6.4

$$E(\hat{\theta}_1) = E\left(\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}\right) = \frac{1}{n}E\left(\sum_{i=1}^{n}x_i^2 - n\bar{x}^2\right)$$

$$= \frac{1}{n}(n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2) = \frac{n-1}{n}\sigma^2$$

$$E(\hat{\theta}_2) = E\left(\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}\right) = \frac{1}{n-1}E\left(\sum_{i=1}^{n}x_i^2 - n\bar{x}^2\right)$$

$$= \frac{1}{n-1}(n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2) = \sigma^2$$

$\Rightarrow$ 故：$\hat{\theta}_2 = \sum_{i=1}^{n}(x_i-\bar{x})^2/(n-1)$ 為 σ^2 之不偏估計量

$\hat{\theta}_1 = \sum_{i=1}^{n}(x_i-\bar{x})^2/n$ 為 σ^2 之偏誤估計量

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