AHSANULLAH UNIVERSITY OF SCIENCE AND TECHNOLOGY (AUST)

ME-3105: FLUID MECHANICS-II
(LEC-2: UNDISTORTED MODEL ANALYSIS)

BY

Dr. FAZLAR RAHMAN
Associate Professor, MPE, AUST

Model Analysis or Modeling and Similitude

Definition:

- ✓ The scientific method to predict the performance of hydraulic structure and machines by fabricating or preparing a model of main structures or machines and testing it the laboratory, this procedure is known as model analysis.
- ✓ Developing and designing of model of the prototype (main structures/machines) will be so that both model and prototype will behave or work in a similar fashion, such as shape and geometry.
- ✓ Performance of the prototype can be predicted by testing the model.

Model Analysis or Modeling and Similitude

Model:

- ✓ A model is a representation of a physical system (structure or machine) used to predict the behavior of the physical system in some desired respect. It is also known as scale model or simply model.
- ✓ Mathematical or computer models also need conform to this definition.
- ✓ Our interest will be in physical model.

Prototype:

- ✓ A physical system or structure for which performance will be evaluated through a model study is called the prototype.

Difference Between Model and Prototype

- ✓ Models resemble to prototype with a different size or scale, which may work with different fluid and often operate under different conditions than prototype.
- ✓ Usually, a model is smaller than the prototype.
- ✓ Occasionally, size of the model may larger than the prototype, which provides easier to study. For example, large models is to study the motion of red blood cells.
- ✓ With a successful and valid model, it is possible to predict the behavior of the prototype under a certain set of conditions.
- ✓ It is essentials that model should be designed and tested properly, so the results may be interpreted correctly.

Advantages and Disadvantages of Model Analysis

Advantages:

- ✓ Behavior and working details (power, efficiency, lift, drag, etc.) of a hydraulic structure and machine can be easily predicted from its model or model analysis.
- ✓ Failure mode of prototype can be easily detected from laboratory result of model testing. It results in time and material saving.
- ✓ Most economical, accurate and safe design may be selected from the result of the model testing.
- ✓ Model testing can help to detect and rectify the defect.
- ✓ Performance of complex of hydraulic structures or machines can be easily predicted by model testing which results in total design cost, safe and reliable design.

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Advantages and Disadvantages of Model Analysis

Disadvantages:

- ✓ There is an inherent danger in the use of models because any undetected error in the model or test procedure of the model will lead to inaccurate prediction of performance of the prototype.

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Hydraulic Similarity or Similitude

Definition:

- ✓ There should be a complete similarity between the prototype and its scale model, this similarity is known as hydraulic similitude or hydraulic similarity.
- ✓ There are three types of hydraulic similarities:
  1) Geometric Similarity
  2) Kinematic Similarity
  3) Dynamic Similarity.

Hydraulic Similarity or Similitude

Geometric Similarity:

- ✓ Model and prototype must have same shape. Linear dimensions of the model and prototype are related with a constant scale factor.
- ✓ Assume, L= length; B = width and D = depth ; (p = data for prototype and m = data for model.)
- ✓ Length, area, and volume are related through constant scale factor.

$$L_r = \frac{L_p}{L_m} = \frac{B_p}{B_m} = \frac{D_p}{D_m}$$

$$A_r = \left(\frac{L_p}{L_m}\right)^2 = \left(\frac{B_p}{B_m}\right)^2 = \left(\frac{D_p}{D_m}\right)^2 = (L_r)^2 \qquad V_r = \left(\frac{L_p}{L_m}\right)^3 = \left(\frac{B_p}{B_m}\right)^3 = \left(\frac{D_p}{D_m}\right)^3 = (L_r)^3$$

Hydraulic Similarity or Similitude

Kinematic Similarity:

- ✓ Velocities at corresponding points on model and prototype differ only by a constant scale factor.

$$V_r = \frac{V_{p1}}{V_{m1}} = \frac{V_{p2}}{V_{m2}} = \frac{V_{p3}}{V_{m3}}$$

Dynamic Similarity:

- ✓ Forces at corresponding points on model and prototype differ only by a constant scale factor.

$$F_r = \frac{F_{p1}}{F_{m1}} = \frac{F_{p2}}{F_{m2}} = \frac{F_{p3}}{F_{m3}}$$

- ✓ In a general flow field, complete similarity between a model and prototype is achieved only when there is geometric, kinematic, and dynamic similarity exist.

Procedure for Model Analysis

The general procedure of model analysis involves the following steps:

- ✓ **Selection of a suitable scale for the model**: Depends on type of structures, fluid type, time and desired results.
- ✓ **Construction of the model:** Selection of suitable material for the model and built model precisely with accurate shape, geometry, and scale factor.
- ✓ **Testing of the model:** Two types of test methods are normally use for model testing: Wind tunnel and Water Tunnel method.
- ✓ **Correct prediction:** After obtaining the precise measurements and data of a model in the laboratory, the next step is to correct the prediction of performance and behavior of the prototype.

Theory of Model Analysis

- ✓ The theory of models can be readily developed by using the principles of dimensional analysis.
- ✓ A problem can be described with a set of (pi) terms,

$$\Pi_1 = \phi(\Pi_2, \Pi_3, , , \Pi_n)$$

- ✓ Similar relationship can be written for a model and prototype; where they (both) must have same phenomenon,

$$\Pi_{1m} = \phi(\Pi_{2m}, \Pi_{3m}, , , , \Pi_{nm})$$

- ✓ Model is designed and operated under the following conditions (called design conditions, also called similarity requirements or modelling laws)

$$\Pi_2 = \Pi_{2m} \qquad \Pi_3 = \Pi_{3m} \ldots\ldots \Pi_n = \Pi_{nm}$$

- ✓ The measured of Π_{1m} obtained with the model will be equal to the corresponding Π_1 for the prototype.

$$\Pi_1 = \Pi_{1m}$$ Called prediction equation

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An Example of Model Analysis

Example-1:

Determining the drag force on a thin rectangular plate ($w \times h$ in size)

$$D = f(w, h, \mu, \rho, V) \longrightarrow \frac{D}{w^2 \rho V^2} = \Phi\left(\frac{w}{h}, \frac{\rho V w}{\mu}\right)$$

The prototype and the model must have the same phenomenon.

$$\frac{D_m}{{w_m}^2 \rho_m {V_m}^2} = \Phi\left(\frac{w_m}{h_m}, \frac{\rho_m V_m w_m}{\mu_m}\right) \qquad \left(\frac{D}{w^2 \rho V^2}\right)_{prototype} = \Phi\left(\frac{w}{h}, \frac{\rho V w}{\mu}\right)_{prototype}$$

$$\frac{w_m}{h_m} = \left(\frac{w}{h}\right)_{prototype} \qquad \frac{\rho_m V_m w_m}{\mu_m} = \left(\frac{\rho V w}{\mu}\right)_{prototype}$$

$$\left(\frac{D}{w^2 \rho V^2}\right)_{prototype} = \frac{D_m}{{w_m}^2 \rho_m {V_m}^2}$$

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An Example of Model Analysis

Example-2:

Considering the drag force on a sphere.

$$F = f(D, V, \rho, \mu) \longrightarrow \frac{F}{\rho V^2 D^2} = f_1\left(\frac{\rho V D}{\mu}\right)$$

The prototype and the model must have the same phenomenon.

$$\frac{F_m}{\rho_m V_m{}^2 D_m{}^2} = f_1\left(\frac{\rho_m V_m D_m}{\mu_m}\right) \qquad \frac{F}{\rho V^2 D^2} = f_1\left(\frac{\rho V D}{\mu}\right)_{prototype}$$

Design conditions. $\left(\frac{\rho V D}{\mu}\right)_{model} = \left(\frac{\rho V D}{\mu}\right)_{prototype}$

Then … $\left(\frac{F}{\rho V^2 D^2}\right)_{model} = \left(\frac{F}{\rho V^2 D^2}\right)_{prototype}$

Classification of Model

✓ All the hydraulic models are classified into two types:

- Undistorted model, and
- Distorted models.

Undistorted Models:

A model, which is geometrically similar to the prototype (i.e. having geometric similarity in length, breadth, height and head of water etc.) is known as an undistorted model.

The prediction of an undistorted model is comparatively easy and some of the results obtained from the models can be easily transferred to the prototypes.

Distorted Models:

Sometimes, a model does not have complete geometrical similarity with its prototype such a model is called distorted model.

Classification of Model

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Undistorted Models:

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Distorted Models:

- ✓ Sometimes, a model does not have complete geometrical similarity with prototype such a model is called distorted model.

Comparison of Undistorted Model and Prototype

- ✓ Undistorted model of prototype is made based on the view of geometric similarity only, and remaining similarities are then compared with the scale ratio (geometric ratio of the prototype and the model).
- ✓ Though the given scale ratio provides a wide range of data of the prototype, but the following are important:
  - Velocity of water in prototype, for given velocity at corresponding point of the model.
  - Discharge of prototype, for given discharge of model.
  - Force of resistance on prototype, for the given force resistance on model.
  - Speed of the prototype, for the given speed of the model.

Comparison of Undistorted Model and Prototype

- ✓ Power developed by the prototype for the given power developed by the model.
- ✓ Time of emptying a prototype for the given time of emptying of the model.
- ✓ Travelling time of a prototype for the given travelling time of the model.

Geometric Ratio of Model and Prototype

Scale Ratio:

It is the ratio of linear dimension of the model to the corresponding linear dimension of the prototype.

$$\text{Scale ratio} = \frac{1}{s} = \frac{\text{Liner dimension of the model}}{\text{Linear dimension of the prototype}}$$

e.g. if the ratio of model to the prototype is 1:10 then,

$$\text{Scale ratio} = \frac{1}{s} = \frac{1}{10} \qquad s = 10$$

Velocity of Water in Prototype and Undistorted Model:

Consider an undistorted model geometrically similar to a proposed prototype like weir, dam, spill way etc.

h_m = Head of water over the model

v_m = Velocity of water at a point in the model

h_p = Head of water over the prototype

v_p = Velocity of water at the corresponding point in the prototype

$\frac{1}{s}$ = Scale ratio of the model to the prototype

We know velocity of water in the model, $v_m = c_{vm}\sqrt{2gh_m}$

Similarly, velocity of water in the prototype, $v_p = c_{vp}\sqrt{2gh_p}$

where c_{vm} and c_{vp} are the coefficient of discharge or flow coefficient for the model and prototype respectively.

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Velocity of Fluid Water in Prototype and Undistorted Model

Consider an undistorted model geometrically similar to a proposed prototype like weir, dam, spill way etc.

$$\frac{v_m}{v_p} = \frac{c_{vm}\cdot\sqrt{2gh_m}}{c_{vp}\cdot\sqrt{2gh_p}}$$

$$= \sqrt{\frac{h_m}{h_p}} = \frac{1}{\sqrt{s}}$$

(taking $c_{vm} = c_{vp}$)

$$v_p = v_m\sqrt{s}$$

Scale ratio model to protype

$$\frac{1}{s} = \frac{h_m}{h_p}$$

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Flow Rate (Q) in Prototype and Undistorted Model

Let A_m = Area of discharge of the model

V_m = Actual velocity of water in the model

Q_m = Discharge of the model

A_p = Area of discharge of the model

V_p = Actual velocity of water in the model

Q_p = Discharge of the model

$\frac{1}{s}$ = Scale ratio of model to prototype

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Flow Rate in Prototype and Undistorted Model

Discharge through the model and prototype,

$$Q_m = A_m \cdot V_m \quad \text{and} \quad Q_p = A_p \cdot V_p$$

$$\frac{Q_m}{Q_p} = \left(\frac{A_m}{A_p}\right) \cdot \left(\frac{V_m}{V_p}\right) \qquad \frac{A_m}{A_p} = \frac{1}{s^2} \text{ and } \frac{V_m}{V_p} = \frac{1}{\sqrt{s}}$$

$$\frac{Q_m}{Q_p} = \frac{1}{s^2} \cdot \frac{1}{\sqrt{s}} = \frac{1}{s^{2.5}} \qquad Q_p = Q_m \times s^{2.5}$$

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Force/Resistance in Prototype and Undistorted Model

Let A_m = Area of model subjected to resistance

V_m = Velocity of the model

A_p = Area of prototype subjected to resistancel

V_p = Velocity of the prototype

$\frac{1}{s}$ = Scale ratio of model to prototype

Force/Resistance in Prototype and Undistorted Model

Force on the model,

$$f_m = \text{Mass} \times \text{Acceleration}$$
$$= (\text{Volume} \times \text{Density}) \times \frac{\text{Velocity}}{\text{Time}}$$
$$= \frac{\text{Volume}}{\text{Time}} \times \text{Density} \times \text{Velocity}$$
$$= (\text{Area} \times \text{Velocity}) \times \text{density} \times \text{Velocity}$$

$$f_m = \text{Area} \times \text{density} \times \text{Velocity}^2 = A_m \times \rho_m \times V_m^2$$

Similarly force on prototype, $f_p = A_p \times \rho_p \times V_p^2$

Force/Resistance in Prototype and Undistorted Model

$$f_m = \text{Area} \times \text{density} \times \text{Velocity}^2 = A_m \times \rho_m \times V_m^{\,2}$$

Similarly force on prototype, $f_p = A_p \times \rho_p \times V_p^{\,2}$

$$\frac{f_m}{f_p} = \left(\frac{A_m}{A_p}\right)\left(\frac{\rho_m}{\rho_p}\right)\left(\frac{V_m}{V_p}\right)^2 = \left(\frac{\rho_m}{\rho_p}\right)\left(\frac{1}{s^2}\right)\left(\frac{1}{\sqrt{s}}\right)^2 = \left(\frac{\rho_m}{\rho_p}\right)\left(\frac{1}{s^3}\right)$$

If model is tested in same fluid of prototype then $\rho_m = \rho_p$ and $f_p = f_m \times s^3$

If model is tested in different fluid of prototype then,

$$f_p = f_m \times \left(\frac{\rho_p}{\rho_m}\right) \times s^3$$

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Speed in Prototype and Undistorted Model

D_m = Diameter of the model impeller

v_m = Tangential velocity of the model impeller

N_m = Speed of the model runner

D_p = Diameter of the of the prototype impeller

v_p = Tangential velocity of the model impeller

N_p = Speed of the prototype runner

$\frac{1}{s}$ = Scale ratio of the model to the prototype

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Speed in Prototype and Undistorted Model

We know tangential velocity of model impeller,

$$v_m = \frac{\pi D_m N_m}{60} \quad \text{and} \quad N_m = \frac{60\, v_m}{\pi D_m}$$

Similarly, speed of runner of the prototype,

$$N_p = \frac{60\, v_p}{\pi D_p}$$

$$\frac{N_m}{N_p} = \left(\frac{D_p}{D_m}\right)\cdot\left(\frac{v_m}{v_p}\right); \quad \frac{D_p}{D_m} = s \ \text{and} \ \frac{v_m}{v_p} = \frac{1}{\sqrt{s}}$$

$$\frac{N_m}{N_p} = s\cdot\frac{1}{\sqrt{s}} = \sqrt{s} \qquad N_p = N_m\cdot\frac{1}{\sqrt{s}}$$

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Power in Prototype and Undistorted Model

Let Q_m = Discharge of the model in $\frac{m^3}{s}$

h_m = Head of water over the model in meters

P_m = Power developed by the model

Q_p = Discharge of the prototype in $\frac{m^3}{s}$

h_p = Head of water over the prototype in meters

P_p = Power developed by the prototype

$\frac{1}{s}$ = Scale ratio of the model to the prototype

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Power in Prototype and Undistorted Model

Power developed by the model, $P_m = \rho g Q_m h_m$

Power developed by the prototype, $P_p = \rho g Q_p h_p$

$$\frac{P_m}{P_p} = \frac{\rho_m g Q_m h_m}{\rho_p g Q_p h_p} = \left(\frac{Q_m}{Q_p}\right)\cdot\left(\frac{h_m}{h_p}\right)\cdot\left(\frac{\rho_m}{\rho_p}\right)$$

$$\frac{Q_m}{Q_p} = \frac{1}{s^{2.5}} \quad \text{and} \quad \frac{h_m}{h_p} = \frac{1}{s}$$

$$\frac{P_m}{P_p} = \frac{1}{s^{2.5}} \times \frac{1}{s}\cdot\left(\frac{\rho_m}{\rho_p}\right) \qquad \text{if } \rho_m = \rho_p \text{ then, } P_p = P_m \times s^{3.5}$$

$$\text{if } \rho_m \neq \rho_p \text{ then } P_p = P_m\left(\frac{\rho_p}{\rho_m}\right) \times s^{3.5}$$

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Time of Emptying a Prototype and Model

Let V_m = Total volume of water in the model,
Q_m = Rate of discharge of water in the model,
V_p = Total volume of water in the prototype,
Q_p = Rate of discharge of water in the prototype,
$\frac{1}{s}$ = Scale ratio of the model to the prototype.

Time of emptying the model,

$$t_m = \frac{\text{Total volume}}{\text{Rate of Discharge}} = \frac{V_m}{Q_m}$$

Similarly time of emptying prototype, $t_p = \frac{V_p}{Q_p}$

$$\frac{t_m}{t_p} = \frac{\frac{V_m}{Q_m}}{\frac{V_p}{Q_p}} = \left(\frac{V_m}{V_p}\right)\cdot\left(\frac{Q_p}{Q_m}\right) \qquad \frac{V_m}{V_p} = \frac{1}{s^3} \text{ and } \frac{Q_m}{Q_p} = \frac{1}{s^{2.5}};$$

$$\frac{t_m}{t_p} = \left(\frac{1}{s^3}\right)\cdot s^{2.5} = \frac{1}{s^{0.5}} \qquad \text{and } t_p = t_m \times s^{0.5}$$

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Traveling of a Prototype and Model

Consider an undistorted model geometrically similar to a proposed prototype like ship and submarine, etc.

Let L_m = Distance travel by the model
v_m = Velocity of the model
L_p = Distance travel by the prototype
v_p = velocity of the prototype
$\frac{1}{s}$ = Scale ratio of the model to prototype

Time taken by the model to travel a distance, $t_m = \frac{L_m}{v_m}$ and time taken by the prototype to travel same distance, $t_p = \frac{L_p}{v_p}$

$$\frac{t_m}{t_p} = \left(\frac{L_m}{L_p}\right)\left(\frac{v_p}{v_m}\right); \quad \frac{v_p}{v_m} = \sqrt{s} \text{ and } \frac{L_m}{L_p} = \frac{1}{s}$$

$$\frac{t_m}{t_p} = \frac{1}{s}\cdot\sqrt{s} \qquad t_p = \sqrt{s} \times t_m$$

Problem-1:

A model of ship of 1:12 scale ratio is tested in fresh water for the prediction of its performance. Find the ratio of speeds of the model to the speed of the prototype operating in sea water for geometrically similar free surface condition. Also calculate the ratio of the powers of the model with that of prototype. Assume mass density of fresh water is 1000 kg/m^3 and sea water is 1030 kg/m^3. **(SEE HAND ANALYIS)**

FORMULA:

$$\frac{1}{s} = \frac{L_m}{L_p} = \frac{w_m}{w_p} = \frac{h_m}{h_p}; \quad \frac{v_m}{v_p} = \frac{1}{\sqrt{s}}; \quad \frac{Q_m}{Q_p} = \frac{1}{s^{2.5}}$$

$$\frac{f_m}{f_p} = \left(\frac{\rho_m}{\rho_p}\right)\cdot\left(\frac{1}{s^3}\right); \quad \frac{N_m}{N_p} = \sqrt{s}; \quad \frac{P_m}{P_p} = \left(\frac{\rho_m}{\rho_p}\right)\cdot\left(\frac{1}{s^{3.5}}\right); \quad \frac{V_m}{V_p} = \frac{1}{s^3}$$

Where 'm' for model and 'p' for prototype; v = velocity or speed;
s = Model to prototype scale; Q = Rate of discharge; f = Force;
N = rpm; V = Volume; P = power; t_{me} = Emptying time;
t_{mt} = travelling time;

Problem-2:

A model of ship of 1:25 scale ratio is towing in water at a velocity of 1.2 m/s ; experience a resistance of 5.0 N. The wetted area of the model is 1.0 m^2.

Calculate for the prototype:

(a). Corresponding speed, and

(b). Drag, if the drag coefficient for model is 0.004 and for prototype is 0.016.

(SEE HAND ANALYSIS)