Solution Manual | Chapter 11 |Belt, Rope and Pulley

Theory of Machines

By Khurmi

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1. An engine shaft running at 120 r.p.m. is required to drive a machine shaft by means of a belt. The pulley on the engine shaft is of 2 m diameter and that of the machine shaft is 1 m diameter. If the belt thickness is 5 mm ; determine the speed of the machine shaft, when 1. there is no slip ; and 2. there is a slip of 3%. [Ans. 239.4 r.p.m. ; 232.3 r.p.m.]

$$\mathrm{N1} = 120\ \mathrm{r.p.m} \qquad \mathrm{d1} = 2\mathrm{m} \qquad \mathrm{t} = 5\mathrm{mm} = 0.005\mathrm{m} \qquad \mathrm{d2} = 1\mathrm{m} \qquad \mathrm{N2} = ?$$

Solution:

1.With no slip

$$\frac{\mathrm{N2}}{\mathrm{N1}} = \frac{\mathrm{d1}+\mathrm{t}}{\mathrm{d2}+\mathrm{t}} \rightarrow \frac{\mathrm{N2}}{120} = \frac{2+0.005}{1+0.005} \rightarrow \mathrm{N2} = \left(\frac{2+0.005}{1+0.005}\right) \times 120 = 239.4\ \mathrm{r.p.m}$$

2.With a slip of 3%

$$\frac{\mathrm{N2}}{\mathrm{N1}} = \frac{\mathrm{d1}+\mathrm{t}}{\mathrm{d2}+\mathrm{t}} \times \left(1 - \frac{\mathrm{s1}+\mathrm{s2}}{100}\right) \rightarrow \frac{\mathrm{N2}}{120} = \frac{2+0.00}{1+0.00} \times \left(1 - \frac{3+0}{100}\right)$$

$$\rightarrow \mathrm{N2} = \left(\frac{2+0.005}{1+0.005}\right) \times \left(1 - \frac{3+0}{100}\right) \times 120 = 232.22\ \mathrm{r.p.m}$$

3. A pulley is driven by a flat belt running at a speed of 600 m/min. The coefficient of friction between the pulley and the belt is 0.3 and the angle of lap is 160°. If the maximum tension in the belt is 700 N ; find the power transmitted by a belt. [Ans. 3.983 kW]

$$\mathrm{v} = 600\ (\mathrm{m/\,min}) \times \left(\frac{1\mathrm{min}}{60\mathrm{s}}\right) = 10\mathrm{m/s} \qquad \mu = 0.3 \qquad \theta = 160° \times \frac{\pi}{180} = 2.8\mathrm{rad}$$

$$\mathrm{T} = 700\mathrm{N} = \mathrm{T1} \qquad \mathrm{P} = ?$$

Solution:

$$\frac{\mathrm{T1}}{\mathrm{T2}} = e^{\mu\theta} = e^{0.3\times 2.8} = 2.31 \rightarrow \mathrm{T2} = \frac{\mathrm{T1}}{2.31}$$

$$\text{When T1} = 700\mathrm{N} \rightarrow \mathrm{T2} = \frac{700}{2.31} = 303\mathrm{N}$$

$$\mathrm{P} = (\mathrm{T1} - \mathrm{T2}) \times \mathrm{v} = (700 - 303) \times 10 = 3983\mathrm{W} = 3.983\mathrm{KW}$$

4. Find the width of the belt, necessary to transmit 7.5 kW to a pulley 300 mm diameter, if the pulley makes 1600 r.p.m and the coefficient of friction between the belt and the pulley is 0.22. Assume the angle of contact as 210° and the maximum tension in the belt is not to exceed 8 N/mm width. [Ans. 67.4 mm]

$b = ?$ $P = 7.5\text{kW} = 7500\text{W}$ $d = 300\text{mm} = 0.3\text{m}$ $N = 1600\text{r.p.m}$ $\mu = 0.22$

$\theta = 210° \times \frac{\pi}{180} = 3.665\text{rad}$ $T' = 8\text{N/mm width}$

Solution:

$$v = \frac{\pi d N}{60} = \frac{\pi \times 0.3 \times 1600}{60} = 25.13\text{m/s}$$

$$\frac{T1}{T2} = e^{\mu\theta} = e^{0.22\times3.665} = 2.2396 \quad \ldots\ldots a$$

$$P = (T1 - T2) \times v \rightarrow 7500 = (T1 - T2) \times 25.13$$

$$\rightarrow T1 - T2 = \frac{7500}{25.13} = 298.44\text{N} \rightarrow T1 = 298.44 + T2 \quad \ldots\ldots b$$

sub a in b :

$$\frac{298.44 + T2}{T2} = 2.2396 \rightarrow T2 = 240.761\text{N}$$

$$T1 = 298.44 + 240.761 = 539.2\text{N}$$

$$b = \frac{T1}{T'} = \frac{539.2}{8} = 67.4\text{mm}$$

5. An open belt 100 mm wide connects two pulleys mounted on parallel shafts with their centers 2.4 m apart. The diameter of the larger pulley is 450 mm and that of the smaller pulley 300 mm. The coefficient of friction between the belt and the pulley is 0.3 and the maximum stress in the belt is limited to 14 N/mm width. If the larger pulley rotates at 120 r.p.m., find the maximum power that can be transmitted. [Ans. 2.39 kW]

$$\mathrm{b} = 100\mathrm{mm} \qquad x = 2.4\mathrm{m} \qquad \mathrm{d1} = 450\mathrm{mm} = 0.45\mathrm{m} \qquad \mu = 0.3$$

$$\mathrm{d2} = 300\mathrm{mm} = 0.3\mathrm{m} \qquad \mathrm{T'} = 14\mathrm{N/m} \qquad \mathrm{N1} = 120\ \mathrm{r.p.m} \qquad \mathrm{P} =?$$

Solution:

$$\mathrm{v} = \frac{\pi \mathrm{d1N1}}{60} = \frac{\pi \times 0.45 \times 120}{60} = 2.827\mathrm{m/s}$$

$$\alpha = \sin^{-1}\left(\frac{\mathrm{d1} - \mathrm{d2}}{2x}\right) = \sin^{-1}\left(\frac{0.45 - 0.3}{2 \times 2.4}\right) = 1.79° \rightarrow\ 2\alpha = 2 \times 1.79° = 3.58°$$

$$\theta = (180 - 2\alpha) \times \frac{\pi}{180} = (180 - 3.58) \times \frac{\pi}{180} = 3.079\ \mathrm{rad}$$

$$\frac{\mathrm{T1}}{\mathrm{T2}} = \mathrm{e}^{\mu\theta} = \mathrm{e}^{0.3 \times 3.079} = 2.518\ \ \ldots \ldots \mathrm{a}$$

$$\mathrm{b} = \frac{\mathrm{T1}}{\mathrm{T'}}\ \rightarrow\ 100 = \frac{\mathrm{T1}}{14} \rightarrow \mathrm{T1} = 100 \times 14 = 1400\mathrm{N} \ldots \ldots \mathrm{b}$$

sub a in b :

$$\frac{\mathrm{T1}}{\mathrm{T2}} = 2.518\ \rightarrow \mathrm{T2} = \frac{\mathrm{T1}}{2.518} = \frac{1400}{2.518} = 555.996\mathrm{N}$$

$$\mathrm{P} = (\mathrm{T1} - \mathrm{T2}) \times \mathrm{v} = (1400 - 555.996) \times 2.827$$

$$\mathrm{P} = 2386\mathrm{W}\ \approx 2.39\mathrm{kW}$$

6. A leather belt 125 mm wide and 6 mm thick, transmits power from a pulley 750 mm diameter which runs at 500 r.p.m. The angle of lap is 150° and μ = 0.3. If the mass of 1 m3 of leather is 1 Mg and the stress in the belt is not to exceed 2.75 MPa, find the maximum power that can be transmitted. [Ans. 19 kW]

$$b = 125\text{mm} = 0.125\text{m} \qquad t = 6\text{mm} = 0.006\text{m} \qquad d = 750\text{mm}\ 0.75\text{m} \qquad \mu = 0.3$$

$$N = 500 \qquad \theta = 150° \times \frac{\pi}{180} = 2.617\text{rad} \qquad \rho = 1\text{Mg/m}^3 = 1000\text{kg/ m}^3$$

$$\sigma = 2.75\text{Mpa} = 2.75 \times 10^6\text{Pa} \qquad P = ?$$

Solution:

$$v = \frac{\pi dN}{60} = \frac{\pi \times 0.75 \times 500}{60} = 19.634\text{m/s} \quad > 10\text{m/s}\ \text{, so we take Tc in the acount}$$

$$\frac{T1}{T2} = e^{\mu\theta} = e^{0.3\times2.617} = 2.1926 \quad \ldots\ldots.a$$

$$Tc = mv^2 = \rho btl \times v^2 = 1000 \times 0.125 \times 0.006 \times 1 \times (19.634)^2 = 289.12N$$

$$T = \sigma bt = 2.75 \times 10^6 \times 0.125 \times 0.006 = 2062.5N/m^2$$

$$T = Tc + T1 \rightarrow T1 = T - Tc = 2062.5 - 289.12 = 1773.38 \ldots\ldots.b$$

sub a in b :

$$\frac{T1}{T2} = 2.1926 \rightarrow T2 = \frac{T1}{2.1926} = \frac{1773.38}{2.1926} = 808.8N$$

$$P = (T1 - T2) \times v = (1773.38 - 808.8) \times 19.634$$

$$P = 18931.1W \approx 19kW$$

7. A flat belt is required to transmit 35 kW from a pulley of 1.5 m effective diameter running at 300 r.p.m. The angle of contact is spread over 11/24 of the circumference and the coefficient of friction between belt and pulley surface is 0.3. Determine, taking centrifugal tension into account, width of the belt required. It is given that the belt thickness is 9.5 mm, density of its material is 1.1 Mg/m3 and the related permissible working stress is 2.5 MPa. [Ans. 143 mm]

$$P = 35kW = 35000W \qquad d = 1.5m \qquad N = 300\ r.p.m \qquad \theta = \frac{11}{24} \times 2\pi = 2.88\ rad$$

$$\mu = 0.3 \qquad t = 9.5mm = 0.095m \qquad \rho = 1.1Mg/m^3 = 1100\ kg/m^3$$

$$\sigma = 2.5Mpa = 2.5 \times 10^6 Pa$$

Solution:

$$v = \frac{\pi dN}{60} = \frac{\pi \times 1.5 \times 300}{60} = 23.56m/s$$

$$\frac{T1}{T2} = e^{\mu\theta} = e^{0.3\times2.88} = 2.37 \quad \ldots\ldots a$$

$$P = (T1 - T2) \times v \rightarrow\ 35000 = (T1 - T2) \times 23.56$$

$$\rightarrow\ T1 - T2 = \frac{35000}{23.56} = 1485.568N \rightarrow T1 = 1485.568 + T2 \ \ldots\ldots b$$

sub a in b :

$$\frac{1485.568 + T2}{T2} = 2.37 \ \rightarrow T2 = 1081.39N$$

$$\rightarrow T1 = 1485.568 + 1081.39 = 2567.96N$$

$$Tc = mv^2 = \rho btl \times v^2 = 1100 \times b \times 0.095 \times 1 \times (23.56)^2 = 5800.5b$$

$$Tc + T1 = \sigma bt \ \rightarrow 5800.5b + 2566.96 = 2.5 \times 10^6 \times b \times 0.095$$

$$2566.96 = 237500b - 5800.5b \rightarrow 2566.96 = 17950b$$

$$b = \frac{2566.96}{17950} = 0.143\ m = 143\ mm$$

$$P = (T1 - T2) \times v = (2566.96 - 17950) \times 23.56$$

$$P = 35920W \ \approx 35.9\ kW$$

8. A blower is driven by an electric motor though a belt drive. The motor runs at 750 r.p.m. For this power transmission, a flat belt of 8 mm thickness and 250 mm width is used. The diameter of the motor pulley is 350 mm and that of the blower pulley 1350 mm. The center distance between these pulleys is 1350 mm and an open belt configuration is adopted. The pulleys are made out of cast iron. The frictional coefficient between the belt and pulley is 0.35 and the permissible stress for the belt material can be taken as 2.5 N/mm² with sufficient factor of safety. The mass of a belt is 2 kg per meter length. Find the maximum power transmitted without belt slipping in any one of the pulleys. [Ans. 35.9 kW]

$$\mathrm{N1} = 750\mathrm{r.p.m} \qquad \mathrm{t} = 8\mathrm{mm} = 0.008\mathrm{m} \qquad \mathrm{b} = 250\mathrm{mm} = 0.25\mathrm{m} \qquad \mu = 0.35$$

$$\mathrm{d1} = 350\mathrm{mm} = 0.35\mathrm{m} \qquad \mathrm{d2} = 1350\mathrm{mm} = 1.35\mathrm{m} \qquad x = 1350\mathrm{mm} = 1.35\mathrm{m}$$

$$\sigma = 2.5\mathrm{N/mm^2} \times 10^6 = 2.5 \times 10^6\mathrm{Pa} \qquad \mathrm{m} = 2\mathrm{kg/m} \qquad \mathrm{P} = ?$$

Solution:

$$\mathrm{v} = \frac{\pi \mathrm{d1N1}}{60} = \frac{\pi \times 0.35 \times 750}{60} = 13.74\mathrm{m/s}$$

$$\mathrm{Tc} = \mathrm{mv}^2 = 2 \times (13.74)^2 = 377.82\mathrm{N}$$

$$\sin\alpha = \left(\frac{\mathrm{d1} - \mathrm{d2}}{2x}\right) \rightarrow \alpha = \sin^{-1}\left(\frac{0.34 - 1.35}{2 \times 1.35}\right) = |-21.73°| = 21.73° \quad \text{Must be + ive}$$

$$\theta = (180 - 2\alpha) \times \frac{\pi}{180} = (180 - (2 \times 21.73)) \times \frac{\pi}{180} = 2.38 \text{ rad}$$

$$\frac{\mathrm{T1}}{\mathrm{T2}} = e^{\mu\theta} = e^{0.35 \times 2.38} = 2.302 \quad \ldots\ldots a$$

$$\mathrm{T} = \sigma \mathrm{bt} = \mathrm{Tc} + \mathrm{T1} \rightarrow 2.5 \times 10^6 \times 0.25 \times 0.008 = 377.82 + \mathrm{T1}$$

$$5000 - 377.82 = \mathrm{T1} \rightarrow \mathrm{T1} = 4622.18\mathrm{N} \quad \ldots\ldots b$$

sub a in b :

$$\frac{4622.18}{\mathrm{T2}} = 2.302 \rightarrow \mathrm{T2} = 2007.89\mathrm{N}$$

$$\mathrm{P} = (\mathrm{T1} - \mathrm{T2}) \times \mathrm{v} = (4622.18 - 2007.89) \times 13.74$$

$$\mathrm{P} = 35920.34\mathrm{W} \approx 35.9\mathrm{kW}$$

9. An open belt drive connects two pulleys 1.2 m and 0.5 m diameter on parallel shafts 3.6 m apart. The belt has a mass of 1 kg/m length and the maximum tension in it is not to exceed 2 kN. The 1.2 m pulley, which is the driver, runs at 200 r.p.m. Due to the belt slip on one of the pulleys, the velocity of the driven shaft is only 450 r.p.m. If the coefficient of friction between the belt and the pulley is 0.3, find : 1. Torque on each of the two shafts, 2. Power transmitted, 3. Power lost in friction, and 4. Efficiency of the drive.
[Ans. 648.6 N-m, 270.25 N-m ; 13.588 kW ; 0.849 kW ; 93.75%]

$$\mathrm{d1} = 1.2\mathrm{m} \qquad \mathrm{d2} = 0.5\mathrm{m} \qquad x = 3.6\mathrm{m} \qquad \mathrm{m} = 1\mathrm{kg/m} \qquad \mathrm{T} = 2\mathrm{kN} = 2000\mathrm{N}$$

$$\mathrm{N1} = 200\mathrm{r.p.m} \qquad \mathrm{N2} = 450\mathrm{r.p.m} \qquad \mu = 0.3 \qquad 1.\,\mathrm{Tq1}\ \&\ \mathrm{Tq2} = ?$$

$$2.\,\mathrm{P} = ? \qquad 3.\,\text{Power lost in friction} = ? \qquad \text{Efficiency} = ?$$

Solution:

$$\mathrm{v} = \frac{\pi \mathrm{d1N1}}{60} = \frac{\pi \times 1.2 \times 200}{60} = 12.566\mathrm{m/s}$$

$$\alpha = \sin^{-1}\left(\frac{\mathrm{d1} - \mathrm{d2}}{2x}\right) = \sin^{-1}\left(\frac{1.2 - 0.5}{2 \times 3.6}\right) = 5.579°$$

$$\theta = (180 - 2\alpha) \times \frac{\pi}{180} = (180 - (2 \times 5.579)) \times \frac{\pi}{180} = 2.946\ \mathrm{rad}$$

$$\frac{\mathrm{T1}}{\mathrm{T2}} = e^{\mu\theta} = e^{0.3 \times 2.946} = 2.42 \quad \ldots\ldots\mathrm{a}$$

$$\mathrm{Tc} = \mathrm{mv}^2 = 1 \times (12.566)^2 = 157.91\mathrm{N}$$

$$\mathrm{T} = \mathrm{Tc} + \mathrm{T1} \rightarrow 2000 = 157.91 + \mathrm{T1} \rightarrow \mathrm{T1} = 2000 - 157.91 = 1842.09\mathrm{N} \ldots\ldots\mathrm{b}$$

sub a in b :

$$\frac{1842.09}{\mathrm{T2}} = 2.42 \rightarrow \mathrm{T2} = \frac{1842.09}{2.42} = 761.194\mathrm{N}$$

2.

$$\mathrm{P} = (\mathrm{T1} - \mathrm{T2}) \times \mathrm{v} = (1842.09 - 761.194) \times 12.556 = 13588\mathrm{W} = 13.588\mathrm{kW}$$

1. $$\mathrm{Tq} = (\mathrm{T1} - \mathrm{T2}) \times \mathrm{r}$$

$$\mathrm{Tq1} = (\mathrm{T1} - \mathrm{T2}) \times \mathrm{r1} = (1842.09 - 761.194) \times \frac{1.2}{2} = 648.53\mathrm{N.m}$$

$$\mathrm{Tq2} = (\mathrm{T1} - \mathrm{T2}) \times \mathrm{r2} = (1842.09 - 761.194) \times \frac{0.5}{2} = 270.22\mathrm{N.m}$$

3. $$\text{Power lost in friction} = \text{input power} - \text{output power}$$

$$\text{input power} = \frac{2\pi \mathrm{N1Tq1}}{60} = \frac{2\pi \times 200 \times 648.53}{60} = 13.582\mathrm{kW}$$

$$\text{output power} = \frac{2\pi \mathrm{N2Tq2}}{60} = \frac{2\pi \times 450 \times 270.22}{60} = 12.733\mathrm{kW}$$

$$\text{Power lost in friction} = 13.582 - 12.733 = 0.848\mathrm{kW}$$

4.

$$\text{Efficiency} = \frac{\text{output power}}{\text{input power}} \times 100\% = \frac{12.733}{13.582} \times 100\% = 93.75\%$$

10. The power transmitted between two shafts 3.5 metres apart by a cross belt drive round the two pulleys 600 mm and 300 mm in diameters, is 6 kW. The speed of the larger pulley (driver) is 220 r.p.m. The permissible load on the belt is 25 N/mm width of the belt which is 5 mm thick. The coefficient of friction between the smaller pulley surface and the belt is 0.35. Determine :
1. necessary length of the belt ; 2. width of the belt, and 3. necessary initial tension in the belt.
[Ans. 8.472 m ; 53 mm ; 888 N]

$x = 3.5\text{m} \qquad \text{d1} = 600\text{mm} = 0.6\text{m} \qquad \text{d2} = 300\text{mm} = 0.3\text{m} \qquad \text{P} = 6\text{kW} = 6000\text{W}$

$\text{N1} = 220\text{r.p.m} \qquad \text{T}' = 25\text{N/m width} \qquad \text{t} = 5\text{mm} = 0.005\text{m} \qquad \mu = 0.35$

$1.\ \text{L} = ? \qquad 2.\ \text{b} = ? \qquad 3.\ \text{T}_o = ?$

Solution:

1. $\text{L} = \frac{\pi}{2}(\text{d1} + \text{d2}) + 2x + \frac{(\text{d1}-\text{d2})^2}{4x} = \frac{\pi}{2}(0.6 + 0.3) + (2 \times 3.5) + \frac{(0.6+0.3)^2}{4\times3.5} = 8.472\text{m}$

2.

$$\text{v} = \frac{\pi \text{d1N1}}{60} = \frac{\pi \times 0.6 \times 220}{60} = 6.911\text{m/s}$$

$$\alpha = \sin^{-1}\left(\frac{0.6 + 0.3}{2 \times 3.5}\right) = \sin^{-1}\left(\frac{0.6 + 0.3}{2 \times 3.5}\right) = 7.387°$$

$$\theta = (180 + 2\alpha) \times \frac{\pi}{180} = (180 + (2 \times 7.387)) \times \frac{\pi}{180} = 3.399 \text{ rad}$$

$$\frac{\text{T1}}{\text{T2}} = e^{\mu\theta} = e^{0.3\times3.399} = 3.286 \quad \dots\dots\text{a}$$

$$\text{P} = (\text{T1} - \text{T2}) \times \text{v} \rightarrow \ 6000 = (\text{T1} - \text{T2}) \times 6.911$$

$$\rightarrow \ \text{T1} - \text{T2} = \frac{6000}{6.911} = 868.181\text{N} \rightarrow \text{T1} = 868.181 + \text{T2} \ \dots\dots.\text{b}$$

sub a in b :

$$\frac{868.181 + \text{T2}}{\text{T2}} = 3.286 \ \rightarrow \text{T2} = 379.781\text{N}$$

$$\text{T1} = 868.181 + 379.781 = 1247.926\text{N}$$

$$\text{b} = \frac{\text{T1}}{\text{T}'} = \frac{1247.926}{25} = 49.9\text{mm}$$

3.

$$\text{T}_o = \frac{\text{T1} + \text{T2}}{2} = \frac{1247.926 + 379.78}{2} = 889.08$$

11. A flat belt, 8 mm thick and 100 mm wide transmits power between two pulleys, running at 1600 m/min. The mass of the belt is 0.9 kg/m length. The angle of lap in the smaller pulley is 165° and the coefficient of friction between the belt and pulley is 0.3. If the maximum permissible stress in the belt is 2 MN/m2, find : 1. maximum power transmitted ; and 2. initial tension in the belt
[Ans. 14.83 kW ; 1002 N]

$$\mathrm{T} = 8\mathrm{mm} = 0.008\mathrm{m} \qquad \mathrm{b} = 100\mathrm{mm} = 0.1\mathrm{m} \qquad \mathrm{v} = 1600\mathrm{m/min} \times \frac{1\mathrm{min}}{60\mathrm{sec}} = 26.67\mathrm{m/s}$$

$$\mathrm{m} = 0.9\mathrm{kg/m} \qquad \theta = 165° \times \frac{\pi}{180} = 2.88\mathrm{rad} \qquad \mu = 0.3 \qquad \sigma = 2\mathrm{MN/m^2} = 2 \times 10^6\mathrm{pa}$$

$$1.\,\mathrm{P} =? \qquad 2.\,\mathrm{T_o} =?$$

Solution:

1.

$$\frac{\mathrm{T1}}{\mathrm{T2}} = e^{\mu\theta} = e^{0.3\times 2.88} = 2.372 \quad \ldots\ldots a$$

$$\mathrm{Tc} = \mathrm{mv^2} = 0.9 \times (26.67)^2 = 640\mathrm{N}$$

$$\mathrm{T} = \sigma \times \mathrm{b} \times \mathrm{t} = \mathrm{Tc} + \mathrm{T1} \rightarrow 2 \times 10^6 \times 0.1 \times 0.008 = 640 + \mathrm{T1}$$

$$\mathrm{T1} = 1600 - 640 \rightarrow \mathrm{T1} = 960\mathrm{N} \quad \ldots\ldots b$$

sub a in b :

$$\frac{960}{\mathrm{T2}} = 2.372 \rightarrow \mathrm{T2} = 404.72\mathrm{N}$$

$$\mathrm{P} = (\mathrm{T1} - \mathrm{T2}) \times \mathrm{v} \rightarrow \mathrm{P} = (960 - 404.72) \times 26.67 = 14809.3\mathrm{W} = 14.8\mathrm{kW}$$

2.

$$\mathrm{T_o} = \frac{\mathrm{T1} + \mathrm{T2} + 2\mathrm{Tc}}{2} = \frac{960 + 404.72 + (2 \times 640)}{2} = 1322.36\mathrm{N}$$

But when we use equation below , we get the same result which given for [T_o]*

$$\mathrm{T_o} = \frac{\mathrm{T1} + \mathrm{T2} + \mathrm{Tc}}{2} = \frac{960 + 404.72 + 640}{2} = 1002.36\mathrm{N}$$

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* see a textbook (theory of machines by r.k.bansal)

12. An open belt connects two flat pulleys. The smaller pulley is 400 mm diameter and runs at 200 r.p.m. The angle of lap on this pulley is 160° and the coefficient of friction between the belt and pulley face is 0.25. The belt is on the point of slipping when 3 kW is being transmitted. Which of the following two alternatives would be more effective in order to increase the power :
1. Increasing the initial tension in the belt by 10 per cent, and
2. Increasing the coefficient of friction by 10 per cent by the application of a suitable dressing to the belt? [Ans. First method is more effective]

$d = 400\text{mm} = 0.4\text{m} \qquad N = 200\text{r.p.m} \qquad \theta = 160° \times \frac{\pi}{180} = 2.79\text{rad} \qquad \mu = 0.25$

$P = 3\text{kW} = 3000\text{W}$

Solution:

$$v = \frac{\pi dN}{60} = \frac{\pi \times 0.4 \times 200}{60} = 4.188\text{m/s}$$

$$\frac{T1}{T2} = e^{\mu\theta} = e^{0.25 \times 2.79} = 2 \ldots\ldots a$$

$$P = (T1 - T2) \times v \rightarrow \; 3000 = (T1 - T2) \times 4.188$$

$$\rightarrow \; T1 - T2 = \frac{3000}{4.188} = 716.33\text{N} \rightarrow T1 = 716.33 + T2 \; \ldots\ldots b$$

sub a in b :

$$\frac{716.33 + T2}{T2} = 2 \; \rightarrow T2 = 716.33\text{N}$$

$$T1 = 716.33 + 716.33 = 1432.66\text{N}$$

$$T_o = \frac{T1 + T2}{2} = \frac{1432.66 + 716.33}{2} = 1079.495\text{N} \ldots\ldots e$$

1st Method , when Increasing the initial tension in the belt by 10%

$$T_o = 1074.495 + \frac{1074.495}{10} = 1181.94N$$

$$T_o = \frac{T1 + T2}{2} \rightarrow 1181.94 = \frac{T1 + T2}{2} \rightarrow T1 + T2 = 2363.896N$$

$$T1 = 2363.896 - T2 \; \ldots \ldots . c$$

sub c in a :

$$\frac{2363.896 - T2}{T2} = 2 \; \rightarrow T2 = 787.965N$$

$$T1 = 2363.896 - 787.965 = 1575.93N$$

$$P = (T1 - T2) \times v \rightarrow \; P = (1575.93 - 787.965) \times 4.188 = 3300W = 3.3kW$$

2nd Method , when Increasing the coefficient of friction in the belt by 10%

$$\mu = 0.25 + \frac{0.25}{10} = 0.275$$

$$\frac{T1}{T2} = e^{\mu\theta} = e^{0.275 \times 2.79} = 2.15 \rightarrow T1 = 2.15 \times T2 \ldots \ldots . d$$

$$T_o = \frac{T1 + T2}{2} \rightarrow \frac{T1 + T2}{2} = 1079.495$$

$$T1 + T2 = 2 \times 1079.495 = 2158.99 \ldots \ldots . f$$

$$T2 = 620.825 \qquad \& \qquad T1 = 1334.744$$

$$P = (T1 - T2) \times v \rightarrow \; P = (1334.744 - 620.825) \times 4.188 = 2989W = 2.99kW$$

P at 1st method is grater than the 2nd So , First method is more effective

14. Power is transmitted between two shafts by a V-belt whose mass is 0.9 kg/m length. The maximum permissible tension in the belt is limited to 2.2 kN. The angle of lap is 170° and the groove angle 45°. If the coefficient of friction between the belt and pulleys is 0.17, find : 1. velocity of the belt for maximum power ; and 2. power transmitted at this velocity. [Ans. 28.54 m/s ; 30.7 kW]

$$\mathrm{m} = 0.9\mathrm{kg/m} \qquad \mathrm{T} = 2.2\mathrm{kN} = 2200\mathrm{N} \qquad \theta = 170° \times \frac{\pi}{180} = 2.967\mathrm{rad} \qquad \mu = 0.17$$

$$2\beta = 45° \qquad 1.\,\mathrm{v} = ? \qquad 2.\,\mathrm{P} = ?$$

Solution:

For maximum power we can write :

1.

$$\mathrm{v} = \sqrt{\frac{\mathrm{T}}{3\mathrm{m}}} = \sqrt{\frac{2200}{3 \times 0.9}} = 28.54\mathrm{m/s}$$

$$\frac{\mathrm{T1}}{\mathrm{T2}} = \mathrm{e}^{\mu\theta \times \frac{1}{\sin\beta}} = e^{0.17 \times 2.967 \times \frac{1}{\sin 22.5}} = 3.73 \ldots \ldots \mathrm{a}$$

$$\mathrm{Tc} = \mathrm{mv}^2 = 0.9 \times (28.54)^2 = 733\mathrm{N}$$

$$\mathrm{T1} = \mathrm{T} - \mathrm{Tc} = 2200 - 733 = 1466.67\mathrm{N} \ldots \ldots \mathrm{b}$$

sub a in b :

$$\frac{1466.67}{\mathrm{T2}} = 3.73 \rightarrow \mathrm{T2} = \frac{1466.67}{3.73} = 393.2\mathrm{N}$$

2.

$$\mathrm{P} = (\mathrm{T1} - \mathrm{T2}) \times \mathrm{v} \rightarrow \mathrm{P} = (1466.67 - 393.2) \times 28.54 = 30636.833\mathrm{W} = 3.7\mathrm{kW}$$

15. Two shafts whose centers are 1 m apart are connected by a V-belt drive. The driving pulley is supplied with 100 kW and has an effective diameter of 300 mm. It runs at 1000 r.p.m. while the driven pulley runs at 375 r.p.m. The angle of groove on the pulleys is 40°. The permissible tension in 400 mm^2 cross-sectional area belt is 2.1 MPa. The density of the belt is 1100 kg/m^3. The coefficient of friction between the belt and pulley is 0.28. Estimate the number of belts required. [Ans. 10]

$$x = 1\text{m} \qquad \text{P} = 100\text{kW} \qquad \text{d1} = 300\text{mm} = 0.3\text{m} \qquad \text{N1} = 1000\ \text{r.p.m}$$

$$\text{N2} = 375\ \text{r.p.m} \qquad 2\beta = 40° \qquad \text{A} = 400\text{mm}^2 = 4 \times 10^{-4}\text{m}^2$$

$$\sigma = 2.1\text{Mpa} = 2.1 \times 10^6\text{Pa} \qquad \rho = 1100\text{kg/m}^3 \qquad \mu = 0.28 \qquad \text{n} = ?$$

Solution:

$$\frac{\text{N2}}{\text{N1}} = \frac{\text{d1}}{\text{d2}} \rightarrow \text{d2} = \frac{\text{N1} \times \text{d1}}{\text{N2}} = \frac{1000 \times 0.3}{375} = 0.8\text{m}$$

$$\alpha = \sin^{-1}\left(\frac{\text{d1} - \text{d2}}{2x}\right) = \sin^{-1}\left(\frac{0.3 - 0.8}{2 \times 1}\right) = |-14.477°| = 14.477°$$

$$\theta = (180 - 2\alpha) \times \frac{\pi}{180} = (180 - (2 \times 14.477°)) \times \frac{\pi}{180} = 2.636\ \text{rad}$$

$$\frac{\text{T1}}{\text{T2}} = e^{\mu\theta \times \frac{1}{\sin\beta}} = e^{0.28 \times 2.636 \times \frac{1}{\sin 20}} = 8.655 \ldots\ldots \text{a}$$

$$\text{T} = \sigma \times \text{A} = 2.1 \times 10^6 \times 4 \times 10^{-4} = 840\text{N}$$

$$\text{v} = \frac{\pi \text{d1N1}}{60} = \frac{\pi \times 0.3 \times 1000}{60} = 15.7\text{m/s}$$

$$\text{Tc} = \rho \text{Av}^2 = 1100 \times 4 \times 10^{-4} \times (15.7)^2 = 108.565\text{N}$$

$$\text{T1} = \text{T} - \text{Tc} = 840 - 108.565 = 731.43\text{N} \ldots\ldots \text{b}$$

sub a in b :

$$\frac{\text{T1}}{\text{T2}} = 8.655 \rightarrow \text{T2} = \frac{731.43}{8.655} = 85.393\text{N}$$

$$\text{P} = (\text{T1} - \text{T2}) \times \text{v} \rightarrow \text{P} = (731.43 - 85.393) \times 15.7 = 10142.7 = 10.14\text{kW}$$

$$\text{n} = \frac{\text{Total power}}{\text{Power per belt}} = \frac{100}{10.14} = 9.86 = 10\ \text{belts}$$

16. A rope drive is required to transmit 230 kW from a pulley of 1 metre diameter running at 450 r.p.m. The safe pull in each rope is 800 N and the mass of the rope is 0.46 kg per metre length. The angle of lap and the groove angle is 160° and 45° respectively. If the coefficient of friction between the rope and the pulley is 0.3, find the number of ropes required. [Ans. 21]

$$P = 230\text{kW} = 230 \times 10^3\text{W} \qquad d = 1\text{m} \qquad N = 450\ \text{r.p.m} \qquad T = 800\text{N}$$

$$m = 0.46\frac{\text{kg}}{\text{meter}} \qquad \theta = 160° \times \frac{\pi}{180} = 2.79\text{rad} \qquad 2\beta = 45° \qquad \mu = 0.3 \qquad n = ?$$

Solution:

$$v = \frac{\pi d N}{60} = \frac{\pi \times 1 \times 450}{60} = 23.56\text{m/s}$$

$$\frac{T1}{T2} = e^{\mu\theta \times \frac{1}{\sin\beta}} = e^{0.3 \times 2.79 \times \frac{1}{\sin 22.5}} = 8.9 \ldots \ldots a$$

$$Tc = mv^2 = 0.46 \times (23.56)^2 = 255.33\text{N}$$

$$T1 = T - Tc = 800 - 255.33 = 544.67\text{N} \ldots \ldots b$$

sub a in b :

$$\frac{T1}{T2} = 8.9 \rightarrow T2 = \frac{544.67}{8.9} = 61.2\text{N}$$

$$P = (T1 - T2) \times v \rightarrow P = (544.67 - 61.2) \times 23.56 = 11390.5\text{W} = 11.39\text{kW}$$

$$n = \frac{\text{Total power}}{\text{Power per belt}} = \frac{230}{11.39} = 20.2 = 21\ \text{belts}$$

17. Power is transmitted between two shafts, 3 metres apart by an open wire rope passing round two pulleys of 3 metres and 2 metres diameters respectively, the groove angle being 40°. If the rope has a mass of 3.7 kg per metre length and the maximum working tension in rope is 20 kN, determine the maximum power that the rope can transmit and the corresponding speed of the smaller pulley. The coefficient of friction being 0.15. [Ans. 400 kW ; 403.5 r.p.m.]

$x = 3\text{m} \qquad \text{d1} = 3\text{m} \qquad \text{d2} = 2\text{m} \qquad 2\beta = 40° \qquad \text{m} = 3.7\text{kg per meter} \qquad \mu = 0.15$

$\text{T} = 20\text{kN} = 20000\text{N} \qquad \text{P} =? \qquad \text{N2} =?$

Solution:

$$\alpha = \sin^{-1}\left(\frac{\text{d1} - \text{d2}}{2x}\right) = \sin^{-1}\left(\frac{3-2}{2\times 3}\right) = 14.477°$$

$$\theta = (180 - 2\alpha) \times \frac{\pi}{180} = (180 - (2 \times 14.477°)) \times \frac{\pi}{180} = 2.636\ \text{rad}$$

$$\frac{\text{T1}}{\text{T2}} = \text{e}^{\mu\theta\times\frac{1}{\sin\beta}} = e^{0.15\times 2.636\times\frac{1}{\sin 20}} = 3.177 \ldots\ldots \text{a}$$

For maximum power we can write :

$$\text{Tc} = \frac{\text{T}}{3} = \frac{20000}{3} = 6666.67\text{N}$$

$$\text{v} = \sqrt{\frac{\text{T}}{3\text{m}}} = \sqrt{\frac{20000}{3\times 3.7}} = 42.447\text{m/s}$$

$$\text{T1} = \text{T} - \text{Tc} = 20000 - 6666.67 = 13333.33\text{N} \ldots\ldots \text{b}$$

sub a in b :

$$\frac{\text{T1}}{\text{T2}} = 3.177 \rightarrow \text{T2} = \frac{13333.33}{3.177} = 4196.8\text{N}$$

$$\text{P} = (\text{T1} - \text{T2}) \times \text{v} \rightarrow \ \text{P} = (13333.33 - 4196.8) \times 42.447 = 387818.28\text{W} = 387.822\text{kW}$$

$$\text{v} = \frac{\pi\text{d2N2}}{60} \rightarrow \text{N2} = \frac{\text{v}\times 60}{\pi\times\text{d2}} = \frac{42.447\times 60}{\pi\times 2} = 405.3\ \text{r.p.m}$$

18. A rope drive transmits 75 kW through a 1.5 m diameter, 45° grooved pulley rotating at 200 r.p.m. The coefficient of friction between the ropes and the pulley grooves is 0.3 and the angle of lap is 160°. Each rope has a mass of 0.6 kg/m and can safely take a pull of 800 N. Taking centrifugal tension into account determine : 1. the number of ropes required for the drive, and 2. initial rope tension. [Ans. 9 ; 510.2 N]

$P = 75\text{kW} = 75000\text{W} \qquad d1 = 1.5\text{m} \qquad 2\beta = 45° \qquad N1 = 200\ \text{r.p.m} \qquad \mu = 0.3$

$\theta = 160° \times \frac{\pi}{180} = 2.79\text{rad} \qquad m = 0.6\text{kg per meter} \qquad T = 800\text{N} \qquad n = ? \qquad T_o = ?$

Solution:

$$v = \frac{\pi d1N1}{60} = \frac{\pi \times 1.5 \times 200}{60} = 15.7\text{m/s}$$

$$\frac{T1}{T2} = e^{\mu\theta \times \frac{1}{\sin\beta}} = e^{0.3 \times 2.79 \times \frac{1}{\sin 22.5}} = 8.91 \ldots\ldots a$$

$$Tc = mv^2 = 0.6 \times (15.7)^2 = 148.04\text{N}$$

$$T1 = T - Tc = 800 - 148.04 = 651.955\text{N} \ldots\ldots b$$

sub a in b :

$$\frac{T1}{T2} = 8.91 \rightarrow T2 = \frac{651.955}{8.91} = 73.056\text{N}$$

$$P = (T1 - T2) \times v \rightarrow P = (651.955 - 73.056) \times 15.7 = 9088.714\text{W} = 9.09\text{kW}$$

$$n = \frac{\text{Total power}}{\text{Power per belt}} = \frac{75}{9.09} = 8.25 = 9 \text{ belts}$$

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