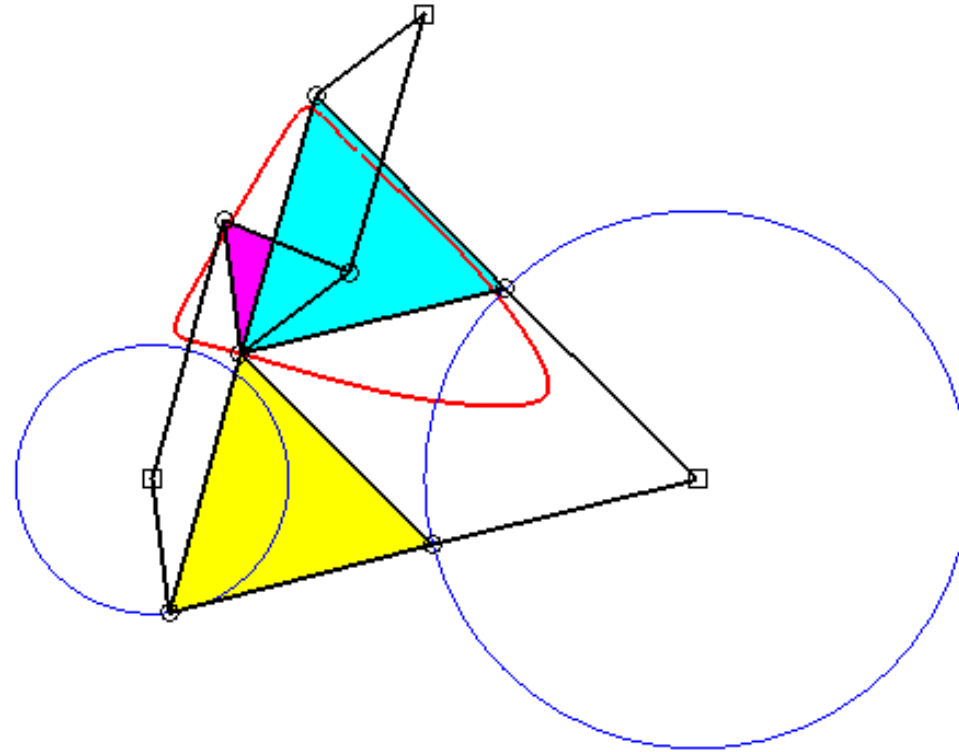
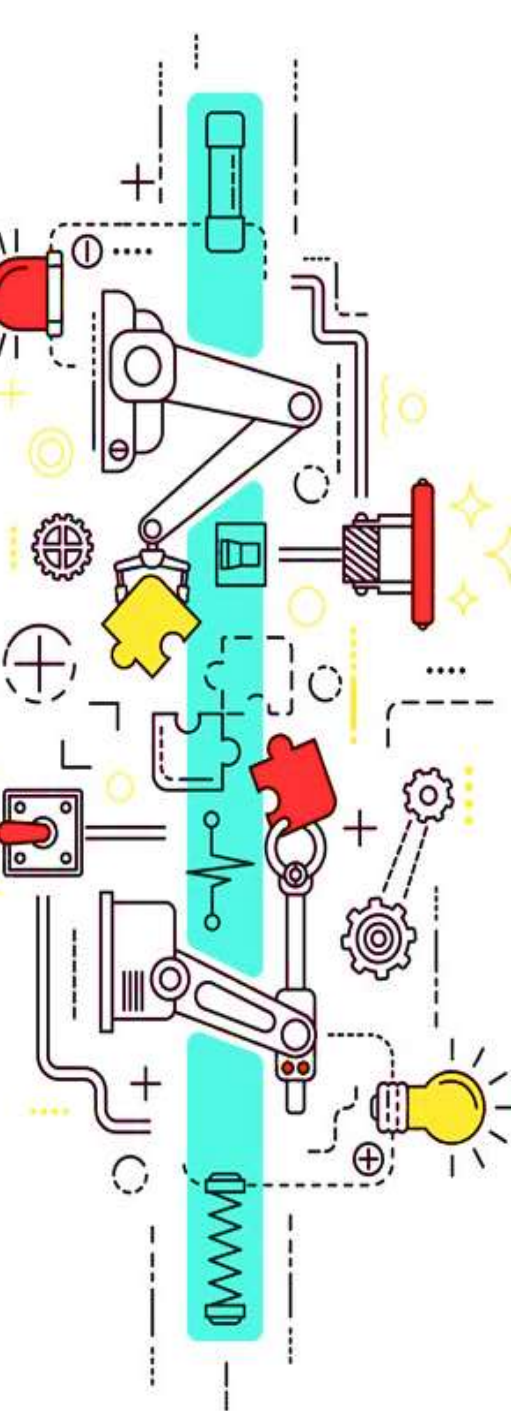


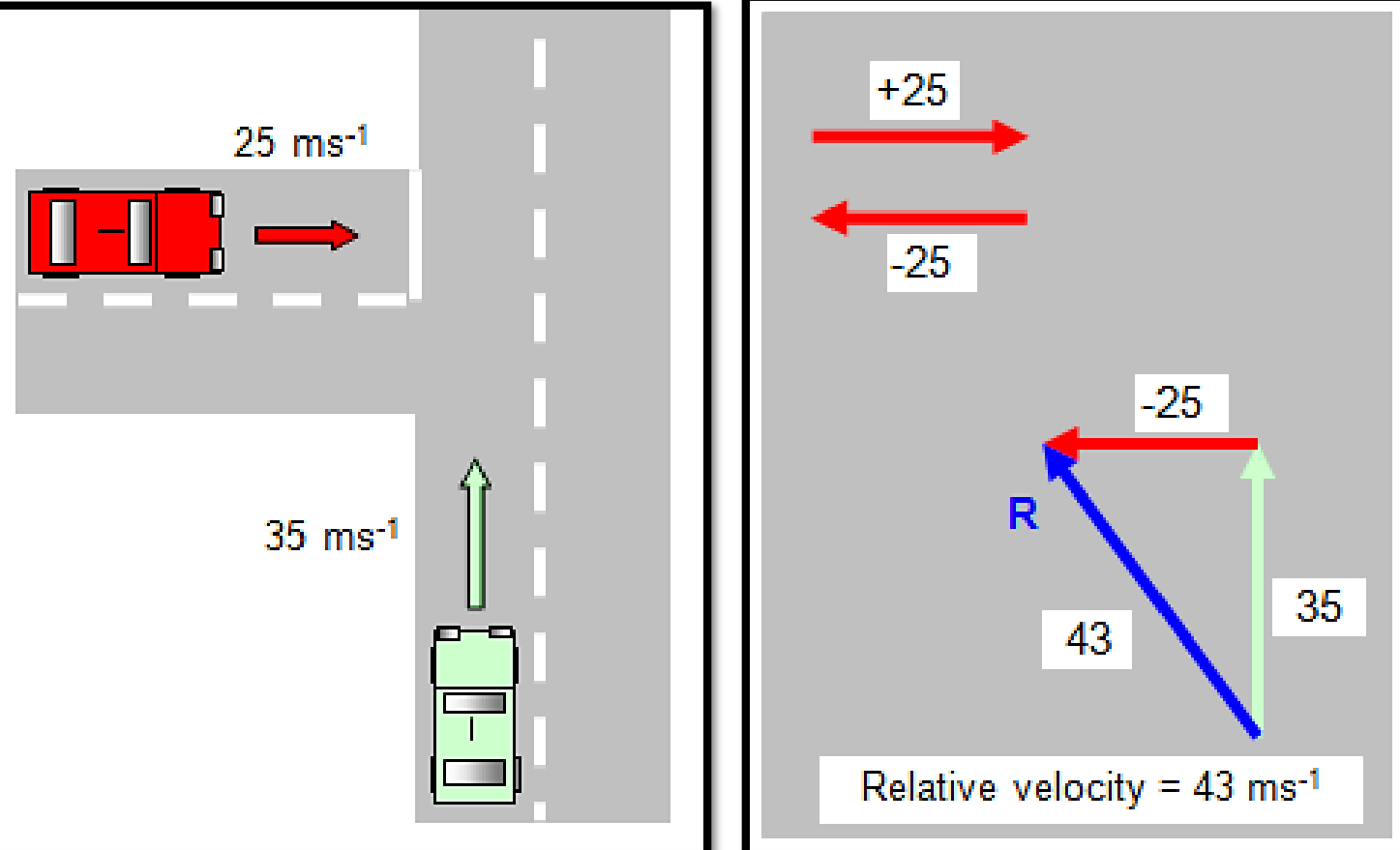
ME 3101: **Mechanics of Machinery**

Velocity & Acceleration Diagram



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RELATIVE VELOCITY



Relative velocity of Green car with respect to Red car = ?

V_A = Absolute velocity of A

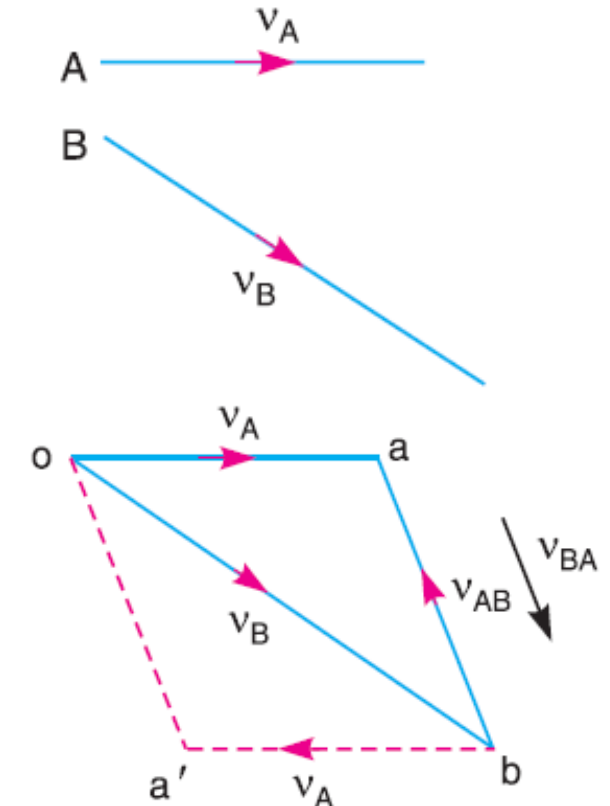
V_B = Absolute velocity of B

V_{AB} = Relative velocity of A with respect to B

V_{BA} = Relative velocity of B with respect to A

$$\overrightarrow{V_A} = \overrightarrow{V_B} + \overrightarrow{V_{AB}} \qquad \overrightarrow{V_B} = \overrightarrow{V_A} + \overrightarrow{V_{BA}}$$

$$\overline{oa} = \overline{ob} + \overline{ba} \qquad \overline{ob} = \overline{oa} + \overline{ab}$$



MOTION OF A LINK

Consider two points A and B on a rigid link AB , as shown in Fig. (a). Let one of the extremities (B) of the link move relative to A , in a clockwise direction. Since the distance from A to B remains the same, therefore there can be no relative motion between A and B , along the line AB . It is thus obvious, that the **relative motion of B with respect to A** must be **perpendicular to AB** .

Velocity of any point on a link with respect to another point on the same link is always perpendicular to the line joining these points on the configuration (or space) diagram

The relative velocity of B with respect to A (i.e. v_{BA}) is represented by the vector ab and is perpendicular to the line AB as shown in Fig. 7.3 (b).

Let ω = Angular velocity of the link AB about A .

We know that the velocity of the point B with respect to A ,

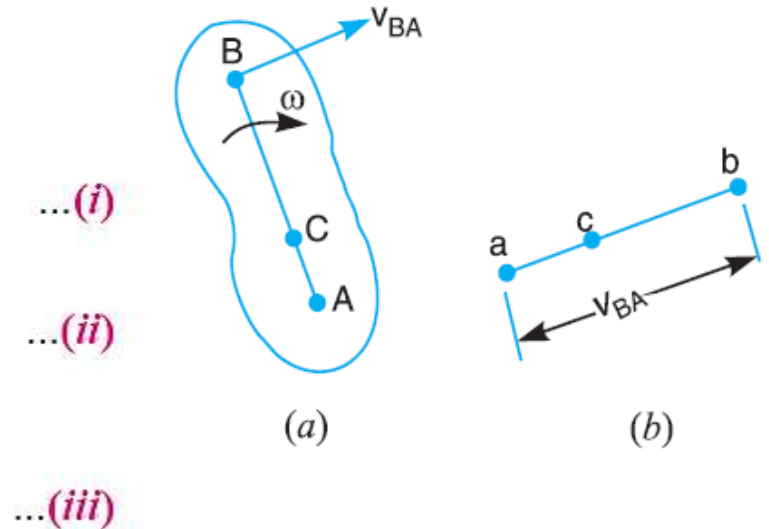
$$v_{BA} = \overline{ab} = \omega \cdot AB$$

Similarly, the velocity of any point C on AB with respect to A ,

$$v_{CA} = \overline{ac} = \omega \cdot AC$$

From equations (i) and (ii),

$$\frac{v_{CA}}{v_{BA}} = \frac{\overline{ac}}{\overline{ab}} = \frac{\omega \cdot AC}{\omega \cdot AB} = \frac{AC}{AB}$$



Thus, we see from equation (iii), that the point c on the vector ab divides it in the same ratio as C divides the link AB

VELOCITY OF A POINT ON A LINK BY RELATIVE VELOCITY METHOD

Steps to draw velocity diagram

1. Take some convenient point o , known as the **pole**
2. Through o , draw oa parallel and equal to v_A , to some suitable scale
3. Through a , draw a line perpendicular to AB of Fig.(a). This line will represent the velocity of B with respect to A , i.e. v_{BA}
4. Through o , draw a line parallel to v_B intersecting the line of v_{BA} at b
5. Measure ob , which gives the required velocity of point B (v_B), to the scale

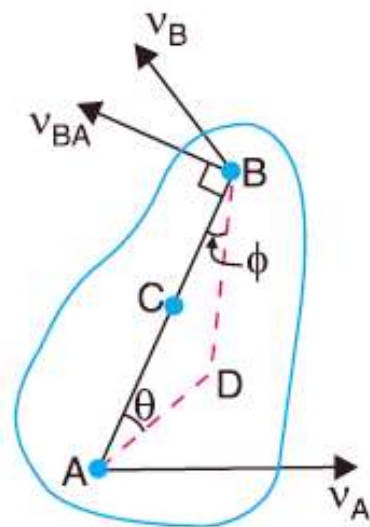
Two points A and B on a link

V_A = Absolute velocity of A
(magnitude & direction known)

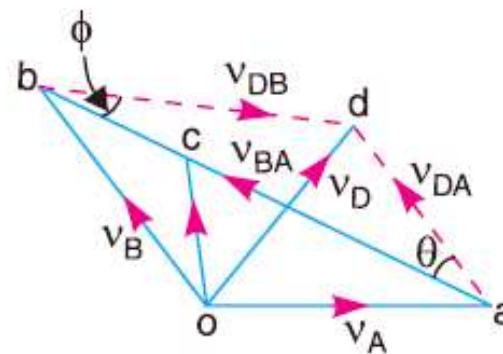
V_B = Absolute velocity of B
(only direction known)

V_{AB} = Relative velocity of A with respect to B

V_{BA} = Relative velocity of B with respect to A



(a) Motion of points on a link.



(b) Velocity diagram.

VELOCITY OF A POINT ON A LINK BY RELATIVE VELOCITY METHOD

Notes :

1. The vector ab which represents the velocity of B with respect to A (v_{BA}) is known as velocity of image of the link AB
2. The absolute velocity of any point C on AB may be determined by dividing vector ab at c in the same ratio as C divides AB in Fig.(a)

In other words,

$$\frac{ac}{ab} = \frac{AC}{AB}$$

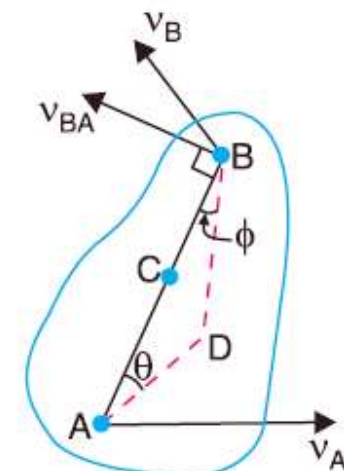
Join oc . The vector oc represents the absolute velocity of point C (v_C) and the vector ac represents the velocity of C with respect to A i.e. v_{CA} .

3. The absolute velocity of any other point D outside AB , as shown in Fig. (a), may also be obtained by completing the velocity triangle abd and similar to triangle ABD , as shown in Fig. (b)

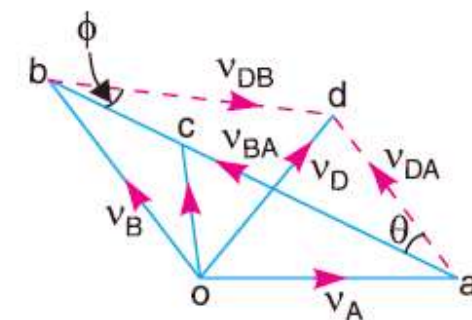
4. The angular velocity of the link AB may be found by dividing the relative velocity of B with respect to A (i.e. v_{BA}) to the length of the link AB

Mathematically, angular velocity of the link AB ,

$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{ab}{AB}$$

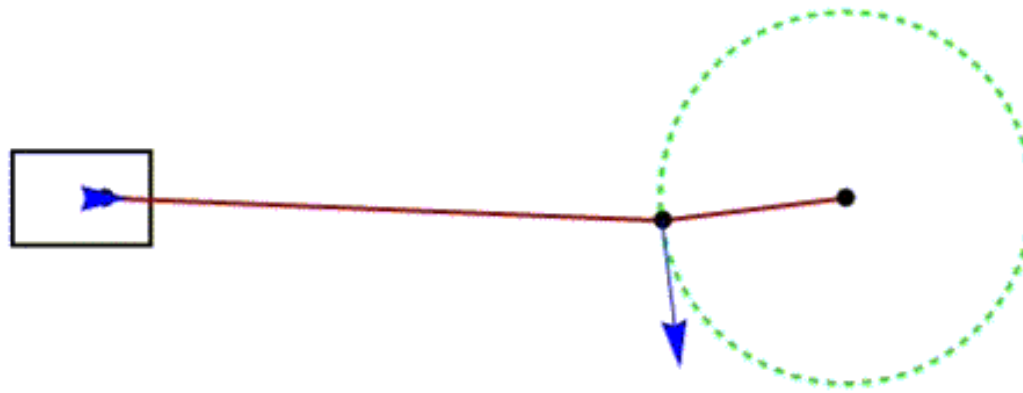


(a) Motion of points on a link.



(b) Velocity diagram.

VELOCITIES IN SLIDER CRANK MECHANISM



VELOCITIES IN SLIDER CRANK MECHANISM

Slider A is attached to the connecting rod AB ; Crank OB rotates in a clockwise direction, about the point O
Radius of crank $OB = r$ (m); Angular velocity of crank = ω rad/s; Velocity of $B = v_B$ (known in magnitude and direction)
Slider reciprocates along the line of stroke AO

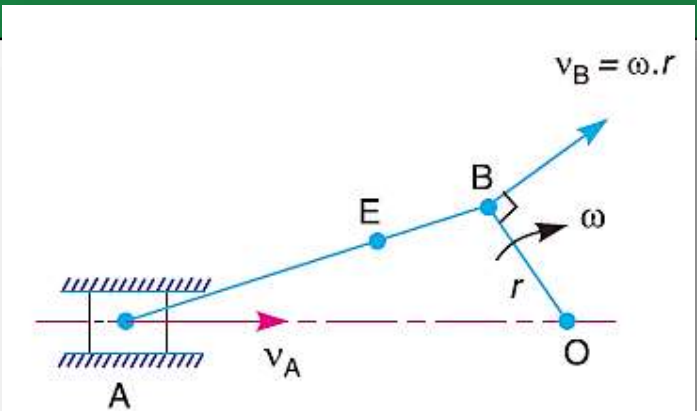
Steps to draw velocity diagram

1. From any point o , draw vector ob parallel to the direction of v_B (or perpendicular to OB) such that $ob = v_B = \omega \cdot r$, to some suitable scale, as shown in Fig. (b)
2. Since AB is a rigid link, therefore the velocity of A relative to B is perpendicular to AB . Now draw vector ba perpendicular to AB to represent the velocity of A with respect to B i.e. v_{AB}
3. From point o , draw vector oa parallel to the path of motion of the slider A (which is along AO only). The vectors ba and oa intersect at a . Now oa represents the velocity of the slider A i.e. v_A , to the scale

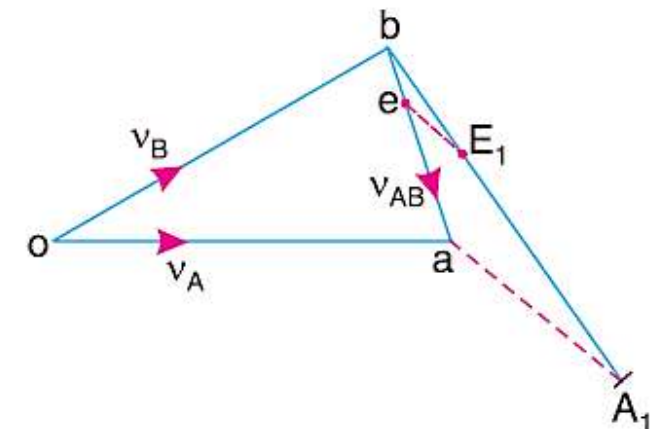
The angular velocity of the connecting rod AB (ω_{AB}) may be determined as follows:

1. The direction of vector ab (or ba) determines the sense of ω_{AB} which shows that it is anticlockwise

Note : The absolute velocity of any other point E on the connecting rod AB may also be found out by dividing vector ba such that $be/ba = BE/BA$. This is done by drawing any line bA_1 equal in length of BA . Mark $bE_1 = BE$. Join a A_1 . From E_1 draw a line E_1e parallel to aA_1 . The vector oe now represents the velocity of E and vector ae represents the velocity of E with respect to A .



(a) Slider crank mechanism.



(b) Velocity diagram.

RUBBING VELOCITY AT A PIN JOINT

- The links in a mechanism are **mostly connected** by means of **pin joints**
- **Rubbing velocity** at a pin joint refers to the relative velocity between the surfaces in contact, typically where one part (such as a pin) rotates or moves against another part (like a bearing or bushing)
- **Rubbing velocity: Algebraic sum between the angular velocities of the two links which are connected by pin joints, multiplied by the radius of the pin.**

Consider two links OA and OB connected by a pin joint at O as shown in Fig.

Let ω_1 = Angular velocity of the link OA or the angular velocity of the point A with respect to O .

ω_2 = Angular velocity of the link OB or the angular velocity of the point B with respect to O , and

r = Radius of the pin.

According to the definition,

Rubbing velocity at the pin joint O

= $(\omega_1 - \omega_2) r$, if the links move in the same direction

= $(\omega_1 + \omega_2) r$, if the links move in the opposite direction

Note :

When the pin connects one sliding member and the other turning member, the **angular velocity of the sliding member is zero**.

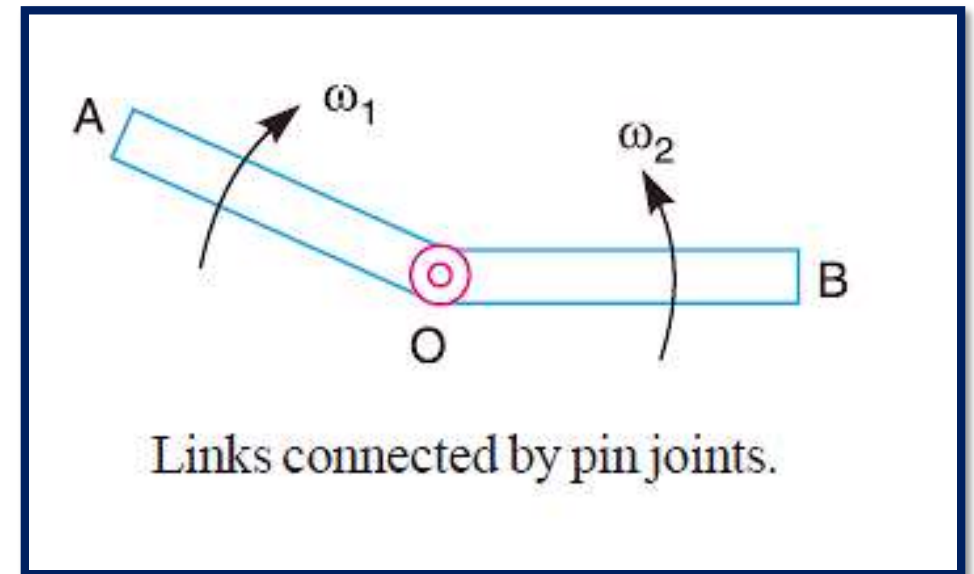
In such cases,

Rubbing velocity at the pin joint = $\omega \cdot r$

Where,

ω = Angular velocity of the turning member

r = Radius of the pin



FORCES ACTING IN A MECHANISM

Consider a mechanism of a four-bar chain, as shown in Fig. Let force F_A newton is acting at the joint A in the direction of the velocity of A (v_A m/s) which is perpendicular to the link DA . Suppose a force F_B newton is transmitted to the joint B in the direction of the velocity of B (i.e. v_B m/s) which is perpendicular to the link CB . If we neglect the effect of friction and the change of kinetic energy of the link (i.e.), (assuming the efficiency of transmission as 100%), then by the principle of conservation of energy,

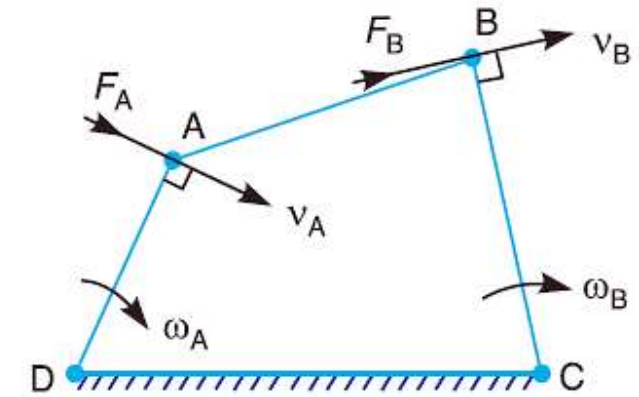
Input work per unit time = Output work per unit time

\therefore Work supplied to the joint A = Work transmitted by the joint B

$$F_A \cdot v_A = F_B \cdot v_B \quad \text{or} \quad F_B = \frac{F_A \cdot v_A}{v_B} \quad \dots (i)$$

If we consider the effect of friction and assuming the efficiency of transmission as η , then

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{F_B \cdot v_B}{F_A \cdot v_A} \quad \text{or} \quad F_B = \frac{\eta \cdot F_A \cdot v_A}{v_B} \quad \dots (ii)$$



Four bar mechanism.

Notes : 1. If the turning couples due to the forces F_A and F_B about D and C are denoted by T_A (known as driving torque) and T_B (known as resisting torque) respectively, then the equations (i) and (ii) may be written as

$$T_A \cdot \omega_A = T_B \cdot \omega_B, \quad \text{and} \quad \eta = \frac{T_B \cdot \omega_B}{T_A \cdot \omega_A} \quad \dots (iii)$$

Where, ω_A and ω_B are the angular velocities of the links DA and CB respectively

2. If the forces F_A and F_B do not act in the direction of the velocities of the points A and B respectively, then the component of the force in the direction of the velocity should be used in the above equations

MECHANICAL ADVANTAGE

- It is defined as the ratio of the load to the effort.
- In a four-bar mechanism, as shown in Fig., the link DA is called the driving link and the link CB as the driven link. The force F_A acting at A is the effort and the force F_B at B will be the load or the resistance to overcome. We know from the principle of conservation of energy, neglecting effect of friction,

$$F_A \times v_A = F_B \times v_B \text{ or } \frac{F_B}{F_A} = \frac{v_A}{v_B}$$

\therefore Ideal mechanical advantage,

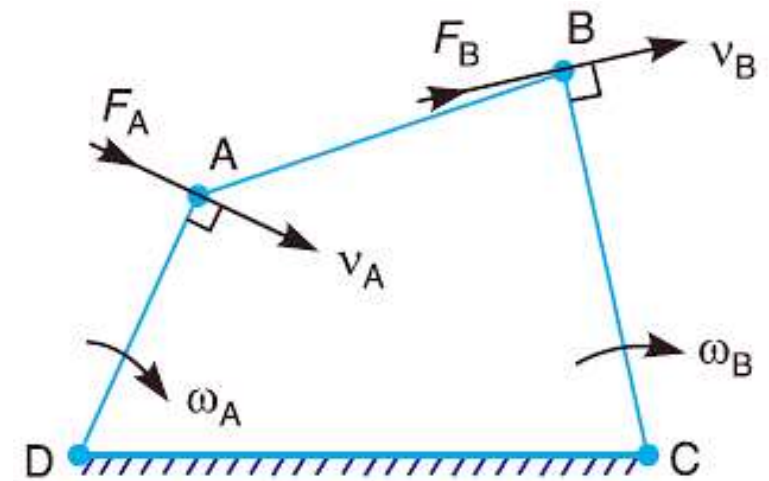
$$M.A._{(ideal)} = \frac{F_B}{F_A} = \frac{v_A}{v_B}$$

If we consider the effect of friction, less resistance will be overcome with the given effort. Therefore, the actual mechanical advantage will be less.

Let η = Efficiency of the mechanism.

\therefore Actual mechanical advantage,

$$M.A._{(actual)} = \eta \times \frac{F_B}{F_A} = \eta \times \frac{v_A}{v_B}$$



Four bar mechanism.

MECHANICAL ADVANTAGE

Note :

The mechanical advantage may also be defined as the ratio of output torque to the input torque.

Let T_A = Driving torque,

T_B = Resisting torque,

ω_A and ω_B = Angular velocity of the driving and driven links respectively.

∴ Ideal mechanical advantage,

$$M.A._{(ideal)} = \frac{T_B}{T_A} = \frac{\omega_A}{\omega_B}$$

... (Neglecting effect of friction)

and actual mechanical advantage,

$$M.A._{(actual)} = \eta \times \frac{T_B}{T_A} = \eta \times \frac{\omega_A}{\omega_B}$$

... (Considering the effect of friction)

ACCELERATION DIAGRAM FOR A LINK

Consider two points A and B on a rigid link as shown in Fig. (a). Let the point B moves with respect to A , with an angular velocity of ω rad/s and let α rad/s² be the angular acceleration of the link AB .

Acceleration of a particle whose velocity changes both in magnitude and direction at any instant has the following two components:

1. The **centripetal or radial component**, which is perpendicular to the velocity of the particle at the given instant.
2. The **tangential component**, which is parallel to the velocity of the particle at the given instant.

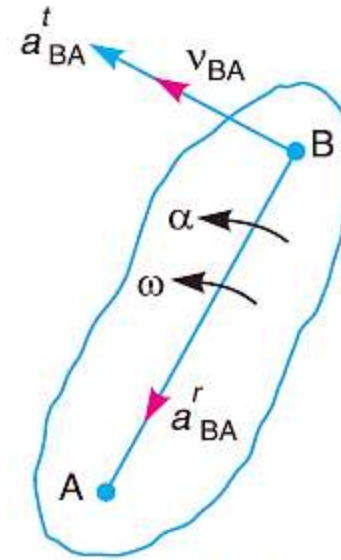
Thus, for a link AB , the velocity of point B with respect to A (i.e. v_{BA}) is perpendicular to the link AB as shown in Fig.(a). Since the point B moves with respect to A with an angular velocity of ω rad/s, therefore centripetal or radial component of the acceleration of B with respect to A ,

$$a_{BA}^r = \omega^2 \times \text{Length of link } AB = \omega^2 \times AB = v_{BA}^2 / AB$$

This radial component of acceleration acts perpendicular to the velocity v_{BA} , In other words, it acts **parallel** to the link AB . We know that tangential component of the acceleration of B with respect to A ,

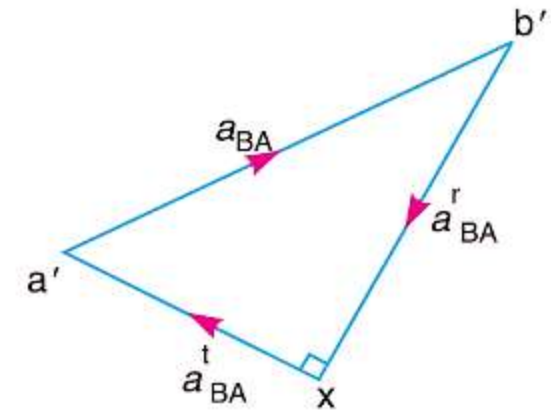
$$a_{BA}^t = \alpha \times \text{Length of the link } AB = \alpha \times AB$$

This tangential component of acceleration acts parallel to the velocity v_{BA} . In other words, it acts **perpendicular** to the link AB .



(a) Link.

$$\dots \left(\because \omega = \frac{v_{BA}}{AB} \right)$$

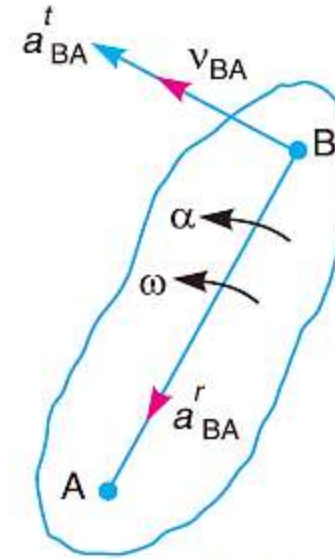


(b) Acceleration diagram.

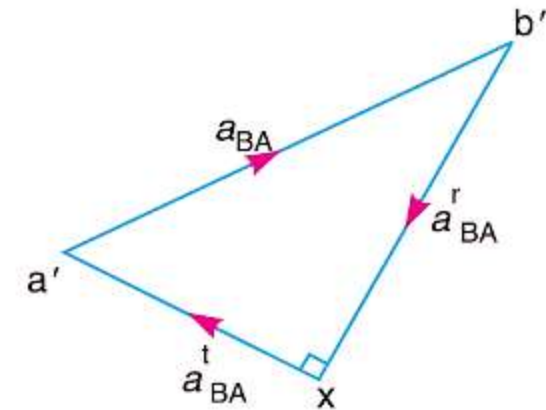
ACCELERATION DIAGRAM FOR A LINK

In order to draw the acceleration diagram for a link AB , as shown in Fig (b),

- From any point b' , draw vector $b'x$ parallel to BA to represent the radial component of acceleration of B with respect to A i.e. a_{BA}^r
- From point x draw vector xa' perpendicular to BA to represent the tangential component of acceleration of B with respect to A i.e. a_{BA}^t .
- **Join $b'a'$.**
- The vector $b'a'$ (known as **acceleration image** of the link AB) represents the total acceleration of B with respect to A (i.e. a_{BA}) and it is the vector sum of radial component a_{BA}^r and tangential component a_{BA}^t at of acceleration.



(a) Link.



(b) Acceleration diagram.

ACCELERATION OF A POINT ON A LINK

Consider two points A and B on the rigid link, as shown in Fig. (a). Let the acceleration of the point A i.e. \mathbf{a}_A is known in magnitude and direction and the direction of path of B is given.

The acceleration of the point B is determined in magnitude and direction by drawing the acceleration diagram as discussed below.

1. From any point o' , draw vector $o'a'$ parallel to the direction of absolute acceleration at point A i.e. \mathbf{a}_A , to some suitable scale, as shown in Fig.(b).

Consider two points A and B on the rigid link, as shown in Fig. 8.2 (a).

2. We know that the acceleration of B with respect to A i.e. \mathbf{a}_{BA} has the following two components:

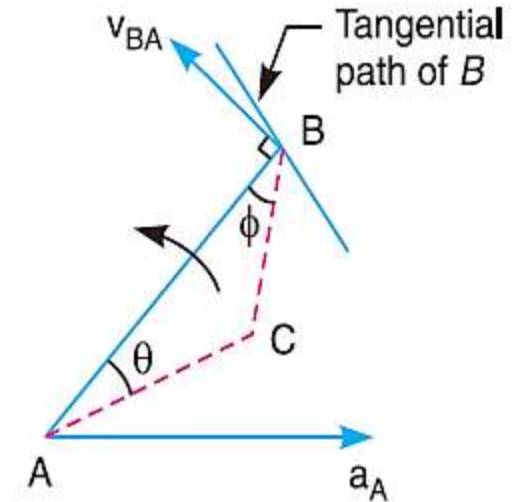
- (i) Radial component of the acceleration of B with respect to A i.e. \mathbf{a}_{BA}^r , and
- (ii) Tangential component of the acceleration B with respect to A i.e. \mathbf{a}_{BA}^t . These two components are mutually perpendicular.

3. Draw vector $a'x$ parallel to the link AB (because radial component of the acceleration of B with respect to A will pass through AB), such that

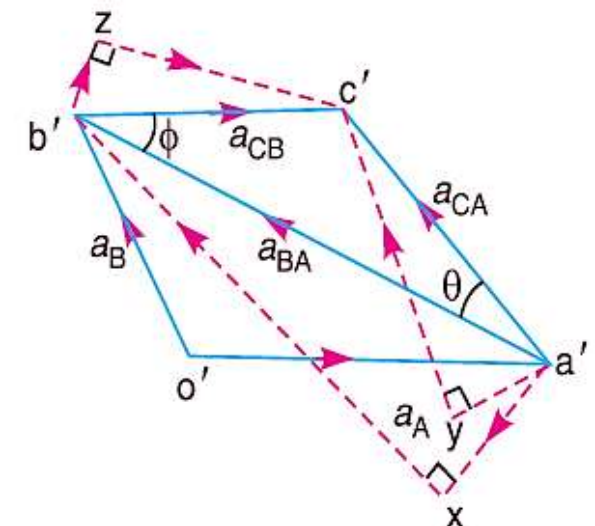
$$\text{vector } a'x = a_{BA}^r = v_{BA}^2 / AB$$

where v_{BA} = Velocity of B with respect to A .

Note: The value of v_{BA} may be obtained by drawing the velocity diagram



(a) Points on a Link.



(b) Acceleration diagram.

ACCELERATION OF A POINT ON A LINK

4. From point x , draw vector xb' perpendicular to AB or vector $a'x$ (because tangential component of B with respect to A i.e. \mathbf{a}_{BA}^t , is perpendicular to radial component \mathbf{a}_{BA}^r) and through o' draw a line parallel to the path of B to represent the absolute acceleration of B i.e. \mathbf{a}_B . The vectors xb' and $o'b'$ intersect at b' . Now the values of \mathbf{a}_B and \mathbf{a}_{BA}^t may be measured, to the scale

5. By joining the points, a' and b' we may determine the total acceleration of B with respect to A i.e. \mathbf{a}_{BA} . The vector $a'b'$ is known as **acceleration image** of the link AB

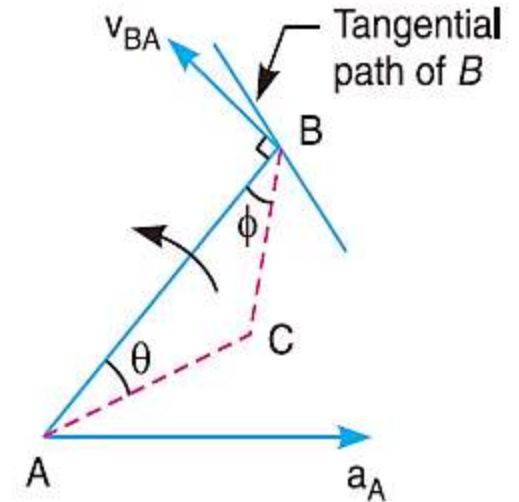
6. For any other point C on the link, draw triangle $a'b'c'$ similar to triangle ABC . Now vector $b'c'$ represents the acceleration of C with respect to B i.e. \mathbf{a}_{CB} , and vector $a'c'$ represents the acceleration of C with respect to A i.e. \mathbf{a}_{CA} . As discussed above, \mathbf{a}_{CB} and \mathbf{a}_{CA} will each have two components as follows :

(i) \mathbf{a}_{CB} has two components; \mathbf{a}'_{CB} and \mathbf{a}''_{CB} as shown by triangle $b'zc'$ in Fig.(b), in which $b'z$ is parallel to BC and zc' is perpendicular to $b'z$ or BC

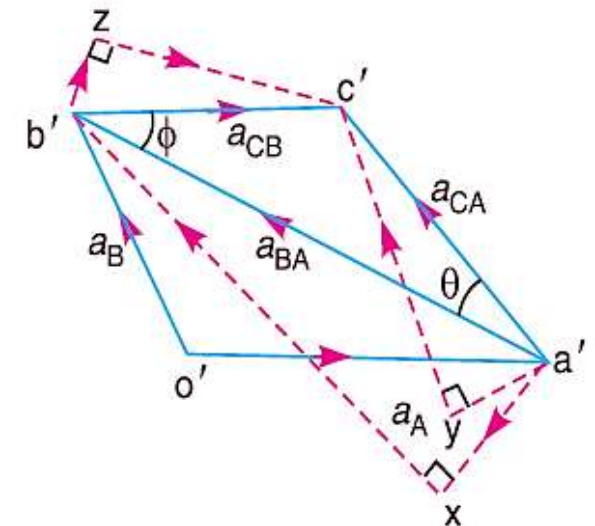
(ii) \mathbf{a}_{CA} has two components ; \mathbf{a}'_{CA} and \mathbf{a}^t_{CA} as shown by triangle $a'yc'$ in Fig.(b), in which $a'y$ is parallel to AC and yc' is perpendicular to $a'y$ or AC

7. The angular acceleration of the link AB is obtained by dividing the tangential components of the acceleration of B with respect to A (\mathbf{a}_{BA}^t) at to the length of the link. Mathematically, angular acceleration of the link AB,

$$\alpha_{AB} = a^t_{BA}/AB$$



(a) Points on a Link.



(b) Acceleration diagram.

ACCELERATION IN THE SLIDER CRANK MECHANISM

Important Points

- Point-1:** Typically, there will be two components of acceleration, radial and tangential, for any link of a mechanism
- Point-2:** The direction of the radial component of acceleration is always parallel to the orientation of the link and towards the **base point** (the point with respect to which relative velocity is calculated)
- Point-3:** The direction of the tangential component is always perpendicular to the orientation of the link
- Point-4:** In case the end of a link is moving at a constant angular velocity (i.e., a link hinged at one end and rotating at constant RPM with respect to the hinged point), the moving end will have only radial acceleration component and no tangential component.
- Point-5:** In case a joint moves along a straight line (i.e., cylinder), it won't have any radial acceleration component. The direction of the total acceleration of the joint, in such cases, will be parallel to the line of movement of the joint.
- Point-6:** If a link has two end points A and B and a length of AB then and the angular acceleration of the link is α_{AB} , the value of the radial and tangential acceleration for the link can be calculated by the following equation:

$$a^r_{AB} = v^2_{AB}/AB$$

$$a^t_{AB} = \alpha_{AB} \times AB$$

ACCELERATION IN THE SLIDER CRANK MECHANISM

A slider crank mechanism is shown in Fig.(a). Let the crank OB makes an angle θ with the inner dead centre (I.D.C.) and rotates in a clockwise direction about the fixed-point O with **uniform** angular velocity ω_{BO} rad/s.

\therefore Velocity of B with respect to O or velocity of B (because O is a fixed point),

$$v_{BO} = v_B = \omega_{BO} \times OB, \text{ acting tangentially at } B$$

We know that centripetal or radial acceleration of B with respect to O or acceleration of B (because O is a fixed point),

$$a_{BO}^r = a_B = \omega_{BO}^2 \times OB = v_{BO}^2 / OB$$

Note : A point at the end of a link which moves with constant angular velocity has no tangential component of acceleration.

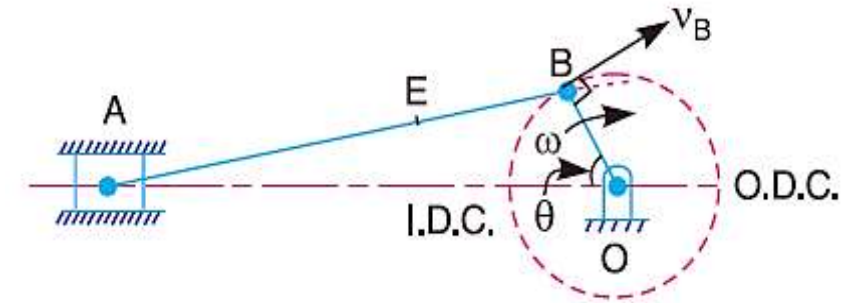
Steps to draw acceleration diagram

1. Draw vector $o'b'$ parallel to BO and set off equal in magnitude of $a_{BO}^r = a_B$, to some suitable scale.

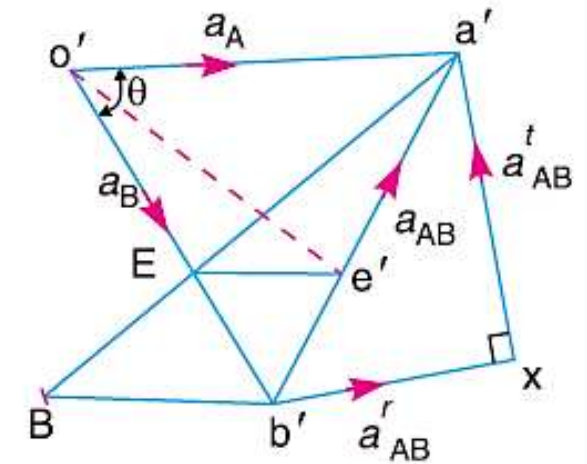
2. From point b' , draw vector $b'x$ parallel to BA . The vector $b'x$ represents the radial component of the acceleration of A with respect to B whose magnitude is given by :

$$a_{AB}^r = v_{AB}^2 / BA$$

Since the point B moves with constant angular velocity, therefore there will be **no tangential** component of the acceleration.



(a) Slider crank mechanism.



(b) Acceleration diagram.

ACCELERATION IN THE SLIDER CRANK MECHANISM

3. From point x , draw vector xa' perpendicular to $b'x$ (or AB). The vector xa' represents the tangential component of the acceleration of A with respect to B i.e. a_{AB}^t .

Note: When a point moves along a straight line, it has **no centripetal or radial** component of the acceleration.

4. Since the point A reciprocates along AO , therefore the acceleration must be parallel to velocity. Therefore, from o' , draw $o'a'$ parallel to AO , intersecting the vector xa' at a' . Now the acceleration of the piston or the slider A (a_A) and a_{AB}^t may be measured to the scale.

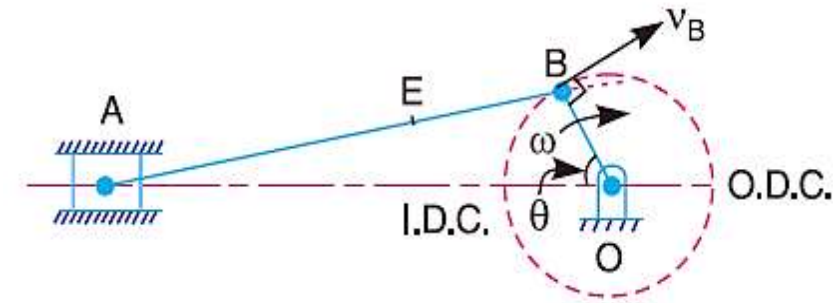
5. The vector $b'a'$, which is the sum of the vectors $b'x$ and xa' , represents the total acceleration of A with respect to B i.e. a_{AB} . The vector $b'a'$ represents the acceleration of the connecting rod AB .

6. The acceleration of any other point on AB such as E may be obtained by dividing the vector $b'a'$ at e' in the same ratio as E divides AB in Fig.(a). In other words

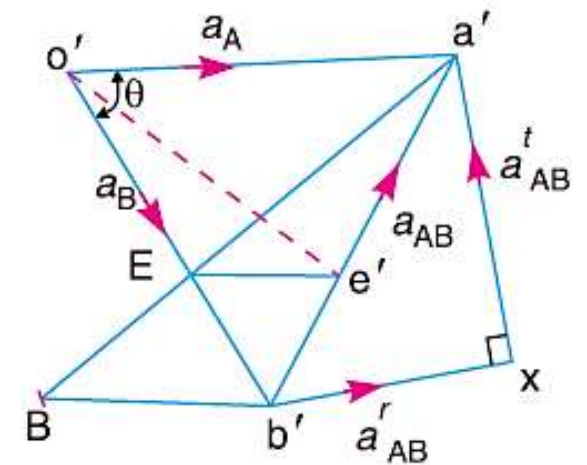
$$a'e' / a'b' = AE / AB$$

7. The angular acceleration of the connecting rod AB may be obtained by dividing the tangential component of the acceleration of A with respect to B (a_{AB}^t) to the length of AB . In other words, angular acceleration of AB ,

$$\alpha_{AB} = a_{AB}^t / AB \text{ (Clockwise about B)}$$



(a) Slider crank mechanism.



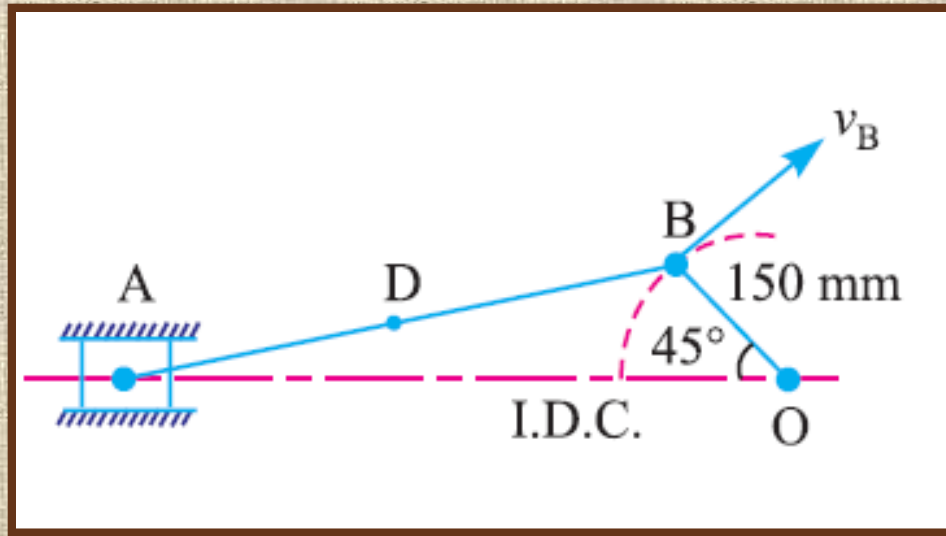
(b) Acceleration diagram.

MATH

The crank of a slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long.

Determine :

1. linear velocity and acceleration of the midpoint of the connecting rod, and
2. angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centre position.



$$v_D = 4.1 \text{ m/s}$$

$$a_D = 117 \text{ m/s}^2$$

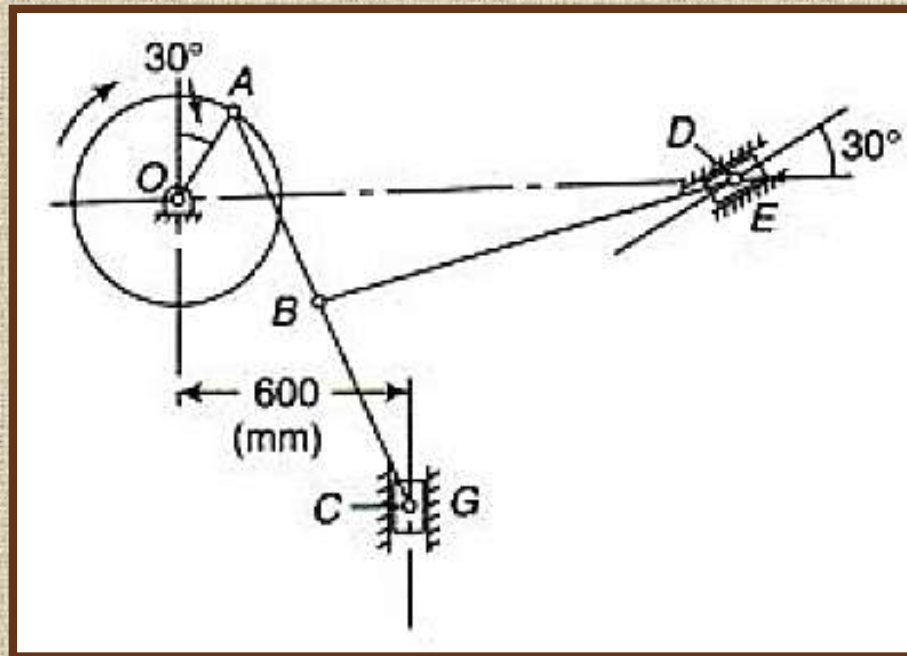
$$\omega_{AB} = 5.67 \text{ rad/s (Anticlockwise about B)}$$

$$\alpha_{AB} = 171.67 \text{ rad/s}^2 \text{ (Clockwise about B)}$$

MATH

Figure shows a mechanism in which $OA = 300$ mm, $AB = 600$ mm, $AC = BD = 1.2$ m. OD is horizontal for the given configuration. If OA rotates at 200 rpm in the clockwise direction, find

1. Linear velocities of C and D
2. Angular velocities of links AC and BD



$$v_c = 5.2 \text{ m/s}$$

$$v_d = 1.55 \text{ m/s}$$

$$\omega_{ac} = 4.75 \text{ rad/s (Clockwise)}$$

$$\omega_{bd} = 4.31 \text{ rad/s (Clockwise)}$$

CORIOLIS EFFECT

In mechanical systems involving rotating links, the Coriolis Effect refers to an apparent force that acts on a moving body within a rotating frame of reference. This force arises due to the combined motion of the link's rotation and the linear motion of a point along the link. It is given by:

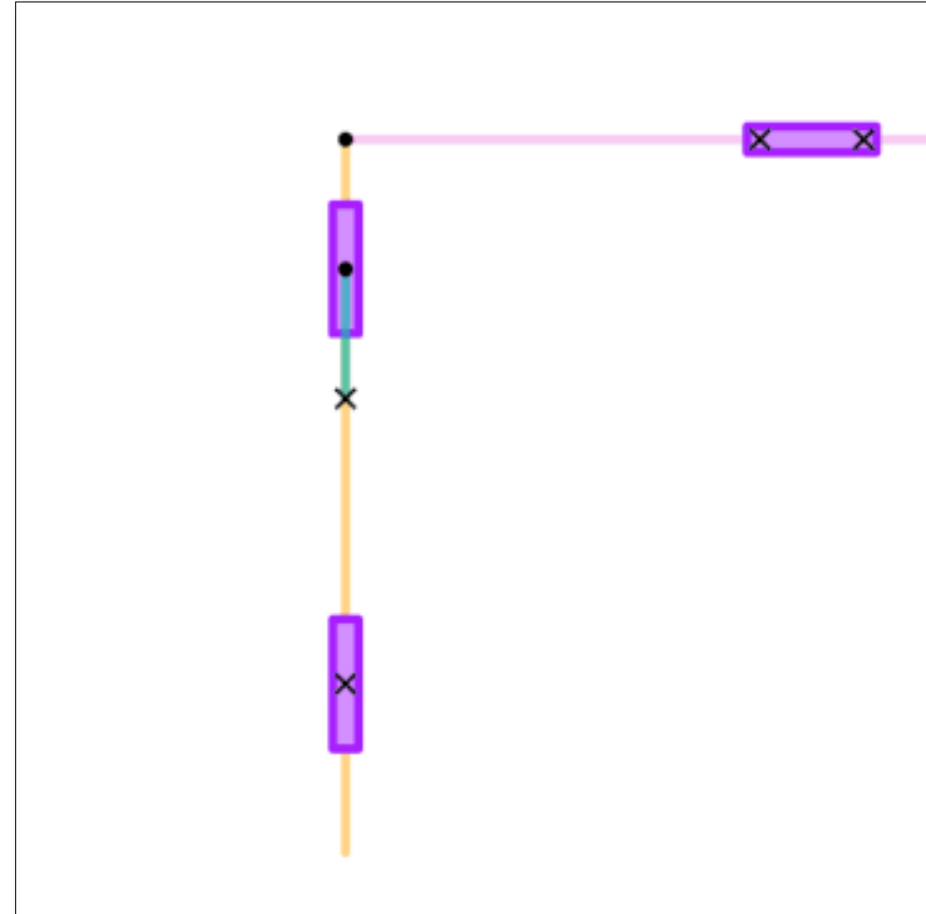
$$F_c = 2m\omega v$$

F_c = Coriolis Force

m = Mass of moving body

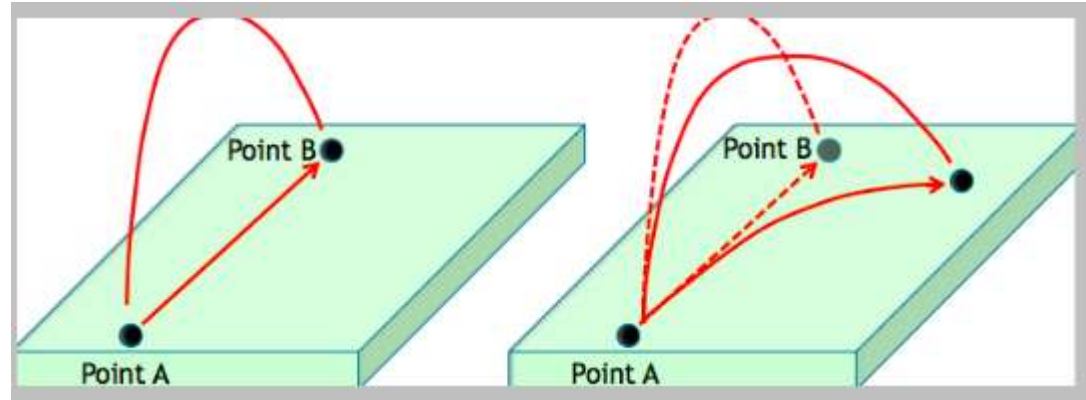
ω = Angular velocity of the rotating link

v = Relative velocity of the moving body along the link



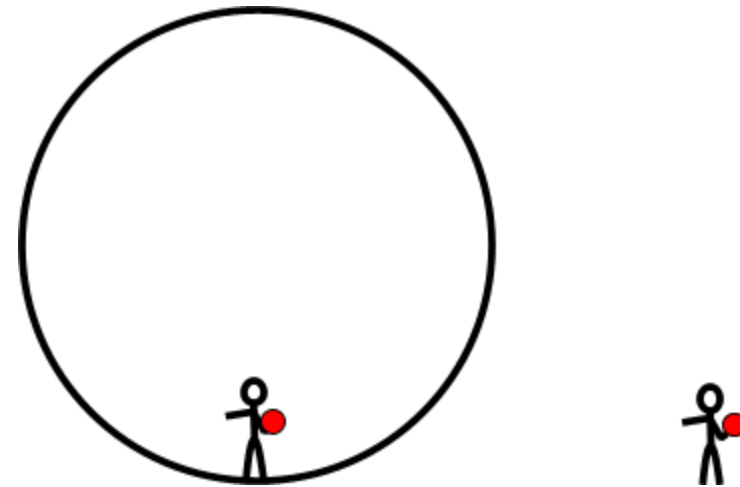
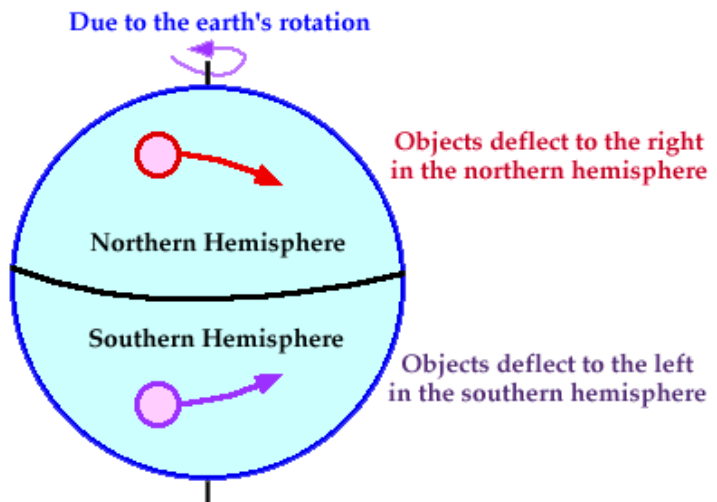
Quick Return Mechanism

CORIOLIS EFFECT



Example in Real Life:

Fire artillery shells, or long-range bullets. We'd expect that the shells moves in a straight line until it hits the ground. Yet when we aim eastward, artillery shells always deflect more and more to one side, as if pushed by some invisible force.



Watch this to understand better: [Coriolis Effect](#)

CORIOLIS COMPONENT OF ACCELERATION

When a point on one link is sliding along another rotating link, such as in quick return motion mechanism, then the Coriolis component of the acceleration must be calculated.

Consider a link OA and a slider B as shown in Fig.(a). The slider B moves along the link OA . The point C is the coincident point on the link OA .

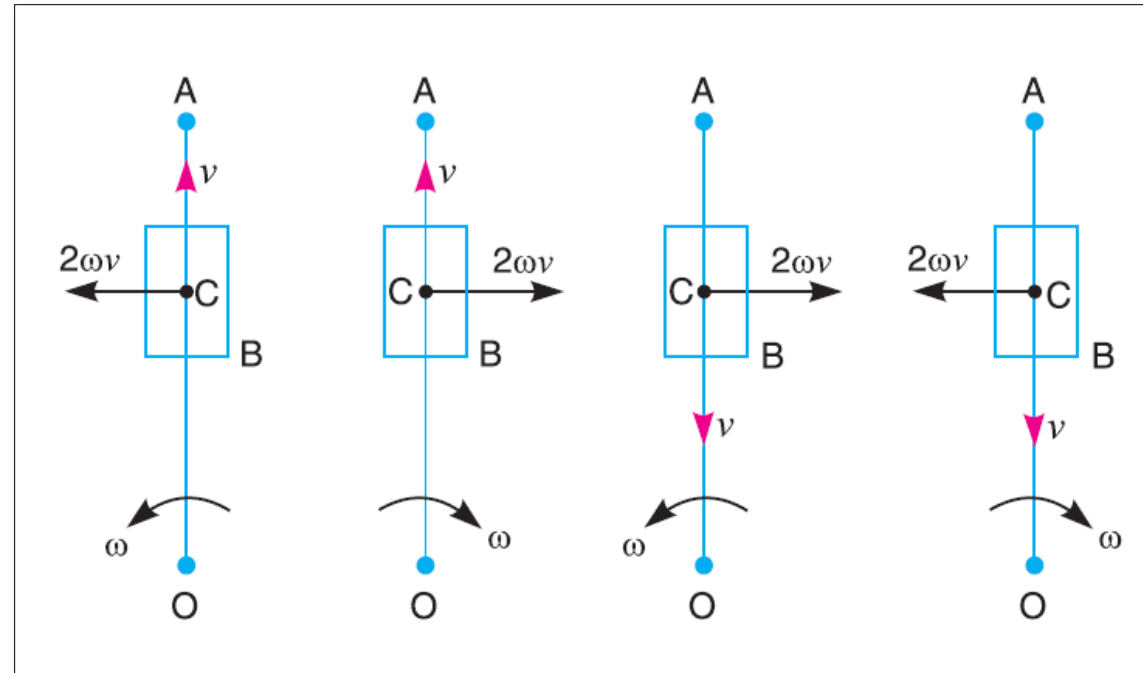
Let ω = Angular velocity of the link OA at time t seconds

v = Velocity of the slider B along the link OA at time t seconds

$\omega.r$ = Velocity of the slider B with respect to O (perpendicular to the link OA) at time t seconds

Coriolis component of the acceleration of B with respect of C ,

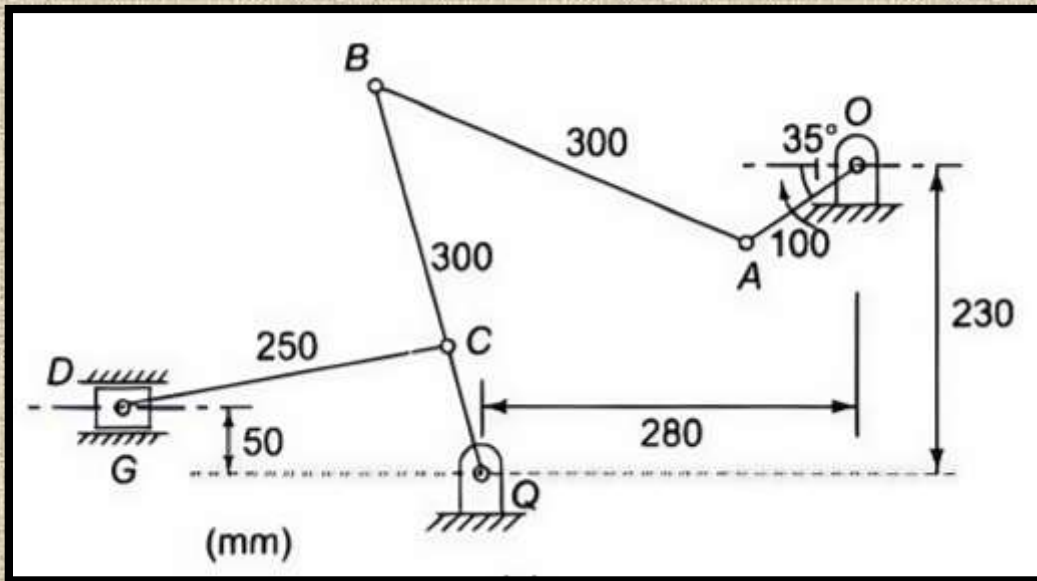
$$a_{BC}^c = a_{BC}^t = 2\omega.v$$



HOMEWORK

Figure shows a mechanism in which $OA = QC = 100$ mm, $AB = QB = 300$ mm and $CD = 250$ mm. The crank OA rotates at 150 rpm in the clockwise direction. Determine the

1. Velocity of the slider at D
2. Angular velocities of links QB and AB
3. Rubbing velocity at the pin B which is 40 mm in diameter
4. Acceleration of slider D
5. Acceleration of point C



$$v_D = 0.56 \text{ m/s}$$

$$\omega_{QB} = 5.63 \text{ rad/s (Anticlockwise)}$$

$$\omega_{AB} = 6.3 \text{ rad/s (Anticlockwise)}$$

$$\text{Rubbing Velocity at point B} = 0.0268 \text{ m/s}$$

$$a_D = \text{Solve}$$

$$a_C = \text{Solve}$$

SOLVE BY YOURSELF

Theory of Machines by Khurmi

Chapter 7

Example 7.1, 7.2, 7.5, 7.7

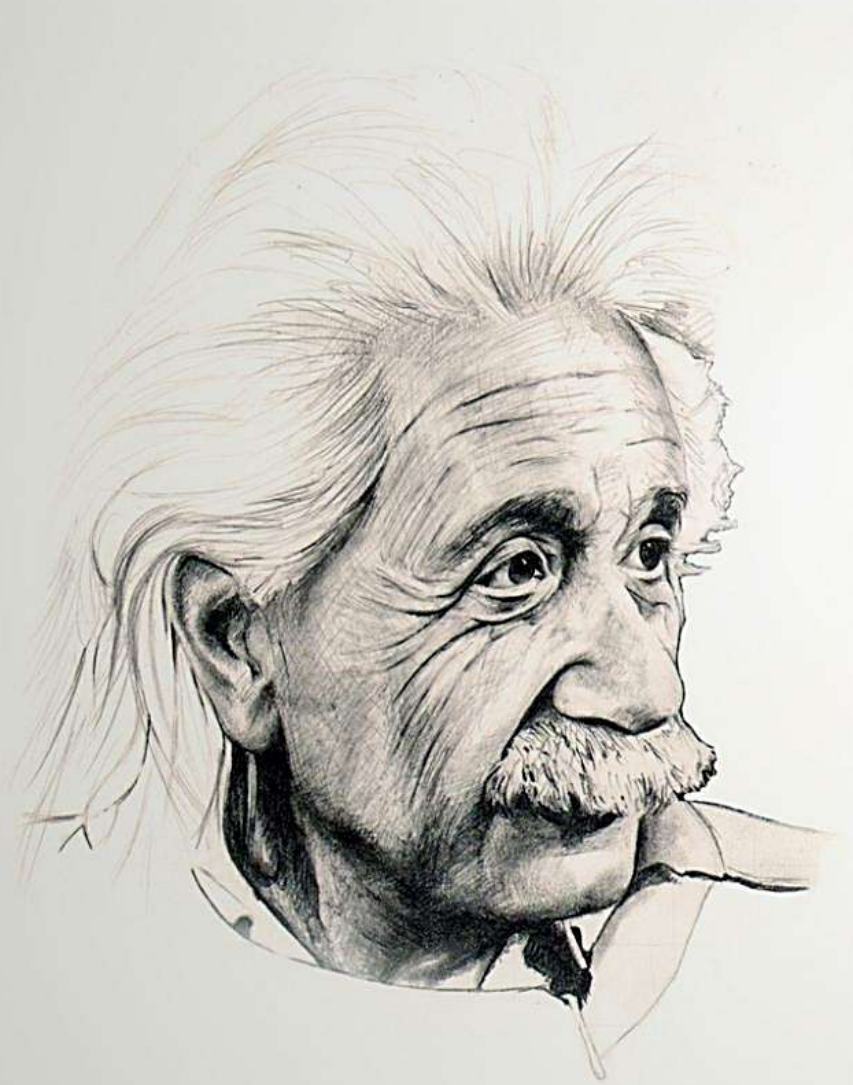
Exercises: 4, 7

Chapter 8

Example 8.1, 8.3, 8.4, 8.6.

8.7, 8.13

Exercises: 4



try not to become a man of **SUCCESS** but
rather try to become a man of **VALUE**