AHSANULLAH UNIVERSITY OF SCIENCE AND TECHNOLOGY (AUST)

ME-3105: FLUID MECHANICS
(LC-2: STATIC PRESSURE)

BY

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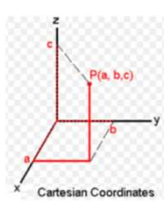
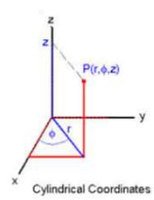
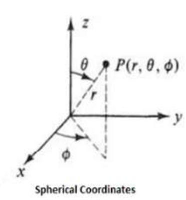
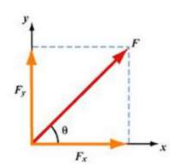
<u>BASIC CONCEPT OF MATHAMETIC IN FLUID MECHANICS (Note)</u>

➢ *Coordinates Systems:*









- Equilibrium of statics body: $\Sigma F_x = 0$; $\Sigma F_y = 0$; $\Sigma F_z = 0$ and $\Sigma M_x = 0$; $\Sigma M_y = 0$; $\Sigma M_z = 0$
- Equilibrium of dynamic body: $\Sigma F_x = m\ a_x$; $\Sigma F_y = m\ a_y$ and $\Sigma F_z = m\ a_z$
- Force Components: $F_x = F^* \ COS\theta$ and $F_y = F^* \ SIN\theta$; $F = Sqrt\ (F_x^2 + F_y^2)$

BASIC CONCEPT OF MATHAMETIC IN FLUID MECHANICS (Note)

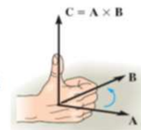
- ***Vector Analysis:***

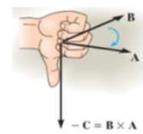
$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$ $\qquad$ $\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$

$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = AB\sin\theta\ \vec{n}$ $\qquad$ $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB\cos\theta$

$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = (A_yB_z - A_zB_y)\hat{\mathbf{i}} + (A_zB_x - A_xB_z)\hat{\mathbf{j}} + (A_xB_y - A_yB_x)\hat{\mathbf{k}}$

$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A_xB_x + A_yB_y + A_zB_z$





- ***Differential Calculus:***

Del Operator, $\nabla = \vec{i}\dfrac{\partial}{\partial x} + \vec{j}\dfrac{\partial}{\partial y} + \vec{k}\dfrac{\partial}{\partial z}$; Del of scalar 'S' $\nabla s \equiv \vec{i}\dfrac{\partial s}{\partial x} + \vec{j}\dfrac{\partial s}{\partial y} + \vec{k}\dfrac{\partial s}{\partial z}$

Divergence of 'V' $\nabla \cdot \vec{v} = \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z}$ $\qquad$ $\vec{v} \equiv \vec{i}u + \vec{j}v + \vec{k}w.$

Curl of A ,
$$\nabla \times \vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \vec{i}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \vec{j}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \vec{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

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BASIC CONCEPT OF SOLID MECHANICS IN FLUID MECHANICS (Note)

- ***Total Differential & Integration:***

Pressure 'P' is function of x and y i.e $P(x, y)$. The change in P, written as dP, between two points in the region separated by the distances dx and dy is given by the total differential

$$dP = \frac{\partial P}{\partial x}dx + \frac{\partial P}{\partial y}dy$$

$\dfrac{d}{dx}x^r = rx^{r-1}$ $\qquad$ $\dfrac{d}{dx}\sin x = \cos x$ $\qquad$ $\dfrac{d}{dx}\cos x = -\sin x$

$\dfrac{d}{dx}\tan x = \sec^2 x$ $\qquad$ $\dfrac{d}{dx}e^x = e^x$ $\qquad$ $\dfrac{d}{dx}\ln x = \dfrac{1}{x}$

d(UV) = U.dV + V. dU $\qquad$ $\displaystyle\int x^r\,dx = \frac{x^{r+1}}{r+1} + C$ $\quad$ where $-1 \neq r \in \mathbb{R}$

$\displaystyle\int \sin x\,dx = -\cos x + C$ $\qquad$ $\displaystyle\int \cos x\,dx = \sin x + C$ $\qquad$ $\displaystyle\int \frac{1}{1+x^2}\,dx = \tan^{-1}x + C$

$\displaystyle\int \sec^2 x\,dx = \tan x + C$ $\qquad$ $\displaystyle\int e^x\,dx = e^x + C$ $\qquad$ $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C$

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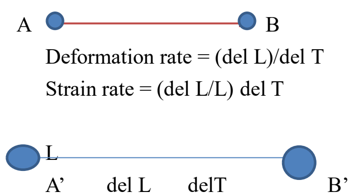
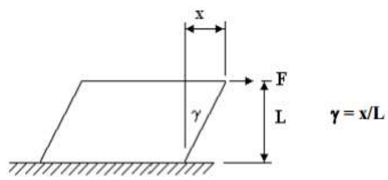
BASIC CONCEPT ON MATHAMETIC IN FLUID MECHANICS (Note)

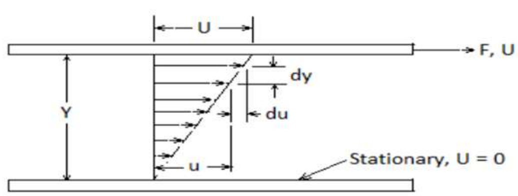
- ***Normal Stress, σ:*** Normal force acting on per unit area of surface is called normal stress $\sigma = P/A$. Normal stress work perpendicular to the surface or perpendicular cross section.
- ***Shear Stress, τ:*** Shear force acting on per unit area of the surface is called shear stress. Shear stress work parallel to the surface or parallel to the cross section. $\tau = F_s/A$.
- ***Deformation rate:*** Rate of change in distance between two neighboring points moving with fluid divided by the distance between the points. That is, "change in length per unit length per unit time.
- Shear Strain, γ or angular deformation:





NEWTON'S LAW OF VISCOSITY

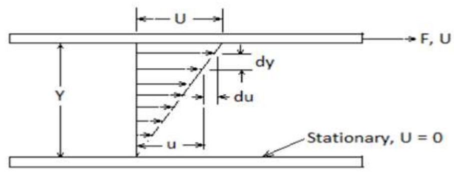
Consider two large plates are placed a small distance Y apart and space between them being filled with fluid. The bottom plate is at rest and upper plate is moved parallel to bottom plate with a velocity U and by application of force F.



From experiment, it is being shown that, $F \propto \frac{AU}{Y}$ $\qquad \frac{F}{A} \propto \frac{U}{Y}$

Shear Stress, $\tau = \frac{F}{A}$ and $\frac{U}{Y} = \frac{du}{dy}$

NEWTON'S LAW OF VISCOSITY (Continue)



$$\frac{F}{A} \propto \frac{U}{Y} \qquad \frac{U}{Y} = \frac{du}{dy} \qquad \tau = \frac{F}{A}$$

$\frac{du}{dy}$ is called velocity gradient

Thus, shear stress is $\tau = \mu \frac{du}{dy}$; where μ is a proportionality constant.

NEWTON'S LAW OF VISCOSITY

The equation $\tau = \mu \frac{du}{dy}$ is called Newton's equation of viscosity. The proportionality constant 'μ' is known as coefficient of viscosity or absolute viscosity or dynamic viscosity or simply viscosity of the fluid. The term $\frac{du}{dy}$ is called rate of strain or shear rate or velocity gradient or rate of shear deformation.

The unit of viscosity 'μ' is $N.s/m^2$ or Poise; 1 $N.s/m^2$ =10 Poise or 1 Poise = 0.10 $N.s/m^2$

Kinematic viscosity 'ν' = μ /ρ (unit m^2/s)

NEWTONIAN LAW OF VISCOSITY (PROBLEM)

Problem -1: A plate 0.0254 mm distant from a fixed plate, moves at 61 cm/sec and requires a force of 0.2 kg(f)/m^2 to maintain this speed. Determine the dynamic viscosity of the fluid between the plate?

SEE HAND CALCULATION !!!!

Y = 0.0254 mm = ( ) m ; U = 61 cm/s = ( ) m/s; F/A = 0.2 kg(f)/m^2 = ??? (τ)

T = (0.2 kg * 9.81 m/s^2) /m^2 = ( ) N/m^2 ; du/dy = U/Y

$$\tau = \mu \frac{du}{dy}$$

NEWTONIAN LAW OF VISCOSITY (PROBLEM)

Problem-2: A cylindrical shaft of 95 mm diameter rotates at a speed of 50 rpm inside a cylinder of 96.4 mm diameter. Both shaft and the cylinder are 0.50 m long. If the torque required to rotate the shaft is equal to 1 N-m, find the viscosity of the oil occupying in between shaft and cylinder.

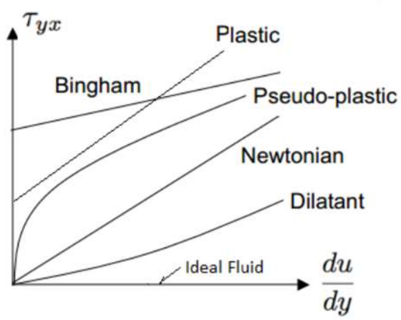
SEE HAND CALCULATION !!!!

NEWTONIAN AND NON-NEWTONIAN FLUID

- Fluids that are obeying the Newton's Law of Viscosity called **Newtonian fluid**; and the fluids that are not obeying the Newton's Law of Viscosity called **Non-Newtonian fluid**.
  - *Newtonian Fluid:* Water, air, oils, glycerin.
  - Non-Newtonian Fluid (example):
    - *Bingham Fluid:* Paint, Ketchup
    - *Peseudo-plastic Fluid:* Styling gel.
    - *Dilatant Fluid:* Coupling fluids used in 2 wheel and 4 wheel drive car.

NEWTONIAN AND NON-NEWTONIAN FLUID

Ideal Fluid: Ideal fluid offer no viscosity to flow (inviscid i.e. 0 viscosity) and incompressible.
Real Fluid: Offer resistance to flow or have viscosity.



Each of these lines can be represented by the equation $\tau = A + B\left(\frac{\delta u}{\delta y}\right)^n$

where A, B and n are constants. For Newtonian fluids A = 0, B = μ and n = 1.

APPLICATION OF TAYLOR'S SERIES IN FLUID MECHANICS)

The Taylor series of a real or complex-valued function $f(x)$ that is infinitely differentiable at a real or complex number a is the power series

$$f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f^{(3)}(a)}{3!}(x-a)^3+\cdots.$$

which can be written in the more compact sigma notation as

$$\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$$

Example: For a function $f(x)$

$$f(x+h)=f(x)+f'(x)h+\frac{f''(x)}{2!}h^2+\frac{f'''(x)}{3!}h^3+\cdots$$

$$\rho)_{x+dx/2}=\rho+\left(\frac{\partial\rho}{\partial x}\right)\frac{dx}{2}+\left(\frac{\partial^2\rho}{\partial x^2}\right)\frac{1}{2!}\left(\frac{dx}{2}\right)^2+\cdots$$

$$\rho)_{x+dx/2}=\rho+\left(\frac{\partial\rho}{\partial x}\right)\frac{dx}{2}\quad\left(\text{Neglecting higher-order terms}\right)$$

APPLICATION OF TAYLOR'S SERIES IN FLUID MECHANICS

Given, $f(x)=\sin(x)$ find the value of $\sin(2)$?

$\sin\left(\frac{\pi}{2}\right)=1 \quad x=\frac{\pi}{2} \quad x+h=2 \quad h=2-x=2-\frac{\pi}{2}=0.42920$

$$f(x+h)=f(x)+f'(x)h+f''(x)\frac{h^2}{2!}+f'''(x)\frac{h^3}{3!}+f''''(x)\frac{h^4}{4!}+\cdots$$

$x=\frac{\pi}{2} \quad h=0.42920$

$f(x)=\sin(x),\ f\left(\frac{\pi}{2}\right)=\sin\left(\frac{\pi}{2}\right)=1 \quad f'(x)=\cos(x),\ f'\left(\frac{\pi}{2}\right)=0 \quad f''(x)=-\sin(x),\ f''\left(\frac{\pi}{2}\right)=-1$

$f'''(x)=-\cos(x),\ f'''\left(\frac{\pi}{2}\right)=0 \quad f''''(x)=\sin(x),\ f''''\left(\frac{\pi}{2}\right)=1$

Hence

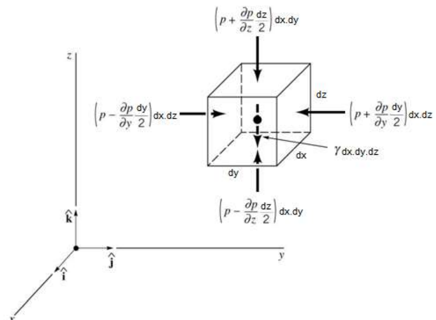
$$f\left(\frac{\pi}{2}+h\right)=f\left(\frac{\pi}{2}\right)+f'\left(\frac{\pi}{2}\right)h+f''\left(\frac{\pi}{2}\right)\frac{h^2}{2!}+f'''\left(\frac{\pi}{2}\right)\frac{h^3}{3!}+f''''\left(\frac{\pi}{2}\right)\frac{h^4}{4!}+\cdots$$

$$f\left(\frac{\pi}{2}+0.42920\right)=1+0(0.42920)-1\frac{(0.42920)^2}{2!}+0\frac{(0.42920)^3}{3!}+1\frac{(0.42920)^4}{4!}+\cdots$$

$$=1+0-0.092106+0+0.00141393+\cdots\cong 0.90931$$

HYDROSTATIC PRESSURE OF FLUID

- The normal force exerted by a fluid per unit area of the surface is called the fluid pressure. It is noted that if an imaginary surface is assumed within a fluid body, the fluid pressure and pressure force on that imaginary surface are exactly the same as acting on any real surface.
- Consider a infinitesimal fluid element of size dx, dy and dz at a point in a static mass of fluid. Let p is the pressure intensity at the center of the element



HYDROSTATIC PRESSURE OF FLUID

The normal force exerted by a fluid per unit area of the surface is called the fluid pressure. It is noted that if an imaginary surface is assumed within a fluid body, the fluid pressure and pressure force on that imaginary surface are exactly the same as acting on any real surface.

Consider a infinitesimal fluid element of size dx, dy and dz at a point in a static mass of fluid. Let p is the pressure intensity at the center of the element.

(DERIVATION BY HAND)

The pressure intensity p at any point in a static mass of fluid does not vary in x and y directions but it varies only in z direction. The equation, $dp/dz = -\rho g$ is valid for both compressible and incompressible fluid.

The minus sign in the equation signifies that the pressure decreases in the direction in which z increases i.e. in the upward direction. Again, if dz = o, then dp is also equal to zero; which means that the pressure remains constant over any horizontal plane in a fluid.

HYDROSTATIC PRESSURE OF FLUID

(DERIVATION p = ρgh BY HAND)

If ρ = constant then, $p = \rho g h = \gamma h$, which is called hydrostatic pressure of stationary and incompressible fluid; and function of vertical depth & density of the fluid.

Pressure Head: The vertical height of the free surface above any point in a liquid at rest is known as Pressure Head. Pressure Head, $h = p/\gamma = p/\rho g$

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HYDROSTATIC PRESSURE OF COMPRESSIBLE FLUID

Gases such as air, oxygen and nitrogen are thought of as compressible, so we must consider the variation of density in the hydrostatic equation:

$\frac{dp}{dz} = -\gamma$ Note: $\gamma = \rho g$ and not a constant, then $\frac{dp}{dz} = -\rho g$

By the Ideal gas law: $p = \rho R T$ Thus, $\rho = \frac{p}{RT}$

R is the Gas Constant
T is the temperature
ρ is the density

Then, $$\frac{dp}{dz} = -\frac{gp}{RT}$$

$$\int_{p_1}^{p_2} \frac{dp}{p} = \ln\frac{p_2}{p_1} = -\frac{g}{R}\int_{z_1}^{z_2}\frac{dz}{T}$$

For Isothermal Conditions, T is constant, T_o

$$p_2 = p_1 \exp\left[-\frac{g(z_2 - z_1)}{RT_0}\right]$$

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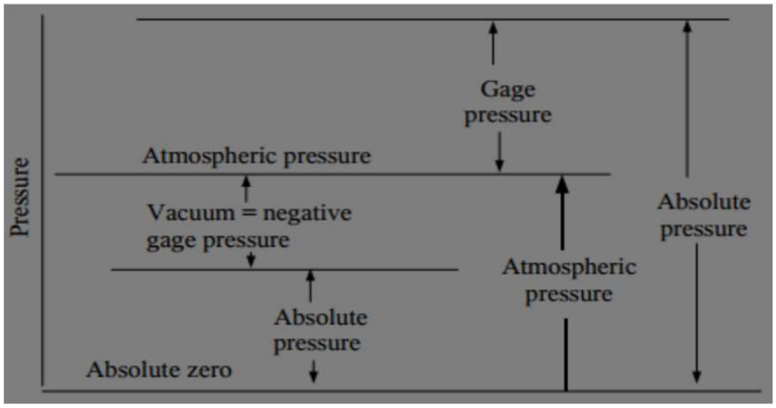
ABSOLUTE AND GAGE PRESSURE



$P_{absolute} = P_{gage} + P_{atmp}$ and $P_{absolute} = P_{atmp} - P_{vacuum}$

Gage Pressure is positive or negative but absolute pressure always positive.

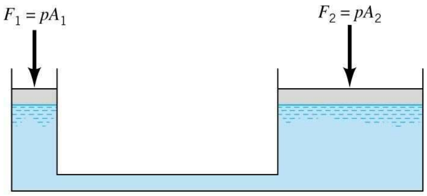
ABSOLUTE AND GAGE PRESSURE UNIT

As we saw, force per unit area is measured in N/m^2 which is the same as a Pascal (Pa). The units used in practice vary:

- 1 kPa = 1000 Pa = 1000 N/m^2
- 1 MPa = 1000 kPa = 1 × 106 N/ m^2
- 1 bar = 10^5 Pa = 100 kPa = 0.1 MPa
- 1 atm = 101,325 Pa = 101.325 kPa = 1.01325 bars

APPLICATION OF HYDROSTATIC FORCE

Transmission of fluid pressure:

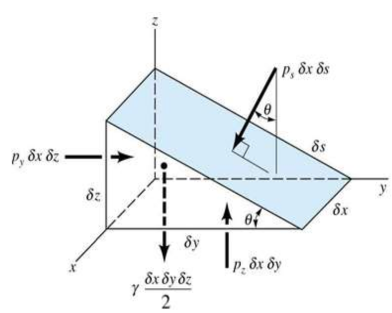


$$F_2 = \frac{A_2}{A_1} F_1 \qquad \frac{A_2}{A_1} > 1 \qquad F_2 > F_1$$

- Mechanical advantage can be gained with equality of pressures.
- A small force applied at the small piston is used to develop a large force at the large piston.
- This is the principle between hydraulic jacks, lifts, presses, and hydraulic controls.
- Mechanical force is applied through jacks action or compressed air for example.

FLUID PRESSURE SAME IN ALL DIRECTION- PASCAL'S LAW

The pressure at any point of fluid at rest has same magnitude in all directions. Consider a free body of an infinitesimal wedge shape of fluid element at rest. Only forces acting on the free body are the normal pressure forces and weight of the fluid element.



$$\sum F_y = p_y\,\delta x\,\delta z - p_s\,\delta x\,\delta s \sin\theta = 0$$

$$p_y\,\delta x\,\delta s \sin\theta - p_s\,\delta x\,\delta s \sin\theta = 0$$

$$p_y = p_s$$

$$(\ \delta y = \delta s \cos\theta \qquad \delta z = \delta s \sin\theta\ )$$

$$\sum F_z = p_z\,\delta x\,\delta y - p_s\,\delta x\,\delta s \cos\theta - \gamma\,\frac{\delta x\,\delta y\,\delta z}{2} = 0$$

$p_z\,\delta x\,\delta y - p_s\,\delta x\,\delta y = 0$ (since the term $\delta x\,\delta y\,\delta z$ is negligible)

$$p_z = p_s$$

$$p_y = p_z = p_s$$

FLUID PRESSURE SAME IN ALL DIRECTION- PASCAL'S LAW

$P_s = P_z = P_y$; So the pressure at a point in a static fluid has same magnitude in all direction. This is known as Pascal's law and applies to fluid at rest.

If fluid in motion so that one layer moves relative to an adjacent layer, shear stress occur and the normal stress are no longer same in all direction. The pressure at a point is then defined as average pressure of any three mutually perpendicular planes at that point.

$$p = (p_x + p_y + p_z)/3$$

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CAPILLARY RISE OF FLUID

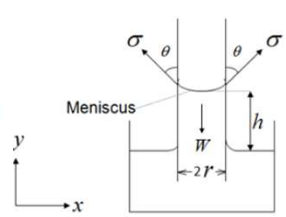
The capillary rise or depression of liquid can be determined by considering the conditions of equilibrium in a circular tube of small diameter inserted in a liquid. Consider a fine tube of radius r is inserted in the liquid, the density of the liquid is ρ and the capillary rise is h as shown in the figure below.

Equilibrium in *y-direction* yields: $\sigma \cos\theta (2\pi r) - W = 0$

$$W = \gamma \pi r^2 h$$

$$\gamma \pi r^2 h = \sigma \cos\theta (2\pi r)$$

$$h = \frac{2\sigma\cos\theta}{\gamma r}$$



Where σ is the surface tension of the fluid.

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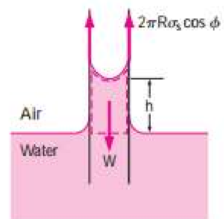
CAPILLARITY PROBLEM

A 0.6-mm-diameter glass tube is inserted into water at 20°C in a cup. Determine the capillary rise of water in the tube

Surface tension of some fluids in air at 1 atm and 20°C (unless otherwise stated)

Fluid	Surface Tension σ_s, N/m*
Water:	
0°C	0.076
20°C	0.073
100°C	0.059
300°C	0.014
Glycerin	0.063
SAE 30 oil	0.035
Mercury	0.440
Ethyl alcohol	0.023
Blood, 37°C	0.058
Gasoline	0.022
Ammonia	0.021
Soap solution	0.025
Kerosene	0.028



SEE HAND ANALYSIS