AHSANULLAH UNIVERSITY OF SCIENCE AND TECHNOLOGY (AUST)

ME-3105: FLUID MECHANICS-II
(LEC-1: DIMENSIONAL ANALYSIS)

BY

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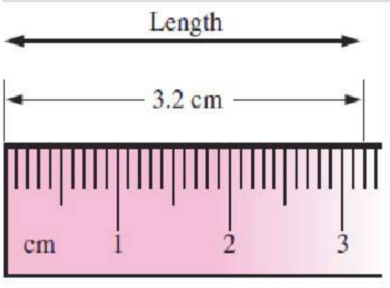
OBJECTIVE OF DIMENSIONAL ANALYSIS

- ✓ Develop a better understanding of dimensions, units, and dimensional homogeneity of equations.
- ✓ Understand the numerous benefits of dimensional analysis.
- ✓ To know how to use the method of repeating variables to identify non-dimensional parameters.
- ✓ Understand the concept of dynamic similarity and how to apply it to experimental modelling.

What is Dimension and Unit?

Dimension:
A measure of a physical quantity (without numerical values).
Unit:
A way to assign a *number to the dimension.*



A *dimension* is a measure of a physical quantity without numerical values, while a *unit* is a way to assign a number to the dimension. For example, length is a dimension, but centimeter is a unit.

Fundamental and Derived Dimensions

Fundamental Dimension:

- ✓ The seven primary dimensions: mass, length, time, temperature, electric current, amount of light, and amount of matter are called fundamental or basic dimension.
- ✓ There is no direct relationship between these dimensions, so they are called fundamental dimensions or fundamental quantities.
- ✓ Two systems for fundamental dimensions: MLT (Mass, Length and Time) and FLT (Force, Length and Time).

Derived Dimensions: All non-primary dimensions can be formed by combination of the seven primary dimensions; they are called derived dimensions, derived quantities or secondary quantities. Such as velocity, force and volume etc.

Fundamental and Derived Dimensions

Primary dimensions and their associated primary SI and English units

Dimension	Symbol*	SI Unit	English Unit
Mass	m	kg (kilogram)	lbm (pound-mass)
Length	L	m (meter)	ft (foot)
Time†	t	s (second)	s (second)
Temperature	T	K (kelvin)	R (rankine)
Electric current	I	A (ampere)	A (ampere)
Amount of light	C	cd (candela)	cd (candela)
Amount of matter	N	mol (mole)	mol (mole)

Dimensions of force: $\{\text{Force}\} = \left\{\text{Mass}\dfrac{\text{Length}}{\text{Time}^2}\right\} = \{\text{mL/t}^2\}$

Dimensions of surface tension: $\{\sigma_s\} = \left\{\dfrac{\text{Force}}{\text{Length}}\right\} = \left\{\dfrac{\text{m}\cdot\text{L/t}^2}{\text{L}}\right\} = \{\text{m/t}^2\}$

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LIST OF FUNDAMENTAL AND DERIVED DIMENSIONS

Quantity	Dimensions in terms of	
	MLT system	FLT system
Length (l)	L	L
Area (A)	L^2	L^2
Volume (V)	L^3	L^3
Time (t)	T	T
Velocity (v)	LT^{-1}	LT^{-1}
Acceleration (a)	LT^{-2}	LT^{-2}
Gravitational acceleration (g)	LT^{-2}	LT^{-2}
Frequency (N)	T^{-1}	T^{-1}
Discharge (Q)	L^3T^{-1}	L^3T^{-1}
Force or weight (F, W)	MLT^{-2}	F

LIST OF FUNDAMENTAL AND DERIVED DIMENSIONS

Power (P)	ML^2T^{-3}	FLT^{-1}
Work or Energy (E)	ML^2T^{-2}	FL
Pressure (p)	$ML^{-1}T^{-2}$	FL^{-2}
Mass (m)	M	FT^2L^{-1}
Mass density (ρ)	ML^{-3}	FT^2L^{-4}
Specific weight (w)	$ML^{-2}T^{-2}$	FL^{-3}
Dynamic viscosity (μ)	$ML^{-1}T^{-1}$	FTL^{-2}
Kinematic viscosity (ν)	L^2T^{-1}	L^2T^{-1}
Surface tension (σ)	MT^{-2}	FL^{-1}
Shear stress (τ)	$ML^{-1}T^{-2}$	FL^{-2}
Bulk modulus (K)	$ML^{-1}T^{-2}$	FL^{-2}

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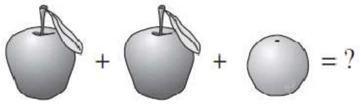
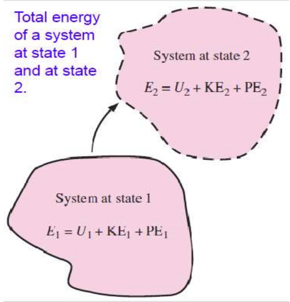
DIMENSIONAL HOMOGENEITY

- ✓ The law of dimensional homogeneity: Every additive term in an equation must have the same dimensions.
- ✓ An equation is called dimensionally homogeneous, if the fundamental dimensions have identical powers of M-L-T or F-L-T on both sides of the equation.

You can't add apples and oranges!





$$\Delta U = m(u_2 - u_1) \quad \Delta \mathrm{KE} = \frac{1}{2}m(V_2^2 - V_1^2) \quad \Delta \mathrm{PE} = mg(z_2 - z_1)$$

EXAMPLE OF DIMENSIONAL HOMOGENEITY

$$\{\Delta E\} = \{\text{Energy}\} = \{\text{Force} \cdot \text{Length}\} \rightarrow \{\Delta E\} = \{\text{mL}^2/\text{t}^2\}$$

$$\{\Delta U\} = \left\{\text{Mass}\,\frac{\text{Energy}}{\text{Mass}}\right\} = \{\text{Energy}\} \rightarrow \{\Delta U\} = \{\text{mL}^2/\text{t}^2\}$$

$$\{\Delta \text{KE}\} = \left\{\text{Mass}\,\frac{\text{Length}^2}{\text{Time}^2}\right\} \rightarrow \{\Delta \text{KE}\} = \{\text{mL}^2/\text{t}^2\}$$

$$\{\Delta \text{PE}\} = \left\{\text{Mass}\,\frac{\text{Length}}{\text{Time}^2}\,\text{Length}\right\} \rightarrow \{\Delta \text{PE}\} = \{\text{mL}^2/\text{t}^2\}$$

- ✓ An equation that is not dimensionally homogeneous is a sign of an error in the equation.
- ✓ Always check the homogeneity in the equation.

EXAMPLE OF DIMENSIONAL HOMOGENEITY

Problem-1:

Bernoulli Equation, $P + \frac{1}{2}\rho V^2 + \rho g z = C$.

(a) Verify that each additive term in the Bernoulli equation has the same dimensions.
(b) What is the dimension of constant 'C'.

EXAMPLE OF DIMENSIONAL HOMOGENEITY

Solution of Problem-1:

Analysis (*a*) Each term is written in terms of primary dimensions,

$$\{P\} = \{\text{Pressure}\} = \left\{\frac{\text{Force}}{\text{Area}}\right\} = \left\{\text{Mass}\,\frac{\text{Length}}{\text{Time}^2}\,\frac{1}{\text{Length}^2}\right\} = \left\{\frac{\text{m}}{\text{t}^2\text{L}}\right\}$$

$$\left\{\frac{1}{2}\rho V^2\right\} = \left\{\frac{\text{Mass}}{\text{Volume}}\left(\frac{\text{Length}}{\text{Time}}\right)^2\right\} = \left\{\frac{\text{Mass}\times\text{Length}^2}{\text{Length}^3\times\text{Time}^2}\right\} = \left\{\frac{\text{m}}{\text{t}^2\text{L}}\right\}$$

$$\{\rho g z\} = \left\{\frac{\text{Mass}}{\text{Volume}}\,\frac{\text{Length}}{\text{Time}^2}\,\text{Length}\right\} = \left\{\frac{\text{Mass}\times\text{Length}^2}{\text{Length}^3\times\text{Time}^2}\right\} = \left\{\frac{\text{m}}{\text{t}^2\text{L}}\right\}$$

Indeed, **all three additive terms have the same dimensions.**

(*b*) From the law of dimensional homogeneity, the constant must have the same dimensions as the other additive terms in the equation. Thus,

Primary dimensions of the Bernoulli constant: $\{C\} = \left\{\frac{\text{m}}{\text{t}^2\text{L}}\right\}$

NON-DIMENSIONALIZATION OF EQUATIONS

- ✓ Nondimensional equation: If we *divide each term in the equation by a collection of variables and constants whose product has those same dimensions,* the equation is rendered as nondimensional. Each terms in a nondimensional equation is dimensionless.
- ✓ Normalized equation: If nondimensional terms in the equation are of order unity, the equation is called normalized.
- ✓ Nondimensional parameters: In the process of nondimensionalizing an equation, the nondimensional parameters often appear, the most of which are named after a notable scientist or engineer. For example, Reynolds number and Froude number. This process is referred to by some authors as inspectional analysis.

NON-DIMENSIONALIZATION OF EQUATIONS

Example:

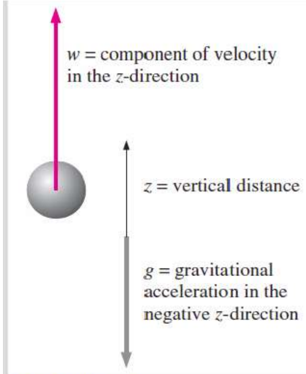
The nondimensionalized Bernoulli equation

$$\underbrace{\frac{P}{P_\infty}}_{\{1\}} + \underbrace{\frac{\rho V^2}{2P_\infty}}_{\{1\}} + \underbrace{\frac{\rho g z}{P_\infty}}_{\{1\}} = \underbrace{\frac{C}{P_\infty}}_{\{1\}}$$

A *nondimensionalized* form of the Bernoulli equation is formed by dividing each additive term by a pressure (here we use P_∞). Each resulting term is *dimensionless* (dimensions of {1}).

VARIABLES, PARAMETER, AND CONSTANT

Dimensional result: $z = z_0 + w_0 t - \frac{1}{2} g t^2$ *Equation of motion:* $\frac{d^2 z}{dt^2} = -g$

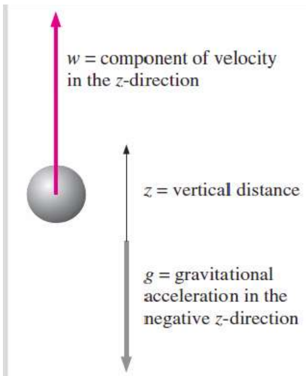


Object falling in a vacuum. Vertical velocity is drawn positively, so $w < 0$ for a falling object.

- ✓ Dimensional variables: Dimensional quantities that change or vary in the problem. Examples: Z (dimension of length) and t (dimension of time).
- ✓ Nondimensional (or dimensionless) variables: Quantities that change or vary in the problem, but have no dimensions. Example: Angle of rotation (measured in degrees or radians, a dimensionless units)
- ✓ Dimensional constant: Gravitational constant g, while dimensional remains constant.

VARIABLES, PARAMETER, AND CONSTANT

Dimensional result: $z = z_0 + w_0 t - \frac{1}{2} g t^2$ *Equation of motion:* $\frac{d^2 z}{dt^2} = -g$



Object falling in a vacuum. Vertical velocity is drawn positively, so $w < 0$ for a falling object.

- ✓ Parameters: Refer to the combined set of dimensional variables, nondimensional variables, and dimensional constant in the problem.
- ✓ Pure constants: The constant ½ and exponent 2 in equation. Other common examples are fixed value constant such as π and e.
- ✓ Dimensional constant: Gravitational constant g, while dimensional remains constant.

DIMENSIONAL ANALYSIS

- ✓ Dimensional analysis is a mathematical technique, which deals with the dimensions of the physical quantities involved in the phenomenon.
- ✓ It helps to determine how variables are related and the phenomenon can be expressed by a dimensionally homogeneous equation with certain variables.
- ✓ *(Dimensional analysis is a method for reducing the number and complexity of experimental variables that affect a given physical phenomena.)*
- ✓ Dimensional analysis alone does not give the exact form of an equation, but it can lead to a significant reduction of the number of variables.

DIMENSIONAL ANALYSIS

- ✓ It is based on two assumptions:
  - Physical quantities have dimensions (fundamental are mass M, length L, and time T). Any physical quantity has a dimension which is a product of powers of the basic dimensions M, L and T.
  - Physical laws are unaltered when changing the units measuring the dimensions.

WHY DIMENSIONAL ANALYSIS

Advantage of Dimensional analysis:

- ✓ Reduce the number of variables by forming non-dimensional parameter.
- ✓ Non-dimensional equations provides the insight on controlling parameters and the nature of the problem.
- ✓ By Scaling, it allows to test the model instead of expensive full-scale prototypes.
- ✓ Bring simplicity of complex analytic phenomenon.

Disadvantage:

- ✓ Error in the model analysis leads to predict inaccurate and poor performance of the prototype.

Example: Find the pressure drop per unit length that develops along the pipe as a result of friction of an incompressible Newtonian fluid, is flowing steadily through a long, smooth- walled, horizontal and circular pipe. Since designing a pipeline, the first step in the planning of an experiment is to study this problem and the factors, or variables, that will have an effect on the pressure drop.

WHY DIMENSIONAL ANALYSIS

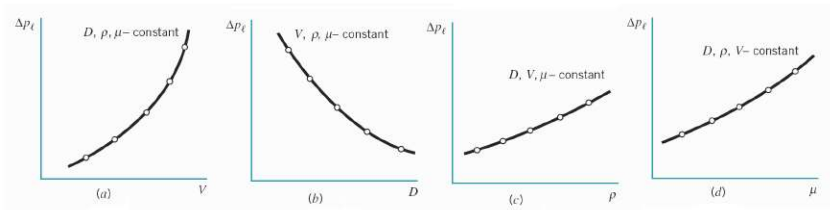
Solution:

Pressure drop per unit length depends on 4 variables: Pipe Diameter (D); Fluid Velocity (V); fluid density (ρ); fluid viscosity (μ).

Pressure drop per unit length $\Delta p_{\ell} = f(D, \rho, \mu, V)$

- ✓ To perform the experiments in a meaningful and systematic manner, it would be necessary to change one of the variable, such as the velocity, and holding all other constant, then measure the corresponding pressure drop.
- ✓ It is difficult to determine the functional relationship between the pressure drop and the various facts that influence it.

WHY DIMENSIONAL ANALYSIS



- ✓ Fortunately, there is a much simpler approach to the problem that will eliminate the difficulties described above.
- ✓ Collecting these variables into two non-dimensional combinations of the variables (called dimensionless product or dimensionless groups).

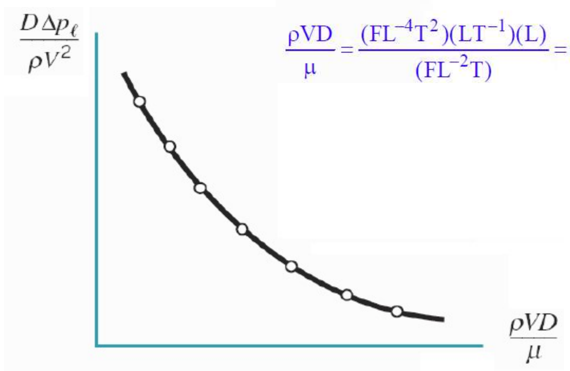
$$\frac{D\Delta p_\ell}{\rho V^2} = \phi\left(\frac{\rho V D}{\mu}\right)$$

- ✓ Only one dependent and one independent variable.
- ✓ Easy to set up experiments to determine dependency.
- ✓ Easy to present results (one graph).

WHY DIMENSIONAL ANALYSIS

$$\frac{D\Delta p_\ell}{\rho V^2} = \frac{L(F/L^3)}{(FL^{-4}T^2)(FT^{-1})^2} = F^0L^0T^0$$

$$\frac{\rho V D}{\mu} = \frac{(FL^{-4}T^2)(LT^{-1})(L)}{(FL^{-2}T)} = F^0L^0T^0$$



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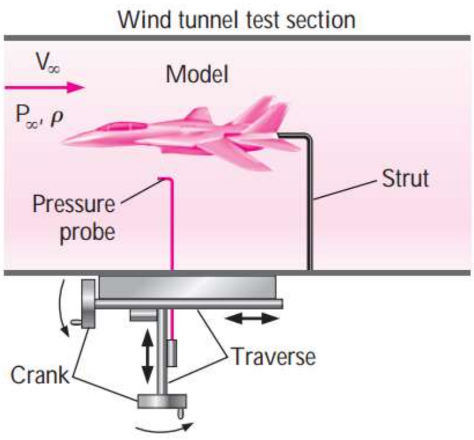
EXAMPLE OF DIMENSIONAL ANALYSIS IN REAL WORLD



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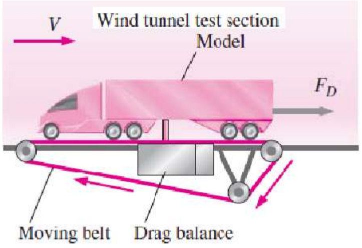

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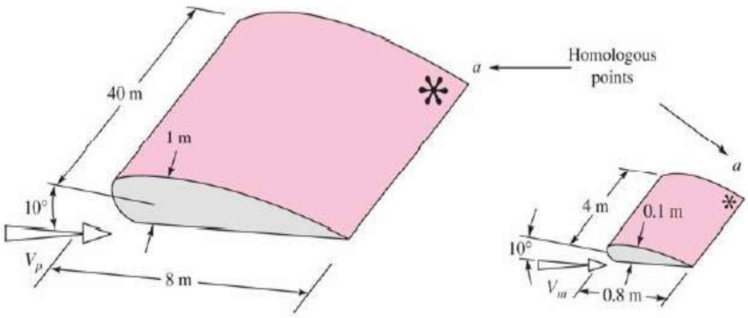

METHOD OF DIMENSIONAL ANALYSIS

Two methods are used in dimensional analysis:

- ✓ Rayleigh's Method and
- ✓ Buckingham's π- theorem

Rayleigh's Method:

The functional relationship of some variables is expressed in the form of an exponential equation, which must be dimensionally homogenous. If 'Y' is function of independent variables X_1, X_2, X_3 . . . etc. and then functional relationship may be written as

$$Y = f(X_1, X_2, X_3 \ldots\ldots)$$

Steps:

- ✓ Write the functional relationship with all given data.
- ✓ Write equation in terms of constant with exponents (i.e. Power) a, b, c.....

$$Y = K \cdot X_1^{a} \cdot X_2^{b} \cdot X_3^{c} \cdot \ldots\ldots$$

- ✓ Find the value of a, b, c... with the help of dimensional homogeneity.
- ✓ Substitute the values of the exponents in the main equation and simplify it.

PROBLEM OF DIMENSIONAL ANALYSIS

Unit of Force, Velocity, Energy, etc.:

Mass = M   Time = T   Length = L

$$\text{Velocity (V)} = \frac{m}{s} = L \cdot T^{-1} \qquad \text{Acceleration (a)} = \frac{V}{t} = \frac{m}{s^2} = L \cdot T^{-2}$$

$$\text{Force (F)} = Kg \cdot \frac{m}{s^2} = N = M \cdot L \cdot T^{-2} \quad \text{(N = Newton, unit of force)}$$

$$\text{Work (W)} = \text{Energy (E)} = N \cdot m = \frac{Kg \cdot m}{s^2} \cdot m = M \cdot L^2 \cdot T^{-2}$$

$$(\text{Work (W)} = \text{Energy} \cdot \text{(E)} = N \cdot m = \text{Joule} = J)$$

$$\text{Power (P)} = \frac{J}{s} = \frac{N \cdot m}{s} = M \cdot L^2 \cdot T^{-3} \quad \text{(Power = Watt= Joule per sec)}$$

$$\text{Pressure (p)} = Pa = \frac{N}{m^2} = M \cdot L \cdot T^{-2} \cdot L^{-2} = M \cdot L^{-1} \cdot T^{-2}$$

(Unit of pressure is Pa = Pascal)

PROBLEM OF DIMENSIONAL ANALYSIS

Unit of Force, Velocity, Energy, etc.:

Dynamic Viscosity (μ) unit = $\frac{N\cdot s}{m^2} = MLT^{-2}\cdot T\cdot L^{-2} = M\cdot L^{-1}\cdot T^{-1}$

(Dynamic Viscostiy or Viscosity of fluid = μ)

Kinematic Viscosity (ν) = $\frac{m^2}{s} = L^2\cdot T^{-1}$

Specific Weight (γ) unit = $\rho\ g = \frac{N}{m^3} = MLT^{-2}\cdot L^{-3} = M\cdot L^{-2}\cdot T^{-2}$

Surface Tension, $\sigma = \frac{\text{Force(F)}}{\text{unit Length}} = \frac{N}{m} = MLT^{-2}\cdot L^{-1} = MT^{-2}$

Shear Stress, Bulk Modulus and Pressure has the same unit (N/m^2)

Force, Resistance, Thrust has the same unit (Newton =N)

Freequency, $N = T^{-1}$; Discharge, $Q = L^3\cdot T^{-1}$

PROBLEM OF DIMENSIONAL ANALYSIS

PROBLEMS-1 (Rayleigh's Method)

Show that the resistance 'R' to the motion of a sphere of diameter 'D' is moving with a uniform velocity 'V' through a real fluid having mass density 'ρ' and viscosity 'μ' is given by

$$R = \rho\ D^2\ V^2\ f\left(\frac{\mu}{\rho\ V\ D}\right)$$

(See Hand Analysis)

PROBLEM OF DIMENSIONAL ANALYSIS

PROBLEMS-2 (Rayleigh's Method)

The thrust (P) of a propeller depends upon the diameter (D), speed (V), mass density (ρ), revolution per minute (N) and coefficient of viscosity (μ), show that

$$P = \rho D^2 V^2 \cdot f\left(\frac{\mu}{\rho D V}, \frac{D N}{V}\right)$$

(See Hand Analysis)

PROBLEM OF DIMENSIONAL ANALYSIS

PROBLEMS-3 (Rayleigh's Method)

Show by the dimensional analysis, that the power P developed by a hydraulic turbine is given by

$$P = \rho N^3 D^5 \cdot f\left(\frac{N^2 D^2}{g H}\right)$$

where, ρ is the mass density of liquid, N is the rotational speed of the turbine in rpm, D is the diameter of runner, H is the working head and g is the gravitational acceleration.

(See Hand Analysis)

DIMENSIONAL ANALYSIS

Buckingham's π-Theorem:

- ✓ If an equation involving k variables is dimensionally homogeneous, it can be reduced to a relationship among k-r independent dimensionless products, where r is the minimum number of reference or primary dimensions required to describe the variables.
- ✓ Given a physical problem in which the dependent variable is a function of k-1 numbers of independent variables.

  $u_1 = f(u_2, u_3, \ldots\ldots, u_k)$
- ✓ Mathematically, we can express the functional relationship in the equivalent form $g(u_1, u_2, u_3, \ldots\ldots, u_k) = 0$; where g is an unspecified function different from 'f'.
- ✓ The Buckingham π (Pi) theorem states that: Given a relation among k variables of the form $g(u_1, u_2, u_3, \ldots\ldots, u_k) = 0$
- ✓ The k variables may be grouped into k-r independent dimensionless products, or Π terms, expressible in functional form by

$$\Pi_1 = \phi(\Pi_2.\Pi_3....\Pi_{k-r}) \quad \text{or} \quad \phi(\Pi_1.\Pi_2.\Pi_3....\Pi_{k-r}) = 0$$

DIMENSIONAL ANALYSIS

Buckingham's π-Theorem:

- ✓ Number 'r' is (but not always) equal to the minimum number of independent dimensions required to specify dimensions of all parameters. Usually, the reference dimensions required to describe the variables will be the basic dimensions M, L, and T or F, L, and T.
- ✓ The theorem does not predict the functional form of ϕ . The functional relation among the independent dimensionless products Π must be determined experimentally.
- ✓ The k-r dimensionless products Π terms obtained from the procedure are independent.
- ✓ A Π term is not independent if it can be obtained from a product or quotient of the other dimensionless products (or Π term) of the problem. For example, if

$$\Pi_5 = \frac{2\Pi_1}{\Pi_2\Pi_3} \quad \text{or} \quad \Pi_6 = \frac{\Pi_1^{3/4}}{\Pi_3^{2}}$$

then neither Π_5 nor Π_6 is independent of the other dimensionless products or dimensionless groups.

DIMENSIONAL ANALYSIS

Steps of Buckingham's π-Theorem:

- ✓ List all the variables: Assume 'k' is the number of variables in the problem or phenomenon.
- ✓ Express each of variables in terms of basic dimensions. Find the minimum number of reference dimensions or primary dimension. Assume 'r' is the minimum number of primary dimensions in MLT or FLT system.
- ✓ Determine the required number of 'π' (PI) term. The number of 'π' (PI) term will be 'k-r' .
- ✓ Select a number of repeating variables, where the number required is equal to the number of reference dimensions. Then select a set of r dimensional variables that includes all the primary dimensions (repeating variables). These repeating variables will all be combined with each (or one) of the remaining parameters.
- ✓ No repeating variables should have dimensions that are power of the dimensions of another repeating variable. Normally, one from geometry, one from fluid property and one from flow characteristics.

DIMENSIONAL ANALYSIS

Steps of Buckingham's π-Theorem:

- ✓ Form a PI (π) term by multiplying one of the non-repeating variables by the product of the repeating variables, each raised to an exponent that will make the combination dimensionless. There will be k –r equations for $\boldsymbol{\pi}$ (PI) terms**.**
- ✓ Check all the resulting π (PI) terms to make sure they are dimensionless.
- ✓ Express the final form as a relationship among the π (PI) terms, and think about what is means.
- ✓ Express the result of the dimensional analysis.

$$\Pi_1 = \phi(\Pi_2, \Pi_3,,,, \Pi_{k-r})$$

DIMENSIONAL ANALYSIS

Example of Buckingham's π-Theorem:

Pressure drop per unit length (ΔP_l)depends on four variables: Pipe diameter (D); flow velocity (V); fluid density (ρ) ; fluid viscosity (μ). Show the following expression by Buckingham Π (PI) theorem using both FLT and MLT system.

$$\frac{D\Delta p_\ell}{\rho V^2} = \phi\left(\frac{\rho V D}{\mu}\right)$$

Solution Steps:

- ✓ First step to find all variables, $(\Delta p_\ell, D, \mu, \rho, \text{ and } V)$
- ✓ Find number of variables, K = 5 and
- ✓ Find the minimum no. of primary dimensions for each variables either MLT or FLT system. (Let assume in FLT system)

Δp_ℓ = Pressure drop per unit length of pipe

DIMENSIONAL ANALYSIS

Example of Buckingham's π-Theorem:

Solution Steps:

1) First step to find all variables,
2) Find number of variables, k = 5 and
3) Find the minimum no. of primary dimensions for each variables either MLT or FLT system. (Let assume in FLT system)

   FLT:

   $\Delta p_\ell = FL^{-3}$ $\quad D = L$ $\quad \rho = FL^{-4}T^2$

   $\mu = FL^{-2}T$ $\quad V = LT^{-1}$

4) Find minimum no. of primary dimensions is required to express all terms. Here minimum of primary dimensions required is 3 i.e. r = 3.
5) Number of Π (PI) terms will be k- r = 5-3 = 2. So, we need two Π terms.
6) Now find no. of repeated variables, which is as same number of r i.e. 3

DIMENSIONAL ANALYSIS

Example of Buckingham's π-Theorem:

Solution Steps:

7) Select repeated variables (at least one from geometry, one from fluid property and one from flow characteristics). Assume, repeated variable will be ρ (density), V (velocity) and D (pipe diameter).
8) Now make each Π terms with three repeated variables and one non repeated variable. Need to put exponent (power on each repeated variable)

$$\Pi_1 = \Delta p_\ell D^a V^b \rho^c \qquad \Pi_2 = \mu D^a V^b \rho^c$$

9) Substitute FLT for each variable in each Π equation and find value of a, b , c.

DIMENSIONAL ANALYSIS

Example of Buckingham's π-Theorem:

Solution Steps:

9) Substitute FLT for each variable in each Π equation and find value of a, b , and c.

$$\Pi_1 = \Delta p_\ell D^a V^b \rho^c \qquad \Pi_2 = \mu D^a V^b \rho^c$$

$$\Pi_1 = \Delta p_\ell D^a V^b \rho^c$$

$$(FL^{-3})(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

$$F: 1 + c = 0$$

$$L: -3 + a + b - 4c = 0$$

$$T: -b + 2c = 0$$

$$a = 1, b = -2, c = -1$$

$$\Pi_1 = \frac{\Delta p_\ell D}{\rho V^2}$$

DIMENSIONAL ANALYSIS

Example of Buckingham's π-Theorem:

Solution Steps:

9) Substitute FLT for each variable in each Π equation and find value of a, b , and c.

$$\Pi_2 = \mu D^a V^b \rho^c$$

$$(FL^{-2}T)(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

$$F: 1 + c = 0$$

$$L: -2 + a + b - 4c = 0$$

$$T: 1 - b + 2c = 0$$

$$a = -1, b = -1, c = -1$$

Substitute value of a, b, c in second equation:

$$\Pi_2 = \frac{\mu}{DV\rho}$$

10) Check each Π term to make sure that each Π term should dimensionless.

11) Finally find expression by using formula, $\Pi_1 = \phi(\Pi_2, \Pi_3, ,, \Pi_{k-r})$

$$\Pi_1 = \phi(\Pi_2) \quad \Big| \quad \frac{\Delta p_\ell D}{\rho V^2} = \phi\left(\frac{\mu}{DV\rho}\right)$$

DIMENSIONAL ANALYSIS

Problem-2 (Buckingham's π-Theorem):

A thin rectangular plate having a width 'w' and a height 'h' is located so that it is normal to a moving stream of fluid. Assume that the drag 'D' of the fluid exerts on the plate is function of 'w' and 'h', the fluid viscosity μ, and density ρ, and the velocity V respectively. Determine a suitable set of pi terms and show that,

$$\frac{D}{w^2V^2\rho} = \phi\left(\frac{h}{w}, \frac{\mu}{wV\rho}\right)$$

(See Hand Analysis)

DIMENSIONAL ANALYSIS

Problem-3:
Prove that the discharge over a spillway is given by the relation:

$$Q = VD^2 f\left[\frac{\sqrt{gD}}{V}, \frac{H}{D}\right]$$

Where V = Velocity of flow, D = Depth at the throat, H = Head of water and g = Acceleration due to gravity.

(See Hand Analysis)

DIMENSIONAL ANALYSIS

Problem-4:
An open cylindrical tank having a diameter D, is supported around its bottom circumference and is filled with liquid up to a depth h, liquid having a specific weight γ. The vertical deflection δ of the cylinder at the bottom center is function of D, h, d, γ, and E, where d is the thickness of cylinder at bottom and E is the modulus of elasticity of cylinder material.

Find (a). all Π terms by Buckingham Π (PI) theorem in both MLT and FLT system; and (b). Find suitable expression of deflection δ.

(See Hand Analysis)