In [1]:

```
import numpy as np

import matplotlib
import matplotlib.pyplot as plt
```

Task 1

Find the dimensions (height - h, radius - r) that will minimize the surface areaof the metal to manufacture a circular cylindrical can of volume V.

Solution

$$\begin{cases} V = \pi r^2 h = const \\ min_{h,r} S(h,r) = 2\pi rh + 2\pi r^2 \end{cases} \Longrightarrow \begin{cases} h = \dfrac{V}{\pi r^2} \\ min_r A(r) = \dfrac{2V}{r} + 2\pi r^2 \end{cases}$$

Point that minimize $A(r)$ must satisfy to the condition

$$dA(h,r) = 0, d^2A(r) > 0$$

$$\frac{dA}{dr} = -\frac{2V}{r^2} + 4\pi r = \frac{-2V + 4\pi r^3}{r^2} = 0$$

$$\begin{cases} -2V + 4\pi r^3 = 0 \\ r \neq 0 \\ h = \dfrac{V}{\pi r^2} \end{cases} \Longrightarrow \begin{cases} r = \left(\dfrac{V}{2\pi}\right)^{\dfrac{1}{3}} \\ h = \dfrac{V}{\pi\left(\dfrac{V}{2\pi}\right)^{\dfrac{2}{3}}} \end{cases} \Longrightarrow \begin{cases} r = \left(\dfrac{2V}{4\pi}\right)^{\dfrac{1}{3}} \\ h = \dfrac{V^{\dfrac{1}{3}}}{\pi\left(\dfrac{1}{2\pi}\right)^{\dfrac{2}{3}}} \\ \textbf{For all } V \geq 0 \end{cases}$$

Task 2

Consider the unconstrained optimization problem to minimize the function

$$f(x) = \frac{3}{2}(x_1^2 + x_2^2) + (1+a)x_1x_2 - (x_1 + x_2) + b$$

, where a and b are real-valued parameters. Find all values of a and b such that the problem has a unique optimal solution.

Solution

$f(x_1, x_2) = \frac{3}{2}(x_1^2 + x_2^2) + (1+a)x_1x_2 - (x_1 + x_2) + b$ Criteria for minimum:

$$\begin{cases} df(x_1, x_2) = 0 \\ d^2f(x_1, x_2) \geq 0 \end{cases} \implies \begin{cases} df(x_1, x_2) = (3x_1 + (1+a)x_2 - 1)\,dx_1 + (3x_2 + (1+a)x_1 - 1)\,dx_2 \\ d^2f(x_1, x_2) = 7 + a \geq 0 \end{cases}$$

In matrix form:

$$\begin{bmatrix} 3 & 1+a \\ 1+a & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \implies$$

Cramer's rule

$$\begin{cases} det = 9 - (1+a)^2 \neq 0 \\ det_{x_1} = 3 - (a+1) \\ det_{x_2} = 3 - (a+1) \end{cases} \implies x_1 = x_2 = \frac{1}{4+a} \implies a \neq -4 \implies \begin{cases} a \geq -7 \\ a \neq 2 \\ a \neq -4 \\ \textbf{For all } b \end{cases} \implies$$

Task 3

In [65]:

```python
def f(args):

    global p

    p += 1

    if np.square(np.linalg.norm(args + 5)) > 25:
        return float('inf')

    x = args[0]
    y = args[1]

    total = np.sin(y) * np.exp(np.square(1 - np.cos(x))) +\
            np.cos(x) * np.exp(np.square(1 - np.sin(y))) +\
            np.square(x - y)

    return total
```

Contour plot

In [76]:

```python
x = np.linspace(-10, 0, 101)
y = np.linspace(-10, 0, 101)
xx, yy = np.meshgrid(x, y, indexing='xy')
```

In [77]:

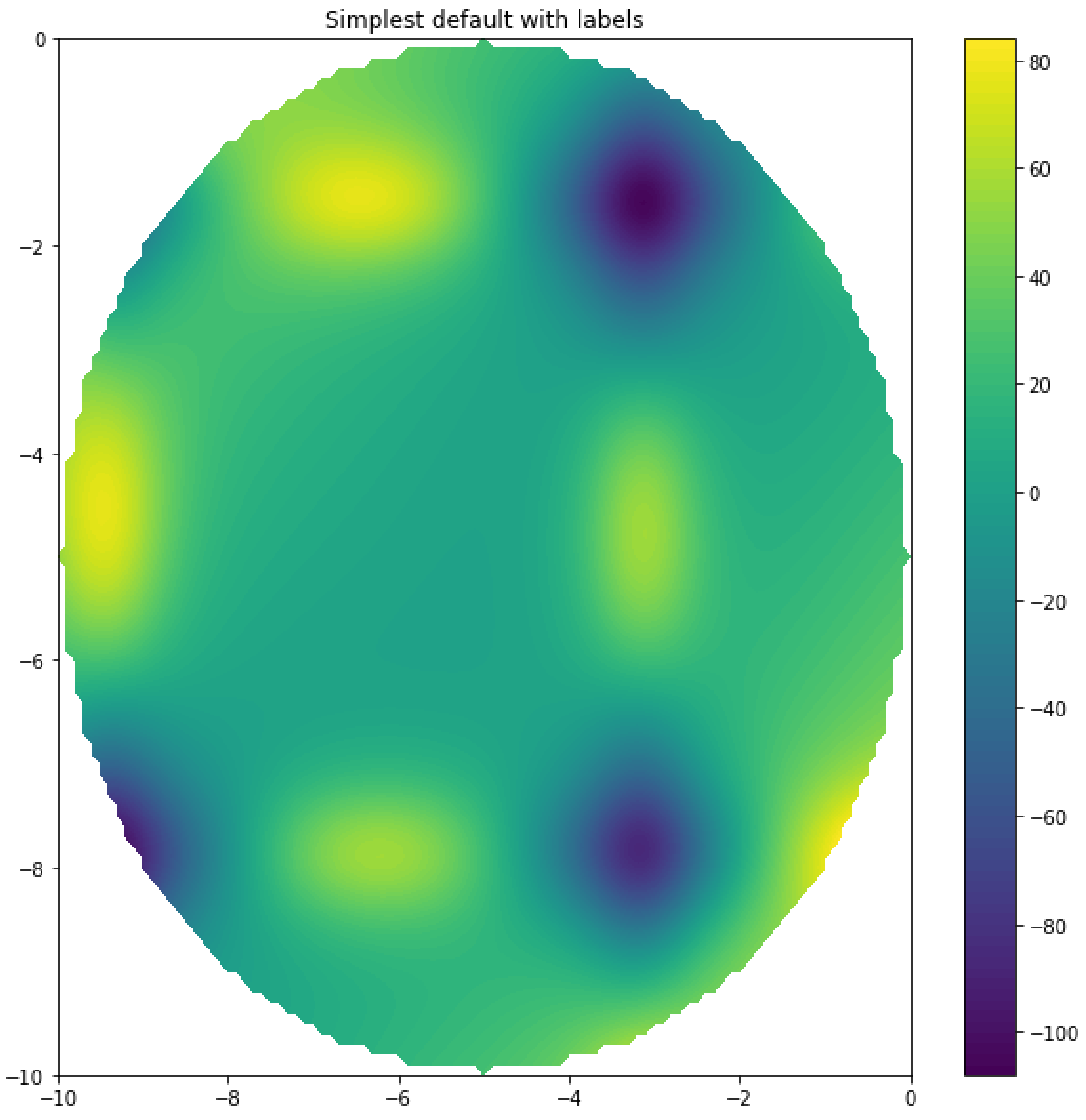
```python
points_set = np.empty((0, 2))
for i in range(len(xx)):
    points_set = np.append(points_set, np.array(list(zip(xx[i], yy[i]))), axis=0
)
```

In [83]:

```
zz = np.apply_along_axis(f, 1, points_set)

X = np.reshape(points_set[:, 0], (101, 101))
Y = np.reshape(points_set[:, 1], (101, 101))
Z = np.reshape(zz, (101, 101))

plt.figure(figsize=(10,10))
plt.contourf(X, Y, Z, levels=100)
plt.colorbar()
plt.title('Simplest default with labels');
```



In [89]:

```
def nelder_mead(func : callable, x_0 : list,
                alpha=1, beta=0.5, gamma=2, max_iter=-1,
                step=0.01, ro=0.5, tol=1e-9):

    hist = []

    points = np.zeros((len(x_0) + 1, len(x_0)))

    for i in range(len(x_0)):
        points[i, :] = x_0
        points[i, :] += step

    scored = sorted([ (point, func(point)) for point in points ],
                    key=lambda x: x[1])

#     print(scored)

    if max_iter == -1:
        max_iter = 25

    iter_num = 0

    while iter_num < max_iter and np.linalg.norm(scored[-1][0] - scored[-2][0])
> tol:

        scored = sorted([ [point, func(point)] for point in points ],
                        key=lambda x: x[1])

        hist.append(scored)

        iter_num += 1

        # Sort (1)
        x_h = scored[-1][0]
        f_h = scored[-1][1]

#         print(x_h, f_h)

        x_l = scored[0][0]
        f_l = scored[0][1]

        x_g = scored[-2][0]
        f_g = scored[-2][1]

        # Center of gravity (2)
        x_c = sum([p[0] for p in scored[:-1]]) / (len(scored) - 1)

        # Reflection (3)
        x_r = x_c + alpha * (x_c - x_h)
        f_r = func(x_r)

        # Comparison (4)
        x_e = x_c + gamma * (x_r - x_c)
        f_e = func(x_e)
        row_h = np.where((points == x_h).all(axis=1))[0][0]

        # 4(a)
        if f_r < f_l:
            if f_e < f_l:
                x_h = x_e
                points[row_h, :] = x_e
                continue

            elif f_e > f_l:
                x_h = x_r
                points[row_h, :] = x_r
                continue

        # 4(b)
        if f_l < f_r < f_g:
            x_h = x_r
            points[row_h, :] = x_r
            continue

        # 4(c)
        if f_h > f_r > f_g:
            x_h = x_r
            points[row_h, :] = x_r

        # Contraction (5)
        x_s = x_c + beta * (x_h - x_c)
        f_s = func(x_s)

        # (6)
        if f_s < f_h:
            x_h = x_s
            points[row_h, :] = x_s
            continue

        # (7)
        if f_s < f_h:
            for i in range(points.shape[0]):
                points[i, :] = x_l + (points[i, :] - x_l) / 2


    scored = sorted([ (point, func(point)) for point in points ],
                    key=lambda x: x[1])
    return scored[-1][0], hist
```

Starts from different points

In [128]:

```
global p
p = 0
x_opt, hist = nelder_mead(func=f, x_0=np.array([-3, -2]), step=0.1, max_iter=125
)
f_opt = f(x_opt)
```

In [129]:

```
print("x_opt : {} \nf_opt : {} \nOracle calls : {}".format(x_opt, f_opt, p - 1))
```

```
x_opt : [-2.92845816 -1.918645  ]
f_opt : -88.03652366117646
Oracle calls : 206
```

In [130]:

```
global p
p = 0
x_opt, hist = nelder_mead(func=f, x_0=np.array([-3, -1]), step=0.1, max_iter=125
)
f_opt = f(x_opt)
```

In [131]:

```
print("x_opt : {} \nf_opt : {} \nOracle calls : {}".format(x_opt, f_opt, p - 1))
```

```
x_opt : [-3.28347604 -1.13341578]
f_opt : -80.31254749659469
Oracle calls : 658
```