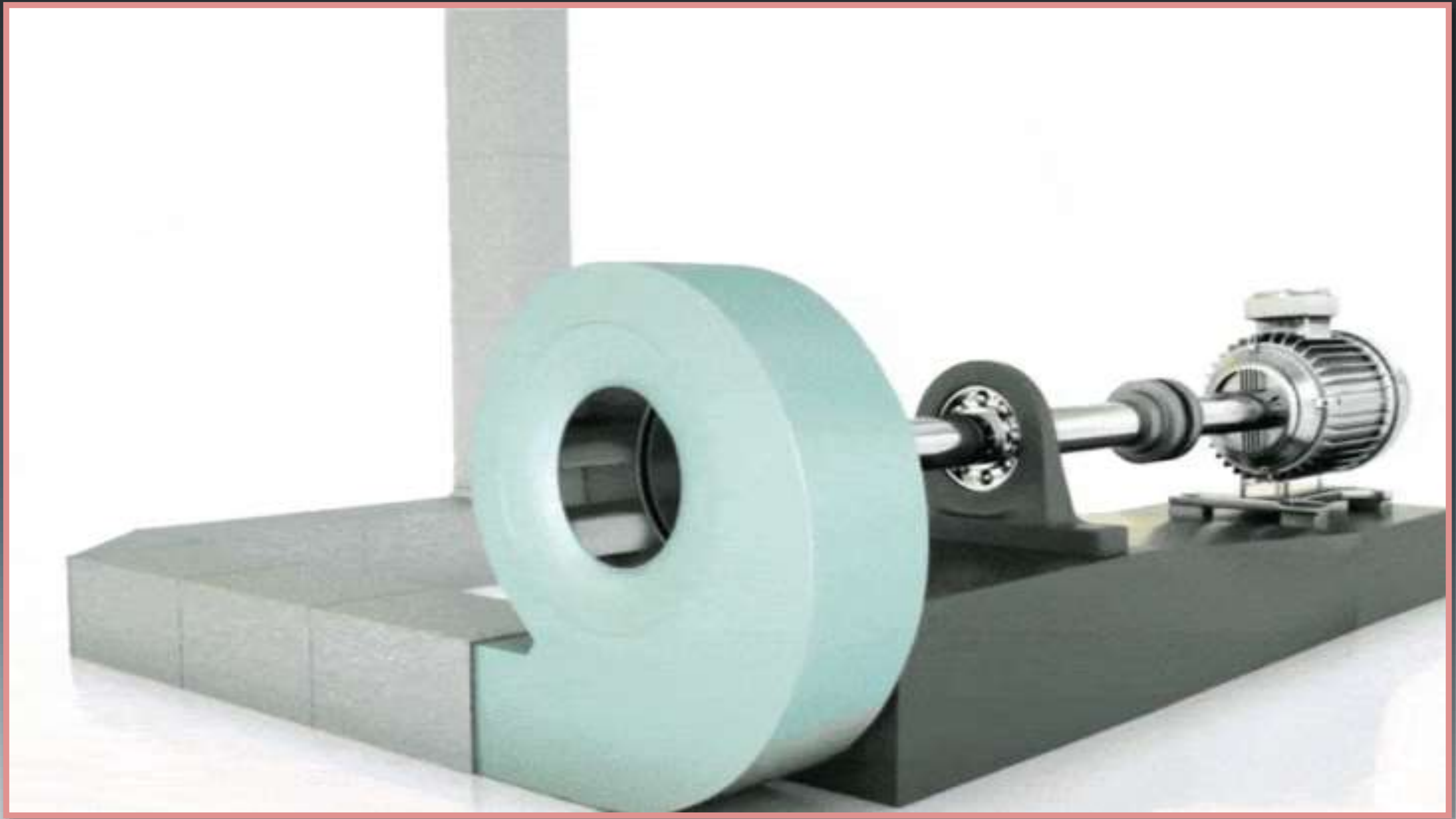
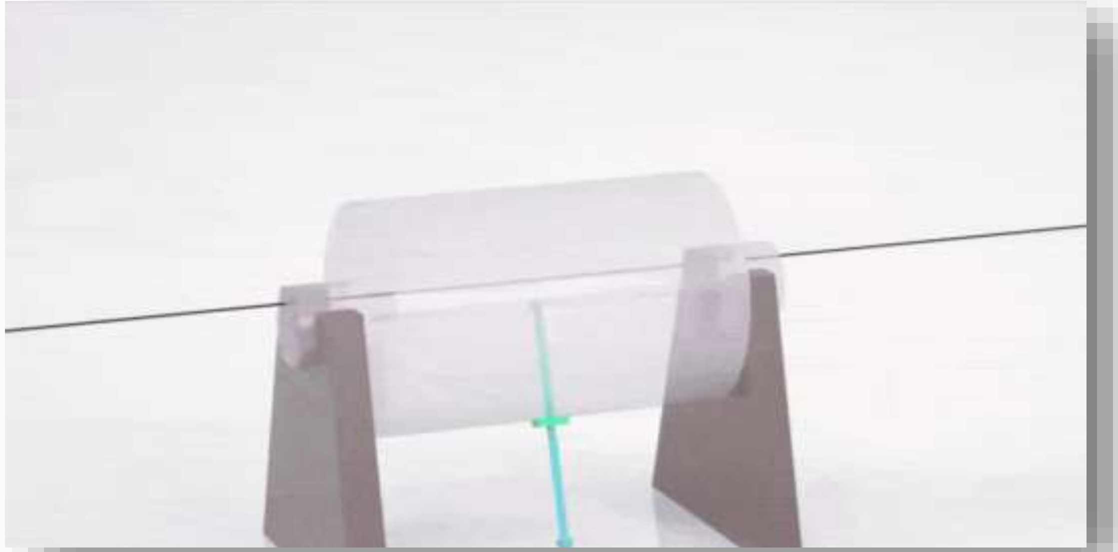
ME 3101: Mechanics of Machinery
Balancing of Rotating Masses

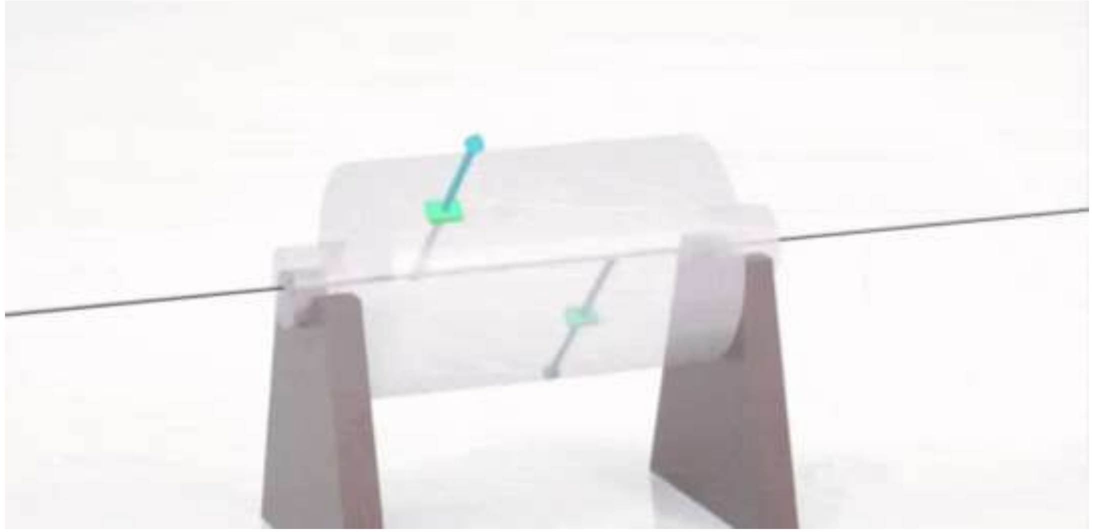


Prepared by
Muhammad Ifaz Shahriar Chowdhury
Lecturer, MPE Dept

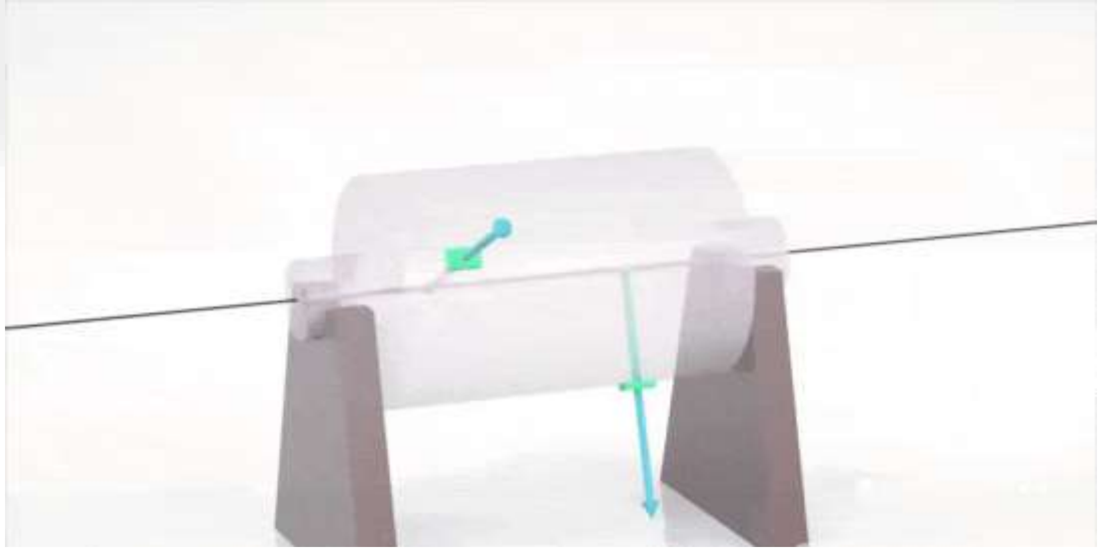
U N B A L A N C E



Static Unbalance



Couple Unbalance



Dynamic Unbalance

[Concepts of balancing: Types of Unbalance]

U
N
B
A
L
A
N
C
E

1. Static Unbalance

- **Definition**: Occurs when the mass center of a rotor does not coincide with its axis of rotation, causing an uneven distribution of mass.
- **Key Characteristics**:
  - The centrifugal force causes the rotor to vibrate in the plane of rotation
  - The unbalance can be corrected by adding or removing weight in a single plane perpendicular to the axis of rotation.
- **Examples**:
  - Unbalanced ceiling fans.

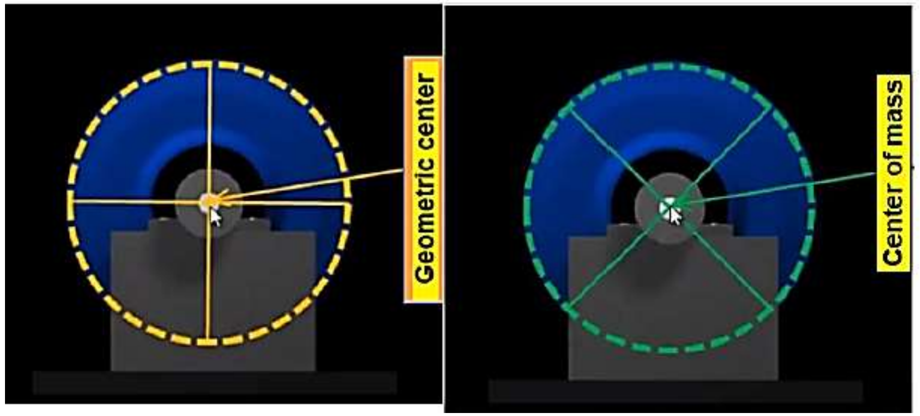
2. Couple Unbalance

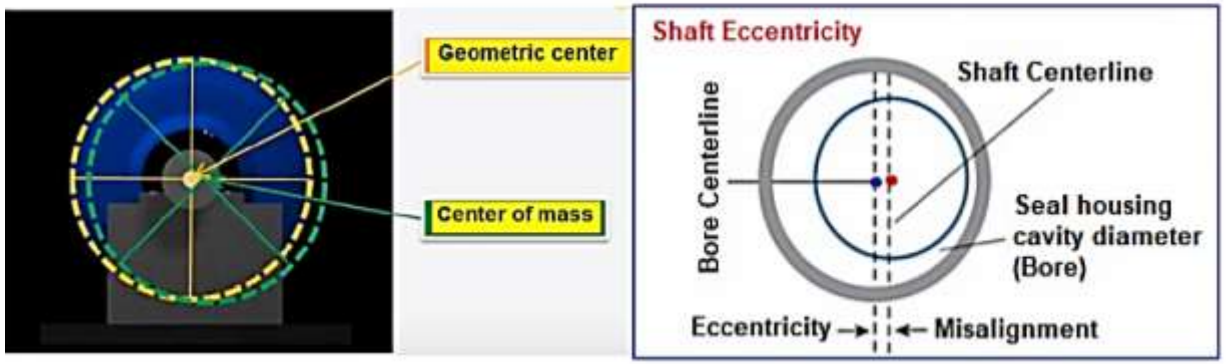
- **Definition**: Occurs when equal masses are positioned diametrically opposite to each other at equal distances from the axis of rotation, but they are not in the same plane.
- **Key Characteristics**:
  - Correction requires balancing weights in two separate planes.
  - Centrifugal forces in different planes generate a rocking effect.
- **Examples**:
  - Rotors with weights attached on opposite sides in different planes.

3. Dynamic Unbalance

- **Definition**: A combination of static and couple unbalance. It occurs when the mass center is not on the axis of rotation and when the rotor has a distributed unbalance across its length.
- **Key Characteristics**:
  - Requires balancing in two or more planes to correct.
  - Both vibrations and rocking motions occur, making it the most severe and complex to manage.
- **Examples**:
  - Crankshafts.

U N B A L A N C E





- In revolving rotors of rotating machinery like motors and engines, machining tools, industrial turbomachinery, etc., the centrifugal force remains unbalanced if the center of mass of rotor does not lie on the axis of rotation of the shaft i.e. there is eccentricity. This unbalance results bending of shaft, vibration, noise etc.

Centre of mass doesn’t lie on axis of rotation of shaft → **Unbalance** → **Bending, Vibration, Noise**

Causes of Unbalance

- Dirty build-up
- Corrosion
- Deformation from material tension
- Loss of material due to wear, cavitation etc.
- Improper manufacture due to poor casting, incorrect roundness etc.
- Loss of part due to balance weight, fasteners etc.

Reducing unbalance is very important because it

- ❑ Increases stress: reduces life
- ❑ Damages of structure
- ❑ Damages the bearing and seals
- ❑ Amplifies resonances and exacerbates looseness

B A L A N C I N G

What is Balancing?

Balancing is the process of designing or modifying machinery so that the unbalance is reduced to an acceptable level and if possible is eliminated entirely

The objective of the balancing an engine to ensure:

- ✓ That the **center of gravity** of the system **remains stationary** during a complete revolution of the crank shaft
- ✓ That the **couple** involved in acceleration of different moving parts **balance each other**

Balancing of Rotating Masses

- Whenever a certain mass is attached to a rotating shaft, it exerts some centrifugal force, whose effect is to bend the shaft and to produce vibrations in it.
- In order to prevent the effect of centrifugal force, another mass is attached to the opposite side of the shaft, at such a position so as to balance the effect of the centrifugal force of the first mass.
- This is done in such a way that the centrifugal force of both the masses are made to be equal and opposite.
- The process of providing the second mass in order to counteract the effect of the centrifugal force of the first mass, is called ***balancing of rotating masses***.

Static Balancing

- ✓ An object is said to be in static balance when the centre of gravity is on the axis of rotation.
- ✓ In this scenario the rotating mass has no tendency to rotate due to the influence of gravity, and it will rest without turning at any angular position on its bearings.

Dynamic Balancing

- ✓ Dynamic balancing is when the rotation does not produce any resultant centrifugal force or couple.
- ✓ Dynamic balancing is a way to balance out machines by rotating parts quickly and then measuring the imbalance using electronic equipment.
- ✓ The imbalance calculated can then be added or subtracted from the weight until the vibration of the parts is reduced.

BALANCING *of* ROTATING MASS

- **Balancing of Single Rotating Mass**
  - *By a Single Mass Rotating in the Same Plane*
  - *By Two Masses Rotating in Different Planes*
- **Balancing of Different Masses Rotating in the Same Plane**
  - *Graphical Method*
  - *Analytical Method*
- **Balancing of Different Masses Rotating in Different Planes**
  - *Dalby's Method*
  - *Analytical Method*

Single Rotating Mass by a Single Mass rotating in Same Plane

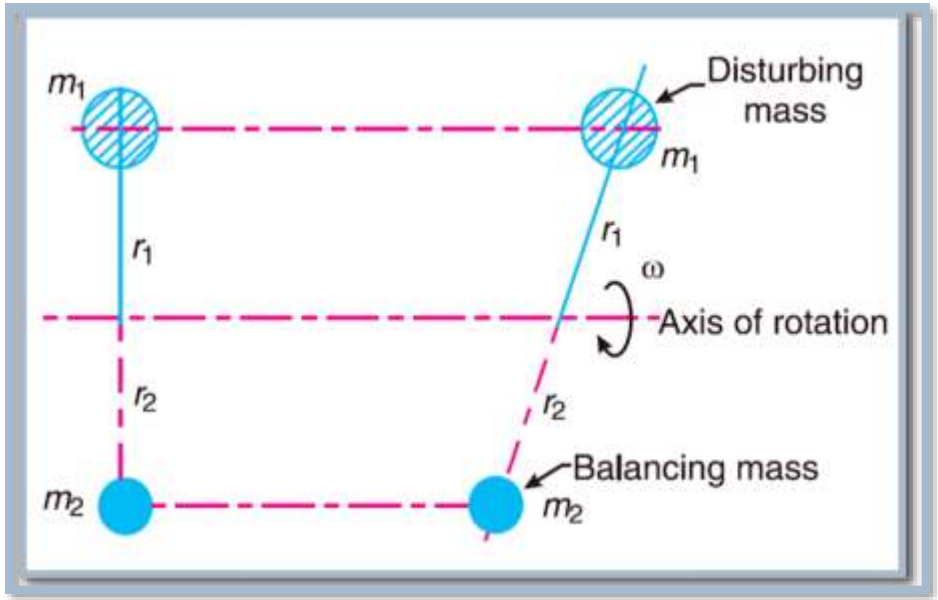
m_1 = disturbing mass (kg)
m_2 = balancing mass (kg)
r_1 = radius of rotation of the mass m_1 (m)
(*i.e.* distance between the axis of rotation of the shaft and the centre of gravity of the mass m_1)
r_2 = radius of rotation of the balancing mass m_2 (m)
ω = angular velocity of shaft (rad/s)

Centrifugal force acts radially outwards and thus produces bending moment on the shaft. Centrifugal force exerted by the mass m_1 on the shaft

$$F_{C1} = m_1 \omega^2 r_1$$

A balancing mass (m_2) may be attached in the same plane of rotation as that of disturbing mass (m_1) to counteract the effect of this force. Centrifugal force exerted by the mass m_1 on the shaft

$$F_{C2} = m_2 \omega^2 r_2$$



Centrifugal forces due to the two masses must be equal and opposite

$$m_1 \omega^2 r_1 = m_2 \omega^2 r_2$$
or
$$m_1 r_1 = m_2 r_2$$

The radius of rotation of the balancing mass **(r_2)** is generally made **large** in order to reduce the balancing mass m_2

Problem of Balancing

Single Rotating Mass by a Single Mass rotating in Same Plane

Gives rise to ***A Couple*** which tends to ***Rock the Shaft*** in its bearings

Solution Steps:

- ✓ Two balancing masses are placed in two different planes
- ✓ Planes are parallel to the plane of rotation of the disturbing mass
- ✓ The following two conditions of equilibrium are satisfied:

Net dynamic force acting on the shaft = 0 or $\Sigma\, mr = 0$

- This requires that the line of action of three centrifugal forces must be the same
- In other words, the centre of the masses of the system must lie on the axis of rotation

Static Balancing

Net couple or moment due to the dynamic forces acting on the shaft = 0 or $\Sigma\, mrl = 0$

- l = distance between different planes

Dynamic Balancing

The following two possibilities may arise while attaching the two balancing masses

1. The plane of the disturbing mass may be **in between** the planes of the two balancing masses

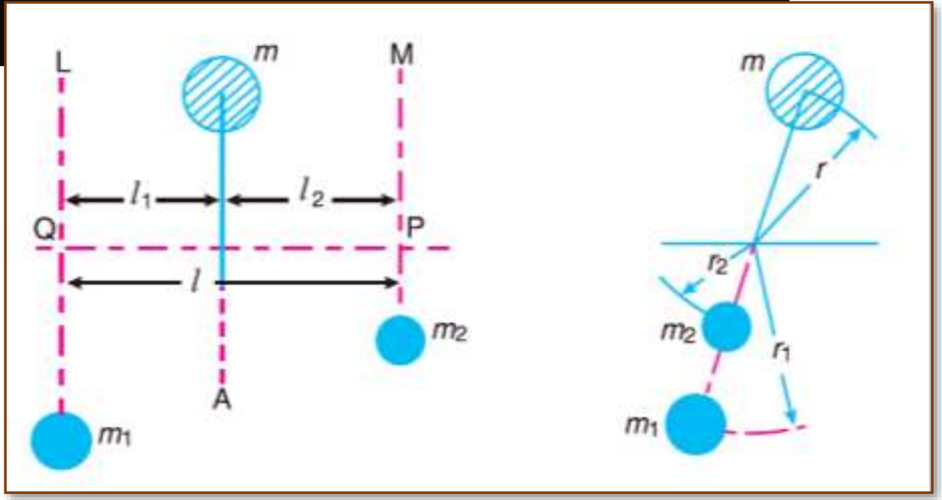
2. The plane of the disturbing mass may lie on the **left** or **right** of the two planes containing the balancing masses

Single Rotating Mass by Two Masses rotating in Different Planes

1. When the plane of the disturbing mass lies in between the planes of the two balancing masses

m = disturbing mass (kg) in plane A
m_1 = balancing mass (kg) in plane L
m_2 = balancing mass (kg) in plane M
r = radius of rotation of the mass m (m) in plane A
r_1 = radius of rotation of the mass m_1 (m) in plane L
r_2 = radius of rotation of the mass m_2 (m) in plane M

l = Distance between the planes L and M (m)
l_1 = Distance between the planes A and L (m)
l_2 = Distance between the planes A and M (m)
ω = angular velocity of shaft (rad/s)



The centrifugal force exerted by the mass m in the plane A $\quad F_C = m\,\omega^2\,r$

Similarly, centrifugal force exerted by the mass m_1 in the plane L $\quad F_{C1} = m_1\,\omega^2\,r_1$

And centrifugal force exerted by the mass m_2 in the plane M $\quad F_{C2} = m_2\,\omega^2\,r_2$

Equating, $\quad F_C = F_{C1} + F_{C2} \Rightarrow \quad m\,\omega^2\,r = m_1\,\omega^2\,r_1 + m_2\,\omega^2\,r_2$

$$mr = m_1 r_1 + m_2 r_2$$ **Static Balancing**

Taking moments about P which is the point of intersection of the plane M and the axis of rotation

$$F_{C1}\,.\,l = F_C\,.\,l_2 \Rightarrow m_1\,\omega^2\,r_1\,.\,l = m\,\omega^2\,r\,.\,l_2 \quad \Rightarrow \quad m_1\,r_1 = m\,r\,\frac{l_2}{l}$$

Taking moments about Q which is the point of intersection of the plane L and the axis of rotation.

$$F_{C2}\,.\,l = F_C\,.\,l_1 \Rightarrow m_2\,\omega^2\,r_2\,.\,l = m\,\omega^2\,r\,.\,l_1 \quad \Rightarrow \quad m_2\,r_2 = m\,r\,\frac{l_1}{l}$$

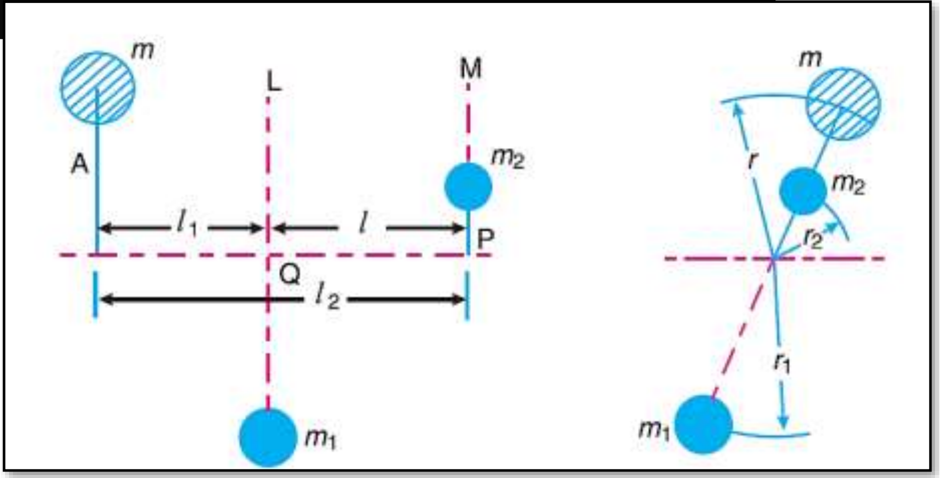
Dynamic Balancing

Single Rotating Mass by Two Masses rotating in Different Planes

2. When the plane of the disturbing mass lies on one end of the planes of the balancing masses

m = disturbing mass (kg) in plane A
m_1 = balancing mass (kg) in plane L
m_2 = balancing mass (kg) in plane M
r = radius of rotation of the mass m (m) in plane A
r_1= radius of rotation of the mass m_1 (m) in plane L
r_2= radius of rotation of the mass m_2 (m) in plane M

l = Distance between the planes L and M (m)
l_1 = Distance between the planes A and L (m)
l_2 = Distance between the planes A and M (m)
ω = angular velocity of shaft (rad/s)



The centrifugal force exerted by the mass *m* in the plane *A* $F_C = m\,\omega^2\,r$

Similarly, centrifugal force exerted by the mass m_1 in the plane *L* $F_{C1} = m_1\,\omega^2\,r_1$

And centrifugal force exerted by the mass m_2 in the plane *M* $F_{C2} = m_2\,\omega^2\,r_2$

Equating, $F_C + F_{C2} = F_{C1} \Rightarrow m\,\omega^2\,r + m_2\,\omega^2\,r_2 = m_1\,\omega^2\,r_1$

$$mr + m_2 r_2 = m_1 r_1$$ **Static Balancing**

Taking moments about *P* which is the point of intersection of the plane *M* and the axis of rotation

$$F_{C1} \,.\, l = F_C \,.\, l_2 \Rightarrow m_1\,\omega^2\,r_1 \,.\, l = m\,\omega^2\,r \,.\, l_2$$ $$m_1 r_1 = m\,r\,\frac{l_2}{l}$$

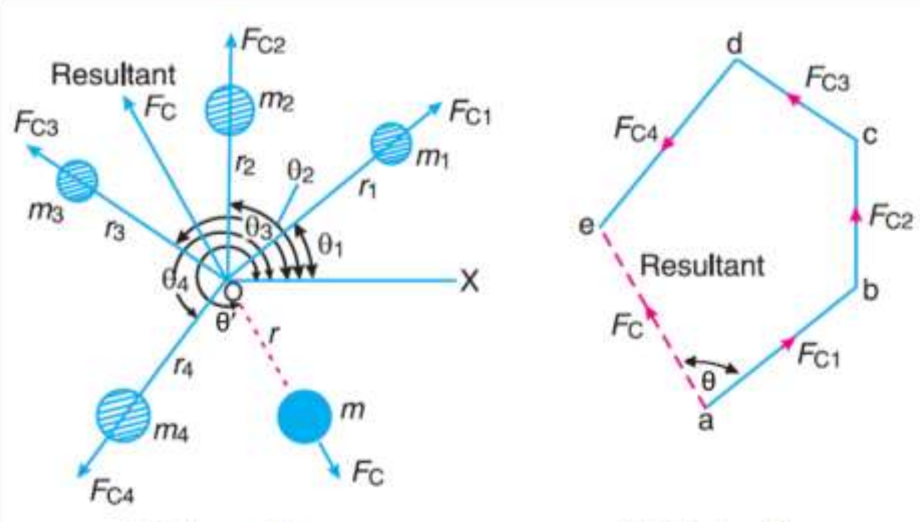
Taking moments about *Q* which is the point of intersection of the plane *L* and the axis of rotation.

$$F_{C2} \,.\, l = F_C \,.\, l_1 \Rightarrow m_2\,\omega^2\,r_2 \,.\, l = m\,\omega^2\,r \,.\, l_1$$ $$m_2 r_2 = m\,r\,\frac{l_1}{l}$$

Dynamic Balancing

Different Masses rotating in Same Plane

m_1, m_2, m_3, m_4 = out of balance masses (kg)
r_1, r_2, r_3, r_4 = radii of rotation of the mass m_1, m_2, m_3, m_4 respectively (m)
$\theta_1, \theta_2, \theta_3, \theta_4$ = angles of mass m_1, m_2, m_3, m_4 with horizontal line *OX* (degree)
m = balancing mass (kg)
r = radius of rotation of the balancing mass m (m)
θ = angle the resultant force makes with horizontal line *OX* (degree)
ω = angular velocity of shaft about an axis through O (rad/s)



(*a*) Space diagram. (*b*) Vector diagram.

1. *Graphical Method*

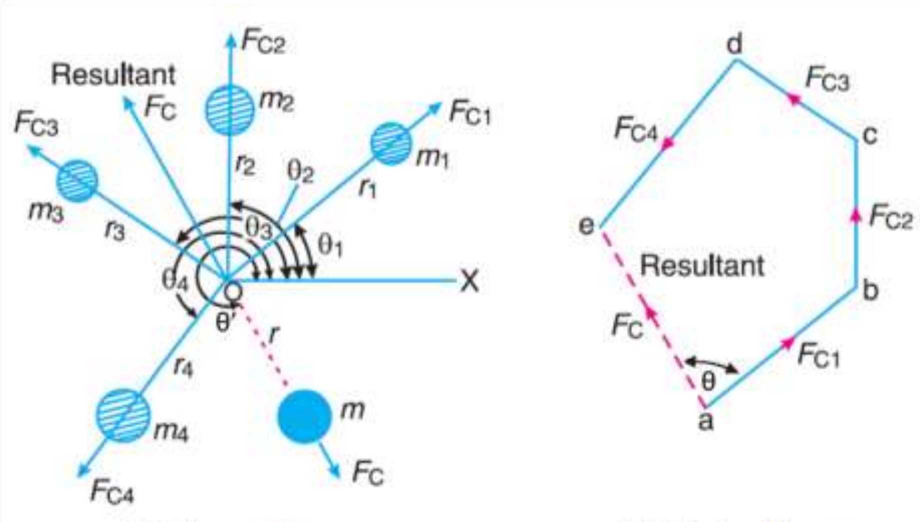
- Draw the space diagram with the positions of the several masses
- Find out the all centrifugal force (or product of the mass and radius of rotation)
- Draw the vector diagram with the obtained centrifugal forces, such that *ab* represents the centrifugal force exerted by the mass m_1 (or $m_1.r_1$) in magnitude and direction to some suitable scale. Similarly, draw *bc*, *cd* and *de* to represent centrifugal forces of other masses m_2, m_3 and m_4 (or $m_2.r_2$, $m_3.r_3$ and $m_4.r_4$)
- As per polygon law of forces, the closing side *ae* represents the resultant force in magnitude and direction
- The balancing force is, then, equal to the resultant force, but in ***opposite direction***
- Find out the magnitude of the balancing mass (m) at a given radius of rotation (r)

$m.r$ = Resultant of $m_1.r_1$, $m_2.r_2$, $m_3.r_3$ and $m_4.r_4$ ⇒ m = (Resultant of $m_1.r_1$, $m_2.r_2$, $m_3.r_3$ and $m_4.r_4$)/ r

- Find the angle that balancing mass makes with horizontal from the vector diagram

Different Masses *rotating in* Same Plane

m_1, m_2, m_3, m_4 = out of balance masses (kg)
r_1, r_2, r_3, r_4 = radii of rotation of the mass m_1, m_2, m_3, m_4 respectively (m)
$\theta_1, \theta_2, \theta_3, \theta_4$ = angles of mass m_1, m_2, m_3, m_4 with horizontal line *OX* (degree)
m = balancing mass (kg)
r = radius of rotation of the balancing mass m (m)
θ = angle the resultant force makes with horizontal line *OX* (degree)
ω = angular velocity of shaft about an axis through O (rad/s)



(*a*) Space diagram. (*b*) Vector diagram.

2. Analytical Method

- Find out all $m \times r$ i.e. centrifugal forces
- Sum of horizontal components of the centrifugal forces

$$\Sigma H = m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + \ldots\ldots$$

- Sum of vertical components of the centrifugal forces

$$\Sigma V = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + \ldots\ldots$$

- Magnitude and angle of the resultant centrifugal force

$$F_C = \sqrt{(\sum H)^2 + (\sum V)^2}$$

$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

- Balancing Force

$$F_B = -F_C$$

Magnitude $|F_B| = |-F_C|$

Angle $\theta' = 180° + \theta$

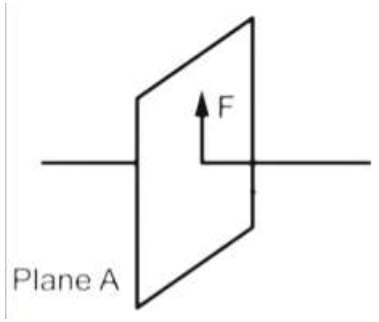
- Balancing Mass

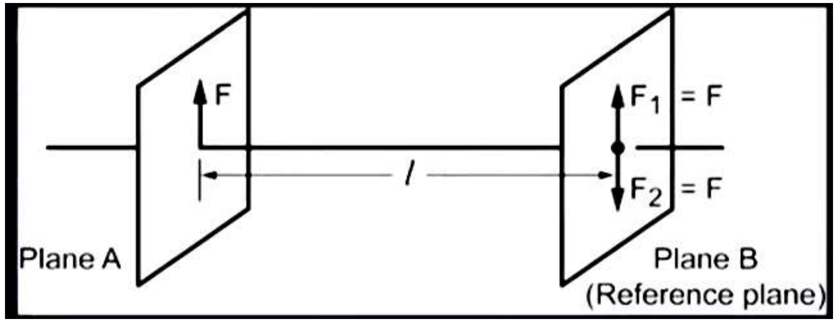
$$F_B = m.r$$

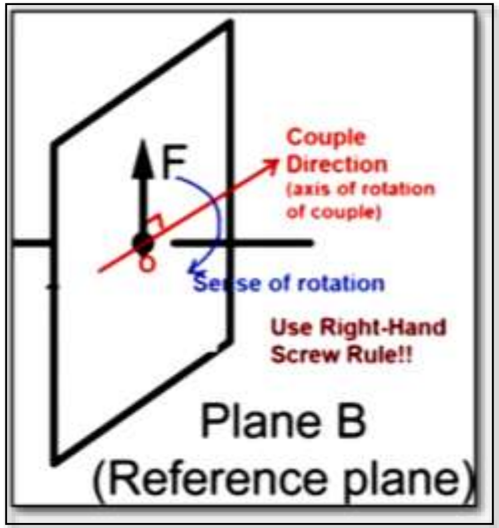
$$m = F_B / r$$

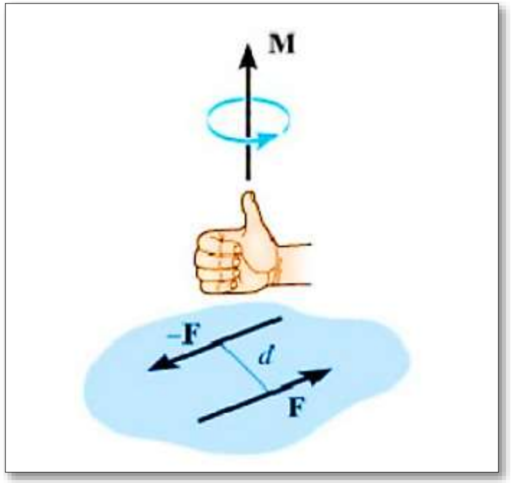
Different Masses rotating in Different Planes

Equivalent Force System









Right Hand Rule

Required Step:

Transfer the centrifugal force acting in each plane to a single parallel plane termed as **Reference Plane (R.P)**

- The effect of transferring ***a force F*** acting in one plane to another plane (reference plane) is equivalent to transfer of the ***same force F in magnitude*** and direction in the reference plane accompanied by ***a couple of magnitude F x l***
- Couple vector is ***perpendicular (90 degrees) to force vector***.
- In balancing problems, it is convenient if couple vectors are drawn by ***turning them through 90 degrees*** (i.e., by drawing them parallel to force vectors). This does not affect their relative positions

Different Masses rotating in Different Planes

1. Dalby's Graphical Method

Reference Plane: Plane passing through a point on the axis of rotation and perpendicular to it

In order to have a complete balance of the several revolving masses in different planes, the following two conditions must be satisfied :

- The forces in the reference plane must balance, i.e. ***the resultant force must be zero*** $\Sigma mr = 0$
- The couples about the reference plane must balance, i.e. ***the resultant couple must be zero*** $\Sigma mrl = 0$

Step 1: Using given data, draw linear and angular positions of planes

Step 2: Take any one of the planes, say X, as the reference plane (R.P)
Distances to the **left** of this reference plane are taken with **negative sign** and those to **right** with **positive sign**

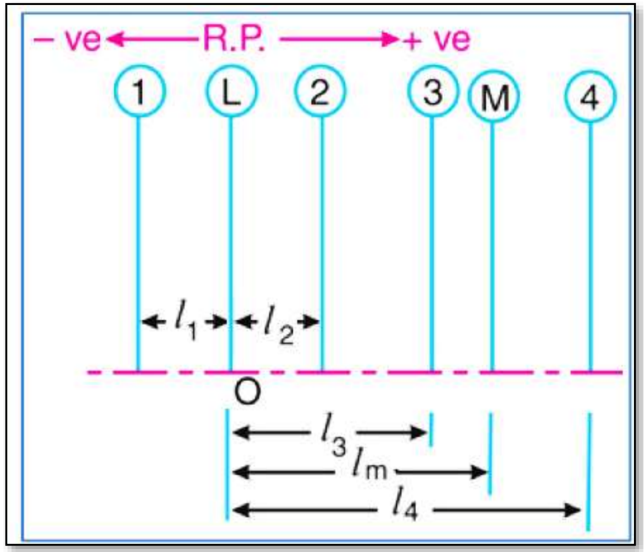
Step 3: Tabulate the forces and couples with respect to the reference plane. Draw couple vectors and **rotate them by 90 degrees** (i.e., by drawing them parallel to force vectors) for convenience.

Step 4: Draw couple polygon using Dalby's method
For dynamic balancing, **couple polygon must be closed**
Using the closing side of the couple polygon, **a set of required (unknown) values can be found**

Step 5: Draw force polygon
For dynamic balancing, **force polygon must be closed**
Using the closing side of the force polygon, **the remaining required (unknown) values can be found**

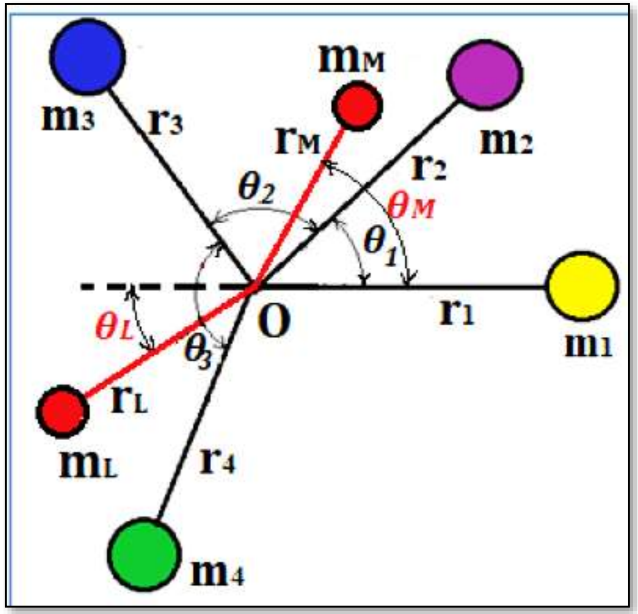
Different Masses rotating in Different Planes

m_1, m_2, m_3, m_4 = out of balance masses in planes 1,2,3,4 respectively (kg)
r_1, r_2, r_3, r_4 = radii of rotation of the mass m_1, m_2, m_3, m_4 respectively (m)
$\theta_1, \theta_2, \theta_3$ = angle between mass m_1 & m_2; m_2 & m_3; m_3 & m_4 (degree)
m_L, m_M = balancing masses in planes L & M respectively (kg)
r_L, r_M = radii of rotation of the balancing masses m_L, m_M respectively (m)
θ_L, θ_M = angles of mass m_L and m_M with horizontal (degree)
ω = angular velocity of shaft about an axis through O (rad/s)

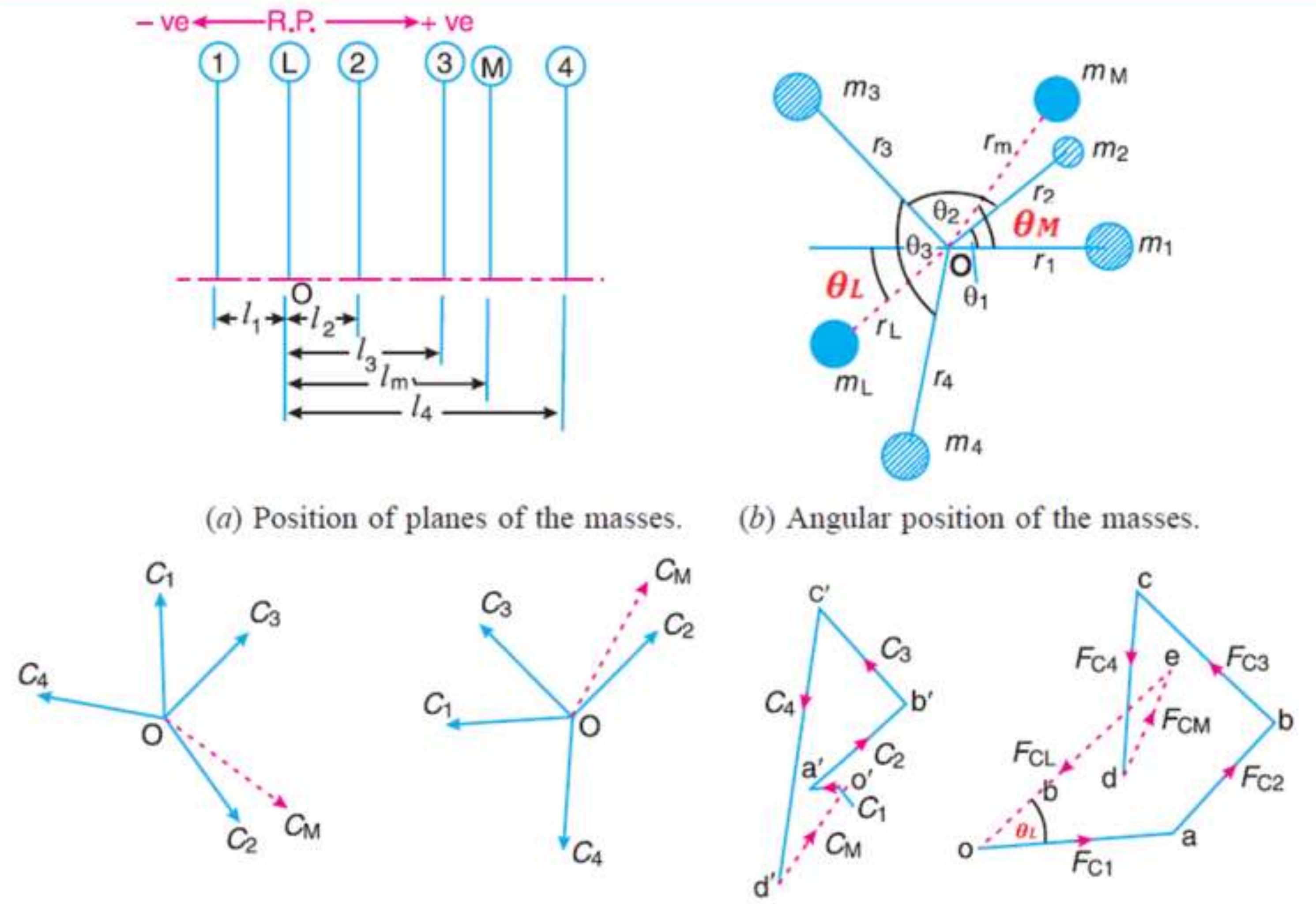


- Take one of the planes, say **L** as the **reference plane (R.P.)**
- Consider left of the reference plane may be regarded as **negative**, and those to the right as **positive**
- ***Now draw the couple polygon and force to find the balanced condition***

Plane (1)	Mass (m) (2)	Radius(r) (3)	Cent.force ÷ ω^2 (m.r) (4)	Distance from Plane L (l) (5)	Couple ÷ ω^2 (m.r.l) (6)
1	m_1	r_1	$m_1.r_1$	$-l_1$	$-m_1.r_1.l_1$
L(R.P.)	m_L	r_L	$m_L.r_L$	0	0
2	m_2	r_2	$m_2.r_2$	l_2	$m_2.r_2.l_2$
3	m_3	r_3	$m_3.r_3$	l_3	$m_3.r_3.l_3$
M	m_M	r_M	$m_M.r_M$	l_M	$m_M.r_M.l_M$
4	m_4	r_4	$m_4.r_4$	l_4	$m_4.r_4.l_4$



Different Masses rotating in Different Planes



(*a*) Position of planes of the masses. (*b*) Angular position of the masses.

(*c*) Couple vector. (*d*) Couple vectors turned counter clockwise through a right angle. (*e*) Couple polygon. (*f*) Force polygon.

$C_M = m_M \cdot r_M \cdot l_M$ = vector d'o'

$$m_M = \frac{vector\ d'0'}{r_M l_M}$$

$m_L \cdot r_L$ = vector eo

$$m_L = \frac{vector\ eo}{r_L}$$

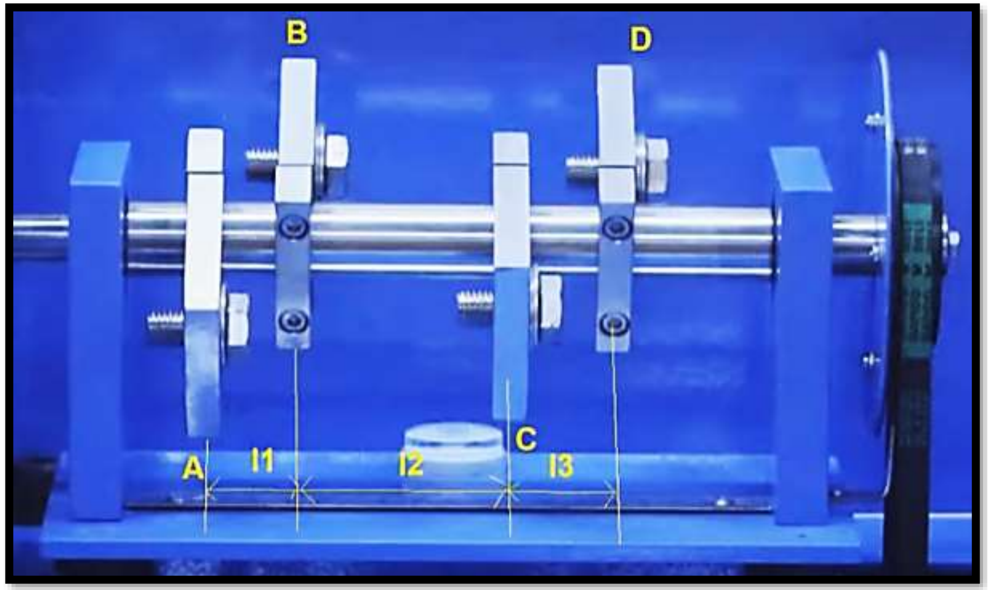
Different Masses rotating in Different Planes

2. Analytical Method

If m_L *and* m_M be the balance forces at radii r_L and r_M respectively, then for the **balance of couples** about plane L

$$\left[\left(\sum m_i r_i l_i \cos\theta_i\right)^2 + \left(\sum m_i r_i l_i \sin\theta_i\right)^2\right]^{0.5} = m_M r_M l_M$$

$$\tan\theta_M = \frac{-\sum m_i r_i l_i \sin\theta_i}{-\sum m_i r_i l_i \cos\theta_i}$$



For balance the forces;

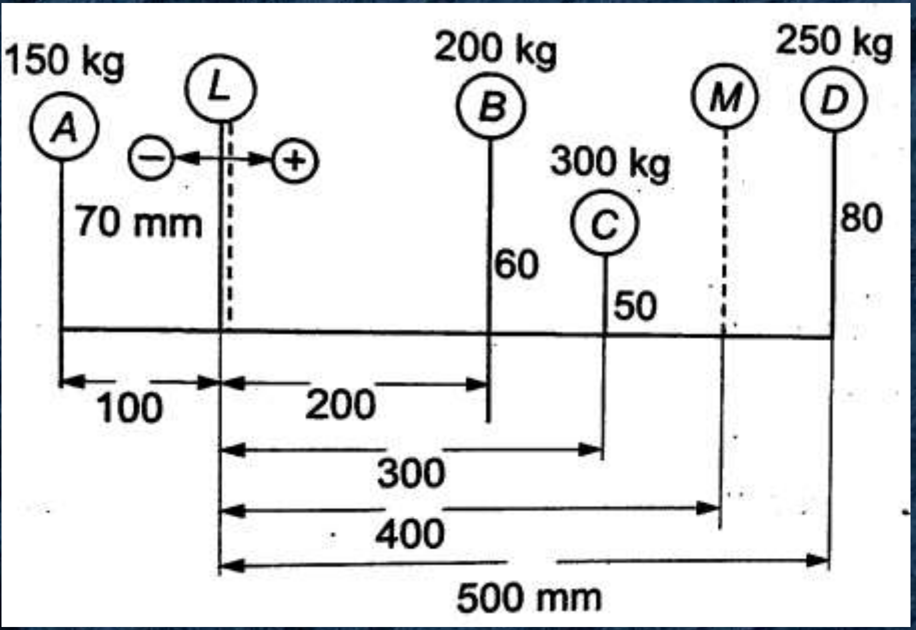
$$\left[\left(\sum m_i r_i \cos\theta_i\right)^2 + \left(\sum m_i r_i \sin\theta_i\right)^2\right]^{0.5}$$
$$= \left[\left(\sum m_L r_L \cos\theta_L\right)^2 + \left(\sum m_M r_M \sin\theta_M\right)^2\right]^{0.5}$$

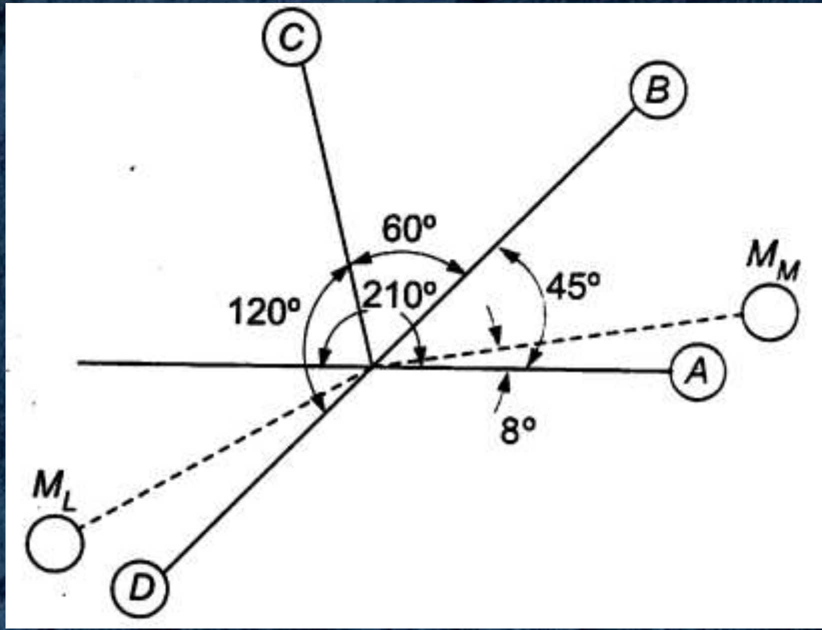
$$\tan\theta_L = \frac{-\left(\sum m_i r_i \sin\theta_i + m_M r_M \sin\theta_M\right)}{-\left(m_i r_i \cos\theta_i + m_M r_M \cos\theta_M\right)}$$

$$m_L r_L = \left[\left(\sum m_i r_i \cos\theta_i + m_M r_M \cos\theta_M\right)^2 + \left(\sum m_i r_i \sin\theta_i + m_M r_M \sin\theta_M\right)^2\right]^{0.5}$$

PROBLEM

- A shaft carries four masses as show in the figure below. The balancing masses are to be placed in planes L and M. If the balancing masses revolve at a radius of 100 mm, find their ***magnitude*** and ***angular positions***.





Ans:

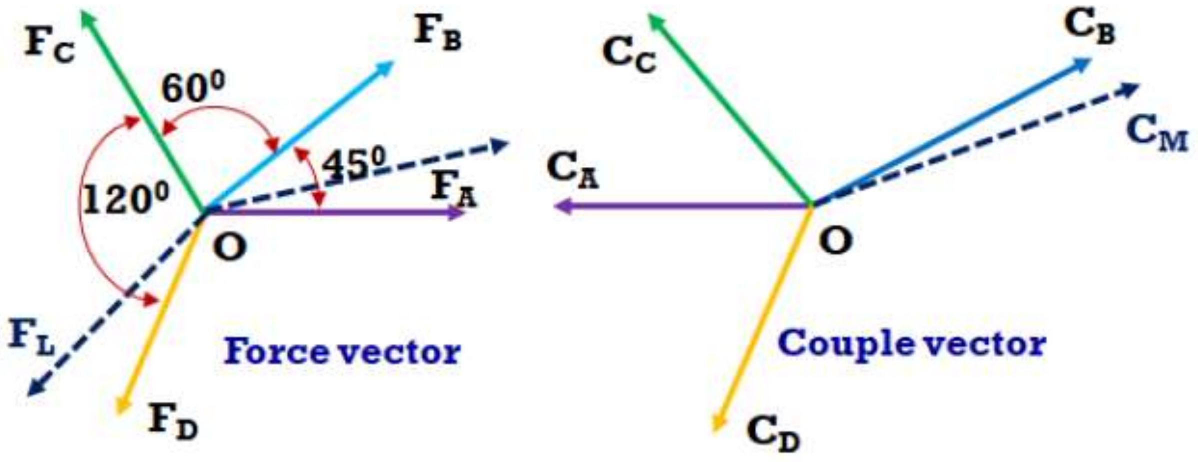
M_M = 192.5 kg

θ_M = 8°

M_L = 235 kg

θ_M = 210°

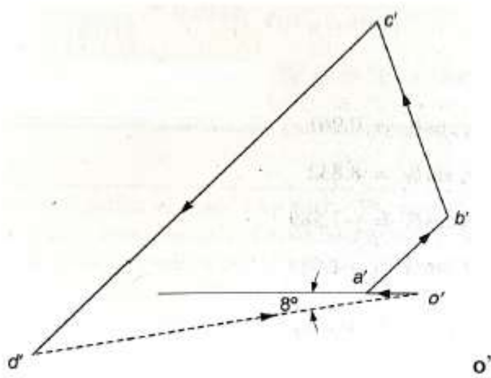
Solution:



Plane	Mass, M kg	Radius, r m	Mr kgm	Distance from plane L, l m	Mrl kgm^2
A	150	0.07	10.5	–0.1	–1.05
L	M_L	0.10	0.1 M_L	0	0
B	200	0.06	12.0	0.2	2.40
C	300	0.05	15.0	0.3	4.50
M	M_M	0.10	0.1 M'_M	0.4	0.04 M_M
D	250	0.08	20.0	0.5	10.0

Draw the couple polygon:

Take scale:1 cm = 1 kgm^2



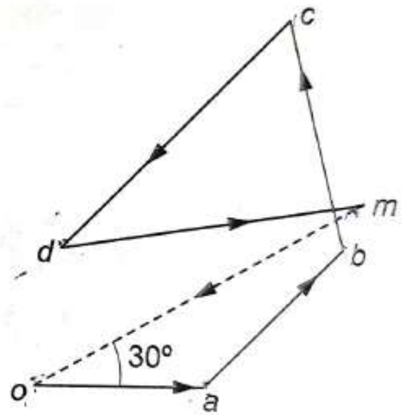
o'd' = 7.7 cm = $0.04M_M$

M_M=192.5 kg

$\theta_M = 8^0$

Draw the couple polygon:

Take scale:1 cm = 5kgm



om = 4.7 cm = 4.7 × 5 = $0.1M_L$

M_L = 235 kg

$\theta_L = 30^0 + 180^0 = 210^0$ Angular position of M_L.

PROBLEM

Four masses A, B, C and D as shown below are to be completely balanced.

	A	B	C	D
Mass (kg)	—	30	50	40
Radius (mm)	180	240	120	150

The planes containing masses B and C are 300 mm apart. The angle between planes containing B and C is 90°. B and C make angles of 210° and 120° respectively with D in the same sense.
Find :

1. **The magnitude and the angular position of mass A**
2. **The position of planes A and D.**

Ans:
m_A = 20 kg
θ_A = 236°
Plane A from Plane B = 1 m
Plane D from Plane B = 0.383 m

Solution

Four masses A, B, C and D as shown below are to be completely balanced.

	A	B	C	D
Mass (kg)	—	30	50	40
Radius (mm)	180	240	120	150

The planes containing masses B and C are 300 mm apart. The angle between planes containing B and C is 90°. B and C make angles of 210° and 120° respectively with D in the same sense. Find :

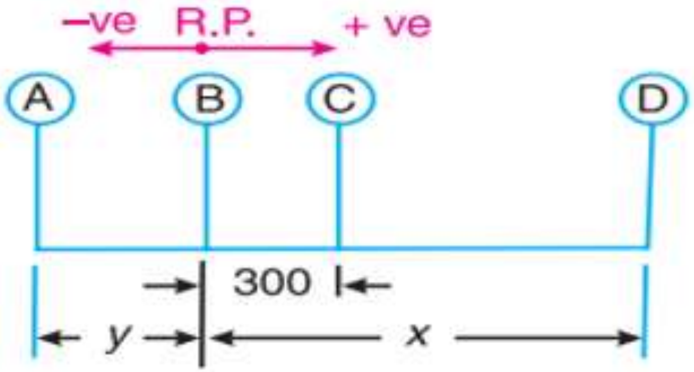
1. *The magnitude and the angular position of mass A ; and*
2. *The position of planes A and D.*

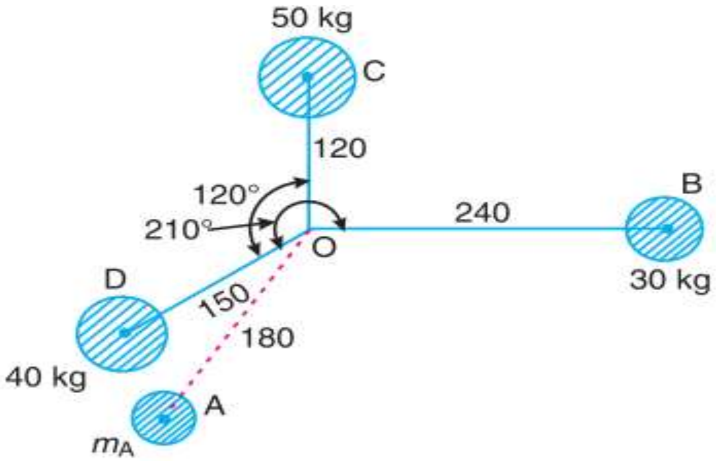
Solution. Given : r_A = 180 mm = 0.18 m ; m_B = 30 kg ; r_B = 240 mm = 0.24 m ; m_C = 50 kg ; r_C = 120 mm = 0.12 m ; m_D = 40 kg ; r_D = 150 mm = 0.15 m ; $\angle BOC = 90°$; $\angle BOD = 210°$; $\angle COD = 120°$

m_A = Magnitude of Mass A,

x = Distance between the planes B and D, and

y = Distance between the planes A and B.

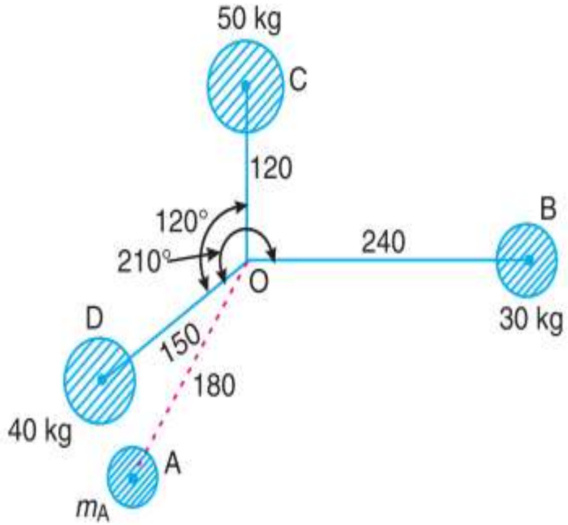


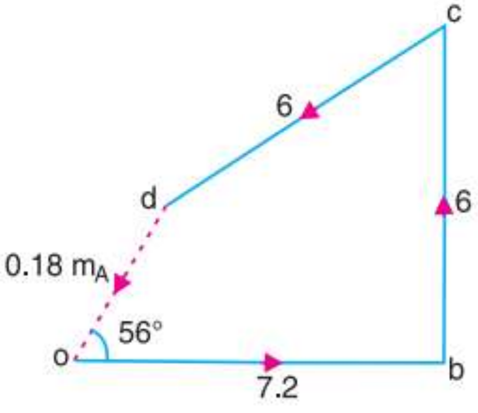


All dimensions in mm.

Plane (1)	*Mass* (m) kg (2)	*Radius* (r) m (3)	*Cent.force* $\div \omega^2$ (m.r) kg-m (4)	*Distance from plane B* (l) m (5)	*Couple* $\div \omega^2$ (m.r.l) kg-m^2 (6)
A	m_A	0.18	0.08 m_A	$-y$	$-0.18\, m_A y$
B (R.P)	30	0.24	7.2	0	0
C	50	0.12	6	0.3	1.8
D	40	0.15	6	x	$6x$

Plane (1)	Cent.force ÷ ω^2 (m.r) kg-m (4)
A	$0.08\ m_A$
B (R.P)	7.2
C	6
D	6





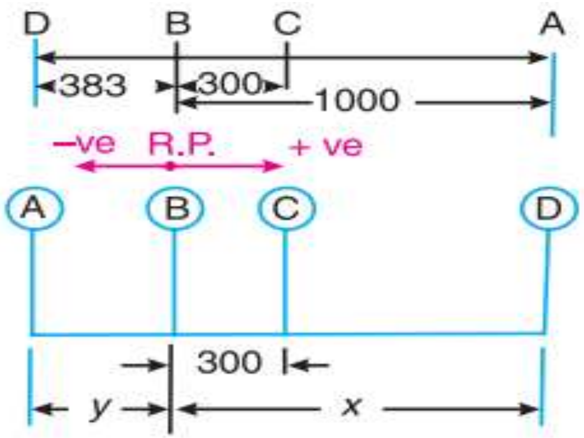
(c) Force polygon.

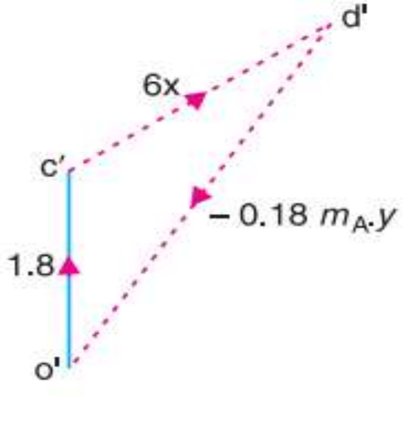
By measurement , we find that

$$0.18\ m_A = \text{Vector } do = 3.6 \text{ kg-m} \quad \text{or} \quad m_A = 20 \text{ kg} \textbf{ Ans.}$$

$\angle AOB =$ **236° Ans.**

Plane (1)	*Couple* $\div \omega^2$ (m.r.l) kg-m^2 (6)
A	$-0.18\, m_A.y$
B (*R.P*)	0
C	1.8
D	$6x$





(*d*) Couple polygon.

By measurement, We find that

$$6\,x = \text{vector } c'd' = 2.3 \text{ kg-m}^2 \text{ or } x = 0.383 \text{ m}$$

Again by measurement from couple polygon,

$$-0.18\, m_A.y = \text{vector } o'd' = 3.6 \text{ kg-m}^2$$

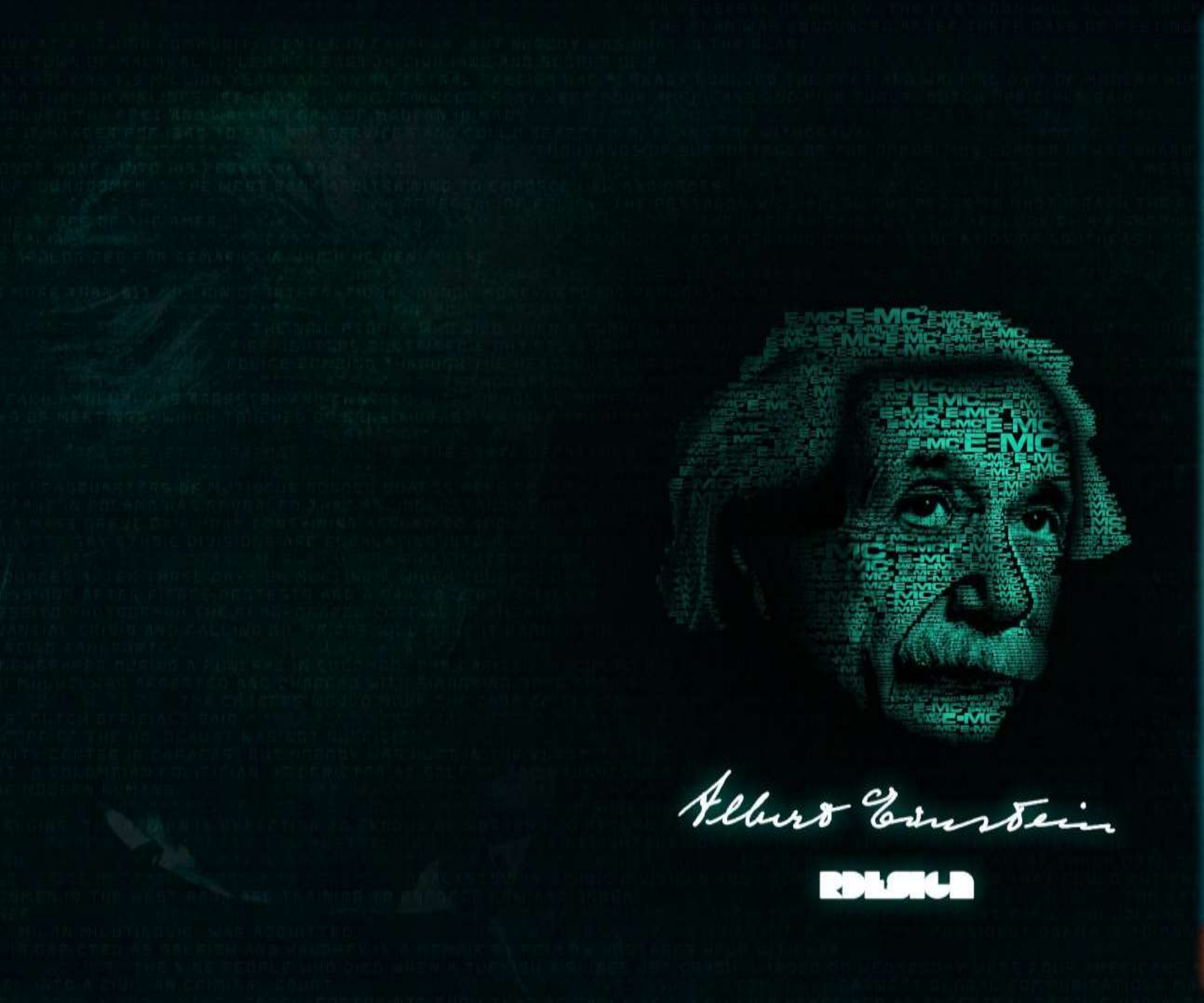
$$-0.18 \times 20\, y = 3.6 \quad \text{or} \quad y = -1 \text{ m}$$

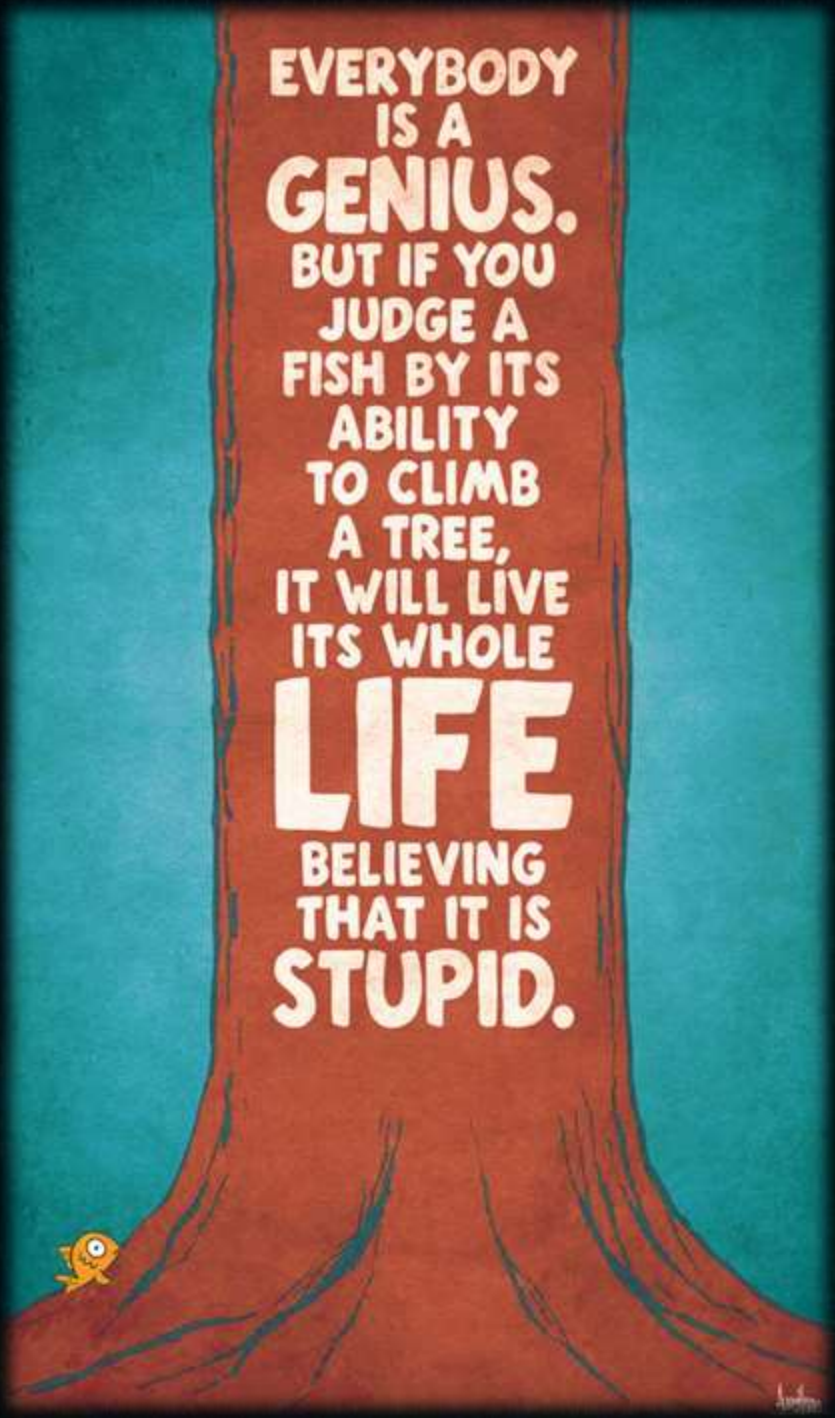
The negative sign indicates that the plane A is not towards left of B , It is 1 m towards right of plane B

Solve by Yourself

Book: Theory of Machine by R S Khurmi
Chapter 21

Example: 21.1, 21.2, 21.4, 21.5, 21.6, 21,7
Exercise: 2,3,4,5


Albert Einstein


EVERYBODY IS A GENIUS. BUT IF YOU JUDGE A FISH BY ITS ABILITY TO CLIMB A TREE, IT WILL LIVE ITS WHOLE LIFE BELIEVING THAT IT IS STUPID.