AHSANULLAH UNIVERSITY OF SCIENCE AND TECHNOLOGY (AUST)

ME-3105: FLUID MECHANICS

(LC-8: Bernoulli's Equation)

BY

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BERNOULLI'S EQUATION OR BERNOULLI'S ENERGY EQUATION

The Bernoulli equation is an approximate relation between pressure, velocity, and elevation, and it is valid in regions of steady, incompressible flow where net frictional forces are negligible. Despite its simplicity, it has proven to be a very powerful tool in fluid mechanics.

We will **derive the Bernoulli equation by applying Newton's second law to a fluid element along a streamline** and demonstrate its use in a variety of applications.

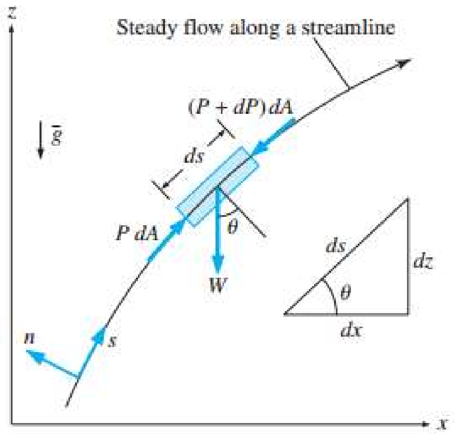
BERNOULLI’S EQUATION OR BERNOULLI’S ENERGY EQUATION

The following **assumptions** is being considered to derive Bernoulli’s equation:

- Steady flow.
- Incompressible and irrotational flow.
- Frictionless fluid or negligible viscous effects.
- No addition of shaft work to the fluid.
- Negligible heat transfer.
- Flow along a streamline.

DERIVATION OF BERNOULLI’S EQUATION (CONT’D)

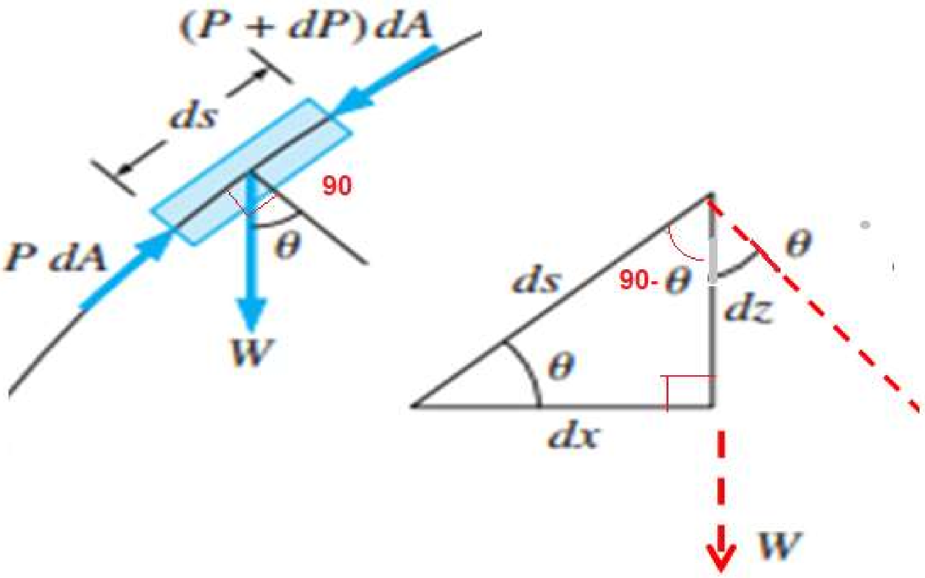
Consider the motion of a very small cylindrical fluid element in a steady flow field. Applying Newton’s second law (which is referred to as the linear momentum equation in fluid mechanics) in the s-direction on a particle moving along a streamline as shown in the figure below.



$$\sum F_s = ma_s$$

P = F/A

WHO ARE CONFUSED WITH ANGLE



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DERIVATION OF BERNOULLI'S EQUATION (CONT'D)

$f = f(x, y, z, t)$ (value of f depends on four variable)

$$df = \frac{\delta f}{\delta x} \cdot dx + \frac{\delta f}{\delta y} \cdot dy + \frac{\delta f}{\delta z} \cdot dz + \frac{\delta f}{\delta t} \cdot dt$$

The velocity V of a fluid particle to be a function of s and t. i.e. V = f(s, t)

$$dV = \frac{\partial V}{\partial s} ds + \frac{\partial V}{\partial t} dt$$

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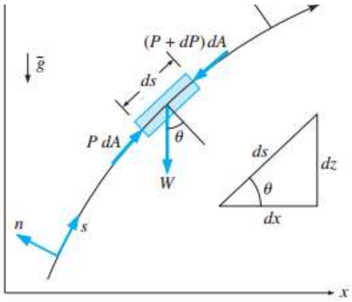
DERIVATION OF BERNOULLI'S EQUATION (CONT'D)

The velocity V of a fluid particle to be a function of s and t. Taking the total differential of V(s, t) and dividing both sides by dt yield,

$$dV = \frac{\partial V}{\partial s} ds + \frac{\partial V}{\partial t} dt \quad \text{and} \quad \frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} + \frac{\partial V}{\partial t}$$

In steady flow $\partial V/\partial t = 0$ and thus $V = V(s)$, and the acceleration in the s-direction becomes

$$a_s = \frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} = \frac{\partial V}{\partial s} V = V\frac{dV}{ds} \quad (\text{where } V = ds/dt)$$



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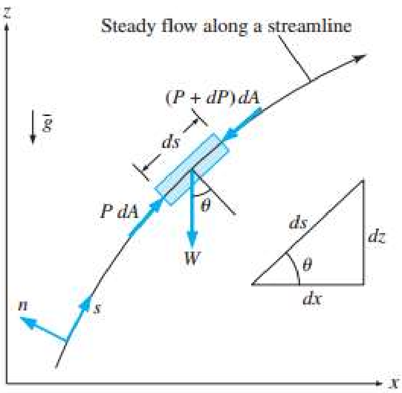
DERIVATION OF BERNOULLI'S EQUATION (CONT'D)

In regions of flow where net frictional forces are negligible, there is no pump or turbine, and there is no heat transfer along the streamline. *The significant forces acting in the s-direction are the pressure (acting on both sides) and the component of the weight of the particle in the s-direction.*



P = F/A, Wt. = Mass * g
Area = dA, Length = ds
Mass = ρ * Vol.
Vol. = dA* ds
W = ρ * dA* ds *g

$$P\,dA - (P + dP)\,dA - W\sin\theta = ma_s = mV\frac{dV}{ds}$$

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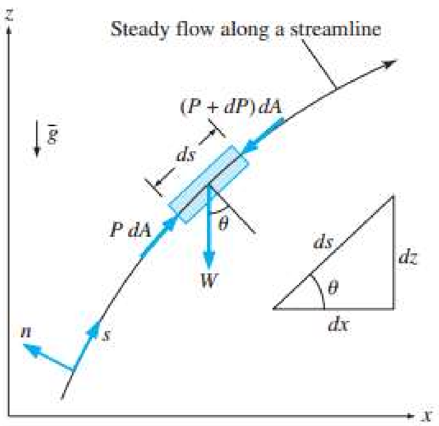
DERIVATION OF BERNOULLI'S EQUATION (CONT'D)

$$P\,dA - (P + dP)\,dA - W \sin\theta = ma_s = mV\frac{dV}{ds}$$

where θ is the angle between the normal of the streamline and the vertical z-axis at that point, $m = \rho V = \rho\, dA\, ds$ is the mass, $W = mg = \rho g\, dA\, ds$ is the weight of the fluid particle, and $\sin\theta = dz/ds$. Substituting,

$$-dP\,dA - \rho g\,dA\,ds\frac{dz}{ds} = \rho\,dA\,ds\,V\frac{dV}{ds}$$



DERIVATION OF BERNOULLI'S EQUATION (CONT'D)

$$-dP\,dA - \rho g\,dA\,ds\frac{dz}{ds} = \rho\,dA\,ds\,V\frac{dV}{ds}$$

Canceling dA from each term and simplifying,

$$-dP - \rho g\,dz = \rho V\,dV$$

$$\frac{dP}{\rho} + \rho V\,dV + g\,dz = 0 \quad (\text{dividing each term by } \rho\,)$$

The above equation is known as Euler's equation of flow.
By integration above equation,

$$\int \frac{1}{\rho}\,dP + \int V\,dV + \int g\,dz = \text{constant}$$

DERIVATION OF BERNOULLI'S EQUATION (CONT'D)

$$\int \frac{1}{\rho}\,dP + \int V\,dV + \int g\,dz = \text{constant}$$

$$\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant} \quad (\rho = \text{constant for incompressible flow})$$

$$\frac{P}{\gamma} + \frac{V^2}{2g} + z = \text{constant} \quad (\gamma = \rho g)$$

$$\frac{P_1}{\gamma} + \frac{V_1^{\,2}}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^{\,2}}{2g} + z_2 = \text{constant} = H(\text{total head})$$

The above equation is known as Bernoulli's equation.

DERIVATION OF BERNOULLI'S EQUATION (CONT'D)

$$\frac{P}{\gamma} + \frac{V^2}{2g} + z = \text{constant}$$

In Bernoulli's equation the terms,

$\frac{P}{\gamma}$ = Static pressure head or pressure energy per unit weight of fluid

$\frac{V^2}{2g}$ = Kinetic head or kinetic energy per unit weight of fluid

z = Potential head or potential energy per unit weight of fluid.

STATIC, DYNAMIC, AND STAGNATION PRESSURES

The Bernoulli equation states that the sum of the Pressure energy, Kinetic energy, and Potential energy of a fluid particle along a streamline is constant. Therefore, any changes in the Kinetic or and Potential energies of the fluid can cause the change of fluid pressure energy during flow and vice versa.

Bernoull's equation, $\frac{P}{\gamma} + \frac{V^2}{2g} + z = \text{constant}$

Multiplying both sides by $\gamma = \rho g$

$$P + \rho\frac{V^2}{2} + \rho g z = \text{constant}$$

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STATIC, DYNAMIC, AND STAGNATION PRESSURES

Multiplying both sides by $\gamma = \rho g$

$$P + \rho\frac{V^2}{2} + \rho g z = \text{constant}$$

Each term in the above equation has pressure units and each term represents some kind of pressure,

where P = Static pressure or actual thermodynamic pressure of the fluid.

$\rho\frac{V^2}{2}$ = Dynamic pressure and it represents pressure rise when fluid in motion is brought to stop isentropically.

$\rho g z$ = Hydrostatic pressure which depends on reference level selected.

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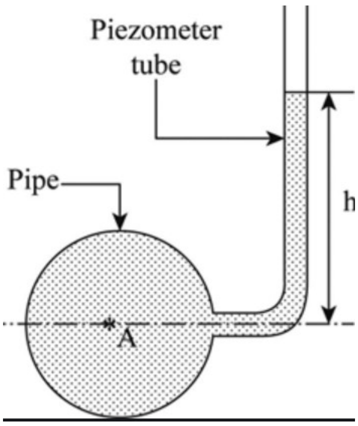
STATIC PRESSURE TAP, PITOT TUBE AND PIEZOMETER TUBE

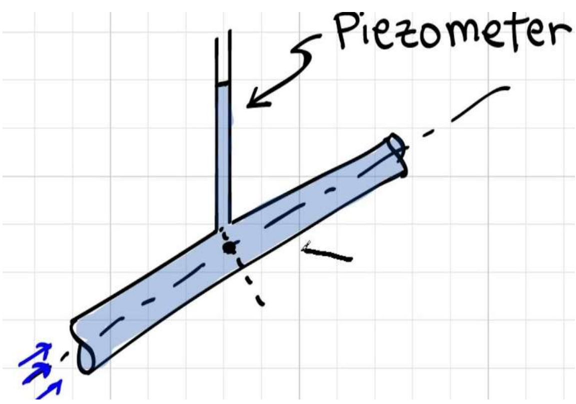
Static Pressure Tap: A static pressure tap is simply a small hole drilled into a wall such that the plane of the hole is parallel to the flow direction. It measures the static pressure of the fluid flow.

Pitot Tube: It is a hollow tube that placed longitudinally in the direction of the fluid flow, allowing the flow of fluid to brought into rest at end where fluid velocity approaches. It is used to measure fluid flow velocity and stagnation pressure of the fluid.

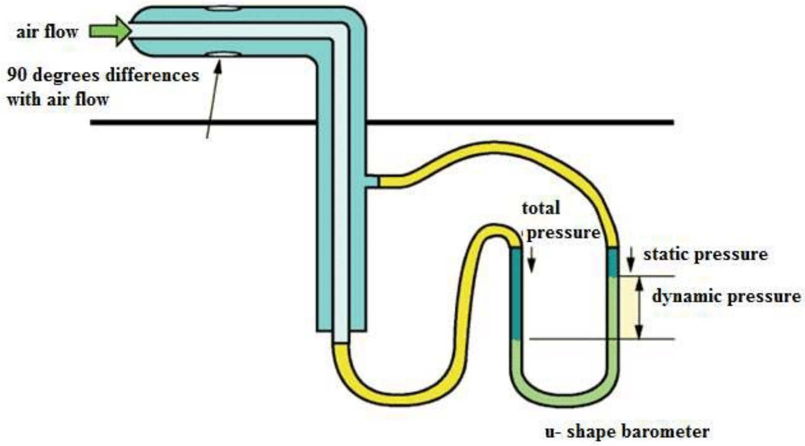
Piezometer: ***A vertical transparent tube is called Piezometer, which is used in situations where the static and stagnation pressure of a flowing liquid are greater than atmospheric pressure.*** *The liquid rises in the Piezometer tube to a column height is proportional to the pressure being measured.*

STATIC PRESSURE TAP, PITOT TUBE AND PIEZOMETER TUBE





STATIC PRESSURE TAP, PITOT TUBE AND PIEZOMETER TUBE



(Figure: Pitot Tube)

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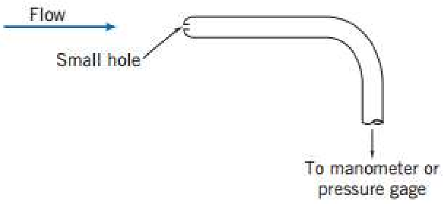
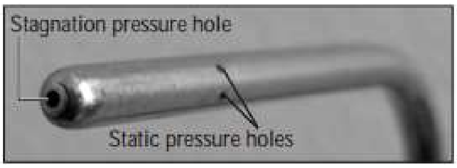
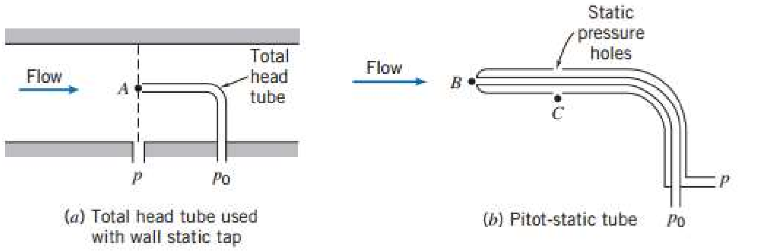
STATIC PRESSURE TAP, PITOT TUBE AND PIEZOMETER TUBE



Measurement of stagnation pressure.





Simultaneous measurement of stagnation and static pressures.

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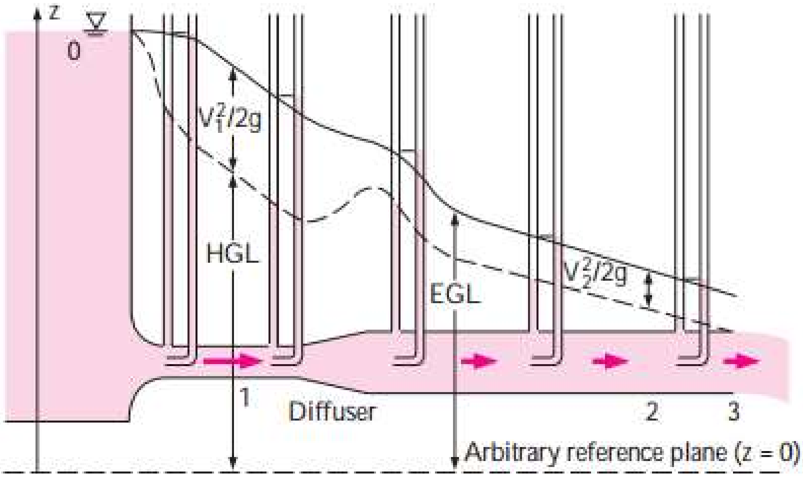
HYDRAULIC GRADE LINE (HGL) AND ENERGY GRADE LINE (EGL)

Hydraulic Grade Line (HGL): The line that represents the sum of the static pressure and the elevation heads, $P/\gamma + z$, is called the hydraulic grade line.



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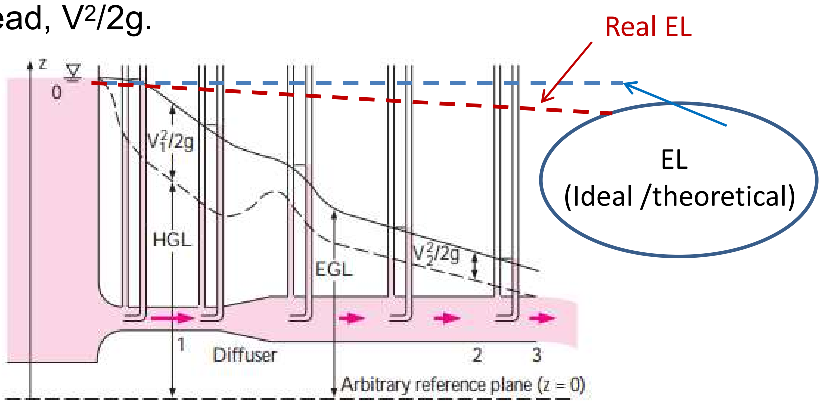
HYDRAULIC GRADE LINE (HGL) AND ENERGY GRADE LINE (EGL)

Energy Grade Line (EGL) or Energy Line (EL): *The line that represents the total head of the fluid, $P/\gamma + V^2/2g + z$, is called the energy grade line or energy line.*

The difference between the heights of EGL and HGL is equal to the dynamic head, $V^2/2g$.



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STATIC, DYNAMIC, AND STAGNATION PRESSURES

The sum of the static, dynamic and hydrostatic pressure is called the total pressure. Therefore, the Bernoull's equation states that the total pressue along a streamline is constant.

The sum of the static and dynamic pressures is called the stagnation pressure, and it is expressed as

$$P_{stag} = P + \rho\frac{V^2}{2}$$

The stagnation pressure represents the pressure at a point where the fluid is brought to a complete stop isentropically.

STATIC, DYNAMIC, AND STAGNATION PRESSURES

$$P_{stag} = P + \rho\frac{V^2}{2}$$

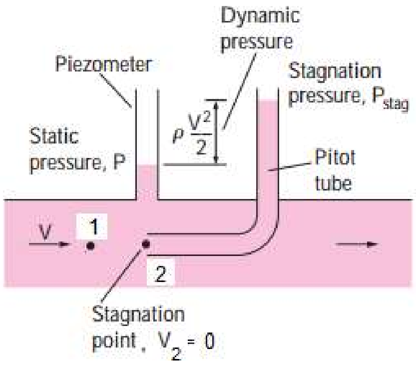
The stagnation pressure represents the pressure at a point where the fluid is brought to a complete stop isentropically.

$V_1 = V \qquad z_1 = z_2 = z$

$V_2 = 0 \qquad P_2 = P_{stag} \qquad P_1 = P$

$$P_1 + \rho\frac{V_1^{\,2}}{2} + z_1\rho g = P_2 + \rho\frac{V_2^{\,2}}{2} + z_2\rho g$$

$$V_1 = V = \sqrt{\frac{2(P_{stag} - P)}{\rho}}$$



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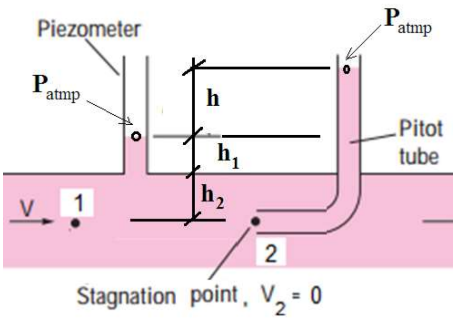
STATIC, DYNAMIC, AND STAGNATION PRESSURES

$$P_s = P_{atmp} + h\cdot\rho\cdot g + h_1\cdot\rho_f\cdot g + h_2\cdot\rho_f\cdot g$$

$$P_1 = P_{atmp} + h_1\cdot\rho_f\cdot g + h_2\cdot\rho_f\cdot g$$

$$P_s - P_1 = h\cdot\rho\cdot g$$



$$\frac{P_1}{\gamma} + \frac{V_1^{\,2}}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^{\,2}}{2g} + Z_2$$

$Z_1 = Z_2 \qquad V_2 = 0 \qquad P_2 = P_s \qquad \gamma = \rho\cdot g$

$$\frac{P_1}{\gamma} + \frac{V_1^{\,2}}{2g} = \frac{P_s}{\gamma} \qquad P_s - P_1 = h\cdot\rho\cdot g$$

$$\frac{V_1^{\,2}}{2\cdot g} = \frac{P_s - P_1}{\gamma} = \frac{P_s - P_1}{\rho\cdot g} = \frac{h\cdot\rho\cdot g}{\rho\cdot g} = h$$

$$V_1 = \sqrt{2g\cdot h}$$

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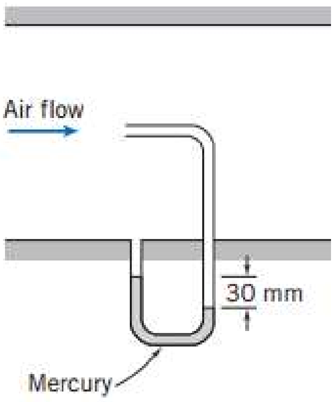
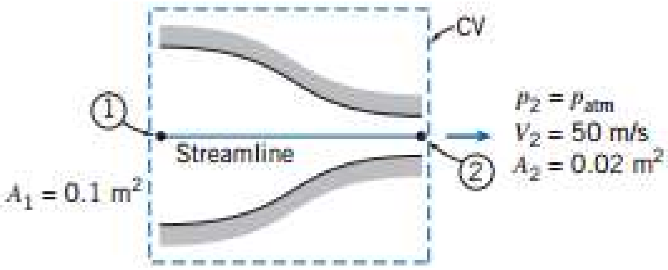
PITOT TUBE PROBLEM

Problem-1: A Pitot tube is inserted in an air flow (at STP) to measure the flow speed. The tube is inserted so that it points upstream into the flow and the pressure sensed by the tube is the stagnation pressure. The static pressure is measured at the same location in the flow, using a wall pressure tap. If the pressure difference is 30 mm of mercury, determine the flow speed.



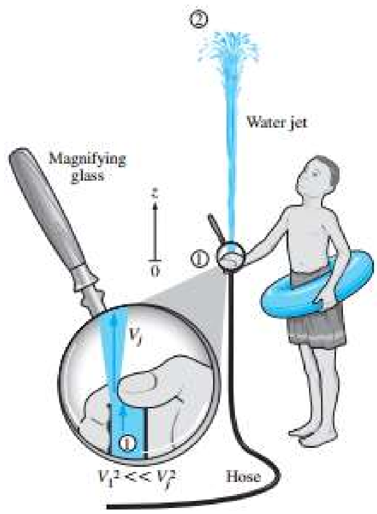
PROBLEM ON BERNULLI'S EQUATION

Problem-2: Air flows steadily at low speed through a horizontal nozzle (by definition a device for accelerating a flow), discharging to atmosphere. The area at the nozzle inlet is 0.1 m². At the nozzle exit the area is 0.02 m². Determine the gage pressure required at the nozzle inlet to produce an outlet speed of 50 m/s.



PROBLEM ON BERNULLI'S EQUATION

Problem-3: Water is flowing from a garden hose (as shown in Figure below). A child places his thumb to cover most of the hose outlet, causing a thin jet of high-speed water to emerge. The pressure in the hose just upstream of his thumb is 400 kPa. If the hose is held upward, what is the maximum height that the jet could achieve?



Do it your self !!! Apply B. equation between points 1 and 2, V2=0, P2 = Ptamp =0; Z2 = h =?; P1 = 400* 1000 Pa; V1=0; Z1 =0;

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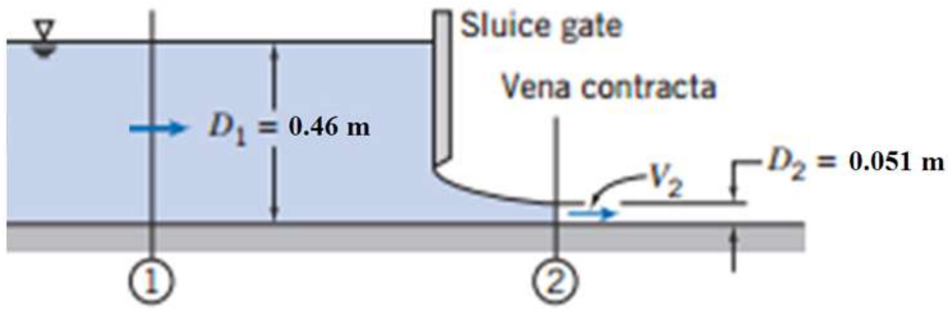
PROBLEM ON BERNULLI'S EQUATION

Problem-4: Water flows under a sluice gate on a horizontal bed at the inlet to a flume. Upstream from the gate, the water depth is **0.460 m** and the speed is negligible. At the vena contracta downstream from the gate, the flow streamlines are straight and the depth is **0.051 m**. Determine the **flow speed** downstream from the gate and the **discharge** in cubic meter per second per meter of width.

Q = A * V **V**



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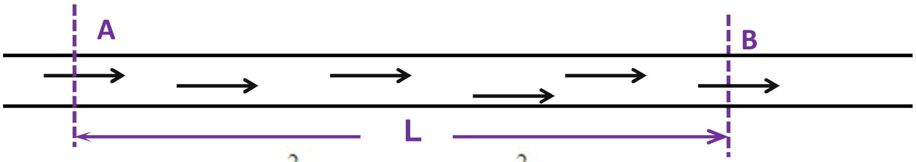
Is Bernoulli's Equation 100% Correct for Real Fluid !!!

In reality, the energy of the fluid doesn't remain constant due to the frictional loss in the conduit or pipe. So, need a correction in the equation.



$$\frac{P_A}{\gamma} + \frac{V_A^2}{2 \cdot g} + Z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2 \cdot g} + Z_B + h_{Lf}$$

where h_{Lf} is the fictional loss in the pipe. So, we need to determine the value of h_{Lf}. If the size or diameter of the pipe is constant, then $V_A = V_B$ and also for the same elevation, $Z_A = Z_B$. The frictional loss or frictional head loss becomes,

$$h_{Lf} = \frac{P_A - P_B}{\gamma}$$

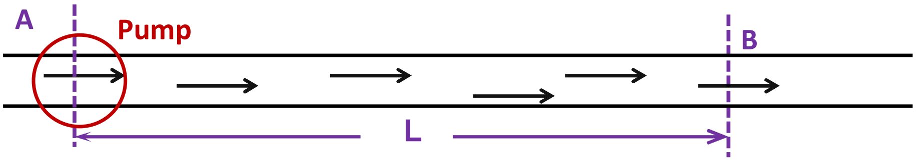
if the pressure difference between two points or location is known, then we canculate the frictional head loss in the pipe. In reality, frictional loss occurs due to surface friction in the pipe, viscostiy of the fluid and types of flow i.e. flow either laminar or turbulent.

How to Calculate the Frictional Head Loss in the Pipe

- There are some other equations to calculate the frictional head loss in the pipe. The frictional head in the pipe depends on type flow such as Laminar flow or Turbulent flow.
- Whether the flow in turbulent or laminar can be determined by the Reynolds's number.
- Those equations are
  - Hagen-Poisellie's euation and
  - Darcy-Weisbach Equation also
  - Darcy friction factor
- We also need to what is Reynolds's number??
- Finding flow criteria by Reynolds's number (whether flow is turbulent or laminar).

Application of Bernoulli's Equation (Situation-2)

Pumps are used in fluid system to ensure the flow. Still Bernoulli's equation is useful in addition of pump in the fluid system.

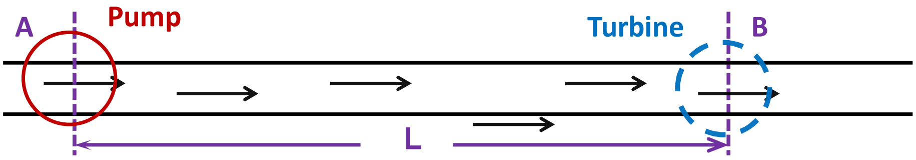


- What is the function of pump?? It added some energy to the fluid.
- *Fluid entering at point 'A' with some energy, then it is gaining some additional energy from the pump.*
- How much energy will be added to fluid by pump? It is equal to the head of the pump, h_{pump}.
- Apply Bernoulli's equation between point 'A' and 'B'.

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2 \cdot g} + Z_A + h_{pump} = \frac{P_B}{\gamma} + \frac{V_B^2}{2 \cdot g} + Z_B + h_{Lf}$$

Application of Bernoulli's Equation (Situation-3)

Pump and turbine both are added in the fluid system. Pump added energy to the fluid and Turbine absorb energy of the fluid to produce mechanical energy.



- The energy produces in the turbine is denoted by head of the turbine, $h_{turbine}$

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2 \cdot g} + Z_A + h_{pump} = \frac{P_B}{\gamma} + \frac{V_B^2}{2 \cdot g} + Z_B + h_{Lf} + h_{turbine} + e_{mech\_loss}$$

Efficiency of Pump and Turbine are given by,

$$\eta_{pump} = \frac{Q \cdot \rho \cdot g \cdot h_{pump}}{P_{power\_in}} \qquad \eta_{turbine} = \frac{P_{power\_out}}{Q \cdot \rho \cdot g \cdot h_{turbine}}$$