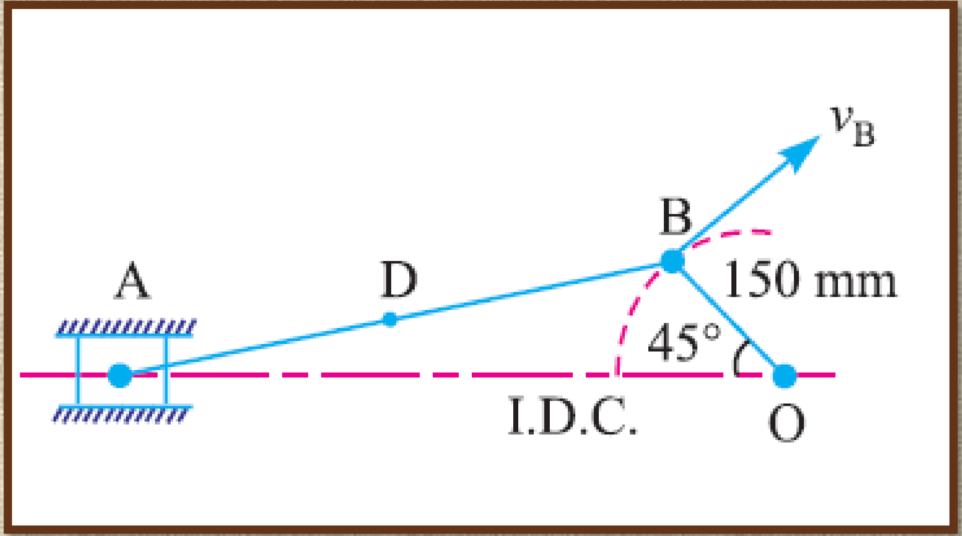
MATH

The crank of a slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long.

Determine :

1. linear velocity and acceleration of the midpoint of the connecting rod, and
2. angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centre position.



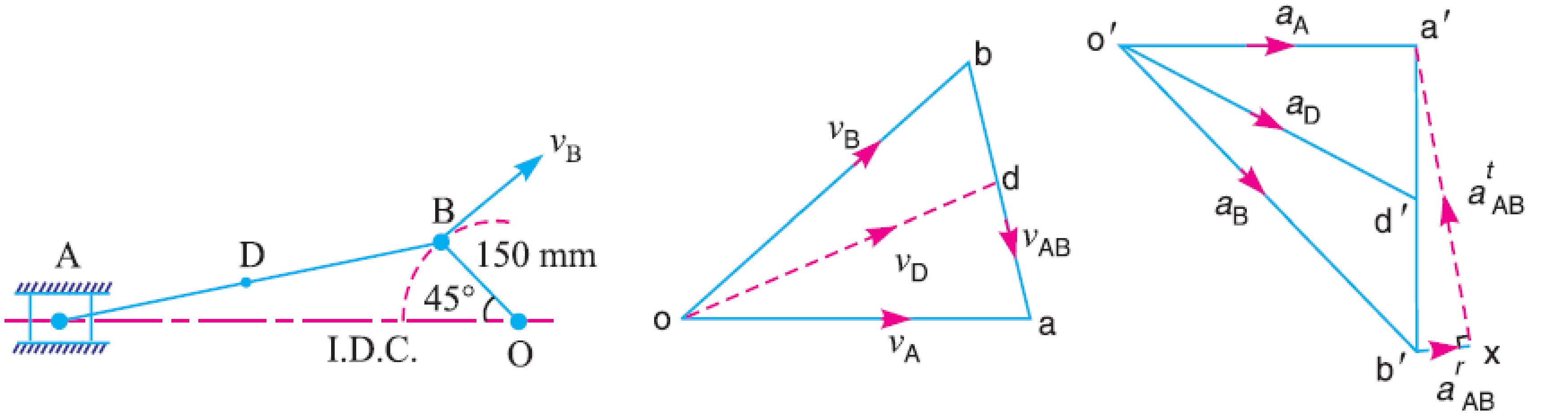
v_D = 4.1 m/s
a_D = 117 m/s^2
ω_{AB} = 5.67 rad/s (Anticlockwise about B)
α_{AB} = 171.67 rad/s^2 (Clockwise about B)

SOLUTION

Given : N_{BO} = 300 r.p.m. or $\omega_{BO} = 2\pi \times 300/60 = 31.42$ rad/s; OB = 150 mm = 0.15 m ; BA = 600 mm = 0.6 m

We know that linear velocity of B with respect to O or velocity of B,

$$v_{BO} = v_B = \omega_{BO} \times OB = 31.42 \times 0.15 = 4.713 \text{ m/s} \quad \text{...(Perpendicular to } BO\text{)}$$



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

SOLUTION

1. *Linear velocity of the midpoint of the connecting rod*

First of all draw the space diagram, to some suitable scale; as shown in Fig.(*a*). Now the velocity diagram, as shown in Fig. (*b*), is drawn as discussed below:

1. Draw vector *ob* perpendicular to *BO*, to some suitable scale, to represent the velocity of *B* with respect to *O* or simply velocity of *B i.e.* v_{BO} or v_B, such that vector $ob = v_{BO} = v_B = 4.713$ m/s

2. From point *b*, draw vector *ba* perpendicular to *BA* to represent the velocity of *A* with respect to *B i.e.* v_{AB} , and from point *o* draw vector *oa* parallel to the motion of *A* (which is along *AO*) to represent the velocity of *A i.e.* v_A. The vectors *ba* and *oa* intersect at *a*.

By measurement, we find that velocity of *A* with respect to *B*,

$$v_{AB} = \text{vector } ba = 3.4 \text{ m/s}$$

and

$$\text{Velocity of } A,\ v_A = \text{vector } oa = 4 \text{ m/s}$$

3. In order to find the velocity of the midpoint *D* of the connecting rod *AB*, divide the vector *ba* at *d* in the same ratio as *D* divides *AB*, in the space diagram. In other words, $bd / ba = BD/BA$

Note: Since *D* is the midpoint of *AB*, therefore *d* is also midpoint of vector *ba*.

4. Join *od*. Now the vector *od* represents the velocity of the midpoint *D* of the connecting rod *i.e.* v_D.

By measurement, we find that

v_D = vector $od = 4.1$ m/s **Ans.**

SOLUTION

Acceleration of the midpoint of the connecting rod

We know that the radial component of the acceleration of B with respect to O or the acceleration of B,

$$a^r_{BO} = a_B = \frac{v^2_{BO}}{OB} = \frac{(4.713)^2}{0.15} = 148.1 \text{ m/s}^2$$

and the radial component of the acceleraiton of A with respect to B,

$$a^r_{AB} = \frac{v^2_{AB}}{BA} = \frac{(3.4)^2}{0.6} = 19.3 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. 8.4 (c) is drawn as discussed below:

1. Draw vector $o'b'$ parallel to BO, to some suitable scale, to represent the radial component of the acceleration of B with respect to O or simply acceleration of B *i.e.* a^r_{BO} or a_B, such that

$$\text{vector } o'b' = a^r_{BO} = a_B = 148.1 \text{ m/s}^2$$

Note: Since the crank OB rotates at a constant speed, therefore there will be no tangential component of the acceleration of B with respect to O.

2. The acceleration of A with respect to B has the following two components:

(a) The radial component of the acceleration of A with respect to B *i.e.* a^r_{AB}, and

(b) The tangential component of the acceleration of A with respect to B *i.e.* a^t_{AB}. These two components are mutually perpendicular.

Therefore from point b', draw vector $b'x$ parallel to AB to represent $a^r_{AB} = 19.3 \text{ m/s}^2$ and from point x draw vector xa' perpendicular to vector $b'x$ whose magnitude is yet unknown.

SOLUTION

3. Now from o', draw vector $o'a'$ parallel to the path of motion of A (which is along AO) to represent the acceleration of A *i.e.* a_A. The vectors xa' and $o'a'$ intersect at a'. Join $a'b'$.

4. In order to find the acceleration of the midpoint D of the connecting rod AB, divide the vector $a'b'$ at d' in the same ratio as D divides AB. In other words

$$b'd' / b'a' = BD / BA$$

Note: Since D is the midpoint of AB, therefore d' is also midpoint of vector $b'a'$.

5. Join $o'd'$. The vector $o'd'$ represents the acceleration of midpoint D of the connecting rod *i.e.* a_D.

By measurement, we find that

$$a_D = \text{vector } o'd' = 117 \text{ m/s}^2 \quad \textbf{Ans.}$$

2. Angular velocity of the connecting rod

We know that angular velocity of the connecting rod AB,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{3.4}{0.6} = 5.67 \text{ rad/s}^2 \text{ (Anticlockwise about } B) \quad \textbf{Ans.}$$

Angular acceleration of the connecting rod

From the acceleration diagram, we find that

$$a^t_{AB} = 103 \text{ m/s}^2 \qquad \text{...(By measurement)}$$

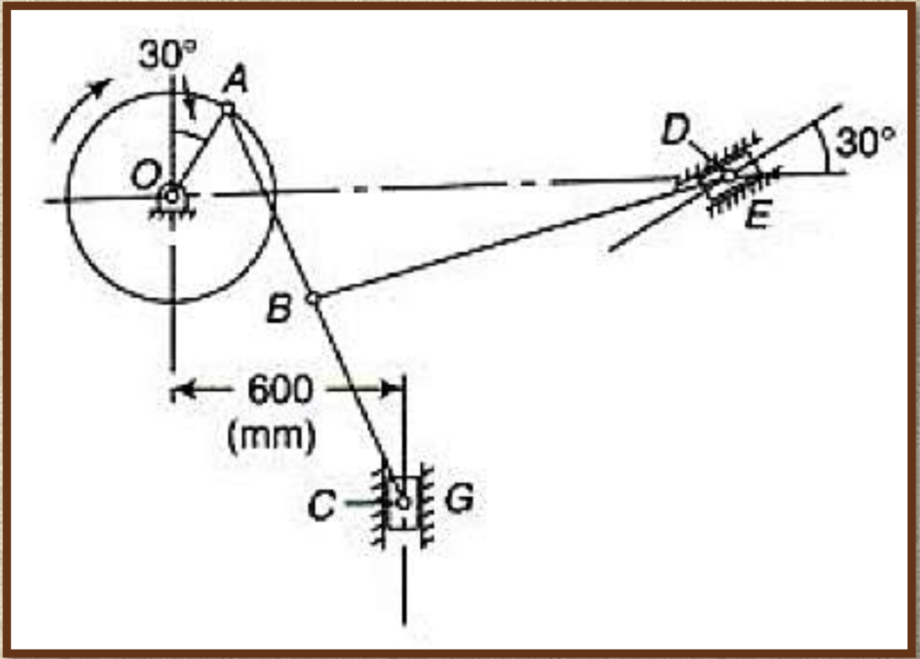
We know that angular acceleration of the connecting rod AB,

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{103}{0.6} = 171.67 \text{ rad/s}^2 \text{ (Clockwise about } B) \quad \textbf{Ans.}$$

MATH

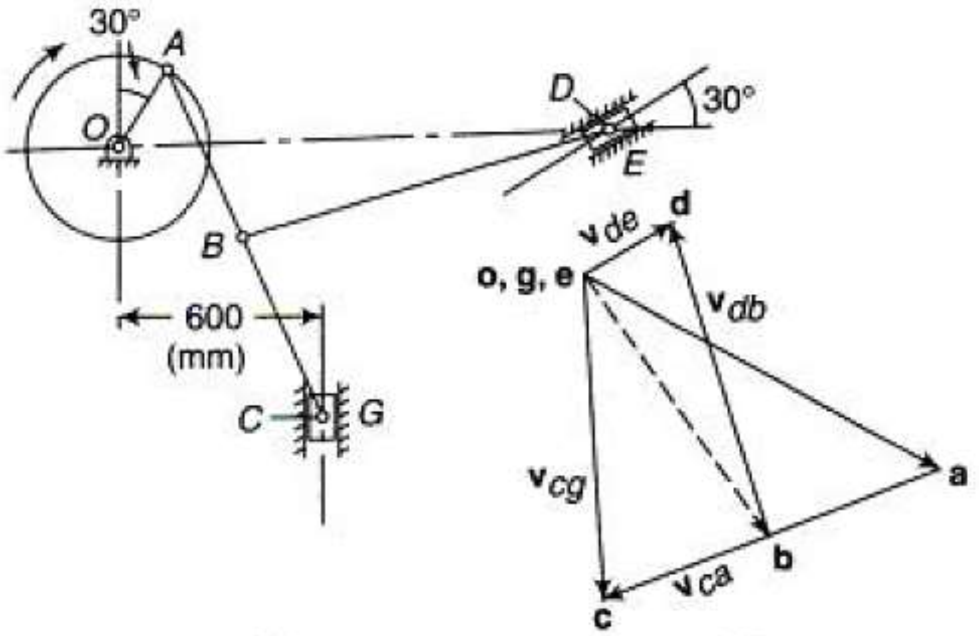
Figure shows a mechanism in which OA = 300 mm, AB = 600 mm, AC = BD = 1.2 m. OD is horizontal for the given configuration. If OA rotates at 200 rpm in the clockwise direction, find

1. Linear velocities of C and D
2. Angular velocities of links AC and BD



v_c = 5.2 m/s
v_d = 1.55 m/s
ω_{ac} = 4.75 rad/s (Clockwise)
ω_{bd} = 4.31 rad/s (Clockwise)

SOLUTION



(a) (b)

Solution: $\omega_a = \dfrac{2\pi \times 200}{60} = 20.94$ rad/s

$$v_a = \omega_a OA = 20.94 \times 0.3 = 6.28 \text{ m/s}$$

Writing the vector equation for the mechanism *OAC*,

$$\mathbf{v}_{co} = \mathbf{v}_{ca} + \mathbf{v}_{ao}$$

or

$$\mathbf{v}_{cg} = \mathbf{v}_{ao} + \mathbf{v}_{ca}$$

or

$$\mathbf{gc} = \mathbf{oa} + \mathbf{ac}$$

SOLUTION

Take the vector $\mathbf{v}_{ao}$ to a convenient scale [Fig. 2.13(b)].

$\mathbf{v}_{ca}$ is $\perp AC$, draw a line $\perp AC$ through **a**;

$\mathbf{v}_{cg}$ is vertical, draw a vertical line through **g** (or **o**).

The intersection of the two lines locates the point **c**. Locate the point **b** on **ac** as usual. Join **ob** which gives $\mathbf{v}_{bo}$. Writing the vector equation for the mechanism *OABD*,

$$\mathbf{v}_{do} = \mathbf{v}_{db} + \mathbf{v}_{bo}$$

or

$$\mathbf{v}_{de} = \mathbf{v}_{bo} + \mathbf{v}_{db}$$

or

$$\mathbf{ed} = \mathbf{ob} + \mathbf{bd}$$

$\mathbf{v}_{db}$ is $\perp BD$, draw a line $\perp BD$ through **b;**

For $\mathbf{v}_{de}$, draw a line through **e**, parallel to the line of stroke of the piston in the guide *E*.

The intersection locates the point **d**.

$$v_c = \mathbf{oc} = \underline{5.2 \text{ m/s}}$$

$$v_d = \mathbf{od} = \underline{1.55 \text{ m/s}}$$

$$\omega_{ac} = \omega_{ca} = \frac{v_{ca}}{AC} = \frac{5.7}{1.20} = \underline{4.75 \text{ rad/s}} \text{ clockwise}$$

$$\omega_{bd} = \omega_{db} = \frac{v_{db}}{BD} = \frac{5.17}{1.20} = \underline{4.31 \text{ rad/s}} \text{ clockwise}$$