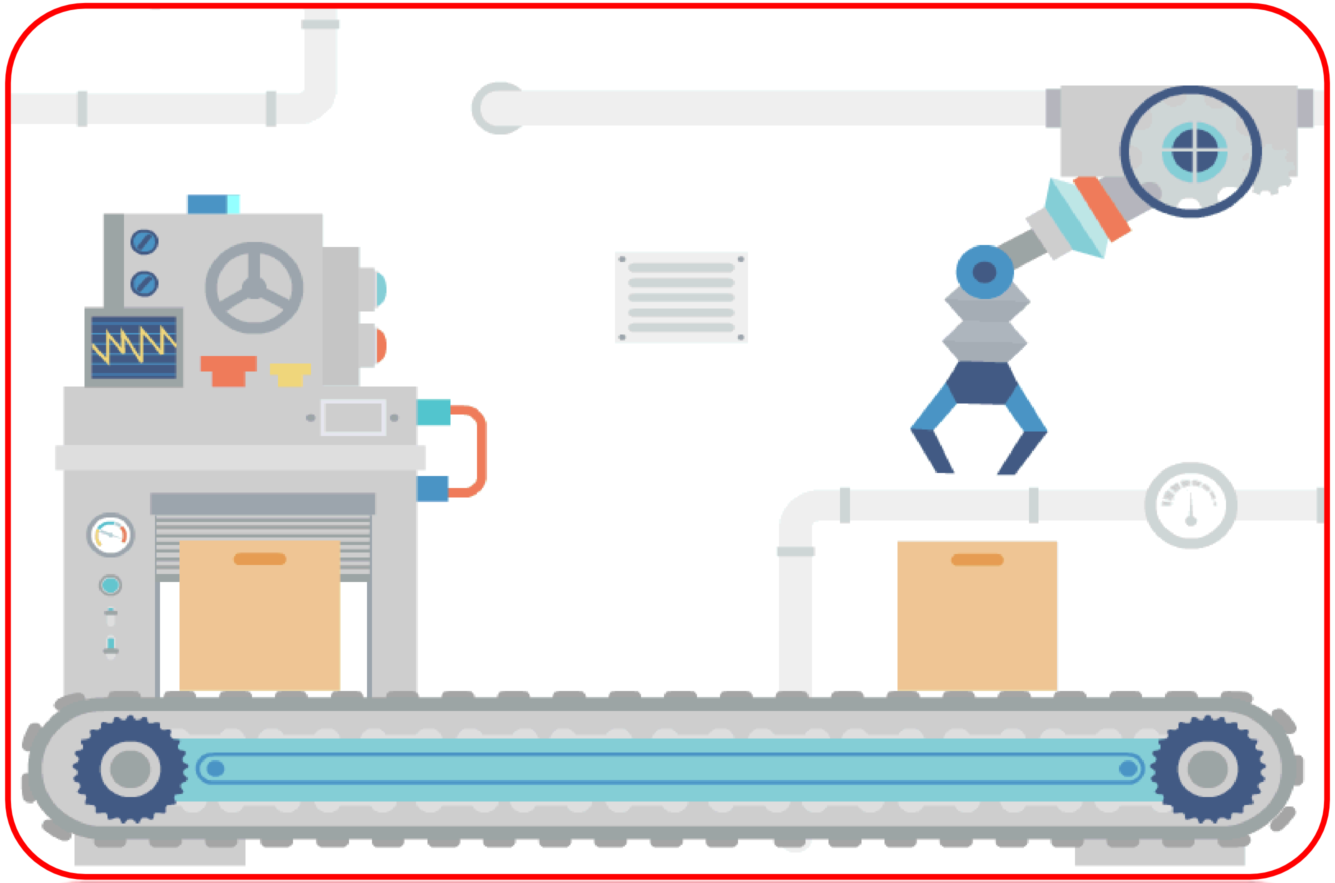
ME 3101: Mechanics of Machinery
Belt and Rope Drives



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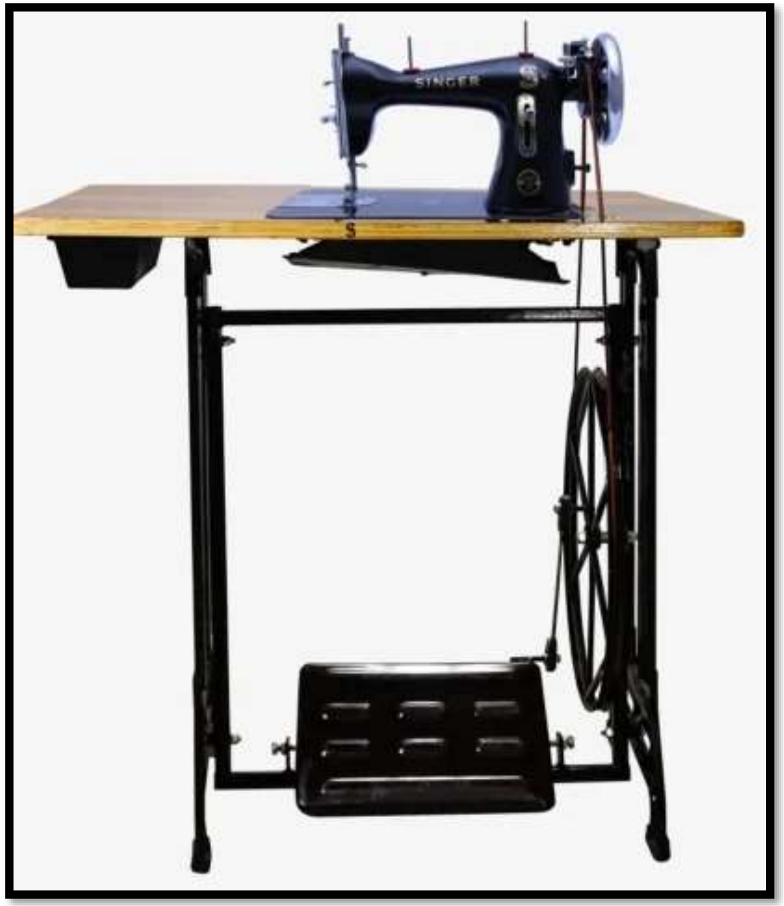
BELT DRIVE MECHANISM



BELT DRIVE

- The belts or ropes are used to transmit power from one shaft to another by means of pulleys which rotate at the same speed or at different speeds.

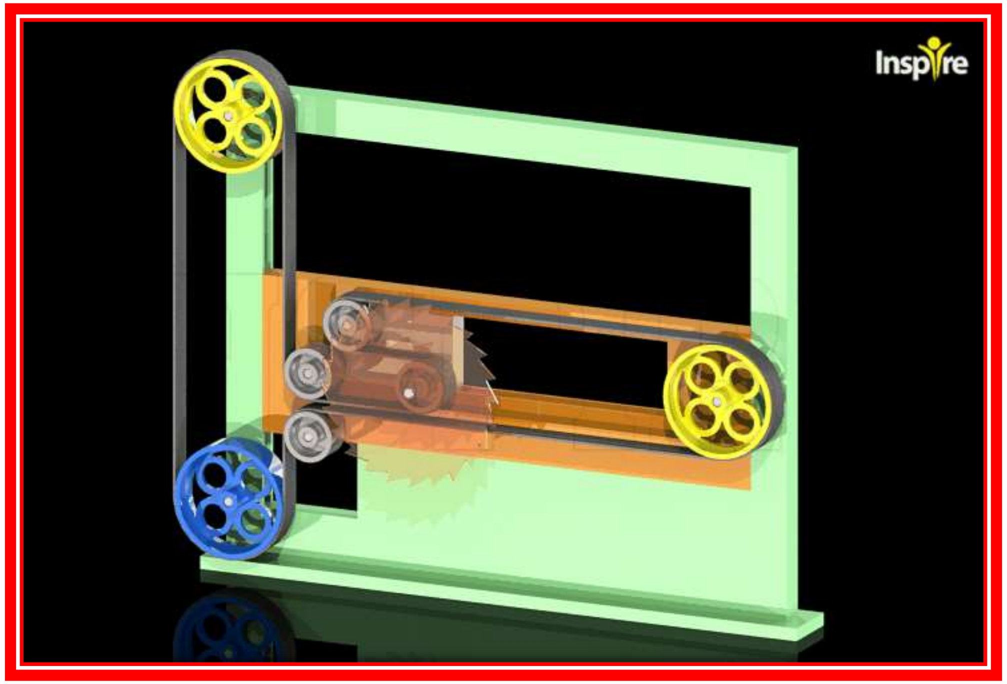
The ***amount of power*** transmitted depends upon the following factors :

1. The **velocity** of the belt

2. The **tension** under which the belt is placed on the pulleys

3. The **arc of contact** between the belt and the smaller pulley

4. The **conditions** under which the belt is used



SELECTION OF A BELT DRIVE

- Following are the various important factors upon which the selection of a belt drive depends:
  1. Speed of the driving and driven shafts, **2.** Speed reduction ratio,
  3. Power to be transmitted, **4.** Centre distance between the shafts,
  5. Positive drive (slip doesn't occur) requirements, **6.** Shafts layout, **7.** Space available, and
  8. Service conditions

TYPE OF BELT DRIVES

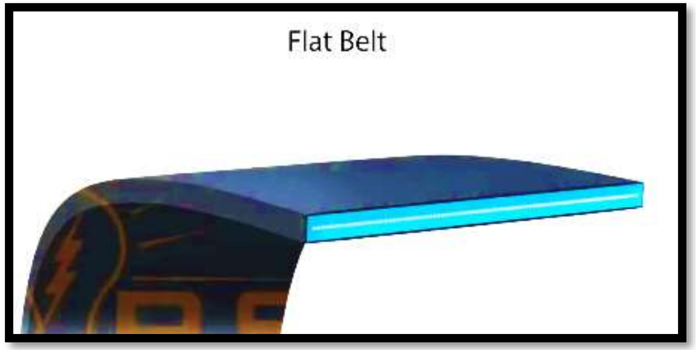
1. ***Light drives:*** These are used to transmit **small powers** at belt speeds **upto about 10 m/s**, as in agricultural machines and small machine tools.
2. ***Medium drives:*** These are used to transmit **medium power** at belt speeds **over 10 m/s but up to 22 m/s**, as in machine tools.
3. ***Heavy drives:*** These are used to transmit **large powers** at belt speeds **above 22 m/s**, as in compressors and generators.

TYPE OF BELTS

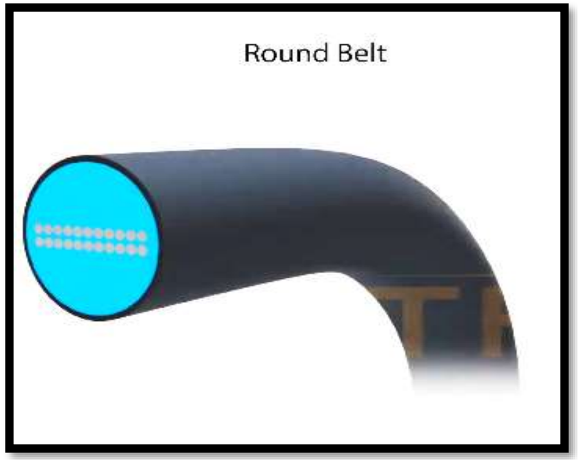
Belt Type	Power	Distance between any two pulleys	Cross-Section	Groove in Pulley
Flat	Moderate	Large but not more than 8m	Rectangular	Not Required
V	Moderate	Relatively Short	Trapezoidal	Required
Circular/ Rope/ Round	High	More than 8m	Round	Required
Ribbed/ Poly-V Belt	High	Relatively Short	Longitudinal Ribs	Required
Timing/ Synchronous	High	Relatively Short	Teeth Like (in transverse direction or from side view)	Required

TYPE OF BELTS

Flat Belt Used in factories, workshops, agriculture, etc.



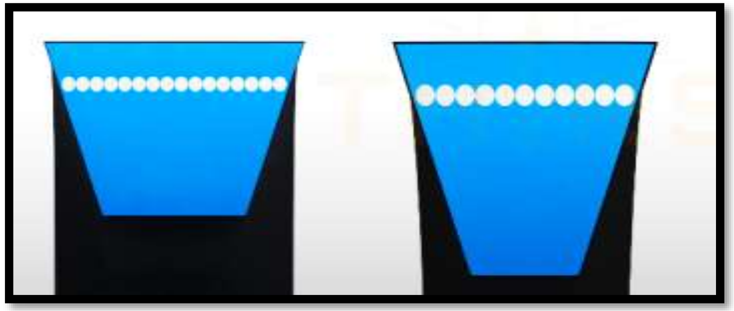
Circular/Rope/Round Belt Used in elevators, material transportation machinery, etc.



Ref: www.thors.com

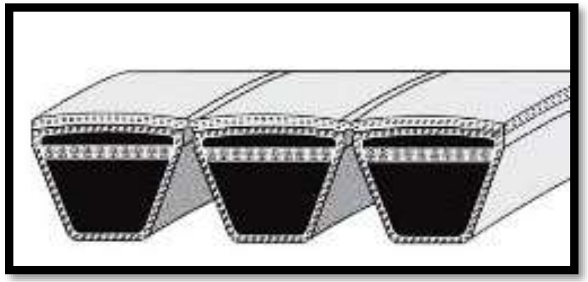
TYPE OF BELTS

V-belt Used in factories, agriculture, automotive, etc.



Classical Narrow/Wedged

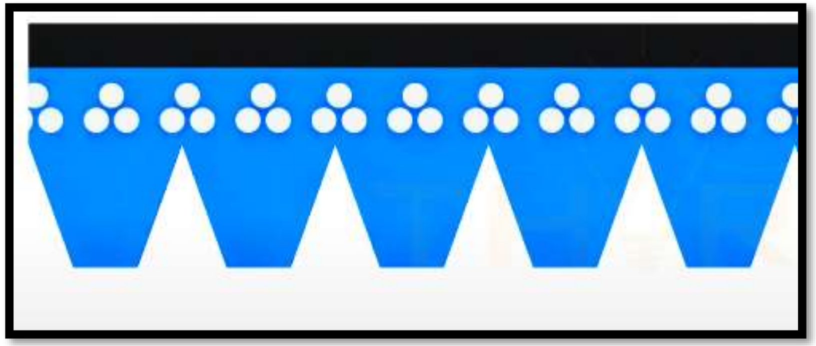
Cogged



Joined/Banded

Banded V-belt with grooved pulley

Ribbed/ Poly V-belt Used in factories, automotive, etc.



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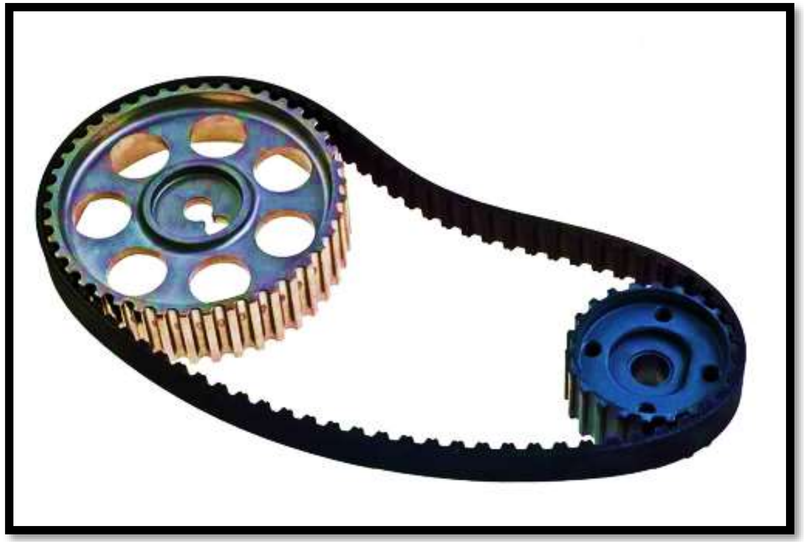
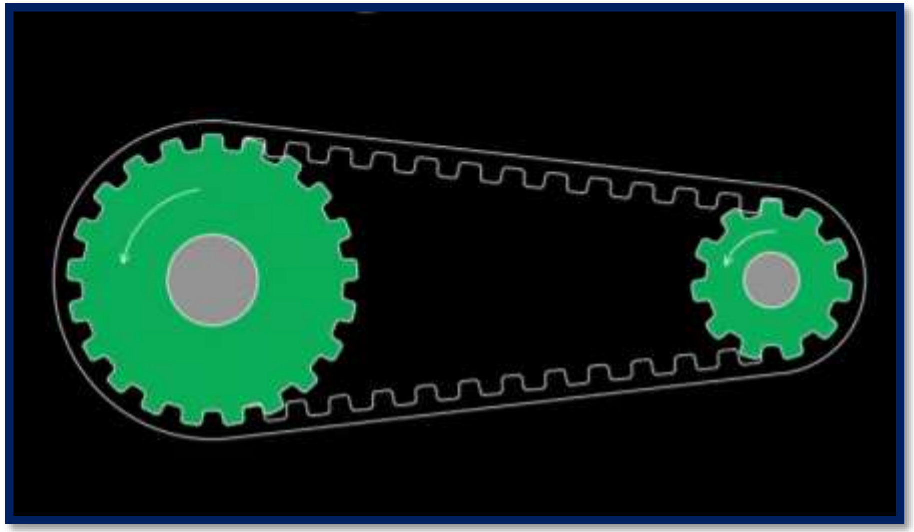
TYPE OF BELTS

Timing/Synchronous/Toothed/Positive Drive belt Used where precision and constant speed are essential. Used in CNC machines, automotive engines, ATMs, printers, robotics, etc.



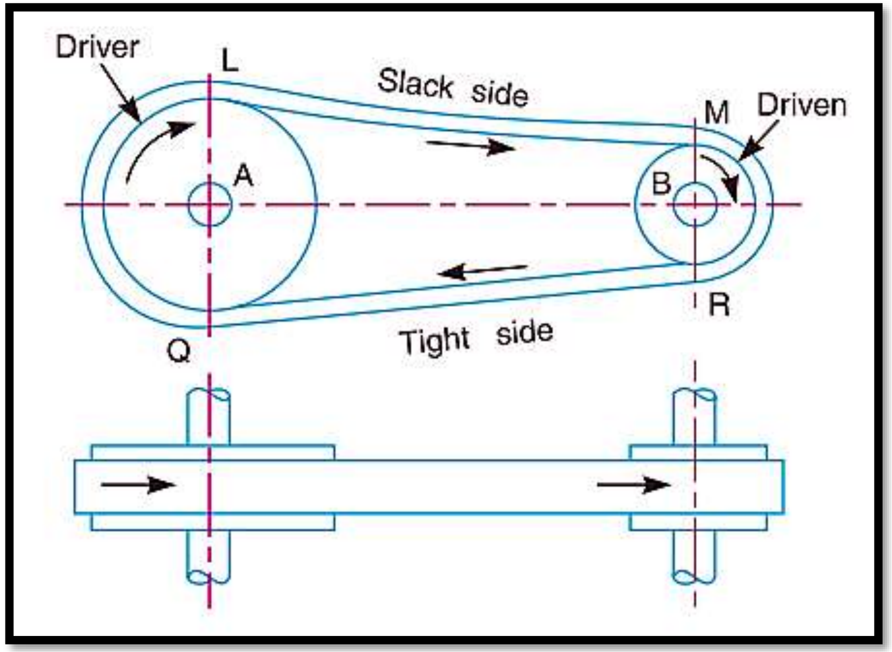


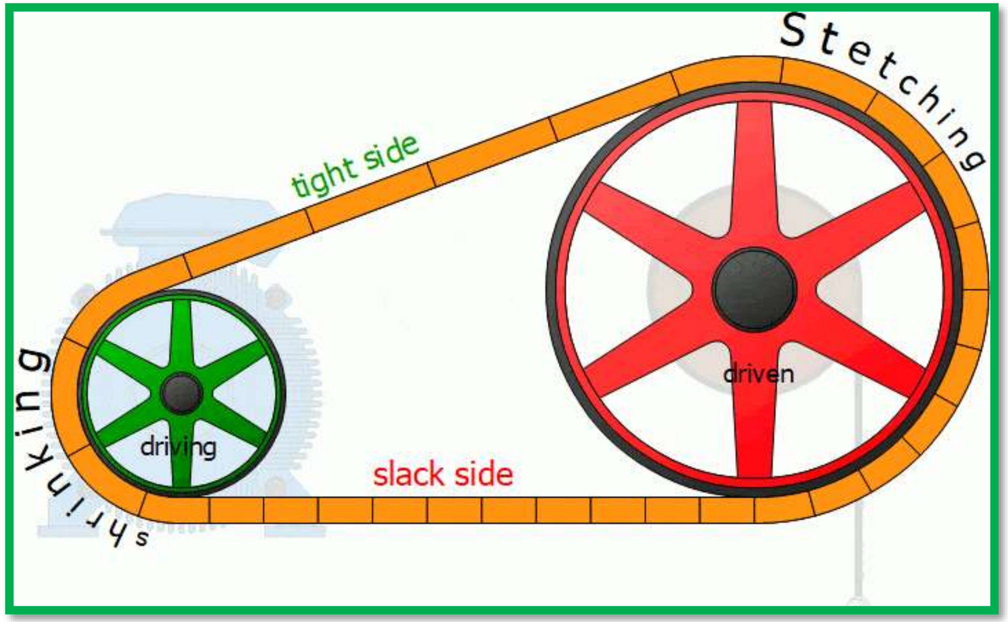
If a huge amount of power is to be transmitted, then a single belt may not be sufficient. In such a case, wide pulleys (for V-belts or circular belts) with a number of grooves are used

TYPE OF FLAT BELT DRIVE

Open Belt Drive

Shafts Arrangement	Shafts Rotation	Tensions in Belt
Parallel	Same Direction	More tension on the tight side and less tension on the slack side

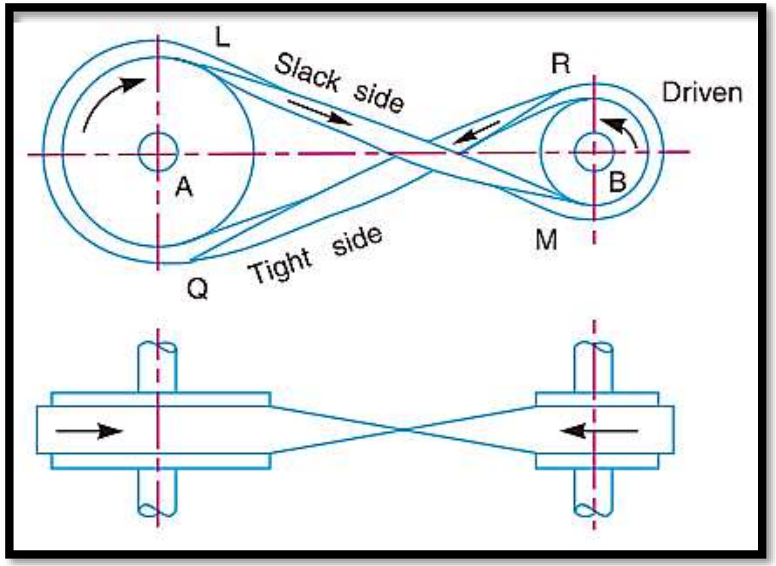


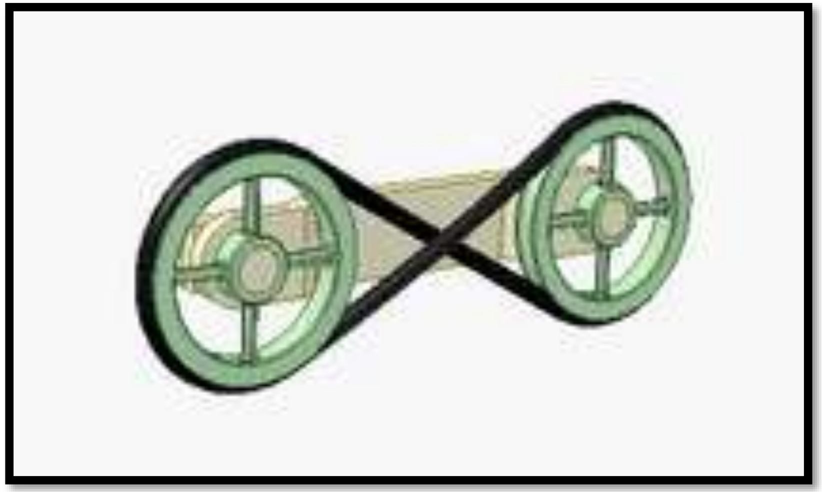


TYPE OF FLAT BELT DRIVE

Crossed/ Twist Belt Drive

Shafts Arrangement	Shafts Rotation	Tensions in Belt
Parallel	**Opposite Direction**	**More tension on the tight side and less tension on the slack side**



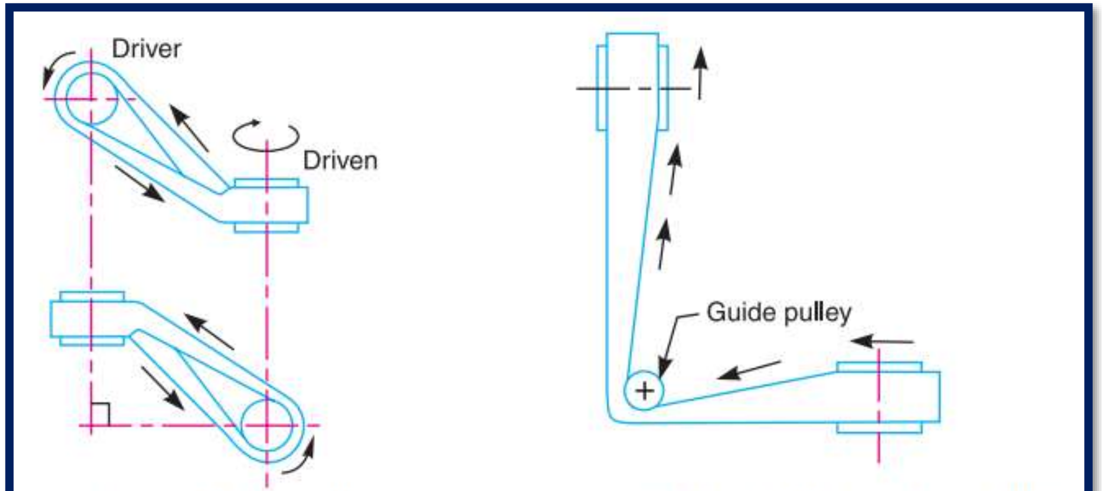


- Wear and tear occurs in belts due to rubbing at the crossing point
- Should be placed at a maximum distance of 20b, b = width of the belt,
- Speed of the belt < 15 m/s.

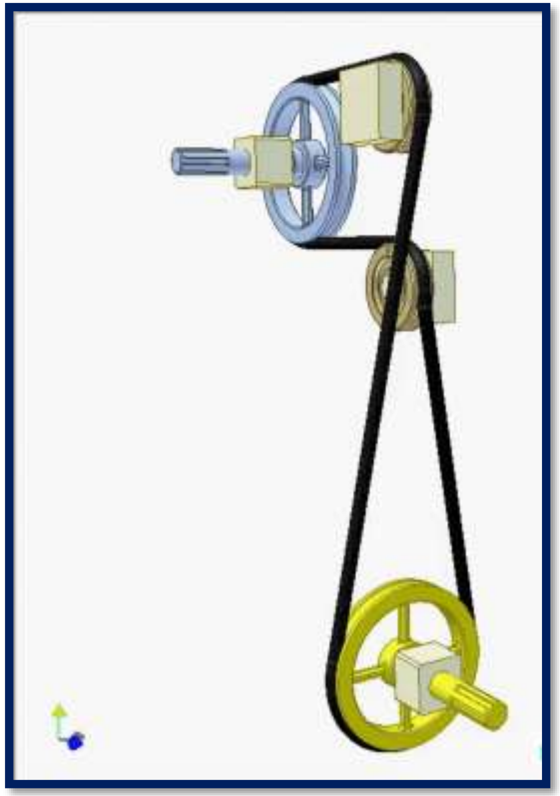
TYPE OF FLAT BELT DRIVE

Quarter Turn/ Right Angle Belt Drive

Shafts Arrangement
Right Angle
Shafts Rotation
In a Definite Direction



(a) Quarter turn belt drive.
(b) Quarter turn belt drive with guide pulley.



- ➢ To prevent the belt from leaving the pulley, the width of the face of the pulley should be greater or equal to 1.4 *b*, *b* = width of the belt.
- ➢ In case the pulleys cannot be arranged, or when the reversible motion is desired, then a quarter-turn belt drive with a guide pulley, may be used

TYPE OF FLAT BELT DRIVE

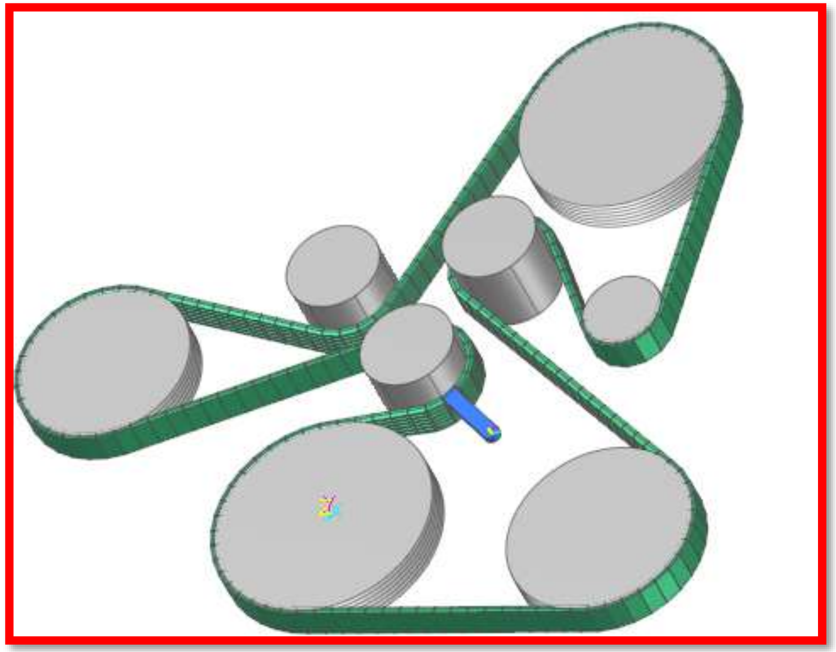
Belt Drive with Idler Pulley

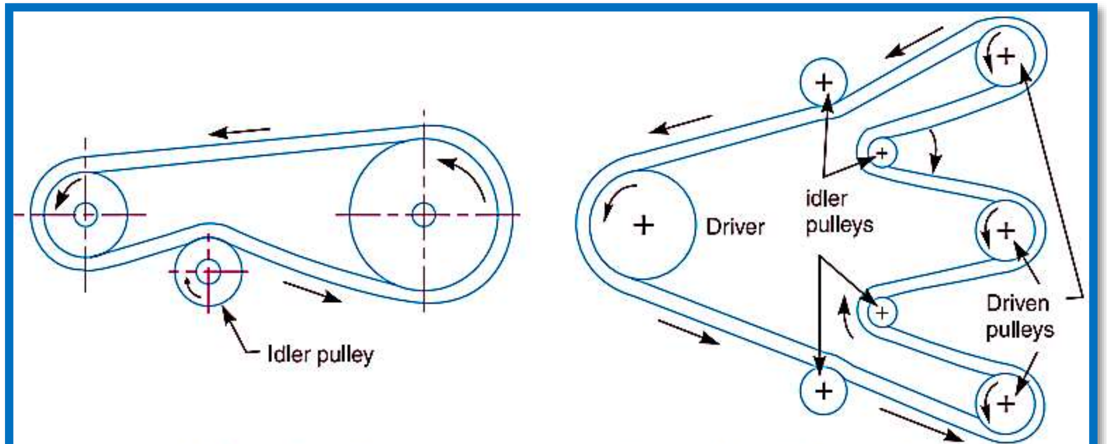
❖ Shafts arranged parallel

❖ Used when,

- an open belt drive cannot be used due to a small angle of contact on the smaller pulley
- it is desired to transmit motion from one shaft to several shafts
- the required belt tension cannot be obtained by other mean

❖ Provides high-velocity ratio





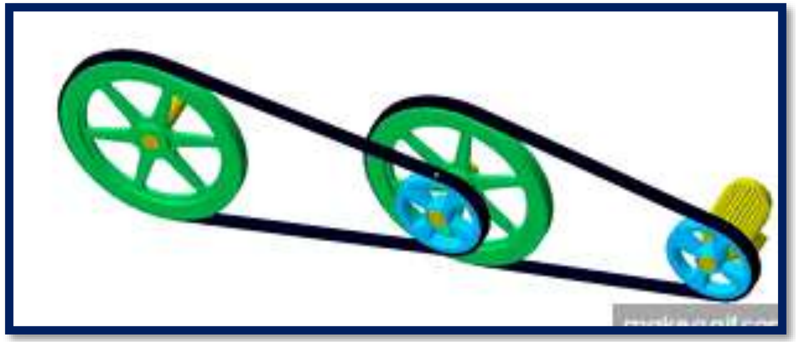
(*a*) Belt drive with single idler pulley. (*b*) Belt drive with many idler pulleys.

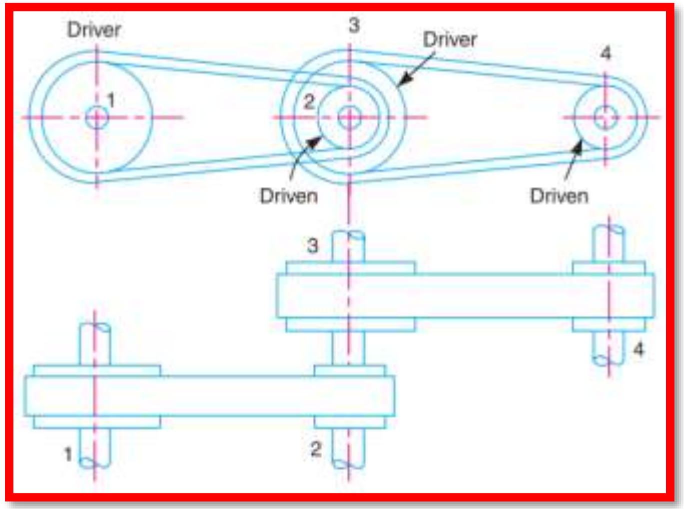
TYPE OF FLAT BELT DRIVE

Compound Belt Drive

> Shafts arranged parallel

- Used when power is transmitted from one shaft to another through several pulleys



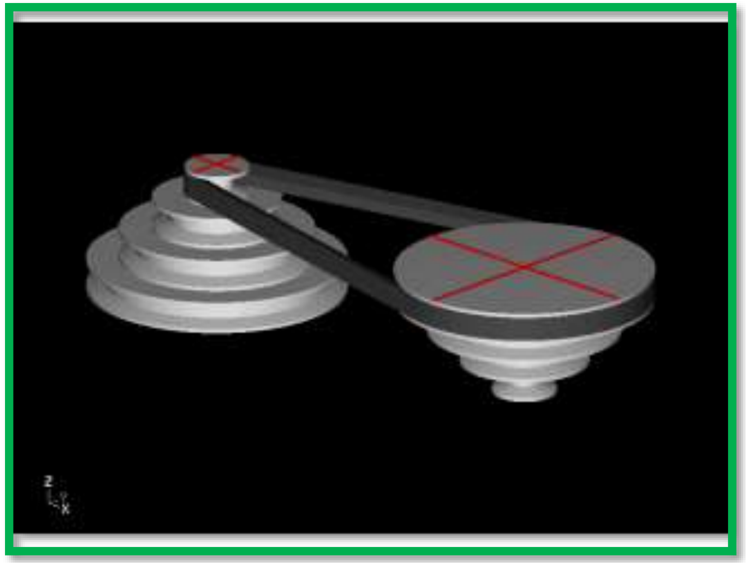


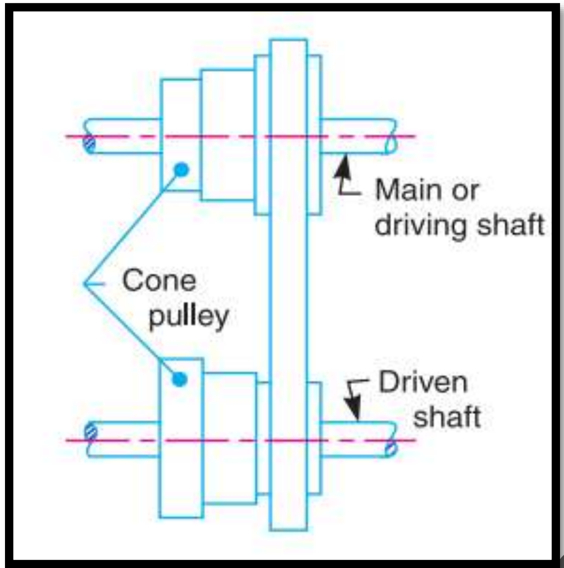
Compound Belt Drive

Stepped or Cone Pulley drive

> Used for changing the speed of the driven shaft while the main or driving shaft runs at a constant speed

> This is accomplished by shifting the belt from one part of the steps to the other.

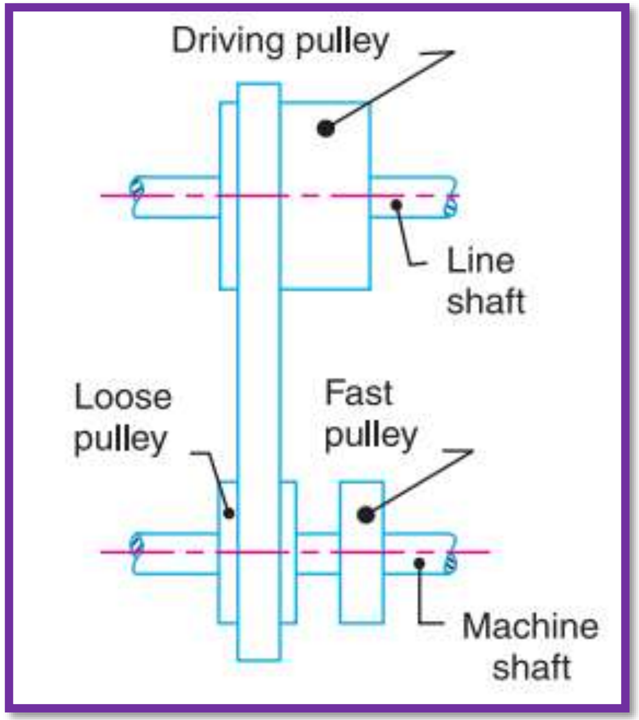


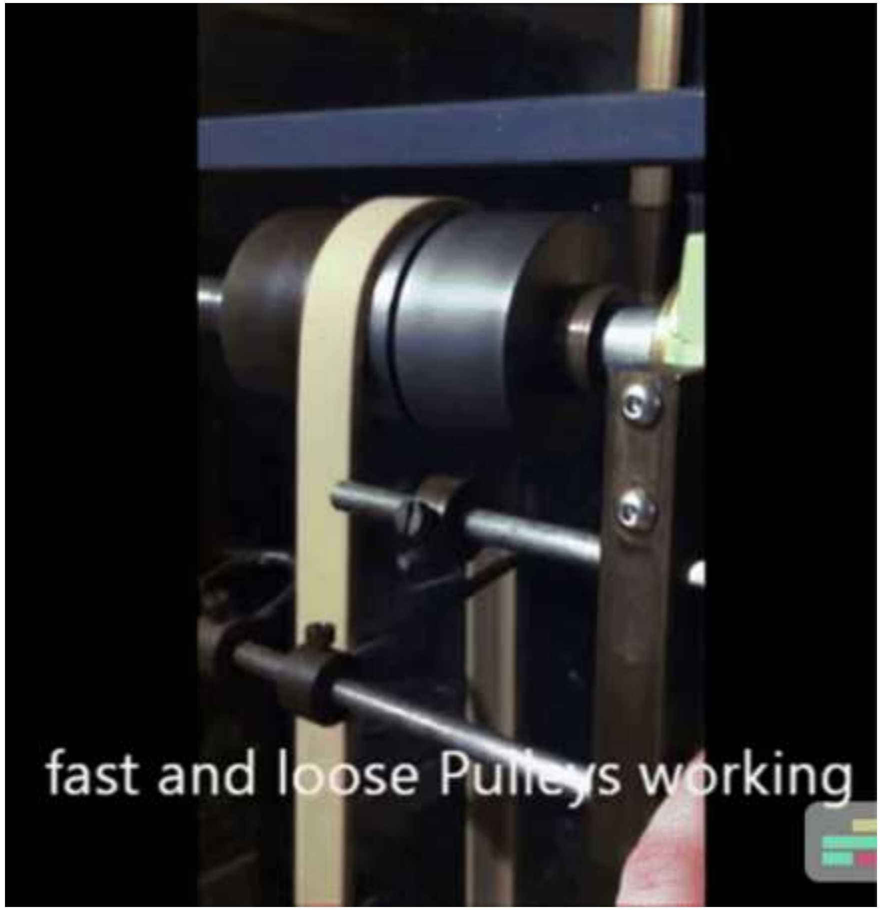


Cone Pulley Drive

Fast and Loose Pulley Drive

- Used when the driven or machine shaft is to be started or stopped when ever desired **without interfering with the driving shaft.**
- A pulley keyed to the machine shaft is called ***fast pulley*** and runs at the same speed as that of the machine shaft.
- A ***loose pulley*** runs freely over the machine shaft and is incapable of transmitting any power.
- When the driven shaft is required to be stopped, the belt is pushed onto the loose pulley by means of a sliding bar having belt forks.





Video

IMPORTANT PARAMETERS

- **Velocity Ratio**
  - *Single Belt Drive*
  - *Compound Belt Drive*
- **Slip of Belt**
- **Creep of Belt**
- **Length**
  - *Open Belt Drive*
  - *Cross Belt Drive*
- **Power Transmitted by a Belt Drive**
- **Ratio of Driving Tensions**
  - *Flat Belt Drive*
  - *V Belt Drive*
  - *Rope Drive*
- **Angle of Contact**
  - *Open Belt*
  - *Cross Belt*
- **Initial Tension in the Belt**

VELOCITY RATIO (SINGLE BELT DRIVE)

$$\text{Velocity Ratio} = \frac{\textit{Velocity of Driven Pulley}}{\textit{Velocity of Driving Pulley}}$$

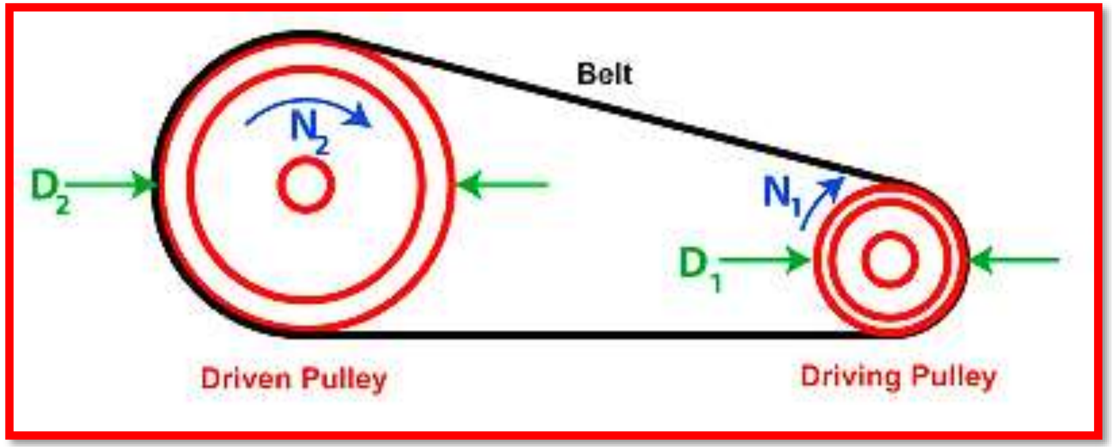
d_1 = Diameter of the driver or driving pulley
d_2 = Diameter of the follower or driven pulley
N_1 = Speed of the driver in r.p.m.
N_2 = Speed of the follower in r.p.m.
t = Thickness of the belt

Peripheral velocity of the belt on the driving pulley

$$v_1 = \frac{\pi d_1 N_1}{60} \text{ m/s}$$

and peripheral velocity of the belt on the driven pulley,

$$v_2 = \frac{\pi d_2 N_2}{60} \text{ m/s}$$



When there is no slip, then, $v_1 = v_2$

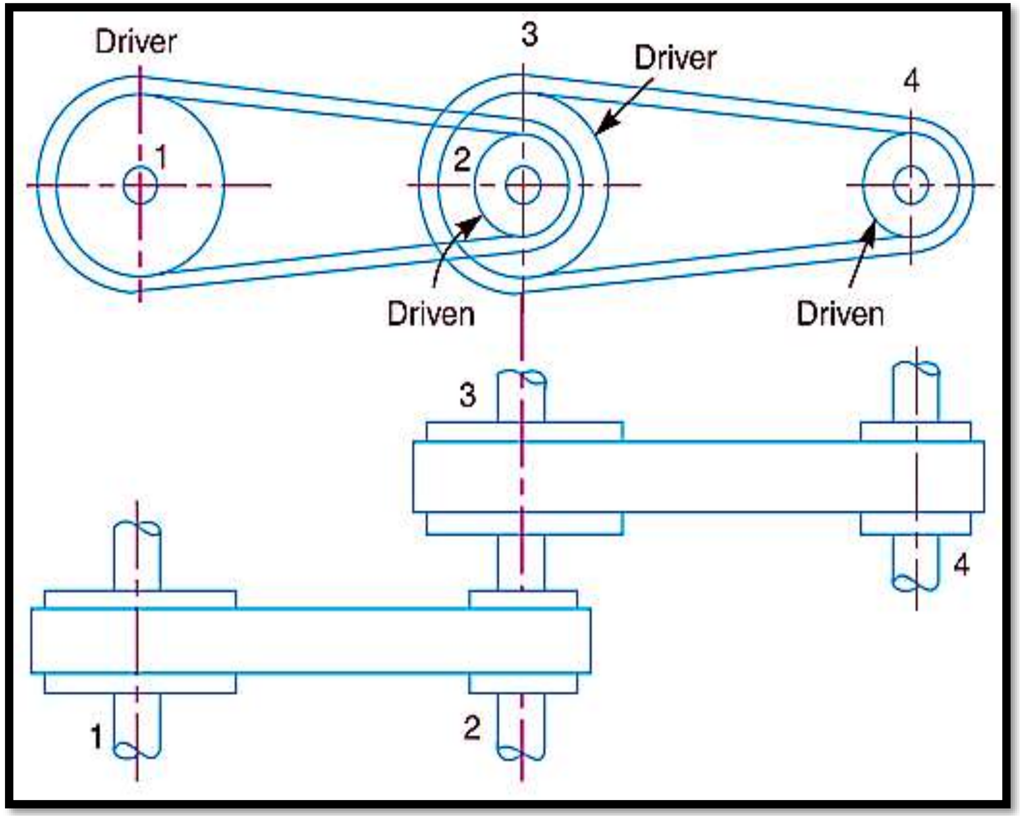
$$\textit{Velocity Ratio}, \frac{N_2}{N_1} = \frac{d_1}{d_2}$$

When the thickness of the belt (t) is considered,

$$\textit{Velocity Ratio}, \frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}$$

VELOCITY RATIO (COMPOUND BELT DRIVE)

d_1 = Diameter of the pulley 1,
N_1 = Speed of the pulley 1 in r.p.m.,
d_2, d_3, d_4, and N_2, N_3, N_4 = Corresponding values for pulleys 2, 3 and 4



Pulleys 1 and 2,

$$Velocity\ Ratio, \frac{N_2}{N_1} = \frac{d_1}{d_2}$$

Pulleys 3 and 4,

$$Velocity\ Ratio, \frac{N_4}{N_3} = \frac{d_3}{d_4}$$

Multiplying equations **(*i*) and (*ii*)** and simplifying,

$$Velocity\ Ratio, \frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}$$

($\because N_2 = N_3$, being keyed to the same shaft)

A little consideration will show, that if there are six pulleys, then,

$$Velocity\ Ratio, \frac{N_4}{N_1} = \frac{d_1 \times d_3 \times d_5}{d_2 \times d_4 \times d_6}$$

$$Velocity\ Ratio = \frac{Speed\ of\ the\ last\ driven}{Speed\ of\ the\ first\ driver} = \frac{Product\ of\ diameters\ of\ drivers}{Product\ of\ diameters\ of\ drivens}$$

SLIP OF BELT

Friction is insufficient → **Forward motion of the driver without carrying the belt with it** + **Forward motion of the belt without carrying the driven pulley with it** → ***Slip of the belt* (expressed in %)**

Belt should never be used where a definite velocity ratio is of importance (as in the case of hour, minute and second arms in a watch).

s_1% = Slip between the driver and the belt
s_2% = Slip between the belt and the follower

$$Velocity\ Ratio, \frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{s}{100}\right)$$

Where, $s = s_1 + s_2$, i.e., total percent of slip

If thickness of the belt (t) is considered,

$$Velocity\ Ratio, \frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left(1 - \frac{s}{100}\right)$$

CREEP OF BELT

When the belt passes from the slack side to the tight side, a certain portion of the belt extends and it contracts again when the belt passes from the tight side to slack side. Due to these changes of length, there is a relative motion between the belt and the pulley surfaces. This relative motion is termed as **creep**. **The total effect of creep is to reduce slightly the speed of the driven pulley or follower.**

$$Velocity\ Ratio, \frac{N_2}{N_1} = \frac{d_1}{d_2}\left(\frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}\right)$$

Where,

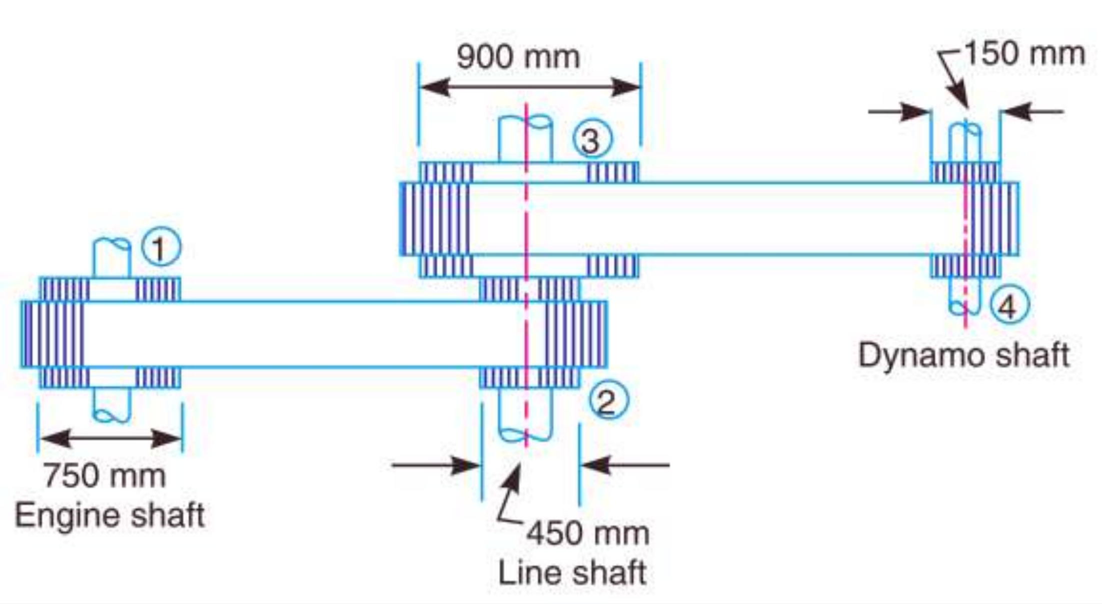
σ_1 and σ_2 = Stress in the belt on the tight and slack side respectively

E = Young's modulus for the material of the belt.

PROBLEM

- An engine, running at 150 r.p.m., drives a line shaft by means of a belt. The engine pulley is 750 mm diameter and the pulley on the line shaft being 450 mm. A 900 mm diameter pulley on the line shaft drives a 150 mm diameter pulley keyed to a dynamo shaft. Find the speed of the dynamo shaft, when 1. there is no slip, and 2. there is a slip of 2% at each drive.

Given :
N_1 = 150 r.p.m. ;
d_1 = 750 mm ;
d_2 = 450 mm ;
d_3 = 900 mm ;
d_4 = 150 mm



Ans:
1. 1500 r.p.m
2. 1440 r.p.m

1. *When there is no slip:*

We know that $$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}$$

$$\frac{N_4}{150} = \frac{750 \times 900}{450 \times 150} = 10$$

$$N_4 = 150 \times 10 = 1500 \text{ r.p.m.}$$ Ans.

2. *When there is a slip of 2% at each drive:*

We know that $$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\frac{N_4}{150} = \frac{750 \times 900}{450 \times 150}\left(1 - \frac{2}{100}\right)\left(1 - \frac{2}{100}\right) = 9.6$$

$\therefore$ $$N_4 = 150 \times 9.6 = 1440 \text{ r.p.m.}$$ Ans.

PROBLEM

- The power is transmitted from a pulley 1 m diameter running at 200 r.p.m. to a pulley 2.25 m diameter by means of a belt. Find the speed lost by the driven pulley as a result of creep, if the stress on the tight and slack side of the belt is 1.4 MPa and 0.5 MPa respectively. The Young's modulus for the material of the belt is 100 Mpa.

Given : $d_1 = 1$ m ;
$N_1 = 200$ r.p.m. ;
$d_2 = 2.25$ m ;
$\sigma_1 = 1.4$ MPa $= 1.4 \times 106$ N/m^2;
$\sigma_2 = 0.5$ MPa $= 0.5 \times 106$ N/m^2;
$E = 100$ MPa $= 100 \times 106$ N/m^2

Ans:
0.2 r.p.m

Solution:

Given : d_1 = 1 m ; N_1 = 200 r.p.m. ; d_2 = 2.25 m ;
σ_1 = 1.4 MPa = 1.4 × 106 N/m^2; σ_2 = 0.5 MPa = 0.5 × 106 N/m^2;
E = 100 MPa = 100 × 106 N/m^2

Let, N_2 = Speed of the driven pulley.

Neglecting creep, we know that,

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \quad \text{or} \quad N_2 = N_1 \times \frac{d_1}{d_2} = 200 \times \frac{1}{2.25} = 88.9 \text{ r.p.m.}$$

Considering creep, we know that,

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}$$

$$N_2 = 200 \times \frac{1}{2.25} \times \frac{100 \times 10^6 + \sqrt{0.5 \times 10^6}}{100 \times 10^6 + \sqrt{1.4 \times 10^6}} = 88.7 \text{ r.p.m.}$$

∴ Speed lost by driven pulley due to creep

= 88.9 – 88.7 = 0.2 r.p.m. **Ans.**

LENGTH OF BELT DRIVE

- **Length**

Open Belt Drive

$$L = \pi\,(r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x}$$

(in terms of pulley radii)

$$L = \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 - d_2)^2}{4x}$$

(in terms of pulley diameters)

Cross Belt Drive

$$L = \pi\,(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$

(in terms of pulley radii)

$$L = \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x}$$

(in terms of pulley diameters)

d_1 = Diameter of larger pulley
d_2 = Diameter of smaller pulley
r_1 = radius of larger pulley
r_2 = radius of smaller pulley
x = Distance between the centres of two pulleys
L = Total length of the belt

POWER TRANSMITTED THROUGH A BELT DRIVE

T_1 and T_2 = Tensions in the tight and slack side of the belt respectively in newtons
r_1 and r_2 = Radii of the driver and follower respectively, and
v = Velocity of the belt in m/s.

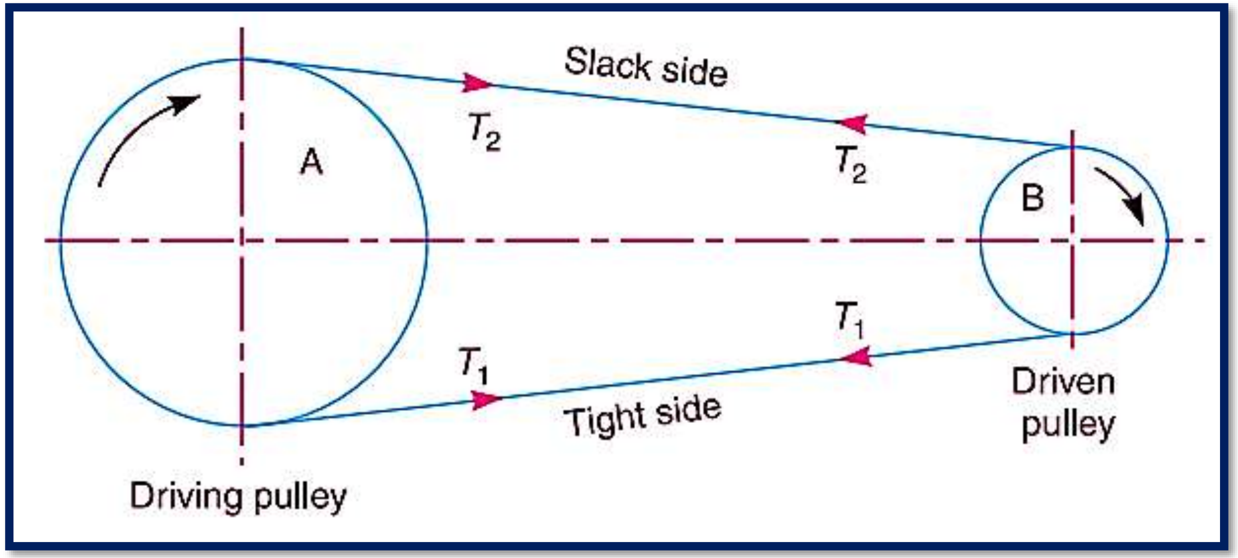
The effective turning (driving) force at the circumference of the follower is the difference between the two tensions (*i.e.* $T_1 - T_2$).

Work Done per second, $(T_1 - T_2)v$ N-m/s

Power Transmitted, $P = (T_1 - T_2)v$ W

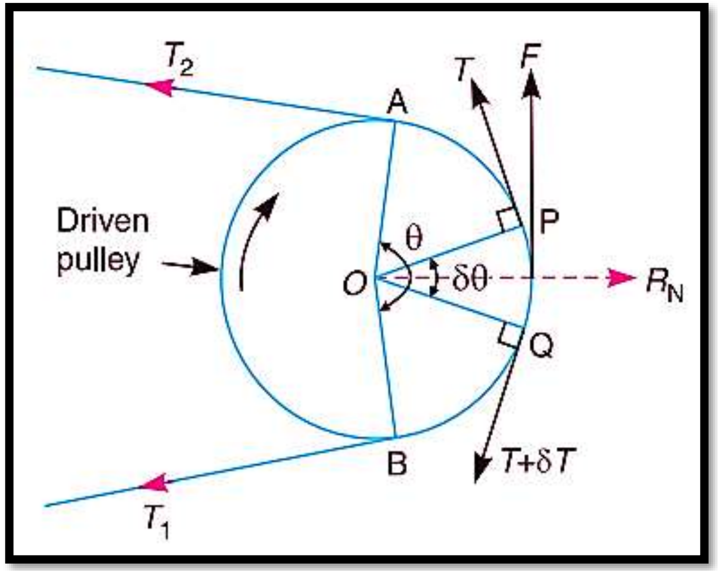
Torque exerted on the driving pulley = $(T_1 - T_2)\, r_1$

Torque exerted on the driven pulley = $(T_1 - T_2)\, r_2$



RATIO OF DRIVING TENSIONS FOR FLAT BELT DRIVE

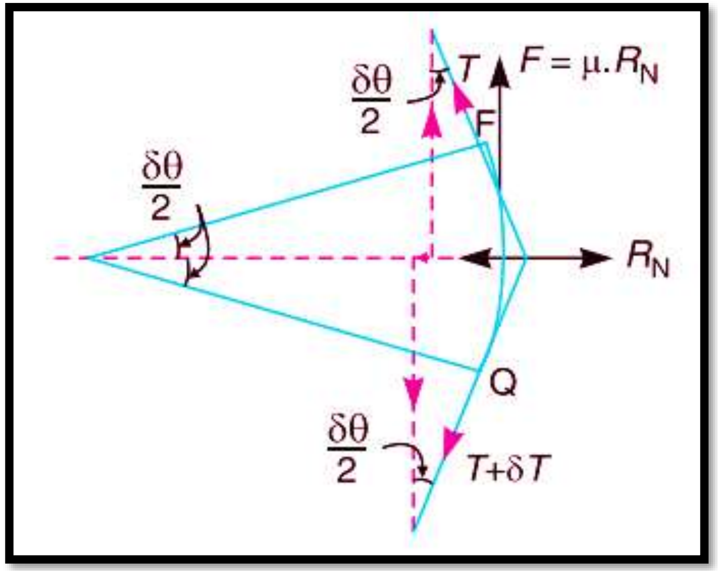
T_1 = Tension in the belt on the tight side
T_2 = Tension in the belt on the slack side
θ = Angle of contact / lap in radians



Frictional force, $F = \mu \times R_N$, where μ is the coefficient of friction between the belt and pulley.

Resolving all the forces horizontally and equating the same,

$$\boxed{R_N = (T + \delta T)\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2}} \quad \text{............(i)}$$



Since the angle δθ is very small, therefore putting $\sin \delta\theta/2 = \delta\theta/2$ in equation **(*i*)**,

$$\boxed{R_N = (T + \delta T)\frac{\delta\theta}{2} + T \times \frac{\delta\theta}{2} = \frac{T.\delta\theta}{2} + \frac{\delta T.\delta\theta}{2} + \frac{T.\delta\theta}{2} = T.\delta\theta} \quad \text{.....(ii)}$$

RATIO OF DRIVING TENSIONS FOR FLAT BELT DRIVE

Now resolving the forces vertically, we have

$$\mu \times R_N = (T + \delta T)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2} \qquad \text{...................(iii)}$$

Since the angle $\delta\theta$ is very small, therefore putting $\cos \delta\theta/2 = 1$ in equation ***(iii)***,

$$\mu \times R_N = T + \delta T - T = \delta T \quad \text{or} \quad R_N = \frac{\delta T}{\mu} \qquad \text{...................(iv)}$$

Equating the values of R_N from equations **(*ii*) and (*iv*),**

$$T.\delta\theta = \frac{\delta T}{\mu} \quad \text{or} \quad \frac{\delta T}{T} = \mu.\delta\theta$$

Integrating both sides between the limits T_2 and T_1 and from 0 to θ respectively,

$$\int_{T_2}^{T_1} \frac{\delta T}{T} = \mu \int_0^{\theta} \delta\theta$$

$$\log_e\left(\frac{T_1}{T_2}\right) = \mu.\theta \quad \text{or} \quad \frac{T_1}{T_2} = e^{\mu.\theta} \qquad \text{.....(v)}$$

$$\left(\frac{T_1}{T_2}\right) = e^{\mu\theta}$$

or

$$2.3\,\log\left(\frac{T_1}{T_2}\right) = \mu.\,\theta$$

Equation (*v*) can be expressed in terms of corresponding logarithm to the base 10, *i.e.*

ANGLE OF CONTACT (OPEN BELT)

When the two pulleys of different diameters are connected by means of an open belt as shown in Fig.(*a*), then the angle of contact or lap (θ) at the smaller pulley must be taken into consideration.

Let r_1 = Radius of larger pulley,

r_2 = Radius of smaller pulley, and

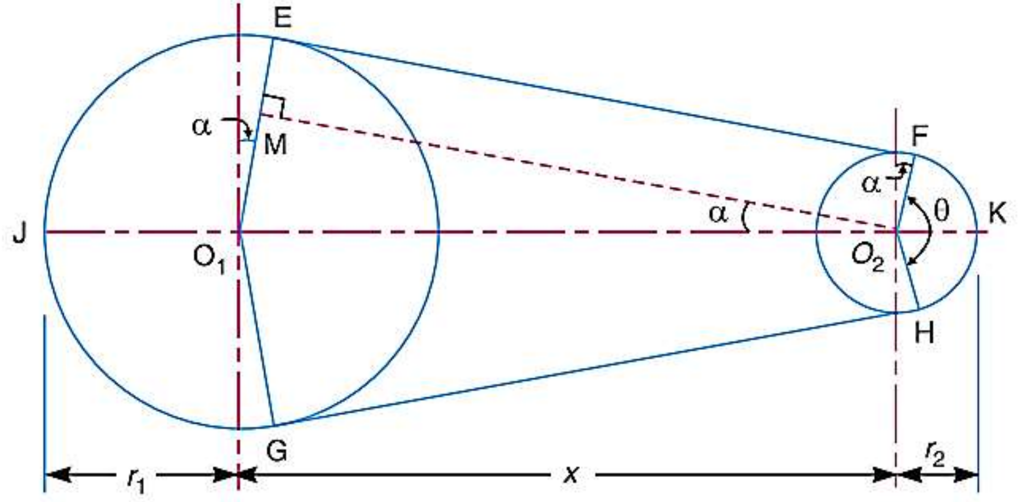
x = Distance between centres of two pulleys (*i.e.* O_1O_2).

From Fig.(*a*),

$$\boxed{\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E - ME}{O_1O_2} = \frac{r_1 - r_2}{x}}$$

$\therefore$ Angle of contact or lap,

$$\theta = (180° - 2\alpha)\frac{\pi}{180}\ \text{rad}$$



(*a*) Open belt drive.

ANGLE OF CONTACT (CROSSED BELT)

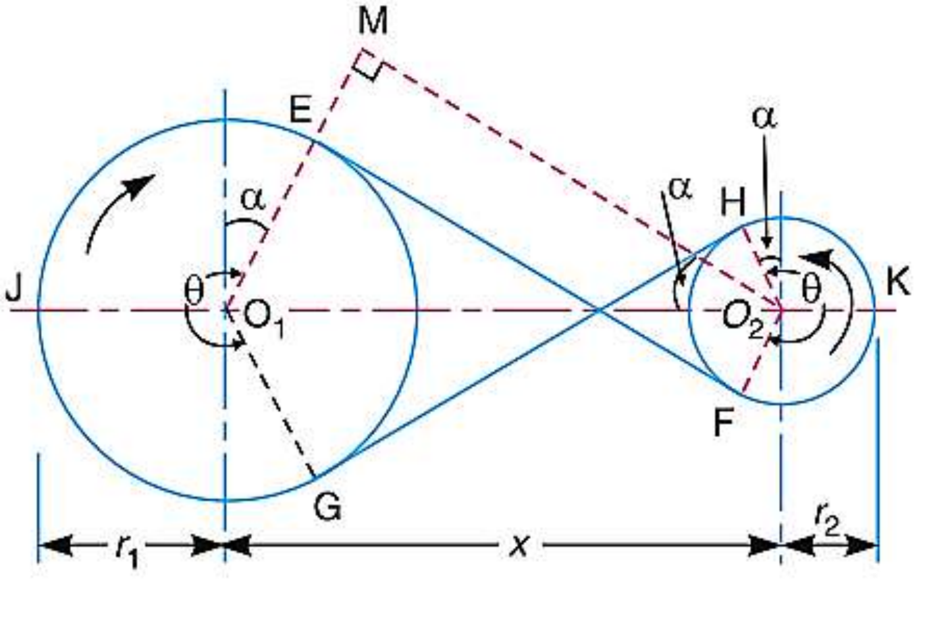
A little consideration will show that when the two pulleys are connected by means of a crossed belt as shown in Fig.(*b*), then the angle of contact or lap (θ) on both the pulleys is same.

From Fig.(*b*),

$$\sin\alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E + ME}{O_1 O_2} = \frac{r_1 + r_2}{x}$$

∴ Angle of contact or lap,

$$\theta = (180° + 2\alpha)\frac{\pi}{180} \text{ rad}$$



(*b*) Crossed belt drive.

PROBLEM

- Two pulleys, one 450 mm diameter and the other 200 mm diameter are on parallel shafts 1.95 m apart and are connected by a crossed belt. (i) Find the length of the belt required and (ii) the angle of contact between the belt and each pulley.
- (iii) What power can be transmitted by the belt when the larger pulley rotates at 200 rev/min, if the maximum permissible tension in the belt is 1 kN, and the coefficient of friction between the belt and pulley is 0.25 ?

Given :

d_1 = 450 mm = 0.45 m or r_1 = 0.225 m ;

d_2 = 200 mm = 0.2 m or r_2 = 0.1m ;

x = 1.95 m ;

N_1 = 200 r.p.m. ;

T_1 = 1 kN = 1000 N ;

μ = 0.25

Ans:

i. 4.975m

ii. 3.477 rad

iii. 2.74 kW

Solution. Given : $d_1 = 450$ mm $= 0.45$ m or $r_1 = 0.225$ m ; $d_2 = 200$ mm $= 0.2$ m or $r_2 = 0.1$ m ; $x = 1.95$ m ; $N_1 = 200$ r.p.m. ; $T_1 = 1$ kN $= 1000$ N ; $\mu = 0.25$

We know that speed of the belt,

$$v = \frac{\pi d_1 . N_1}{60} = \frac{\pi \times 0.45 \times 200}{60} = 4.714 \text{ m/s}$$

Length of the belt

We know that length of the crossed belt,

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(0.225 + 0.1) + 2 \times 1.95 + \frac{(0.225 + 0.1)^2}{1.95} = 4.975 \text{ m}$$ **Ans.**

Angle of contact between the belt and each pulley

Let θ = Angle of contact between the belt and each pulley.

We know that for a crossed belt drive,

$$\sin \alpha = \frac{r_1 + r_2}{x} = \frac{0.225 + 0.1}{1.95} = 0.1667 \quad \text{or} \quad \alpha = 9.6°$$

$\therefore$ $\theta = 180° + 2\,\alpha = 180° + 2 \times 9.6° = 199.2°$

$$= 199.2 \times \frac{\pi}{180} = 3.477 \text{ rad}$$ **Ans.**

Power transmitted

Let T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.25 \times 3.477 = 0.8692$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.8692}{2.3} = 0.378 \quad \text{or} \quad \frac{T_1}{T_2} = 2.387 \qquad \text{...(Taking antilog of 0.378)}$$

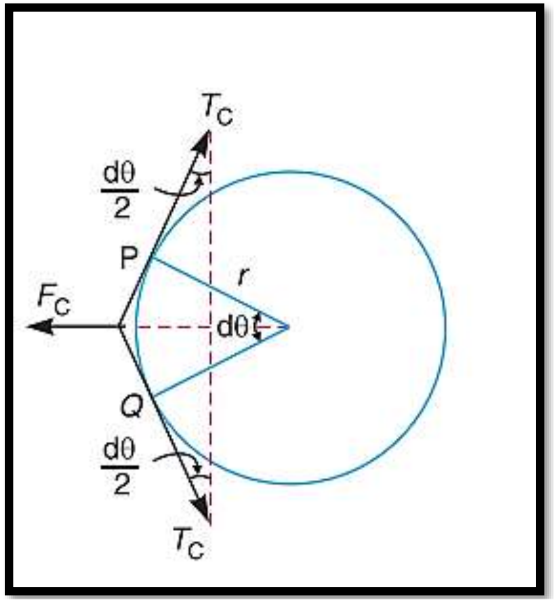
$$\therefore \qquad T_2 = \frac{T_1}{2.387} = \frac{1000}{2.387} = 419 \text{ N}$$

We know that power transmitted,

$$P = (T_1 - T_2)\, v = (1000 - 419)\, 4.714 = 2740 \text{ W} = 2.74 \text{ kW} \quad \textbf{Ans.}$$

CENTRIFUGAL TENSION

- Belt continuously runs over the pulleys
- Some centrifugal force is caused
- Increases the tension on both, tight as well as the slack sides
- The tension caused by centrifugal force is called **centrifugal tension**
- At lower belt speeds (less than 10 m/s), the centrifugal tension is very small, but at higher belt speeds (more than 10 m/s), its effect is considerable and thus should be taken into account.



m = Mass of the belt per unit length (kg),
v = Linear velocity of the belt in (m/s),
r = Radius of the pulley over which the belt runs in (m),
T_C = Centrifugal tension acting tangentially at P and Q (N)

$$F_C = (m.r.d\theta)\frac{v^2}{r} = m.d\theta.v^2$$

$$T_C \sin\left(\frac{d\theta}{2}\right) + T_C \sin\left(\frac{d\theta}{2}\right) = F_C = m.d\theta.v^2$$

$$T_C = m.v^2$$

When the centrifugal tension is taken into account,
Total tension in the tight side, $T_{t1} = T_1 + T_C$
Total tension in the slack side, $T_{t2} = T_2 + T_C$

Power transmitted, $P = (T_1 - T_2)\, v$ ...(same as before, no effect of centrifugal tension)

The ratio of driving tensions

$$2.3 \log\left(\frac{T_{t1} - T_C}{T_{t2} - T_C}\right) = \mu.\theta$$

MAXIMUM TENSION IN THE BELT

Maximum tension in the belt (**T**) is equal to the total tension in the tight side of the belt (**T_{tl}**)

σ = Maximum safe stress in N/mm^2,
b = Width of the belt in mm,
t = Thickness of the belt in mm.

Maximum tension in the belt, T = Maximum stress × Cross-sectional area of belt

Maximum tension in the belt , $T_{max} = \sigma . b . t$

When centrifugal tension is neglected,
T_{max} or T or $T_{tl} = T_l$,
i.e. Tension in the tight side of the belt

When centrifugal tension is considered,
T_{max} or T or $T_{tl} = T_l + T_C$

CONDITION FOR THE TRANSMISSION OF MAXIMUM POWER

T_1 = Tension in the belt on the tight side,
T_2 = Tension in the belt on the slack side,
T = Maximum tension in N,
T_C = Centrifugal tension in N,
v = Velocity of the belt in m/s

Power transmitted by a belt,

$$P = (T_1 - T_2)\,v$$

The ratio of driving tensions is

$$\frac{T_1}{T_2} = e^{\mu.\theta} \quad \text{or} \quad T_2 = \frac{T_1}{e^{\mu.\theta}}$$

Substituting the value of T_2

$$P = \left(T_1 - \frac{T_1}{e^{\mu.\theta}}\right) v = T_1\left(1 - \frac{1}{e^{\mu.\theta}}\right) v = T_1.v.C$$

Where

$$C = 1 - \frac{1}{e^{\mu.\theta}}$$

We know that

$$T_1 = T - T_C$$

Substituting the value of T_1 in equation

$$\begin{aligned} P &= (T - T_C)\,v.C \\ &= (T - m.v^2)\,v.C = (T.v - m\,v^3)\,C \quad \text{... (Substituting } T_C = m.\ v^2) \end{aligned}$$

For maximum power, differentiate the above expression with respect to v and equate to zero

$$\frac{dP}{dv} = 0 \quad \text{or} \quad \frac{d}{dv}(T.v - mv^3)\,C = 0$$

Simplifying,

$$\begin{aligned} T - 3\,m\,.\,v^2 &= 0 \\ T - 3\,T_C &= 0 \end{aligned}$$

Maximum Power Condition, $T = 3\,T_C$

*It shows that when the power transmitted is maximum, **1/3rd** of the maximum tension is absorbed as centrifugal tension*

1. We know that $T_1 = T - T_C$ and for maximum power, $T_C = \frac{T}{3}$.

$\therefore$
$$T_1 = T - \frac{T}{3} = \frac{2T}{3}$$

2. From equation (*iv*), the velocity of the belt for the maximum power,

$$v = \sqrt{\frac{T}{3m}}$$

PROBLEM

- A shaft rotating at 200 r.p.m. drives another shaft at 300 r.p.m. and transmits 6 kW through a belt. The belt is 100 mm wide and 10 mm thick. The distance between the shafts is 4m. The smaller pulley is 0.5 m in diameter. Calculate the stress in the belt, if it is i. an open belt drive, and ii. a cross belt drive. Take $\mu = 0.3$.

Given : $N_1 = 200$ r.p.m. ;
$N_2 = 300$ r.p.m. ;
$P = 6 \text{ kW} = 6 \times 10^3 \text{ W}$;
$b = 100$ mm ; $t = 10$ mm ;
$x = 4$ m ;
$d_2 = 0.5$ m ;
$\mu = 0.3$

Ans:
i. 1.267 MPa
ii. 1.184 MPa

Solution. Given : $N_1 = 200$ r.p.m. ; $N_2 = 300$ r.p.m. ; $P = 6$ kW $= 6 \times 10^3$ W ; $b = 100$ mm ; $t = 10$ mm ; $x = 4$ m ; $d_2 = 0.5$ m ; $\mu = 0.3$

Let σ = Stress in the belt.

1. *Stress in the belt for an open belt drive*

First of all, let us find out the diameter of larger pulley (d_1). We know that

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \quad \text{or} \quad d_1 = \frac{N_2 . d_2}{N_1} = \frac{300 \times 0.5}{200} = 0.75 \text{ m}$$

and velocity of the belt,

$$v = \frac{\pi d_2 . N_2}{60} = \frac{\pi \times 0.5 \times 300}{60} = 7.855 \text{ m/s}$$

Now let us find the angle of contact on the smaller pulley. We know that, for an open belt drive,

$$\sin \alpha = \frac{r_1 - r_2}{x} = \frac{d_1 - d_2}{2x} = \frac{0.75 - 0.5}{2 \times 4} = 0.03125 \quad \text{or} \quad \alpha = 1.8°$$

$\therefore$ Angle of contact, $\quad \theta = 180° - 2\,\alpha = 180 - 2 \times 1.8 = 176.4°$

$= 176.4 \times \pi / 180 = 3.08$ rad

Let $\quad T_1$ = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 3.08 = 0.924$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{0.924}{2.3} = 0.4017 \quad \text{or} \quad \frac{T_1}{T_2} = 2.52 \qquad ...(i)$$

...(Taking antilog of 0.4017)

We also know that power transmitted (P),

$$6 \times 10^3 = (T_1 - T_2)\,v = (T_1 - T_2)\,7.855$$

$$\therefore \quad T_1 - T_2 = 6 \times 10^3 / 7.855 = 764 \text{ N} \qquad ...(ii)$$

From equations **(*i*)** and **(*ii*)**,

$$T_1 = 1267 \text{ N, and } T_2 = 503 \text{ N}$$

We know that maximum tension in the belt (T_1),

$$1267 = \sigma\,.\,b.\,t = \sigma \times 100 \times 10 = 1000\,\sigma$$

$$\therefore \quad \sigma = 1267 / 1000 = 1.267 \text{ N/mm}^2 = 1.267 \text{ MPa} \quad \textbf{Ans.}$$

...[$\because$ 1 MPa = 1 MN/m^2 = 1 N/mm^2]

Stress in the belt for a cross belt drive

We know that for a cross belt drive,

$$\sin\alpha = \frac{r_1 + r_2}{x} = \frac{d_1 + d_2}{2x} = \frac{0.75 + 0.5}{2 \times 4} = 0.1562 \quad \text{or} \quad \alpha = 9^\circ$$

∴ Angle of contact,

$$\theta = 180^\circ + 2\alpha = 180 + 2 \times 9 = 198^\circ$$

$$= 198 \times \pi / 180 = 3.456 \text{ rad}$$

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 3.456 = 1.0368$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{1.0368}{2.3} = 0.4508 \quad \text{or} \quad \frac{T_1}{T_2} = 2.82 \qquad \text{...}(iii)$$

...(Taking antilog of 0.4508)

From equations *(ii)* and *(iii)*,

$$T_1 = 1184 \text{ N and } T_2 = 420 \text{ N}$$

We know that maximum tension in the belt (T_1),

$$1184 = \sigma.\, b.\, t = \sigma \times 100 \times 10 = 1000\,\sigma$$

∴ $$\sigma = 1184 / 1000 = 1.184 \text{ N/mm}^2 = 1.184 \text{ MPa}$$ **Ans.**

INITIAL TENSION IN THE BELT

When a belt is wound round the two pulleys, its two ends are joined together ; so that the belt may continuously move over the pulleys, since the motion of the belt from the driver and the follower is governed by a firm grip, due to friction between the belt and the pulleys. In order to increase this grip, the belt is tightened up. At this stage, even when the pulleys are stationary, the belt is subjected to some tension, called ***initial tension.***

When the driver starts rotating, it pulls the belt from one side (increasing tension in the belt on this side) and delivers it to the other side (decreasing the tension in the belt on that side). The increased tension in one side of the belt is called ***tension in tight side*** and the decreased tension in the other side of the belt is called ***tension in the slack side***.

Let, T_0 = Initial tension in the belt,
T_1 = Tension in the tight side of the belt,
T_2 = Tension in the slack side of the belt, and
α = Coefficient of increase of the belt length per unit force

INITIAL TENSION IN THE BELT

A little consideration will show that the increase of tension in the tight side

$= T_1 - T_0$

and increase in the length of the belt on the tight side

$$= \alpha\,(T_1 - T_0) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

Similarly, decrease in tension in the slack side

$$= T_0 - T_2$$

and decrease in the length of the belt on the slack side

$$= \alpha\,(T_0 - T_2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

Assuming that the belt material is perfectly elastic such that the length of the belt remains constant, when it is at rest or in motion, therefore increase in length on the tight side is equal to decrease in the length on the slack side. Thus, equating equations **(*i*) and (*ii*),**

$$\boxed{\alpha\,(T_1 - T_0) = \alpha\,(T_0 - T_2) \quad \text{or} \quad T_1 - T_0 = T_0 - T_2}$$

Initial Tension, $T_0 = \dfrac{T_1 + T_2}{2}$ (Neglecting centrifugal tension)

Initial Tension, $T_0 = \dfrac{T_1 + T_2 + 2T_C}{2}$ (Considering centrifugal tension)

PROBLEM

- An open belt connects two flat pulleys. The smaller pulley is 400 mm diameter and runs at 200 r.p.m. The angle of lap on this pulley is 160° and the coefficient of friction between the belt and pulley face is 0.25. The belt is on the point of slipping when 3 kW is being transmitted. Which of the following two alternatives would be more effective in order to increase the power :
- 1. Increasing the initial tension in the belt by 10 per cent, and
- 2. Increasing the coefficient of friction by 10 per cent by the application of a suitable dressing to the belt?

Ans:
First method is more effective

Solution. Given : $d_1 = 400$ mm $= 0.4$ m ; $d_2 = 250$ mm $= 0.25$ m ; $x = 2$ m ; $\mu = 0.4$; $T = 1200$ N ; $v = 10$ m/s

Power transmitted

We know that for an open belt drive,

$$\sin\alpha = \frac{r_1 - r_2}{x} = \frac{d_1 - d_2}{2x} = \frac{0.4 - 0.25}{2 \times 2} = 0.0375 \quad \text{or} \quad \alpha = 2.15°$$

∴ Angle of contact,

$$\theta = 180° - 2\alpha = 180° - 2 \times 2.15° = 175.7°$$

$$= 175.7 \times \pi / 180 = 3.067 \text{ rad}$$

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

Neglecting centrifugal tension,

$$T_1 = T = 1200 \text{ N} \qquad \text{...(Given)}$$

We know that

$$2.3\log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.4 \times 3.067 = 1.2268$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{1.2268}{2.3} = 0.5334 \quad \text{or} \quad \frac{T_1}{T_2} = 3.41$$

...(Taking antilog of 0.5334)

and

$$T_2 = \frac{T_1}{3.41} = \frac{1200}{3.41} = 352 \text{ N}$$

We know that power transmitted,

$$P = (T_1 - T_2)\, v = (1200 - 352)\, 10 = 8480 \text{ W} = 8.48 \text{ kW}$$ **Ans.**

Power transmitted when initial tension is increased by 10%

We know that initial tension,

$$T_0 = \frac{T_1 + T_2}{2} = \frac{1200 + 352}{2} = 776 \text{ N}$$

∴ Increased initial tension,

$$T_0' = 776 + \frac{776 \times 10}{100} = 853.6 \text{ N}$$

Let T_1 and T_2 be the corresponding tensions in the tight side and slack side of the belt respectively.

$$\therefore \qquad T_0' = \frac{T_1 + T_2}{2}$$

or $$T_1 + T_2 = 2\,T'_0 = 2 \times 853.6 = 1707.2 \text{ N} \qquad ...(i)$$

Since the ratio of tensions is constant, therefore

$$\frac{T_1}{T_2} = 3.41 \qquad ...(ii)$$

From equations *(i)* and *(ii)*,

$$T_1 = 1320.2 \text{ N} \; ; \text{ and } T_2 = 387 \text{ N}$$

$\therefore$ Power transmitted, $\quad P = (T_1 - T_2)\, v = (1320.2 - 387)\, 10 = 9332 \text{ W} = 9.332 \text{ kW}$

Power transmitted when coefficient of friction is increased by 10%

We know that coefficient of friction,

$$\mu = 0.4$$

$\therefore$ Increased coefficient of friction,

$$\mu' = 0.4 + 0.4 \times \frac{10}{100} = 0.44$$

Let T_1 and T_2 be the corresponding tensions in the tight side and slack side respectively.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu'.\theta = 0.44 \times 3.067 = 1.3495$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{1.3495}{2.3} = 0.5867 \quad \text{or} \quad \frac{T_1}{T_2} = 3.86 \qquad ...(iii)$$

... (Taking antilog of 0.5867)

Here the initial tension is constant, *i.e.*

$$T_0 = \frac{T_1 + T_2}{2} \quad \text{or} \quad T_1 + T_2 = 2\,T_0 = 2 \times 776 = 1552 \text{ N} \qquad ...(iv)$$

From equations *(iii)* and *(iv)*,

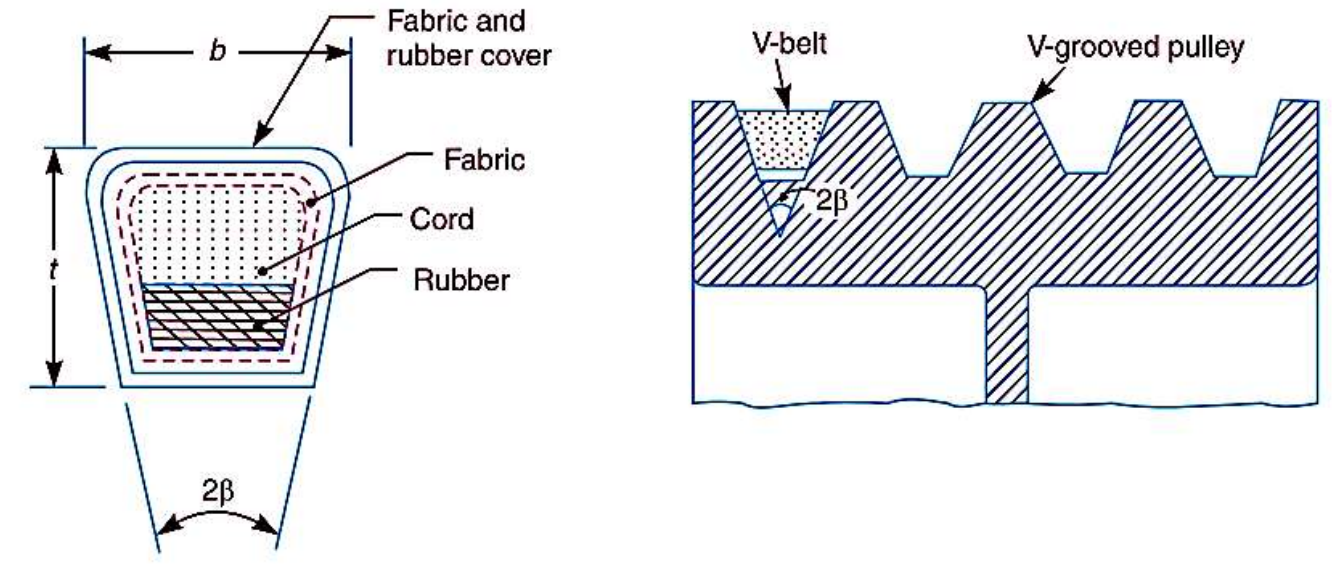
$$T = 1232.7 \text{ N and } T_2 = 319.3 \text{ N}$$

$\therefore$ Power transmitted,

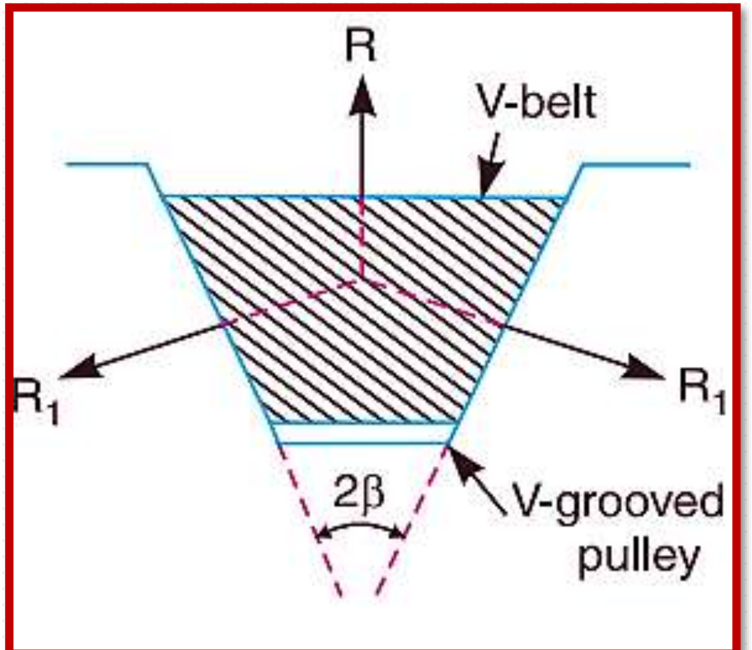
$$P = (T_1 - T_2)\, v = (1232.7 - 319.3)\, 10 = 9134 \text{ W} = 9.134 \text{ kW}$$

Since the power transmitted by increasing the initial tension is more, therefore in order to increase the power transmitted we shall adopt the method of increasing the initial tension. **Ans.**

RATIO OF DRIVING TENSIONS FOR V-BELT DRIVE



(a) Cross-section of a V-belt. (b) Cross-section of a V-grooved pulley.



$$2.3\ \log\left(\frac{T_1}{T_2}\right) = \mu.\theta.\mathrm{cosec}\beta$$

R_1 = Normal reaction between the belt and sides of the groove
R = Total reaction in the plane of the groove
2 β = Angle of the groove in degree
μ = Coefficient of friction between the belt and sides of the groove

PROBLEM

- A belt drive consists of two V-belts in parallel, on grooved pulleys of the same size. The angle of the groove is 30°. The cross-sectional area of each belt is 750 mm^2 and μ. = 0.12. The density of the belt material is 1.2 Mg/m^3 and the maximum safe stress in the material is 7 MPa. i. Calculate the power that can be transmitted between pulleys 300 mm diameter rotating at 1500 r.p.m. ii. Find also the shaft speed in r.p.m. at which the power transmitted would be maximum.

Given : $2\beta = 30°$ or $\beta = 15°$;
$A = 750\ mm^2 = 750 \times 10^{-6}\ m^2$;
$\mu = 0.12$; $\rho = 1.2\ Mg/m^3 = 1200\ kg/m^3$;
$\sigma = 7\ MPa = 7 \times 10^6\ N/m^2$;
$d = 300\ mm = 0.3\ m$;
$N = 1500$ r.p.m.

Ans:
i. 171.752 kW
ii. 2809 r.p.m

Solution: Given : $2\beta = 30°$ or $\beta = 15°$; $\alpha = 750$ mm^2 = 750×10^{-6} m^2 ; μ = 0.12 ; $\rho = 1.2$ Mg/m^3 = 1200 kg/m^3 ; $\sigma = 7$ MPa = 7 × 106 N/m^2 ; d = 300 mm = 0.3 m ; N = 1500 r.p.m.

Power transmitted

We know that velocity of the belt,

$$v = \frac{\pi d.N}{60} = \frac{\pi \times 0.3 \times 1500}{60} = 23.56 \text{ m/s}$$

and mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = 750 \times 10^{-6} \times 1 \times 1200 = 0.9 \text{ kg/m}$$

∴ Centrifugal tension,

$$T_C = m.v^2 = 0.9\,(23.56)^2 = 500 \text{ N}$$

We know that maximum tension in the belt,

$$T = \text{Maximum stress} \times \text{cross-sectional area of belt} = \sigma \times a$$

$$= 7 \times 10^6 \times 750 \times 10^{-6} = 5250 \text{ N}$$

∴ Tension in the tight side of the belt,

$$T_1 = T - T_C = 5250 - 500 = 4750 \text{ N}$$

T_2 = Tension in the slack side of the belt.

Since the pulleys are of the same size, therefore angle of contact, θ = 180° = π rad. We know that

$$2.3\log\left(\frac{T_1}{T_2}\right) = \mu.\theta \operatorname{cosec}\beta = 0.12 \times \pi \times \operatorname{cosec} 15° = 1.457$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{1.457}{2.3} = 0.6334 \quad \text{or} \quad \frac{T_1}{T_2} = 4.3$$

$$T_2 = \frac{T_1}{4.3} = \frac{4750}{4.3} = 1105 \text{ N}$$

We know that power transmitted,

$$P = (T_1 - T_2)\, v \times 2 \qquad \text{...(}\because \text{ No. of belts} = 2)$$

$$= (4750 - 1105)\, 23.56 \times 2 = 171\ 752 \text{ W} = 171.752 \text{ kW} \quad \textbf{Ans.}$$

Shaft speed

N_1 = Shaft speed in r.p.m., and

v_1= Belt speed in m/s.

We know that for maximum power, centrifugal tension,

$$T_C = T/3 \quad \text{or} \quad m\,(v_1)^2 = T/3 \quad \text{or} \quad 0.9\,(v_1)^2 = 5250/3 = 1750$$

$$(v_1)^2 = 1750/0.9 = 1944.4 \quad \text{or} \quad v_1 = 44.1 \text{ m/s}$$

We know that belt speed (v_1),

$$44.1 = \frac{\pi d . N_1}{60} = \frac{\pi \times 0.3 \times N_1}{60} = 0.0157\,N_1$$

$$N_1 = 44.1 / 0.0157 = 2809 \text{ r.p.m.} \quad \textbf{Ans.}$$

ROPE DRIVE

Terms	Fibre	Wire
Constituents	hemp, manila and cotton	steel, iron, stainless steel, and bronze
Distance between Pulleys	About 60m	Upto 150m
Quality	smooth, steady and quiet service	Lighter, durable and quiet service
Efficiency	High	High

The ratio of driving tensions for the rope drive may be obtained in the similar way as V-belts:

$$\boxed{2.3\ \log\left(\frac{T_1}{T_2}\right) = \mu.\,\theta.\,\mathrm{cosec}\beta}$$

θ = Angle of contact / lap in radians
2 β = Angle of the groove in degree
μ = Coefficient of friction between the belt and sides of the groove

PROBLEM

- A rope drive transmits 600 kW from a pulley of effective diameter 4 m, which runs at a speed of 90 r.p.m. The angle of lap is 160°; the angle of groove 45°; the coefficient of friction 0.28; the mass of rope 1.5 kg/m and the allowable tension in each rope 2400 N. Find the number of ropes required.

Given : P = 600 kW ;
d = 4 m ; N = 90 r.p.m. ;
$\theta = 160° = 160 \times \pi / 180 = 2.8$ rad;
$2\beta = 45°$ or $\beta = 22.5°$;
$\mu = 0.28$; m = 1.5 kg / m ;
T = 2400 N

Ans: 20 ropes

Example: A rope drive transmits 600 kW from a pulley of effective diameter 4 m, which runs at a speed of 90 r.p.m. The angle of lap is 160°; the angle of groove 45°; the coefficient of friction 0.28; the mass of rope 1.5 kg/m and the allowable tension in each rope 2400 N. Find the number of ropes required.

Solution: Given : P = 600 kW ; d = 4 m ; N = 90 r.p.m. ; $\theta = 160° = 160 \times \pi / 180 = 2.8$ rad; $2\beta = 45°$ or $\beta = 22.5°$; $\mu = 0.28$; m = 1.5 kg / m ; T = 2400 N

We know that velocity of the rope,

$$v = \frac{\pi d \cdot N}{60} = \frac{\pi \times 4 \times 90}{60} = 18.85 \text{ m/s}$$

∴ Centrifugal tension,

$$T_C = m.v^2 = 1.5\,(18.85)^2 = 533 \text{ N}$$

and tension in the tight side of the rope,

$$T_1 = T - T_C = 2400 - 533 = 1867 \text{ N}$$

T_2 = Tension in the slack side of the rope.

We know that,

$$2.3\log\left(\frac{T_1}{T_2}\right) = \mu.\theta\ \text{cosec}\beta = 0.28 \times 2.8 \times \text{cosec}\,22.5° = 2.05$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{2.05}{2.3} = 0.8913 \quad \text{or} \quad \frac{T_1}{T_2} = 7.786$$

$$T_2 = \frac{T_1}{7.786} = \frac{1867}{7.786} = 240\ \text{N}$$

We know that power transmitted per rope

$$= (T_1 - T_2)\, v = (1867 - 240)\, 18.85 = 30\ 670\ \text{W} = 30.67\ \text{kW}$$

$$\text{Number of ropes} = \frac{\text{Total power transmitted}}{\text{Power transmitted per rope}} = \frac{600}{30.67} = 19.56 \text{ or } 20$$

SOLVE BY YOURSELF

- **Book: Theory of Machine by R S Khurmi**
- ***Chapter 11***

- **Example: 11.9, 11.10, 11.11, 11.15, 11.16, 11.17, 11.19**
- **Exercise: 7, 9, 14, 18**


"YESTERDAY IS HISTORY,
TOMORROW IS A MYSTERY, BUT
TODAY IS A GIFT. THAT
IS WHY IT'S CALLED THE
PRESENT."
-Master Oogway