AHSANULLAH UNIVERSITY OF SCIENCE AND TECHNOLOGY (AUST)

# ME-3105: FLUID MECHANICS

(LC-3: PRESSURE MEASUREMENT)

BY

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## MEASUREMENT OF FLUID PRESSURE

Various *devices* adopted for measuring fluid pressure is classified into *two categories: (1). Manometers and (2). Mechanical Gages.*

**Manometers:** Manometer is a device which measures fluid pressure based on the principle of balancing the column of liquid (whose pressure is to be found) by the same or another column of liquid. Mercury, oil, alcohol and water can be use as Manometric fluid.

*Manometers are classified as*

*(a). Simple Manometers and*

*(b). Differential Manometers.*

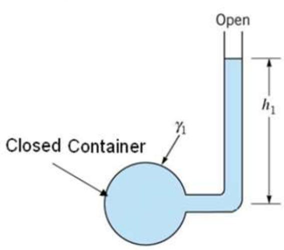
## MEASUREMENT OF FLUID PRESSURE

- ✓ *Simple Manometers* are used to measure pressure *at a point* in a fluid contain in a pipe or a vessel.
  - o Most common Simple Manometer are: (1). Piezometer, (2). U-tube Manometer and (3). Single Column Manometer.
- ✓ *Differential Manometers are used to measure the difference of pressure between any two points in a fluid contain in a pipe or a vessel.*
  - o Most common types are: (1). Two-Piezometer, (2). Inverted U-tube Manometer, (3). U-Tube Differential Manometer and (4). **Micro manometer.**

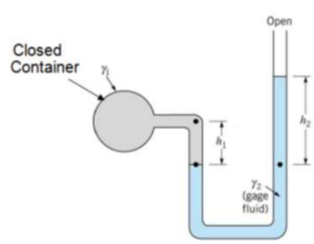


## MEASUREMENT OF FLUID PRESSURE

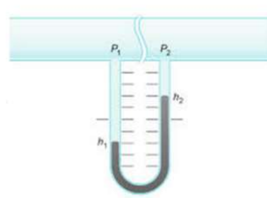
***Example of Manometer:***




Piezometer Tube



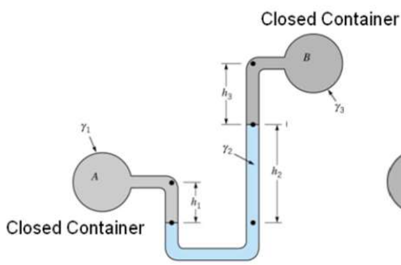

U-Tube Manometer



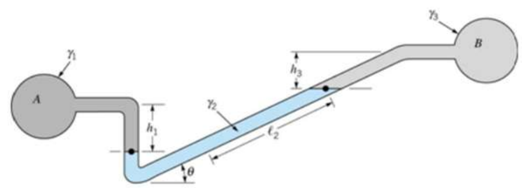

Differential Manometer

*MEASUREMENT OF FLUID PRESSURE*

**Example of Manometer:**




Differential U-Tube Manometer




Inclined-Tube Manometer

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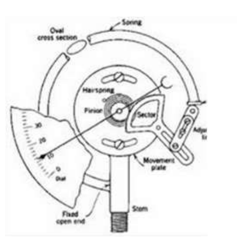
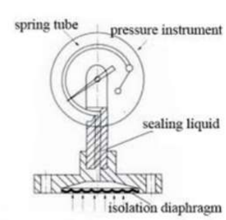
5

**MEASUREMENT OF FLUID PRESSURE**

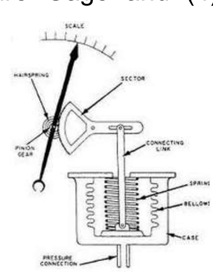
**Mechanical Gage:** Some of mechanical pressure gages which are commonly used are: (1). Bourdon Tube Pressure Gage; (2). Diaphragm Pressure Gage; (3). Bellows Pressure Gage and (4). Dead-weight Pressure Gage.



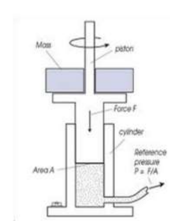
Bourdon Tube Pressure Gage




Diaphragm Pressure Gage




Bellows Pressure Gage



Dead-weight Pressure Gage

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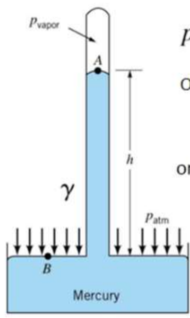
6

## MEASUREMENT OF ATMOSPHERIC PRESSURE

***Pressure:*** Barometer is used to measure atmospheric pressure. The first mercury barometer was constructed in 1643-1644 by Torricelli. He showed that height of mercury in a column was 1/14 that of a barometer, due to the fact that the *mercury is 14 times denser than water*. He also noticed that *level of mercury varied from day to day due to weather changes, and that at the top of the column there is a vacuum.*




$$p_{atm} = p_B = \gamma h + p_{vapor}$$

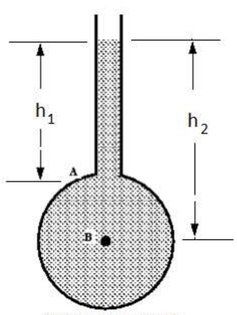
Often $p_{vapor}$ is very small, thus

$$p_{atm} \approx \gamma h = \rho g h$$

or, $p_B - \gamma h = p_{vapor}$ (going upward direction)



## MEASUREMENT OF FLUID PRESSURE BY MANOMETER

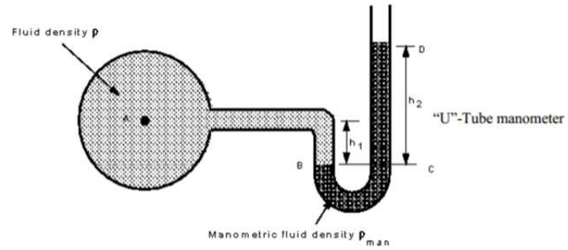



Simple Piezometer

Pressure at A, $p_A$ = pressure due to column of liquid above A $= \rho g h_1 + P_{atmp}$

Pressure at B, $p_B$ = pressure due to column of liquid above B $= \rho g h_2 + P_{atmp}$

## MEASUREMENT OF FLUID PRESSURE BY MANOMETER




Pressure in a continuous static fluid is the same at any horizontal level so,

Pressure at B = Pressure C i.e. $p_B = p_C$

Pressure at B, $p_B$ = Pressure at A + Pressure due to height $h_1$ of fluid $= p_A + \rho g h_1$

Pressure at C, $p_C$ = Pressure at D + Pressure due to height $h_2$ of manometric fluid $= p_{\text{Amospheric}} + \rho_{\text{man}} g h_2$

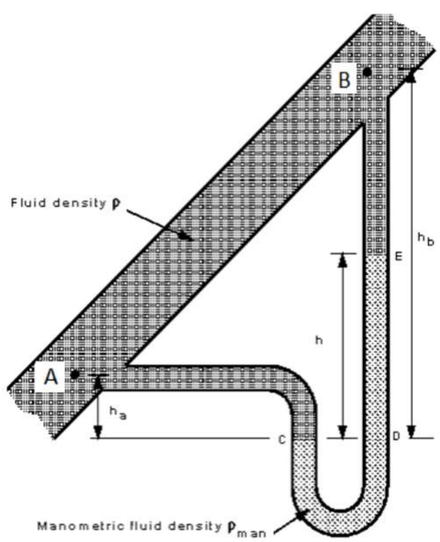
$$p_A + \rho g h_1 = p_{\text{Amospheric}} + \rho_{\text{man}} g h_2 \qquad p_A = \rho_{\text{man}} g h_2 - \rho g h_1 + p_{\text{Amospheric}}$$

Second approach, $p_A + \rho g h_1 - \rho_{\text{man}} g h_2 = p_{\text{Amospheric}}$ $\quad p_A + \rho g h_1 = p_{\text{Amospheric}} + \rho_{\text{man}} g h_2$

As we are measuring *gauge pressure* we can subtract $p_{\text{Atmospheric}}$ giving $\underline{p_A = \rho_{\text{man}} g h_2 - \rho g h_1}$



## MEASUREMENT OF FLUID PRESSURE DIFF. BY MANOMETER




$$p_C = p_A + \rho g h_a \qquad p_D = p_B + \rho g(h_b - h) + \rho_{\text{man}} g h$$

$$p_C = p_D \qquad p_A + \rho g h_a = p_B + \rho g(h_b - h) + \rho_{\text{man}} g h$$

Giving the pressure difference

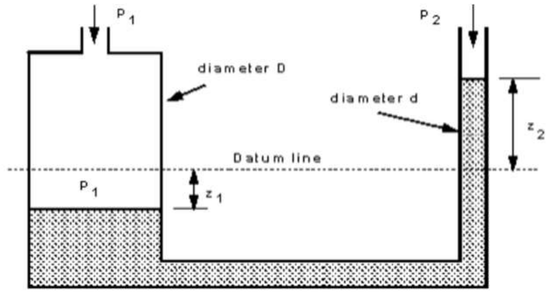
$$p_A - p_B = \rho g(h_b - h_a) + (\rho_{\text{man}} - \rho) g h$$

Second approach,

$$p_A + \rho g h_a - \rho_{\text{man}}\, g h - \rho g(h_b - h) = p_B$$

$$p_A - p_B = \rho g(h_b - h_a) + (\rho_{\text{man}} - \rho) g h$$

## MEASUREMENT OF FLUID PRESSURE BY MANOMETER (ONE READING)




The volume of liquid transferred from the left side to the right $= z_2 \times (\pi d^2/4)$

And the fall in level of the left side is $z_1 = \dfrac{\text{Volume moved}}{\text{Area of left side}} = \dfrac{z_2(\pi d^2/4)}{\pi D^2/4} = z_2\left(\dfrac{d}{D}\right)^2$

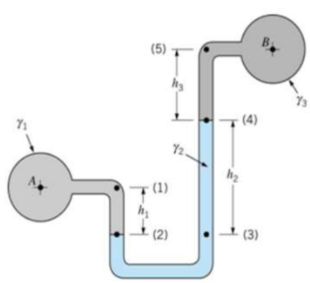
$p_1 - \rho g\,(z_1 + z_2) = p_2 \qquad p_1 - p_2 = \rho g\left[z_2 + z_2\left(\dfrac{d}{D}\right)^2\right] = \rho g z_2\left[1 + \left(\dfrac{d}{D}\right)^2\right]$

Clearly if D is very much larger than $d$ then $(d/D)^2$ is very small so $p_1 - p_2 = \rho g z_2$

So only one reading need be taken to measure the pressure difference.



## MEASUREMENT OF FLUID PRESSURE DIFF. BY MANOMETER

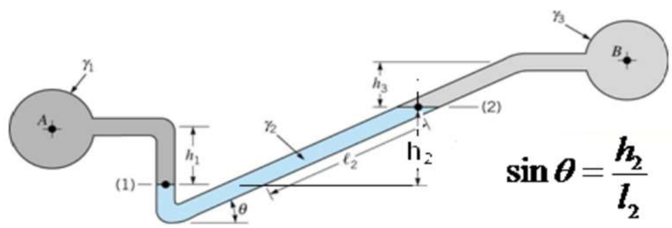



$$p_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = p_B$$
$$p_A - p_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1$$

Second approach,

$$P_2 = p_A + \gamma_1 h_1 \qquad P_3 = p_B + \gamma_3 h_3 + \gamma_2 h_2$$
$$P_2 = P_3 \qquad p_A + \gamma_1 h_1 = p_B + \gamma_3 h_3 + \gamma_2 h_2$$
$$p_A - p_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1$$

## MEASUREMENT OF FLUID PRESSURE DIFF. BY MANOMETER




$$\sin\theta = \frac{h_2}{l_2}$$

$$p_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = p_B$$

$$p_A + \gamma_1 h_1 - \gamma_2 \ell_2 \sin\theta - \gamma_3 h_3 = p_B$$

$$p_A - p_B = \gamma_2 \ell_2 \sin\theta + \gamma_3 h_3 - \gamma_1 h_1$$

**Pressure measuring by Manometer is very simple, accurate and no calibration is required.

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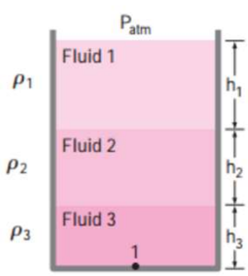
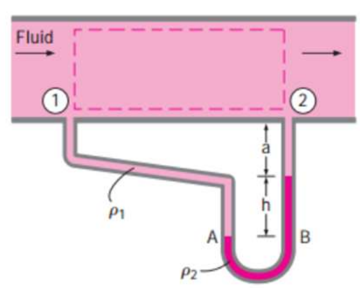
13

## MEASUREMENT OF FLUID PRESSURE DIFF. BY MANOMETER




$$P_{atm} + \rho_1 g h_1 + \rho_2 g h_2 + \rho_3 g h_3 = P_1$$




$$P_1 + \rho_1 g(a + h) - \rho_2 g h - \rho_1 g a = P_2$$

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14

## MEASUREMENT OF FLUID PRESSUE (PROLEM-1)

The water in a tank is pressurized by air, and the pressure is measured by a multifluid manometer as shown in Fig . The tank is located on a mountain at an altitude of 1400 m where the atmospheric pressure is 85.6 kPa. Determine the air pressure in the tank if $h_1 = 0.1$ m, $h_2 = 0.2$ m, and $h_3 = 0.35$ m. Take the densities of water, oil, and mercury to be 1000 kg/m³, 850 kg/m³, and 13,600 kg/m³, respectively.

**SOLUTION:**

Given,

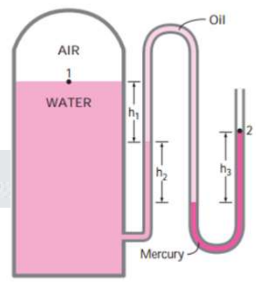
$\rho_{oil} = 850 \text{ kg/m}^3$

$\rho_{mercury} = 13{,}600 \text{ kg/m}^3$

$\rho_{water} = 1000 \text{ kg/m}^3$

$h_1 = 0.1$ m, $h_2 = 0.2$ m, $h_3 = 0.35$ m.

$P_{atm} = 85.6$ kPa $\quad P_1 = ?$






## MEASUREMENT OF FLUID PRESSUE (PROBLEM-1)

$$P_1 + \rho_{water}gh_1 + \rho_{oil}gh_2 - \rho_{mercury}gh_3 = P_{atm}$$

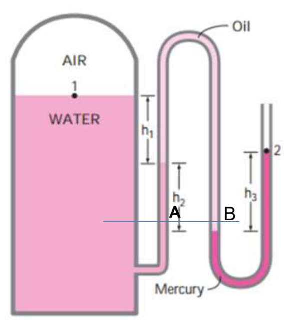
$$P_1 = P_{atm} - \rho_{water}gh_1 - \rho_{oil}gh_2 + \rho_{mercury}gh_3$$

$$= P_{atm} + g(\rho_{mercury}h_3 - \rho_{water}h_1 - \rho_{oil}h_2)$$

$$= 85.6 \text{ kPa} + (9.81 \text{ m/s}^2)[(13{,}600 \text{ kg/m}^3)(0.35 \text{ m}) - (1000 \text{ kg/m}^3)(0.1 \text{ m})$$

$$= 85.6 \text{ kPa} + (9.81 \text{ m/s}^2)[(13{,}600 \text{ kg/m}^3)(0.35 \text{ m}) - (1000 \text{ kg/m}^3)(0.1 \text{ m}) - (850 \text{ kg/m}^3)(0.2 \text{ m})]\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right)\left(\frac{1 \text{ kPa}}{1000 \text{ N/m}^2}\right)$$

$$= 130 \text{ kPa}$$

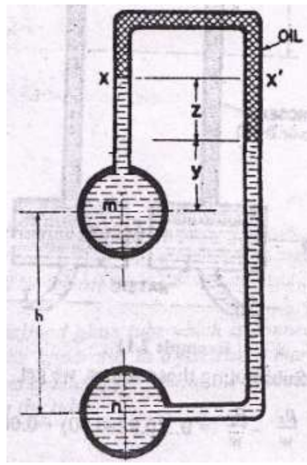



$P_A = P_1 + \rho_{water}gh_1 \quad P_B = P_{atm} + \rho_{mercury}gh_3 - \rho_{oil}gh_2$ (alternate method)

$1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2 \quad 1 \text{ kPa} = 1000 \text{ N/m}^2$

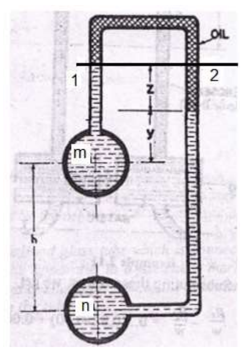
## MEASUREMENT OF FLUID PRESSUE (PROLEM-2)

Water fills the vessels shown in the figure below. Specific gravity of manometric liquid is 0.9 ; (a). Find the difference in pressure intensity at m and n when h = 1.25 m and z = 0.3 m; (b). Instead of water mercury in the vessel and manometric liquid has specific gravity of 1.6; find in the pressure intensity at m and n if h = 0.6 m and z = 1.0 m.






## MEASUREMENT OF FLUID PRESSUE (PROBLEM-2)




Problem (a):

Given, $h = 1.25\,m$ $z = 0.3\,m$ $SG_m = 0.9$

$\rho_w = 1000\,\frac{kg}{m^3}$ $\Delta P_{mn} = ?$

Solution:

$SG_m = \frac{\rho_m}{\rho_w}$ $\rho_m = 0.9\,(1000)\,\frac{kg}{m^3} = 900\,\frac{kg}{m^3}$

$$P_1 = P_m - \rho_w g\,(z + y)$$

$$P_2 = P_n - \rho_w g\,h - \rho_w g\,y - \rho_m g\,z$$

$$P_1 = P_2$$

$$P_m - \rho_w g\,(z + y) = P_n - \rho_w g\,h - \rho_w g\,y - \rho_m g\,z$$

$$P_m - P_n = \rho_w g\,z + \rho_w g\,y - \rho_w g\,h - \rho_w g\,y - \rho_m g\,z$$

## MEASUREMENT OF FLUID PRESSUE (PROBLEM-2)

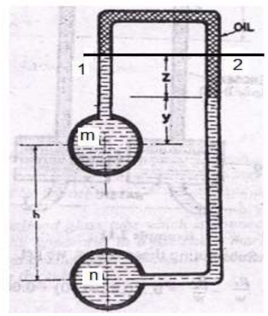
Problem (a) Cont'd

$$P_m - P_n = \rho_w g(z-h) - \rho_m g z$$

$$P_m - P_n = \left(1000\,\frac{kg}{m^3}\right)\left(9.81\,\frac{m}{s^2}\right)(0.3\,m - 1.258\,m) - \left(900\,\frac{kg}{m^3}\right)\left(9.81\,\frac{m}{s^2}\right)(0.3\,m)$$

$$P_m - P_n = -12.05 \times 1000 \cdot \left(\frac{kg \cdot m}{s^2}\right) \cdot \frac{1}{m^2} \qquad \left(1N = \frac{kg \cdot m}{s^2}\right)$$

$$P_n - P_m = 12.05 \times 1000\,\frac{N}{m^2} = 12\,\frac{KN}{m^2}$$






## MEASUREMENT OF FLUID PRESSUE (PROBLEM-2)

Problem (b):

Given, $h = 0.60\,m$ $\quad z = 1.0\,m$ $\quad SG_m = 1.6$

$\rho_w = 1000\,\frac{kg}{m^3}$ $\quad SG_{mc} = 13.6$ (mercury)

$SG = \frac{\rho_{fluid}}{\rho_w}$ $\quad \Delta P_{mn} = ?$

Solution:

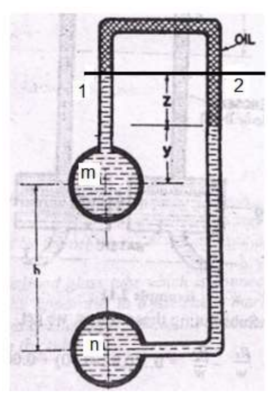
$$SG_m = \frac{\rho_m}{\rho_w} \qquad \rho_m = 1.6\,(1000)\,\frac{kg}{m^3} = 1600\,\frac{kg}{m^3}$$

$$SG_{mc} = \frac{\rho_{mc}}{\rho_w} \qquad \rho_{mc} = 13.6\,(1000)\,\frac{kg}{m^3} = 13600\,\frac{kg}{m^3}$$

$$P_1 = P_m - \rho_{mc} g (z+y)$$

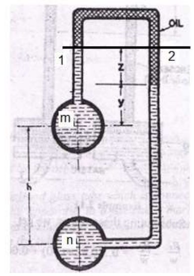
$$P_2 = P_n - \rho_{mc} g h - \rho_{mc} g y - \rho_m g z$$

$$P_1 = P_2$$



### MEASUREMENT OF FLUID PRESSUE (PROBLEM-2)

$$P_1 = P_m - \rho_{mc}\, g\,(z+y)$$
$$P_2 = P_n - \rho_{mc}\, g\, h - \rho_{mc}\, g\, y - \rho_m\, g\, z$$
$$P_1 = P_2$$




$$P_m - \rho_{mc}\, g\,(z+y) = P_n - \rho_{mc}\, g\, h - \rho_{mc}\, g\, y - \rho_m\, g\, z$$
$$P_m - P_n = \rho_{mc}\, g\, z + \rho_{mc}\, g\, y - \rho_{mc}\, g\, h - \rho_{wc}\, g\, y - \rho_m\, g\, z$$
$$P_m - P_n = \rho_{mc}\, g\,(z-h) - \rho_m\, g\, z$$
$$P_m - P_n = \left(13600\,\frac{kg}{m^3}\right)\left(9.81\,\frac{m}{s^2}\right)(1.0\,m - 0.60\,m) - \left(1600\,\frac{kg}{m^3}\right)\left(9.81\,\frac{m}{s^2}\right)(1.0\,m)$$
$$P_m - P_n = 37.67 \times 1000 \cdot \left(\frac{kg\cdot m}{s^2}\right)\cdot\frac{1}{m^2}$$
$$P_m - P_n = 37.67 \times 1000\,\frac{N}{m^2} = 38\,\frac{KN}{m^2}$$

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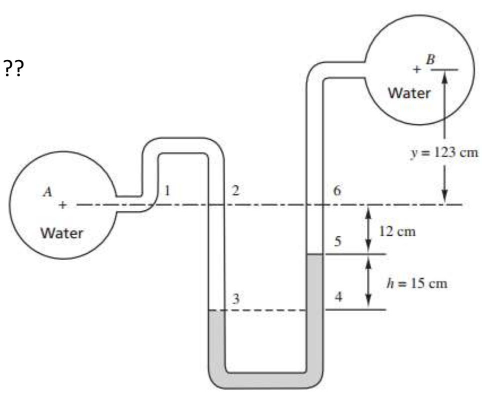
### MEASUREMENT OF FLUID PRESSUE (PROBLEM-3)

A mercury manometer (sp. gr. = 13.6) is used to measure the pressure difference in vessels A and B, as shown in the Figure below. Determine the pressure difference in pascals.

Write Manometric Equation ??

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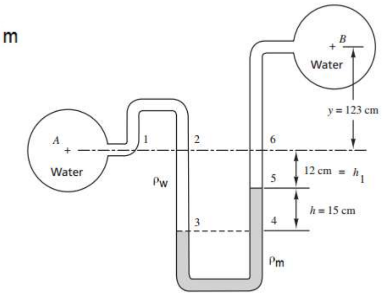
## MEASUREMENT OF FLUID PRESSUE (MANOMETER PROBLEM-3)

Given, $h = 15.0\,\text{cm} = 0.15\,\text{m}$ $\quad y = 123\,\text{cm} = 1.23\,\text{m}$

$h_1 = 12.0\,\text{cm} = 0.12\,\text{m}$ $\quad \rho_w = 1000\,\dfrac{\text{kg}}{\text{m}^3}$

$SG_m = 13.6 = \dfrac{\rho_m}{\rho_w}$ $\quad \Delta P_{AB} = ?$




Solution:

$$P_3 = P_A + \rho_w g\left(h + h_1\right)$$

$$P_4 = P_B + \rho_w g\left(y + h_1\right) + \rho_m g\,h$$

$$P_3 = P_4$$

$$P_A = P_1 = P_2$$

$$SG_m = \frac{\rho_m}{\rho_w} = 13.6 \qquad \rho_m = 13.6\,\rho_w$$

$$P_A + \rho_w g\left(h + h_1\right) = P_B + \rho_w g\left(y + h_1\right) + \rho_m g\,h$$



## MEASUREMENT OF FLUID PRESSUE (PROBLEM-3)

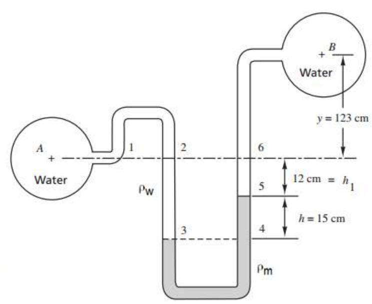
$$P_A + \rho_w g\left(h + h_1\right) = P_B + \rho_w g\left(y + h_1\right) + \rho_m g\,h$$

$$P_A - P_B = \rho_w g\left(y + h_1\right) + \rho_m g\,h - \rho_w g\left(h + h_1\right)$$

$$\Delta P_{AB} = \rho_w g\,(y - h) + \rho_m g\,h$$

$$\Delta P_{AB} = \rho_w g\,(y - h) + \left(13.6\rho_w\right) g\,h$$

$$\Delta P_{AB} = \rho_w g\,(y - h + 13.6\,h) = \rho_w g\,(y + 12.6\,h)$$




$$\Delta P_{AB} = \left(1000\,\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)(1.23\,\text{m} + 12.6 \times 0.15\,\text{m})$$

$$\Delta P_{AB} = 30.61 \times 1000 \cdot \left(\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\right) \cdot \frac{1}{\text{m}^2} \qquad \left(1\text{N} = \frac{\text{kg}\cdot\text{m}}{\text{s}^2}\right)$$

$$P_{AB} = 30.61 \times 1000\,\frac{\text{N}}{\text{m}^2} = 30.61 \times 1000\,\frac{\text{N}}{\text{m}^2} \qquad \left(1\text{Pa} = \frac{\text{N}}{\text{m}^2}\right)$$

$$P_{AB} = 30.61 \times 1000\,\text{Pa} = 30.61\,\text{Kpa}$$