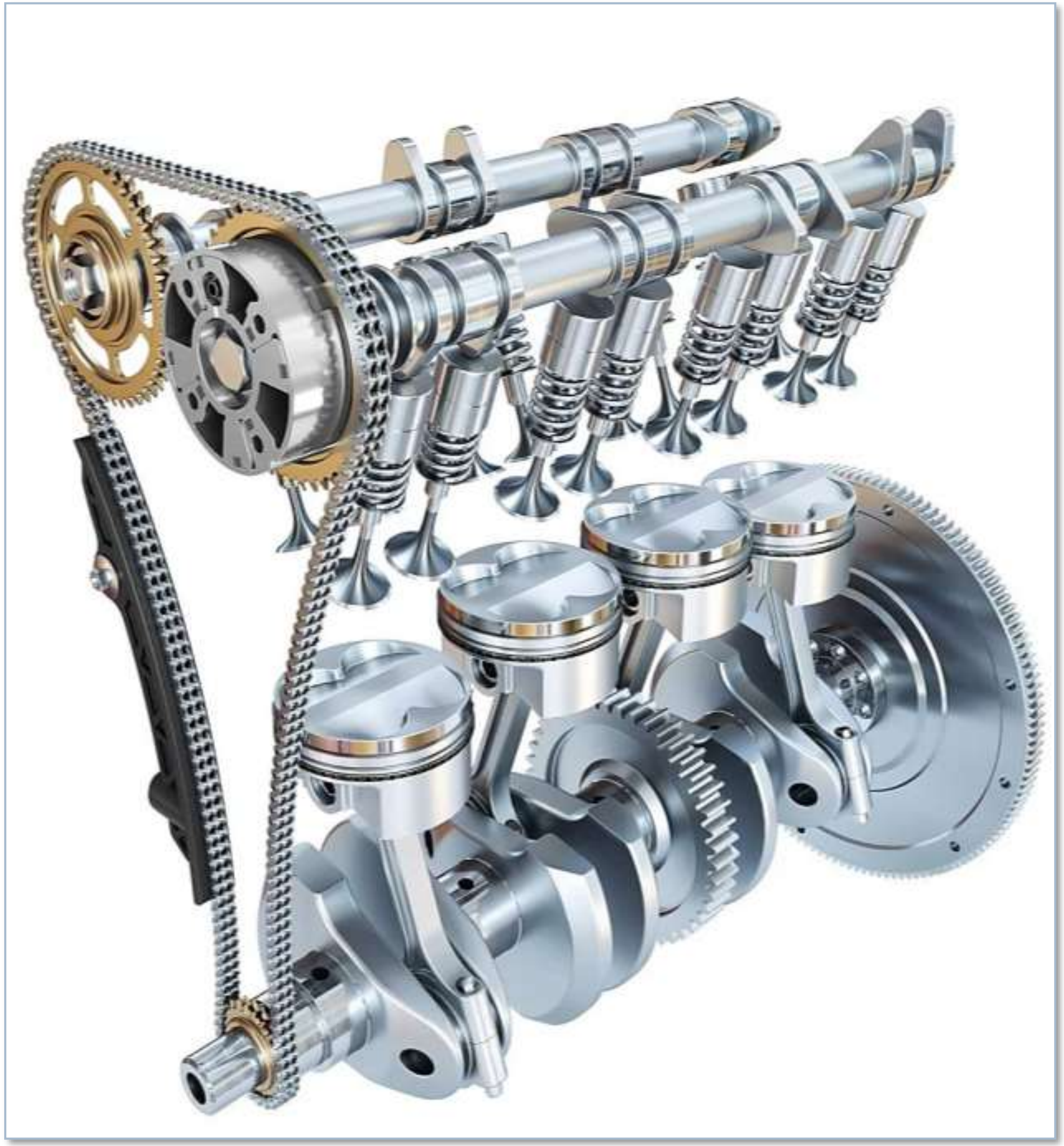


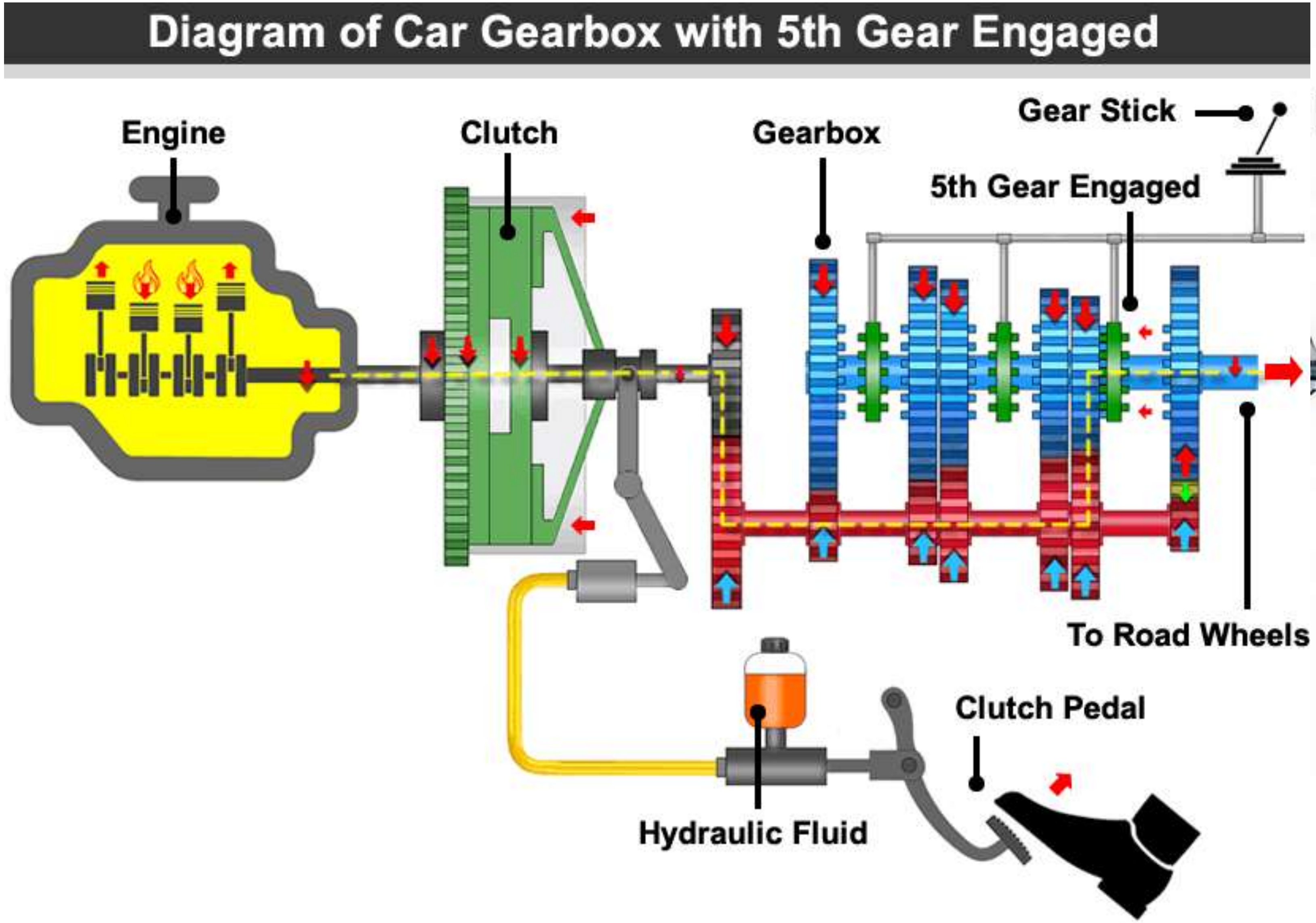
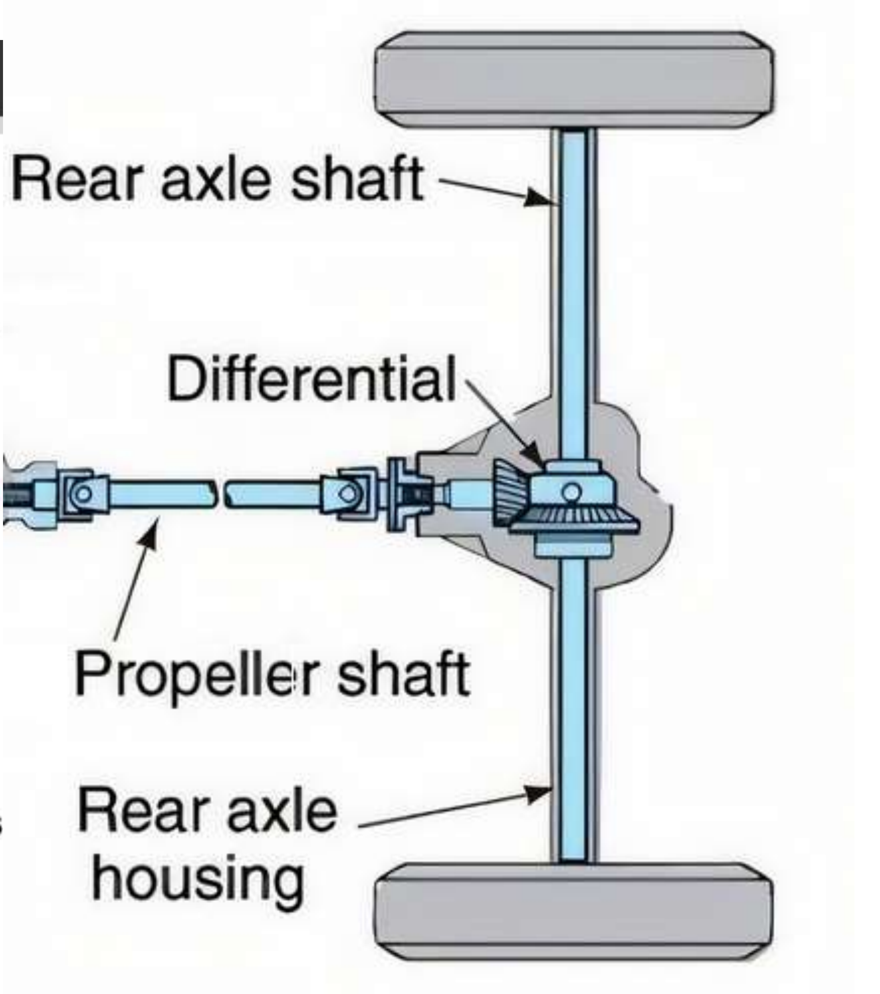
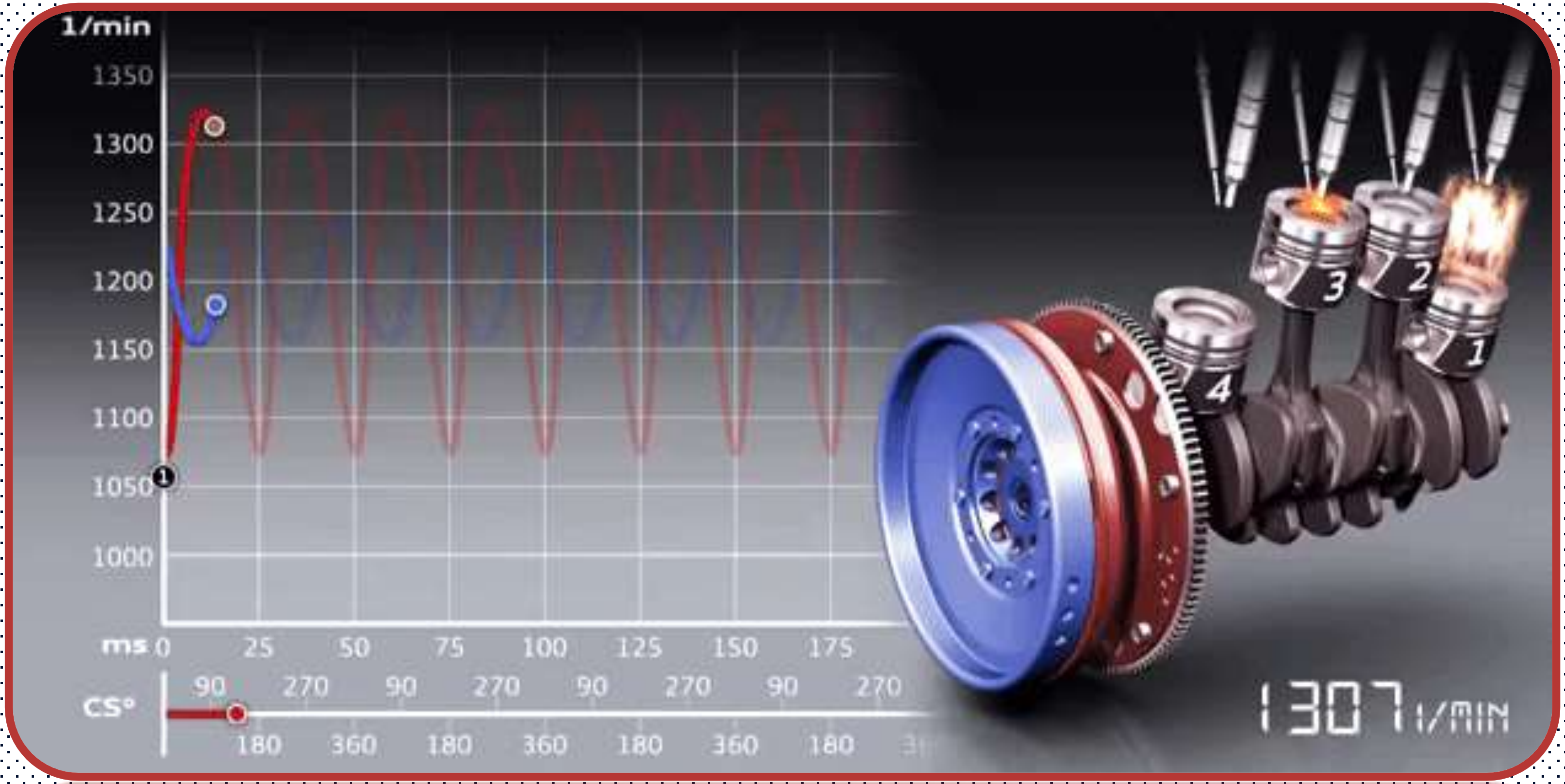


Diagram of Car Gearbox with 5th Gear Engaged
Engine
Clutch
Gearbox
Gear Stick
5th Gear Engaged
To Road Wheels
Clutch Pedal
Hydraulic Fluid


Rear axle shaft
Differential
Propeller shaft
Rear axle housing

ME 3101: Mechanics of Machinery
Turning Moment Diagram & Flywheel



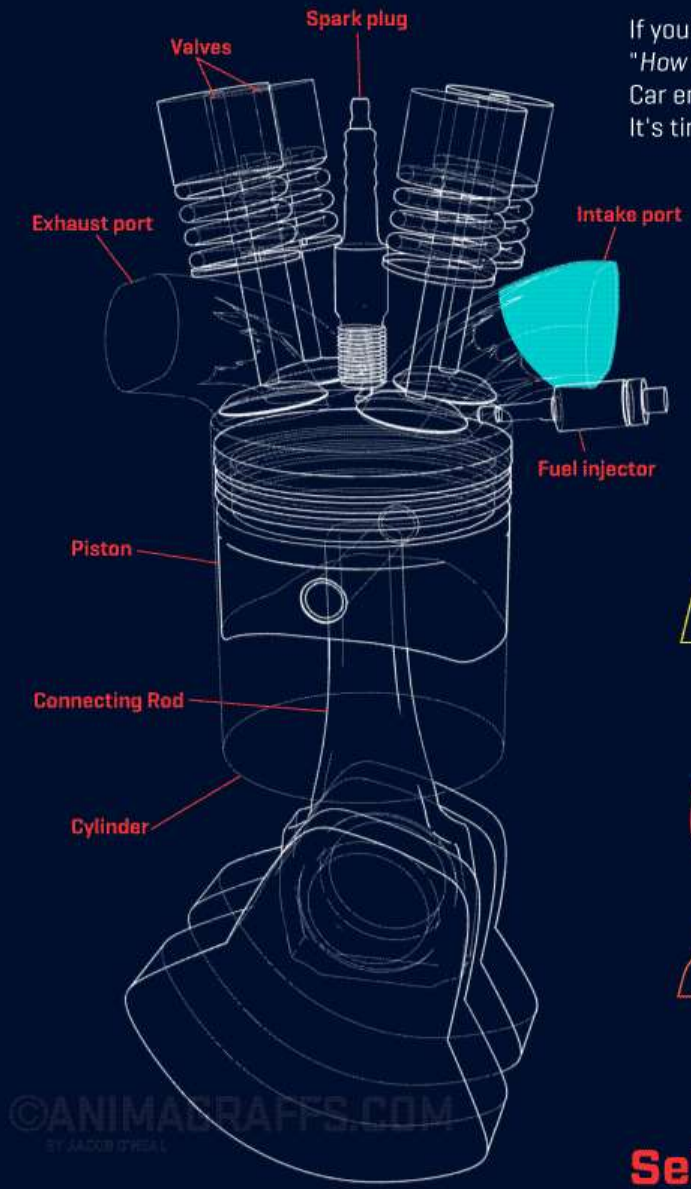
Prepared by
Muhammad Ifaz Shahriar Chowdhury
Lecturer, MPE Dept

Engine Starting System

DESIGN & RESEARCH BY **JACOB O'NEAL** NEED DESIGN? VISIT **JACOBONEAL.COM**

HOW A CAR ENGINE WORKS

[And a note about hybrid gas-electric cars too]



If your only experience with a car engine's inner workings is "*How much is that going to cost to fix?*" this graphic is for you! Car engines are astoundingly awesome mechanical wonders. It's time you learned more about the magic under the hood!

The 4 Stroke Cycle

Let's take a look inside just one cylinder.

1

INTAKE STROKE

The piston descends, sucking air into the cylinder through open intake valves as fuel is injected.

2

COMPRESSION STROKE

With all valves closed, the piston comes back up, compressing the fuel-air mixture. Compressing the mixture delivers better power and efficiency.

3

POWER STROKE

A spark ignites the compressed fuel-air mixture, and the resulting combustion forces the piston to the bottom of the cylinder again.

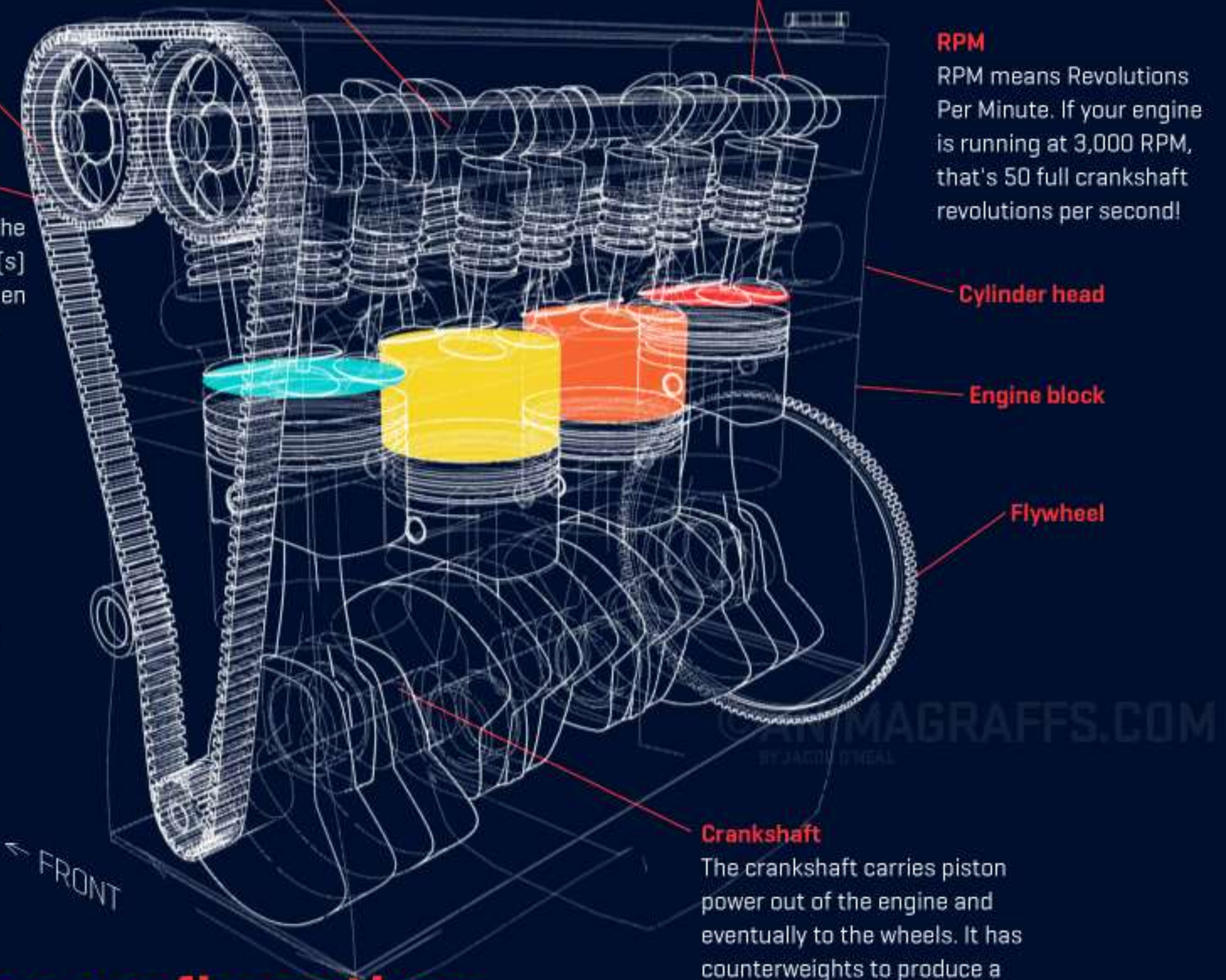
4

EXHAUST STROKE

The piston comes back up, pushing the spent mixture out through open exhaust valves.

Select engine parts

Unless your main ride is a lawn mower (no disrespect), your engine probably has more than one cylinder. Here's a 4 cylinder setup.



Timing belt (or chain)
A timing belt or chain synchronizes the rotation of the crankshaft and the camshaft(s) so that the engine's valves open and close at the proper times during each cylinder's intake and exhaust strokes. It also prevents the piston from striking the valves.

A timing belt (shown) is a rubber belt that usually features teeth on the inside surface, while a timing chain is a metal roller chain.

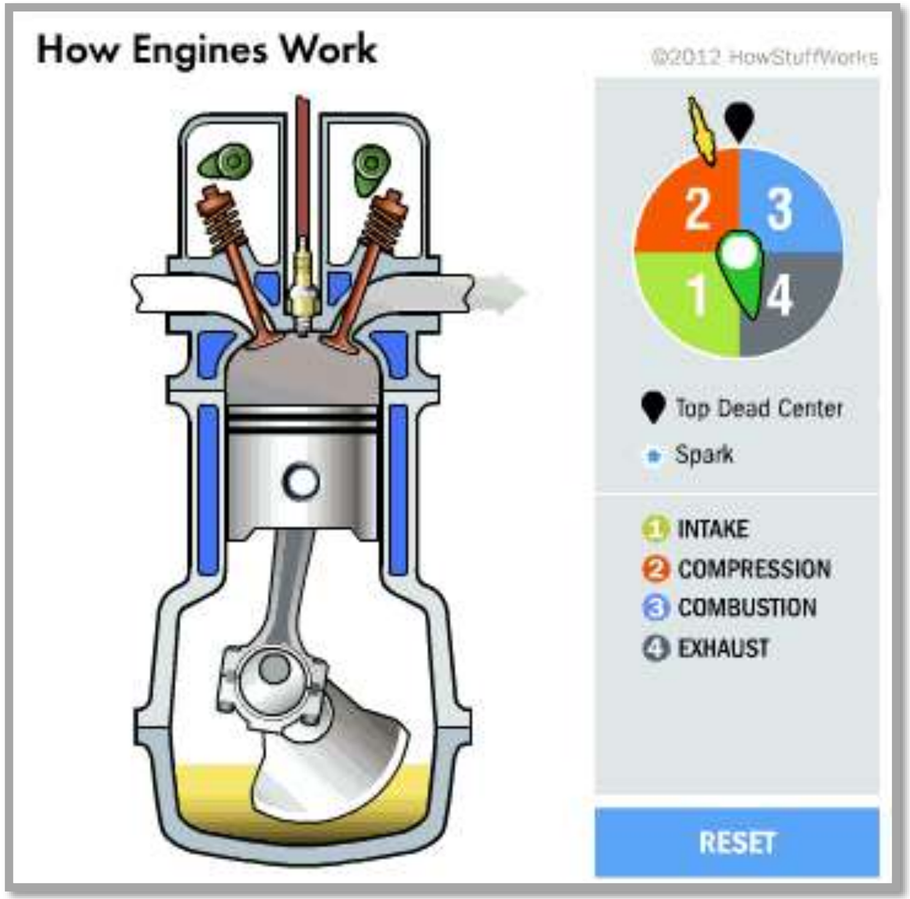
RPM
RPM means Revolutions Per Minute. If your engine is running at 3,000 RPM, that's 50 full crankshaft revolutions per second!

Crankshaft
The crankshaft carries piston power out of the engine and eventually to the wheels. It has counterweights to produce a balanced spin.

Other engine configurations

Turning Moment

- In a crank and connecting-rod mechanism operated by a piston, the axial force on the piston induces a force at the crank pin, perpendicular to the crank
- The product of this force and the crank radius is termed the **crank effort** or **turning moment**
- This turning moment or torque varies on such factors as the **crank position**, the **pressure in the cylinder**, and the inertia of the **moving parts**



4-Stroke Single Cylinder SI Engine

Mathematically

The Turning Moment is given by-

$$T = F_P \times r\left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right]$$

Where,

F_P = Piston effort,

r = Radius of crank,

n = Ratio of the connecting rod length and radius of crank

θ = Angle turned by the crank from inner dead centre.

$T = 0$ when, $(\theta = 0^\circ)$

$T = F_P \times r$ when, $(\theta = 90^\circ)$

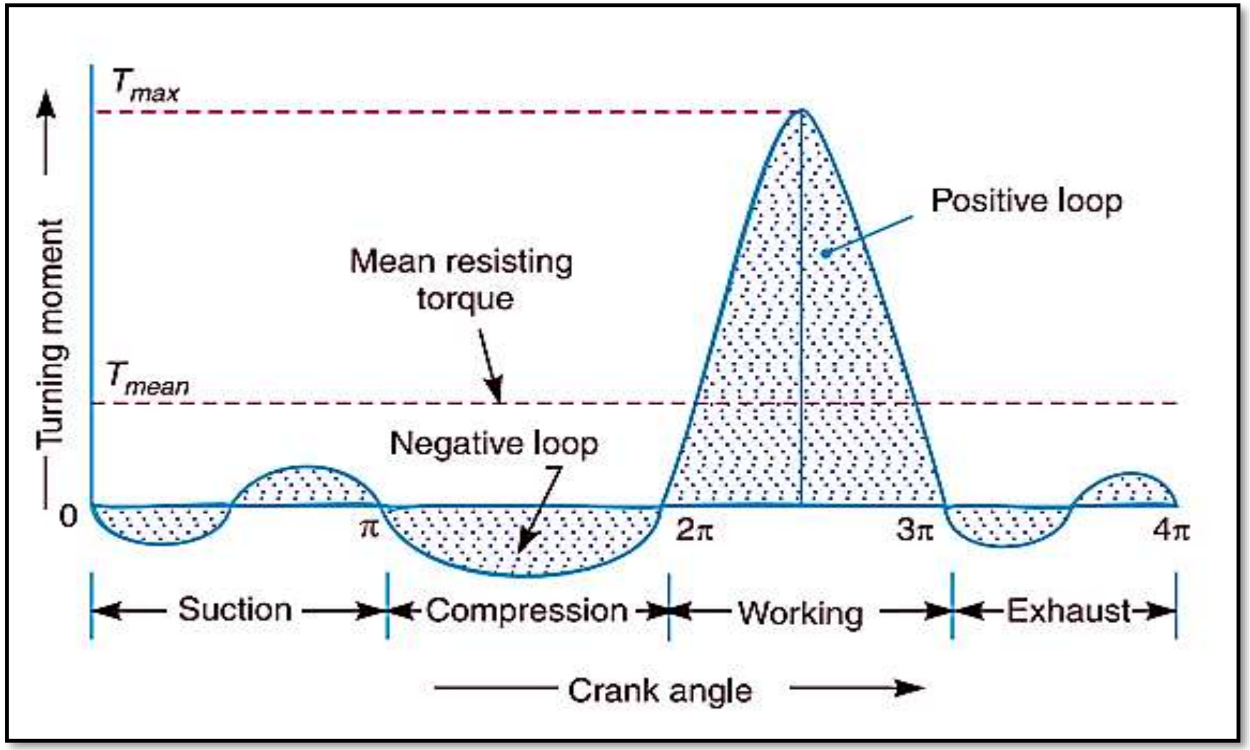
$T = 0$ when, $(\theta = 180^\circ)$

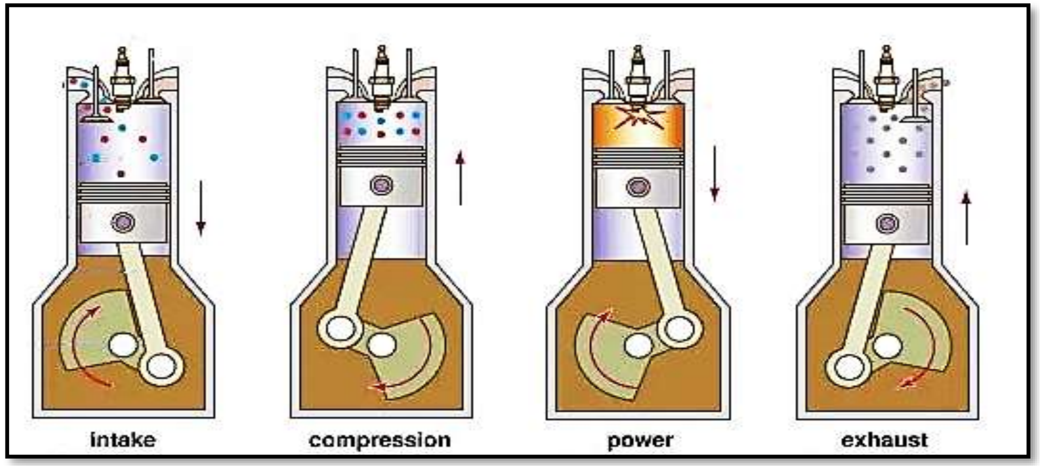
Turning Moment Diagram

The ***turning moment diagram*** (also known as the ***crank effort diagram***) is the graphical representation of the turning moment or crank effort for various positions of the crank. **Turning moment** in **Y-axis**, **Crank angle** in **X-axis**.

Turning moment diagram for a **4-stroke cycle internal combustion engine**. We know that in a 4-stroke cycle internal combustion engine, there is one working stroke after the crank has turned through two revolutions, i.e. **720° (or 4 π radians).**

- In the **suction stroke**, Pr inside the engine cylinder is less than atm. Pr therefore a **negative loop** is formed
- During **the compression stroke**, the work is done on the gases, therefore a **higher negative loop** is obtained.
- During the expansion or **working stroke**, the fuel burns and the gases expand, therefore a **large positive loop** is obtained.
- During the **exhaust stroke**, the work is done on the gases, therefore a **negative loop** is formed.





Turning Moment Diagram for a Single Cylinder Double-Acting Steam Engine

Mathematically

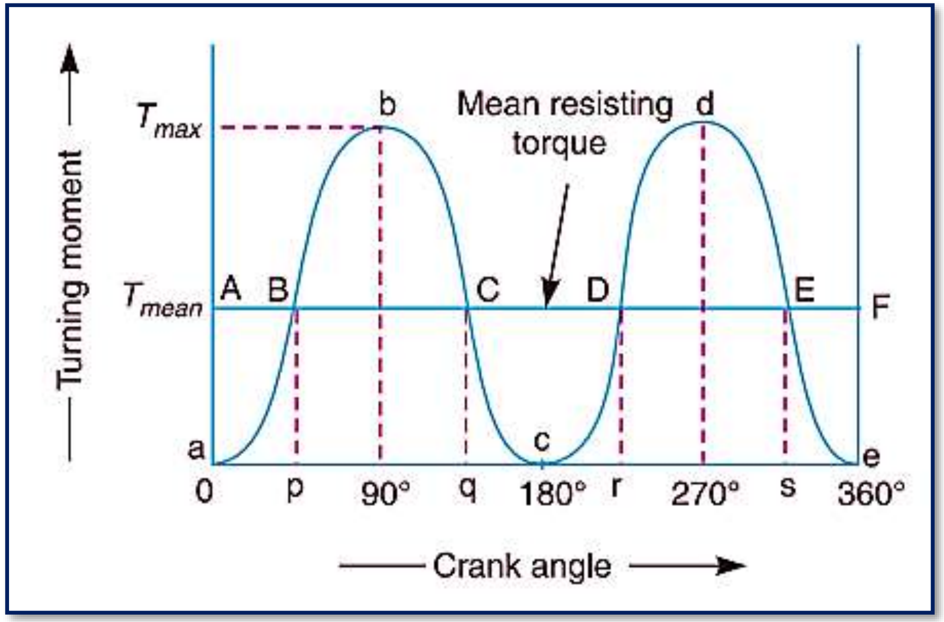
The Turning Moment is given by-

$$T = F_P \times r\left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right]$$

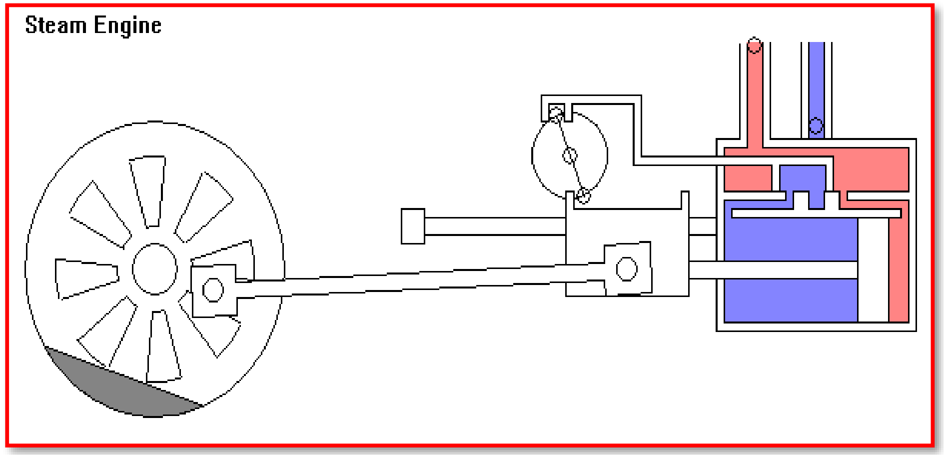
$T = 0$ when, $(\theta = 0^\circ)$

$T = F_P \times r$ when, $(\theta = 90^\circ)$

$T = 0$ when, $(\theta = 180^\circ)$



- The **curve abc** in represents the turning moment diagram for **outstroke**. The **curve cde** is the turning moment diagram for **instroke** and is somewhat similar to the curve abc
- Since the work done is the product of the turning moment and the angle turned, therefore **the area of the turning moment diagram** represents the **work done per revolution**
- In actual practice, the engine is assumed to work against the **mean resisting torque**, as shown by a **horizontal line AF**. The height of the ordinate aA represents the mean height of the turning moment diagram
- Since it is assumed that the work done by the turning moment per revolution is equal to the work done against the mean resisting torque, therefore **the area of the rectangle aAFe is proportional to** the **work done against the mean resisting torque**



Turning Moment Diagram for a Single Cylinder Double-Acting Steam Engine

Mathematically

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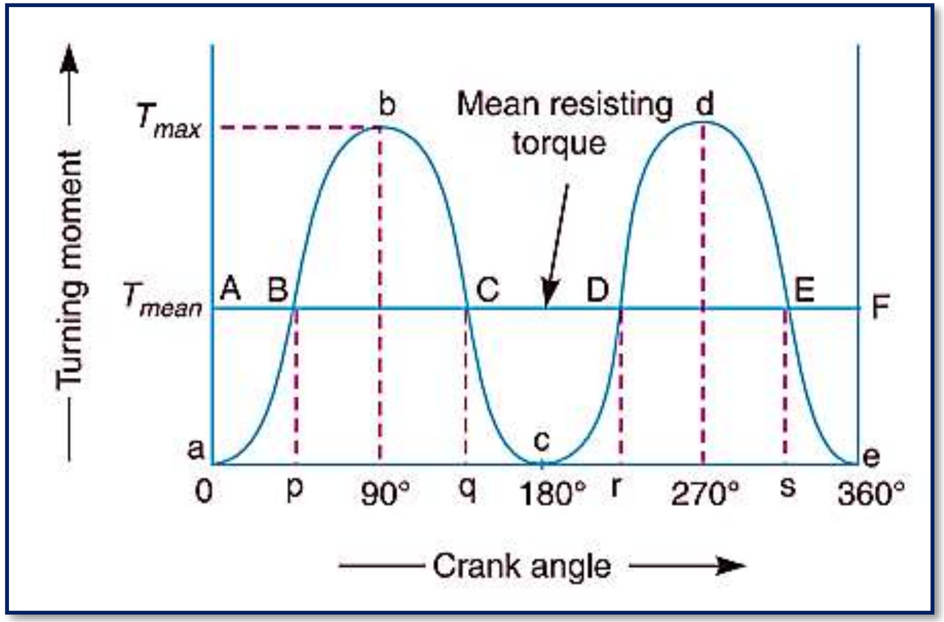
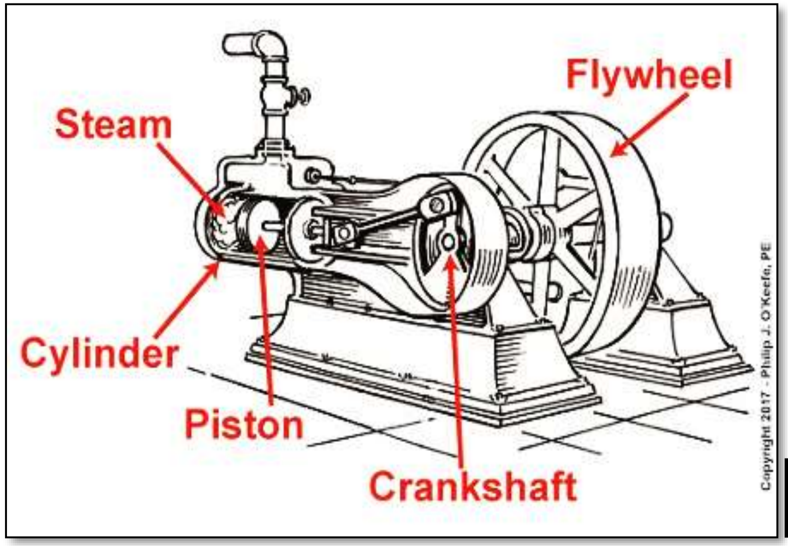
$T = 0$ when, $(\theta = 180^\circ)$



- When the **turning moment is positive** (i.e. **the engine torque is more than the mean resisting torque**) as shown between points B and C (or D and E) in Fig., the **crankshaft accelerates** and the work is done by the steam.
- When the **turning moment is negative** (i.e. **the engine torque is less than the mean resisting torque**) as shown between points C and D in Fig., the **crankshaft retards** and the work is done on the steam.
- If T = Torque on the crankshaft at any instant, and T_{mean} = Mean resisting torque. Then accelerating torque on the rotating parts of the engine = $T - T_{mean}$
- If $(T - T_{mean})$ is **positive**, the flywheel **accelerates**, and if $(T - T_{mean})$ is **negative**, then the flywheel **retard**



Turning Moment Diagram for a Multi-cylinder Engine

N.B. The first cylinder is the high-pressure cylinder, second cylinder is the intermediate cylinder and the third cylinder is the low-pressure cylinder

For multi-cylinder engines, the total torque for any crankshaft position is the algebraic sum of the torques exerted by the various cranks

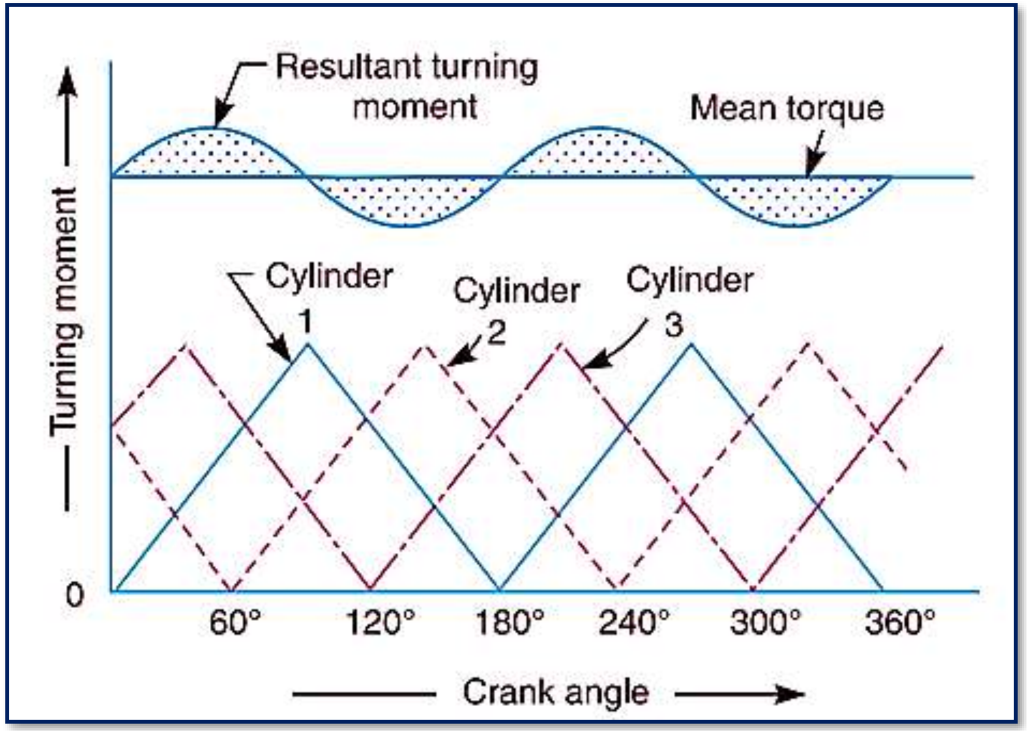
Turning moment diagram for a compound steam engine having three cylinders and the resultant turning moment diagram is shown in figure

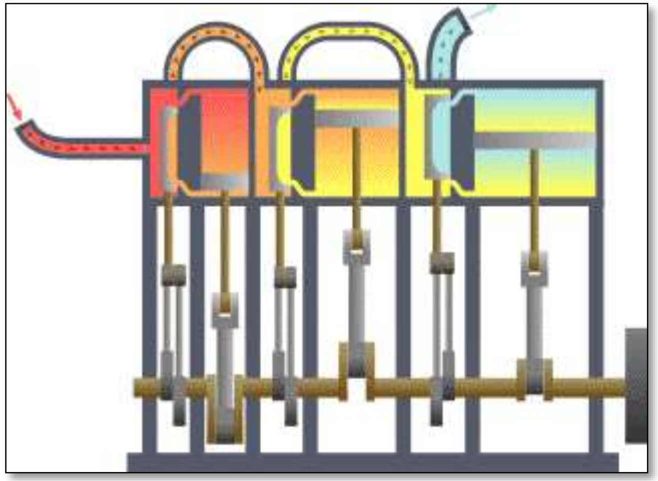
Resultant turning moment diagram

=

Sum of turning moment diagrams for the three cylinders

The cranks, in the case of three cylinders, are usually placed at **120°** to each other





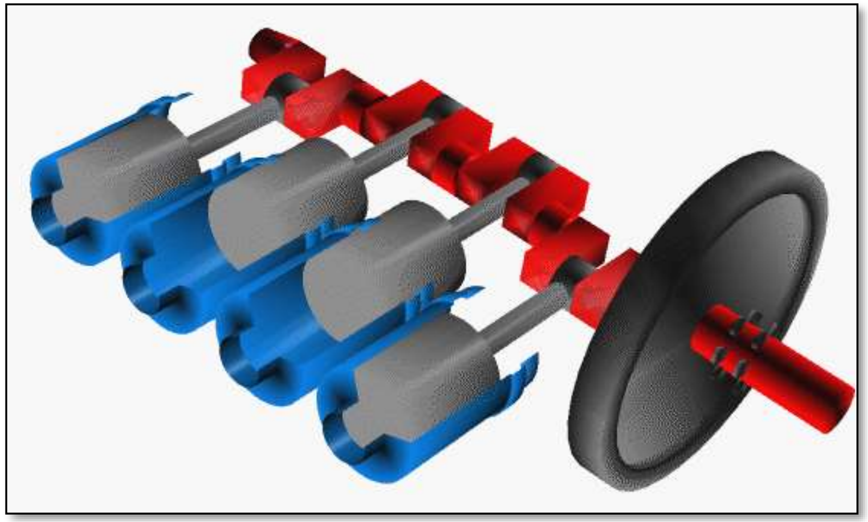
Flywheel

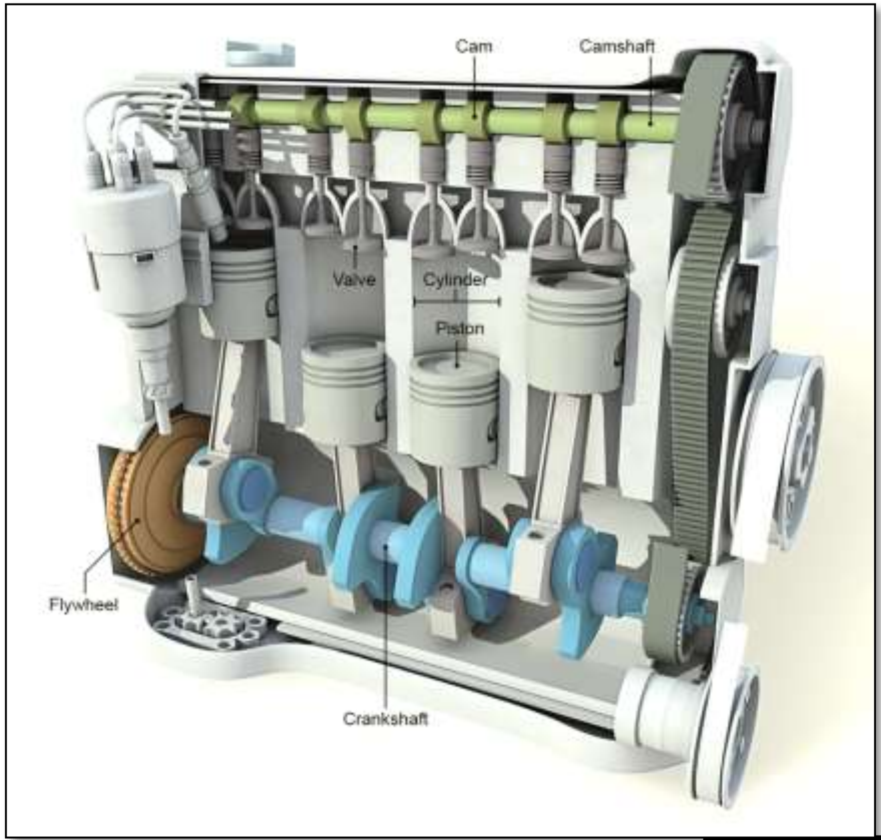
Flywheel

- Energy Reservoir like battery, capacitor etc.
- Stores energy when the supply of energy is more than the requirement, so its speed increases
- Releases energy when the requirement of energy is more than the supply, so its speed decreases
- Does not maintain a constant speed, it simply reduces or controls the fluctuation of speed caused by engine

In case of **steam engines**, **internal combustion engines**, **reciprocating compressors** and **pumps**, the engine is to run for the whole cycle on the energy produced during this power stroke.

The **excess energy** developed during power stroke is **absorbed by the flywheel** and **releases it to the crankshaft** during other strokes in which no energy is developed, thus rotating the **crankshaft at a uniform speed**.



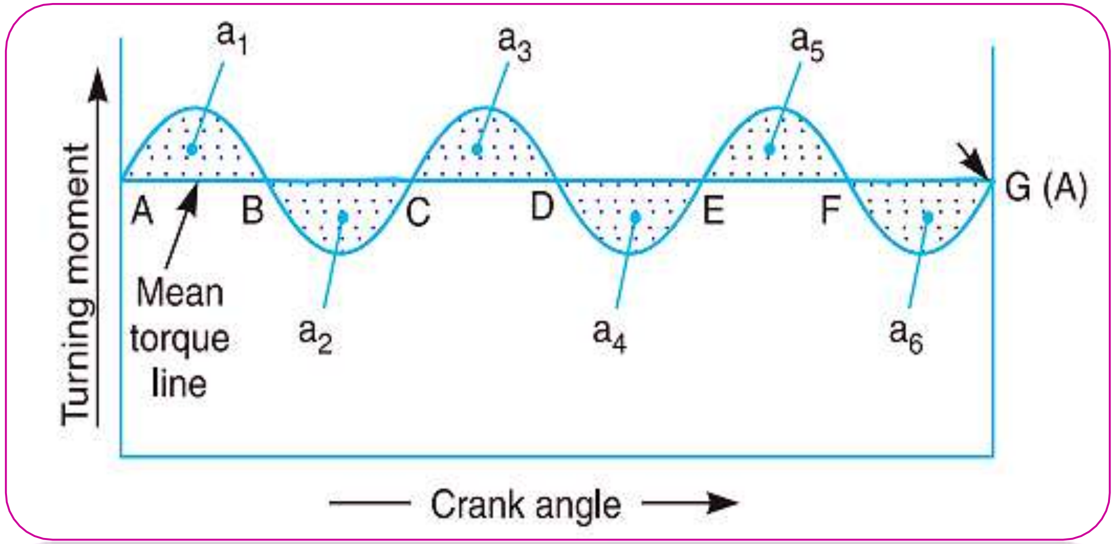


Fluctuation of Energy

- The variations of energy above and below the mean resisting torque line are called ***fluctuations of energy***
- The difference between the maximum and the minimum energies is known as the ***maximum fluctuation of energy***

AG = Mean torque line (MTL)
a_1, a_3, a_5 areas above the mean torque line
a_2, a_4, a_6 areas below the mean torque line
Energy in the flywheel at A = ξ
Let's assume greatest of these energies is at **B** and least at **E**

Over a complete cycle,
Σ Areas of the loops above MTL = **Σ** Areas of the loops below MTL



Turning moment diagram for a multi-cylinder engine

Energy at **B** = $\xi + a_1$
Energy at C = $\xi + a_1 - a_2$
Energy at D = $\xi + a_1 - a_2 + a_3$
Energy at **E** = $\xi + a_1 - a_2 + a_3 - a_4$
Energy at F = $\xi + a_1 - a_2 + a_3 - a_4 + a_5$
Energy at G = $\xi + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$ = Energy at A (i.e. cycle repeats after G)

Assuming, Maximum energy in flywheel = $\xi + a_1$
Minimum energy in the flywheel = $\xi + a_1 - a_2 + a_3 - a_4$

Maximum fluctuation of energy, $\Delta\xi$ = Maximum energy – Minimum energy
$= (\xi + a_1) - (\xi + a_1 - a_2 + a_3 - a_4) = a_2 - a_3 + a_4$

Fluctuation of Energy

Fluctuation of Energy

Excess energy available between the points of ***minimum speed*** and ***maximum speed***

Difference between the ***Kinetic Energies*** of the system at these points

ω = mean angular speed
ω_1 = maximum angular speed
ω_2= minimum angular speeds
I = moment of inertia of the rotating parts
Work done per cycle = area of the rectangle below the mean torque line
T_{mean} = Mean torque
θ = Angle turned (in radians), in one revolution {= 2π for steam engine & 2-stroke IC engines}, { = 4π, for 4-stroke IC engines}

Fluctuation of energy U during the cycle,

$$U = \tfrac{1}{2} I \omega_1^2 - \tfrac{1}{2} I \omega_2^2$$

$$Work\ done\ per\ cycle, W = T_{mean} \times \theta$$

$$Power, P = T_{mean} \times \omega$$

Coefficient of Fluctuation of Speed

The difference between the maximum and minimum speeds during a cycle is called the ***maximum fluctuation of speed*** $(\omega_1 - \omega_2)$.

The ratio of the maximum fluctuation of speed to the mean speed is called the ***coefficient of fluctuation of speed*** (C_s).

$$C_s = \frac{\omega_1 - \omega_2}{\omega}$$

N.B. The reciprocal of the coefficient of fluctuation of speed is known as the ***coefficient of steadiness*** (m).

$$m = \frac{1}{C_s}$$

Coefficient of Fluctuation of Energy

Ratio of the maximum fluctuation of energy to the work done per cycle is called **coefficient of fluctuation of energy**

$$C_E = \frac{Maximum\ fluctuation\ of\ energy}{Work\ done\ per\ cycle}$$

$$C_E = \frac{\frac{1}{2} I(\omega_1^2 - \omega_2^2)}{W}$$

Here, $U = ½ I\omega_1{}^2 - ½ I\omega_2{}^2$

$U = ½ I (\omega_1 + \omega_2)(\omega_1 - \omega_2)$

The fluctuation of speed $(\omega_1 - \omega_2)$ is small in comparison with the mean speed ω and, assuming the variations above and below the mean speed are equal,

$$\omega_1 + \omega_2 \cong 2\omega$$

$$also, \omega_1 - \omega_2 = C_s\ \omega$$

$$Thus, C_E = \frac{\frac{1}{2} I.2\omega.C_s.\omega}{W}$$

$$C_E = C_s \frac{I\omega^2}{W}$$

Problem

The torque exerted on the crankshaft of an engine is given by the equation

$$T(\theta) = 10500 + 1620 \sin 2\theta - 1340 \cos 2\theta \text{ (Nm)}$$

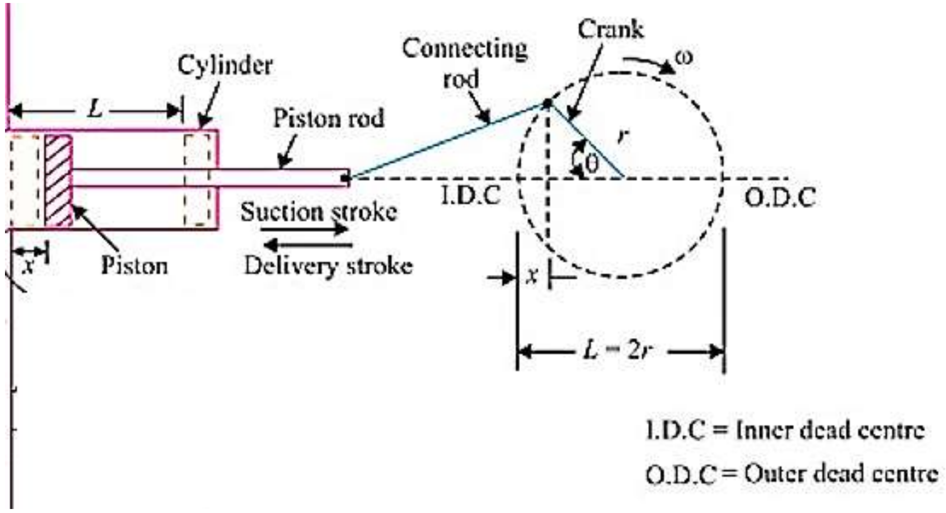
where θ is the crank-angle displacement from the inner dead centre

Assuming the resisting torque to be constant, determine

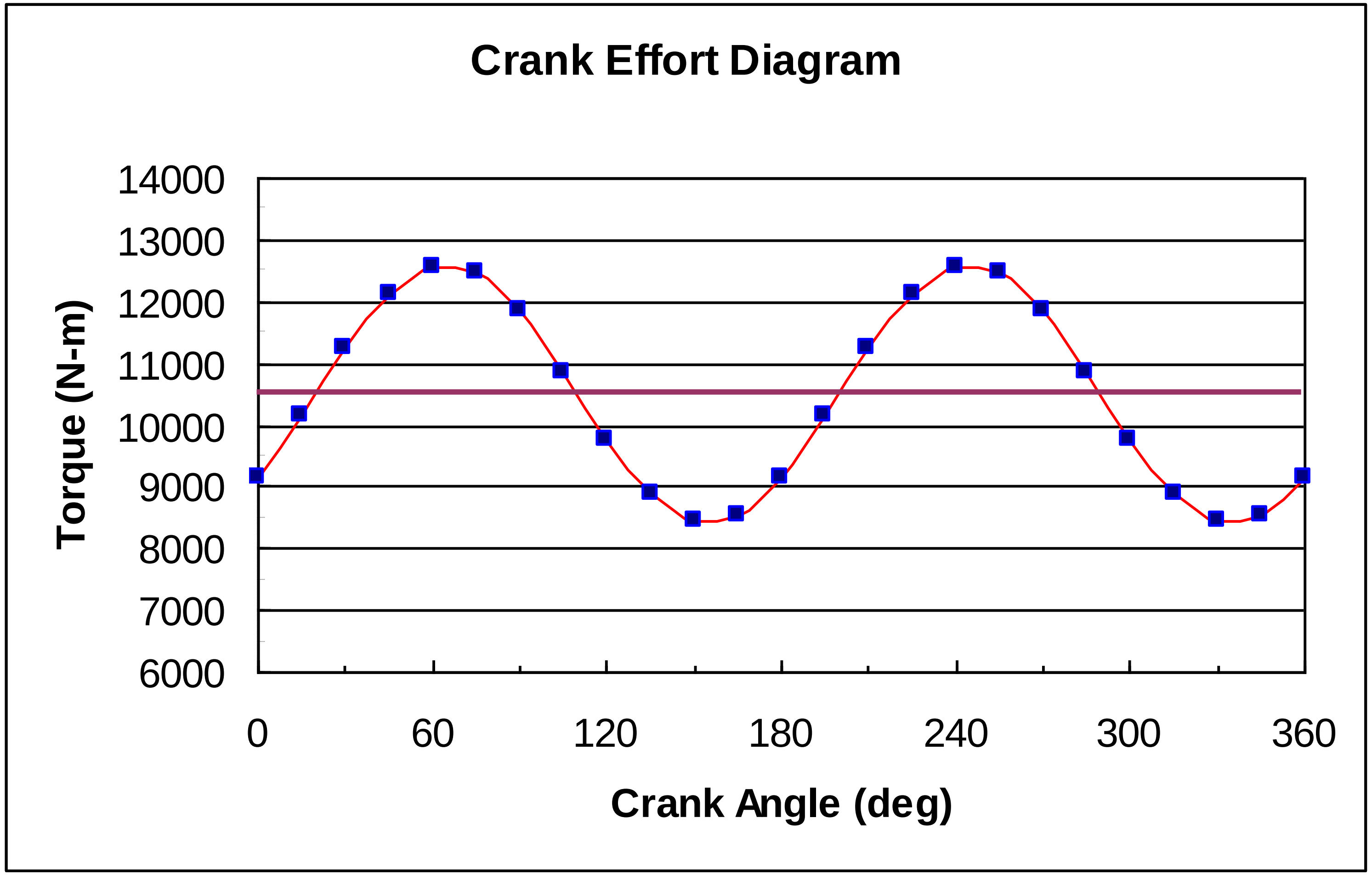
(a) The power of the engine when the speed is $150\ rev/\mathrm{min}$,

(b) The moment of inertia of the flywheel if the speed variation is not to exceed $\pm 0.5\%$ of the mean speed, and

(c) The angular acceleration of the flywheel when the crank has turned through 30° from the inner dead centre.



Solution


Crank Effort Diagram
Torque (N-m)
14000
13000
12000
11000
10000
9000
8000
7000
6000
0
60
120
180
240
300
360
Crank Angle (deg)

Solution

$Work\ done\ per\ revolution$

$$= \int_0^{2\pi} (10500 + 1620 \sin 2\theta - 1340 \cos 2\theta)\, d\theta$$

$$= 21000\pi\ Nm$$

$$T_{mean} = \frac{1}{2\pi} \times 21000\pi = 10500\ Nm$$

$$P = T_{mean} \times \omega$$

$$\Rightarrow P = 165\ kW \quad (\boldsymbol{Ans.})$$

$$Mean\ speed, \omega = 5\pi\ \frac{rad}{s}$$

$$\omega_1 = \frac{201\,\pi}{40}$$

$$\omega_2 = \frac{199\,\pi}{40}$$

Now, Engine torque = Mean torque

$$10500 + 1620 \sin 2\theta - 1340 \cos 2\theta = 10500$$

$$\Rightarrow\ \theta = 19.79°, 109.79°, 199.795°$$

$$U = \int_{19.79°}^{109.79°} \left(T_{Engine} - T_{Mean}\right) d\theta$$

$$\Rightarrow U = \int_{19.79°}^{109.79°} (1620 \sin 2\theta - 1340 \cos 2\theta) d\theta = 2102.37$$

Again,

$$U = \int_{109.79°}^{199.795°} \left(T_{Engine} - T_{Mean}\right) d\theta$$

$$\Rightarrow U = \int_{109.79°}^{199.795°} (1620 \sin 2\theta - 1340 \cos 2\theta) d\theta = -2102.37$$

Solution

Thus,

$$U_{max} = 2102.37$$

$$\Rightarrow \frac{1}{2} I(\omega_1^2 - \omega_2^2) = 2102.37$$

$$\Rightarrow I = 582.05\ kg\ m^2 \quad (\boldsymbol{Ans.})$$

$$When, \theta = 30°$$

$$Net\ Torque = 1620 \sin 2\theta - 1340 \cos 2\theta = 732.96\ Nm$$

$$\alpha = \frac{T_{net}}{I} = 0.8602\ rad\ s^{-2} \quad (\boldsymbol{Ans.})$$

Problem

The turning moment diagram for an engine is given by: $Torque\ (N-m) = 2100\sin\theta + 900\sin 2\theta$ for values of θ, the crank angles, between 0 and π, and by: $Torque\ (N-m) = 375\sin\theta$ for values of θ between π and 2π. This is repeated for every revolution of the engine.

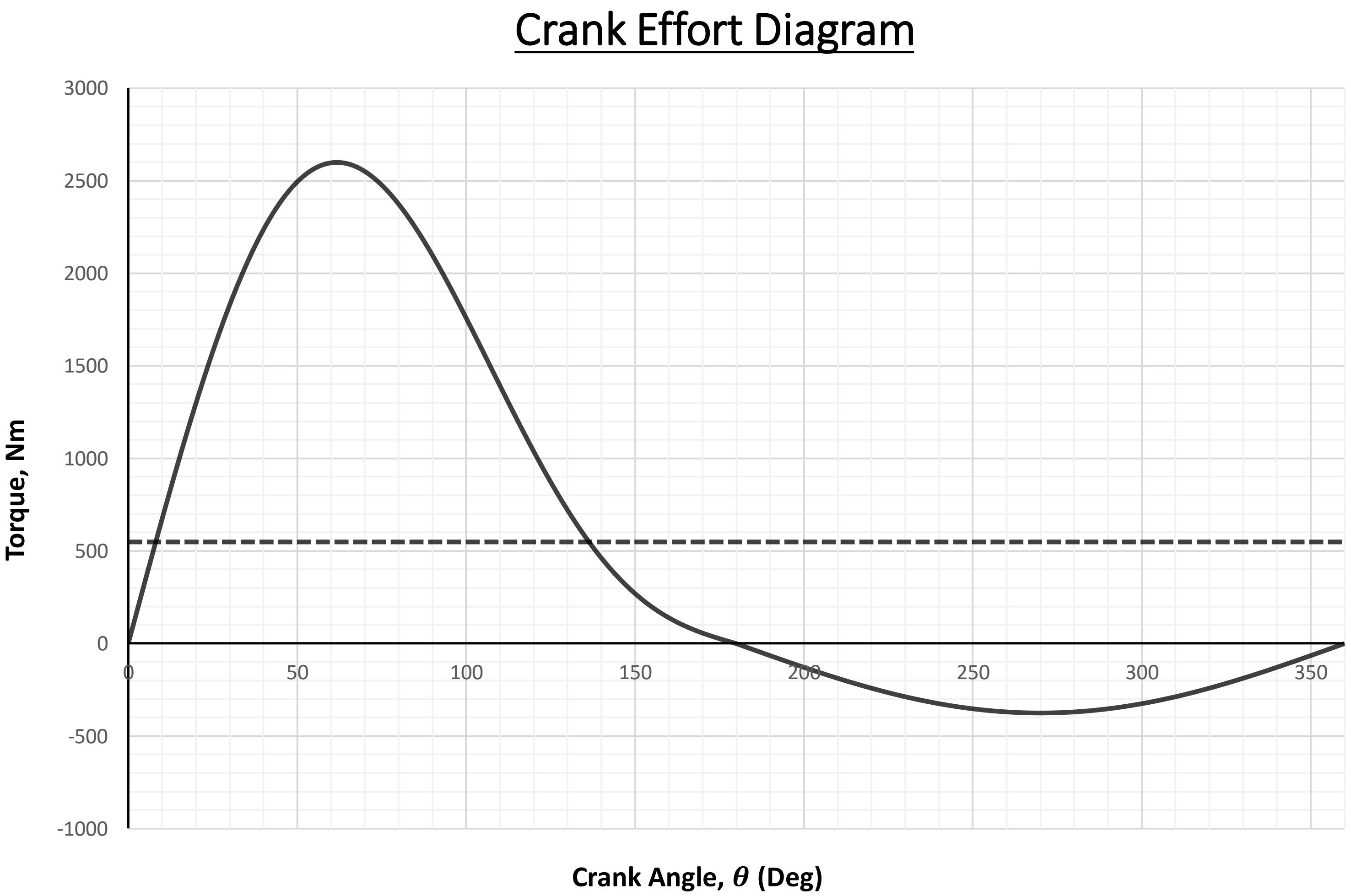
The resisting torque is constant and the speed is $850\ rpm$. The total moment of inertia of the rotating parts of the engine and the driven member is $270\ kg-m^2$. Determine:

(i) The power;

(ii) The fluctuation in speed;

(iii) The maximum instantaneous angular acceleration of the engine, and the value of θ at which it occurs.

Solution


Crank Effort Diagram
3000
2500
2000
1500
1000
500
0
-500
-1000
0
50
100
150
200
250
300
350
Torque, Nm
Crank Angle, θ (Deg)

Solution

Work done per revolution

$$= \int_0^{\pi} (2100 \sin\theta + 900 \sin 2\theta)\, d\theta + \int_{\pi}^{2\pi} 375 \sin\theta \, d\theta$$

$$= 3450 \text{ N m}$$

$$\therefore \text{power} = 3450 \times \left(\frac{2\pi}{60} \times 850\right) = 307\,000 \text{ W} \quad \text{or} \quad \underline{307 \text{ kW}}$$

$$\text{Resisting torque} = \text{mean engine torque} = \frac{3450}{2\pi} = 549{\cdot}3 \text{ N m}$$

Solution

The engine torque/crank angle diagram is shown in Fig. 4.5. The engine and resisting torques are equal when

$$2100 \sin \theta + 900 \sin 2\theta = 549{\cdot}3$$

By trial or plotting, $\theta = 8^\circ\,10'$ and $136^\circ\,30'$

The greatest fluctuation of energy occurs between points A and B,

$$\text{i.e. fluctuation of energy} = \int_{8^\circ\,10'}^{136^\circ\,30'} (2100 \sin \theta + 900 \sin 2\theta)\, d\theta$$

$$- 549{\cdot}3(136^\circ\,30' - 8^\circ\,10') \times \frac{\pi}{180}$$

$$= 2778 \text{ N m}$$

$$\therefore 2778 = \tfrac{1}{2} I(\omega_1{}^2 - \omega_2{}^2) = \tfrac{1}{2} I \,.\, 2\omega \,.\, (\omega_1 - \omega_2)$$

$$= \tfrac{1}{2} \times 270 \times 2 \times 850 \times (N_1 - N_2) \times \left(\frac{2\pi}{60}\right)^2$$

from which $N_1 - N_2 = \underline{1{\cdot}103 \text{ rev/min}}$

Solution

For maximum torque, $$\frac{dT}{d\theta} = 0$$

i.e. $$2100 \cos\theta + 1800 \cos 2\theta = 0$$

or $$12 \cos^2\theta + 7 \cos\theta - 6 = 0$$

from which $$\theta = 61° \, 45'$$

$$\therefore T_{max} = 2100 \sin 61° \, 45' + 900 \sin 123° \, 30'$$

$$= 2600 \text{ N m}$$

$\therefore$ maximum instantaneous angular acceleration

$$= \frac{\text{net accelerating torque}}{I}$$

$$= \frac{2600 - 549{\cdot}3}{270} = \underline{7{\cdot}6 \text{ rad/s}^2}$$

Solve by Yourself

Book: Theory of Machine by R S Khurmi
Chapter 16

Example: 16.5,16.7,16.10,16.13
Exercise: 7, 8,10,11,12

PER

ASPERA

AD

ASTRA