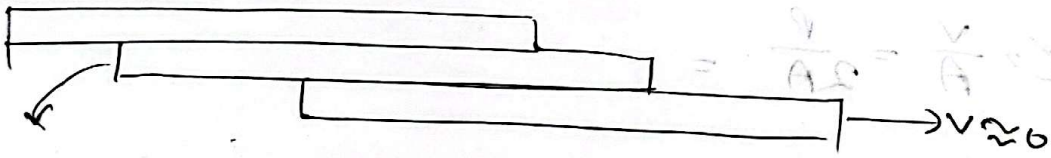
Fluid mechanics

09-12-24

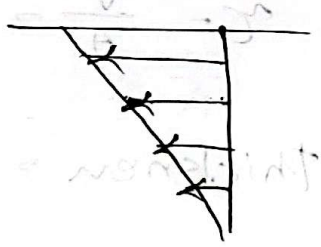
Newton's 2nd law $F = ma$

Static condition.

$\Sigma f_x = 0$	$\Sigma M_x = 0$
$\Sigma f_y = 0$	$\Sigma M_y = 0$
$\Sigma f_z = 0$	$\Sigma M_z = 0$



in river



infinitesimal → very little / small amount.

Cons

$$K = -\frac{dP}{\frac{dv}{v}}$$ = Bulk' modulus of elasticity.

dP = changing pressure

dv = changing volume

v = volume of fluid

K = Psi / Pa unit.

specific volume, $v = \frac{1}{\rho}$

$\Rightarrow v\rho = 1$

$\Rightarrow v\,d\rho + \rho\,dv = 0$

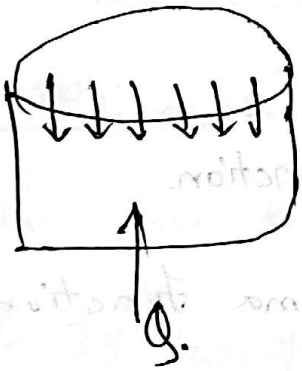
$\Rightarrow \frac{dv}{v} = \frac{d\rho}{\rho}$

$\therefore$ Bulk's modulus of elasticity $K = -\frac{dp}{dv/v} = -\frac{dp}{\frac{d\rho}{\rho}}$

Cohesion → always together

adhesion → adhere with other one

Surface tension (σ)



$PV = nRT$

$P \propto T$

Fluid pressure, $P = F/A = \frac{dF}{dA}$

dF = infinitesimal force acting on infinitesimal area

dA = boundary surface dA.

Fluid

10-12-24

Continuum concpt of fluid;

↳ no empty space

nothing is 100%.

viscosity matters.

$\rho = 1025$ ① $P = P_{atm}$

↓ 2km

② $P_2 = 840$ kg/cm²

Given $P_2 = 840 \frac{kgf}{cm^2}$ (gage pressure)

$K = 29000$ kg(f)/cm²

$\rho_s = 1025$ kg(f)/m³

$K = \frac{dP}{\frac{d\rho}{\rho}}$

$29000 = \frac{840}{d\rho/\rho_1}$

$dP = P_2 - P_1$

$\rho = \rho_1$

$\rho_1 = 1025$

$\frac{d\rho}{\rho_1} = 0.035$

$\Rightarrow P_2 - P_1 = 0.035 P_1$

$\Rightarrow P_2 = 1.035 P_1$

$\Rightarrow P_2 = 1068$ kg/

Shear strain → angular deformation.

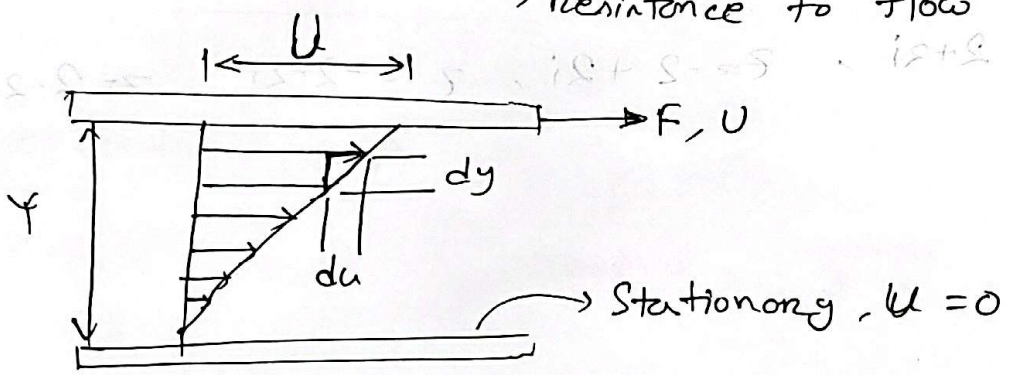
Fluid

17-12-24

Continuum concept of fluid:-

Newton's law of viscosity:-

↳ resistance to flow



U = velocity of fluid

$F \propto \frac{1}{Y}$

$F \propto U$

$F \propto A$

$F \propto \frac{AU}{Y}$

$\Rightarrow \frac{F}{A} \propto \frac{U}{Y}$

$\Rightarrow \tau \propto \frac{U}{Y}$

Shear stress $\tau = F/A$

$\frac{U}{Y} = \frac{du}{dy}$

considering triangle

$\therefore \tau \propto \frac{U}{Y} \left(= \frac{du}{dy}\right)$

$1\ Ns/m^2 = 10$ poise

$\tau = \mu \frac{du}{dy}$ or $\mu \frac{U}{Y}$

$\mu = \frac{\tau Y}{U} = \frac{N/m^2 \cdot m}{m/s}$

$\mu = Ns/m^2$

Problem 2)

$d_s = 95\,mm$

$r_s = 95/2$

$N = 50\ rpm$

$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 50}{60} =$

$d_c = 96.9\ mm$

$r_c = \frac{96.9}{2}$

$L_s = L_c = 0.50\,m$

$T_s = 1.0\ N/m$

Ref. $Y = r_c - r_s$

$\tau = F r_s \Rightarrow F = \frac{T}{r_s}$

$\cancel{\tau = F/A}$

$A = 2\pi r_s \times L_s$

$=$

$F = \frac{T}{r_s}$

$U = \omega r_s =$

$\tau = \mu \frac{U}{Y}$

$\Rightarrow \mu = \frac{F \cdot Y}{A \cdot U}$

$=$

$= N s/m^2$

Fluid mechanics:- 23-12-24

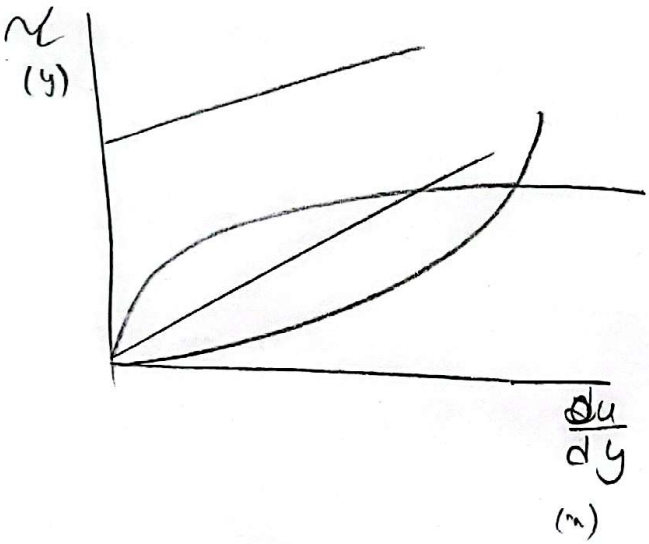
Newtonian & non-newtonian fluid

obeying newton's Low of viscosity.

not obeying newton's law of viscosity.

Ideal fluid means zero viscosity.

Real fluid: viscosity.

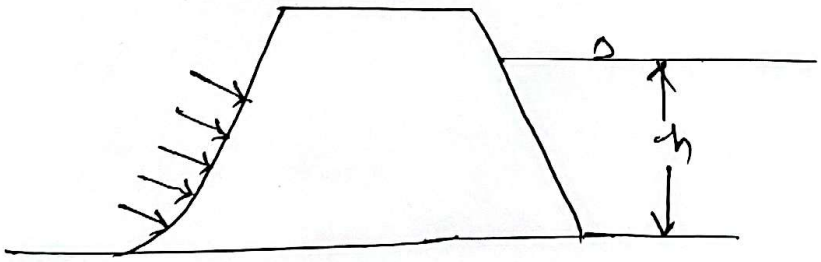


$y = mx$

$\tau = \mu \left(\frac{du}{dy}\right)^n$

$\tau = A + B\left(\frac{du}{dy}\right)^n$

$A \to 0$, $B \to \mu$

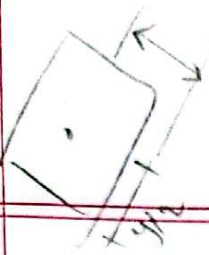


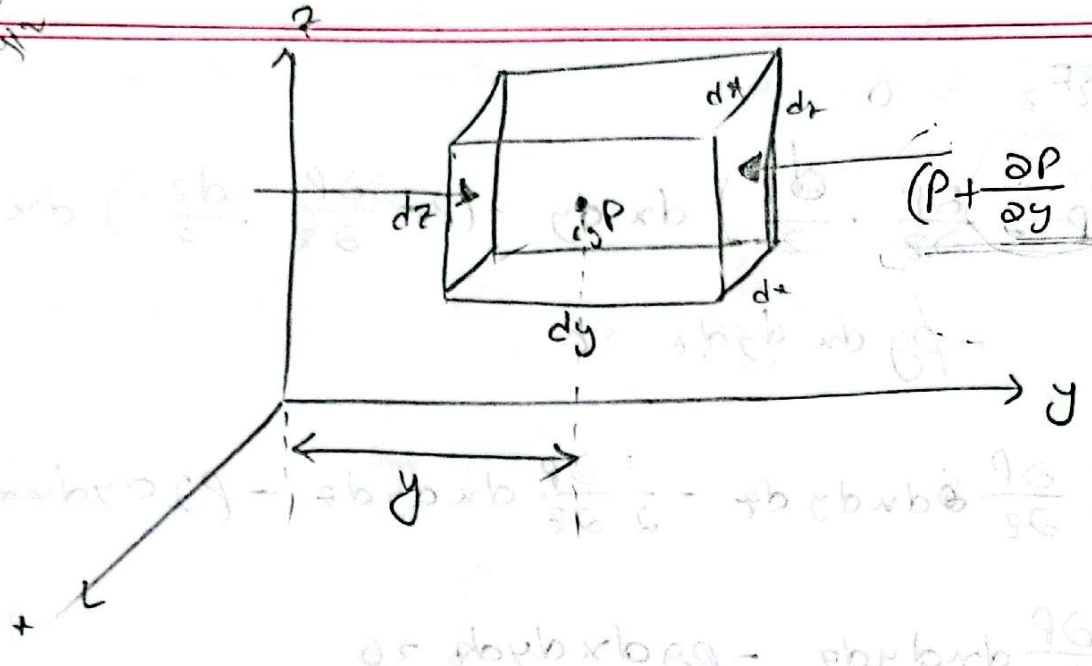
$P = h\rho g \times a$

$a = \rho g$

$\sigma = F/A$

$P = F/A$





$$\left(P+\frac{\partial P}{\partial y}\cdot\frac{dy}{2}\right)\cdot dx\cdot dz$$

$$f(x+h) = f(x) + \frac{\partial f}{\partial x}\cdot\frac{h}{1!} + \frac{\partial^2 f}{\partial x^2}\cdot\frac{h^2}{\lfloor 2} + \cdots$$

$$P_{y+dy/2} = \left(P+\frac{\partial P}{\partial y}\cdot\frac{dy}{2}\right)$$

$$\xrightarrow{+}\Sigma F_y = 0$$

$$\Rightarrow \left(P-\frac{\partial P}{\partial y}\cdot\frac{dy}{2}\right)dx\,dz - \left(P+\frac{\partial P}{\partial y}\cdot\frac{dy}{2}\right)\cdot dx\,dz = 0$$

$$\Rightarrow -\frac{1}{2}\frac{\partial P}{\partial y}\cdot dx\,dy\,dz - \frac{1}{2}\frac{\partial P}{\partial y}\cdot dx\,dy\cdot dz = 0$$

$$\Rightarrow \frac{\partial P}{\partial y}\,dx\,dy\,dz = 0$$

$$\Rightarrow \frac{\partial P}{\partial y}\,\Delta V = 0$$

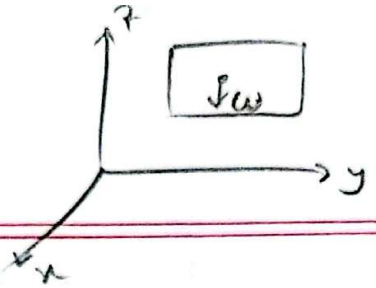
$\Delta V \neq 0$

$$\frac{\partial P}{\partial y} = 0$$

$$\Rightarrow \partial P_y = 0$$

$$\Rightarrow \int P_y = 0$$

$$\Rightarrow P_y = \text{const.}$$



$$+\uparrow \Sigma F_z = 0$$

$$\Rightarrow \left(P - \frac{\partial P}{\partial z}\cdot\frac{dz}{2}\right) dx\,dy - \left(P + \frac{\partial P}{\partial z}\cdot\frac{dz}{2}\right) dx\,dy$$

$$- \rho g\, dx\,dy\,dz = 0$$

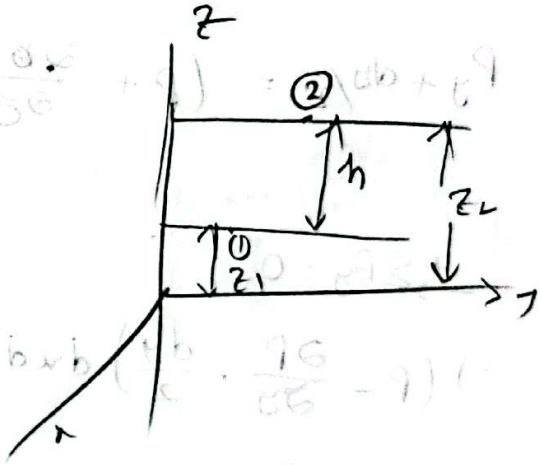
$$\Rightarrow -\frac{1}{2}\frac{\partial P}{\partial z} dx\,dy\,dz - \frac{1}{2}\frac{\partial P}{\partial z} dx\,dy\,dz - \rho g\, dy\,dx\,dz = 0$$

$$\Rightarrow -\frac{\partial P}{\partial z} dx\,dy\,dz - \rho g\, dx\,dy\,dz = 0$$

$$\Rightarrow dx\,dy\,dz\left(+\frac{\partial P}{\partial z} + \rho g\right) = 0$$

$$\Rightarrow \nabla V\left(\frac{\partial P}{\partial z} + \rho g\right) = 0$$

$$\nabla V \neq 0$$



$$\Rightarrow \partial P/\partial z + \rho g = 0$$

$$\Rightarrow dP = -\rho g\, dz$$

$$h = z_2 - z_1$$

$$\Rightarrow \int_1^h dP = -\rho g \int_1^h dz$$

$$\Rightarrow P_2 - P_1 = -\rho g\,(z_2 - z_1)$$

point ② surface pressure mean atm pressure

$$\Rightarrow P_{atm} - P_1 = -\rho g h$$

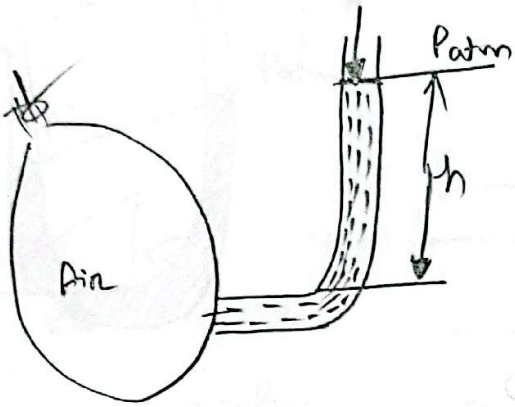
$$\Rightarrow P_1 = \rho g h + P_{atm}. \text{ (absolute)}$$

$$\Rightarrow P = \rho g h \text{ (gage)}$$

in x & y direction pressure is not changing.

$$P = \rho g h = \gamma h \qquad \boxed{\gamma = \rho h}$$

pressure head $h = P/\gamma = \dfrac{P}{\rho g}$



Hydrostatic pressure of compressible fluid

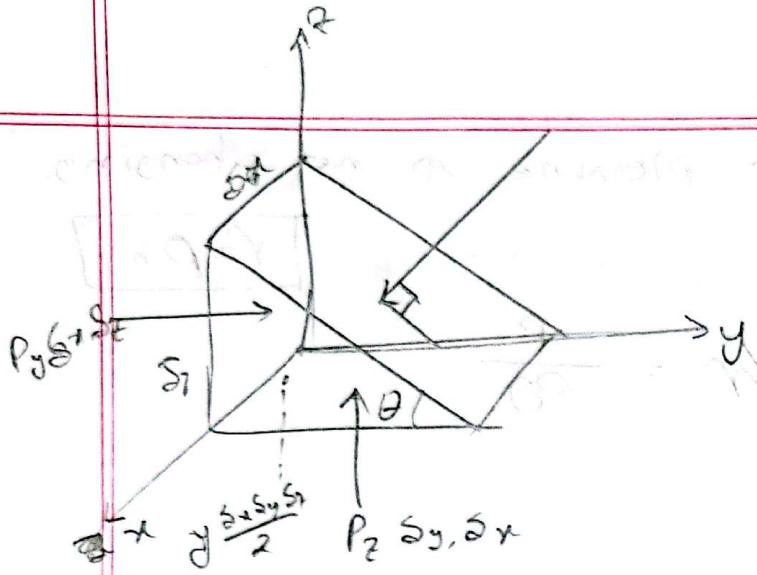
$\dfrac{dP}{dz} = -\gamma$ $\qquad \gamma = \rho g$ not const then $\dfrac{dP}{dz} = -\rho g$

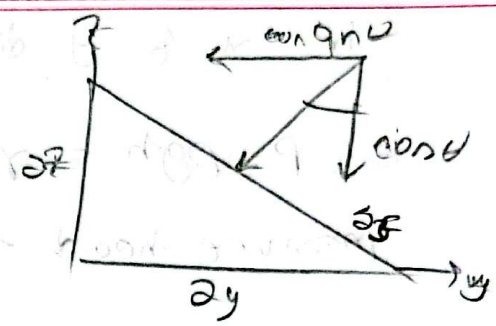
by ideal gas law, $P = \rho R T$ thus, $\rho = \dfrac{P}{RT}$

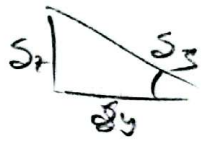
$$\frac{dP}{dz} = -\rho g$$

$$\frac{dP}{dz} = -\frac{P}{RT} \cdot g$$

$$\frac{dP}{}$$







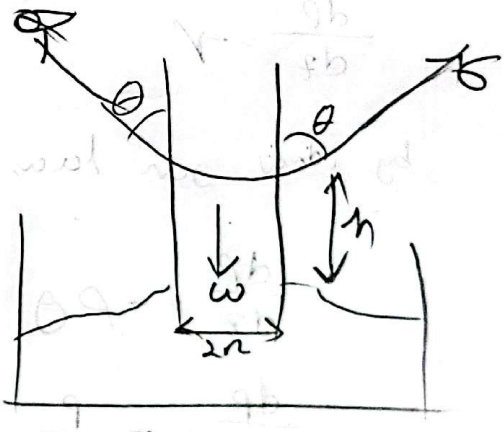
$\sin\theta = \frac{\delta_z}{\delta_s}$

$\Rightarrow \sum F_y = 0$

$\Rightarrow P_y \delta_x \delta_z - P_x \delta_x \delta_s \sin\theta = 0$

$P_y \delta_x \delta_s \sin\theta - P_x \delta_x \delta_s \sin\theta = 0$

$\Rightarrow$



Capillary rise of fluid:

Equilibrium y-direction yields

$$\sigma \cos\theta (2\pi r) - w = 0$$

$$W = \gamma \pi r^2 h$$

$$\gamma \pi r^2 h = \sigma \cos\theta (2\pi r).$$

$$h = \frac{\sigma \cos\theta \, 2\pi r}{\pi r^2 \gamma}$$

$\boxed{\text{math } \theta = 0^\circ}$

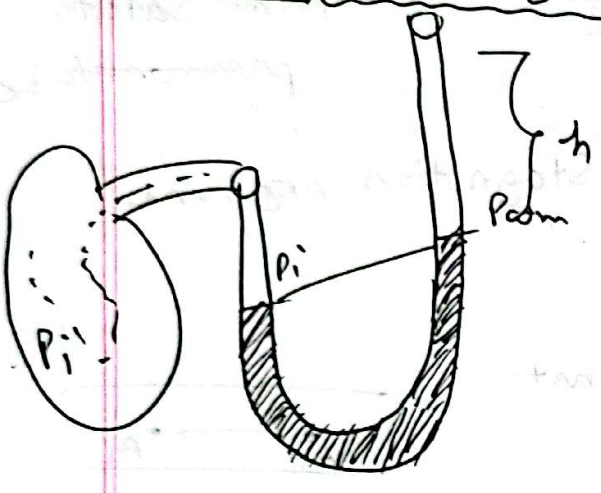
next one.

Fluid

29-12-24

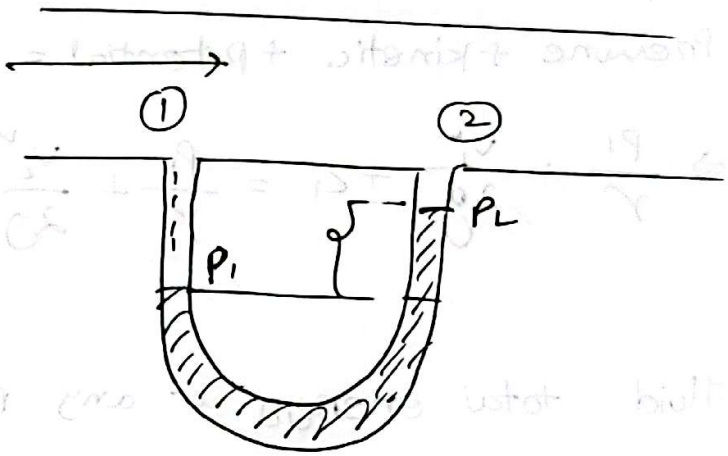
math. $\theta = 0$,

L-03 - Pressure & measurement.



$P_i > P_{atm}$

$P_i = h\rho g$



$P_1 > P_L$

$\Delta P_L = h\rho g$

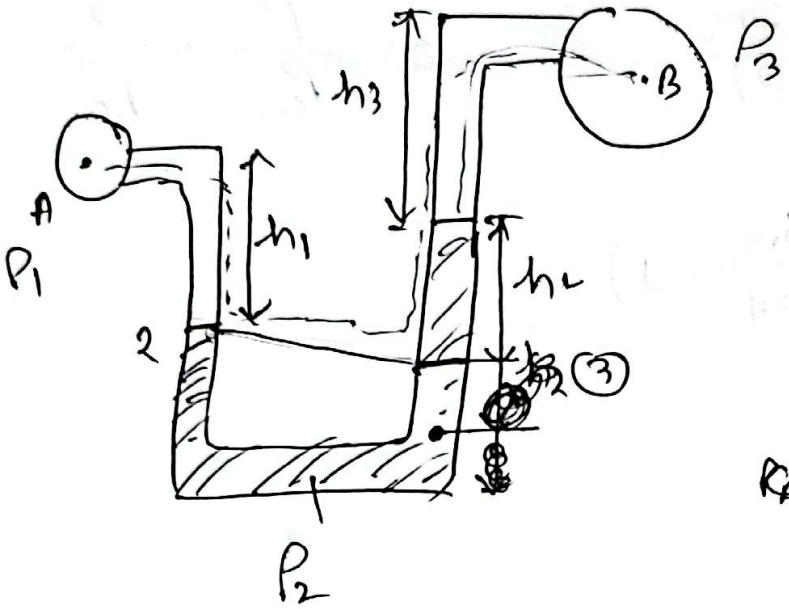
types of monometer → simple → Piezometer
→ U-tube Monometer
→ Single column Monometer

Q. What principle used in monometer
Q. Types of pressure measurement instru.
Q.

Absolute pressure = Gauge pressure + Atm pres
Practical application

101.325 kPa
101325 Pa
76 cm mm

30-12-24



same level.
$P_2 = P_3$

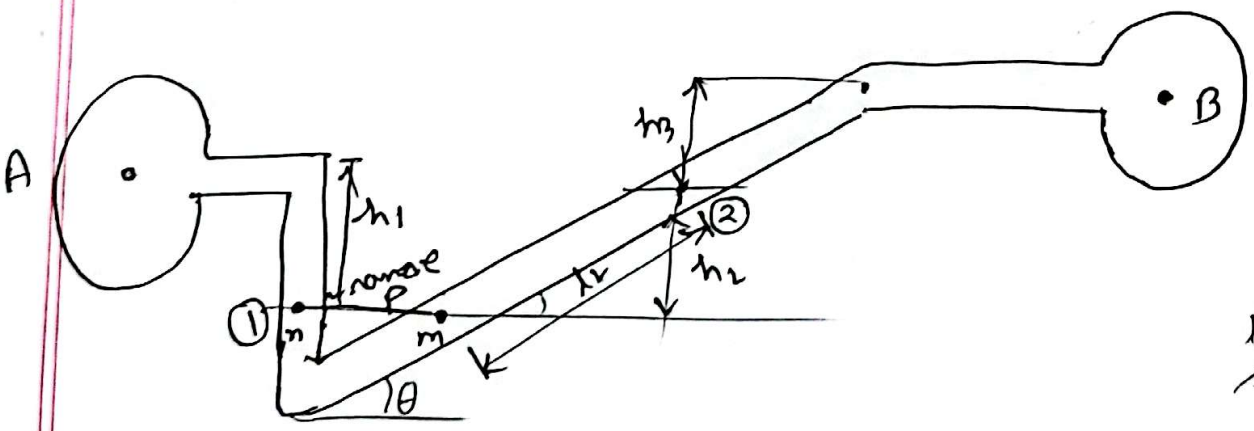
$P_A + h_1 \rho_1 g = P_2$

$\boxed{\text{as } P_2 = P_3}$

$P_B + h_3 \rho_3 g + h_2 \rho_2 g = P_3$

we can write, $P_A + h_1 \rho_1 g = P_B + h_3 \rho_3 g + h_2 \rho_2 g$

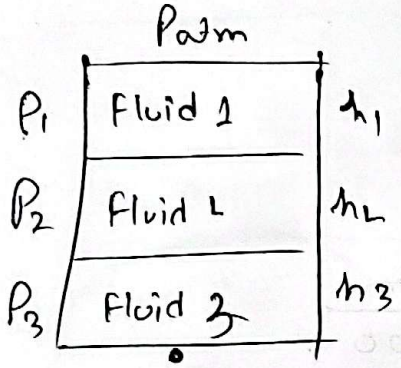
$\Rightarrow P_A - P_B = h_3 \rho_3 g + h_2 \rho_2 g - h_1 \rho_1 g.$



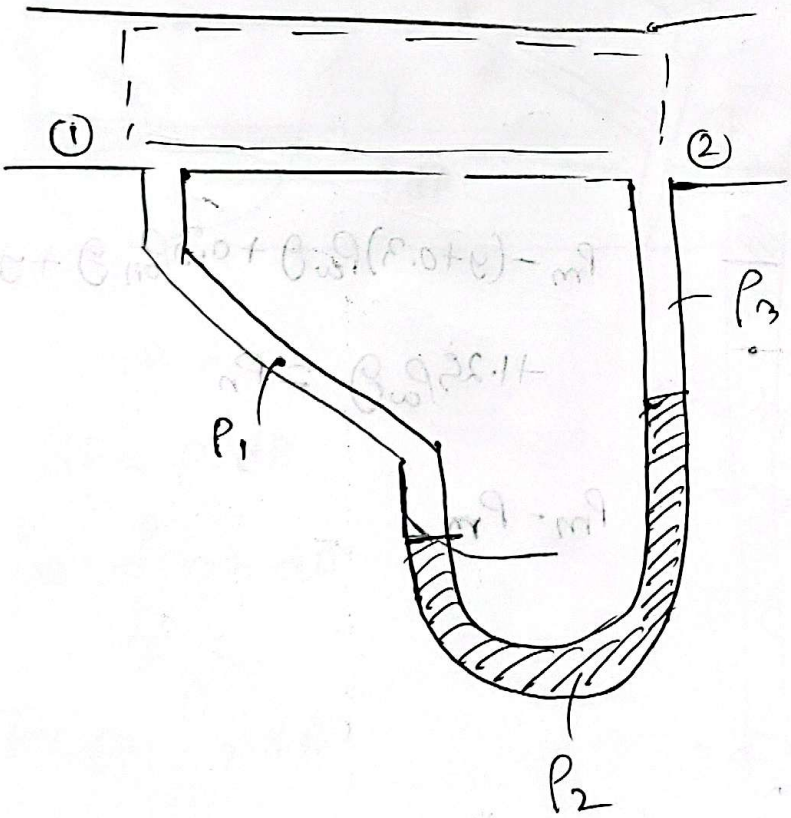
$$P_A + h_1\rho_1 g - \overset{h_2}{\boxed{l_2 \sin\theta}}\,\rho_2 g - h_3\rho_3 g = P_B$$

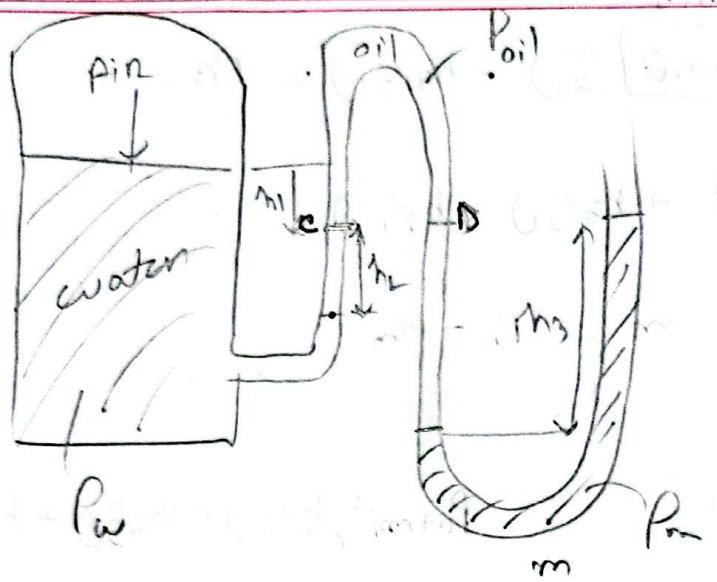
$$P_A - P_B = l_2 \sin\theta + h_3\rho_3 g - h_1\rho_1 g$$

for point n & m, $P_n = P_m$



$$P_{atm} + h_1\rho_1 g + h_2\rho_2 g + h_3\rho_3 g = P_1$$



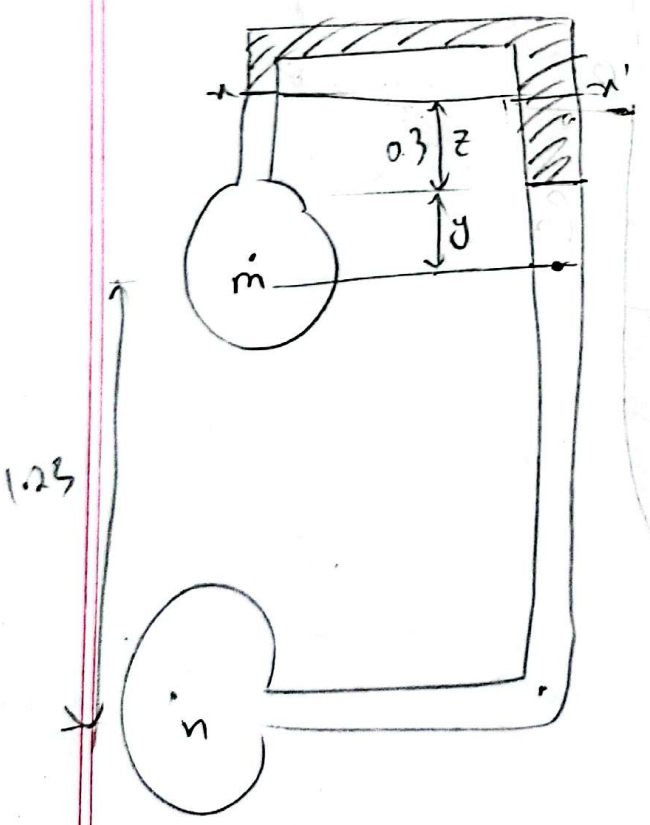


$P_c = P_D$

$P_c' = P_1 + h_1$

Specific. Gravity $= \frac{\rho_m}{\rho_w} = \frac{\rho_m}{1000}$

$\rho_m = 0.9 \times 1000$



$P_m - (y+0.3)\rho_w g + 0.3\rho_{oil} g + y\rho_w g$

$+ 1.25\rho_w g = P_n$

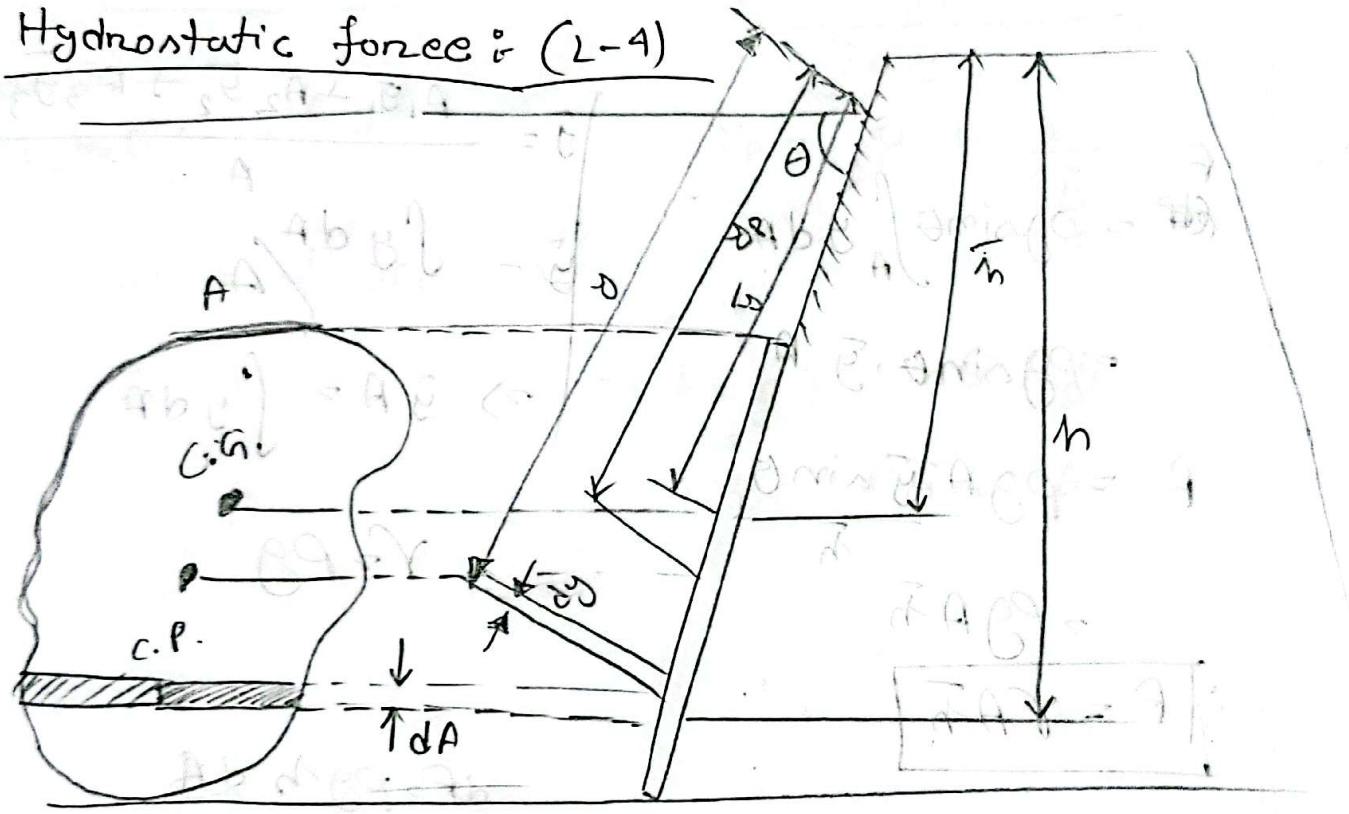
$P_m - P_m$

$P_3 = P_4$

$P_A = P_1 = P_2$

$P_2 + 27\rho_w g - 15\rho_m g - 12\rho_w g - 123\rho_w g = P_B$

Hydrostatic force : (L-4)



$F = P \cdot dA$

$dF = P \cdot dA$

$\int dF = \int_A \rho g h \, dA$

$F = \rho g \int_A h \, dA$

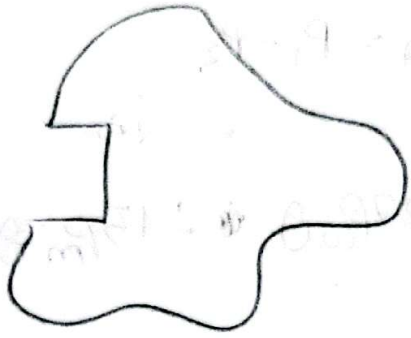
$dF = h \rho g \, dA$

$= y \sin\theta \, \rho g \, dA$

$= \rho g \int_A \sin\theta \, y \cdot dA$

$= \cancel{\rho g \int_A}$

$\sin\theta = \frac{\bar{h}}{\bar{y}}$



$$\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3}{A}$$

$$\bar{y} = \int y\,dA / A$$

$$\Rightarrow \bar{y}A = \int y\,dA$$

$$\gamma = \rho g$$

$$F = \rho g \sin\theta \int_A y\,dA$$

$$= \rho g \sin\theta \cdot \bar{y} A$$

$$F = \rho g A \underbrace{\bar{y}\sin\theta}_{\bar{h}}$$

$$= \rho g A \bar{h}$$

$$\therefore \boxed{F = \gamma A \bar{h}}$$

$$dF = \rho g h\,dA$$

$$= \gamma y \sin\theta\,dA$$

$$F \cdot y_p = \int_A y\,dF$$

$$\Rightarrow y_p = \frac{\int_A y\,dF}{F}$$

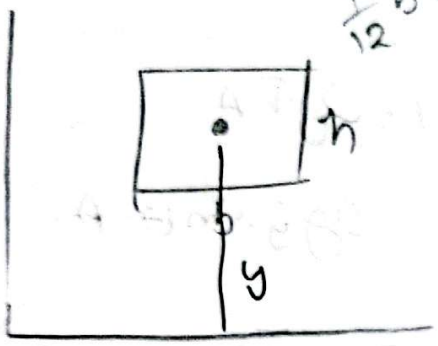
$$= \frac{\int_A y \cdot \gamma \cdot y \sin\theta\,dA}{\gamma \sin\theta\, \bar{y} \cdot A}$$

$$y_p = \frac{\gamma \sin\theta \int_A y^2\,dA}{\gamma \sin\theta\, \bar{y} \cdot A}$$

$$= \frac{\int_A y^2\,dA}{A\bar{y}}$$

$$= \frac{I_x}{A\bar{y}}$$

$\frac{1}{12} b h^3 = I_0$



$$I_x = I_{cg} + \bar{y}^2 A$$

$$\therefore y_p = \frac{I_x}{\bar{y} A} = \frac{I_{cg} + A\bar{y}^2}{\bar{y} A}$$

$$y_p = \bar{y} + \frac{I_0}{A\bar{y}}$$

way)

$I_0 > 0$; $I_0, A, \bar{y} \neq 0$

$\bar{y}$ location of C.G.

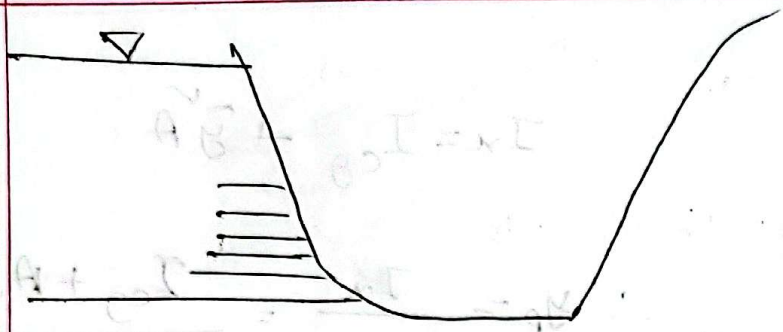
ie, $y_p > \bar{y}$

$$F = \gamma \sin\theta \, A \bar{y}$$
$$= \rho g A \bar{y} \sin\theta$$

$1\ m/k^2$
kg

Fluid

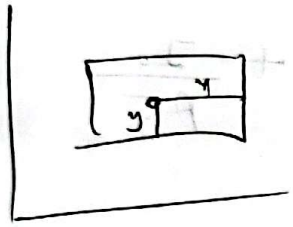
31-12-24



$F = \rho g \bar{h} A$

$= \rho g \bar{y} \sin\theta\ A$

$y_p = \bar{y} + \frac{I_o}{A\bar{y}}$ $\quad y_p > \bar{y}$

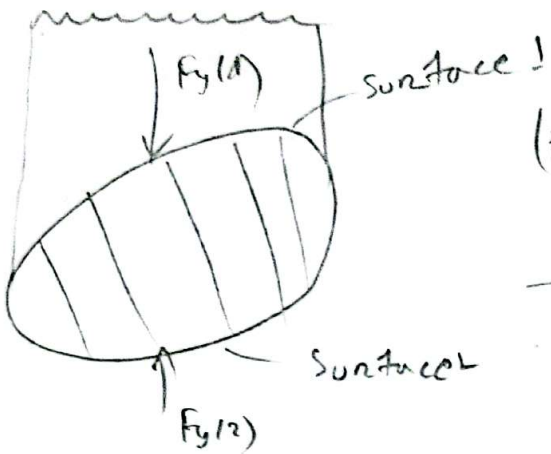


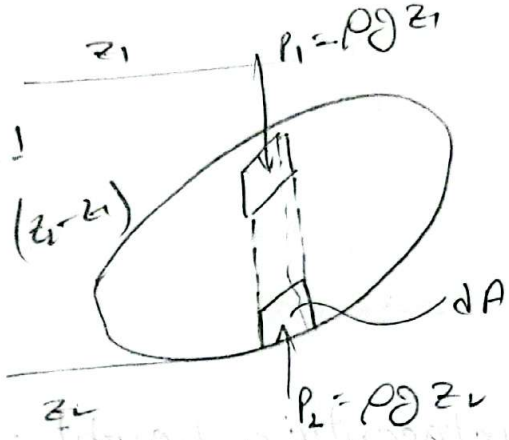
$I_{x=3} = \frac{1}{12} b h^3$

Fluid

06-01-25

Buoyant force (F_B)





$$dF_B = (P_2 - P_1)\,dA$$

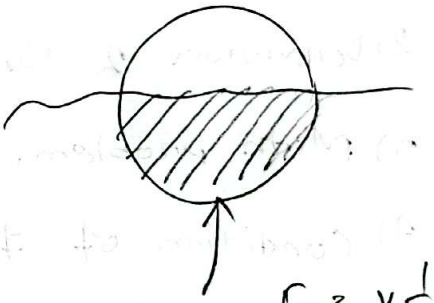
$$= \rho g (z_2 - z_1)\,dA$$

$$\int dF_B = \int \rho g (z_2 - z_1)\,dA$$

$$\Rightarrow F_B = \rho g \int_A h\,dA$$

$$= \rho g \gamma \text{Vol}^m$$

$$= \gamma \text{vol}^m$$



$F_B = V \frac{1}{2} \times \gamma$

Floating body

$F_B = \gamma$ (displa

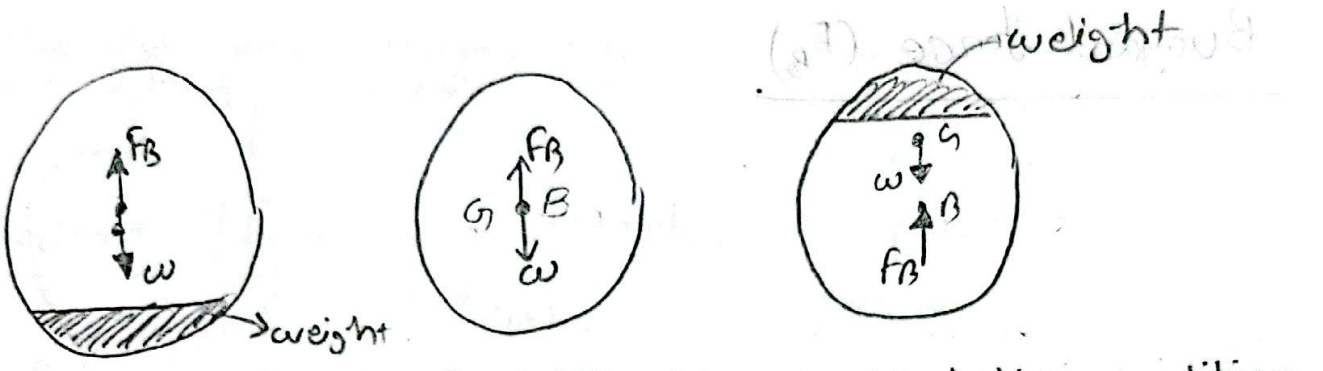
moral of the story:

$\rho_B > \rho_f \rightarrow$ sink

$\rho_B < \rho_f \rightarrow$ float

ρ_B = density of Body

ρ_f = density of fluid



Stable condition Neutrally stable Unstable condition.

Metacentric height.

L-4

1) Defination

2) Derivation & Formulas (Y_p)

3) Math problem. (Force, location of center pressure)

4) Condition of floating of a body [Prove]

5) condition of stability of particle of ship.

6) what is metacentric height

positive metacentric height is better or negative metacentric height.

All Defination important

L-5) Fluid Kinematics

a) Difference between kinematics & kinetics.

b) Difference between Lagrangian & Eulerian method

Lagrangian	Eulerian
→ Looking every point	looking particular point
- getting pathling	→ no pathline.

u = velocity along x-axis
v = " " y-axis
w = " " z-axis.

Steady flow
characteristics of fluid is not changing with time.

Unsteady flow
characteristics of fluid is changing with time

uniform flow:
velocity of ~~flow~~ fluid does not change both in magnitude & direction.

ununiform flow:

Steady uniform flow:

Laminar flow: fluid sliding smoothly over adjacent layer

Turbulent flow: a rapid and continuous mixing of the fluid leading to momentum transfer an flow occurs.

Fluid

07-01-25

Streamline:

Q) Defination Streamline, Streamtube, pathline
Steak-line

Principle of conservation of mass:

Principle of conservation of energy:

Energy neither be created nor destroyed.

$$F = ma = m\left(\frac{V_2 - V_1}{\Delta t}\right)$$

impulse $\boxed{F \cdot \Delta t} = m(v_2 - v_1) = mv_2 - mv_1$

Force working in short period time is called impulse

impulse momentum: impulse force $= \frac{mv_2 - mv_1}{\Delta t}$

$\dot{m}_{in} > \dot{m}_{out}$ some will be inside the system.

Fluid

13-01-25

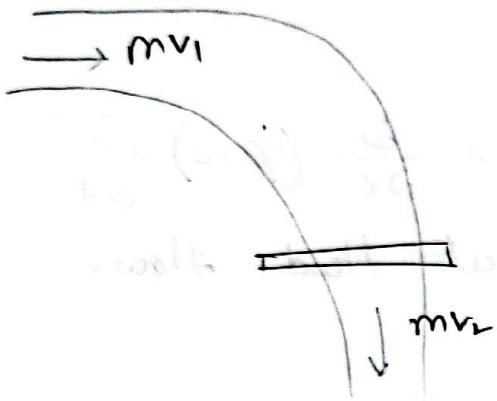
Basic Principles of fluid flow:

Newton's second law $F = ma$

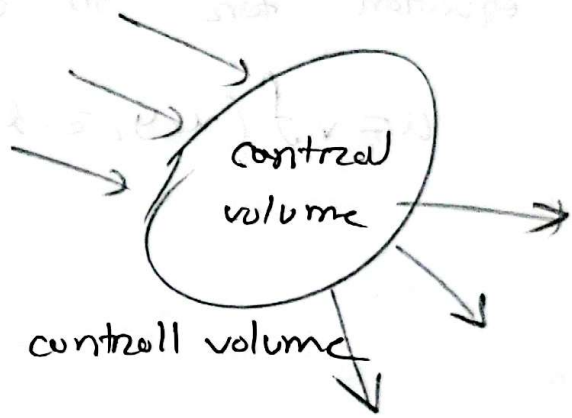
$= m\left(\frac{v_2 - v_1}{\Delta t}\right)$

impulse

$\Rightarrow F\Delta t = m(v_2 - v_1)$ = change of momentum.



Continuity equation



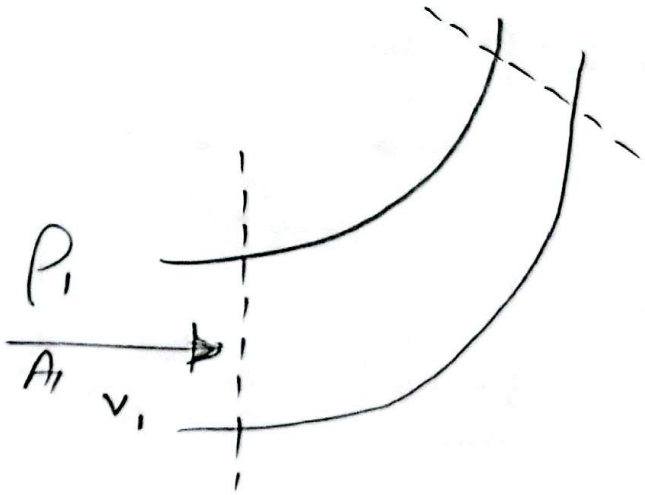
$\dot{m}_{out} < \dot{m}_{in}$

some will be inside the controll volume

Streamtube:

$\dot{m}_{in} = \rho_1 A_1 v_1$

$\dot{m}_{out} = \rho_2 A_2 v_2$

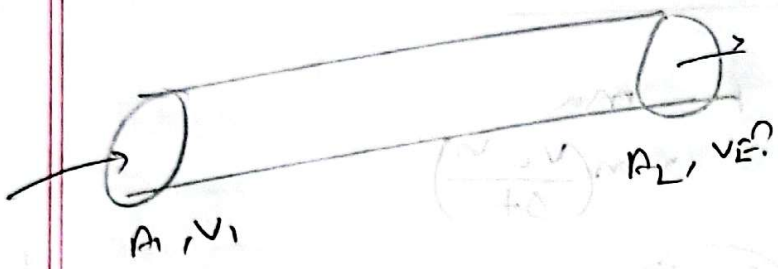


$\dot{m}_{in} = \dot{m}_{out}$

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

$A_1 v_1 = A_2 v_2$

Density same incompressible



~~AV~~

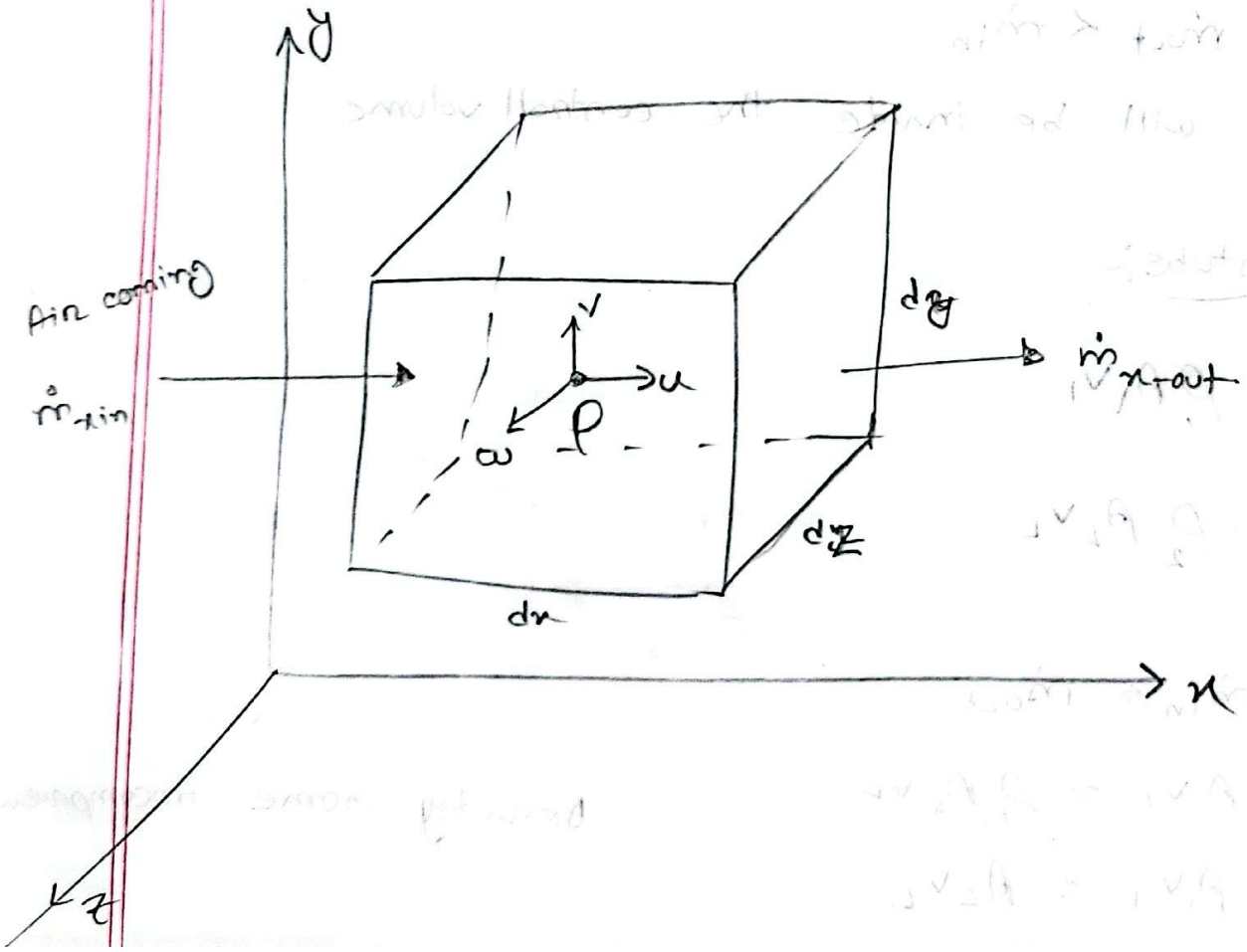
Flow rate, $Q = AV \; m^3/s$

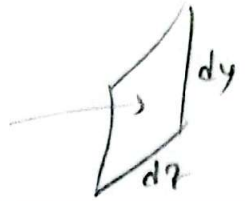
Q| Derive the equation:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) + \frac{\partial \rho}{\partial t} = 0$$

equation for 3D dimensional fluid flow.

$$u = v f(x, y, z, t)$$





mass flow rate $\dot{m} = A\rho V$ $\qquad$ $h \Rightarrow$ step size.

Taylor's series. $\qquad$ $f(x) = \rho$

$$f(x+h) = f(x) + \frac{f'(x)}{1!} h + f''(x)\frac{h^2}{2!} + \dots$$

left side $\rho_{x-\frac{dx}{2}} = \rho + \frac{\partial \rho}{\partial x}\left(-\frac{dx}{2}\right) + \dots$

$$U_{x-\frac{dx}{2}} = U + \frac{\partial u}{\partial x}\left(-\frac{dx}{2}\right) + \dots$$

<u>right side</u>

$$\rho_{x+dx/2} = \rho + \frac{\partial \rho}{\partial x}\left(\frac{dx}{2}\right) + \dots$$

$$U_{x+dx/2} = u + \frac{\partial u}{\partial x}\frac{dx}{2} + \dots$$

$$\dot{m}_{in-x} = \left(\rho - \frac{\partial \rho}{\partial x}\frac{dx}{2}\right)\left(u - \frac{\partial u}{\partial x}\cdot\frac{dx}{2}\right) dy\, dz$$

very small value so ∂x

$$= \left[\rho u - \rho\frac{\partial u}{\partial x}\frac{dx}{2} - u\frac{\partial \rho}{\partial x}\frac{dx}{2} + \frac{\partial \rho}{\partial x}\cancelto{0}{\left(\frac{dx}{2}\right)^2}\cdot\frac{\partial u}{\partial x} + \right] dy\, dz$$

$$\dot{m}_{out-x} = \left(\rho + \frac{\partial \rho}{\partial x}\frac{dx}{2}\right)\left(u + \frac{\partial u}{\partial x}\frac{dx}{2}\right) dy\, dz$$

$$= \left[\rho u + \rho\frac{\partial u}{\partial x}\frac{dx}{2} + u\frac{\partial \rho}{\partial x}\frac{\partial x}{2}\right] dy\, dz$$

$$\dot{m}_{x\text{-net}} = \dot{m}_{x\text{-in}} - \dot{m}_{x\text{-out}}$$

$$= \left[-\rho \frac{\partial u}{\partial x} \cdot \frac{dx}{2} - u \frac{\partial \rho}{\partial x} \cdot \frac{dx}{2}\right] dy\,dz$$

$$- \left[\rho \frac{\partial u}{\partial x} \cdot \frac{dx}{2} - u \frac{\partial \rho}{\partial x} dx/2\right] dy\,dz$$

$$= \left(-\rho \frac{\partial u}{\partial x} dx - u \frac{\partial \rho}{\partial x} dx\right) dy\,dz$$

$$= -\left(\rho \frac{\partial u}{\partial x} + u \frac{\partial \rho}{\partial x}\right) \cdot dx\,dy\,dz$$

$$\therefore \dot{m}_{x\text{-net}} = -\frac{\partial}{\partial x}(\rho u) \cdot dx\,dy\,dz$$

$$\dot{m}_{y\text{-net}} = -\frac{\partial}{\partial y}(\rho v) \cdot dx\,dy\,dz$$

$$\dot{m}_{z\text{-net}} = -\frac{\partial}{\partial z}(\rho w)\, dx\,dy\,dz$$

$m = \rho v$

$\frac{d}{dt}(m) = \frac{d}{dt}(\rho v)$

$\dot{m} = \frac{d\rho}{dt} \cdot v$

mass changing = rate of change of density × volume

$$\dot{m}_{change} = \frac{d\rho}{dt} dx\,dy\,dz$$

$$\Rightarrow \dot{m}_{net\,x} + \dot{m}_{net\,y} + \dot{m}_{net\,z} = \frac{\partial \rho}{\partial t} dx\,dy\,dz$$

$$\Rightarrow -\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] dx \cdot dy \cdot dz = \frac{\partial \rho}{\partial t} dx\,dy\,dz$$

$$\Rightarrow \underbrace{\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)}_{\nabla \rho \vec{V}} + \cancel{\frac{\partial \rho}{\partial t}}^{\,0} = 0.$$

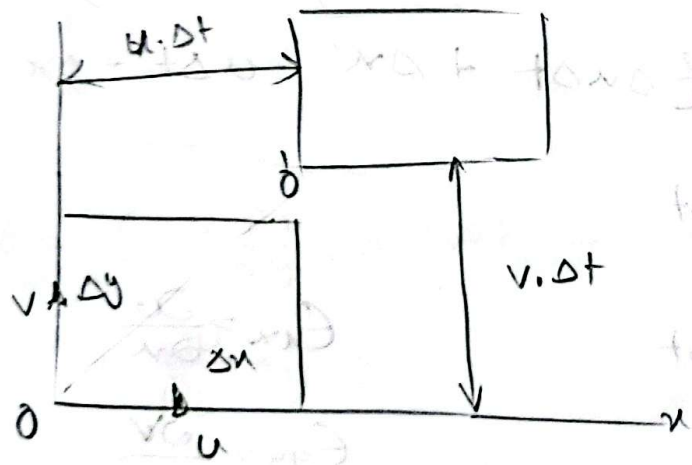
[Proved]

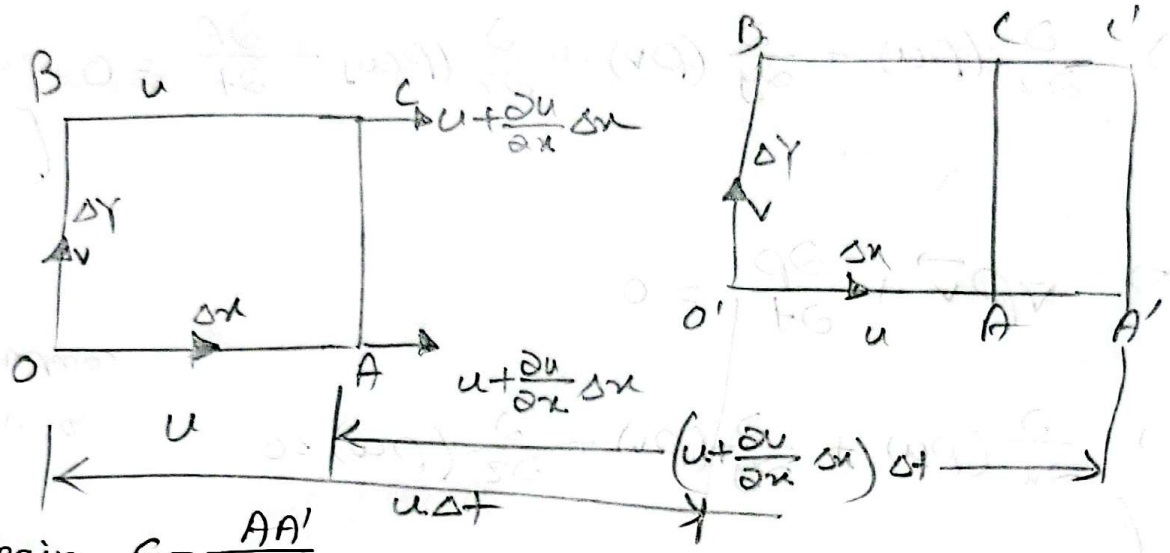
$$\Rightarrow \nabla \rho \vec{V} + \frac{\partial \rho}{\partial t} = 0$$

compressible & steady

$$\Rightarrow \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \longrightarrow$$ if flow is incompressible.





Strain, $\epsilon = \frac{AA'}{\Delta x}$

$$AA' = \left(u+\frac{\partial u}{\partial x}\Delta x\right)\Delta t + \Delta x - (u\Delta t + \Delta x)$$

$$= u\Delta t + \frac{\partial u}{\partial x}\Delta x\Delta t + \Delta x - u\Delta t - \Delta x$$

$$= \frac{\partial u}{\partial x}\Delta x\Delta t$$

$$\Rightarrow \frac{AA'}{\Delta x} = \frac{\partial u}{\partial x}\Delta t$$

$$\Rightarrow \epsilon_x = \frac{\partial u}{\partial x}\Delta t$$

$$\Rightarrow \dot{\epsilon}_x = \frac{\partial u}{\partial x}$$

$$\epsilon_{xx} = \frac{\partial u}{\partial x}$$

$$\epsilon_{yy} = \frac{\partial v}{\partial y}$$

$$\epsilon_{zz} = \frac{\partial w}{\partial z}$$

$$\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} = 0$$

$$\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.$$

$\vec{V} = (x^2 - 3y)\hat{i} + (x^3 - 4y)\hat{j} + (xy - 1)\hat{k}$

$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$

$u = x^2 - 3y$

$v = x^3 - 4y$

$w = xy - 1.$

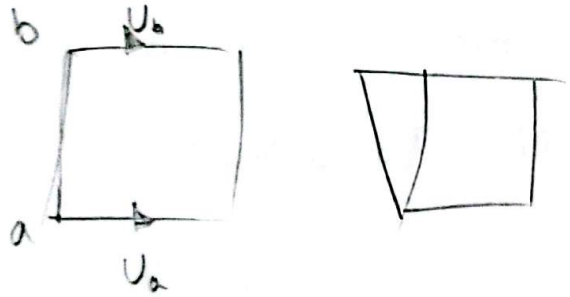
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$= \frac{\partial}{\partial x}(x^2 - 3y) + \frac{\partial}{\partial y}(x^3 - 4y) + \frac{\partial}{\partial z}(xy - 1)$$

$$= 2x - 4 + 0$$

$$= 2x - 4$$

not zero



angular deformation

$\Delta\beta$

$\tan\Delta\beta \simeq \Delta\beta$

$$\Delta\epsilon = \left(u + \frac{\partial u}{\partial y} \cdot \Delta y\right)\Delta t - u\Delta t$$

$$= \frac{\partial u}{\partial y}\Delta y\,\Delta t$$

$$\Rightarrow \frac{\Delta\epsilon}{\Delta Y} = \frac{\partial u}{\partial y}\Delta t$$

$$\Rightarrow \tan\Delta\beta = \frac{\partial u}{\partial y} \cdot \Delta t$$

$$\Rightarrow \Delta\beta = \frac{\partial v}{\partial y}\Delta t$$

$$\Delta\eta = \left(v + \frac{\partial v}{\partial x}\Delta x\right)\Delta t - v\Delta t$$

$$= \frac{\partial v}{\partial x}\Delta x\,\Delta t$$

$$\tan\Delta\beta = \frac{\Delta\epsilon}{\Delta Y}$$

$$\cancel{\Rightarrow \Delta\beta = \tan^{-1}\left(\frac{\Delta\eta}{\Delta\epsilon}\right)}.$$

$$\Rightarrow \Delta\beta = \tan^{-1}\left(\Delta\epsilon / \Delta Y\right).$$

$\Delta t \to 0$ very small

$$\tan \Delta\alpha = \frac{\partial \eta}{\partial x}$$

$$\Rightarrow \frac{\partial \eta}{\partial x} = \frac{\partial v}{\partial x} \Delta t$$

$$\Rightarrow \Delta\alpha = \frac{\partial v}{\partial x} \Delta t$$

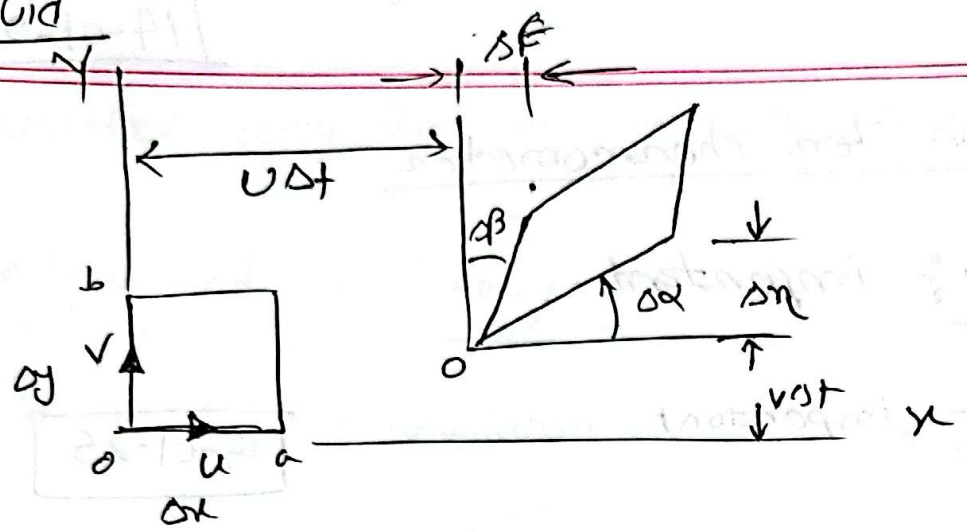
$$\omega_z = \lim_{\Delta t \to 0} \left(\frac{\Delta\beta - \Delta\alpha}{\Delta t} \right) \cdot \frac{1}{2}$$

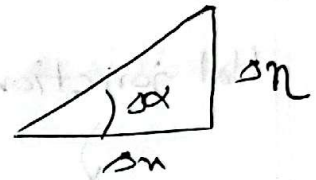
$$= \frac{1}{2}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right)$$

if value zero then no rotation.

Fluid

14-01-25





$$\Delta\eta = \left(v + \frac{\partial v}{\partial x}\Delta x\right)\Delta t - v\Delta t$$

$$= \frac{\partial v}{\partial x}\Delta x\,\Delta t$$

since $\Delta\alpha$ is very small so $\tan\Delta\alpha \cong \Delta\alpha$

$$\Rightarrow \frac{\Delta\eta}{\Delta x} = \frac{\partial v}{\partial x}\Delta t$$

$$\Rightarrow \tan\Delta\alpha = \frac{\partial v}{\partial x}\Delta t$$

$\Delta\alpha$ → rotation (anticlockwise)

$$\Rightarrow \Delta\alpha = \frac{\partial v}{\partial x}\Delta t$$

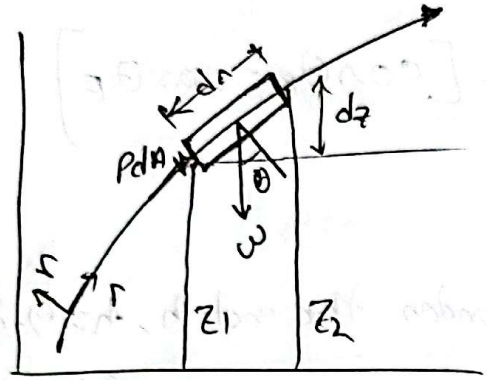
$$\omega_{oa} = \lim_{\Delta t \to 0}\left[\frac{\frac{\partial v}{\partial x}\Delta t}{\Delta t}\right]$$

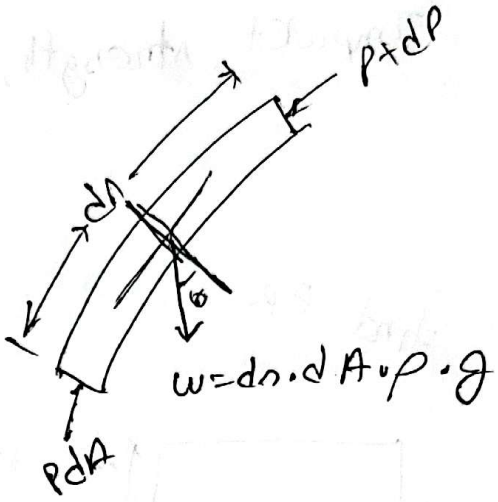
$$= \frac{\partial v}{\partial x}$$

Fluid I

04-02-25

Bernoulli's equation:

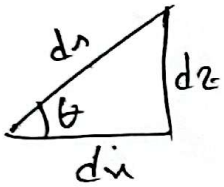




$w = dr \cdot dA \cdot \rho \cdot g$

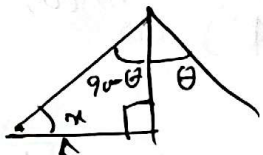
newton 2nd law of motion:- $F = ma$ $\Rightarrow$ $\cancel{F = PA}$

$F = PdA$.



$\sin\theta = \frac{dz}{dr}$

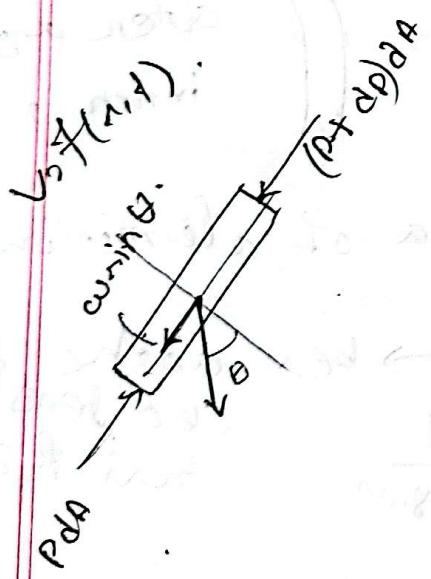
$\cos\theta = \frac{dx}{dr}$.



$x + 90 - \theta + 90 = 0$

$\Rightarrow x - \theta = 0$

$\boxed{\Rightarrow x = \theta}$



$(\rightarrow)$ $w = dr \cdot dA \cdot \rho g$

$\Rightarrow PdA - (p + dp) \cdot dA - w\sin\theta = dr\, dA\, \rho\, a$

$\Rightarrow -dp\, dA - dr \cdot dA\, \rho g \sin\theta = dr\, dA\, \rho \cdot a$

$\Rightarrow -dp - \rho g \cdot dz = dr \cdot \rho \cdot a$

$\Rightarrow -dp - \rho g\, dz = dr \cdot \rho \cdot \frac{dv}{dt}$

$\Rightarrow -dp - \rho g\, dz = dr \cdot v \cdot \frac{\partial v}{\partial r}\, \rho$

$$P = P(x, y)$$

$$\Rightarrow dp = \frac{\partial P}{\partial x} \cdot dx + \frac{\partial P}{\partial y} \cdot dy$$

$$\Rightarrow dv = \frac{\partial v}{\partial s} \cdot ds + \frac{\partial v}{\partial t} \cdot dt$$

$$\Rightarrow \frac{dv}{dt} = \frac{\partial v}{\partial s} \frac{ds}{dt} + \cancel{\frac{\partial v}{\partial t}}^{\,0} \cdot d$$

$$\Rightarrow \frac{dv}{dt} = v \cdot \frac{\partial v}{\partial s}$$

$$\Rightarrow \frac{dv}{dt} = v \frac{dv}{ds}.$$

$$\Rightarrow -dp - \rho g\, dz = ds \cdot v \cdot \frac{dv}{ds} \cdot \rho$$

$$\Rightarrow +dp + \rho g\, dz + \rho v\, dv = 0$$

$$\Rightarrow \frac{dp}{\rho g} + \frac{1}{g} v\, dv + dz = 0$$

$$\Rightarrow \frac{dp}{\gamma} + \frac{v\, dv}{g} + dz = 0$$

$$\Rightarrow \frac{1}{\gamma}\int dp + \frac{1}{g}\int v\, dv + \int dz = \text{const}$$

$$\Rightarrow \frac{P}{\gamma} + \frac{v^2}{2g} + z = \text{const.}$$

$$v_1 = v = \sqrt{\frac{2(P_{atm} - P)}{\rho}}$$

$$P_1 = P_{atm_p} + h_1 \rho_f g + h_2 \rho_f g$$

$$P_2 = P_{atm} + h \rho_f^* g + h_1 \rho_f g + h_2 \rho_f g$$

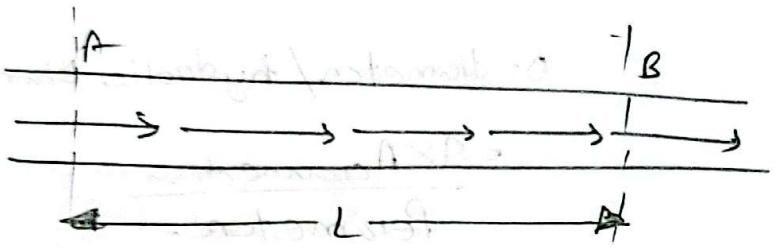
$$P_2 - P_1 = h \rho_f^* g \simeq h \rho_m g.$$

(Fluid) Slide 8, 9, 10 — 10-02-25

Bit

$$\frac{P}{\gamma} + \frac{v^2}{2g} + z = \text{const}$$

$$P + \rho\frac{v^2}{2} + \rho g z = \text{const} \quad \text{divied by } \gamma g$$



$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{LT}$$

→ friction loss

$$h_{LT} = \frac{P_A - P_B}{\gamma}$$

↳ Depends (L, v, friction, viscosity (μ)) ↑

→ velocity high → Turbulent

→ volcity low → laminar ok

for all thin h_{LT} will high

$Q = AV$

$Q/A = V$

if A same V will same at both end.

① Hagen Poiselle's Equation:

$$Q = \frac{\Delta P \pi D}{128 \mu \cdot L} \longrightarrow \text{only for laminar flow.}$$

② Darcy - Weinbach eqn:

$$h_{LT} = \frac{f L V_{avg}^2}{2 g D}$$

$$V_{avg} = \frac{Q}{A}$$

f = Darcy friction.

will given from the manufacturer / in question.

for Laminar flow; $f = \frac{64}{Re}$

if Turbulent flow, $\frac{1}{f} = -2 \cdot \log_{10}\left(\frac{\frac{\epsilon}{D}}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$

$\frac{Re}{M_{Re}} = \frac{\rho V D}{\mu}$

D = diameter / hydrolic Diameter

$= \frac{4 \times A_{cross\ section}}{Perimeter.}$

$= \frac{4 \times A}{P}$ → wetted perimeter

$Re = \frac{\text{Inertia force}}{\text{viscous force}} = \frac{ma}{\tau \cdot A} = \frac{\rho \cdot L^3 \cdot \frac{V}{t}}{\mu \cdot \frac{V}{L} L^2}$

$= \frac{\rho \cdot L^4 \cdot V^2}{\mu \cdot V \cdot L} = \frac{\rho V \cdot L}{\mu}$

frictional headloss two components

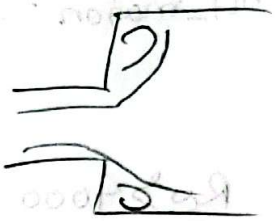
→ major head loss.

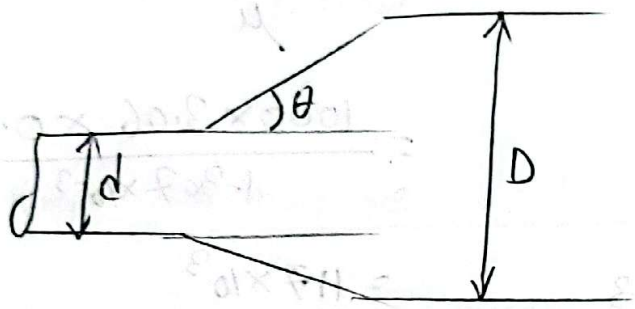
→ minor head loss → fittings

↳ $h_{lf} = \frac{f \cdot L}{D}$

$h_{L\,total} = h_{L\,major} + h_{L\,minor} = \Sigma h_{L\,major} + \Sigma h_{L\,minor}$

Sudden expansion, $K_L = \left(1 - \frac{d^2}{D^2}\right)^2$





$1L = 1000\,cm^3$

$= 1000 \times \frac{1}{100 \times 100 \times 100}\,m^3$

$= 10^{-3}\,m^3$

② $D = 5\,cm = 0.05\,m$ $\quad \therefore z_2 = 4\,m$

$Q = 6\,L/s = 6 \times 10^{-3}\,m^3/s$

$V_{avg} = \frac{Q}{A} = \frac{6 \times 10^{-3}}{\frac{\pi}{4} \times (0.05)^2} = 3.06\,m/s$

$\epsilon = 0.26\,mm = 2.6 \times 10^{-2}\,m$

$\mu = 1.307 \times 10^{-3}\,N.s/m^2$

Length of pipe = 89 m

at

① $\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_{L\,t}$ — ①

velocity at top surface zero.

$V_1 = 0$; $V_2 = 0$.

$P_1 = P_{atm}$; $P_2 = P_{atm}$

2 components major & minor

$$z_1 = 4 + h_{Lf_total} \quad \text{—— (2)}$$

$$h_{Lf_major} = \frac{f L V_{avg}^2}{2gD}$$

to find f we need R_e

$$R_e = \frac{\rho V D_H}{\mu}$$

$$= \frac{1000 \times 3.06 \times 0.05}{1.307 \times 10^{-3}}$$

$$= 117 \times 10^3$$

$$= 1.17 \times 10^5$$

$Re > 4000$ turbulent

relative roughness, $\frac{\epsilon}{D}$

$$= \frac{0.26 \times 10^{-3}}{0.05}$$

$$= 5.2 \times 10^{-3} \text{ m}$$

$$= 0.0052$$

find f from formula or moody diagram

$$f = 0.0315$$

$$h_{Lf_major} = \frac{0.0315 \times 89 \times (3.06)^2}{2 \times 9.81 \times 0.05}$$

$$h_{Lf_minor} = \Sigma K_L \left(\frac{V^2}{2g}\right)$$

$$K_L = 0.5 + 0.3 + 0.2 + 0.3 + 1.06$$

$$= 2.306$$

$$= 2.306 \times \frac{(3.06)^2}{2 \times 9.8}$$

$$=$$

$h_{Lt_total} = h_{Lt_major} + h_{Lt_minor}$

$= 27.89\ m.$

$z_1 = 4 + h_{Lt_total} = 31.89\ m$

Fluid-Lec-15

11.02.25

Pumps and turbine in Fluid system:-

$h_{pump} \uparrow$ means pump is more powerfull.

$P_{pump} = \rho g Q h_{pump}$

$$\eta_{eff} = \frac{P_{output}}{P_{input}} = \frac{\rho g Q h_{pump}}{P_{input}}$$

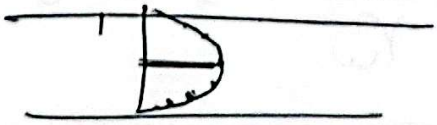
if turbine added

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_{pump} = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{Lt} + h_{turbine}$$

turbine absorbs energy pump output = turbine input

$= \rho g Q h_{turbine}$

$V_{avg} = V = \frac{Q}{A}$ in real life is velocity is not same velocity varying according to dis



Given, $Q = 0.15$ $\qquad P_{pump} = \rho g Q h_{pump}$.

$D_1 = .23\,m$ $\qquad V_1 = \frac{Q}{A_1} = \frac{Q}{\pi/4 (0.23)^2} = 3.61$

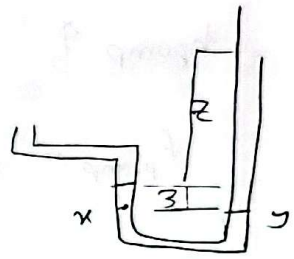
$D_2 = .18$ $\qquad V_2 = \frac{Q}{\pi/4 (0.18)^2} = .$

Ben,

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{z_1} + h_{pump} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_{L7}$$

$$h_{pump} = \frac{P_2}{\gamma} - \frac{P_1}{\gamma} + \square + z_2$$

$$= \frac{P_2 - P_1}{\gamma_w} + \square + z_2 \quad \text{—} (1)$$



$P_x = P_y \Rightarrow P_1 + z\rho_w g + 3g \times \rho_{Hg} = P_2 + z_2 \rho_w g + \cancel{z\rho_w g} + 3\rho_w g$

$\cancel{\Rightarrow P_2 - P_1 = 3 \times 9.8 \times 13600 - (z_2 + 3) \times 1000 \times 9.8}$

$\Rightarrow P_2 - P_1 = 3\rho_{Hg} g - z_2 \rho_w g - 3\rho_w g$

$\Rightarrow \frac{P_2 - P_1}{\rho_w g} = 3\left(\frac{\rho_{Hg}}{\rho_w}\right) - z_2 - 3$

$\Rightarrow \frac{P_2 - P_1}{\gamma_w} = 3 \times 13.6 - z_2 - 3$

$\Rightarrow \frac{P_2 - P_1}{\gamma_w} = 37.8 - z_2 \quad \text{——} (1)$

$$h_{pump} = \frac{P_2 - P_1}{\gamma_w} + \square + z_2$$

$$= 37.8 - z_2 + \square + z_2$$

$$h_{pump} = 42\ m$$

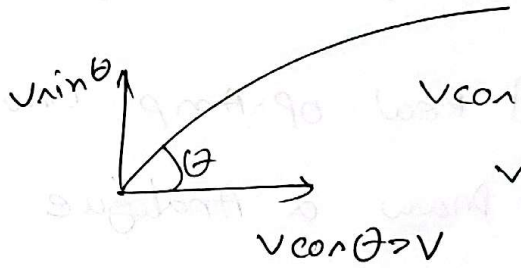
$$\therefore P_{pump} = \rho g Q \cdot h_{pump} = 1000 \times 9.8 \times 0.15 \times 42 =$$

(2) Tom = 22071

$Q = A V$

$= \frac{\pi}{4} \times (0.05)^2 \times 21.21$

$= 0.042\ m^3/s$



$V\cos\theta = 15$

$V = \frac{15}{\cos 45} = 21.21.$

applying Bernoulli in point 2, 3.

at datum $z_2 = 0$

$z_3 = 20$

$$\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + \cancel{z_2}^{\,0} = h_{L7} + \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + z_3 + h_{turbine}$$

Thursday

Fluid

13-02-25

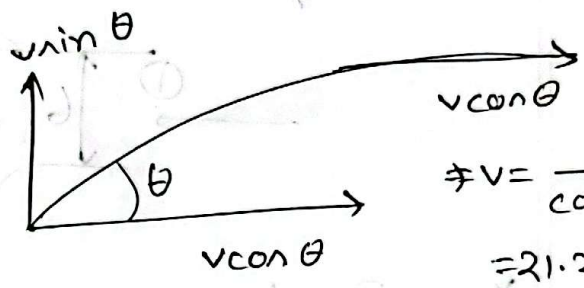
$P_{turbine} = \rho . g . Q . h_{turbine}$; $\eta_{turbine} = \dfrac{P_{output}}{\rho . g . Q . h_{turbine}}$.

2) $70\,m = 220\,ft$

$Q = AV$

$= \frac{\pi}{4} \times (0.05)^2 \times 21.21$

$= 0.042\,m^3/s$



$\Rightarrow V = \dfrac{15}{\cos 45}$

$= 21.21$

Bernoulli's eqn

$$\frac{P_2}{\gamma} + \alpha_2\left(\frac{V_2^2}{2g}\right) + z_2 = \frac{P_3}{\gamma} + \alpha_3\left(\frac{V_3^2}{2g}\right) + z_3 + h_{Turbine} + h_{Lt}$$

$$\Rightarrow \frac{P_2}{\gamma} + \frac{V_2^2}{2g} = \frac{P_{atm}}{\gamma} + \frac{(21.21)^2}{2 \times 9.81} + 20 + h_{turbine} \quad \text{— (I)}$$

$V_3 = 21.21$; $h_{Lt} = 5m$

$z_2 = 0$

$z_3 = 20m$

$\alpha_1 = \alpha_2 = 1$.

$P_3 = P_{atm}$.

For point 1, 2

$$\frac{P_1}{\gamma} + \left(\frac{V_1^2}{2g}\right)\alpha_1 + z_1 = \frac{P_2}{\gamma} + \left(\frac{V_2^2}{2g}\right)\alpha_2 + \cancel{z_2}^{\,0} + \cancel{h_{Lt}}^{\,0}$$

$\alpha_1 = \alpha_2 = 1$

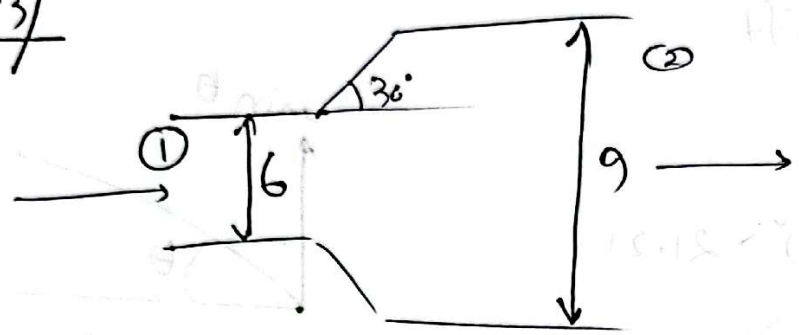
$V_1 = 0$

$P_1 = P_{atm}$.

$$\Rightarrow \frac{P_2}{\gamma} + \frac{V_2^2}{2g} = \frac{P_{atm}}{\gamma} + 0 + 70 \quad \text{— (II)}$$

From eq^n ① & ⑪ $h_{turbine} = 22.06\,m$

Problem - 3/



$V_{avg} = 7\,m/s$ find head loss

$P = 150\,kPa$

$= 150 \times 10^3\,Pa$

$d_1 = 0.06\,m$

$d_2 = 0.09\,m$

$Q = A_1 V_1 = A_2 V_2$

$$\Rightarrow V_2 = \frac{A_1 V_1}{A_2} = \frac{\frac{\pi}{4} \times (0.06)^2 \times 7}{\frac{\pi}{4} \times (0.09)^2}$$

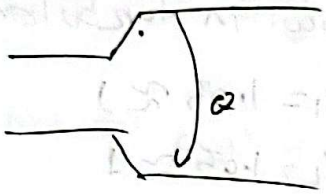
$\Rightarrow V_2 = 3.11\,m/s$

$$\frac{P_1}{\gamma} + \alpha_1 \left(\frac{V_1^2}{2g}\right) + z_1 = \frac{P_2}{\gamma} + \alpha_2 \left(\frac{V_2^2}{2g}\right) + z_2 + \cancel{h_{L\,f\text{-major}}} + h_{L\,f\text{-minor}}$$

0

$Re_1 = \frac{\rho v_1 D_1}{\mu}$ $\quad Re_2 = \frac{\rho V_2 D_2}{\mu}$ $\quad \alpha > 2$ (Laminar)

$\alpha > 1.4$ (Turb



$\theta = 60 \quad k_L = 0.07$

$$h_{L\text{-minor}} = K_L \frac{v_1^2}{2g} = 0.07 \times \frac{7^2}{2 \times 9.81}$$

<u>Impulse momentum law:</u> Newton's 2nd law

$$F = ma = m\left(\frac{\Delta v}{\Delta t}\right) = m\left(\frac{v_2 - v_1}{\Delta t}\right) = \frac{m \cdot v_2 - m v_1}{\Delta t}$$

Rate of change in momentum $= \frac{m v_2 - m v_1}{\Delta t}$

$\frac{m}{\Delta t}$ = mass flow rate.

$\dot{m} = \rho Q$

$$F = \frac{m}{\Delta t}(v_2 - v_1)$$
$$= \dot{m}(v_2 - v_1)$$
$$= \rho Q (v_2 - v_1)$$

if flow is compressive

$$F = \dot{m}_2 v_2 - \dot{m}_1 v_1 = \rho_2 Q_2 v_2 - \rho_1 Q_1 v_1$$

if force is zero, $\quad F = \rho_2 Q_2 v_2 - \rho_1 Q_1 v_1$

$$0 = \rho_2 Q_2 v_2 - \rho_1 Q_1 v_1$$

$$\therefore \rho_1 Q_1 v_1 = \rho_2 Q_2 v_2$$

pipe length is not mentioned
so $h_{Lt\text{-}major} = 0$.
no fitting so $h_{Lt\text{-}minor} = 0$

(1) $P_{pump} = \rho \cdot g \cdot Q \cdot h_{pump}$

$Q = A_1 V_1 = A_L V_L$

$\Rightarrow \frac{\pi d_1^2}{4} V_1 = \frac{\pi}{4} d_L^2 V_L$

$\Rightarrow d_1^2 V_1 = d_L^2 V_L$

$\Rightarrow V_1 = \left(\frac{d_L}{d_1}\right)^2 V_L$

lets assume,
flow in turbulent
$\alpha_1 = 1.05 \approx 1$
$\alpha_L = 1.05 \approx 1$.

$z_2 = 0$
$z_3 = 3.5$

$$\frac{P_L}{\gamma} + \alpha_2 \left(\frac{V_L^2}{2g}\right) + z_L = \frac{P_3}{\gamma} + \alpha_3 \left(\frac{V_3^2}{2g}\right) + z_3 + h_{pump} + \cancel{h_{Lt\text{-}major}}^{0} + \cancel{h_{Lt\text{-}minor}}^{0}$$

$$\Rightarrow \frac{P_{atm}}{\gamma} + 1.5 + 0 + 0 = \cancel{\frac{P_{atm}}{\gamma}} + \frac{V_3^2}{2g} + 3.5 + h_{pump}$$

$V_2 = 0$

$$\Rightarrow \frac{V_3^2}{2g} + 2 + h_{pump} = 0 \quad \text{—— (1)}$$

$P_2 = P_{atm} + 1.5 g \cdot \rho$
$= P_{atm} + 1.5\gamma$

$$h_{pump} = \frac{P_{pump}}{\rho g Q} = \frac{P_{pump}}{\rho g A_3 V_3} \quad \text{—— (11)}$$

use, $\frac{\boxed{}}{V_3} = h_{pump}$ in eqn (1).

Fluid

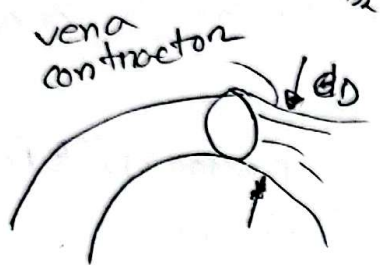
17-02-25

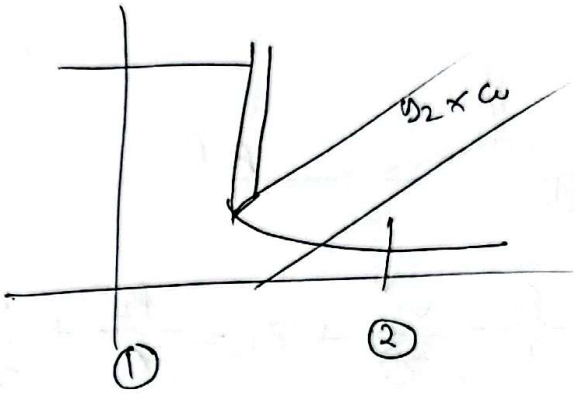
Problem 4)

$D_1 = 0.46m$

$D_2 = 0.051\ m$

maintaining constant hieight thath why speed is negligible.





$Q = A \cdot V$

$= D_2 \times \omega \times V_L$

$\Rightarrow \frac{Q}{\omega} = D_2 V_L$ —— (1)

Bernoulli's eqn:

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

$V_1 = 0$

$z_1 = 0.46m$

$z_2 = 0.051\ m.$

$P_1 = P_{atm}$

$P_2 = P_{atm}$

$\Rightarrow \frac{V_2^2}{2g} = z_1 - z_2$

$\Rightarrow V_2 = \sqrt{(z_1 - z_2) \times 2g}$

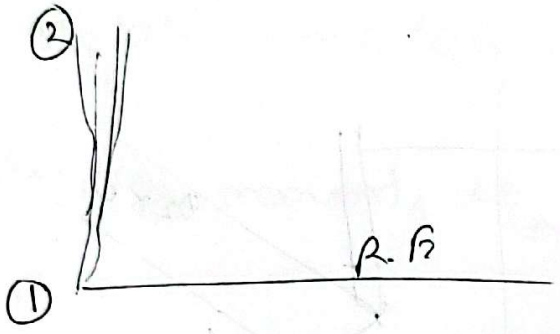
$\Rightarrow V_2 = 2.833\ m/s$

using value of V_2 in eqn (1) $\frac{Q}{\omega} = 0.051 \times 2.833$

$\Rightarrow \frac{Q}{\omega} = 0.14\ \frac{m^3/s}{m}$

In fluid area decreased mean velocity increased.

In datum line or reference line height is zero $z_1 = 0$



$P_1 = 400 \times 10^3$ kPa

$z_1 = 0$

$z_2 = h$

$V_1 = 0$ below the thumb.

$P_2 = P_{atm} = 0$

for simplification putting $P_{atm} \approx 0$

$P_2 = 101.3$ kPa

$$\frac{P_1}{\gamma} + \cancel{\frac{V_1^2}{2g}} + \cancel{z_1} = \cancel{\frac{P_2}{\gamma}} + \cancel{\frac{V_2^2}{2g}} + z_2$$

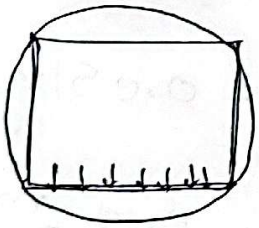
$$\Rightarrow z_2 = \frac{400 \times 1000}{1000 \times 9.8} = 40.78 \text{ m}$$

2) Difference between ideal & real fluid.

3) Difference " Duct, Pipe & tube.
& which condition you will use there.

Duct → rectangular cross section.

tube → circular



high pressure stress will be at the corner

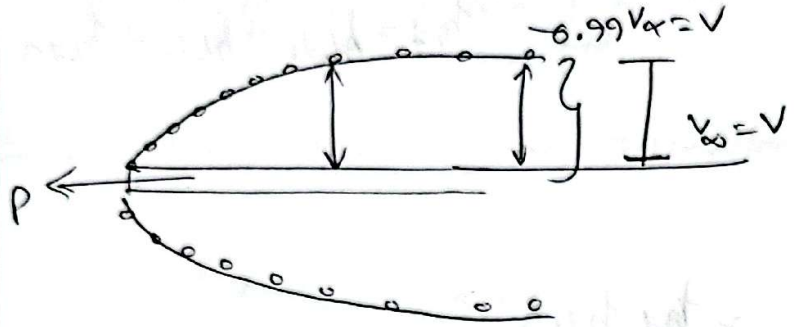
inside pressure > atm pressure

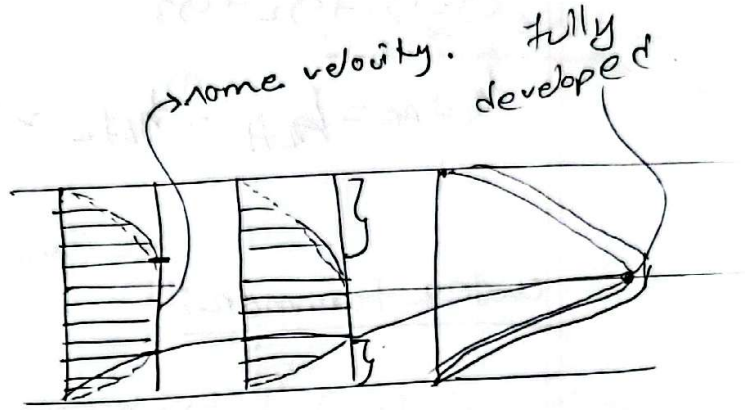
than use circular cross section

P_o > inside pressure.

Boundary layer in pipe flow:-

V_∞ = Free stream velocity.





laminer flow, $L_h = 0.05 Re D$

turbulent flow, $L_h = 1.359 D Re^{1/4}$ $> 10D$ (approx). D = Diameter

Re =

Kinetic energy correction Factor:-

$$\frac{P_1}{\gamma} + \alpha_1\left(\frac{V_1^2}{2g}\right) + z_1 = \frac{P_2}{\gamma} + \left(\frac{V_2^2}{2g}\right)\alpha_2 + z_2 + h_{L1}$$

if laminer flow, $\alpha = 2$

Turbulent flow, $\alpha = 1.05 \approx 1$.

if flow in Turbulent less loss will occur.

Pipe in Series:-

→ mass flow will same.

$$q = q_3 = q_2 = q_1$$

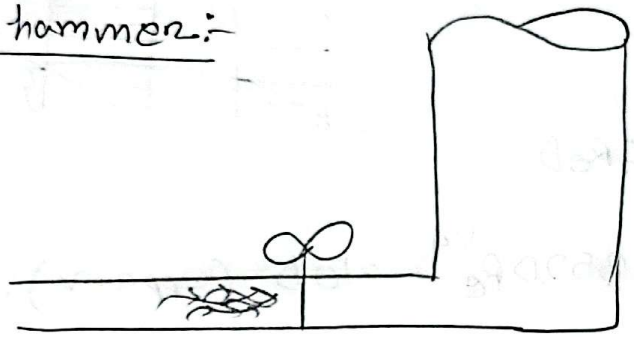
$$h_{Lf} = h_{Lf_1} + h_{Lf_2} + h_{Lf_3}.$$

Pipe in parallel:-

$$q = q_1 + q_2 + q_3$$

$$h_{Lf_{AB}} = h_{Lf_1} = h_{Lf_2} = h_{Lf_3}$$

water hammer:-



9 water hammer:-

9 write the name of 10 common types of pipes we mostly used

9 valve types

Part - B

Fluid

18-02-23

Dynamic analysis:

$$P_{turbine} = f(N, n, h, \rho).$$

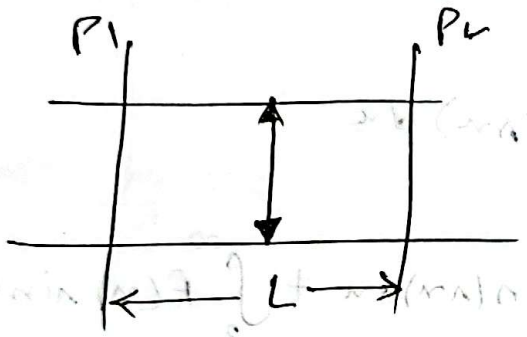
$$\frac{P}{\gamma} + \frac{v^2}{2g} + z = \text{const}$$

$m(L)$ $m(L)$ $m(L)$

$J = N \cdot m$

$= MLT^{-2} \cdot LT^{-}$

$= ML^{2}T^{-2}$



$$\Delta P = f(L, D, V, \rho, \mu)$$

Fluid

24-02-25

$\Delta P = f(D, L, \mu, \rho, v)$ MLT

$\Delta P = K D^a L^b \mu^c \rho^d v^e$ FLT

Problem 1

$R = f(D, v, \rho, \mu)$

$R_e = \frac{\rho v D}{\mu}$

$D = L$

$R = MLT^{-2}$

$v = LT^{-1}$

$\rho = ML^{-3}$

$\mu = \frac{N.S}{m^2} = \frac{MLT^{-2}.T}{L^2}$

$= ML^{-1}T^{-1}$

$R = K[D^a . v^b . \rho^c . \mu^D]$

$\Rightarrow MLT^{-2} = K[L^a (LT^{-1})^b (ML^{-3})^c . (ML^{-1}T^{-1})^D]$

for M

$1 = c + d$

$c = 1 - d$

for L

$1 = a + b - 3c - d$

$\Rightarrow 1 = a + 2 - d - 3 + 3d - d$

$= a - 1 + d$

$\Rightarrow a = 2 - d$

for T

$-2 = -b - d$

$b = 2 - d$

$R = K[D^{2-d} v^{2-d} \rho^{1-d} \mu^d]$

$= K\left[\frac{D^2 v^2}{(Dv)^d} \frac{\rho \mu^d}{\rho^d}\right]$

$= K\left[\rho v^2 D^2 \left(\frac{\mu}{Dv\rho}\right)^d\right]$

$$R = \rho v^{v} D^{b} \cdot f\left(\frac{\mu}{\rho v D}\right).$$

(3)

$$P = f(\rho, N, D, H, g)$$

$$= K\left[\rho^{a} \cdot N^{b} \cdot D^{c} \cdot H^{d} \cdot g^{e}\right]$$

$$ML^{2}T^{-3} = K\left[(ML^{-3})^{a} \cdot (T^{-1})^{b} \cdot L^{c} \cdot L^{d} \cdot (LT^{-2})^{e}\right].$$

for M

$1 = a$ $\qquad 2 = -3a + c + d + e \qquad -3 = -b - 2e$

$\qquad b = 3 - 2e$

$\Rightarrow 2 = -3 + c + d + e$

$\Rightarrow c = 5 - d - e$

$$P = K\left[\rho^{a} \cdot N^{b} \cdot D^{c} \cdot H^{d} \cdot g^{e}\right]$$

$$= K\left[\rho^{a} \cdot N^{3-2e} \cdot D^{5-d-e} \cdot H^{d} \cdot g^{e}\right]$$

$$= K\left[\rho N^{3} D^{5} \left(N^{-2e} D^{-e} \cdot g^{e} \cdot D^{-d} \cdot g^{e} \cdot H^{d}\right)\right]$$

$$= K\left[\rho N^{3} D^{5} \cdot \left(\frac{g}{N^{2} D}\right)^{e} \cdot \left(\frac{H}{D}\right)^{d}\right]$$

$$= K\left[\rho N^3 D^5 \cdot \left(\frac{\partial H}{N^2 D^2}\right)^{c_1}\right] \quad c_1 = de$$

$$= \rho N^3 D^5 f\left(\frac{N^2 D^2}{\partial H}\right)$$

<u>Buckingham's π - Theorem:</u>

if $\Delta P = f(v, D, L, \rho, N)$ $\quad$ k = no of variable = 6

$\Rightarrow f(\Delta P, v, D, L, \rho, \mu) = 0$ $\quad$ n = 3

No. of function on π-term = k-n = 3

$\pi_1 = f$ (geometry, flow characteristic).

$\pi_1 = f(D^a, \rho^b, v^c) \cdot \Delta P$

$\pi_2 = f(D^a, \rho^b, v^c)\, L$

$\pi_3 = f(D^a, \rho^b, v^c)\, \mu$

normal to a moving stream fluid → fluid moving vertically

Drag = force

(2) $D = f(\omega, h, v, \mu, \rho)$

$f(D, \omega, h, v, \mu, \rho) = 0$

No. of primary / fundamental dimension

$n = 3$

No of pi-terms $= k - n$
$= 6 - 3$
$= 3$

Geometry,
fluid property,
flow characteristic.

$k = 6$.

FLT	MLT
$D = F$	$D = MLT^{-2}$
$\omega = L$	$\omega = L$
$h = L$	$h = L$
$v = LT^{-1}$	$v = LT^{-1}$
$\mu = F.T.L^{-2}$	$\mu = ML^{-1}T^{-1}$
$\rho = \gamma = FL^{-3}$	$\rho = ML^{-3}$

$\rho = \frac{kg}{m^3} \times \frac{m}{s^2}$

$= \frac{N}{m^3}$

The repeated variables $= \omega, \rho, v$

(1)

$\pi_1 = D\ \omega^a . v^b . \rho^c$

$\pi_2 = h\ \omega^a . v^b . \rho^c$

$\pi_3 = \mu\ \omega^a . v^b . \rho^c$

(2)

$\pi_1 = D\ \omega^a . v^b . \rho^c$

$M^0L^0T^0 = (MLT^{-2}).L^a.(LT^{-1})^b.(ML^{-3})^c$

$0 = 1 + c$ $\quad$ $0 = 1 + a + b - 3c$

$0 = -2 - b$

$b = 2$
$c = -1$
$a = -2$

(3)

$\pi_1 = D\omega^{-2} v^{-2} . \rho^{-1}$

$= \frac{D}{\rho \omega^2 v^2}$

For π_L

$$M^0 L^0 T^0 = L^a \cdot (LT^{-1})^b (ML^{-3})^c$$

$$c = 0, \quad b = 0, \quad a = -1$$

$$\pi_2 = h \cdot \omega^{-1}$$

$$\pi_2 = h/\omega$$

$$\pi_3 = \frac{\mu}{\omega v \rho}$$

$$\pi_1 = f(\pi_2, \pi_3)$$

$$\frac{D}{\omega^2 v^2 \rho} = f\left(\frac{h}{\omega}, \frac{\mu}{\omega v \rho}\right)$$

L-2 Undistorted model analysis

Fluid

25-02-25

Hydraulic similarity:

$$\frac{A_p}{A_m} = \frac{L_p \times W_p}{L_m \times W_m}$$

$$= (L_r)^2$$

$$\frac{V_p}{V_m} = (L_r)^3$$

$$\frac{1}{S} = \frac{L_m}{L_p} = \frac{h_m}{h_p}$$

$$S = \frac{L_p}{L_m}$$

$$\frac{L_p}{L_m} = L_r$$

$$\frac{W_p}{W_m} = L_r$$

$$S_h = \frac{W_m}{W_p} = \frac{L_m}{L_p}$$

$$S_v = \frac{H_m}{H_p}$$

scale ratio = λ

$$\frac{Q_m}{Q_p} = \frac{A_m}{A_p} \cdot \frac{V_m}{V_p}$$

$$= \left(\frac{1}{\lambda}\right)^2 \cdot \frac{1}{\sqrt{\lambda}}$$

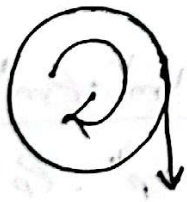
$$= \left(\frac{1}{\lambda}\right)^{2.5}$$

volume / time = flow rate = Area × velocity

$$\frac{F_m}{F_p} = \frac{A_m}{A_p} \cdot \frac{\rho_m}{\rho_p} \left(\frac{V_m}{V_p}\right)^2$$

$$= \frac{1}{\lambda^2} \cdot \frac{\rho_m}{\rho_p} \left(\frac{1}{\sqrt{\lambda}}\right)^2$$

$$\frac{F_m}{F_p} = \frac{1}{\lambda^3} \frac{\rho_m}{\rho_p}$$



linear velocity $v = \omega \cdot r$

$$\omega = \frac{2\pi N}{60} \qquad V = \omega . r = \frac{2\pi N r}{60} = \frac{\pi D N}{60}$$

$$\frac{N_m}{N_p} = \frac{V_m}{D_m} \times \frac{D_p}{V_p}$$

$$= \frac{D_p}{D_m} \times \frac{V_m}{V_p}$$

$$= n \cdot \frac{1}{\sqrt{n}}$$

$$= \sqrt{n}$$

Pro2) $n = 25$

$V_m = 1.2$

$F_m = 5N$

$A_m = 1.0 m^2$

$$\frac{P}{\gamma} + \frac{v^2}{2g} + z = \text{const}$$

$$\Rightarrow P + \frac{v^2}{2}\rho + z\rho g = \text{const}$$

term of pressure

$$f = \frac{1}{2} v^2 \rho . K_{pm}$$

$$F_m = C_{dm} \frac{1}{2} V_m^2 \rho_m^2$$

$$F_p = C_{dp} . \frac{1}{2} . V_p^2 \rho_p^2$$

wk, $\frac{V_m}{V_p} = \frac{1}{\sqrt{25}}$

$\therefore V_p = \sqrt{25} \times 1.2 = 60$

Drag = Drag force × Area

$$\frac{F_m}{F_p} = \frac{C_{dm} . \frac{1}{2} V_m^2 . \rho_m^2}{C_{dp} . \frac{1}{2} V_p^2 . \rho_p^2}$$

$$\frac{5}{F_p} = \frac{0.08}{0.016} \times \left(\frac{V_m}{V_p}\right)^2$$

$$\Rightarrow F_p = 500 N.$$

$F_m = C_{dm} \frac{1}{2} v_m^2 \cdot \rho_m^2 = 2.88N$

Drag force $= F - F_m = 5 - 2.88 = 2.12$ coming down wave generated.

Fluid

10-03-25

~~in compressive flow~~

$$\frac{dp}{\rho} + v\,dv = 0$$

$$pv^k = \text{const}$$

$$= \frac{p}{\rho^k} = \text{const}$$

$$= \frac{p}{\rho^k} = \frac{p_1}{\rho_1^k}$$

$$\frac{1}{\rho^k} = \frac{p_1}{\rho_1^k} \cdot \frac{1}{p}$$

$$\Rightarrow \frac{1}{\rho^k} = \left(\frac{p_1}{p}\right)^{1/k} \cdot \frac{1}{\rho_1}$$

$$pv^k = \text{const.}$$

$$\cancel{p = \rho^k}$$

$$p = \rho^k$$

$$\frac{dp}{\rho} + v\,dv = 0$$

$$\Rightarrow \frac{p_1^{1/k}}{\rho_1} \int_1^2 \frac{dp}{p^{1/k}} + \int_1^2 v\,dv = 0$$

$$\Rightarrow \frac{p_1^{1/k}}{\rho_1} \left[\frac{p^{-1/k+1}}{-\frac{1}{k}+1}\right]_1^{1/2} + \frac{1}{2}(v_2^2 - v_1^2) = 0$$

$$\Rightarrow \frac{k}{k-1} \frac{p_1^{1/k}}{\rho_1} \left[p_2^{\frac{k-1}{k}} - p_1^{\frac{k-1}{k}}\right] + \frac{1}{2}(v_2^2 - v_1^2) = 0$$

$$\Rightarrow \frac{1}{2}(v_2^2 - v_1^2) = \frac{k}{k-1}\left[\frac{p_1^{\frac{k-1}{k}} \cdot p_1^{1/k}}{\rho_1} - p_2^{\frac{k-1}{k}} \cdot \frac{p_1^{1/k}}{\rho_1}\right]$$

$$\Rightarrow \quad = \frac{k}{k-1}\left[\frac{p_1}{\rho_1} - \frac{1}{\rho_1} p_2^{\frac{k-1}{k}} p_1^{1/k}\right]$$

$P = \rho R T$

$P_1 = \rho_1 R T_1$

$P_2 = \rho_2 R T_2$

$$= \frac{K}{K-1}\left[\frac{P_1}{\rho_1} - P_2^{\frac{K-1}{K}} \cdot \frac{P_2^{1/K}}{\rho_2}\right]$$

$$\frac{1}{2}(v_2^2 - v_1^2) = \frac{K}{K-1}\left[\frac{P_1}{\rho_1} - \frac{P_2}{\rho_2}\right] \quad - (1)$$

$$\Rightarrow \frac{1}{2}(v_2^2 - v_1^2) = \frac{KR}{K-1}(T_1 - T_2)$$

$$\Rightarrow \frac{1}{2}(v_2^2 - v_1^2) = \frac{K}{K-1} \cdot \frac{P_1}{\rho_1}\left[1 - \frac{P_2}{\rho_2} \cdot \frac{\rho_1}{P_1}\right]$$

$$\frac{P_1}{\rho_1^K} = \frac{P_2}{\rho_2^K}$$

$$\Rightarrow \left(\frac{\rho_1}{\rho_2}\right)^K = \frac{P_2}{P_1}$$

$$\Rightarrow \frac{\rho_1}{\rho_2} = \left(\frac{P_1}{P_2}\right)^{1/K}$$

$$= \left(\frac{P_2}{P_1}\right)^{-1/K}$$

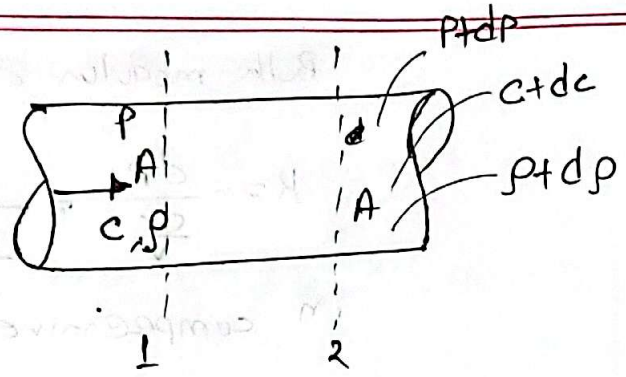
$$\Rightarrow \frac{1}{2}(v_2^2 - v_1^2) = \frac{K}{K-1} \cdot \frac{P_1}{\rho_1}\left[1 - \frac{P_2}{P_1} \cdot \frac{\rho_1}{\rho_2}\right]$$

$$\Rightarrow \frac{1}{2}(v_2^2 - v_1^2) = \frac{K}{K-1} \cdot \frac{P_1}{\rho_1}\left[1 - \frac{P_2}{P_1}\left(\frac{P_2}{P_1}\right)^{-1/K}\right]$$

$$\Rightarrow \frac{1}{2}(v_2^2 - v_1^2) = \frac{K}{K-1} \cdot \frac{P_1}{\rho_1}\left[1 - \left(\frac{P_2}{P_1}\right)^{1-1/K}\right]$$

$$\Rightarrow \frac{1}{2}(v_2^2 - v_1^2) = \frac{K}{K-1} \frac{P_1}{\rho_1}\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}}\right]$$

[Proved]



compressive flow density will not same

$\dot{m} = A v \rho$

an mass flow rate is same so,

$\dot{m}_1 = \dot{m}_2$

$\Rightarrow A.C.\rho = A(C+dc)\cdot(\rho+d\rho)$

$\Rightarrow \cancel{AC\rho} = \cancel{AC\rho} + A.C.d\rho + A.dc.\rho + \cancel{A.dc.d\rho}$ — very small value

for pressure difference fluid is moving.
point 1 P. High
point 2 P. low
fluid ~~low~~ high P - low P

$\Rightarrow A.C.d\rho + A\rho dc = 0$

$\Rightarrow C d\rho + \rho dc = 0$

$\Rightarrow dc = -C\dfrac{d\rho}{\rho}$ —— (I)

we can apply

$F = ma = m.\dfrac{dv}{dt}$

$P.A = \dfrac{m}{dt}.dv$

$\boxed{P.A = \dot{m}.dv}$

fluid is moving left to right,

$[P-(P+dP)].A = \rho.C.A[(C+dc)-C]$

$\Rightarrow P - P - dP = \rho C.dc$

$\Rightarrow dc = -\dfrac{dP}{\rho.C}$ —— (II)

from equation (I) & (II)

$$-e\frac{d\rho}{\rho} = -\frac{dp}{\rho \cdot c}$$

$$\Rightarrow c^2 d\rho = dp$$

$$\Rightarrow c = \sqrt{\frac{dp}{d\rho}}$$

velocity for compressive flow

$$\Rightarrow c = \sqrt{E/\rho}$$

$$\Rightarrow c = \sqrt{\frac{E}{\rho}}$$

Bulk modulus of elasticity

$$K = -\frac{dp}{\frac{dv}{v}} = \frac{dp}{\frac{d\rho}{\rho}}$$

in compressive.

$$E = \frac{dp}{\frac{d\rho}{\rho}}$$

in pai

$$\Rightarrow E = \frac{dp}{d\rho} \cdot \rho$$

$$\Rightarrow \frac{dp}{d\rho} = E/\rho$$

if $p = \rho RT$

$$\rho = \frac{p}{RT}$$

For isothermal condition;

$$pv = \text{const}$$

$$\Rightarrow p \cdot dv + v \cdot dp = 0$$

$$\Rightarrow p = -v\frac{dp}{dv}$$

$$\Rightarrow p = -\frac{dp}{dv/v} = \frac{dp}{\frac{d\rho}{\rho}}$$

$$\Rightarrow p = E$$

$$c = \sqrt{\frac{p}{\rho}}$$

$$= \sqrt{\frac{p}{p/RT}}$$ [using $p = \rho RT$ for isothermal]

$$\boxed{c = \sqrt{RT}}$$ velocity of sound for isothermal condition.

Adiabatic condition;

$$pv^K = \text{const}$$

$$\Rightarrow dp.v^K + Kv^{K-1}.dv = 0$$

$$\Rightarrow dp + \frac{K}{v}.dv = 0$$

$$\Rightarrow$$

$$pv^K = \text{const}$$

$$\Rightarrow dpv^K + p.Kv^{K-1}dv = 0$$

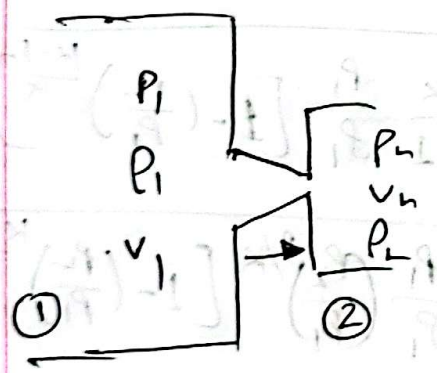
$$\Rightarrow dp + p.\frac{K}{v}.dv = 0$$

$$\Rightarrow Kp = -\frac{dp}{dv/v}$$

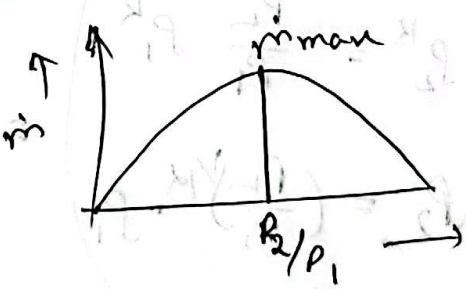
$$\Rightarrow K p = E$$

velocity of sound in adiabatic condition; $\cancel{C = \sqrt{KRT}}$

$$C = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{Kp}{\rho}} = \sqrt{\frac{Kp}{\frac{p}{RT}}} = \sqrt{KRT}$$



$$\frac{P_T}{P_2} \uparrow = \dot{m} \uparrow$$



Point 1

max flow rate is zero

cuz inlet nozzle velocity $\boxed{v_1 = 0}$

from energy eqn:

$$\frac{1}{2}(v_2^2 - v_1^2) = \frac{K}{K-1}\frac{P_1}{\rho_1}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}}\right]$$

velocity of inlet nozzle, $v_1 = 0$

$$\therefore v_2 = \sqrt{\frac{2K}{K-1}\frac{P_1}{\rho_1}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}}\right]}$$

for point @ max flow rate

$$\dot{m} = v_2 \rho_2 A_2$$

$$= A_2 \rho_2 \left[\frac{2K}{K-1}\frac{P_1}{\rho_1}\left[1-\left(\frac{P_2}{P_1}\right)^{K-1/K}\right]\right]^{1/2} \quad \text{—— (1)}$$

$$\frac{P_1}{\rho_1^K} = \frac{P_2}{\rho_2^K}$$

$$\Rightarrow \rho_2^K = \frac{P_2}{P_1}\rho_1^K$$

$$\Rightarrow \rho_2 = \left(\frac{P_2}{P_1}\right)^{1/K}\cdot\rho_1$$

$$\cancel{= A_2\rho_2}$$

$$= A_2\left(\frac{P_2}{P_1}\right)^{1/K}\cdot\rho_1\sqrt{\frac{2K}{K-1}\frac{P_1}{\rho_1}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}}\right]}$$

$$= A_2\rho_1\sqrt{\frac{2K}{K-1}\frac{P_1}{\rho_1}\left(\frac{P_2}{P_1}\right)^{2/K}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}}\right]}$$

$$= A_2\sqrt{\frac{2K}{K-1}\rho_1^2\cdot\frac{P_1}{\rho_1}\left[\left(\frac{P_2}{P_1}\right)^{2/K}-\left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}+\frac{2}{K}}\right]}$$

$$\dot{m}_2 = A_2\sqrt{\frac{2K}{K-1}P_1\rho_1\left[\left(\frac{P_2}{P_1}\right)^{2/K}-\left(\frac{P_2}{P_1}\right)^{\frac{K+1}{K}}\right]}$$

$$\dot{m}_2 = A_2 \sqrt{\frac{2K}{K-1} P_1 \rho_1 \left[x^{2/K} - x^{K+1/K} \right]}$$ Here $x = \frac{P_2}{P_1}$

if $\frac{d\dot{m}}{dx} = 0$

$$\Rightarrow A_2 \cdot \frac{1}{2} \left[\frac{2K}{K-1} \cdot P_1 \rho_1 \right]^{1/2} \cdot \left(x^{2/K} \cdot x^{K+1/K} \right)^{-1/2} \left[\frac{2}{K} x^{\frac{2}{K}-1} - \frac{K+1}{K} x^{\frac{K+1}{K}-1} \right] = 0$$

$$\Rightarrow \frac{2}{K} x^{2/K - 1} - \frac{K+1}{K} x^{\frac{K+1}{K} - 1} = 0$$ since $x^{2/K} \cdot x^{\frac{K+1}{K}} \neq 0$

$$\Rightarrow \left[\frac{2}{K} x^{\frac{2-K}{K}} - \frac{K+1}{K} x^{\frac{K+1-K}{K}} \right] = 0$$

$$\Rightarrow 2 \cdot x^{2-K/K} - (K+1)\, x^{1/K} = 0$$

$$\Rightarrow x = \left(\frac{2}{K+1} \right)^{K/K-1}$$

Fluid

11-03-25

Critical pressure ratio:-

$$\boxed{\frac{P_2}{P_1} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}} \qquad k = 1.4$$

$$= 0.528$$

$$\dot{m} = A_2 \rho_1 \sqrt{\frac{2k}{k+1}} \sqrt{\frac{P_1}{\rho_1}\left[\left(\frac{P_2}{P_1}\right)^{2/k} - \left(P_2/P_1\right)^{k+1/k}\right]}$$

$$\dot{m}_{(max)} = A_2 \sqrt{\frac{2k}{k-1} P_1 \rho_1 \left[\left(\frac{2}{k+1}\right)^{\frac{k}{k-1}\cdot\frac{2}{k}} - \left(\frac{2}{k+1}\right)^{\frac{k+1}{k}\cdot\frac{k}{k-1}}\right]}^{1/2}$$

$$= A_2 \sqrt{\frac{2k}{k-1} P_1 \rho_1 \left[\left(\frac{2}{k+1}\right)^{\frac{2}{k-1}} - \left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\right]}^{1/2}$$

$$= A_2 \sqrt{\frac{2k}{k-1} P_1 \rho_1} \sqrt{\left(\frac{2}{k+1}\right)^{2/k-1}\left[1 - \left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\left(\frac{2}{k+1}\right)^{-2/k-1}\right]}$$

$$= A_2 \sqrt{\frac{2k}{k-1} P_1 \rho_1} \left(\frac{2}{k+1}\right)^{1/k-1}\left[1 - \frac{2}{k+1}\right]^{1/2}$$

$$= A_2 \sqrt{\frac{2k}{k-1} P_1 \rho_1} \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \sqrt{\frac{k-1}{k+1}}$$

$\sqrt{k-1}$

$\sqrt{k-1}$

$$= \cancel{A_2 \sqrt{\frac{2k}{k+1} P_1 \rho_1} \left(\frac{2}{k+1}\right)^{1/k-1} \left(\frac{2}{k+1}\right)^{1/2}}$$

$$= \cancel{A_2 \sqrt{\left(\frac{k}{k+1}\right) P_1 \rho_1} \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}+\frac{1}{2}}}$$

$$\cancel{= A_2 \left(\frac{k}{k+1}\right)^{\frac{1}{2}} P_1 \rho_1 \left(\frac{2}{k+1}\right)^{\frac{2+k-1}{2(k-1)}}}$$

$$= A_2 \sqrt{2k P_1 \rho_1} \left(\frac{2}{k+1}\right)^{1/k-1} \frac{1}{\sqrt{k+1}}$$

$$= A_2 \sqrt{k P_1 \rho_1} \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \left(\frac{2}{k+1}\right)^{1/2}$$

$$= A_2 \sqrt{k P_1 \rho_1} \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}+\frac{1}{2}}$$

$$\frac{2+k-1}{2(k-1)}$$

$$= A_2 \sqrt{k P_1 \rho_1} \left(\frac{2}{k+1}\right)^{\frac{k+1}{2(k-1)}}$$

<u>from energy eqn:</u>

$$\frac{1}{2}(v_2^2 - v_1^2) = \frac{kR}{k-1}(T_1 - T_2)$$

$$\Rightarrow \frac{1}{2} v_2^2 = \frac{kR}{k-1} T_2 \left(\frac{T_1}{T_2} - 1\right) \quad : \quad [v_1 = 0]$$

again $\frac{P_1}{\rho_1^k} = \frac{P_2}{\rho_2^k}$

$$\Rightarrow \frac{\rho_2}{\rho_1} = \left(\frac{P_2}{P_1}\right)^{1/k}$$

$$P_1 = \rho_1 R T_1$$

$$P_2 = \rho_2 R T_2$$

$$\frac{T_1}{T_2} = \frac{P_1 \rho_2}{P_2 \rho_1}$$

$$\Rightarrow \frac{T_1}{T_2} = \boxed{\frac{\rho_2}{\rho_1}} \cdot \frac{P_1}{P_2}$$

$$\frac{T_1}{T_2} = \left(\frac{P_2}{P_1}\right)^{1/k} \cdot \left(\frac{P_2}{P_1}\right)^{-1}$$

$$\frac{T_1}{T_2} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{k}-1}$$

$$\Rightarrow \frac{1}{2}V_2^2 = \frac{kR}{k-1} T_2 \left[\left(\frac{P_2}{P_1}\right)^{\frac{1-k}{k}} - 1 \right]$$

from critical pressure ratio

$$\frac{-(k-1)}{k-1}$$

$$= \frac{kR}{k-1} T_2 \left[\left(\frac{2}{k+1}\right)^{\frac{k}{k-1} \cdot \frac{1-k}{k}} - 1 \right]$$

$$= \frac{kR}{k-1} T_2 \left[\frac{k+1}{2} - 1 \right]$$

$$\frac{1}{2}V_2^2 = \frac{kR}{k-1} T_2 \left(\frac{k-1}{2}\right)$$

$$V_2 = \sqrt{kRT_2} = c$$

Area velocity relationship for one dimensional compressible flow:-

$$\frac{dp}{\rho} + v\,dv = 0 \quad \text{--- (1)}$$

$$E = \frac{dp}{\frac{d\rho}{\rho}} = kp$$

$$dp = kp \cdot \frac{d\rho}{\rho} \quad \text{--- (A)}$$

$$c = \sqrt{\frac{kp}{\rho}}$$

$$\Rightarrow kp \cdot \frac{d\rho}{\rho} \cdot \frac{1}{\rho} + v\,dv = 0$$

$$\Rightarrow kp \frac{d\rho}{\rho^2} + v\,dv = 0$$

$$\Rightarrow c^2 \cdot \rho \cdot \frac{d\rho}{\rho^2} + v\,dv = 0$$

$$\Rightarrow c^2 \frac{d\rho}{\rho} + v\,dv = 0$$

$$\Rightarrow \frac{c^2}{v^2}\frac{d\rho}{\rho} + \frac{dv}{v} = 0$$

Mach number $= M = \frac{v}{c}$ $\quad \frac{\text{velocity of flow}}{\text{velocity of sound}}$

$$\Rightarrow \frac{1}{M^2}\frac{d\rho}{\rho} + \frac{dv}{v} = 0$$

$$\Rightarrow \frac{d\rho}{\rho} = -M^2 \cdot \frac{dv}{v} \quad \text{——— (III)}$$

$$A v \rho = \text{const}$$

$$\Rightarrow \log A + \log v + \log \rho = \log c$$

$$\Rightarrow \frac{dA}{A} + \frac{dv}{v} + \frac{d\rho}{\rho} = 0$$

$$\Rightarrow \frac{dA}{A} + \frac{dv}{v} - M^2 \cdot \frac{dv}{v} = 0$$

$$\Rightarrow \frac{dA}{dv} = \frac{A}{v}(M^2 - 1)$$

i) if Subsonic flow, $M < 1$; $\frac{dA}{dv} = (-)ve$

$dA = +$ $\quad dv = -$

$dA = -$ $\quad dv = +$

Divergen

v_1 $\quad v_2$

$v_2 < v_1$

convergen

v_1 $\quad v_2$

$v_2 > v_1$

(ii) Supersonic flow if $M > 1$; $\frac{dA}{dv} = +$

divergen $dA = +$; $dv = +$

v_1, A_1 — A_2, v_2

$A_2 > A_1$

$v_2 > v_1$

$dA = -$; $dv = -$

A_1, v_1 — A_2, v_2

$A_2 < A_1$

$v_2 < v_1$

For compressive fluid / flow we use divergen nozzle

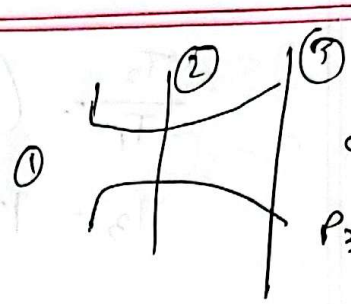
iii) $M = 1$ $\frac{dA}{dv} = 0$.

$\Rightarrow dA = 0$ [No change in Area]

Problem 3/ $P_1 = 690\,kPa$

$T_1 = 38°C = 38 + 273\,K$

$= 311\,K$



$d_3 = 50\,mm$

$P_3 = P_{atm}$

$= 101.325\,kPa$

$\frac{P_2}{P_1} = 0.528$ [critical pressure ratio].

$\Rightarrow P_2 = 0.528 \times 690 = 364.32\,kPa$

velocity at the throat, $v_2 = \sqrt{KRT_2}$

$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}} = (0.528)^{\frac{1.4-1}{1.4}}$

$P_1 = \rho_1 R T_1$

$P_2 = \rho_2 R T_2$

$\Rightarrow T_2 = 0.833 T_1 = 259.13\,K$

$\frac{\rho_1}{\rho_2} = \frac{T_1 P_2}{T_2 P_1}$

$v_2 = \sqrt{1.4 \times R \times 259.13} = 322.67\,m/s$

$P = \rho R T \Rightarrow \rho_2 = \frac{P_2}{RT_2}$ $\rho_3 = \frac{P_3}{RT_3}$

$\Rightarrow \rho_2 = 4.90\,kg/m^3$

$\dot{m}$

$$\frac{T_3}{T_1} = \left(\frac{P_3}{P_1}\right)^{\frac{K-1}{K}} \qquad \rho_3 = \frac{P_3}{RT_3}$$

$T_3 = 179.81\,K$ $\qquad = 1.965\ kg/m^3$

$$\frac{1}{2}(V_3^2 - V_2^2) = \frac{KR}{K-1}(T_2 - T_3)$$

$\Rightarrow V_3 = 513.29\ m/s$

$\dot{m} = A_3 V_3 \rho_3$

$= \frac{\pi}{4}(0.05)^2 \times 513.29 \times 1.965$

$\dot{m} = 1.98\ kg/s$

in throat $\dot{m} = A_2 \rho_2 V_2$

$\Rightarrow 1.98 = A_2 \times \square$

$\Rightarrow \pi/4\ d_2^2 = \frac{1.98}{\square}$

$\Rightarrow d_2 = \square\ m = 40\ mm$

8.8.

(2) $v_1 = 350\ m/s$ $\quad P_2 = 120\ kPa$ $\quad v_2 = ?$

$T_1 = 40 + 273 = 313\ K$ $\quad R = 287\ J/kg\,K$ $\quad T_2 = ?$

$P_1 = 80\ kPa$ $\quad k = 1.4$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

$$T_2 = 351.44\ K$$

$$\frac{1}{2}(v_2^2 - v_1^2) = \frac{kR}{k-1} \cdot T_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{k-1/k}\right]$$

$$\therefore v_2 = 212.76\ m/s$$

for section (1), Mach number $M_1 = \frac{v_1}{c_1} = \frac{v_1}{\sqrt{kRT_1}} = 0.99$

" " (2), " " $M_2 = \frac{v_2}{c_2} = \frac{v_2}{\sqrt{kRT_2}} \approx 0.57$

* Part-2/ 2.5 question compressible fluid flow } mendatory Set.

* Part 1/ Bernoulli's equation

1. what is hydraulic jump

2. Describe boundary layer thickness with water.

Derivation

Bernoullin

continuity

$P = h\rho g$

viscosity - Newton's low.

force in sub merge plate in water.

All math in slide

solve all math.