AHSANULLAH UNIVERSITY OF SCIENCE AND TECHNOLOGY (AUST)

ME-3105: FLUID MECHANICS

(LC-4: HYDROSTATIC FORCE)

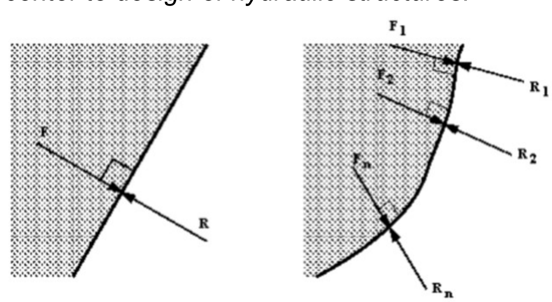
BY

Dr. FAZLAR RAHMAN
Associate Professor, MPE, AUST

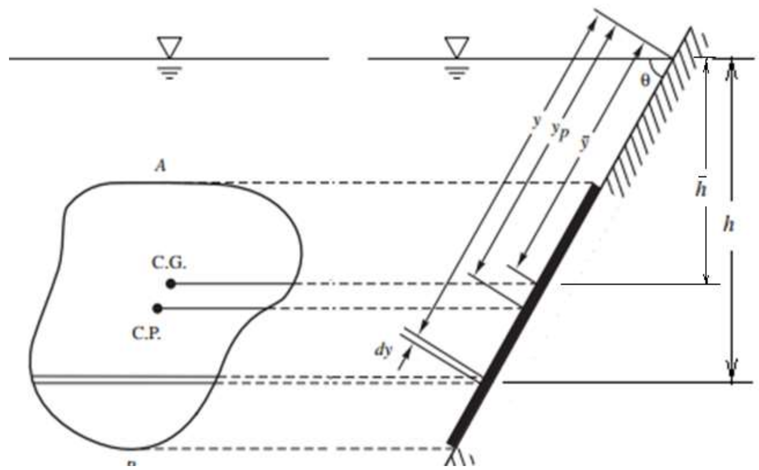
HYDROSTATIC FORCES ON SURFACES

Total Pressure and Center of Pressure: When a static mass of fluid comes contact with a surface, either plane or curved, a force is exerted by the fluid on the surface. This force is known as ***Total Pressure***. Total Pressure always acts in the *direction normal to the surface*.

- The point of application of total pressure on the surface is known as Center of Pressure.
- *It is needed to find the magnitude of total pressure and location of pressure center to design of hydraulic structures.*



HYDROSTATIC FORCES ON FLAT SURFACES



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HYDROSTATIC FORCES ON FLAT SURFACES

Assume an arbitrary area AB on the face of a dam that inclines at an angle θ . *The x-axis on the line where the surface of the water intersects with the dam surface (i.e. into the page) with the y-axis running downward along the surface or face of the dam.*

Assume that the plane surface AB is made up of an infinite number of horizontal strips, each having a width of dy and an area of dA. The hydrostatic pressure on each strip is considered constant because the width of each strip is very small. For a strip at depth h below the free surface, the pressure is

$$P = \gamma h = \gamma y \sin\theta$$

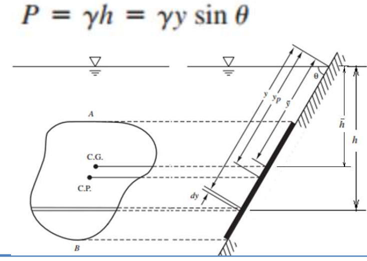
The total pressure force on the strip

$$dF = P\,dA = \gamma y \sin\theta\, dA$$

$$F = \int_A dF = \int_A \gamma y \sin\theta\, dA = \gamma \sin\theta \int_A y\,dA$$

$$= \gamma \sin\theta\, A\bar{y} \qquad \text{where} \quad \bar{y} = \frac{\int_A y\, dA}{A}$$

$$F = \gamma \bar{h} A \qquad (\bar{h} = \bar{y}\sin\theta)$$



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The total pressure force on the strip

$$dF = P\,dA = \gamma y \sin\theta\, dA$$

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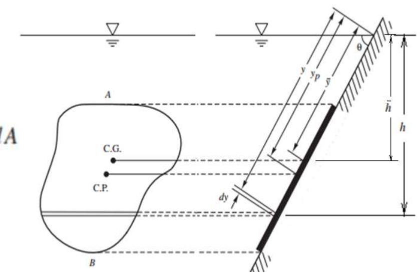

$$P = \gamma h = \gamma y \sin\theta$$

This equation states that the *total hydrostatic pressure force on any submerged plane surface is equal to the product of the surface area and the pressure* acting at the centroid of the plane surface.

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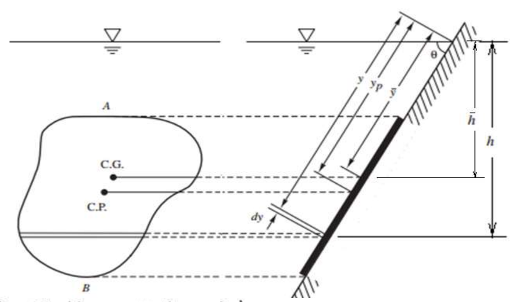
The distributed forces can be replaced by the single resultant force at the pressure center without altering any reactions or moments in the system. Designating y_p as the distance measured from the x-axis to the center of pressure,



$$Fy_p = \int_A y\,dF \qquad y_p = \frac{\int_A y\,dF}{F}$$

$$dF = \gamma y \sin\theta\, dA \text{ and } F = \gamma \sin\theta\, A\bar{y}$$

$$y_p = \frac{\int_A y^2 dA}{A\bar{y}} \qquad y_p = \frac{I_x}{A\bar{y}}$$

$$\int_A y^2 dA = I_x \quad (\text{moment of inertia of surface } AB \text{ with respect to the } x\text{-axis})$$

$$I_x = I_0 + A\bar{y}^2 \qquad y_p = \frac{I_0 + A\bar{y}^2}{A\bar{y}} = \frac{I_0}{A\bar{y}} + \bar{y}$$

where I_0 is the moment of inertia of the plane with respect to its own centroid, A is the plane surface area and y is the distance between the centroid and the x-axis.

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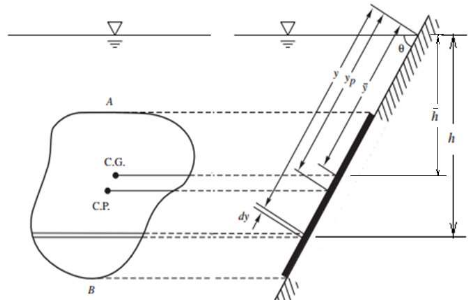
HYDROSTATIC FORCES ON FLAT SURFACES

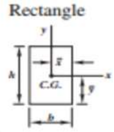
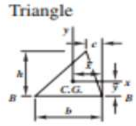
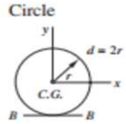
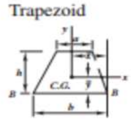


$$I_x = I_0 + A\bar{y}^2 \qquad y_p = \frac{I_0 + A\bar{y}^2}{A\bar{y}} = \frac{I_0}{A\bar{y}} + \bar{y}$$

$$F = \gamma \sin\theta\, A\bar{y} = \gamma\bar{h}A \; (\bar{h} = \bar{y}\sin\theta)$$

The first term on the right-hand side is positive and always $y_p > y$. So, the center of pressure of any submerged plane surface is always below the centroid of the surface area .

HYDROSTATIC FORCES ON FLAT SURFACES

Shape	Area	Centroid	Moment of Inertia About the Neutral *x*-Axis
Rectangle 	bh	$\bar{x} = \frac{1}{2}b$ $\bar{y} = \frac{1}{2}h$	$I_0 = \frac{1}{12}bh^3$
Triangle 	$\frac{1}{2}bh$	$\bar{x} = \frac{b+c}{3}$ $\bar{y} = \frac{h}{3}$	$I_0 = \frac{1}{36}bh^3$
Circle 	$\frac{1}{4}\pi d^2$	$\bar{x} = \frac{1}{2}d$ $\bar{y} = \frac{1}{2}d$	$I_0 = \frac{1}{64}\pi d^4$
Trapezoid 	$\frac{h(a+b)}{2}$	$\bar{y} = \frac{h(2a+b)}{3(a+b)}$	$I_0 = \frac{h^3(a^2+4ab+b^2)}{36(a+b)}$

HYDROSTATIC FORCES ON FLAT SURFACES

Shape	Area	Centroid	Moment of Inertia About the Neutral x-Axis
Semi-ellipse	$\frac{\pi}{2}bh$	$\bar{x} = b$, $\bar{y} = \frac{4h}{3\pi}$	$I_0 = \frac{(9\pi^2 - 64)}{72\pi}bh^3$
Parabolic section ($y = h\left(1 - \frac{x^2}{b^2}\right)$)	$\frac{2}{3}bh$	$\bar{y} = \frac{2}{5}h$, $\bar{x} = \frac{3}{8}b$	$I_0 = \frac{8}{175}bh^3$
Semicircle	$\frac{1}{2}\pi r^2$	$\bar{y} = \frac{4r}{3\pi}$	$I_0 = \frac{(9\pi^2 - 64)r^4}{72\pi}$
Ellipse	πbh	$\bar{x} = b$, $\bar{y} = h$	$I_0 = \frac{\pi}{4}bh^3$

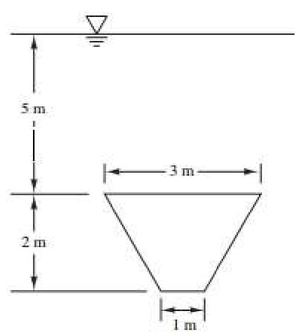
HYDROSTATIC FORCES ON FLAT SURFACES (PROBLEM-1)

A *vertical trapezoidal* gate with its upper edge located below the free surface of water is shown below. Determine the total pressure force and the center of pressure on the gate.

$$I_x = I_0 + A\bar{y}^2 \qquad y_p = \frac{I_0 + A\bar{y}^2}{A\bar{y}} = \frac{I_0}{A\bar{y}} + \bar{y}$$

$$F = \gamma \sin\theta\, A\bar{y} = \gamma\bar{h}A \;(\bar{h} = \bar{y}\sin\theta)$$

(vertical plane $\theta = 90$ deg, $\bar{h} = \bar{y}$)



HYDROSTATIC FORCES ON FLAT SURFACES (PROBLEM-1)

Given, $h_{depth} = 5\,m$ $\rho_{water} = 1000\,\frac{kg}{m^3}$

Trapezoid Gemometry:

$a = 1.0\,m$ $b = 3.0\,m$ $h = 2\,m$

Hydrostatic force, P = ? Center of pressure y_p = ?

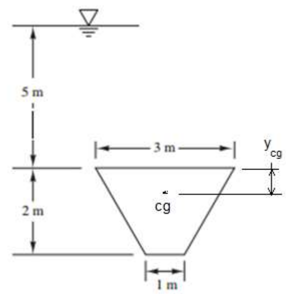
Solution:

$$A = \frac{h(a+b)}{2} = \frac{2\,m(1.0\,m + 3.0\,m)}{2} = 4\,m^2$$

$$y_{cg} = \frac{h(2a+b)}{3(a+b)} = \frac{2\,m(2\times 1\,m + 3\,m)}{3(1\,m + 2\,m)} = 0.83\,m$$

$$I_o = \frac{h^3\left(a^2 + 4ab + b^2\right)}{36(a+b)} = \frac{(2m)^3\left[(1m)^2 + 4\cdot(1m)\cdot(3m) + (3m)^2\right]}{36(1m + 3m)}$$

$I_o = 1.22\,m^4$ $h_{bar} = h_{depth} + y_{cg} = 5\,m + 0.83\,m = 5.83\,m$



HYDROSTATIC FORCES ON FLAT SURFACES (PROBLEM-1)

Hydrostatic force,

$$P = \gamma\cdot(y_{bar}\cdot SIN\theta)\cdot A = \rho\, g\, h_{bar}\, A \quad \left(h_{bar} = y_{bar}\cdot SIN\theta \quad \theta = 90\cdot deg\right)$$

$$P = \left(1000\,\frac{Kg}{m^3}\right)\cdot\left(9.81\,\frac{m}{s^2}\right)\cdot(5.83\,m)\left(4.0\,m^2\right)$$

$$P = (228.77)\cdot(1000)\,\frac{kg\cdot m}{s^2} = (228.77)\cdot(1000)\,N = 229\,KN$$

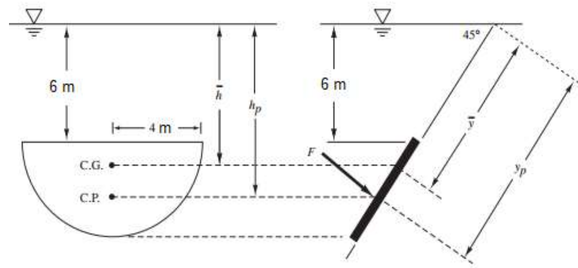
Center of pressure,

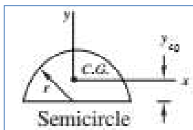
$$y_p = \frac{I_o}{A\cdot y_{bar}} + y_{bar} = \frac{1.22\,m^4}{\left(4.0\,m^2\right)(5.83\,m)} + 5.83\,m = 5.88\,m$$

(Below water surface to center of pressure)

HYDROSTATIC FORCES ON FLAT SURFACES (PROBLEM-2)

An inverted semicircular gate (shown below) is installed at 45° with respect to the free water surface. The top of the gate is 6 m below the water surface in the vertical direction. Determine the hydrostatic force and the center of pressure on the gate.



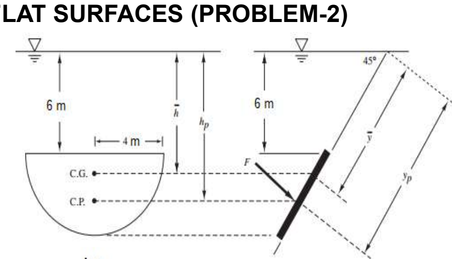


$A = \frac{1}{2}\pi r^2$ $\quad y_{cg} = \frac{4r}{3\pi}$ $\quad I_0 = \frac{(9\pi^2 - 64)r^4}{72\,\pi}$

HYDROSTATIC FORCES ON FLAT SURFACES (PROBLEM-2)

$$F = \gamma \sin\theta\, A\bar{y} = \gamma\bar{h}A \;(\bar{h} = \bar{y}\sin\theta)$$

$$I_x = I_0 + A\bar{y}^2 \qquad y_p = \frac{I_0 + A\bar{y}^2}{A\bar{y}} = \frac{I_0}{A\bar{y}} + \bar{y}$$



Given, $r = 4\cdot m$ $\quad \theta = 45\cdot deg$ $\quad h = 6\cdot m$ $\quad \rho_{water} = 1000\cdot\frac{kg}{m^3}$

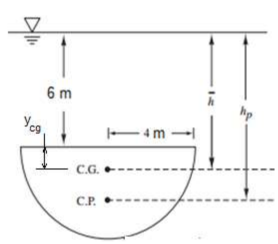
Hydrostatic force, P = ? Center of pressure y_p = ?

$A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(4\,m)^2 = 25.13\,m^2$ $\qquad y_{cg} = \frac{4r}{3\,\pi} = \frac{4\,(4\,m)}{3\,\pi} = 1.70\,m$

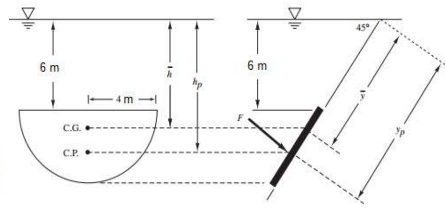
$I_o = \left(\frac{9\pi^2 - 64}{72\pi}\right)r^4 = \left(\frac{9\pi^2 - 64}{72\pi}\right)(4\,m)^4 = 28.10\,m^4$

$h_{bar} = h + y_{cg} = 6\,m + 1.70\,m = 7.70\,m$

$SIN(45deg) = \frac{h_{bar}}{y_{bar}}$ $\qquad y_{bar} = \frac{h_{bar}}{SIN(45deg)} = 10.88\,m$



HYDROSTATIC FORCES ON FLAT SURFACES (PROBLEM-2)



Hydrostatic force,

$$P = \gamma \cdot (y_{bar} \cdot \mathrm{SIN}\theta) \cdot A = \rho g h_{bar} A$$

$$P = \left(1000 \frac{\mathrm{Kg}}{\mathrm{m}^3}\right) \cdot \left(9.81 \frac{\mathrm{m}}{\mathrm{s}^2}\right) \cdot (7.70\,\mathrm{m})(25.13\,\mathrm{m}^2)$$

$$P = (1898,.24) \cdot (1000) \frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}^2} = (1898.24) \cdot (1000)\,\mathrm{N} = 1898\,\mathrm{KN}$$

Center of pressure,

$$y_p = \frac{I_o}{A \cdot y_{bar}} + y_{bar} = \frac{28.1\,\mathrm{m}^4}{(25.13\,\mathrm{m}^2)(10.88\,\mathrm{m})} + 10.88\,\mathrm{m} = 10.98\,\mathrm{m}$$

(Inclined distance from water surface to center of pressure)

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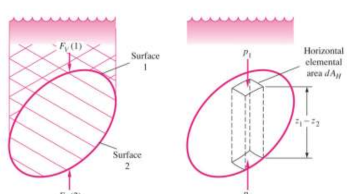
BUOYANCY, BUOYANT FORCE AND CENTER OF BUOYANCY

Buoyant Force: When a body is immersed in a fluid either fully or partially, it experiences a vertical upward force to the opposite direction of action of gravity, this phenomenon is known as Buoyancy and the vertical force acts on body by the fluid is called Buoyant Force (or Force of Buoyancy). Buoyant Force is denoted by F_B.

Center of Buoyancy: The point through which the buoyant force acts on the body is called Center of Buoyancy.

Archimedes's Laws of Buoyancy: A body immersed in a fluid experiences a vertical buoyant force equal to the weight of the fluid it displaces.



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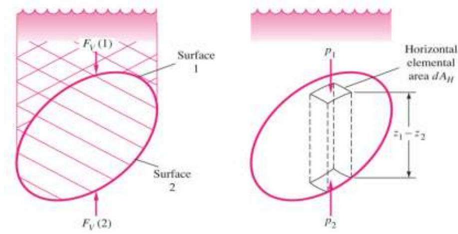
BUOYANCY, BUOYANT FORCE AND CENTER OF BUOYANCY



Both liquids and gases exert buoyancy force on immersed bodies.

$$F_B = \int_{body} (p_2 - p_1)\, dA_H = -\gamma \int (z_2 - z_1)\, dA_H = \gamma(body\ volume)$$

This equation assumes that the body has a uniform specific weight.

<u>A floating body</u> displaces its own weight in the fluid in which it floats.

$F_B = \gamma(displaced\ volume)$ = weight of the floating body

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CONDITION OF FLOATING OF A BODY

"The *buoyant force* acting on a body immersed in a fluid is equal to the weight of the fluid displaced by the body, and it acts upward through the centroid of the displaced volume" *this is known as Archimedes' principle*.

For floating bodies, the weight of the entire body, W must be equal to the buoyant force, F_B .

F_B = Buoyant Force = Weight of fluid whose volume is equal to the submerged portion of the floating body i.e. $F_B = W$.

$$F_B = \rho_f g V_{sub} \qquad W = \rho_b g V_{total}$$

$$\rho_f g V_{sub} = \rho_b g V_{total}$$

$$\frac{V_{total}}{V_{sub}} = \frac{\rho_f}{\rho_b} \qquad \frac{V_{total}}{V_{sub}} > 1$$

$$\frac{\rho_f}{\rho_b} > 1 \qquad \rho_b < \rho_f$$

A solid body dropped into a fluid will sink, float or remain at rest at any point in the fluid, depending on its density relative to the density of the fluid.

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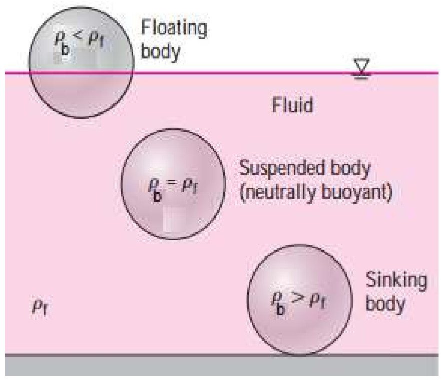
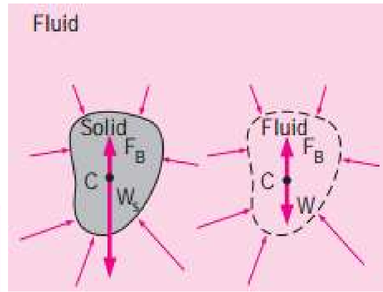
CONDITION OF FLOATING OF A BODY





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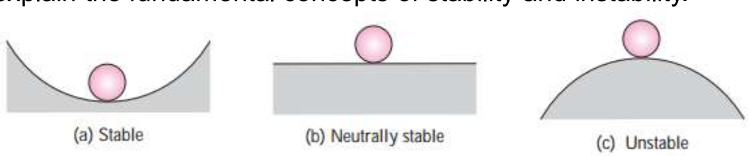
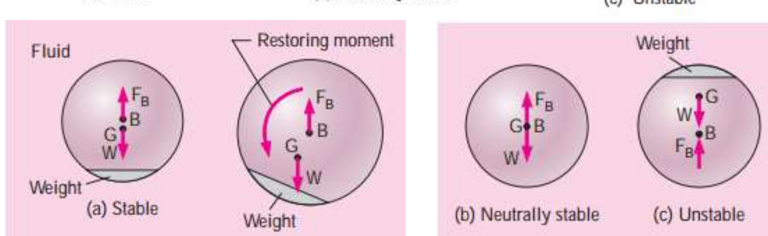
CONDITION OF STABILITY OF FLOATING BODIES

An important application of the buoyancy concept is the assessment of the stability of immersed and floating bodies with no external attachments. This is one of great importance in the design of ships and submarines. A ' Ball on the floor' analogy to explain the fundamental concepts of stability and instability.





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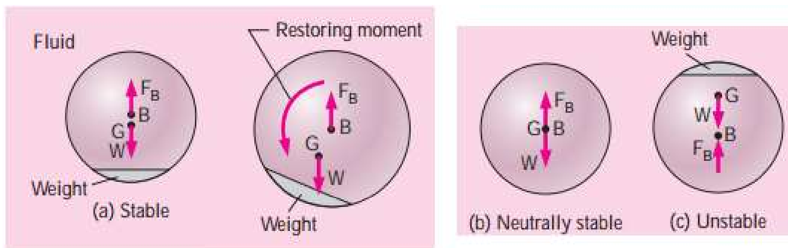
CONDITION OF STABILITY OF FLOATING BODIES

Stable Condition: The rotational stability of an immersed body depends on the relative locations of the center of gravity G of the body and the center of buoyancy B, which is the centroid of the displaced volume. An immersed body is stable if the body is bottom-heavy and thus point G is directly below point B. A rotational disturbance of the body in such cases produces a restoring moment to return the body to its original stable position.



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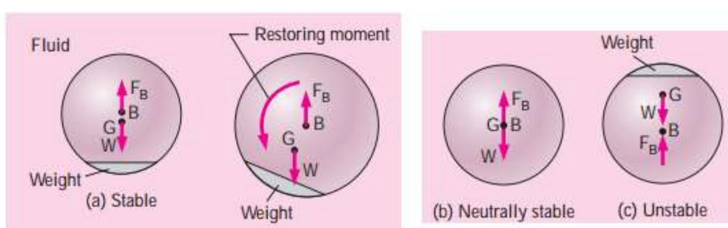
CONDITION OF STABILITY OF FLOATING BODIES



Neutrally Stable Condition: An immersed neutrally buoyant body is neutrally stable if center of gravity G and center of buoyancy B are coincident. Body may not return to its original stable condition after small angular displacement and no restoring moment exists.
Unstable Condition: An immersed neutrally buoyant body is unstable if center of gravity G is directly above center of buoyancy B. A small angular displacement causes further increase in angular displacement by over turning moment.

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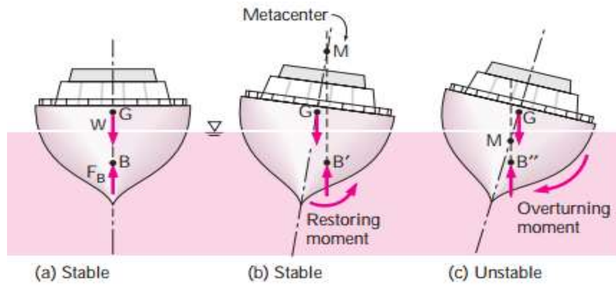
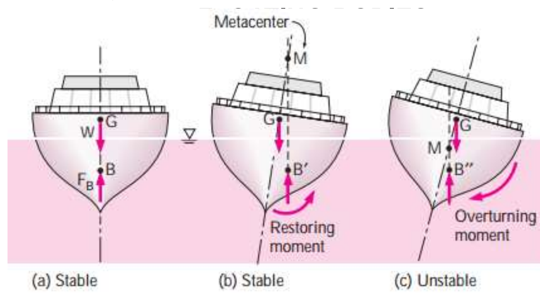
METACENTER, METACENTRIC HEIGHT AND STABILITY OF FLOATING BODIES

A measure of stability for floating bodies is the **metacentric height GM**, which is the distance between the center of gravity G and the **metacenter M** —the intersection point of the lines of action of the buoyant force through the body before and after rotation as shown in the figure below.



METACENTER, METACENTRIC HEIGHT AND STABILITY OF



A floating body is *stable if point M is above point G* and thus GM is positive, and *unstable if point M is below point G* and thus GM is negative. In the latter case, the weight and the buoyant force acting on the tilted body generate an overturning moment instead of a restoring moment, causing the body to capsize. *The length of the metacentric height GM above G is a measure of the stability: the larger it is, the more stable is the floating body.*