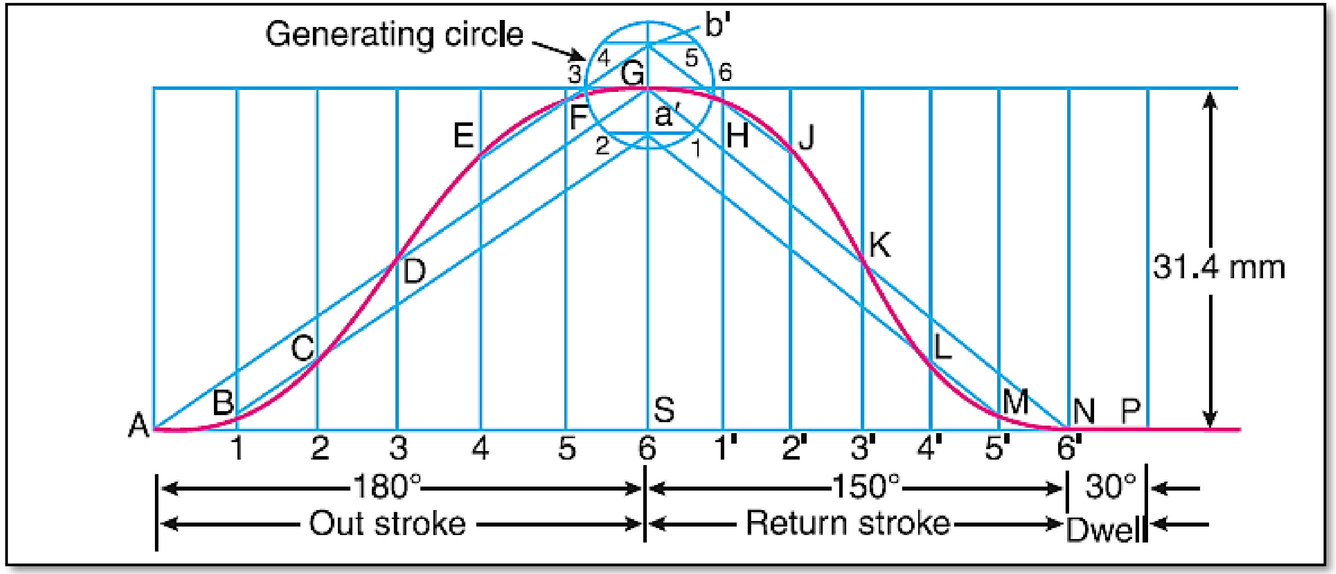
4. PROBLEM

Draw the profile of the cam when the roller follower moves with cycloidal motion during out stroke and return stroke, as given below :

1. *Out stroke with maximum displacement of 31.4 mm during 180° of cam rotation,*
2. *Return stroke for the next 150° of cam rotation,*
3. *Dwell for the remaining 30° of cam rotation.*

The minimum radius of the cam is 15 mm and the roller diameter of the follower is 10 mm. The axis of the roller follower is offset by 10 mm towards right from the axis of cam shaft.

Solution



Construction

First of all, the displacement diagram, as shown in Fig., is drawn as discussed in the following steps :

1. Draw horizontal line *ASP* such that *AS* = 180° to represent the out stroke, *SN* = 150° to represent the return stroke an *NP* = 30° to represent the dwell period.

2. Divide *AS* and *SN* into any number of even equal parts (say six).

3. From the points 1, 2, 3 . . . etc. draw vertical lines and set-off equal to the stroke of the follower.

4. From a point *G* draw a generating circle of radius,

$$r = \frac{\text{Stroke}}{2\pi} = \frac{31.4}{2\pi} = 5 \text{ mm}$$

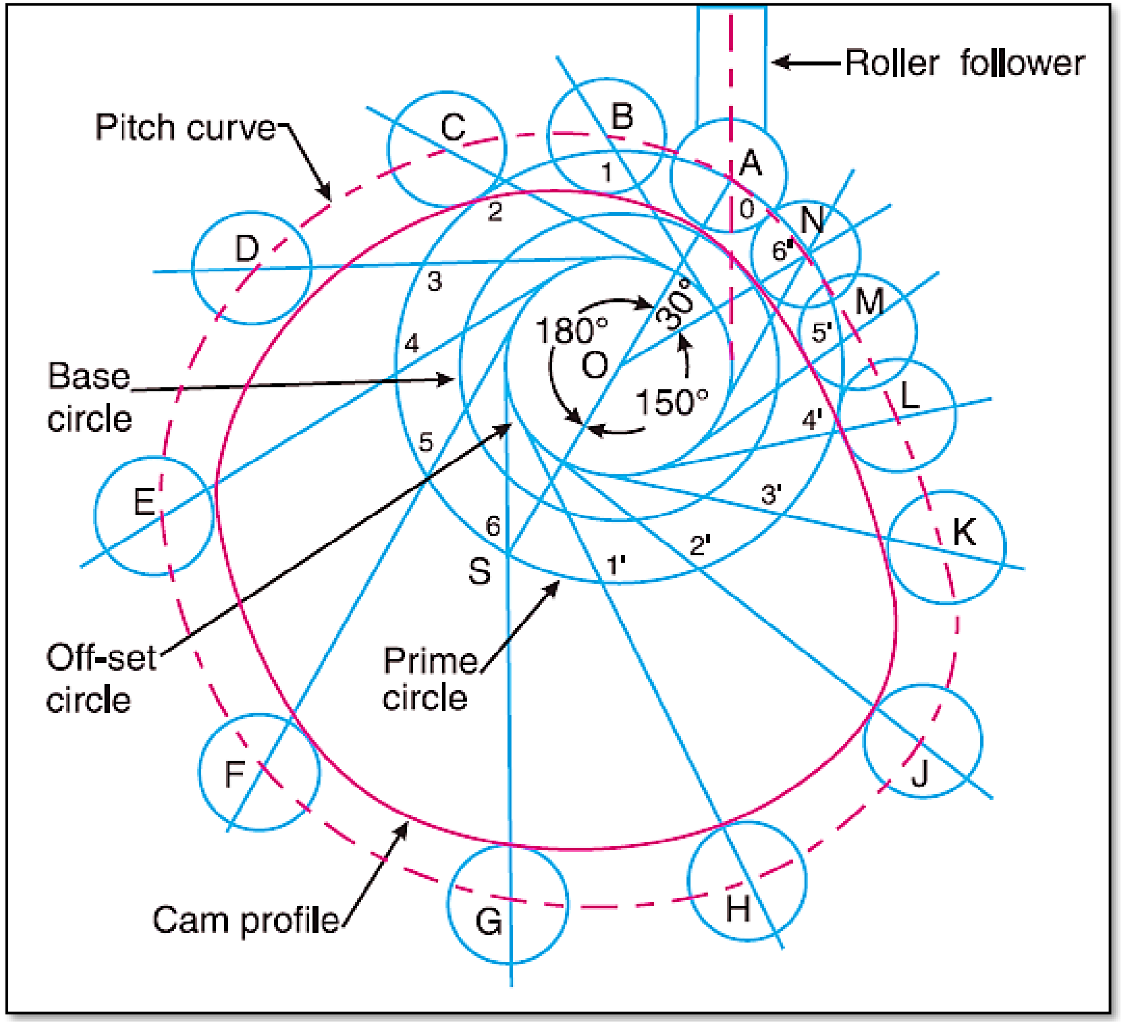
5. Divide the generating circle into six equal parts and from these points draw horizontal lines to meet the vertical diameter at *a′* , *G* and *b′* .

6. Join *AG* and *GN*. From point *a′* , draw lines parallel to *AG* and *GN* to intersect the vertical lines drawn through 1, 2, 4′ and 5′ at *B*, *C*, *L* and *M* respectively. Similarly draw parallel lines from *b′* intersecting the vertical lines through 4, 5, 1′ and 2′ at *E*, *F*, *H* and *J* respectively.

7. Join the points *A*, *B*, *C* . . . *L*, *M*, *N* with a smooth curve.

8. The curve *A B C* . . . *L M N* is the required displacement diagram.

Solution



1. Draw a base circle with centre *O* and radius equal to the least radius of cam (*i.e.* 15 mm)

2. Draw a prime circle with centre *O* and radius, *OA* = Least radius of cam + radius of roller = 15 + 5/2 = 17.5 mm

3. Draw an offset circle with centre *O* and radius equal to 10 mm

4. Join *OA*. From *OA* draw angular displacements of the cam, *i.e.* draw angle *AOS* = 180° to represent out stroke or lift of the follower, angle *SON* = 150° to represent return stroke or fall, angle *AON* = 30° to represent dwell of the follower

5. Divide the angular displacements during lift and fall (*i.e.* angle *AOS* and *SON*) into the same number of equal even parts (*i.e.* six parts) as in the displacement diagram

6. From points 1, 2, 3 . . . etc. and 0′, 1′, 2′, 3′ . . . etc. on the prime circle, draw tangents to the off-set circle

8. Set off 1*B*, 2*C*, 3*D* . . . etc. equal to the displacements as measured from the displacement diagram

9. By joining the points *A*, *B*, *C* . . . *M*, *N*, *P* with a smooth curve, we get a pitch curve

10. Now with *A*, *B*, *C* . . . etc. as centre, draw circles with radius equal to the radius of roller.

11. Join the bottoms of the circles with a smooth curve as shown in Fig. This is the required profile of the cam

Solve by Yourself

Book: Theory of Machine by R S Khurmi
Chapter 20

Example: 20.3, 20.4, 20.7, 20.8, 20.9, 20.12


BE LESS CURIOUS ABOUT PEOPLE
AND MORE CURIOUS ABOUT IDEAS.
- Marie Curie