# Hierarchical Bayesian models in accounting: A tutorial

Bullipe R. Chintha and Sanjay Kallapur*

**Abstract**

Accounting parameters such as earnings response coefficients (ERC) are generally heterogeneous across firms. When panel data is available, the parameters are typically estimated using OLS with either pooled data which ignores parameter heterogeneity, or using firm-specific observations which tends to give noisy estimates. An alternative is to use Bayesian hierarchical models which preserve parameter heterogeneity but have the advantage of being less noisy than firm-specific OLS. Their advantage stems from the use of data about all other firms to form an informative prior about each firm's parameter. In this paper, using a sample of 301 firms we compare the results from three Bayesian hierarchical models to OLS-based ERCs. Our results show that the Bayesian models produce ERCs that reduce the number of negative ERCs from 48 to 6 and lower mean squared error in a hold-out sample by more than 90%. We show the formulation and estimation of these models step by step, with a discussion of the Stata commands involved. The Stata as well as R code is available online. We conclude by discussing the potential for such models to be applied to other contexts in accounting research.

**Keywords**: Hierarchical Bayesian models, Earnings Response Coefficients, Empirical priors

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* The authors are at the Indian School of Business. They can be reached at bullipereddy_chintha@isb.edu, and sanjay_kallapur@isb.edu.

## 1 Introduction

Chapter 4 of the monograph (Kallapur 2022) explains that hierarchical Bayesian models are useful for panel data where one has several observations on each of multiple units which are heterogeneous, e.g., multiple days' returns for each of several different firms to estimate their *firm-specific* stock betas. In such cases, the estimates from other units can be used as informative priors for Bayesian estimation of unit-specific parameters, i.e., beta estimates of other firms can be used as informative priors for the given firm's beta. The use of such information can substantially improve the efficiency of the estimates.

The monograph describes the hierarchical Bayesian estimation of betas by Jorion (1986) and Karolyi (1992). Hierarchical Bayesian models are flexible enough to allow the firm-specific estimates to depend on firm and time-specific determinants—e.g., Cosemans et al. (2016) model the firm-specific betas as depending on firm-specific variables such as size, time-specific business cycle variables, as well as an interaction of the two. Several Accounting contexts lend themselves well to hierarchical Bayesian estimation--Landsman and Damodaran (1989a; 1989b) make this point in the context of James-Stein shrinkage estimation, which is a precursor to hierarchical Bayesian estimation. The purpose of this online appendix is to illustrate the technique, along with the computation involved, in the context of a typical case in accounting, namely estimation of firm-specific earnings response coefficients (ERCs).[1]

We start with the simplest hierarchical model, and then progressively introduce refinements. We explain the purpose of each model, the estimation details and interpretation of the results. Snippets of the computer code in Stata are given here and the full code is available on https://github.com/skallapur/AAA_monograph. The analogous R code for building these models is also available there, but we do not discuss it in this paper.

## 2 The Frequentist approach: Firm-specific ERCs based on OLS estimation

We begin by downloading financial statement data from Compustat and restricting the sample to the 1980 - 2020 period. We filter out firms with book assets of less than $1.5 million, common shareholders' equity of less than $0.5 million and a fiscal-end share price of less than $3. Finally, we restrict our observations to non-financial, non-utility firms with December fiscal ends and with at least 30 years of uninterrupted time-series data. For this sample, we obtain returns (over the financial year) and earnings levels and changes deflated by lagged market cap. The purpose of restricting the sample

[1] The results of a simple hierarchical Bayesian estimation of ERC (same as Model HBM1 below) have been included in the monograph. For this appendix, our sample is different; hence, some numbers do not match exactly.

to firms with at least 30 years is to ensure that we have enough observations to estimate firm-level OLS ERCs after keeping aside enough observations to serve as a hold-out sample.

Our final sample consists of 301 firms. For this list of firms, we create a test sample using the first 15 years' data and a holdout sample of the remaining years' observations. For this example, we calculate ERC as the coefficient obtained by regressing returns ($ret$) on earnings change deflated by lagged market cap, $\Delta E$, as in the classic models such as Beaver, Lambert and Morse (1980).[2]

Since each firm's ERC could differ, we begin by calculating separate ERCs as $b\_ols_j$ in firm-specific OLS regressions $ret_{jt} = a_j + b\_ols_j \Delta E_{jt}$. The firm-level OLS ERCs for the test sample (ERC_ols) and the hold-out sample (ERC_ols_post) are presented in columns 2 and 3 of Table 6. Consistent with previous research, the firm-specific OLS estimates are noisy: e.g., although they are expected to be positive, 48 of the 301 ERC_ols values, and 43 ERC_ols_post values are negative. The distribution and some characteristics of these coefficients are presented in the following table.

**Table 1: Descriptive statistics of firm specific ERC estimates in the test and holdout periods**

| | ERC_ols | ERC_ols_post |
|---|---|---|
| Number of firms | 301 | 301 |
| Mean | 4.79 | 2.61 |
| Median | 2.00 | 1.60 |
| Maximum | 184.80 | 24.83 |
| Minimum | -11.82 | -4.65 |
| Standard Deviation | 14.22 | 3.86 |
| # negative values | 48 | 43 |
| MSE using ERC_OLS as predictor for ERC_ols_post | 210.28 | |

Taken at face value, the firm whose calculated ERC is 184.8 (maximum value), has a 50% chance that its actual ERC is even greater than 184.8 (assuming symmetric distribution, given that 184.8 is an unbiased estimate of the mean), which is implausible because all other firms' ERCs are much lower. In other words, the ERC of other firms gives us some information about this particular firm's ERC. Kallapur (2022, 32-33) shows that the James-Stein estimator shrinks each firm's estimators towards the grand mean, and Landsman and Damodaran (1989a,b) use this method in the accounting context. The amount of shrinkage is a function of the precision of each firm's ERC estimate (as computed from the regression equation), and the 'between-firms' variance which is

[2] These days it is more common to estimate ERCs using both earnings levels and changes as explanatory variables, but to keep the model simple for pedagogical purposes, we use only earnings changes.

calculated by partitioning the total sum of squares into 'within' and 'between' sum of squares. In the following models however, we will compute the firm shrinkage ERCs in the Bayesian framework which is more flexible as well as more justifiable theoretically.

In the last row of the table, we present the mean squared forecast error (MSE) between the OLS ERC estimated in our test period and the holdout period. The MSE is 210.28. In the following sections, we demonstrate how Bayesian models are able to tremendously improve on the forecast error by incorporating the firm shrinkage discussed in the previous paragraph.

## 3 The Bayesian approach: ERCs based on hierarchical Bayesian Model estimation

### 3.1 Hierarchical Bayesian Model 1: Shrinking firm-level ERCs towards a grand mean

In our first Bayesian model, we assume that the estimated OLS ERCs, $\widehat{b\_ols_j}$ are distributed with unknown firm-specific means $b_j$ and known firm-specific variances $\sigma_j^2$. In fact, we do not know $\sigma_j^2$, but do know the sample standard error $se\left(\widehat{b\_ols_j}\right)$ = $s_j$ from the OLS regression which we will substitute for $\sigma_j^2$. If each firm's OLS regression had 30 or more observations, the normal distribution assumption would be okay. But in our case, each firm's number of observations is 15, so the degrees of freedom are 13, which is much less than 30, so we will instead assume that $\widehat{b\_ols_j}$ has a t-distribution.

The unknown $b_j$s are themselves assumed to be distributed normally with an unknown grand mean $\mu_b$ and unknown standard deviation $\sigma_b$. This assumption of a common distribution for $b_j$s is the key that enables other firms' data to be used to 'shrink' each firm's $b_j$ towards the grand mean. It is justified by what is known as 'exchangeability,' explained next.

For now, all we know about the firms is their Compustat identifier gvkey (we will relax the assumption later and bring in information about determinants of ERCs), e.g., the firm with gvkey 1078 has $b\_\widehat{ols_{j=1}} = 28.09$, and the one with gvkey 1186 has $b\_\widehat{ols_{j=2}} = 0.5$. At least for those of us who cannot make out the firm's name and other details from only its Compustat identifier, the gvkey labels are exchangeable—we would not be able to tell if the values were interchanged. Making the Bayesian assumption that any unknown quantity can be considered to have a distribution, exchangeability implies that the unknown $b_j$ values are random drawings from a *common distribution*, as modeled. If we know more details relevant to each firm's ERC then they will not be random drawings, but they can still be modeled as being random *conditional* on the relevant details, i.e., as functions of the relevant details plus error terms that are exchangeable, as we will do later.

The parameters to be estimated in the model are a vector of 301 $b_j$ values, one for each firm, and the parameters of their prior distribution, $\mu_b$ and $\sigma_b$. Although we have some priors about ERCs based on theoretical considerations (e.g., that they cannot be negative), we will let the data determine the values of these parameters, by giving uninformative prior distributions for $\mu_b$ and $\sigma_b$. We are interested in the posterior distributions of parameters $b_j$. The data for this estimation are the 301 $\widehat{b\_ols_j}$ values and their associated standard errors, $s_j$.

In some cases, such as with normal priors and normal likelihoods, the posterior distributions can be worked out analytically; in most other cases, however, analytical derivation of the posteriors is not possible and numeric simulations must be used instead. Markov Chain Monte Carlo (MCMC) sampling is a widely used simulation-based method for estimating these posterior distributions. The methodology produces a "chain" of simulated values for the model parameters, which represents a sample from the posterior distribution as long as the chain "converges".

Broadly, it works as follows. Representing the parameters by a vector $\theta$, and the data by $y$, we wish to determine the density function $p(\theta|y)$. By Bayes theorem it equals $p(\theta) \cdot p(y|\theta)/p(y)$. We have specified the priors for $p(\theta)$, which is therefore known to start with. We have also specified the functional form of the distribution for $y$ given $\theta$ (i.e., for b_ols as the t-distribution, discussed above), so for any given $\theta$ value it can be calculated. The whole issue is with $p(y)$, which needs to be calculated by integrating $\int_\theta \; p(y|\theta)d\theta$ over all possible values of $\theta$. Fortunately, however, because $\theta$ is integrated out, the result does not depend on it, so for two different values of $\theta$, say $\theta_{t-1}$ and $\theta_t$, the likelihood $p(\theta_t|y)/p(\theta_{t-1}|y)$ does not depend on $p(y)$–the latter cancels out.

This property is exploited as follows for numerical simulation (which gives the *Monte Carlo* part of MCMC's name). The user supplies initial values for each parameter, i.e., element of the vector $\theta_t \; for \; t = 0$. The program then randomly chooses a new value to come up with a candidate $\theta^*_{t+1}$, and computes the likelihood ratio $p(\theta_{t+1}|y)/p(\theta_t|y)$, which works out to, say, $r$. With probability $r$ (or 1 if $r > 1$), it sets $\theta_{t+1} = \theta^*_{t+1}$, and with probability $1 - r$ it keeps $\theta$ unchanged, i.e., $\theta_{t+1} = \theta_t$ (but that still counts as an iteration). The whole process is iterated for as many times as the user specifies. The *Markov chain* part of MCMC's name comes from the fact that $\theta_{t+1}$ depends only on $\theta_t$, i.e., the immediately preceding value, through sampling from a jump distribution. It has been proved that this process converges to the true distribution of $p(\theta|y)$, whereupon the $\theta$ values generated are in proportion to their probabilities $p(\theta|y)$. These $\theta$ values therefore provide its empirical distribution, and we can determine any parameter of interest, such as mean values and quantiles, simply by calculating them from the vector of generated values.

In this paper, we demonstrate how to program the hierarchical Bayesian models using the `bayesmh` command of Stata. The `bayesmh` command performs the above discussed MCMC estimation using an adaptive random-walk Metropolis-Hastings (MH) algorithm with optional blocking of parameters (Andrieu and Thoms, 2008). It also supports Gibbs sampling in scenarios where a block of parameters has a conjugate pair. For example, in normal-normal or normal-inverse gamma likelihood-prior configurations. Detailed documentation about `bayesmh` can be found at https://www.stata.com/manuals/bayesbayesmh.pdf. To assist exposition, we provide relevant snippets of Stata code as we go about building the models. The complete Stata code to input Compustat and risk-free data and output ERC estimates based on all the models discussed in the paper is available at https://github.com/skallapur/AAA_monograph.

We begin with our first model, which we refer to as Hierarchical Bayesian Model 1 (HBM1). The model is hierarchical because hyperpriors for $\mu_b$ and $\sigma_b$ determine $b_j$, which in turn becomes a prior for the data $b\_ols_j$. The prior and the data enable us to derive a posterior distribution for $b_j$. Although $b_j$'s are random *conditional* on $\mu_b$ and $\sigma_b$, the latter are determined by all firms' $b_i$'s for $i \neq j$. Thus, the data for other firms gets used to determine an informative prior for $b_j$. As mentioned above, the input data for HBM1 consists of a vector of estimated OLS ERCs (ERC_ols) and their sampling variance, $s_j^2$ (ERC_var).

We start by importing the input processed data file into Stata using

```
. import delimited "Temp/Input_for_HBM1.csv", clear case(preserve)
```

The input file consists of firm-level identifiers (gvkey), and their OLS ERCs (ERC_ols) and sample variances (ERC_var), estimated on the test sample i.e., the data of the first 15 years in our sample dataset. The below compound command is used to estimate our HBM1.

```
. bayesmh ERC_ols, noconstant ///
reffects(gvkey) ///
likelihood(t(ERC_var,13)) ///
prior({ERC_ols:i.gvkey}, normal({mu},{sig2})) ///
prior({mu}, normal(0, 1e6)) ///
prior({sig2}, igamma(0.001, 0.001)) ///
block({mu}) block({sig2}) adapt(tolerance(0.002)) ///
showreffects({ERC_ols:i.gvkey}) ///
nchains(3) ///
initall({ERC_ols:} rnormal(0, 100) {mu} rnormal(0, 100) {sig2} runiform(0,
100)) ///
mcmcsize(5000) burnin(5000) rseed(149)
```

We break down the above syntax into seven arguments. First, the `bayesmh` command is immediately followed by the name of our dependent variable "ERC_ols". The `noconstant` option specifies that the algorithm suppress the intercept term from the regression model (The `bayesmh` command by default includes the model intercept parameter). Second, the `reffects()` option tells the algorithm how the model parameters are grouped. We use our Compustat identifier "gvkey" here. In this model, all the gvkey-level parameters are treated as random-effects parameters. There are two ways to define random effects in `bayesmh`. Here, we use the global `reffects` option which allows us to have different random effects at each observation level. In HBM2 and HBM3, we define random effects using the `block()` sub option.

Third, we define the likelihood function as a t-distribution with variance ERC_var and 13 degrees of freedom. When defining likelihood functions using `bayesmh`, we do not need to specify the mean in the likelihood function. Fourth, the next three lines define our priors. The prior of each firm ERC is defined using the `prior()` option as a normal distribution with an unknown mean mu and unknown variance sig2, representing $\mu_b$ and $\sigma_b^2$. These terms need to be defined, by giving them their own priors. So, using a hierarchical structure, we define uninformative priors for the hyper-parameter mu as a normal with a zero mean and a high variance 1e6. Similarly, we define an uninformative prior for sig2 as an igamma distribution with parameters 0.001 and 0.001.

Fifth, the `block` function is used to specify a block of model parameters for the MH algorithm, and the adapt function is used to define the adaptation tolerance level for the MH algorithm. Blocking and adaptation are functional aspects of the MH algorithm.[3] Optimal "blocking", consisting of specifying approximately independent model parameters in separate blocks, can improve the efficiency of the MH algorithm. In this specification, we specify two blocks - one each for mu and sig2.

Sixth, the `showreffects` option is used to display all the random effects parameters in the output. It ensures that the output displays the individual firm-level (gvkey-level) Bayesian ERCs. Finally, we provide several arguments to input details about the MCMC simulation procedure. In this example, we are employing three separate chains using the `nchains` function[4] and specify random initial states for these chains using `initall()`. As discussed before, we dispose of some early MCMC simulations to allow for convergence. In this example, we "burn" the first 5000 simulations and use the next 5000 simulations for parameter estimation. The `rseed()` option is used to set a random seed for replication.

The above `bayesmh` command will output the model summary, simulation details including number of iterations and MCMC efficiency and the distributional statistics for each parameter as shown below.[5]

**Commented [SK1]:** Please show this--not all 301 values, but their summary.

**Commented [BC2R1]:** I have added the output

[3] For a detailed treatment of how blocking and adaptation settings can affect MCMC procedure in the MH algorithm, refer to https://www.stata.com/manuals/bayesbayesmh.pdf.

[4] Note that running multiple chains happens sequentially in Stata and may result in substantial computation times. The user contributed package `bayesparallel` may be used to simulate chains simultaneously.

[5] To translate this output from Stata to word, we use the `dyndoc` command of Stata. Further, for brevity the output below suppresses all firm-level random effects parameters except those pertaining to gvkeys 1078 and 1186.

**Table 2: HBM1 Output**

```
Model summary
------------------------------------------------------------------------------
Likelihood:
  ERC_ols ~ t(xb_ERC_ols,ERC_var,13)

Prior:
  {ERC_ols:i.gvkey} ~ normal({mu},{sig2})                                  (1)

Hyperpriors:
    {mu} ~ normal(0,1e6)
  {sig2} ~ igamma(0.001,0.001)
------------------------------------------------------------------------------
(1) Parameters are elements of the linear form xb_ERC_ols.

Bayesian t regression                          Number of chains    =         3
Random-walk Metropolis-Hastings sampling       Per MCMC chain:
                                                   Iterations      =    10,000
                                                   Burn-in         =     5,000
                                                   Sample size     =     5,000
                                               Number of obs       =       301
                                               Avg acceptance rate =     .3652
                                               Avg efficiency: min =    .01086
                                                               avg =     .1189
                                                               max =     .1847
Avg log marginal-likelihood =  -892.9405      Max Gelman-Rubin Rc =      1.01

------------------------------------------------------------------------------
             |                                                Equal-tailed
             |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
ERC_ols      |
       gvkey |
       1078  |  1.661373   1.232934   .024597   1.668004  -.7865177    4.27916
       1186  |  .8648708   .7455379   .019547   .8552456  -.5802389   2.314417
[Other firms data omitted]
-------------+----------------------------------------------------------------
          mu |  1.577402   .1441721   .009281    1.57198   1.308562   1.870329
        sig2 |  1.476278   .3912515   .030656   1.440775   .8370693   2.342603
------------------------------------------------------------------------------
```

The above output shows that, for firm with gvkey 1078 (for which ERC_ols is an implausible 28), the Bayesian point estimate of ERC (based on the mean of the posterior distribution) is 1.66 and the 95% credible interval is (-0.8, 4.3) (2.5 and 97.5 percentiles). The outputted mean HBM1 ERC coefficients for all firms are listed in column 4 of Table 6. The output also provides the mean estimates of the hyperparameters mu and sig2. The displayed distributional statistics themselves were estimated from the MCMC simulations that can be saved using the below command.

```
. bayesmh, saving (Simdata/simdata_model1.dta, replace)
```

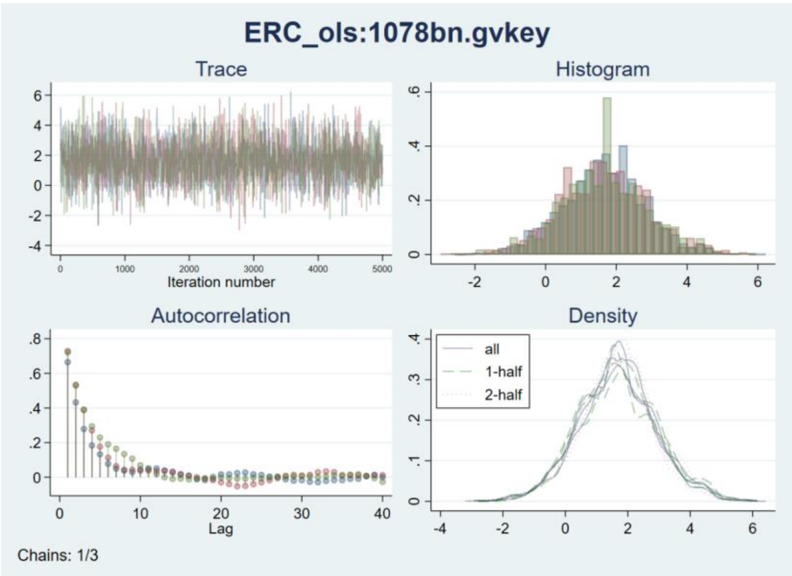
It is useful to save the simulations as such data may be used to construct transformations of the distribution or estimate other summary statistics like 25th percentile parameter which is not displayed by default. We use this saved simulation data to compare the HBM1 ERC estimates with those of OLS ERCs and other Bayesian models. But before we do this, we need to be convinced about the convergence of the MCMC simulations based on which the reliability of the posterior distributions can be assumed. First, we want to check whether there is convergence between the three MCMC chains. If the chains do not sufficiently converge, the parameter estimates could be biased and highly inaccurate.  A popular methodology to investigate convergence is through the Gelman-Rubin convergence diagnostic, which is a measure of the ratio of variance between chains to that within chains, and should be close to 1, below a suggested critical value of 1.1. The following Stata command estimates the Gelman-Rubin convergence ($R_c$) for our model.

```
. bayesstats grubin
```

The output presents $R_c$ for all 301 firm-level parameters and for mu and sig2.  Among these, the maximum Gelman-Rubin $R_c$ is 1.0096,[6] which is well below the critical value of 1.1 indicating convergence. Second, for checking the convergence and diagnostics of a parameter of interest graphically, we can use the command `bayesgraph diagnostic`. For instance, if we want to look at the diagnostics of the Bayesian ERC estimate of the earlier discussed firm with gvkey 1078, we use the below command.

```
. bayesgraph diagnostic {ERC_ols: 1078.gvkey}
```

**Figure 1: Diagnostic plots- HBM1 ERC parameter estimate for firm with gvkey 1078**



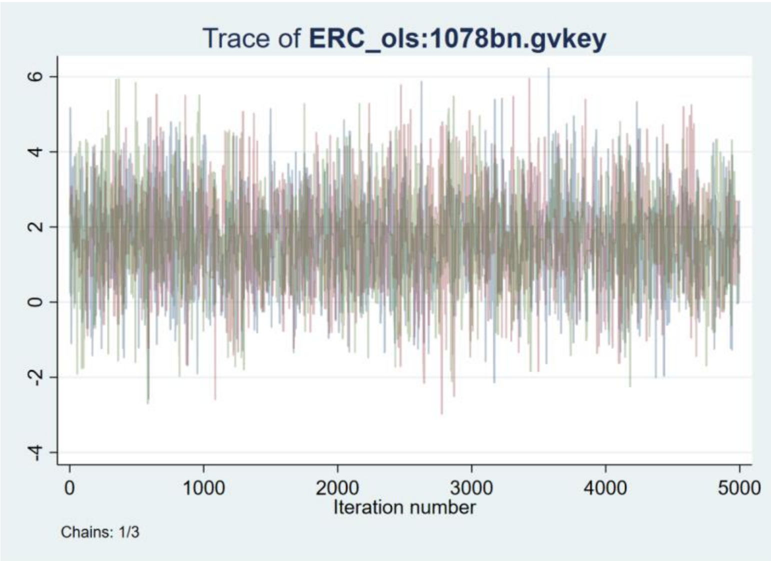

[6] It was shown in the Table 2 output as equaling 1.01. The `grubin` command helps whenever the maximum value is high, and one wants to check which parameters are not converging.

The output consists of four plots based on all three MCMC chains with each chain being represented by a different colour. On the top left is a trace plot which connects the consecutively simulated parameter values and graphs them against the iteration number. For instance, in our plot, because we throw away the initial 5000 runs out of a total 10,000 simulations, the x-axis has a scale between 0 to 5000 (representing observations 5,001 to 10,000). A lack of sparseness, unidentifiable trends, and a dense plotting indicates that the parameter of interest is a well-mixing parameter. In other words, the MCMC chain has a probability of jumping from any point to any point (i.e., it is not going to only nearby points), and it returns to the starting point in finite time, which is a condition for the Markov chain to converge.

On the bottom left is the autocorrelation plot which shows that the autocorrelation is dramatically dropping with increasing lags suggesting good convergence. Finally, the histogram on the top right suggests that the distribution is approximately normal, and the density plot suggests that there is demonstrable overlap between the first half, second half and the overall set of simulations of the parameter for all the multiple simulated chains. If the first half and the second half densities are largely different, we may further increase the burn size to allow for convergence. For examining any of these individual plots, such as the trace plot closely, one may use the below command.[7]

```
. bayesgraph trace {ERC_ols: 1078.gvkey}
```

**Figure 2: Trace plot- HBM1 ERC parameter estimate for firm with gvkey 1078**

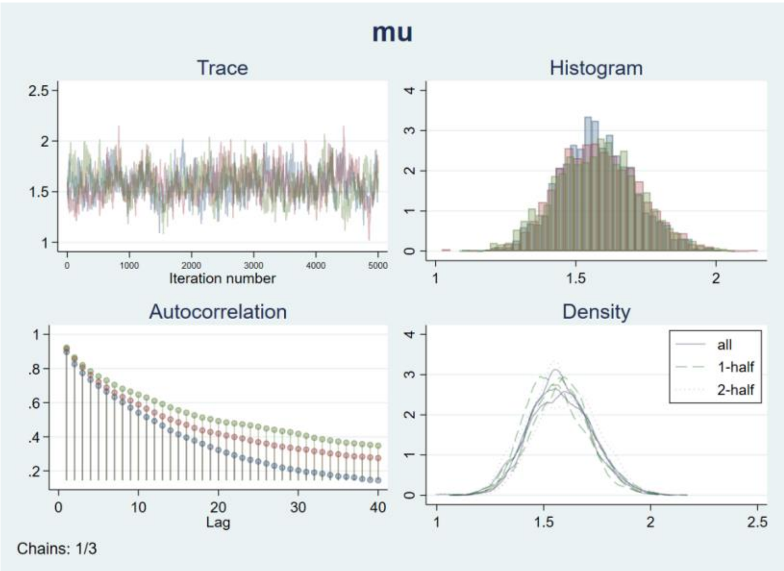



[7] Similarly, for an individual density plot, use `bayesgraph kdensity {mu}`

We may also study the diagnostic plots of the hyperparameters that we defined in our `bayesmh` framework. For instance, the following command outputs the diagnostics for the hyper-parameter $\mu_b$. The diagnostic plots in Figure 3 suggest that there is decent convergence and no major issues with the HBM1 estimation of the hyperparameter $\mu_b$, even though the autocorrelation drop in all chains is not as rapid as in the case of figure 1.

```
. bayesgraph diagnostic {mu}
```

**Figure 3: Diagnostic plots for hyperparameter $\mu_b$**




The final diagnostic that we would like to look at before interpreting the results is the MCMC efficiency, estimated as the ratio of the effective sample size to the MCMC sample size. The effective sample size is a measure of the independent observations in the MCMC sample size. As a rule of thumb, an efficiency of greater than 0.10 is considered as good and anything smaller than 0.01 may be re-evaluated. In our `bayesmh` output in Table 2, the average MCMC efficiency is displayed as 0.1189 which is comfortably higher than the 0.01 cut-off. Even the minimum exceeds that level. We may also examine the MCMC efficiency of all parameter levels using the using the command

```
. bayesstats ess.
```

Having convinced ourselves that the MCMC simulation has run satisfactorily, we can examine whether and to what extent the Bayesian estimates are better than OLS estimates. In Table 3, we compare the descriptive statistics based on the OLS and Bayesian ERC point estimates of the 301 firms.

**Table 3: Descriptive statistics – firm specific OLS ERCs vs. HBM1 ERCs**

| | ERC_ols | ERC_ols_post | ERC_HBM1 |
|---|---|---|---|
| Mean | 4.79 | 2.61 | 1.58 |
| Median | 2.00 | 1.60 | 1.67 |
| Maximum | 184.80 | 24.83 | 3.65 |
| Minimum | -11.82 | -4.65 | -0.57 |
| Standard Deviation | 14.22 | 3.86 | 0.69 |
| # negative values | 48 | 43 | 6 |
| MSE with ERC_ols_post | 210.28 | 0.00 | 15.96 |

First, we note that the mean of the Bayesian estimates is lower than that of OLS ERCs because the latter is affected by a few extreme positive values. These extreme values have high OLS standard errors, so they are shrunk considerably. For example, the ERC estimate at maximum (184.80) has a standard error of 60.3. Since shrinkage uses weights approximately[8] equalling $1/60.3^2$, its mean Bayesian ERC estimate is as low as 1.67. This shrinkage is apparent across all the firm-level parameter estimates and manifests as the much lower range of the Bayesian estimates. The HBM1 ERC estimates for all 301 firms are listed in column 4 of Table 6.

Second, we saw earlier that 48 of the 301 OLS ERC coefficients are negative. The corresponding number for the Bayesian estimates is only 6, a much smaller number. Overall, the Bayesian ERC estimates are a-priori much more plausible compared to our expectations based on theory. Third, we can compare the mean squared errors of predicting ERCs for the holdout period.[9] They are respectively 210.28 and 15.9 using test-period OLS and Bayesian estimates as predictors. That is, Bayesian estimates' mean squared prediction error is lower by 92 percent, a substantial improvement. To reiterate, this improvement results simply from using the information contained in other firms' ERCs, no *subjective* priors are used (the only priors used are uninformative).

[8] With normally distributed priors and data, the weights are exact, but we have assumed a t-distribution of the data here.

[9] They are themselves the OLS estimates for the holdout period, so they will have their own error and perfect prediction should not be expected.

## 3.2 Hierarchical Bayesian Model 2: Varying intercept, varying slope model using inverse-Wishart prior

In HBM1 we modeled the $b\_ols_j$ estimates from OLS regressions. We did not model $a_j$, the firm-specific intercepts. In fact, the intercepts could be correlated with the $b\_ols_j$ estimates, so there could be some benefits to modelling them simultaneously. To do this, we directly model the panel data of firm annual returns and change in annual earnings of the firm in the Bayesian Hierarchical Model. Specifically, we estimate a varying-intercept, varying slope model sometimes referred to as random coefficients model in the frequentist context. As there are a total of 301 firms with 15 years of observations each in our test sample, the total number of observations that we use to build the model is 4,515.

To begin, consider the estimation of earnings response coefficients in a varying-intercept and varying-slope specification as in the equation $ret_{jt} = a_j + b_j \Delta E_{jt}$. In this equation, the constant term ($a_j$) and the slope term ($b_j$), which is a measure of the ERC are assumed to be distinct for each firm in the sample.[10] This contrasts with a pooled regression approach where all the 4,515 observations are used to estimate only one slope and one constant coefficient.

The vectors $a_j$ and $b_j$ can be stacked together as matrix $U_j$ (where the dimension of each matrix $U_j$ is $2 \times 1$ because we are using two coefficients which are varying in our model). To build the Bayesian Model, the $U_j$ is assigned a hierarchical multivariate normal prior as below

$$\begin{pmatrix} a_j \\ b_j \end{pmatrix} \sim N\,(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

$$\text{Where, } \boldsymbol{\mu} = \begin{pmatrix} \mu_a \\ \mu_b \end{pmatrix} \text{ and } \boldsymbol{\Sigma} = \begin{pmatrix} \sigma_a^2 & \rho\sigma_a\sigma_b \\ \rho\sigma_a\sigma_b & \sigma_b^2 \end{pmatrix}$$

Further, $\mu_a, \mu_b$ and $\boldsymbol{\Sigma}_{2X2}$ are the hyper-parameters to which we assign hyper-priors. We assign a normal prior to $\mu_a$ and $\mu_b$, and there are several approaches to define the prior for the variance-covariance matrix $\boldsymbol{\Sigma}_{2X2}$ which we discuss below.

Inspection of matrix $\boldsymbol{\Sigma}_{2X2}$ shows that it has three parameters, $\sigma_a$ , $\sigma_b$ , and $\rho$. We could assign prior distributions for each of them, but the number of parameters that must be coded separately increases as the square of the number of regression variables. A convenient way to get around this is to use the inverse-Wishart matrix as a prior for $\boldsymbol{\Sigma}$. Its advantage is that the posterior distribution is also an inverse Wishart distribution, given normally distributed data. In the following

[10] As discussed earlier, the Bayesian Hierarchical modelling helps us improve the estimates of ERCs for a firm k ($b_k$) using information of all other firms where $j \neq k$.

**Commented [SK3]:** I think the dimension is 2 x 1.

**Commented [BC4R3]:** Going ahead with 2 x j for now

**Commented [SK5R3]:** I have changed it to 2 x1. You cannot have a 2x1 mu and 2x2 sigma matrix for a 2 x j.

**Commented [BC6R3]:** You are right Professor. I confused myself unnecessarily.

code, we model the prior of **Σ** as an inverse-Wishart distribution. As discussed earlier, our input data for HBM2 and HBM3 consists of firm-year level data of returns (return) and change in earnings (delta_E).

```
***HBM2*********************************************************************

*Input data and suppress base level of “gvkey” to use factor notation
. import delimited "Temp/Input_for_HBM2_HBM3.csv", clear case(preserve)
. fvset base none gvkey

*Input raw data of return and delta_E
. bayesmh return i.gvkey i.gvkey#c.delta_E, noconstant ///
likelihood(normal({var_0})) ///
prior({return:i.gvkey i.gvkey#c.delta_E}, mvnormal(2, {return:_cons},
{return:delta_E}, {covar,m})) ///
prior({var_0}, igamma(0.01, 0.01)) ///
prior({covar,m}, iwishart(2, 3, I(2))) ///
prior({return:delta_E}, normal(0, 100)) ///
prior({return:_cons}, normal(0, 100)) ///
block({return: i.gvkey}, reffects) ///
block({return: i.gvkey#c.delta_E}, reffects) ///
block({var_0}, gibbs) ///
block({covar,m}, gibbs) ///
block({return:_cons}) ///
block({return:delta_E}) adapt(tolerance(0.002)) ///
nchains(3) ///
initall({return: i.gvkey} rnormal(0, 100) {return: i.gvkey#c.delta_E}
rnormal(0, 100) {var_0} runiform(0, 100) {return:delta_E} rnormal(0, 100)
{return:_cons} rnormal(0, 100) {covar,m} I(2)) ///
mcmcsize(5000) burnin(5000) rseed(149)

*Save the simulations
. bayesmh, saving (Temp/simdata_model2.dta, replace)

****************************************************************************
```

The HBM2 `bayesmh` command differs from that of HBM1 in several respects. First, immediately following `bayesmh`, we have a different regression equation – return is equal to firm-level constant and firm-level slope times delta_E. Second, our likelihood function (for returns) in this model is a normal distribution with unknown variance, which we have named *var_0*. Similar to HBM1, we do not need to define the mean for the likelihood function as `bayesmh` automatically picks it up from the regression specification above. However, we define the unknown variance as {*var_0*} to indicate that it is a new variable that we are defining, which will have its own prior defined later. This contrasts with the corresponding HBM1 syntax, where we inputted the known variance as variable--ERC_var.

Third, along with other prior functions, we also define a multivariate normal distribution prior for the matrix $U_j$. We define the multivariate normal distribution using the command `mvnormal`. We

**Commented [SK7]:** If this is the same as SIGMA, then be consistent in the notation.

**Commented [BC8R7]:** Addressed

**Commented [SK9]:** Please write this in a way that is exactly parallel to HBM1 so as to make it easy to see the similarities and differences.

**Commented [SK10]:** Previously normal() had 2 parameters. Now it seems to have only 1. Why? What does it stand for--variable or variance? Reffects() came immediately after bayesmh. A detailed explanation would help readers to *use* it, which is what will get us citations.

**Commented [BC11R10]:** When defining a normal likelihood function for use with the bayesmh package, you typically need to define only the variance of the distribution (or variance and degrees of freedom in case of t-distribution) because the mean is typically a parameter of the model that you are trying to infer and bayesmh automatically picks this from the first statement of the code. You can see this in Table 2 third line. {var_0} is a variable (for variance measure) that we are defining and give it a diffuse igamma prior. Note that in HBM1, we directly input the variable measure already available in the data-so without the flower brackets. There are two ways to define reffects in bayesmh: global reffects option which we use in the HBM1, which allows us to have different random effects at each observation level and as a sub-option within option block which I believe is useful when you want all observations to be affected by the same set of random effects.

**Commented [SK12R10]:** Okay, thanks. Can you explain this here (as you have already done in HBM1) also?

**Commented [SK13R10]:** Okay. Please explain covar,m--what is m? What is block()? Explain the priors etc. even if it is similar to HBM1. This explanation should be self-contained--you should not assume that they remember from before, or ask them to go back and forth.

**Commented [BC14R10]:** "m" just refers to the covar as matrix. I explained it. I explained the priors as well. I provided a simple explanation for the block () in HBM 1 present three arguments to `mvnormal`. The number "2" denotes the dimension of the matrix, `{return:_cons}` and `{return:delta_E}` denote $\mu_a$ and $\mu_b$ respectively and `{covar,m}` denotes the covariance matrix **Σ** (the "m" denotes that "covar" is a matrix). Then, we define uninformative hyperpriors for $\mu_a$ and $\mu_b$. As discussed above, we define a two-dimensional inverse Wishart prior for **Σ**. Fourth, unlike HBM1 where we used the global `reffects()` to specify random-effect parameters, here we use the `reffects` sub option in the `block()` option. Note that we specify "reffects" for intercept terms and slope terms of the specified regression equation. Finally, we request for Gibbs sampling to update parameters in certain blocks with conjugate pairs. The output is similar to HBM1 and not shown here.

In Table 4, we compare our model 2 Bayesian ERCs with model 1 and OLS ERCs. All 301 ERCs are listed in column 5 of Table 6.

**Table 4: Descriptive statistics – firm specific OLS ERCs vs. HBM1 ERCs vs. HBM2 ERCs**

| | ERC_ols | ERC_ols_post | ERC_HBM1 | ERC_HBM2 |
|---|---|---|---|---|
| Mean | 4.79 | 2.61 | 1.58 | 2.13 |
| Median | 2.00 | 1.60 | 1.67 | 2.12 |
| Maximum | 184.80 | 24.83 | 3.65 | 7.92 |
| Minimum | -11.82 | -4.65 | -0.57 | -0.64 |
| Standard Deviation | 14.22 | 3.86 | 0.69 | 1.27 |
| # negative values | 48 | 43 | 6 | 6 |
| MSE with ERC_ols_post | 210.28 | 0.00 | 15.96 | 15.40 |

We see that the number of negative ERCs is six, which is the same as that of HBM1 but continue to be a much smaller number compared to OLS ERCs. Like HBM1 estimates, the HBM2 ERC estimates also outperform firm-specific OLS estimates in terms of their mean squared error with the hold out sample ERC. However, this number is 15.4, which is only a modest improvement over the 15.9 value for HBM1.

### 3.3 Hierarchical Bayesian Model 3: Varying intercepts and varying slopes model using information from the determinants of ERCs

It is possible to further improve our measures of ERCs by employing information of determinants of ERCs in our Bayesian hierarchical model. For illustrative purposes, we use a previously documented determinant of ERCs--risk-free rate (Collins and Kothari 1989)--to demonstrate how the temporal variation in ERCs captured by risk-free rate can be used in our Bayesian Hierarchical Model specification to improve our ($b_j$) estimates.

Formally, if we assume that risk-free rate is a determinant of $(b_j)$ as documented in extant literature, $(b_j)$ can be written as $b_j = c_j + \delta\, rf_t$ where $rf_t$ is the risk-free rate defined as 10-year government bond yields.[11] When we incorporate this information into our ERC estimation, the equation changes to $ret_{jt} = a_j + c_j \Delta E_{jt} + \delta \Delta E_{jt} rf_t$. Next, we proceed to build a varying-intercepts, varying-slopes Bayesian hierarchical model similar to HBM2, but this time matrix $U_j$ has components $a_j$ and $c_j$. As in HBM2, we assign an inverse Wishart hyper-prior for the variance-covariance matrix for the random coefficients $a_j$ and $c_j$.

We further assume a normal prior for the coefficient $\delta$, which captures the association between firm ERCs, and risk-free rates documented in the prior literature. Ex-ante, we would expect $\delta$ to be a negative real number indicating that firm ERCs are lower in years in which risk-free rates are higher. Note that we have already merged risk-free rate data (rf) to our input data file. We estimate a new variable ratedelta_E based on the last term of our new model equation $ret_{jt} = a_j + c_j \Delta E_{jt} + \delta \Delta E_{jt} rf_t$. The Stata code is provided below.

```
***HBM3**************************************************************************

*Input data and suppress base level of "gvkey" to use factor notation
. import delimited "Temp/Input_for_HBM2_HBM3.csv", clear case(preserve)
. fvset base none gvkey

*Create a variable to estimate the product of risk-free rate and delta_E
. gen ratedelta_E = rf*delta_E

*Input raw data of return, delta_E and ratedelta_E
. bayesmh return i.gvkey i.gvkey#c.delta_E ratedelta_E, noconstant ///
likelihood(normal({var_0})) ///
prior({return:i.gvkey i.gvkey#c.delta_E}, mvnormal(2, {return:_cons},
{return:delta_E}, {covar,m})) ///
prior({var_0}, igamma(0.01, 0.01)) ///
prior({covar,m}, iwishart(2, 3, I(2))) ///
prior({return: delta_E}, normal(0, 100)) ///
prior({return: _cons}, normal(0, 100)) ///
prior({return: ratedelta_E}, normal(0, 100)) ///
block({return: i.gvkey}, reffects) ///
block({return: i.gvkey#c.delta_E}, reffects) ///
block({var_0}, gibbs) ///
block({covar,m}, gibbs) ///
block({return: delta_E}) ///
block({return: _cons}) ///
block({return: ratedelta_E}) adapt(tolerance(0.002)) ///
nchains(3) ///
initall({return: i.gvkey} rnormal( 0, 100) {return: i.gvkey#c.delta_E}
rnormal(0, 100) {var_0} runiform(0, 100) {return: delta_E} rnormal(0, 100)
```

Commented [SK15]: Make similar to HBM2

[11] Data downloaded from https://fred.stlouisfed.org/series/IRLTLT01USM156N

```
{return: _cons} rnormal(0, 100) {return: ratedelta_E} rnormal(0, 100)
{covar,m} I(2)) ///
mcmcsize(5000) burnin(5000) rseed(149)

*Save the simulations
. bayesmh, saving (Temp/simdata_model3.dta, replace)

***************************************************************************
```

Note the change in the regression equation in the `bayesmh` command in HBM3 compared to HBM2 -- we have two independent variables instead of one. This is because of the inclusion of risk-free rate information for this estimation. Further, we also define an uninformative prior for $\delta$ in the highlighted step ($\delta$ is the coefficient of the independent variable "ratedelta_E", which is the product of risk-free rate and earnings change measure). Similar to earlier models, we save the simulation to a new file and compare the characteristics of these estimates to those of the earlier models.

Commented [SK16]: Show where it is defined. Maybe use highlights of different colors within the code to refer to them

Commented [BC17R16]: addressed

The output (not shown) displayed the estimate of $\delta$ as -6.02, agreeing with the ex-ante expectation of a negative association between risk-free rates and firm ERCs. With respect to firm-level coefficients, the estimates outputted by the model are that of $c_j$ and not of $b_j$ which is the measure of ERC. In order to compare our ERC estimates from model 3 (with the earlier models), we estimate $c_j$ from $b_j$, using the equation $b_j = c_j + \delta r f_t$ where $\delta$ is the parameter estimate provided by the BHM3 output.[12] In this equation, we use the average risk-free rate through our time-period of each firm as a measure of $rf_t$. In Table 5, we compare the ERC estimates from HBM3 with the earlier estimates.

Commented [SK18]: Show the command and the output

Commented [BC19R18]: Output shown in HBM1.

Commented [SK20R18]: Okay

**Table 5: Descriptive statistics – OLS ERCs vs. HBM1 ERCs vs. HBM2 ERCs vs. HBM3 ERCs**

| | ERC_ols | ERC_ols_post | ERC_HBM1 | ERC_HBM2 | ERC_HBM3 |
|---|---|---|---|---|---|
| Mean | 4.79 | 2.61 | 1.58 | 2.13 | 2.14 |
| Median | 2.00 | 1.60 | 1.67 | 2.12 | 2.11 |
| Maximum | 184.80 | 24.83 | 3.65 | 7.92 | 7.99 |
| Minimum | -11.82 | -4.65 | -0.57 | -0.64 | -0.62 |
| Standard Deviation | 14.22 | 3.86 | 0.69 | 1.27 | 1.28 |
| # negative values | 48 | 43 | 6 | 6 | 5 |
| MSE with ERC_ols_post | 210.28 | 0.00 | 15.96 | 15.40 | 15.38 |

[12] Note that there is a high correlation between the variables $\Delta E_{jt}$ and $\Delta E_{jt} rf_t$ because $rf_t$ varies only across years. As in the frequentist approach, high correlation between independent variables in a regression equation can reduce precision of the estimated coefficients in the Bayesian context too. This multicollinearity issue has a negligible effect on our outcome prediction and on our ERC_HBM3 estimate. The latter is because our ERC estimation is based on a *combination* of $c_j$ and $\delta$ parameter estimations. For robustness, we use different random initial states of all parameters and hyperparameters in running HBM3 and find that the results in column 6 of Table 5 are robust.

The HBM3 model continues to exhibit superior performance with respect to the OLS estimates by a wide margin. The count of negative ERCs is now 5, which is the best among all the models compared in the table. The MSE of the ERC_HBM3 improves marginally when compared with ERC_HBM2 and is again, the best among all the models. In summary, the ERCs estimated based on a Bayesian hierarchical model incorporating information of the determinants of ERCs seems to outperform the ERC estimation by our other Bayesian models as well as their OLS counterparts.

We may extend this application to a general parameter estimation problem when the parameter has several ex-ante firm-level determinants. Consider the common regression problem of firm-level parameter estimation: $y_{jt} = p_j + q_j x_{jt} + e_{jt}$, where *y* and *x* represent the dependent and independent variables respectively, and *j* and *t* denote the firm and time respectively. Our objective is to estimate $q_j$ (which is ERC in this paper, and firm-specific stock beta in Cosemans et al. 2017). Further, $q_j$ could have its own ex-ante determinants. For example, in case of the stock beta, a rich asset pricing literature documents several firm characteristics like firm size and book-to-market determine beta. Assume that $q_j$ has two firm-level determinants denoted by $r_{jt}$ and $s_{jt}$. Further, suppose $r_{jt}$ affects each firm differently, while $s_{jt}$ affects each firm uniformly. That is, $q_j = \delta_{0j} + \delta_{1j} r_{jt} + \delta_2 s_{jt}$. In our new estimation equation after substituting for $q_j$ we get $y_{jt} = p_j + \delta_{0j} x_{jt} + \delta_{1j} x_{jt} r_{jt} + \delta_2 x_{jt} s_{jt} + \epsilon_{jt}$, where $p_j$, $\delta_{0j}$ and $\delta_{1j}$ are firm-level parameter estimates and $\delta_2$ is a global parameter for the whole sample of firms capturing the relationship between $q_j$ and $s_{jt}$.

The Stata code is along the lines of HBM3: we can define the regression equation in the `bayesmh` command as `bayesmh y i.gvkey i.gvkey#c.x i.gvkey#c.xr xs`, where `xr` is equal to $x_{jt} \times r_{jt}$ , `xs` is equal to $x_{jt} \times s_{jt}$ and `gvkey` is the firm-level identifier. We further define `reffects` using `block ()` for the parameters $p_j$, $\delta_{0j}$ and $\delta_{1j}$ and a multivariate normal prior for the vector containing these random coefficients as discussed in section 3.2. Finally, we define uninformative priors for the means of these parameters as well as for the parameter $\delta_2$. In this way, we can solve a general parameter estimation problem by accommodating information of firm-level determinants of the coefficient of interest.

**Commented [SK21]:** Write it in stata language (just this line) instead of an equation.

## 4 Discussion and Conclusion

The three models above were meant to illustrate the mechanics of hierarchical Bayesian models. These models are well-suited for modeling populations with multiple units with a shared structure between the units along with unit-specific differences. Estimation of ERCs shown here, and stock betas (Cosemans et al. 2016) are such examples where firms constitute these units.

Hierarchical Bayesian models are especially useful when we have limited data for each unit or subpopulation, and we want to use information from other units to inform our estimates. In this case, the hierarchical structure allows us to use data from other units to make more precise estimates for units with limited data. ERCs and stock betas are good examples of such parameter estimation because of the limited time-series data available for an average firm in Compustat and CRSP. In Marketing, hierarchical Bayesian models are used extensively and have been used to model even person-occasion level parameters, i.e., an individual engaged in a specific instance of an activity such as purchasing a can of beer (Allenby et al. 2005).

The technique has been used in a few Accounting papers. For instance, Du et al. (2020) propose a structural Markov model of a firm's transitioning between states High and Low, and parameters to determine whether it reports signal high or low as a function of its true unobservable state. The transition probabilities as well as the parameters determining the relation between true state and the signal are firm-specific, and are estimated using a hierarchical Bayesian model.

Other areas of application could be in parameter estimations such as earnings persistence. Even the classic accrual models which are based on OLS regressions at industry-levels maybe reimagined as firm-level models based on hierarchical Bayesian modeling. The parameters need not be restricted to regression coefficients—they could also include the Dechow-Dichev accruals quality measure which represents the standard deviation from a regression.

In this paper, we illustrate the usefulness of hierarchical Bayesian models in the context of estimating firm specific ERCs. Using a sample set of 301 firms, we show that a frequentist approach-based ordinary least squares (OLS) method to estimating ERCs results in a high count of negative ERCs (48 out of 301). Further, when we break the firm-level time-series into first 15 years and the rest of the period (assuming a time-series of greater than 30) and estimate the ERCs separately for each period, we observe that there is considerably high mean square error between the two ERC estimates of the same firm. Finally, OLS-based ERCs displayed significant outliers and high standard deviation within their distribution. These problems are considerably alleviated using hierarchical Bayesian approaches.

In the first Bayesian hierarchical model (HBM1), we shrink the OLS ERCs to an assumed grand mean using a Bayesian model where the ERCs are considered to be random drawings from a population with an overall grand mean ERC and an unknown variance. The grand mean and variance are given uninformative priors and the model gives a posterior distribution based on the prior and the data consisting of the 301 firm-specific OLS ERCs. This posterior becomes an informative prior for each firm, resulting in a firm-specific posterior that is determined by this prior and the firm-specific OLS ERC. Since the informative prior was developed using data from all other firms, the effect is to use data from other firms to estimate each firm-specific ERC. This model produces ERCs which have just six out of 301 negative values compared to the 48 negative OLS ERCs. Also, these ERCs cut down the mean squared error (MSE) with their holdout set ERCs by more than 90% compared to OLS ERCs.

We progressively refine our model in HBM2 and HBM3. HBM2 jointly models both the firm-specific beta and a firm-specific market model intercept, which could be correlated, instead of using OLS ERCs and known sample standard deviations as inputs to the algorithm, we build varying-intercepts and varying-slopes models inputting information of firm-year level return and earnings information. Additionally, in HBM3, we also incorporate information of risk-free rates, a documented determinant of ERC. These models produce ERCs that slightly improve upon HBM1, and largely improve upon the OLS model based on the validation tests of negative counts and the MSE measure. Using the example of the ERC estimation, we demonstrate how a Bayesian approach using hierarchical models can improve traditional parameter estimation in accounting literature.

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**Table 6: ERC lists from firm-specific OLS and hierarchical Bayesian models**

This table lists the firm-level ERCs estimated using different approaches discussed in this chapter using the Stata command `bayesmh`. gvkey refers to the firm-level identifier in Compustat. ERC_ols and ERC_ols_post are ERC estimations based on the first 15-years data and the hold-out data respectively. HBM1 is the Bayesian hierarchical ERC estimated by shrinking OLS ERCs to a grand mean. HBM2 and HBM3 are varying intercept, varying slope models without and with incorporating information of risk-rate rates in the ERC estimation respectively.

| gvkey | ERC_ols | ERC_ols_post | HMB1 | HBM2 | HBM3 |
|---|---|---|---|---|---|
| 1078 | 28.09 | 3.12 | 1.66 | 2.30 | 2.31 |
| 1186 | 0.50 | 3.94 | 0.86 | 0.55 | 0.50 |
| 1230 | 0.67 | 0.94 | 0.89 | 0.83 | 0.76 |
| 1234 | 0.73 | 3.14 | 0.92 | 0.93 | 0.88 |
| 1254 | 5.37 | 0.28 | 2.53 | 3.10 | 3.00 |
| 1262 | 2.11 | 0.23 | 2.03 | 2.04 | 1.99 |
| 1300 | -0.21 | 1.95 | 0.42 | 0.21 | 0.25 |
| 1356 | 0.42 | 0.57 | 0.71 | 0.92 | 0.95 |
| 1380 | 0.18 | 0.58 | 0.36 | 0.47 | 0.43 |
| 1408 | -0.23 | 0.87 | 1.17 | 1.84 | 1.76 |
| 1585 | 2.65 | 6.88 | 1.85 | 2.62 | 2.68 |
| 1598 | 3.56 | 9.84 | 2.05 | 2.44 | 2.39 |
| 1613 | 0.37 | 0.01 | 0.61 | 0.64 | 0.72 |
| 1678 | 0.35 | 0.45 | 0.61 | 0.47 | 0.52 |
| 1728 | 1.13 | 10.37 | 1.41 | 1.57 | 1.55 |
| 1773 | 0.09 | 1.05 | 0.78 | 0.30 | 0.29 |
| 1906 | -1.93 | 0.83 | -0.31 | -0.57 | -0.57 |
| 1968 | 0.16 | 8.94 | 0.44 | 0.62 | 0.76 |
| 1974 | 0.51 | 2.08 | 0.97 | 0.74 | 0.73 |
| 1979 | 9.41 | 12.24 | 2.07 | 3.95 | 4.08 |
| 1988 | 1.86 | -0.17 | 1.69 | 1.94 | 1.86 |
| 2044 | 24.78 | 3.83 | 2.01 | 3.18 | 3.22 |
| 2049 | 2.00 | 0.17 | 1.74 | 1.95 | 1.97 |
| 2055 | 1.29 | 1.12 | 1.54 | 1.67 | 1.68 |
| 2086 | 2.27 | -0.48 | 1.83 | 2.14 | 2.09 |
| 2136 | -3.25 | 0.03 | 0.94 | 1.35 | 1.48 |
| 2137 | 0.66 | 5.62 | 1.08 | 1.44 | 1.39 |
| 2154 | 0.58 | 1.87 | 1.39 | 2.10 | 2.03 |
| 2176 | 5.17 | 1.17 | 1.87 | 2.92 | 2.97 |
| 2220 | 7.33 | 2.44 | 2.64 | 4.92 | 4.92 |
| 2290 | 0.75 | 0.36 | 0.78 | 0.85 | 0.77 |
| 2312 | 5.84 | 3.74 | 3.44 | 4.77 | 4.88 |
| 2403 | 4.12 | 0.84 | 1.73 | 2.30 | 2.27 |
| 2408 | 1.41 | 6.39 | 1.57 | 2.07 | 2.09 |
| 2410 | 2.65 | 1.21 | 2.17 | 2.45 | 2.41 |
| 2444 | 1.29 | 1.96 | 1.42 | 1.48 | 1.54 |
| 2448 | 0.90 | 5.21 | 1.17 | 1.22 | 1.19 |
| 2574 | -1.66 | 2.19 | -0.57 | -0.64 | -0.62 |

| gvkey | ERC_ols | ERC_ols_post | HMB1 | HBM2 | HBM3 |
|---|---|---|---|---|---|
| 2577 | -0.45 | -0.48 | -0.02 | 0.08 | 0.05 |
| 2698 | 0.79 | 3.49 | 0.94 | 1.02 | 1.02 |
| 2703 | 6.34 | 12.85 | 1.98 | 2.45 | 2.43 |
| 2721 | 3.73 | 0.98 | 1.87 | 2.26 | 2.31 |
| 2771 | 0.24 | 4.83 | 1.16 | 1.54 | 1.55 |
| 2811 | 2.41 | 2.00 | 2.07 | 2.34 | 2.36 |
| 2817 | 2.22 | 0.43 | 2.09 | 2.12 | 2.20 |
| 2884 | 0.10 | 0.23 | 1.45 | 2.02 | 1.98 |
| 2953 | 5.83 | 2.23 | 2.52 | 3.86 | 3.88 |
| 2960 | 2.31 | 2.37 | 1.76 | 2.16 | 2.07 |
| 2991 | -0.68 | 1.44 | 0.30 | 0.78 | 0.73 |
| 2999 | -0.07 | 2.89 | 0.36 | 1.15 | 1.17 |
| 3026 | -2.11 | 4.09 | 1.19 | 1.30 | 1.28 |
| 3138 | 3.94 | 5.20 | 2.07 | 2.81 | 2.79 |
| 3144 | 16.17 | 1.63 | 1.80 | 2.62 | 2.57 |
| 3158 | 7.19 | 1.85 | 2.89 | 4.77 | 4.71 |
| 3170 | 1.12 | 9.92 | 1.23 | 1.72 | 1.74 |
| 3275 | 9.30 | 2.40 | 1.98 | 3.99 | 4.01 |
| 3342 | 0.64 | 0.69 | 0.92 | 0.77 | 0.72 |
| 3416 | 2.78 | 0.07 | 2.05 | 2.23 | 2.23 |
| 3465 | 0.77 | 0.75 | 1.22 | 1.03 | 1.03 |
| 3497 | 4.73 | 2.34 | 2.02 | 2.48 | 2.54 |
| 3502 | 13.10 | 2.31 | 1.82 | 3.75 | 3.70 |
| 3505 | 1.36 | 2.33 | 1.46 | 1.60 | 1.64 |
| 3532 | 3.30 | 0.69 | 1.86 | 2.42 | 2.43 |
| 3538 | 12.58 | 8.71 | 2.13 | 3.23 | 3.29 |
| 3580 | 2.04 | 0.15 | 1.89 | 2.09 | 2.13 |
| 3650 | 0.09 | 1.60 | 0.31 | 0.22 | 0.28 |
| 3662 | -0.07 | 3.89 | 0.12 | 0.24 | 0.21 |
| 3735 | 2.09 | 7.16 | 1.79 | 2.42 | 2.49 |
| 3946 | 9.38 | 0.39 | 2.23 | 2.84 | 2.92 |
| 4040 | 4.99 | 2.33 | 2.22 | 2.65 | 2.54 |
| 4058 | 6.72 | 4.01 | 2.16 | 2.42 | 2.32 |
| 4060 | 0.31 | 2.60 | 1.03 | 1.20 | 1.18 |
| 4066 | 0.24 | 2.85 | 1.01 | 1.52 | 1.54 |
| 4087 | 0.66 | 1.31 | 1.32 | 1.88 | 1.79 |
| 4108 | 3.91 | 1.88 | 3.13 | 2.90 | 2.89 |
| 4145 | 4.62 | 6.35 | 2.55 | 3.05 | 3.01 |
| 4186 | 1.20 | 2.46 | 1.31 | 1.65 | 1.66 |
| 4199 | 1.13 | 1.43 | 1.21 | 1.44 | 1.54 |
| 4213 | -2.02 | 5.59 | 0.90 | 1.40 | 1.55 |
| 4367 | 3.60 | 1.40 | 1.87 | 3.60 | 3.55 |
| 4423 | -11.82 | 0.48 | 0.85 | 0.66 | 0.65 |
| 4439 | 5.47 | 2.50 | 2.02 | 3.71 | 3.70 |
| 4494 | 16.53 | 13.34 | 1.80 | 3.11 | 3.24 |
| 4503 | 2.13 | 2.64 | 1.78 | 2.17 | 2.12 |
| 4608 | 6.09 | 0.24 | 1.87 | 2.33 | 2.31 |

| gvkey | ERC_ols | ERC_ols_post | HMB1 | HBM2 | HBM3 |
|---|---|---|---|---|---|
| 4622 | 0.71 | 1.09 | 1.35 | 1.46 | 1.51 |
| 4669 | 3.43 | 4.53 | 2.00 | 2.49 | 2.51 |
| 4881 | 0.25 | 5.53 | 0.69 | 0.68 | 0.71 |
| 4941 | -0.24 | -0.93 | -0.12 | 0.01 | 0.22 |
| 4988 | 0.57 | 0.26 | 1.56 | 2.12 | 2.04 |
| 5046 | 0.97 | 0.70 | 1.01 | 1.02 | 0.97 |
| 5047 | -0.31 | 1.80 | 1.54 | 2.19 | 2.11 |
| 5087 | 1.77 | 3.08 | 1.67 | 1.91 | 1.82 |
| 5116 | 28.16 | 18.42 | 1.82 | 5.42 | 5.51 |
| 5125 | 10.07 | 0.54 | 1.99 | 2.22 | 2.12 |
| 5179 | 0.33 | -0.15 | 0.66 | 0.75 | 0.70 |
| 5229 | -0.12 | -2.53 | -0.05 | -0.05 | 0.02 |
| 5237 | 2.11 | 15.10 | 1.63 | 2.03 | 1.98 |
| 5252 | 2.91 | 2.43 | 2.02 | 2.49 | 2.51 |
| 5256 | -8.84 | 4.59 | 1.23 | 1.78 | 1.79 |
| 5379 | 1.64 | 1.33 | 1.60 | 1.70 | 1.69 |
| 5439 | 0.62 | -0.36 | 0.95 | 0.98 | 0.98 |
| 5496 | 0.85 | -1.49 | 1.20 | 1.66 | 1.66 |
| 5518 | 6.14 | 4.97 | 2.68 | 5.01 | 5.04 |
| 5523 | 4.27 | 3.81 | 2.14 | 3.19 | 3.22 |
| 5539 | 3.67 | 21.66 | 1.87 | 2.36 | 2.30 |
| 5597 | 8.11 | 2.88 | 2.10 | 2.49 | 2.50 |
| 5690 | 3.46 | -0.78 | 1.73 | 2.41 | 2.37 |
| 5764 | 2.73 | -0.40 | 1.68 | 2.13 | 2.16 |
| 5783 | 2.78 | 9.43 | 1.66 | 2.22 | 2.26 |
| 5860 | 0.29 | 1.12 | 0.96 | 0.99 | 0.90 |
| 5878 | 1.54 | -4.65 | 1.53 | 2.17 | 2.16 |
| 5903 | 5.63 | 2.10 | 2.57 | 2.53 | 2.57 |
| 5959 | -0.35 | 2.07 | 0.45 | 0.40 | 0.47 |
| 5978 | 8.64 | 0.36 | 3.65 | 7.60 | 7.65 |
| 6008 | 4.59 | 1.85 | 2.10 | 3.77 | 3.77 |
| 6034 | 4.12 | 3.22 | 2.00 | 2.56 | 2.53 |
| 6066 | 1.47 | 4.30 | 1.51 | 1.53 | 1.54 |
| 6078 | -1.80 | 8.76 | 1.50 | 2.10 | 2.04 |
| 6104 | 0.21 | 2.74 | 0.56 | 0.83 | 0.89 |
| 6115 | 1.69 | 0.05 | 1.65 | 1.73 | 1.86 |
| 6136 | 7.29 | -2.75 | 1.87 | 2.44 | 2.44 |
| 6158 | 5.43 | -0.19 | 2.46 | 3.99 | 4.01 |
| 6196 | -0.51 | 0.07 | 0.96 | 0.86 | 0.90 |
| 6266 | 2.94 | -2.09 | 1.69 | 2.22 | 2.18 |
| 6326 | 0.38 | 1.43 | 0.80 | 0.81 | 0.72 |
| 6335 | 1.17 | -1.14 | 1.39 | 1.39 | 1.36 |
| 6375 | 9.77 | -0.99 | 1.75 | 2.45 | 2.40 |
| 6379 | 6.54 | 1.39 | 1.87 | 2.57 | 2.55 |
| 6435 | -3.58 | 4.79 | 0.15 | 1.10 | 1.00 |
| 6574 | 3.37 | 3.79 | 1.97 | 2.32 | 2.24 |
| 6617 | 9.22 | 0.50 | 2.14 | 2.28 | 2.26 |

| gvkey | ERC_ols | ERC_ols_post | HMB1 | HBM2 | HBM3 |
|---|---|---|---|---|---|
| 6649 | 6.55 | 4.12 | 2.17 | 3.18 | 3.17 |
| 6672 | 5.55 | 6.29 | 2.80 | 3.61 | 3.63 |
| 6730 | 4.60 | -0.93 | 1.77 | 2.44 | 2.38 |
| 6771 | 5.99 | -1.22 | 2.08 | 2.65 | 2.69 |
| 6845 | 1.91 | 0.91 | 1.82 | 2.00 | 2.12 |
| 6867 | 4.62 | 1.25 | 2.69 | 2.53 | 2.53 |
| 7085 | 3.35 | 1.68 | 1.77 | 2.21 | 2.23 |
| 7116 | 1.44 | 2.51 | 1.47 | 1.52 | 1.63 |
| 7154 | 31.19 | 9.22 | 1.64 | 2.26 | 2.21 |
| 7161 | 11.41 | 1.91 | 2.40 | 2.97 | 2.98 |
| 7163 | -0.23 | -2.65 | 0.87 | 1.36 | 1.33 |
| 7241 | 0.54 | 1.26 | 0.86 | 0.86 | 0.75 |
| 7257 | 23.36 | 0.87 | 1.72 | 2.57 | 2.62 |
| 7420 | 4.44 | 13.29 | 2.40 | 2.70 | 2.72 |
| 7435 | -5.26 | 5.64 | 1.22 | 1.97 | 1.99 |
| 7481 | 20.58 | 5.89 | 2.00 | 3.82 | 3.82 |
| 7585 | 1.24 | 2.39 | 1.53 | 2.04 | 1.99 |
| 7620 | 0.70 | 1.06 | 0.83 | 0.95 | 0.91 |
| 7636 | 6.81 | 1.23 | 2.44 | 3.29 | 3.25 |
| 7691 | 1.40 | 0.32 | 1.59 | 2.00 | 1.94 |
| 7741 | 1.72 | 7.54 | 1.64 | 1.84 | 1.86 |
| 7762 | 3.84 | 7.50 | 1.78 | 2.24 | 2.24 |
| 7799 | 6.91 | 5.04 | 2.39 | 5.45 | 5.54 |
| 7866 | -0.50 | 0.22 | 1.12 | 1.57 | 1.51 |
| 7875 | 1.83 | 0.32 | 1.59 | 2.59 | 2.57 |
| 7912 | 1.26 | 0.79 | 1.40 | 1.51 | 1.61 |
| 7923 | -0.79 | 3.47 | 0.28 | 0.78 | 0.85 |
| 7938 | 1.83 | 0.41 | 1.62 | 1.86 | 1.86 |
| 7985 | 0.15 | 1.36 | 0.50 | 0.46 | 0.48 |
| 8020 | 26.44 | 24.83 | 1.85 | 2.76 | 2.79 |
| 8030 | 2.98 | 1.76 | 1.70 | 2.25 | 2.22 |
| 8068 | 0.60 | 1.51 | 0.65 | 0.74 | 0.74 |
| 8123 | -0.05 | -0.07 | 0.06 | 0.12 | 0.11 |
| 8133 | 8.68 | 1.20 | 1.94 | 3.46 | 3.44 |
| 8169 | 0.88 | 1.74 | 1.32 | 1.10 | 1.08 |
| 8213 | 2.79 | 0.95 | 1.67 | 2.35 | 2.37 |
| 8247 | -0.70 | 2.30 | 0.99 | 1.63 | 1.59 |
| 8253 | 0.04 | -0.10 | 0.90 | 0.98 | 0.98 |
| 8336 | 0.26 | 0.90 | 0.80 | 0.52 | 0.49 |
| 8463 | 4.76 | 0.79 | 2.03 | 2.93 | 2.92 |
| 8479 | 5.98 | -2.88 | 2.20 | 2.65 | 2.66 |
| 8530 | -3.49 | -0.61 | 1.43 | 1.90 | 2.00 |
| 8543 | 9.09 | 1.71 | 2.05 | 2.90 | 2.74 |
| 8546 | 0.78 | 2.09 | 0.85 | 0.83 | 0.79 |
| 8549 | 1.07 | 1.57 | 1.17 | 1.36 | 1.37 |
| 8692 | 0.17 | 1.69 | 1.04 | 1.32 | 1.33 |
| 8823 | 11.38 | 0.82 | 2.32 | 7.92 | 7.99 |

| gvkey | ERC_ols | ERC_ols_post | HMB1 | HBM2 | HBM3 |
|---|---|---|---|---|---|
| 8850 | 0.92 | 2.68 | 1.26 | 1.53 | 1.49 |
| 8870 | 0.08 | -0.25 | 0.25 | 0.20 | 0.17 |
| 8901 | 12.95 | 4.11 | 2.49 | 5.89 | 5.99 |
| 8972 | 10.74 | 1.10 | 2.06 | 2.22 | 2.24 |
| 9016 | 5.86 | 1.05 | 2.22 | 3.07 | 3.08 |
| 9216 | 0.99 | 1.68 | 1.05 | 1.02 | 0.91 |
| 9225 | -0.15 | 20.86 | 0.42 | 0.70 | 0.72 |
| 9258 | 1.71 | 0.07 | 1.58 | 1.81 | 1.93 |
| 9299 | 1.48 | 0.84 | 1.54 | 1.92 | 1.83 |
| 9302 | 0.62 | 1.69 | 1.28 | 1.00 | 0.92 |
| 9372 | -2.29 | 6.24 | 0.93 | -0.05 | -0.08 |
| 9465 | 0.12 | 0.52 | 0.58 | 0.70 | 0.61 |
| 9538 | 2.54 | 5.86 | 2.12 | 2.36 | 2.37 |
| 9647 | 6.80 | 1.57 | 2.82 | 4.17 | 4.25 |
| 9667 | 15.77 | 1.26 | 2.29 | 3.08 | 3.11 |
| 9699 | 31.32 | 2.58 | 1.67 | 2.35 | 2.36 |
| 9771 | 1.84 | 2.00 | 1.79 | 1.85 | 2.03 |
| 9778 | 4.42 | -1.11 | 1.96 | 2.21 | 2.18 |
| 9815 | 2.57 | -1.38 | 1.71 | 2.25 | 2.19 |
| 9882 | 4.11 | 3.96 | 1.78 | 2.72 | 2.74 |
| 9899 | -0.80 | 1.15 | 1.48 | 2.00 | 2.04 |
| 10000 | 9.45 | 2.46 | 2.75 | 4.23 | 4.18 |
| 10016 | -6.14 | -2.51 | 0.48 | 1.03 | 0.98 |
| 10056 | 4.29 | 2.74 | 2.37 | 2.49 | 2.56 |
| 10115 | 184.80 | -4.41 | 1.62 | 2.83 | 2.85 |
| 10124 | 5.30 | 4.94 | 2.14 | 2.83 | 2.86 |
| 10156 | 0.56 | 1.73 | 0.79 | 1.00 | 0.94 |
| 10195 | 1.90 | 0.18 | 1.72 | 2.14 | 2.26 |
| 10198 | 4.89 | 5.60 | 2.32 | 2.80 | 2.75 |
| 10236 | 1.21 | 1.28 | 1.41 | 1.31 | 1.41 |
| 10385 | 3.15 | 4.10 | 2.26 | 2.53 | 2.53 |
| 10386 | 2.40 | -0.15 | 1.91 | 2.16 | 2.15 |
| 10390 | 32.53 | -3.41 | 1.65 | 2.34 | 2.32 |
| 10405 | 1.76 | 0.05 | 1.67 | 1.85 | 1.80 |
| 10407 | 8.63 | 0.40 | 1.98 | 2.31 | 2.29 |
| 10411 | -5.42 | 0.95 | 1.27 | 1.27 | 1.19 |
| 10420 | 17.59 | 0.93 | 2.00 | 5.24 | 5.21 |
| 10441 | -0.77 | 1.11 | 0.66 | 1.62 | 1.63 |
| 10453 | 5.94 | 2.79 | 2.79 | 4.52 | 4.49 |
| 10499 | 1.13 | 9.25 | 1.24 | 1.29 | 1.25 |
| 10519 | 3.70 | 0.39 | 2.01 | 2.46 | 2.45 |
| 10530 | -11.39 | 5.16 | 0.97 | 1.04 | 1.07 |
| 10540 | -0.09 | 0.59 | 1.07 | 1.66 | 1.61 |
| 10581 | 0.92 | 2.14 | 1.04 | 1.19 | 1.20 |
| 10609 | 9.93 | 17.50 | 2.10 | 3.22 | 3.23 |
| 10631 | 134.94 | 14.61 | 1.68 | 3.20 | 3.14 |
| 10846 | 1.65 | -0.06 | 1.57 | 2.16 | 2.13 |

| gvkey | ERC_ols | ERC_ols_post | HMB1 | HBM2 | HBM3 |
|---|---|---|---|---|---|
| 10867 | -1.57 | 1.08 | -0.31 | -0.21 | -0.17 |
| 10983 | -0.33 | 6.40 | 0.09 | 0.12 | 0.13 |
| 11032 | 7.48 | 4.79 | 2.67 | 3.95 | 3.88 |
| 11051 | 24.34 | 2.83 | 1.87 | 3.50 | 3.66 |
| 11060 | 8.35 | -0.28 | 2.90 | 4.68 | 4.69 |
| 11065 | 1.84 | 3.22 | 1.74 | 1.91 | 1.88 |
| 11094 | 2.23 | 4.57 | 1.96 | 2.27 | 2.30 |
| 11191 | 1.39 | 0.59 | 1.48 | 1.68 | 1.63 |
| 11217 | 1.10 | 0.42 | 1.12 | 1.15 | 1.17 |
| 11228 | 2.21 | 7.77 | 1.77 | 2.05 | 2.06 |
| 11258 | 5.04 | 1.38 | 2.54 | 2.95 | 2.97 |
| 11300 | 4.98 | 2.33 | 2.11 | 2.58 | 2.48 |
| 11313 | 8.27 | 9.46 | 1.95 | 3.16 | 3.21 |
| 11343 | 28.39 | 3.45 | 1.84 | 2.43 | 2.38 |
| 11376 | 0.92 | 1.64 | 1.37 | 1.72 | 1.60 |
| 11443 | 3.84 | -0.49 | 1.98 | 2.47 | 2.43 |
| 11455 | 3.67 | 5.18 | 1.94 | 2.18 | 2.22 |
| 11456 | -0.33 | 1.25 | 0.54 | 1.04 | 0.97 |
| 11465 | 9.24 | 0.64 | 2.46 | 3.37 | 3.31 |
| 11566 | 1.42 | -0.58 | 1.45 | 1.57 | 1.55 |
| 11636 | -0.26 | 6.33 | 1.02 | 0.89 | 0.95 |
| 11749 | 1.95 | -0.93 | 1.81 | 1.95 | 1.95 |
| 11781 | 4.81 | 0.79 | 2.55 | 2.52 | 2.49 |
| 11923 | 4.28 | 0.26 | 2.67 | 3.13 | 3.15 |
| 12215 | 3.97 | 3.61 | 2.45 | 3.67 | 3.65 |
| 12262 | 1.22 | 2.41 | 1.31 | 1.25 | 1.29 |
| 12383 | 1.40 | 4.21 | 1.48 | 1.53 | 1.61 |
| 12389 | 4.00 | -0.68 | 1.80 | 3.24 | 3.27 |
| 12544 | -3.52 | 0.97 | 0.93 | 1.24 | 1.39 |
| 12625 | 1.80 | 8.03 | 1.72 | 1.88 | 1.91 |
| 12635 | -0.65 | 3.42 | 0.82 | 1.27 | 1.25 |
| 12796 | 1.66 | 4.36 | 1.62 | 1.71 | 1.75 |
| 12840 | 10.49 | 8.73 | 1.80 | 2.99 | 3.10 |
| 12850 | 9.95 | 3.98 | 2.19 | 6.87 | 6.90 |
| 13421 | 6.18 | 0.37 | 2.30 | 4.37 | 4.39 |
| 13526 | -2.21 | -0.12 | 1.28 | 1.96 | 1.96 |
| 13623 | 7.89 | 0.46 | 2.82 | 6.24 | 6.26 |
| 13683 | 3.50 | 3.70 | 2.34 | 2.93 | 2.92 |
| 13709 | -0.18 | 0.72 | 0.78 | 0.66 | 0.70 |
| 13710 | 3.39 | 1.25 | 1.66 | 2.27 | 2.44 |
| 13714 | 3.05 | 0.37 | 2.17 | 2.60 | 2.62 |
| 13839 | 4.48 | 0.69 | 3.52 | 3.73 | 3.88 |
| 13994 | 13.48 | 0.68 | 2.35 | 3.95 | 3.92 |
| 13997 | 3.86 | 0.29 | 2.43 | 2.93 | 3.03 |
| 14042 | 2.55 | 0.99 | 2.33 | 2.46 | 2.47 |
| 14062 | 0.42 | 4.04 | 0.90 | 0.96 | 0.96 |
| 14065 | -0.05 | 1.96 | 0.61 | 0.34 | 0.28 |

| gvkey | ERC_ols | ERC_ols_post | HMB1 | HBM2 | HBM3 |
|---|---|---|---|---|---|
| 14084 | 3.66 | 3.45 | 1.88 | 2.39 | 2.50 |
| 14108 | 5.16 | 4.43 | 2.08 | 4.79 | 4.74 |
| 14225 | 8.64 | 2.93 | 1.61 | 2.51 | 2.60 |
| 14311 | 2.61 | 6.28 | 2.01 | 2.62 | 2.66 |
| 14316 | 1.49 | -0.31 | 1.53 | 1.72 | 1.67 |
| 14330 | 1.83 | 1.10 | 1.74 | 1.94 | 1.96 |
| 14352 | 1.46 | -0.36 | 1.46 | 1.51 | 1.58 |
| 14359 | 0.42 | -4.29 | 0.54 | 0.45 | 0.48 |
| 14369 | 6.87 | 1.93 | 1.88 | 2.98 | 3.10 |
| 14418 | -0.79 | 0.17 | 1.05 | 0.58 | 0.63 |
| 14447 | 8.33 | 1.68 | 1.80 | 3.06 | 3.20 |
| 14538 | 17.89 | 0.17 | 1.73 | 3.41 | 3.43 |
| 14620 | 1.52 | 7.91 | 1.56 | 1.68 | 1.74 |
| 14934 | 1.86 | 0.65 | 1.80 | 1.77 | 1.79 |
| 15014 | 2.91 | -1.10 | 2.17 | 2.17 | 2.20 |
| 15020 | 4.95 | 3.88 | 2.40 | 3.31 | 3.31 |
| 15172 | 0.31 | -0.68 | 0.36 | 0.36 | 0.34 |
| 15247 | 0.56 | 3.00 | 0.77 | 0.59 | 0.64 |
| 15319 | -0.45 | 0.58 | 1.01 | 1.37 | 1.44 |
| 15334 | -1.35 | 0.66 | 1.11 | 1.48 | 1.57 |
| 15444 | 8.70 | -0.06 | 2.03 | 2.78 | 2.87 |
| 15459 | 1.42 | 4.55 | 1.48 | 1.52 | 1.59 |
| 16188 | 5.82 | 4.17 | 1.76 | 3.12 | 3.16 |
| 16478 | -1.90 | 0.84 | 1.01 | -0.07 | -0.07 |
| 16582 | 2.48 | 0.38 | 2.18 | 2.38 | 2.41 |
| 19965 | 0.79 | 2.10 | 1.08 | 1.08 | 1.13 |
| 20676 | 4.46 | 0.61 | 2.07 | 3.41 | 3.44 |
| 20860 | 9.87 | 8.40 | 1.95 | 2.63 | 2.75 |