Control Engineering Practice 17 (2009) 88–96

Contents lists available at ScienceDirect

Control Engineering Practice

journal homepage: www.elsevier.com/locate/conengprac

Time-scale decomposition of an optimal control problem in greenhouse climate management

E.J. Van Henten [a,b,*], J. Bontsema [a]

[a] Wageningen UR Greenhouse Horticulture, P.O. Box 16, NL-6700 AA Wageningen, The Netherlands

[b] Farm Technology Group, Wageningen University, P.O. Box 17, NL-6700 AA Wageningen, The Netherlands

ARTICLE INFO

Article history:
Received 16 January 2007
Accepted 22 May 2008
Available online 9 July 2008

Keywords:
Hierarchical control
Singular perturbation
Maximum principle
Climate control
Lettuce

ABSTRACT

Based on differences in dynamic response times in the crop production process, a hierarchical decomposition of greenhouse climate management is proposed. To a large extent the proposed decomposition builds on the time-scale decomposition of singularly perturbed systems commonly found in the literature. Main difference with these existing theoretical concepts is that the proposed decomposition is able to deal with rapidly fluctuating deterministic external inputs or disturbances acting on the fast sub-processes. For an example of economic optimal greenhouse climate management during one lettuce production cycle, the decomposition was successfully evaluated in simulations. Using these favourable results, a hierarchical concept for economic optimal greenhouse climate management is derived and discussed in view of application in horticultural practice.

1. Introduction

In horticultural practice, greenhouse climate control is considered to be an important tool to control crop growth and production both in a quantitative and a qualitative sense. Because the cost of operating modern, sophisticated greenhouses is high, optimal use of their potential is required. Energy consumption, for example, amounts to approximately 15% of total production costs and as such ranks amongst the three most important cost factors for a horticultural firm in the Netherlands. In addition, the consumption of natural gas for horticultural crop production amounts to 10% of the total consumption in the Netherlands. Therefore, any gain in energy efficiency may contribute significantly to an improvement in the economic performance of greenhouse crop production and will be in line with governmental policy aiming at efficient application of natural resources and the reduction of emissions to the environment.

Efficient greenhouse climate management requires a continuous trade-off between the benefits associated with the marketable product against the operating costs of the climate conditioning equipment, taking into account the current state of the process and its future evolution as well as the future evolution of the outdoor climate. Climate control based on explicitly balancing economic costs and benefits is a typical example of an optimal control problem (see e.g. Bryson, 1999; Kirk, 1970).

In the literature, optimal greenhouse climate management systems are commonly presented and analysed as hierarchical systems (Challa & Van Straten, 1991; Tantau, 1991; Udink ten Cate, Bot, & Van Dixhoorn, 1978). Also in other application fields, hierarchical decomposition of the control system is applied, for instance in waste water treatment (Brdys, Grochowski, Gminski, Konarczak, & Drew, 2008). One reason for the hierarchical decomposition of greenhouse climate management is the inherent complexity of the crop production process in which both physical and physiological phenomena take place. Another reason for the decomposition is the fact that in the crop production process, significant differences in response times do exist. For instance crop dry matter production responds rather slowly to changes in the environmental conditions compared with the relatively fast dynamic response of the greenhouse climate to changes in the control inputs and weather conditions. Though offering a means to structure the control of a complex system, main limitations of these hierarchical schemes were that (i) in some cases, not at all levels, control of dynamic system responses was emphasised, (ii) the choice of the number of control levels was relatively arbitrary and not based on an analysis of systems dynamics, (iii) the control objectives used at each level did not have a clear relationship with each other nor with the main high-level objective of greenhouse climate management, (iv) the interaction between the individual control layers was not very clearly defined.

The theory of singularly perturbed systems offers a way to tackle these issues and to produce a sound decomposition based

* Corresponding author at: Farm Technology Group, Wageningen University, P.O. Box 17, NL-6700 AA Wageningen, The Netherlands. Tel.: +31 317483328; fax: +31 317484819.

E-mail address: eldert.vanhenten@wur.nl (E.J. Van Henten).

0967-0661/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.conengprac.2008.05.008

on the differences in dynamic response times present in the system. This technique has been used quite a number of times in various engineering applications (see e.g. Kokotović, Khalil, & O'Reilly, 1986; Subbaram Naidu, 2002). Besides the preliminary results presented by Van Henten and Bontsema (1992), in this paper, for the first time, singular perturbation theory will be applied to an optimal control problem in greenhouse climate management. Based on the time-scale decomposition obtained, a new hierarchical structure for optimal greenhouse climate management will be presented.

The paper is organised as follows. First, time-scale issues in optimal greenhouse climate management will be described in a formal way in Section 2. Then, in Section 3, the time-scale decomposition is proposed. It will be shown that this decomposition yields a performance criterion at each level which is consistent with the overall objective of the optimisation problem and has a crisp interpretation. Then, in Section 3, the time-scale decomposition is applied and evaluated in a simulation example concerning economic optimal control of a full lettuce production cycle. Based on the time-scale decomposition, in Section 4, a new hierarchical concept for greenhouse climate management will be presented and discussed in view of application in horticultural practice. Finally, Section 5 contains some concluding remarks and some implications for future research on optimal greenhouse climate control will be addressed.

2. Time-scale issues in optimal greenhouse climate control

For a single harvest crop, such as lettuce, the net economic return of the crop production process can be defined as

$$J(u) = \Phi(x(t_f)) - \int_{t_b}^{t_f} L(x, z, u, v, t)\,\mathrm{d}t, \tag{1}$$

where x and z are state variables like crop dry matter, indoor temperature, humidity and carbon dioxide concentration, u are control inputs like energy input by the heating system, ventilation flux through the vents and carbon dioxide supply flux, v are external disturbances like solar radiation, outdoor temperature, humidity and wind speed, t denotes time, t_b is the planting date, t_f is the harvest date, $\Phi(x(t_f))$ represents the gross economic return of the production process by selling the harvested product at the auction at the harvest date $t = t_f$ and $L(x, z, u, v, t)$ represents the operating costs of the climate conditioning equipment. A typical growing period for a lettuce crop lasts approximately 60 days depending on the season. For crops like tomatoes, cucumbers and sweet pepper, the term $\Phi(x(t_f))$ in the performance criterion has to be modified to account for multiple harvests in the interval $t \in [t_b, t_f)$.

In the crop production process, significant differences in response times do exist. For instance crop dry matter production responds rather slowly to changes in the environmental conditions compared with the relatively fast dynamic response of the greenhouse climate to changes in the control inputs and weather conditions. It is on the basis of these differences in response times, that the state variables have been divided into two groups, the slowly responding state variables x, representing crop growth and evolution, and the fast state variables z, representing the indoor greenhouse climate conditions.

The dynamics of these state variables are described by the following set of differential equations:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = f(x, z, u, v, t), \quad x(t_b) = x_b, \tag{2a}$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = G(x, z, u, v, t), \quad z(t_b) = z_b. \tag{2b}$$

The control inputs are subject to technical limits, represented by the following inequality constraints:

$$u_{\min} \leqslant u(t) \leqslant u_{\max}. \tag{3}$$

Also, limits are imposed on the state variables z

$$z_{\min} \leqslant z(t) \leqslant z_{\max}. \tag{4}$$

These inequality constraints should prevent the greenhouse climate variables from being driven into conditions unfavourable for crop growth. For instance very high and very low temperatures are lethal for most production crops. Also, some crops become sensitive to fungal diseases when grown under high relative humidity levels.

Then the optimal control problem is to find

$$u^*(t) = \max_u J(u), \quad t \in [t_b, t_f), \tag{5}$$

subject to Eqs. (2)–(4).

This and related optimal control problems in greenhouse climate management have received considerable attention in agricultural engineering research. Refer to Chalabi (1992), Chalabi, Biro, Bailey, Aikman, and Cockshull (2002a, 2002b), Critten (1991), Reinisch, Arnold, Markert, and Puta (1989), Seginer, Angel, Gal, and Krantz (1986), Seginer, Shina, Albright, and Marsh (1991), Seginer (1991), Van Henten and Bontsema (1991), to mention a few examples. In these researches, the dynamics of the fast state variables were commonly neglected. Implicitly or explicitly, Eq. (2b) was written as

$$0 = G(x, z, u, v, t), \tag{6}$$

from which z was numerically or analytically solved as

$$z = h(x, u, v, t) \tag{7}$$

and then substituted into Eq. (2a) to yield a reduced order system description

$$\frac{\mathrm{d}x}{\mathrm{d}t} = f(x, u, v, t), \quad x(t_b) = x_b. \tag{8}$$

For this reduced order system, the optimal control problem was to find $u^*(t) = \max_u J(u)$, $t \in [t_b, t_f)$, subject to Eq. (3), a redefined Eq. (4) and Eq. (8). Or, alternatively, u was written as

$$u = h'(x, z, v, t), \tag{9}$$

which substituted into Eq. (2a) yielded a reduced order system description

$$\frac{\mathrm{d}x}{\mathrm{d}t} = f(x, z, v, t), \quad x(t_b) = x_b. \tag{10}$$

Also the performance criterion of Eq. (1) and the control constraints of Eq. (3) were suitably redefined in terms of z, i.e.

$$J(z) = \Phi(x(t_f)) - \int_{t_b}^{t_f} L'(x, z, v, t)\,\mathrm{d}t \tag{11}$$

and

$$u_{\min} \leqslant h'(x, z, v, t) \leqslant u_{\max}. \tag{12}$$

Then, the optimal control problem was to find $z^*(t) = \max_z J(z)$, $t \in [t_b, t_f)$, subject to Eqs. (4), (10) and (12). The objective of this approach was not to generate optimal trajectories for the control inputs u, but to generate optimal set-point trajectories for the greenhouse climate variables z to be realised in the greenhouse environment by the commonly available feed-back control systems.

If the outdoor climate is only slowly varying, it may be plausible to neglect the dynamics of the fast state variables since in such a situation they do not affect the solution of the optimal control problem to a large extent and/or the overall system just follows slowly varying trajectories in time. After a short initial boundary layer transient at $t = t_b$, the dynamics of z rapidly diminish in time and Eq. (7) is a sufficiently accurate description of z. Under slowly varying outdoor conditions, the boundary layer will be in the order of an hour since within an hour the climate variables in a greenhouse will have achieved a steady-state. Compared to the length of the optimisation interval in the order of weeks or months, this time span is negligible. Also, the impact of this boundary layer transient on the overall performance of the optimal controlled process can then be neglected (Van Henten & Bontsema, 1992).

However, it was shown by Tap, Van Willigenburg, Van Straten, and Van Henten (1993) that under Dutch circumstances it is not feasible to neglect the greenhouse climate dynamics, since they significantly affect the performance of the optimal controlled process. The origin of this effect lies in the rapid fluctuations of the outdoor climate conditions (e.g. solar radiation, temperature, wind speed) and the fact that these outdoor conditions strongly influence the net economic return of the crop production process. Basically, crop growth relies on the light induced (photosynthetic) conversion of carbon dioxide into sugars. Therefore, crop growth and the gross economic return will be enhanced if the carbon dioxide concentration is raised during high radiation levels. On the other hand, due to the poor isolation between indoor and outdoor climates, raising the indoor concentration of carbon dioxide or the temperature above the ambient level will result in losses to the environment and therefore affect the operating cost of the climate conditioning equipment. Rapid anticipation to the outdoor climate was found to be profitable. But, these rapid responses lie within the dynamic response time of the greenhouse climate variables z in which case Eq. (7) is no longer an accurate description and the dynamics of z have to be explicitly accounted for. But calculating optimal controls for such a system on, let us say, a minute-by-minute basis over an interval of several weeks or months is impractical from a computational point of view. Not to mention the need of having to tackle the stiffness in the system equations due to the considerable differences in response time involved.

To circumvent both the computational and stiffness problems mentioned above, in this paper, a decomposition of the optimal control problem is presented. Based on the time-scale properties of the system, the control problem is decomposed into a sub-problem accounting for the slow dynamic responses and a sub-problem that accounts for the fast dynamic responses. Using a simulation example, it will be shown that this two-stage solution of the optimal control problem accurately approximates the solution of the full optimisation problem.

3. Proposition of the time-scale decomposition

For the full optimal control problem stated in Eqs. (1)–(5), the Hamiltonian is

$$H(x,\lambda,z,\eta,u,v,t) = -L(x,z,u,v,t) + \lambda^T f(x,z,u,v,t) + \eta^T G(x,z,u,v,t). \tag{13}$$

If penalties are added to $L(\cdot)$ to deal with the state constraints of Eq. (4), the optimal control problem can be solved numerically using the Maximum Principle of Pontryagin employing the necessary conditions: $\dot{x} = \partial H/\partial\lambda$ with $x(t_b) = x_b$, $\dot{z} = \partial H/\partial\eta$ with $z(t_b) = z_b$, $\dot{\lambda} = -\partial H/\partial x$ with $\lambda(t_f) = \partial\Phi(x(t_f))/\partial x$, $\dot{\eta} = -\partial H/\partial z$ with $\eta(t_f) = \partial\Phi(x(t_f))/\partial z$ and $H(x^*,\lambda^*,z^*,\eta^*,u,v,t) \leqslant H(x^*,\lambda^*,z^*,\eta^*,u^*,v,t)$ (Pontryagin, Boltyanski, Gamkrelidze, & Mishchenko, 1962).

For the decomposition of the optimal control problem, a decomposition is used which very much resembles a time-scale decomposition based on the theory of singularly perturbed systems (e.g. Kokotović et al., 1986). The system Eq. (2) is rewritten as

$$\frac{dx}{dt} = f(x,z,u,v,t), \quad x(t_b) = x_b, \tag{14a}$$

$$\varepsilon\frac{dz}{dt} = g(x,z,u,v,t), \quad z(t_b) = z_b. \tag{14b}$$

For $\varepsilon \to 0$, this reduces to

$$\frac{d\overline{x}}{dt} = f(\overline{x},\overline{z},\overline{u},v,t), \quad \overline{x}(t_b) = x_b, \tag{15a}$$

$$0 = g(\overline{x},\overline{z},\overline{u},v,t), \tag{15b}$$

in which $\overline{x}$, $\overline{z}$ and $\overline{u}$ refer to the responses of the slow dynamics. For the optimal control problem concerning these slow system dynamics, referred to as the slow sub-problem, the Hamiltonian is

$$H(\overline{x},\overline{\lambda},\overline{z},\overline{\eta},\overline{u},v,t) = -L(\overline{x},\overline{z},\overline{u},v,t) + \overline{\lambda}^{\mathrm{T}} f(\overline{x},\overline{z},\overline{u},v,t) + \overline{\eta}^{\mathrm{T}} g(\overline{x},\overline{z},\overline{u},v,t), \tag{16}$$

and necessary conditions for optimality require that $\dot{\overline{x}} = \partial H/\partial\overline{\lambda}$ with $\overline{x}(t_b) = x_b$, $\varepsilon\dot{\overline{z}} = \partial H/\partial\overline{\eta}$ which is reduced to $0 = \partial H/\partial\overline{\eta}$ when $\varepsilon \to 0$, $\dot{\overline{\lambda}} = -\partial H/\partial\overline{x}$ with $\overline{\lambda}(t_f) = \partial\Phi(\overline{x}(t_f))/\partial\overline{x}$, $\varepsilon\dot{\overline{\eta}} = -\partial H/\partial\overline{z}$ which reduces to $0 = -\partial H/\partial\overline{z}$ when $\varepsilon \to 0$ and $H(\overline{x}^*,\overline{\lambda}^*,\overline{z}^*,\overline{\eta}^*,\overline{u},v,t) \leqslant H(\overline{x}^*,\overline{\lambda}^*,\overline{z}^*,\overline{\eta}^*,\overline{u}^*,v,t)$. So for both z and η a quasi stationary description is obtained. Both variables may change in time, but they achieve a steady-state infinitely fast. This Hamiltonian is associated with the problem of finding $\overline{u}^*(t) = \max_{\overline{u}} J(\overline{u})$, $t \in [t_b, t_f)$ subject to the constraints of Eq. (15), $z_{\min} \leqslant \overline{z}(t) \leqslant z_{\max}$ and $u_{\min} \leqslant \overline{u}(t) \leqslant u_{\max}$ and with the performance measure

$$J(\overline{u}) = \Phi(\overline{x}(t_f)) - \int_{t_b}^{t_f} L(\overline{x},\overline{z},\overline{u},v,t)\,dt. \tag{17}$$

In the standard singular perturbation theory concerning optimal control systems, equations are derived for boundary layers at $t = t_b$ and $t = t_f$. These approximations will also suffice if the system is excited by slowly varying inputs. But following the arguments in the introduction, if the system is excited with external inputs v exhibiting changes within the dynamic response time of z, corrections should be made over the whole interval $t = [t_b, t_f)$ because $\overline{z}$ is then no longer an accurate approximation of z (Kokotović et al., 1986).

To approximate the fast dynamic responses, zero-th order approximations of the states, costates and control inputs are used: $x(t) \cong \hat{x}(t) = \overline{x}(t) + \tilde{x}(\varepsilon\tau)$, $z(t) \cong \hat{z}(t) = \overline{z}(t) + \tilde{z}(\varepsilon\tau)$, $\lambda(t) \cong \hat{\lambda}(t) = \overline{\lambda}(t) + \tilde{\lambda}(\varepsilon\tau)$, $\eta(t) \cong \hat{\eta}(t) = \overline{\eta}(t) + \tilde{\eta}(\varepsilon\tau)$ and $u(t) \cong \hat{u}(t) = \overline{u}(t) + \tilde{u}(\varepsilon\tau)$, where $\tilde{x}$, $\tilde{z}$, $\tilde{\lambda}$, $\tilde{\eta}$ and $\tilde{u}$ refer to the fast corrections. Then, the Hamiltonian is approximated by

$$\begin{aligned} H(x,\lambda,z,\eta,u,v,t) \cong\ & H(\overline{x},\overline{\lambda},\overline{z},\overline{\eta},\overline{u},v,t) \\ & + H(\overline{x}+\tilde{x},\overline{\lambda}+\tilde{\lambda},\overline{z}+\tilde{z},\overline{\eta}+\tilde{\eta},\overline{u}+\tilde{u},v,t) \\ & - H(\overline{x},\overline{\lambda},\overline{z},\overline{\eta},\overline{u},v,t). \end{aligned} \tag{18}$$

The first term on the right-hand side equals Eq. (16) and was used to solve the slow sub-problem of Eqs. (15) and (17). So, the fast sub-problem deals with the Hamiltonian system

$$\begin{aligned} & \tilde{H}(\overline{x},\tilde{x},\overline{\lambda},\tilde{\lambda},\overline{z},\tilde{z},\overline{\eta},\tilde{\eta},\overline{u},\tilde{u},v,t) \\ & = H(\overline{x}+\tilde{x},\overline{\lambda}+\tilde{\lambda},\overline{z}+\tilde{z},\overline{\eta}+\tilde{\eta},\overline{u}+\tilde{u},v,t) \\ & - H(\overline{x},\overline{\lambda},\overline{z},\overline{\eta},\overline{u},v,t) \\ & = -L(\overline{x}+\tilde{x},\overline{z}+\tilde{z},\overline{u}+\tilde{u},v,t) + L(\overline{x},\overline{z},\overline{u},v,t) + \overline{\lambda}^{\mathrm{T}} \\ & \times (f(\overline{x}+\tilde{x},\overline{z}+\tilde{z},\overline{u}+\tilde{u},v,t) - f(\overline{x},\overline{z},\overline{u},v,t)) + \tilde{\lambda}^{\mathrm{T}} \\ & \times f(\overline{x}+\tilde{x},\overline{z}+\tilde{z},\overline{u}+\tilde{u},v,t) + \overline{\eta}^{\mathrm{T}}(g(\overline{x}+\tilde{x},\overline{z}+\tilde{z},\overline{u} \\ & + \tilde{u},v,t) - g(\overline{x},\overline{z},\overline{u},v,t)) + \tilde{\eta}^{\mathrm{T}} g(\overline{x}+\tilde{x},\overline{z}+\tilde{z},\overline{u}+\tilde{u},v,t). \end{aligned} \tag{19}$$

Since z responds much faster than x, the fast corrections $\tilde{x}$ and $\tilde{\lambda}$ can be neglected. This is common practice in singular perturbation theory. Then reformulating the Hamiltonian of Eq. (19) with $\hat{z} = \overline{z} + \tilde{z}$, $\hat{\eta} = \overline{\eta} + \tilde{\eta}$ and $\hat{u} = \overline{u} + \tilde{u}$ the following result is obtained:

$$\begin{aligned}\hat{H}(\overline{x}, \overline{\lambda}, \hat{z}, \hat{\eta}, \hat{u}, v, t) &= -L(\overline{x}, \hat{z}, \hat{u}, v, t) + L(\overline{x}, \overline{z}, \overline{u}, v, t) \\ &+ \overline{\lambda}^{\mathrm{T}}(f(\overline{x}, \hat{z}, \hat{u}, v, t) - f(\overline{x}, \overline{z}, \overline{u}, v, t)) + \hat{\eta}^{\mathrm{T}} g(\overline{x}, \hat{z}, \hat{u}, v, t) \\ &- \overline{\eta}^{\mathrm{T}} g(\overline{x}, \overline{z}, \overline{u}, v, t).\end{aligned} \tag{20}$$

Since both $L(\overline{x}, \overline{z}, \overline{u}, v, t)$, $f(\overline{x}, \overline{z}, \overline{u}, v, t)$ and $g(\overline{x}, \overline{z}, \overline{u}, v, t)$ are not a function of $\hat{u}$, these terms can be omitted from Eq. (20) to obtain:

$$\begin{aligned}&\hat{H}(\overline{x}, \overline{\lambda}, \hat{z}, \hat{\eta}, \hat{u}, v, t) \\ &= -L(\overline{x}, \hat{z}, \hat{u}, v, t) + \overline{\lambda}^{\mathrm{T}} f(\overline{x}, \hat{z}, \hat{u}, v, t) + \hat{\eta}^{\mathrm{T}} g(\overline{x}, \hat{z}, \hat{u}, v, t).\end{aligned} \tag{21}$$

This Hamiltonian is associated with the optimal control problem of finding $\hat{u}^* = \max_{\hat{u}} J(\hat{u})$, with

$$J(\hat{u}) = \int_{t_b}^{t_f} -L(\overline{x}, \hat{z}, \hat{u}, v, t) + \overline{\lambda}^{\mathrm{T}} f(\overline{x}, \hat{z}, \hat{u}, v, t)\,\mathrm{d}t, \tag{22}$$

subject to the fast system equations

$$\varepsilon \dot{\hat{z}} = g(\overline{x}, \hat{z}, \hat{u}, v, t), \quad \hat{z}(t_b) = z_b, \tag{23}$$

and the state and control inequality constraints $z_{\min} \leqslant \hat{z}(t) \leqslant z_{\max}$ and $u_{\min} \leqslant \hat{u}(t) \leqslant u_{\max}$. In this research, for practical purposes, the time scaling parameter ε was set at $\varepsilon = 1$.

Observe the difference between the performance criterion of the slow sub-problem in Eq. (17) which is equivalent to the original criterion of Eq. (1) and the criterion derived for the fast sub-problem in Eq. (22) which differs from Eq. (1) but directly follows from the original performance criterion. In the fast sub-problem, $\overline{x}$ and $\overline{\lambda}$ act as slowly varying external (reference) inputs. Eq. (22) has an interesting interpretation. With $f(\overline{x}, \hat{z}, \hat{u}, v, t)$ describing crop growth and development and λ representing the marginal value of a unit growth, Eq. (22) shows that the fast subsystem, i.e. the greenhouse climate, is controlled on the basis of a trade-off between the additional costs on one hand and the revenues of increasing crop growth and development on the other hand, at any moment during the growing period $[t_b, t_f)$. This is an interesting feature, because growers usually find it difficult to translate the long-term tactical objective of maximising net economic return into short term operational decisions and actions. So, this hierarchical decomposition bridges the gap between tactical planning and operational control.

4. Application of the time-scale decomposition to optimal greenhouse climate control

4.1. The optimal control problem

The research reported in this paper focuses on economic optimal greenhouse climate management during the production of a lettuce crop. The objective is to maximise the net economic return of the crop production process. The net economic return of the lettuce production process, expressed in Dutch guilders per square metre greenhouse (Hfl m^{-2}), is described by the equation:

$$J = \Phi(x(t_f)) - \int_{t_b}^{t_f} (c_q u_q(t) + c_{CO_2} u_{CO_2}(t))\,\mathrm{d}t, \tag{24}$$

where $\Phi(x(t_f))$ in Hfl m^{-2} is the gross income obtained at harvest time t_f when selling the harvested product at the auction, and $c_q u_q(t) + c_{CO_2} u_c(t)$ is the running costs of the climate conditioning equipment in Hfl m^{-2} s^{-1}. Analysis of the auction price of lettuce in the period 1985–1990 revealed a linear relationship $\Phi(x(t_f)) = c_{pri,1} + c_{pri,2} x_d(t_f)$, parameterised by $c_{pri,1}$ in Hfl m^{-2} and $c_{pri,2}$ in Hfl kg^{-1} m^{-2}, between the auction price and the harvest weight of lettuce x_d in kg m^{-2}. As shown by Van Henten (1994), this is a reasonable equation to use in the Dutch case, especially in winter if there is no competition from lettuce crops grown in the open field. The running costs of the climate conditioning equipment is assumed to be linearly related with the amount of energy u_q in W m^{-2} and the amount of carbon dioxide u_c in kg m^{-2} s^{-1}, put into the system. These running costs are parameterised by the energy price c_q in Hfl J^{-1} and the price of carbon dioxide c_{CO_2} in Hfl kg^{-1}, respectively. Their values are given in Table 1. It is assumed that no costs were associated with natural ventilation used for cooling and dehumidification. The contributions of the electrical equipment used for greenhouse climate conditioning, such as pumps and valves, to the operating costs are ignored. Furthermore, it is assumed that other production factors, such as the nutrient and water supply, screening and those not directly related to greenhouse climate control, such as labour input, pest and disease control, do not affect the control strategies. Consequently, they are not included in the performance criterion. The running costs are integrated over the whole growing period starting at the planting date t_b and ending at harvest time t_f. Then, subtraction of the integrated operating costs from the gross income yields the net economic return of the crop production process to be optimised.

The greenhouse crop production process is described by a four state variable dynamic model. The model describes the evolution in time of the dry matter content of the crop x_d in kg m^{-2},

Table 1
Model parameters

Parameter	Value
$c_{\alpha\beta}$	0.544
$c_{ai,ou}$	6.1 W m^{-2} °C^{-1}
$c_{cap,c}$	4.1 m
$c_{cap,h}$	4.1 m
$c_{cap,q}$	30,000 J m^{-2} °C^{-1}
$c_{cap,q,v}$	1290 J m^{-3} °C^{-1}
c_{CO_2}	42×10^{-2} Hfl kg^{-1}
$c_{CO_{2,1}}$	5.11×10^{-6} m s^{-1} °C^{-2}
$c_{CO_{2,2}}$	2.30×10^{-4} m s^{-1} °C^{-1}
$c_{CO_{2,3}}$	6.29×10^{-4} m s^{-1}
c_{leak}	0.75×10^{-4} m s^{-1}
$c_{pl,d}$	53 m^2 kg^{-1}
$c_{pri,1}$	1.8 Hfl m^{-2}
$c_{pri,2}$	16 Hfl kg^{-1}
c_q	6.35×10^{-9} Hfl J^{-1}
c_R	8314 J K^{-1} kmol^{-1}
$c_{rad,phot}$	3.55×10^{-9} kg J^{-1}
$c_{rad,q}$	0.2
$c_{resp,d}$	2.65×10^{-7} s^{-1}
$c_{resp,c}$	4.87×10^{-7} s^{-1}
$c_{T,abs}$	273.15 K
$c_{v,pl,ai}$	3.6×10^{-3} m s^{-1}
$c_{v,1}$	9348 J m^{-3}
$c_{v,2}$	17.4
$c_{v,3}$	239 °C
$c_{v,4}$	10,998 J m^{-3}
c_Γ	5.2×10^{-5} kg m^{-3}
$u_{c,\min}$	0 kg m^{-2} s^{-1}
$u_{c,\max}$	1.2×10^{-6} kg m^{-2} s^{-1}
$u_{q,\min}$	0 W m^{-2}
$u_{q,\max}$	150 W m^{-2}
$u_{v,\min}$	0 m s^{-1}
$u_{v,\max}$	7.5×10^{-3} V$_w$ m s^{-1}
$z_{T,\min}$	6.5 °C
$z_{T,\max}$	40 °C
$z_{c,\min}$	0 kg m^{-3}
$z_{c,\max}$	2.75×10^{-3} kg m^{-3}
$Rz_{h,\min}$	0%
$Rz_{h,\max}$	90%

the carbon dioxide concentration in the greenhouse z_c in $\mathrm{kg\,m^{-3}}$, the air temperature in the greenhouse z_T in °C and the humidity content of the greenhouse air z_h in $\mathrm{kg\,m^{-3}}$, with the equations:

$$\frac{\mathrm{d}x_d}{\mathrm{d}t} = c_{\alpha\beta}\varphi_{phot,c} - c_{resp,d}x_d 2^{(0.1z_T-2.5)}, \tag{25}$$

where $c_{\alpha\beta}$ is a yield factor, $\varphi_{phot,c}$ is the gross canopy photosynthesis rate in $\mathrm{kg\,m^{-2}\,s^{-1}}$, $c_{resp,d}$ in $\mathrm{s^{-1}}$ is the respiration rate expressed in terms of the amount of respired dry matter and z_T is the air temperature in the greenhouse in °C,

$$\frac{\mathrm{d}z_c}{\mathrm{d}t} = \frac{1}{c_{cap,c}}[-\varphi_{phot,c} + c_{resp,c}x_d 2^{(0.1z_T-2.5)} + u_c - \varphi_{vent,c}], \tag{26}$$

where $c_{cap,c}$ is the greenhouse volume in $\mathrm{m^3\,m^{-2}}$, $c_{resp,c}$ in $\mathrm{s^{-1}}$ is the respiration coefficient expressed in terms of the amount of carbon dioxide produced, u_c is the supply rate of carbon dioxide in $\mathrm{kg\,m^{-2}\,s^{-1}}$ and $\varphi_{vent,c}$ is the mass exchange of carbon dioxide through the vents in $\mathrm{kg\,m^{-2}\,s^{-1}}$,

$$\frac{\mathrm{d}z_T}{\mathrm{d}t} = \frac{1}{c_{cap,q}}[u_q - Q_{vent,q} + Q_{rad,q}], \tag{27}$$

where $c_{cap,q}$ is the heat capacity of the greenhouse air in $\mathrm{J\,m^{-2}\,{}^\circ C^{-1}}$, u_q is the energy supply by the heating system in $\mathrm{W\,m^{-2}}$, $Q_{vent,q}$ is the energy exchange with the outdoor air by means of ventilation and transmission through the cover in $\mathrm{W\,m^{-2}}$ and $Q_{rad,q}$ is the heat load by solar radiation in $\mathrm{W\,m^{-2}}$,

$$\frac{\mathrm{d}z_h}{\mathrm{d}t} = \frac{1}{c_{cap,h}}[\varphi_{transp,h} - \varphi_{vent,h}], \tag{28}$$

where $c_{cap,h}$ is the greenhouse volume in $\mathrm{m^3\,m^{-2}}$, $\varphi_{transp,h}$ is the canopy transpiration in $\mathrm{kg\,m^{-2}\,s^{-1}}$ and $\varphi_{vent,h}$ is the mass exchange of water vapour through the vents in $\mathrm{kg\,m^{-2}\,s^{-1}}$.

The gross photosynthesis rate $\varphi_{phot,c}$ in $\mathrm{kg\,m^{-2}\,s^{-1}}$, is described by:

$$\varphi_{phot,c} = (1 - \mathrm{e}^{-c_{pl,d}x_d}) \times \frac{c_{rad,phot}v_{rad}(-c_{CO_{2,1}}z_T^2 + c_{CO_{2,2}}z_T - c_{CO_{2,3}})(z_c - c_\Gamma)}{c_{rad,phot}v_{rad} + (-c_{CO_{2,1}}z_T^2 + c_{CO_{2,2}}z_T - c_{CO_{2,3}})(z_c - c_\Gamma)}, \tag{29}$$

where $c_{pl,d}$ is the effective canopy surface in $\mathrm{m^2\,kg^{-1}}$, $c_{rad,phot}$ is the light use efficiency in $\mathrm{kg\,J^{-1}}$, v_{rad} is the solar radiation outside the greenhouse in $\mathrm{W\,m^{-2}}$, $c_{CO_{2,1}}$ in $\mathrm{m\,s^{-1}\,{}^\circ C^{-2}}$, $c_{CO_{2,2}}$ in $\mathrm{m\,s^{-1}\,{}^\circ C^{-1}}$ and $c_{CO_{2,3}}$ in $\mathrm{m\,s^{-1}}$ parameterise the temperature influence on gross canopy photosynthesis, c_Γ is the carbon dioxide compensation point in $\mathrm{kg\,m^{-3}}$. The mass transfer of carbon dioxide due to ventilation and leakage $\varphi_{vent,c}$ in $\mathrm{kg\,m^{-2}\,s^{-1}}$, is defined by:

$$\varphi_{vent,c} = (u_v + c_{leak})(z_c - v_c), \tag{30}$$

where u_v is the ventilation rate through the vents in $\mathrm{m\,s^{-1}}$, c_{leak} is the leakage through the cover in $\mathrm{m\,s^{-1}}$ and v_c is the carbon dioxide concentration outside the greenhouse in $\mathrm{kg\,m^{-3}}$. The energy transfer between the indoor environment and the outdoor environment due to ventilation and transmission $Q_{vent,q}$ in $\mathrm{W\,m^{-2}}$, is covered by the equation:

$$Q_{vent,q} = (c_{cap,q,v}u_v + c_{ai,ou})(z_T - v_T), \tag{31}$$

in which $c_{cap,q,v}$ is the heat capacity per volume unit of greenhouse air in $\mathrm{J\,m^{-3}\,{}^\circ C^{-1}}$, $c_{ai,ou}$ in $\mathrm{W\,m^{-2}\,{}^\circ C^{-1}}$ parameterises the overall heat transfer through the cover, v_T in °C stands for the outside air temperature. The energy input to the greenhouse system by solar radiation $Q_{rad,q}$ in $\mathrm{W\,m^{-2}}$, is described by:

$$Q_{rad,q} = c_{rad,q}v_{rad} \tag{32}$$

where $c_{rad,q}$ is the heat load coefficient due to solar radiation. Canopy transpiration $\varphi_{transp,h}$ in $\mathrm{kg\,m^{-2}\,s^{-1}}$, is governed by the equation:

$$\varphi_{transp,h} = (1 - \mathrm{e}^{-c_{pl,d}x_d})c_{v,pl,ai} \times \left(\frac{c_{v,1}}{c_R(z_T + c_{T,abs})}\,\mathrm{e}^{(c_{v,2}z_T)/(z_T+c_{v,3})} - z_h\right), \tag{33}$$

in which the term $c_{v,1}/c_R(z_T + c_{T,abs})\,\mathrm{e}^{c_{v,2}z_T/z_T+c_{v,3}}$ in $\mathrm{kg\,m^{-3}}$ represents the saturated water vapour content at canopy temperature z_T, $c_{v,pl,ai}$ is the mass transfer coefficient in $\mathrm{m\,s^{-1}}$, $c_{v,1}$ in $\mathrm{J\,m^{-3}}$, $c_{v,2}$ and $c_{v,3}$ in °C parameterise the saturation water vapour pressure, c_R is the gas constant in $\mathrm{J\,K^{-1}\,kmol^{-1}}$ and $c_{T,abs}$ parameterises the conversion of temperature from °C to K. The mass transfer of water vapour by means of ventilation $\varphi_{vent,h}$ in $\mathrm{kg\,m^{-2}\,s^{-1}}$, is described by:

$$\varphi_{vent,h} = (u_v + c_{leak})(z_h - v_h) \tag{34}$$

in which v_h in $\mathrm{kg\,m^{-3}}$ is the humidity concentration outside the greenhouse.

Model parameters are listed in Table 1. The model, though being of rather simple structure was found to describe measured data rather well. For a more detailed description and verification of this model is referred to Van Henten (1994).

Physical limitations on the control inputs u_c, u_q and u_v, are represented by the linear inequality constraints $u_{c,\min} \leqslant u_c \leqslant u_{c,\max}$, $u_{q,\min} \leqslant u_q \leqslant u_{q,\max}$, $u_{v,\min} \leqslant u_v \leqslant u_{v,\max}$, respectively. Bounds are also imposed on the temperature in the greenhouse z_T, the carbon dioxide concentration z_c and the humidity level z_h, to prevent the control system from driving the process into unfavourable conditions for crop growth and development. These bounds are represented by the linear inequality constraints $z_{T,\min} \leqslant z_T \leqslant z_{T,\max}$, $z_{c,\min} \leqslant z_c \leqslant z_{c,\max}$ and $z_{h,\min} \leqslant z_h \leqslant z_{h,\max}$. In fact, these bounds represent the limitations of the rather simple crop growth model used in this research, since the adverse effect of unfavourable climate conditions on crop growth and development should have been covered by the dynamic crop growth model. In the example considered, bounds are imposed on the relative humidity instead of the absolute humidity. This requires the transformation

$$z_{h,\min} = \frac{Rz_{h,\min}}{100} z_{h,sat}(z_T)$$

and

$$z_{h,\max} = \frac{Rz_{h,\max}}{100} z_{h,sat}(z_T),$$

with $Rz_{h,\min}$ and $Rz_{h,\max}$ being the lower and upper bound on the relative humidity, respectively, and the saturation water vapour pressure $z_{h,sat}$ in $\mathrm{kg\,m^{-3}}$, is a function of the temperature z_T:

$$z_{h,sat} = \frac{c_{v,4}}{c_R(z_T + c_{T,abs})}\,\mathrm{e}^{(c_{v,2}z_T)/(z_T+c_{v,3})}, \tag{35}$$

where $c_{v,4}$ in $\mathrm{J\,m^{-3}}$ parameterises the saturation water vapour pressure together with the parameters $c_{v,2}$ and $c_{v,3}$.

To deal with the inequality constraints on the state variables, following Pierre (1969), the performance criterion of Eq. (1) is extended with penalty functions $p(t)$ in $\mathrm{Hfl\,m^{-2}\,s^{-1}}$, having the general form:

$$p(z(t)) = c_\sigma \left[\frac{2z(t) - z_{\min} - z_{\max}}{z_{\max} - z_{\min}}\right]^{2k}, \tag{36}$$

where c_σ $\mathrm{Hfl\,m^{-2}\,s^{-1}}$ is a weighting factor, z is a state variable, $z_{\min}$ and $z_{\max}$ are the lower and the upper bounds put on the state variable, respectively, and the exponent k forces the penalty function to attain values near zero between the bounds and very steep slopes close to the bounds when $k = 1, 2, \ldots, \infty$. In this way, the controlled system is prevented from traversing the bounds. To guarantee consistence in the units used, the penalty is expressed in $\mathrm{Hfl\,m^{-2}\,s^{-1}}$. In a way this is a slightly artificial construction, though one may argue that by modifying the weighting parameter c_σ the grower is able to express his attitude towards taking risks, when the health of the crop is considered. After adding penalties for violations of the constraints on the temperature, carbon dioxide concentration and humidity, $p(z_T)$, $p(z_c)$ and $p(z_h)$, respectively, the resulting performance measure has the following form:

$$J = (c_{pri,1} + c_{pri,2}x_d(t_f)) - \int_{t_b}^{t_f} (c_q u_q(t) + c_{CO_2} u_c(t) + p_c(t) + p_T(t) + p_h(t))\,dt. \quad (37)$$

With the preliminaries presented above, the optimal control problem is defined as to find optimal control strategies for the control variables u_c, u_q and u_v over the time-interval $t \in [t_b, t_f)$, maximising the performance criterion of Eq. (37), subject to the differential equation constraints of Eqs. (25)–(28) and the linear inequality constraints on the controlled variables.

For the example considered, the Hamiltonian H has the following form:

$$\begin{aligned} H = &-c_{CO_2}u_c - c_q u_q + \lambda_d\{c_{\alpha\beta}\varphi_{phot,c} - c_{resp,d}x_d 2^{(0.1z_T-2.5)}\} \\ &+ \eta_c\left\{\frac{1}{c_{cap,c}}[-\varphi_{phot,c} + c_{resp,c}x_d 2^{(0.1z_T-2.5)} + u_c - \varphi_{vent,c}]\right\} \\ &+ \eta_T\left\{\frac{1}{c_{cap,q}}[u_q - Q_{vent,q} + Q_{rad,q}]\right\} + \eta_h \\ &\times\left\{\frac{1}{c_{cap,h}}[\varphi_{transp,h} - \varphi_{vent,h}]\right\} - p(z_c) - p(z_T) - p(z_h). \end{aligned} \quad (38)$$

4.2. Solution and verification of the time-scale decomposition

In this example a growing period of 50 days was considered. Two-minute measurements of outdoor climate conditions, including solar radiation, wind speed, temperature and humidity, obtained during a growing experiment in early 1992 were used as external inputs during the solution (Van Henten, 1994). Using the Hamiltonian of Eq. (38), first the full problem was solved to yield trajectories $x^*(t)$, $z^*(t)$, $\lambda^*(t)$, $\eta^*(t)$, $u^*(t)$ and a value for $J(u^*)$. These were used to evaluate the decomposition. Then, secondly, the slow sub-problem was solved to yield trajectories for $\bar{x}_d$ and $\bar{\lambda}_d$. Finally, with these trajectories of the slow states and costates, the fast sub-problem was solved. The decomposition was evaluated using the following criteria:

1. the optimal control trajectories of the fast sub-problem should approximate the optimal control trajectories of the original full optimal control problem, i.e. $\hat{u}^*(t) \cong u^*(t)$,
2. the evolution of the slow state and costate variables simulated with the optimal control trajectories of the fast sub-problem should approximate the state trajectories of the full control problem, i.e . $x(\hat{u}^*) \cong x(u^*)$ and $\lambda(\hat{u}^*) \cong \lambda(u^*)$,
3. the evolution of the fast state and costate variables calculated in the fast sub-problem should approximate the evolution of the fast state variables calculated in the full control problem, i.e. $\hat{z}(t) \cong z(t)$and $\hat{\eta}(t) \cong \eta(t)$, and finally,
4. the performance of the optimal control system using the solution of the fast sub-problem $\hat{u}(t)$should approximate the performance of the full optimal control system, i.e. $J(\hat{u}) \cong J(u)$.

4.3. Results and discussion

In Fig. 1a it is shown that the crop growth trajectory obtained in the slow sub-problem agrees well with the evolution of the crop state obtained in the solution of the full control problem, though an underestimation does occur. However, if the full system dynamics were simulated using the optimal controls of the fast sub-problem a more accurate approximation was obtained. The same can be observed for the slow costate trajectories (see Fig. 1b).

Fig. 2 shows the rapid fluctuations of the solar radiation over 2 days during the 50 days production cycle. Fig. 3 presents, for the same 2 days, the optimal state trajectories of the carbon dioxide concentration in the greenhouse. It is clearly illustrated that the slow sub-problem produces an inaccurate approximation of the fast system dynamics, because the rapid fluctuations in the external inputs are much faster than the dynamics of the fast sub-system (compare Fig. 3a and b). On the other hand, the same figure shows that the trajectories calculated in the fast sub-problem accurately describe the dynamics of the fast sub-system (compare Fig. 3a and c). The same can be observed for the fast costate trajectories associated with the carbon dioxide concentration (see Fig. 4) as well as for the state and costate trajectories of the temperature and humidity in the greenhouse (not shown).

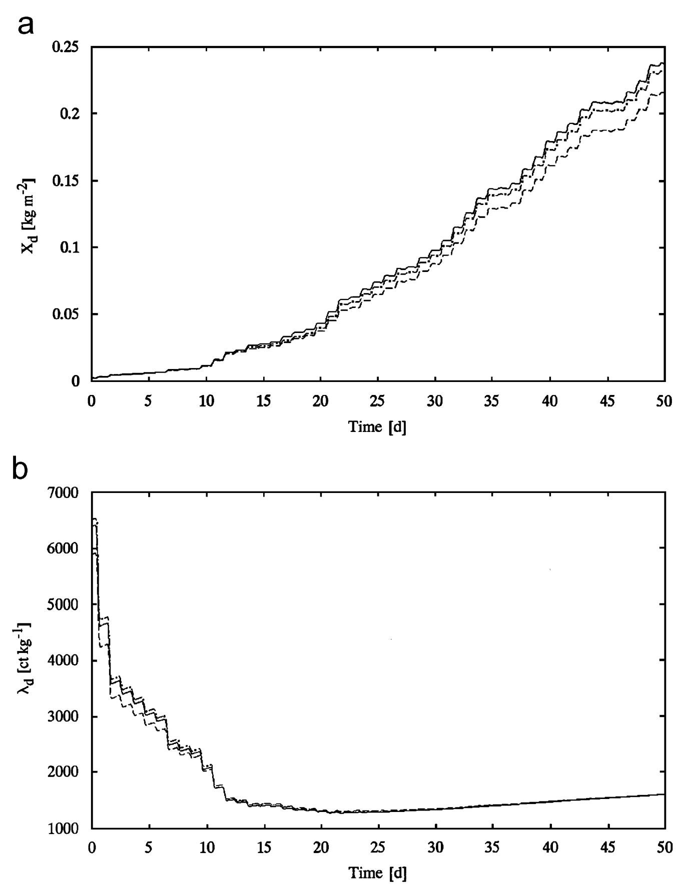

Fig. 1. State trajectories (a) and costate trajectories (b) obtained in the solution of the full control problem (—) and the slow sub-problem (- - -) and a simulation of the full system dynamics using the optimal control trajectories of the fast sub-problem (– · – · –).

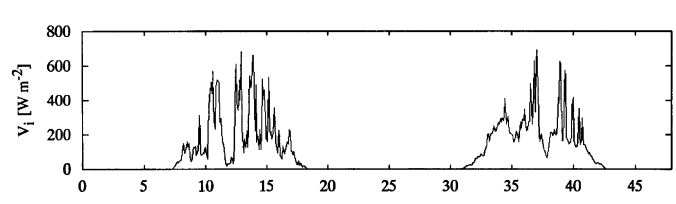

Fig. 2. Solar radiation during 2 days of the 50 days production cycle.

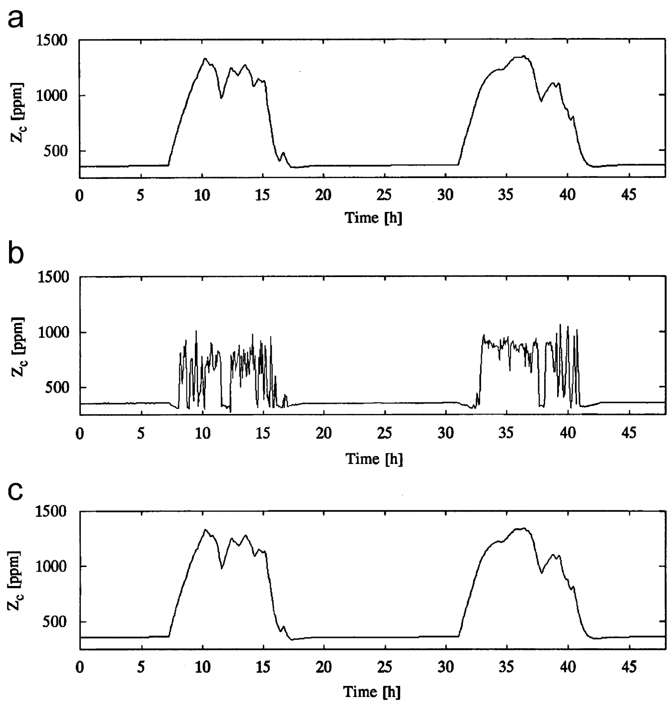

Fig. 3. Optimal carbon dioxide concentration obtained in the solution of the full problem (a), the slow sub-problem (b) and the fast sub-problem (c).

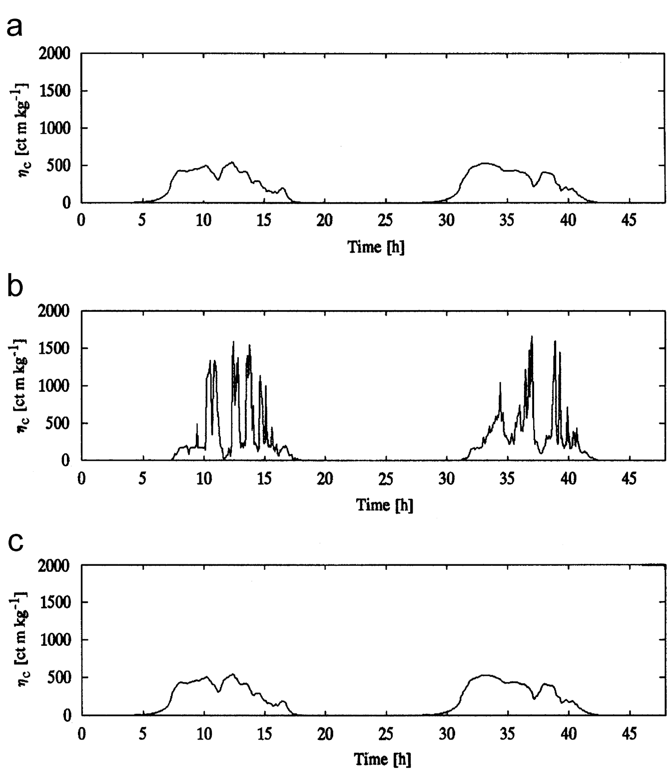

Fig. 4. Costate of carbon dioxide concentration obtained after solving the full problem (a), the slow sub-problem (b) and the fast sub-problem (c).

Fig. 5 shows the trajectories of the carbon dioxide supply rate. Again, the solution of the slow sub-problem is not an accurate approximation of the optimal control of the full problem, but the solution of the fast sub-problem is. Similar observations can be made for the control trajectories of energy supply and ventilation rate (not shown).

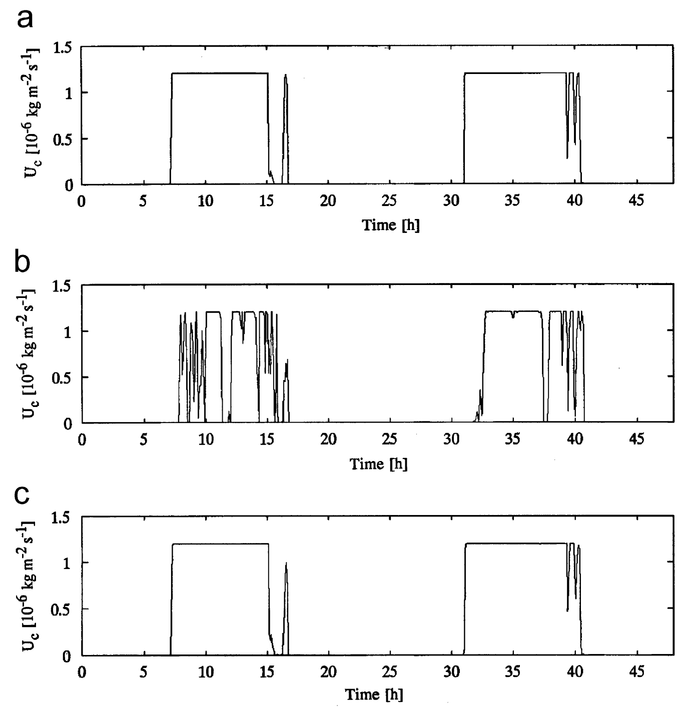

Fig. 5. Optimal carbon dioxide supply rate obtained in the solution of the full problem (a), the slow sub-problem (b) and the fast sub-problem (c).

Finally, evaluation of the performance using the control inputs calculated in the fast sub-problem, yielded only a 2% reduction compared with the performance obtained in the solution of the full optimal control problem. So, an accurate approximation was obtained for this optimal control problem. This result is obtained because as shown in Fig. 1 the slow system dynamics indeed do not respond to a large extent to the rapid fluctuations in the fast system dynamics, external inputs and control inputs, which was the main assumption on which this decomposition was based.

5. A hierarchical concept for greenhouse climate management

It was just shown that an optimal control problem arising in greenhouse climate management during one lettuce production cycle can be hierarchically decomposed into two sub-problems; one dealing with the slow crop growth dynamics and one dealing with the relatively fast greenhouse climate dynamics. But what is the implication of this result for greenhouse climate management in horticultural practice?

Fig. 6 shows in a schematic diagram the procedure a grower uses nowadays to control crop growth and production by means of climate conditioning. Depending on the current state of the crop, the grower decides on the set-points of the greenhouse climate variables such as air temperature, humidity and carbon dioxide concentration. These set-points are usually not defined as fixed values. Following rules defined by the grower based on experience, they may change during actual operation of the climate conditioning equipment in response to changes in the outside climate conditions. Such adaptations of the set-points include, for instance, a solar radiation dependent change of the air temperature set-point, and radiation and ventilation dependent adaptation of the carbon dioxide set-point. The grower may also put bounds on the ventilator's aperture and the temperature of the heating pipes. A minimum temperature of the heating pipes is

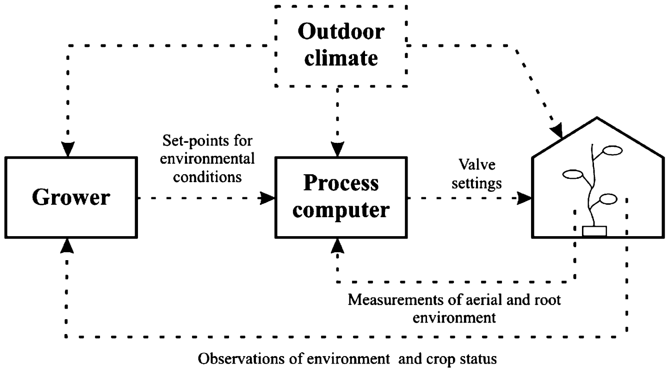

Fig. 6. A schematic diagram of the climate control procedure in current horticultural practice.

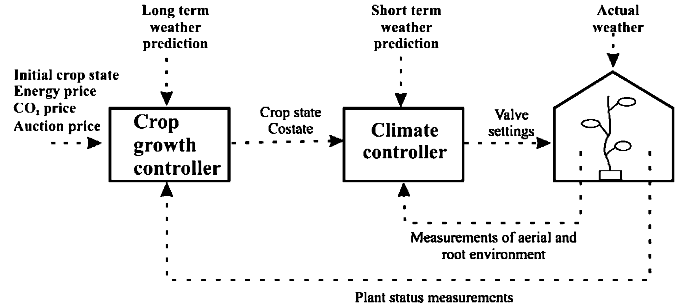

Fig. 7. Optimal greenhouse climate management: a new hierarchical concept.

often used to assure circulation of air within the canopy. All together in a modern greenhouse climate control computer systems, a large number of parameters (>150) need to be specified by the grower. Once the grower has decided on the settings of all these parameters, the greenhouse climate computer aims to achieve the desired climate in the greenhouse using measurements of the indoor climate and standard PI feed-back control techniques. But there is a second indirect feed-back loop from the crop growth process to the grower, in which during the growing season, the grower may decide to modify the settings on the control computer based on observations of the actual state of the crop and detected or expected indoor and outdoor climate conditions.

Optimal control techniques can improve the performance of a crop production cycle as was shown by Van Henten, Bontsema, and Van Straten (1997). But there is the challenging issue of implementing an optimal control system given modelling errors, rapidly fluctuating disturbances that are hard to predict but have a strong impact on the economic performance of the system and finally, large differences in dynamic response times. It is at this point where the above shown decomposition may play a role. Fig. 7 shows a hierarchical scheme for greenhouse climate control based on the previously described and validated decomposition of optimal greenhouse climate management. The hierarchical control scheme contains two control loops, an outer loop controlling the (slow) crop growth dynamics and an inner loop controlling the (fast) greenhouse climate dynamics.

Using a long-term weather prediction (e.g. long-term averages), a prediction of the auction price and a measurement of the initial state of the crop, the slow sub-problem is solved for the outer control loop. Due to modelling errors and errors in the predictions of the weather and the auction price, the actual state and costate may deviate from the pre-calculated trajectories and state feedback is therefore required. Repeated solution of the control problem using new information about the state of the crop, the auction price and the weather is hence needed. Since the optimal control problem uses a fixed final time, the recurrent solution will consider a slowly decreasing time interval as t_b slowly approaches the harvest time t_f. Recalculation should take place once a week, i.e. the time-scale of the slow crop growth dynamics.

Using the state and costate trajectories calculated in the outer loop and a short term weather prediction, the fast sub-problem is solved for the inner control loop to control the greenhouse climate dynamics. This can be achieved in a receding horizon optimal control framework as was demonstrated by Tap (2000). For that purpose the performance criterion of the fast sub-problem has the favourable property that it does not have to be optimised over a whole growing period, but only over that time interval during which a control action affects the greenhouse climate. In horticultural practice this time interval will be in the order of one hour or less.

It is interesting to observe that the schemes shown in Figs. 6 and 7 have a strong resemblance, both showing a hierarchical decomposition, however in the latter, the grower's expert knowledge is at least partially replaced by a quantitative model of crop growth and production.

Hierarchical control schemes for greenhouse climate management have been proposed before by for instance Udink ten Cate et al. (1978), Tantau (1991) and Challa and Van Straten (1991). They were all built on a proven scheme due to Richalet, Rault, Testud, and Papon (1978) that found common application in industrial practice. These schemes used set-point optimisation techniques to improve system performance at the highest level, neglecting the fast system dynamics, and relied on set-point tracking techniques for the lowest control level in which economic performance was not explicitly considered. The interesting feature of the hierarchical structure presented in this paper is the fact that at each level, i.e. both at the highest and the lowest control level, an economic performance criterion is used which has a direct relationship with the main objective of economic optimal greenhouse climate management defined at the highest level.

6. Conclusions

In this paper it has been shown that based on differences in response times, an optimal control problem in greenhouse climate management can be decomposed into two sub-problems, one dealing with the slow system dynamics concerning crop growth and evolution, and one dealing with the faster greenhouse climate dynamics. For this particular problem, the decomposition was found to be sufficiently accurate. To generalise this concept further research is required aiming at deriving a formal proof of this decomposition.

Based on this decomposition, a hierarchical scheme for greenhouse climate management has been proposed. This scheme is characterised by the fact that (i) at each control level, control of the dynamic process responses is emphasised, (ii) at each control level, a performance criterion is used which has a clear and direct relationship with the main objective of economic optimal greenhouse climate management defined at the highest level, (iii) the relation between the control levels is defined in terms of state and costate trajectories, with the costate trajectories expressing the economic value of achieving the reference state trajectory at the lower level, and (iv) it is easy to implement in a receding horizon optimal control framework.

Acknowledgements

The authors gratefully acknowledge the constructive comments of three anonymous referees.

References

Brdys, M. A., Grochowski, M., Gminski, T., Konarczak, K., & Drew, M. (2008). Hierarchical predictive control of integrated wastewater treatment systems. *Control Engineering Practice*, *16*, 751–767.

Bryson, A. E., Jr. (1999). *Dynamic optimization*. Menlo-Park, CA: Addison-Wesley.

Chalabi, Z. S. (1992). A generalized optimisation strategy for dynamic CO_2 enrichment in a greenhouse. *European Journal of Operational Research*, *59*, 308–312.

Chalabi, Z. S., Biro, A., Bailey, B. J., Aikman, D. P., & Cockshull, K. E. (2002a). Optimal control strategies for carbon dioxide enrichment in greenhouse tomato crops—Part 2: Using the exhaust gases of natural gas fired boilers. *Biosystems Engineering*, *81*(3), 323–332.

Chalabi, Z. S., Biro, A., Bailey, B. J., Aikman, D. P., & Cockshull, K. E. (2002b). Optimal control strategies for carbon dioxide enrichment in greenhouse tomato crops—Part 1: Using pure carbon dioxide. *Biosystems Engineering*, *81*(4), 421–431.

Challa, H., & Van Straten, G. (1991). Reflections about optimal climate control in greenhouse cultivation. In Y. Hashimoto, & W. Day (Eds.), *Mathematical and control applications in agriculture and horticulture. IFAC workshop series* (Vol. 1, pp. 13–18).

Critten, D. L. (1991). Optimization of CO_2 concentration in greenhouses: A modelling analysis for a lettuce crop. *Journal of Agricultural Engineering Research*, *48*, 261–271.

Kirk, D. E. (1970). *Optimal control theory*. Englewood Cliffs, NJ: Prentice-Hall.

Kokotović, P., Khalil, H. K., & O'Reilly, J. (1986). *Singular perturbation methods in control: Analysis and design*. Orlando, FL: Academic Press.

Pierre, D. A. (1969). *Optimization theory with applications*. New York: Wiley.

Pontryagin, L. S., Boltyanski, V. G., Gamkrelidze, R. V., & Mishchenko, E. F. (1962). *The mathematical theory of optimal processes*. New York: Inter-science Publishers Inc.

Reinisch, K., Arnold, E., Markert, A., & Puta, H. (1989). Development of strategies for temperature and CO_2 control in greenhouse production of cucumbers and tomatoes based on model building and optimisation. *Acta Horticulturae*, *260*, 67–75.

Richalet, S., Rault, A., Testud, J. L., & Papon, J. (1978). Model predictive heuristic control: Application to industrial process. *Automatica*, *14*, 413–428.

Seginer, I. (1991). Optimal greenhouse temperature trajectories for a multi statevariable tomato model. In Y. Hashimoto, & W. Day (Eds.), *Mathematical and control applications in agriculture and horticulture. IFAC workshop series* (Vol. 1, pp. 73–79).

Seginer, I., Angel, G., Gal, S., & Krantz, D. (1986). Optimal CO_2 enrichment strategy for greenhouses: A simulation study. *Journal of Agricultural Engineering Research*, *34*, 285–304.

Seginer, I., Shina, G., Albright, L. D., & Marsh, L. S. (1991). Optimal temperature setpoints for greenhouse lettuce. *Journal of Agricultural Engineering Research*, *49*, 209–226.

Subbaram Naidu, D. (2002). Singular perturbations and time scales in control theory and applications: An overview. Dynamics of continuous, discrete and impulsive systems, series B. *Applications & Algorithms*, *9*, 233–278.

Tantau, H. J. (1991). Optimal control for plant production in greenhouses. In Y. Hashimoto, & W. Day (Eds.), *Mathematical and control applications in agriculture and horticulture. IFAC workshop series* (Vol. 1, pp. 1–6).

Tap, R. F. (2000). *Economics-based optimal control of greenhouse tomato crop production*. Ph.D. thesis, Wageningen Agricultural University, Wageningen, The Netherlands.

Tap, R. F., Van Willigenburg, L. G., Van Straten, G., & Van Henten, E. J. (1993). Optimal control of greenhouse climate: Computation of the influence of fast and slow dynamics. In G. C. Goodwin, R. J. Evans, J. B. Cruz, Jr., & U. Jaaksoo (Eds.), *Proceedings of the 12th IFAC world congress* (pp. 321–324). The Institution of Engineers, Sydney, Australia.

Udink ten Cate, A. J., Bot, G. P. A., & Van Dixhoorn, J. J. (1978). Computer control of greenhouse climates. *Acta Horticulturae*, *87*, 265–272.

Van Henten, E. J. (1994). *Greenhouse climate management: An optimal control approach*. Ph.D. thesis, Wageningen Agricultural University, Wageningen, The Netherlands.

Van Henten, E. J., & Bontsema, J. (1991). Optimal control of greenhouse climate. In Y. Hashimoto, & W. Day (Eds.), *Mathematical and control applications in agriculture. IFAC workshop series* (Vol. 1, pp. 27–32).

Van Henten, E. J., & Bontsema, J. (1992). Singular perturbation methods applied to a variational problem in greenhouse climate control. In T. Basar (Ed.), *Proceedings of the 31st IEEE congress on decision and control* (pp. 3068–3069). IEEE Control Systems Society, New York.

Van Henten, E. J., Bontsema, J., & Van Straten, G. (1997). Improving the efficiency of greenhouse climate control: An optimal control approach. *Netherlands Journal of Agricultural Science*, *45*, 109–125.