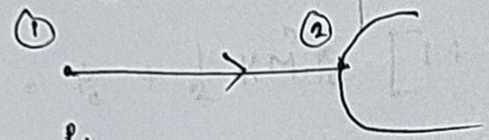
13/03/2025

For incompressible fluid

∴ stagnation pressure:



P_1

V_1

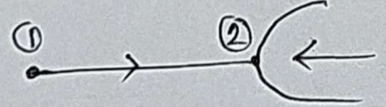
$\left(\begin{array}{l} V_2 = 0 \\ P_2 = P_s \end{array}\right)$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\Rightarrow \frac{P_1}{\rho} + \frac{V_1^2}{2} = \frac{P_s}{\rho}$$

$$\Rightarrow P_s = P_1 + \frac{1}{2}\rho V_1^2$$

For compressible ~~flow~~ fluid



P_1

$V_1 = V_0$

$V_2 = V_s = 0$

P_2

$$\frac{V_0^2}{2} + h_0 = \frac{V_s^2}{2} + h_s$$

$$\Rightarrow \frac{V_0^2}{2} + c_p T_0 = 0 + c_p T_s$$

$$\Rightarrow T_s = T_0 + \frac{V_0^2}{2c_p} \quad \text{--- (1)}$$

$h = c_p \cdot T$

↓

Enthalpy

Again,

$$\frac{c_p}{c_v} = K$$

$$\Rightarrow c_p - c_v = R$$

$$\Rightarrow c_p\left(1 - \frac{c_v}{c_p}\right) = R$$

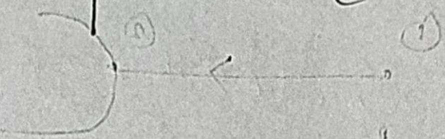
$$\Rightarrow c_p\left(1 - \frac{1}{K}\right) = R$$

$$\Rightarrow c_p = \frac{KR}{K-1}$$

$$c = \sqrt{KRT_0}$$

$$\Rightarrow c^2 = KRT_0$$

$$M = \frac{V_0}{c}$$



From equation ①,

$$T_s = T_0 + \frac{V_0^2}{2c_p}$$

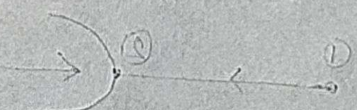
$$\Rightarrow \frac{T_s}{T_0} = 1 + \frac{V_0^2}{2T_0}\left(\frac{K-1}{KR}\right)$$

$$= 1 + \frac{V_0^2}{2c^2}(K-1)$$

$$= 1 + \frac{M^2}{2}(K-1)$$

$$\Rightarrow \left(\frac{P_s}{P_0}\right)^{\frac{K-1}{K}} = 1 + \frac{M^2}{2}(K-1)$$

$$\Rightarrow P_s/P_0 = \left[1 + \frac{M^2}{2}(K-1)\right]^{\frac{K}{K-1}}$$



We know,

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots$$

$$\therefore \frac{P_s}{P_0} = 1 + \left(\frac{K}{K-1}\right)\cdot\frac{M^2}{2}(K-1) + \left(\frac{K}{K-1}\right)\left(\frac{K}{K-1} - 1\right)\cdot\frac{1}{2!}\cdot\frac{M^4}{4}(K-1)^2$$

$$= 1 + K\cdot\frac{M^2}{2} + \left(\frac{K}{K-1}\right)\left(\frac{1}{K-1}\right)(K-1)^2\cdot\frac{M^4}{8}$$

$$= 1 + \frac{M^2}{2}\cdot K + \frac{K}{8}M^4$$

$$= 1 + \frac{KM^2}{2}\left(1 + \frac{M^2}{4}\right)$$

$$= 1 + \frac{KM^2}{2}\left[1 + \left(\frac{M^2}{2}\right)^2\right]$$

$$\Rightarrow \frac{P_S}{P_0} = 1 + \frac{KM^2}{2}\left[1 + \left(\frac{M^2}{2}\right)^2\right]$$

$$\Rightarrow P_S = P_0 + \frac{1}{2} KM^2 P_0 \left[1 + \left(\frac{M^2}{2}\right)^2\right]$$

$$= P_0 + \frac{1}{2}\rho_0 C_0^2 \cdot \frac{V_0^2}{C_0^2}\left(1 + \frac{M^4}{4}\right)$$

$$\Rightarrow P_S = P_0 + \frac{1}{2}\rho_0 V_0^2 \left(1 + \frac{M^4}{4}\right)$$

$$C = \sqrt{\frac{KP}{\rho}}$$

$$\Rightarrow C^2 = \frac{KP}{\rho}$$

$$\Rightarrow KP = \rho_0 C_0^2$$

$$M = \frac{V_0}{C}$$