Problem - 1

$L_p = 240\,m$, ~~$Q_p = 250\,m^3/s$~~

$Q_p = 250\,m^3/s$

$H_p = 1.2\,m$

$L_m = 3\,m$

$H_m = 500\,m$ (maximum can be)

$Q_m = 25\,L/s = 0.025\,m^3/s$

We know,

$$Q_p = Q_m \times S_h \times S_v^{1.5}$$

$$\Rightarrow Q_m = Q_p \times \frac{1}{S_h} \times \frac{1}{S_v^{1.5}}$$

$$\Rightarrow Q_m = Q_p \times \frac{L_m}{L_p} \times \left(\frac{h_m}{h_p}\right)^{1.5}$$

$$\Rightarrow 0.025 = 250 \times \frac{3}{240} \times \left(\frac{h_m}{1.2}\right)^{1.5}$$

$$\therefore h_m = 0.048\,m$$

$$= 48\,mm$$

$\therefore$ Head should be 48 mm

Problem - 2

a) $h_m = 6m$

$h_p = 30m$

$N_p = 428$ rpm

$\frac{1}{S_h} = \frac{1}{8}$

$\therefore S_h = 8$

$\frac{1}{S_v} = \frac{h_m}{h_p}$

$S_v = \frac{h_p}{h_m} = \frac{30}{6} = 5$

$$\frac{N_m}{N_p} = \frac{S_h}{\sqrt{S_v}}$$

$$\Rightarrow N_m = N_p \times \frac{S_h}{\sqrt{S_v}} = 428 \times \frac{8}{\sqrt{5}}$$

$$= 1531.26 \text{ rpm}$$

b)

$P_m = 5kW$

$Q_m = 110 \text{ L/s} = 0.11 \text{ m}^3/\text{s}$

$$\frac{P_m}{P_p} = \frac{\rho_m}{\rho_p} \frac{Q_m}{Q_p} \frac{h_m}{h_p} \times \frac{\eta_m}{\eta_p}$$

$$= \frac{Q_m}{Q_m \times S_h \times S_v^{1.5}} \times \frac{6}{30} \times \frac{100}{103}$$

$$\frac{5000}{P_p} = \frac{0.11}{0.11 \times 8 \times 5^{1.5}} \times 0.97$$

$$\Rightarrow P_p = 461044.94 \text{ W} = 461.044 \text{ kW}$$

Non - Dimensional Constant

Inertia force = mass × acceleration

$= \rho \times L^3 \times \frac{V}{t} = \rho L^2 V^2$

Viscous = Area × Shear Stress

$= L^2 \times \mu \times \frac{V}{L} = \mu V L$

Gravity = mass × g = $\rho \times L^3 \times g = \rho L^3 g$

Pressure force = Pressure × Area = $P L^2$

Elastic Force = Bulk Modulus of Elasticity × area

$= E L^2 = \rho C^2 L^2$

Surface Tension = σL

* Reynolds No → (N_{Re})

It is ratio of inertia force and viscous force.

$$N_{Re} = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho V^2 L^2}{\mu V L} = \frac{\rho V L}{\mu}$$

* Froude No (F_r) →

It is the ratio of inertia force and gravity force

$$F_r = \frac{\text{Inertia force}}{\text{Gravity force}} = \frac{\rho V^2 L^2}{\rho L^3 g} = \frac{V^2}{L.g}$$

* Euler Number (E):

It is the ratio of pressure force and inertia force

$$E = \frac{\text{Pressure force}}{\text{Inertia force}} = \frac{PL^2}{\rho L^2 V^2} = \frac{P}{\rho V^2} = \frac{F}{\rho V^2 L^2}$$

* Mach Number (M):

It is square root of the ratio of inertia force to the elastic force.

$$M = \left(\frac{\text{Inertia force}}{\text{Elastic force}}\right)^{\frac{1}{2}} = \left(\frac{\rho V^2 L^2}{\rho c^2 L^2}\right)^{\frac{1}{2}} = \frac{V}{c}$$

$$\text{Cauchy's Number} = \frac{V^2}{c^2}$$

* Weber Number (W):

It is the ratio of inertia force and surface tension force.

$$W = \frac{\text{Inertia force}}{\text{Surface tension force}} = \frac{\rho V^2 L^2}{\sigma . L} = \frac{\rho L V^2}{\sigma}$$

Problem - 1

$\Delta P_p = 2\ kN/m^2$

$\Delta P_m = 162\ kN/m^2$

$\rho_m = 900\rho_p$

$\mu_m = 90\mu_p$

Here,

$Re_m = Re_p$

$\Rightarrow \frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p}$

$\Rightarrow \frac{900\cancel{\rho_p} \times V_m \times L_m}{90\cancel{\mu_p}} = \frac{\cancel{\rho_p} V_p L_p}{\cancel{\mu_p}}$

$\Rightarrow V_r L_r = 0.1$ ---- (i)

Again,

$E_m = E_p$

$\Rightarrow \frac{P_m}{\rho_m V_m^2} = \frac{P_p}{\rho_p V_p^2}$

$\Rightarrow \frac{162}{900\cancel{\rho_p} \times V_m^2} = \frac{2}{\cancel{\rho_p} \times V_p^2}$

$\Rightarrow V_r = \sqrt{\frac{162}{900 \times 2}} = 0.3$ — (ii) putting in ① $\Rightarrow$

$$0.3 \times L_r = 0.1$$

$$\therefore L_r = \frac{1}{3}$$

$$\Rightarrow \frac{L_m}{L_p} = \frac{1}{3}$$

$$\therefore 1:3$$

Problem - 2

$$\frac{1}{S} = \frac{1}{45} = \frac{L_m}{L_p} \qquad \cancel{L_p = 45 L_m}$$

$$\mu_m = 50 \mu_p \qquad \Rightarrow L_m = \frac{L_p}{45}$$

$$\rho_m = 750 \rho_p$$

Now,

$$Re_m = Re_p$$

$$\Rightarrow \frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p}$$

$$\Rightarrow \frac{750 \cancel{\rho_p} \times V_m \times \frac{\cancel{L_p}}{45}}{50 \cancel{\mu_p}} = \frac{\cancel{\rho_p} \times V_p \times \cancel{L_p}}{\cancel{\mu_p}}$$

$$\Rightarrow V_m = 3 V_p \quad \cancel{\text{—— ①}}$$

$$\Rightarrow \frac{V_m}{V_p} = 3 \quad \ldots\ldots ①$$

Drag Force,

$$F_D = \frac{1}{2} \rho V^2 A C_d$$

$$\therefore \frac{F_{Dm}}{F_{Dp}} = \frac{\rho_m V_m^2 A_m}{\rho_p V_p^2 A_p}$$

$$\Rightarrow \frac{0.98}{F_{Dp}} = 750 \times 3^2 \times \left(\frac{1}{45}\right)^2$$

$$\Rightarrow F_{Dp} = 0.294 \text{ N}$$

Problem - 3

$$\frac{1}{5} = \frac{1}{60} = \frac{L_m}{L_p}$$

$$R_m = 0.36 \text{ N}$$

$$V_m = 1.25 \text{ m/s}$$

Now,

$$\text{Froude No} = \frac{V^2}{Lg}$$

$$\frac{V_m^2}{L_m g} = \frac{V_p^2}{L_p g}$$

$$\Rightarrow V_p = V_m \sqrt{\frac{L_p}{L_m}} = 1.25 \times \sqrt{60} = 9.68 \text{ ms}^{-1}$$

$$\frac{R_m}{R_p}$$

$$\therefore R_p = R_m \left(\frac{\cancel{\rho_p}}{\cancel{\rho_m}} \right) \times 5^3$$

$$= 0.36 \times \cancel{60}\ 60^3$$

$$= 77760\ N$$

$$\therefore P_m = R_m \times V_m = 0.36 \times 1.25 = 0.45\ W$$

$$\therefore P_p = R_p \times V_p = 77760 \times 9.68 = 752716.8\ W$$

$$= 752.716\ kW$$

Compressible Flow

Mach No (M) $= \dfrac{\text{Velocity of fluid or aircraft}}{\text{Velocity of sound}}$

$$= \frac{V}{C}$$

* Energy equation of compressible flow for adiabatic flow →

$\Rightarrow$

$$PV^k = \text{constant}$$

$$\Rightarrow \frac{P}{\rho^k} = \text{constant} \Rightarrow \frac{P}{\rho^k} = \frac{P_1}{\rho_1^k}$$

$$\Rightarrow \frac{1}{\rho^k} = \frac{P_1}{\rho_1^k} \cdot \frac{1}{P} \Rightarrow \frac{1}{\rho} = \left(\frac{P_1}{P}\right)^{\frac{1}{k}} \cdot \frac{1}{\rho_1} \quad \text{———} \ (1)$$

Now,

$$\frac{dp}{\rho} + v\,dv = 0$$

$$\Rightarrow \left(\frac{P_1}{P}\right)^{\frac{1}{k}} \frac{dp}{\rho_1} + v\,dv = 0 \quad [\text{From } (1)]$$

$$\Rightarrow \frac{P_1^{\frac{1}{k}}}{\rho_1} \int_1^2 \frac{dp}{p^{1/k}} + \int_1^2 v\,dv = 0 \quad [\text{Integrating}]$$

$$\Rightarrow \frac{P_1^{1/k}}{\rho_1} \left[\frac{p^{-\frac{1}{k}+1}}{-\frac{1}{k}+1} \right]_1^2 + \frac{1}{2}\left(v_2^2 - v_1^2\right) = 0$$

$$\Rightarrow \frac{K}{K-1}\frac{P_1^{1/K}}{\rho_1}\left[P_2^{\frac{K-1}{K}} - P_1^{\frac{K-1}{K}}\right] + \frac{1}{2}\left(V_2^2 - V_1^2\right) = 0$$

$$\Rightarrow \frac{1}{2}\left(V_2^2 - V_1^2\right) = \frac{K}{K-1}\left[\frac{P_1^{\frac{K-1}{K}} \cdot P_1^{1/K}}{\rho_1} - \frac{P_2^{\frac{K-1}{K}} P_1^{1/K}}{\rho_1}\right]$$

$$\Rightarrow \frac{1}{2}\left(V_2^2 - V_1^2\right) = \frac{K}{K-1}\left[\frac{P_1}{\rho_1} - \frac{1}{\rho_1} P_2^{\frac{K-1}{K}} P_1^{\frac{1}{K}}\right]$$

$$= \frac{K}{K-1}\left[\frac{P_1}{\rho_1} - P_2^{\frac{K-1}{K}} \frac{P_2^{1/K}}{\rho_2}\right]$$

$$P = \rho R T$$
$$P_1 = \rho_1 R T_1$$
$$P_2 = \rho_2 R T_2$$

$$= \frac{K}{K-1}\left[\frac{P_1}{\rho_1} - \frac{P_2}{\rho_2}\right] \quad \cdots\cdots (1)$$

$$= \frac{KR}{K-1}\left(T_1 - T_2\right) = \frac{K}{K-1} R T_1 \left(1 - \frac{T_2}{T_1}\right)$$

$$= \frac{K}{K-1} \cdot \frac{P_1}{\rho_1}\left[1 - \frac{P_2}{\rho_2} \frac{\rho_1}{P_1}\right]$$

$$= \frac{K}{K-1}\frac{P_1}{\rho_1}\left[1 - \frac{P_2}{P_1}\frac{\rho_1}{\rho_2}\right]$$

$$= \frac{K}{K-1}\frac{P_1}{\rho_1}\left[1 - \frac{P_2}{P_1}\left(\frac{P_2}{P_1}\right)^{-\frac{1}{K}}\right]$$

$$= \frac{K}{K-1}\frac{P_1}{\rho_1}\left[1 - \left(\frac{P_2}{P_1}\right)^{1-\frac{1}{K}}\right]$$

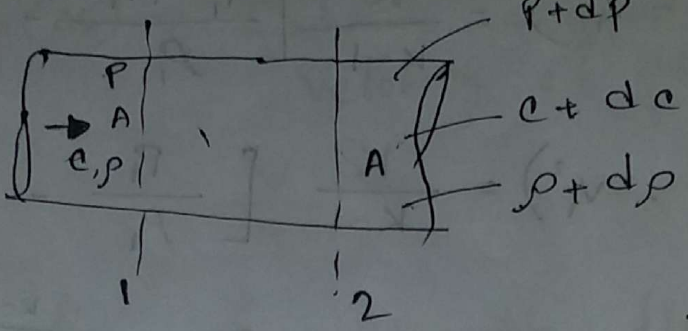
$$\frac{P_1}{\rho_1^K} = \frac{P_2}{\rho_2^K}$$
$$\Rightarrow \left(\frac{\rho_1}{\rho_2}\right)^K = \frac{P_1}{P_2}$$
$$\Rightarrow \frac{\rho_1}{\rho_2} = \left(\frac{P_1}{P_2}\right)^{1/K} = \left(\frac{P_2}{P_1}\right)^{-\frac{1}{K}}$$

$$\Rightarrow \boxed{\frac{1}{2}\left(V_2^2 - V_1^2\right) = \frac{K}{K-1}\frac{P_1}{\rho_1}\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}}\right]}$$

[Proved]

* Prove that velocity of sound in a fluid media fo[r] isentropic condition, $c = \sqrt{KRT}$

$\Rightarrow$



Mass flow rate same -

$$\dot{m}_1 = \dot{m}_2$$

$$\Rightarrow A c \rho = A(c+dc)(\rho+d\rho)$$

$$\Rightarrow \cancel{Ac\rho} = \cancel{Ac\rho} + Ac\,d\rho + A\,dc\rho + \cancel{A\,dc\,d\rho}$$

$$\Rightarrow A c\,d\rho + A\rho\,dc = 0$$

$$\Rightarrow c\,d\rho + \rho\,dc = 0$$

$$\Rightarrow dc = -c\frac{d\rho}{\rho} \text{ ------ (I)}$$

$$\dot{m} = A v \rho$$

$$F = ma = m\frac{dv}{dt}$$

$$\Rightarrow PA = \frac{m}{dt}dv$$

$$PA = \dot{m}\,dv$$

Fluid is moving left to right,

$$[P - (P+dp)]A = Ac\rho[(c+dc) - c]$$

$$\Rightarrow P - P - dp = \rho c\,dc$$

$$\Rightarrow dc = -\frac{dp}{\rho c} \text{ ------ (II)}$$

From (i) & (ii) $\Rightarrow$

$$-c\frac{d\rho}{\rho} = -\frac{dp}{\rho c}$$

$$\Rightarrow c^2 d\rho = dp$$

$$\Rightarrow c = \sqrt{\frac{dp}{d\rho}}$$

$$\Rightarrow c = \sqrt{\frac{E}{\rho}} \quad \text{---} (iii)$$

For isothermal condition,

$$PV = \text{constant}$$

$$\Rightarrow P.dv + v.dp = 0$$

$$\Rightarrow P = -v\frac{dp}{dv}$$

$$\Rightarrow P = -\frac{dp}{\frac{dv}{v}} = \frac{dp}{\frac{d\rho}{\rho}}$$

$$\Rightarrow P = E \quad \text{---} (iv)$$

From (iv) in (iii) $\Rightarrow$

$$c = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{\cancel{P}}{\cancel{P}/RT}}$$

$$\Rightarrow \boxed{c = \sqrt{RT}} \longrightarrow$$ velocity of sound for isothermal condition

Bulk modulus of elastics

$$K = -\frac{dp}{\frac{dv}{v}} = \frac{dp}{\frac{d\rho}{\rho}}$$

in compressive,

$$E = \frac{dp}{\frac{d\rho}{\rho}}$$

$$\Rightarrow E = \frac{dp}{d\rho}.\rho$$

$$\Rightarrow \frac{E}{\rho} = \frac{dp}{d\rho}$$

$$P = \rho RT$$

$$\Rightarrow \rho = \frac{P}{RT}$$

Adiabatic condition,

$$PV^k = \text{constant}$$

$$\Rightarrow dp\, V^k + p . k V^{k-1} dv = 0$$

$$\Rightarrow V^k \left(dp + p . \frac{k}{v} . dv \right) = 0$$

$$\Rightarrow dp + p \frac{k}{v} . dv = 0$$

$$\Rightarrow Kp = - \frac{dp}{\frac{dv}{v}}$$

$$\Rightarrow Kp = E \quad \text{—— (v)}$$

velocity of sound in adiabatic condition,

From (iii) $\Rightarrow$

$$C = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{K\cancel{p}}{\frac{\cancel{p}}{RT}}} \quad [\text{From (v)}]$$

$$\Rightarrow \boxed{C = \sqrt{KRT}}$$

(Proved)

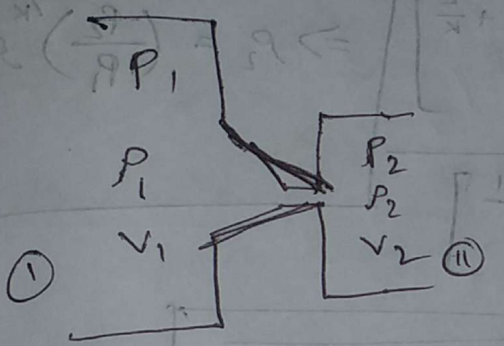
Derive the following equation of mass flow rate through a convergence nozzle for compressible fluid flow,

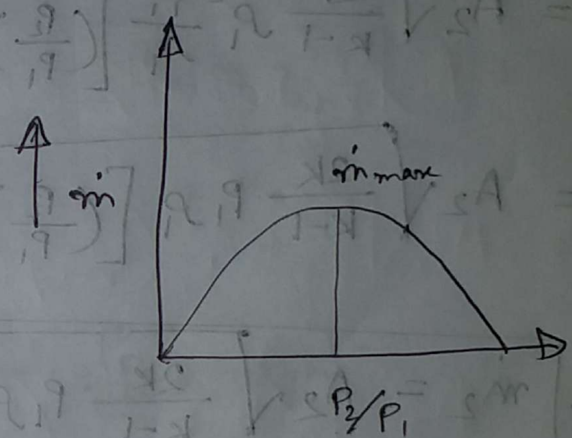
$$\dot{m} = A_2 \rho_1 \sqrt{\frac{2k}{k-1} \cdot \frac{P_1}{\rho_1} \left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{k}} - \left(\frac{P_2}{P_1}\right)^{\frac{k+1}{k}} \right]}$$

and for maximum mass flow rate,

$$\frac{P_2}{P_1} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$

$\Rightarrow$





At point ① →

mass flow rate is zero

cause inlet nozzle velocity, $V_1 = 0$

Now,

From energy eqn,

$$\frac{1}{2}\left(V_2^2 - V_1^2\right) = \frac{k}{k-1} \frac{P_1}{\rho_1} \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]$$

$$\Rightarrow V_2 = \sqrt{\frac{2k}{k-1} \frac{P_1}{\rho_1} \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]} \quad [\because V_1 = 0]$$

——— ①

For point ii, mass flow rate,

$$\dot{m}_2 = V_2 \rho_2 A_2$$

$$= A_2 \rho_2 \left[\frac{2k}{k-1} \frac{P_1}{\rho_1} \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} \right] \right]^{\frac{1}{2}} \quad \text{[From 1]}$$

$$= A_2 \left(\frac{P_2}{P_1} \right)^{1/k} \rho_1 \sqrt{ \frac{2k}{k-1} \frac{P_1}{\rho_1} \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} \right] }$$

$$= A_2 \rho_1 \sqrt{ \frac{2k}{k-1} \frac{P_1}{\rho_1} \left(\frac{P_2}{P_1} \right)^{2/k} \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} \right] }$$

$$= A_2 \sqrt{ \frac{2k}{k-1} \rho_1^2 \frac{P_1}{\rho_1} \left[\left(\frac{P_2}{P_1} \right)^{2/k} - \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k} + \frac{2}{k}} \right] }$$

$$= A_2 \sqrt{ \frac{2k}{k-1} P_1 \rho_1 \left[\left(\frac{P_2}{P_1} \right)^{2/k} - \left(\frac{P_2}{P_1} \right)^{\frac{k+1}{k}} \right] }$$

$$\frac{P_1}{\rho_1^k} = \frac{P_2}{\rho_2^k}$$

$$\Rightarrow \rho_2^k = \frac{P_2}{P_1} \rho_1^k$$

$$\Rightarrow \rho_2 = \left(\frac{P_2}{P_1} \right)^{1/k} \rho_1$$

$$\therefore \boxed{ \dot{m}_2 = A_2 \sqrt{ \frac{2k}{k-1} P_1 \rho_1 \left[\left(\frac{P_2}{P_1} \right)^{2/k} - \left(\frac{P_2}{P_1} \right)^{\frac{k+1}{k}} \right] } }$$

[Proved]

Now, Let, $x = \frac{P_2}{P_1}$

$$m_2 = A_2 \sqrt{ \frac{2k}{k-1} P_1 \rho_1 \left[x^{2/k} - x^{\frac{k+1}{k}} \right] }$$

if $\frac{d\dot{m}}{dx} = 0$,

$$A_2 \cdot \frac{1}{2} \left\{ \left[\frac{2k}{k-1} P_1 \rho_1 \left[x^{2/k} - x^{\frac{k+1}{k}} \right] \right] \right\}^{-\frac{1}{2}} \left[\frac{2}{k} x^{2/k - 1} - \frac{k+1}{k} x^{\frac{k+1}{k} - 1} \right] = 0$$

$$\Rightarrow \frac{2}{k}\, x^{2/k - 1} - \frac{k+1}{k}\, x^{\frac{k+1}{k} - 1} = 0$$

$$\Rightarrow \frac{2}{k}\, x^{\frac{2-k}{k}} - \frac{k+1}{k}\, x^{\frac{k+1-k}{k}} = 0$$

$$\Rightarrow 2 \cdot x^{2-k/k} - (k+1)\, x^{1/k} = 0$$

$$\Rightarrow 2\, x^{2-k/k} = (k+1)\, x^{1/k}$$

$$\cancel{\Rightarrow 2\, x^{2/k}\, x^{-1} = (k+1)\, x^{1/k}}$$

$$\Rightarrow \frac{2}{k+1} = x^{\frac{1}{k} - \frac{2-k}{k}}$$

$$\Rightarrow x^{\frac{k-1}{k}} = \frac{2}{k+1}$$

$$\Rightarrow \boxed{x = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}}$$ ~~[Proved]~~

$$\Rightarrow \boxed{\frac{P_2}{P_1} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}}$$ [Proved]

* Derive the equation for relationship between area, velocity and Mach no. for compressible flow, $\frac{dA}{dV} = \frac{A}{V}(M^2-1)$

$\Rightarrow$

$$\dot{m} = A_2\sqrt{\frac{2k}{k-1}P_1\rho_1\left[\left(\frac{P_2}{P_1}\right)^{2/k} - \left(\frac{P_2}{P_1}\right)^{\frac{k+1}{k}}\right]}$$

$$\boxed{\frac{P_2}{P_1} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}}$$

$$= A_2\sqrt{\frac{2k}{k-1}P_1\rho_1}\left[\left(\frac{2}{k+1}\right)^{\frac{2}{k-1}} - \left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\right]^{\frac{1}{2}}$$

$$= A_2\sqrt{\frac{2k}{k-1}P_1\rho_1}\sqrt{\left(\frac{2}{k+1}\right)^{\frac{2}{k-1}}\left[1 - \left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\left(\frac{2}{k+1}\right)^{-\frac{2}{k-1}}\right]}$$

$$= A_2\sqrt{\frac{2k}{k-1}P_1\rho_1}\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}\left[1 - \frac{2}{k+1}\right]^{\frac{1}{2}}$$

$$= A_2\sqrt{\frac{2k}{k-1}P_1\rho_1}\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}\sqrt{\frac{k-1}{k+1}}$$

$$= A_2\frac{\sqrt{2kP_1\rho_1}}{\cancel{\sqrt{k-1}}}\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}\frac{\cancel{\sqrt{k-1}}}{\sqrt{k+1}}$$

$$= A_2\sqrt{2kP_1\rho_1}\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}\frac{1}{\sqrt{k+1}}$$

$$= A_2\sqrt{kP_1\rho_1}\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}\left(\frac{2}{k+1}\right)^{\frac{1}{2}}$$

$$= A_2\sqrt{kP_1\rho_1}\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}+\frac{1}{2}}$$

$$= A_2 \sqrt{K P_1 \rho_1} \left(\frac{2}{k+1}\right)^{\frac{k+1}{2(k-1)}}$$

From energy eq^n $\rightarrow$

$$\frac{1}{2}(V_2^2 - V_1^2) = \frac{KR}{K-1}(T_1 - T_2)$$

$$\Rightarrow \frac{1}{2} V_2^2 = \frac{KR}{K-1} T_2 \left(\frac{T_1}{T_2} - 1\right) \quad [\because V_1 = 0]$$

$$\Rightarrow \frac{1}{2} V_2^2 = \frac{KR}{K-1} T_2 \left[\left(\frac{P_2}{P_1}\right)^{\frac{1-k}{k}} - 1\right]$$

$$= \frac{KR}{K-1} T_2 \left[\left(\frac{2}{K+1}\right)^{\frac{1-k}{k-1}} - 1\right]$$

$$= \frac{KR}{K-1} T_2 \left[\frac{k+1}{2} - 1\right]$$

~~$\frac{1}{2} V_2 =$~~

$$\Rightarrow \frac{1}{\cancel{2}} V_2^2 = \frac{KR}{\cancel{K-1}} T_2 \left[\frac{\cancel{K-1}}{\cancel{2}}\right]$$

$$\therefore V_2 = \sqrt{KRT_2} = c$$

$$P_1 = \rho_1 R T_1$$

$$P_2 = \rho_2 R T_2$$

$$\frac{T_2}{T_1} = \frac{P_1 \rho_2}{P_2 \rho_1}$$

$$\frac{T_1}{T_2} = \frac{\rho_2}{\rho_1} \cdot \frac{P_1}{P_2}$$

$$= \left(\frac{P_2}{P_1}\right)^{1/k} \left(\frac{P_2}{P_1}\right)^{-1}$$

$$\frac{T_1}{T_2} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{k} - 1}$$

$$\frac{P_1}{\rho_1^k} = \frac{P_2}{\rho_2^k}$$

$$\Rightarrow \frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)^k$$

$$\Rightarrow \frac{\rho_2}{\rho_1} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{k}}$$

* Area velocity relationship for one dimentional compressible flow,

$$\frac{dp}{\rho} + vdv = 0$$

$$\Rightarrow Kp \frac{d\rho}{\rho} \frac{1}{\rho} + vdv = 0$$

$$\Rightarrow KP \frac{d\rho}{\rho^2} + vdv = 0$$

$$\Rightarrow c^2 \rho \frac{d\rho}{\rho^2} + vdv = 0$$

$$\Rightarrow c^2 \frac{d\rho}{\rho} + vdv = 0$$

$$\Rightarrow \frac{c^2}{V^2} \frac{d\rho}{\rho} + \frac{dv}{V} = 0$$

$$\Rightarrow \frac{1}{M^2} \frac{d\rho}{\rho} + \frac{dv}{v} = 0$$

$$\Rightarrow \frac{d\rho}{\rho} = -M^2 \frac{dv}{v}$$

$$E = \frac{dp}{\frac{d\rho}{\rho}} = KP$$

$$dp = Kp \frac{d\rho}{\rho}$$

$$c = \sqrt{KRT}$$

$$\Rightarrow c = \sqrt{\frac{KP}{\rho}}$$

$$\text{Mach No} = \frac{V}{C} = M$$

Now,

$$AV\rho = \text{constant}$$

$$\Rightarrow \log A + \log v + \log \rho = \log c$$

$$\Rightarrow \frac{dA}{A} + \frac{dv}{V} + \frac{d\rho}{\rho} = 0$$

$$\Rightarrow \frac{dA}{A} + \frac{dv}{V} - M^2 \frac{dv}{V} = 0$$

$$\therefore \boxed{\frac{dA}{dV} = \frac{A}{V}(M^2 - 1)}$$ (Proved)