MATH 2425 **Midterm 1(Version A)** **Fall 2015**

Print your name legibly as it appears on your class roll.

Last ________________________________ First ________________________________

ID Number __ __ __ __ __ __ __ __ __ __

Check the appropriate section:

____ 100 - Mr. Ali (MoWeFr 10AM-10:50AM)

____ 200 - Dr. Lacy (MoWeFr 11AM-11:50AM)

____ 300 - Dr. Gornet (TuTh 11AM-12:20PM)

____ 400 - Dr. Adams (TuTh 7PM-8:20PM)

____ 450 - Mr. Choi (MoWeFr 10AM-10:50AM)

On your scantron, write:

Name: Last Name (and circle it), First Name,
Subject: 2425−???, (Fill In Section Number On Scantron)
Test No: M1-Version of Exam,
Date: 25 Sep 2015

Do Not Write Below This Line

Part I (50 Points)	Your Score:
#11 (10 points)	
#12 (10 points)	
#13 (10 points)	
#14 (10 points)	
#15 (10 points)	
Part II Total (50 Points)	Your Score:
Midterm One Total:	

INSTRUCTIONS FOR PART I:Write your answers for these questions on a scantron (forms 882-E or 882-ES) and mark only one answer per question. You may use an approved calculator. You may write on this exam or request scratch paper if needed. **Scantrons will not be returned so mark your answers on your exam paper; however, your score in Part I will be determined solely by what you mark on your scantron.**

1. Assume $f'(x)$ is continuous on the interval $[0, \frac{\pi}{2}]$. Evaluate $\int_0^{\pi/2} \sin x \, f'(x)\, dx$ given that $f(0) = 5$, $f\left(\frac{\pi}{2}\right) = -3$, and $\int_0^{\pi/2} \cos x \, f(x)\, dx = -7$

(a) 2

(b) 4

(c) −2

(d) −4

(e) −8

$u = \sin x \quad dv = f'(x)\,dx$

$du = \cos x\,dx \quad v = f(x)$

$\underbrace{f(x)\sin(x)\Big|_0^{\pi/2}} - \int_0^{\pi/2} f(x)\cos(x)\,dx$

$[(-3)(1) - (5)(0)] - (-7) = 4$

2. How many applications of integration by parts are needed to evaluate $\int x^7 e^{(-x)}\, dx$?

(a) five

(b) six

(c) seven

(d) eight

(e) The technique of integration by parts cannot be used to evaluate this integral.

$\frac{d^8}{dx^8}(x^7) = 0$

3. Evaluate $\int_0^{\pi/2} \cos^3\theta \sqrt{\sin\theta}\, d\theta$.

(a) −8/21

(b) 20/21

(c) 21/8

(d) 8/21

(e) −2

$u = \sin\theta \quad du = \cos\theta\,d\theta \quad u(\pi/2) = 1 \quad u(0) = 0$

$\int_0^1 (1-u^2)u^{1/2}\,du = \int_0^1 u^{1/2} - u^{5/2}\,du$

$= \frac{2}{3}u^{3/2} - \frac{2}{7}u^{7/2}\Big|_0^1 = \frac{2}{3} - \frac{2}{7} = \frac{14-6}{21} = \frac{8}{21}$

4. Using trigonometric substitution correctly to evaluate $\int_4^8 x^2 \underbrace{\sqrt{x^2-16}}dx$, the trigonometric integral you must actually evaluate is

$s^2 + c^2 = 1$
$t^2 + 1 = \sec^2$
$t^2 = \sec^2 - 1$

$x = 4\sec\theta$
$dx = 4\sec\theta\tan\theta\, d\theta$

$\int_{\theta(4)}^{\theta(8)} (16\sec^2\theta)(4\tan\theta)(4\sec\theta\tan\theta)\, d\theta$

$= 256 \int_{\theta(4)}^{\theta(8)} \sec^3\theta \tan^2\theta\, d\theta$

For bounds use $\theta(x) = \operatorname{arcsec}\left(\frac{x}{4}\right)$
$\theta(8) = \frac{\pi}{3}$ and $\theta(4) = 0$

(a) $128\int_{-\pi}^{\pi} \tan^2\theta\, d\theta$

(b) $64\int_0^{\pi/3} \sec^2\theta \tan^2\theta\, d\theta$

(c) $128\int_0^{\pi/2} \sec^3\theta \tan^3\theta\, d\theta$

(d) $128\int_4^8 \sec^3\theta \tan^2\theta\, d\theta$

(e) $256\int_0^{\pi/3} \sec^3\theta \tan^2\theta\, d\theta$

5. The trigonometric substitution $x - 2 = \tan\theta$ is appropriate for evaluating which one of the following integrals?

$\tan^2\theta + 1 = \sec^2\theta$

(a) $\int \sqrt{-x^2+4x-5}\, dx = \int \sqrt{-(x-2)^2 - 1}\, dx$

(b) $\int \sqrt{x^2-4x+5}\, dx = \int \sqrt{(x-2)^2+1}\, dx$

(c) $\int \sqrt{(x-2)^2 - 1}\, dx$

(d) $\int \sqrt{(x-2)^2 + 2}\, dx$

(e) $\int \sqrt{1-(x-2)^2}\, dx$

6. The correct partial fraction decomposition of $\dfrac{x^3-2x^2+x-42}{\underbrace{(x^2+2x+1)}_{(x+1)^2}(x^2+4)^2}$ is

(a) $\dfrac{Ax+B}{x^2+2x+1}+\dfrac{Cx+D}{(x^2+4)^2}$

(b) $\dfrac{Ax+B}{x^2+2x+1}+\dfrac{Cx+D}{x^2+4}+\dfrac{Ex+F}{(x^2+4)^2}$

(c) $\dfrac{A}{x+1}+\dfrac{B}{(x+1)^2}+\dfrac{Cx+D}{x^2+4}+\dfrac{Ex+F}{(x^2+4)^2}$

(d) $\dfrac{A}{x+1}+\dfrac{B}{(x+1)^2}+\dfrac{Cx+D}{x^2+4}$

(e) $\dfrac{A}{x+1}+\dfrac{B}{(x+1)^2}+\dfrac{Cx+D}{(x^2+4)^2}$

$\dfrac{A}{x+1}+\dfrac{B}{(x+1)^2}+\dfrac{Cx+D}{x^2+4}+\dfrac{Ex+F}{(x^2+4)^2}$

7. Evalute $\displaystyle\int_0^2 \frac{1}{x-1}\,dx.$ $\quad x \neq 1$ $\quad \displaystyle\lim_{a\to 1^-}\int_0^a \frac{1}{x-1}\,dx + \lim_{b\to 1^+}\int_b^2 \frac{1}{x-1}\,dx$

(a) The integral diverges.

(b) 0

(c) $\ln 2$

(d) 1

(e) 2

$\displaystyle\lim_{a\to 1^-} \ln\left|\frac{a-1}{-1}\right| + \lim_{b\to 1^+} \ln\left|\frac{2-1}{b-1}\right|$

$\displaystyle\lim_{a\to 1^-} \ln|1-a| - \lim_{b\to 1^+} \ln|b-1|$

$-\infty$

8. The sequence $\left\{\dfrac{\cos^{-1}\left(\frac{1}{n}\right)}{\sqrt{n}}\right\}$ $\quad \displaystyle\lim_{n\to\infty}\frac{\cos^{-1}\left(\frac{1}{n}\right)}{\sqrt{n}}$ $\quad 0 \le \cos^{-1}\left(\frac{1}{n}\right) \le \pi/2$

$= 0$

(a) Converges to ∞

(b) Converges to $\pi/2$

(c) Converges to π

(d) Diverges to 0

(e) Diverges by oscillation

Seqvence converges to 0.

9. The Monotone Convergence Theorem cannot be applied to the sequence $\left\{\frac{\sin n}{n}\right\}$ because

(a) The sequence is not bounded. ×

(b) The sequence does not converge. ×

(c) The sequence is not monotonic.

(d) All of the above.

(e) None of the above

$-\frac{1}{n} \le \frac{\sin(n)}{n} \le \frac{1}{n}$

goes to 0.

10. What is the largest set of real values r such that the sequence $\lim_{n\to\infty} r^n$ converges

(a) $-1 < r \le 1$

(b) $0 \le r \le 1$

(c) $-1 < r < 1$

(d) $-1 \le r < 1$

(e) $-1 \le r \le 1$

$\lim_{n\to\infty} (-1)^n$ oscillates

$\lim_{n\to\infty} (1)^n = 1$

$\lim_{n\to\infty} (r)^n = 0$ $|r| < 1$

INSTRUCTIONS FOR PART II: For these questions, you must write down work, including the justification of any limits, to support your answer; answers without supporting work will receive no credit. Write legibly and carefully label any graphs or pictures. Presentation is a component of your grade. Draw a box around your final answer. Partial credit will be given for those parts of your solution that are correct. Each of the questions in this section counts 10 points, for a total possible score of 50 points.

11. In great detail, evaluate the following integral. Draw a box around your final answer. Work shown is part of your grade.

$$\int e^x \sin x \, dx$$

$u = \sin x$ $\quad dv = e^x$

$du = \cos x \, dx$ $\quad v = e^x \, dx$

$e^x \sin x - \int e^x \cos x \, dx$

$\hat{u} = \cos x$ $\quad d\hat{v} = e^x$

$d\hat{u} = -\sin x$ $\quad \hat{v} = e^x dx$

$e^x \sin x - \left(e^x \cos x + \int e^x \sin x \, dx\right)$

$\int e^x \sin x \, dx = e^x(\sin x - \cos x) - \int e^x \sin x \, dx$

$$\boxed{\int e^x \sin x \, dx = \frac{e^x}{2}(\sin x - \cos x) + C}$$

13. In great detail, evaluate the following integral. Draw a box around your final answer. Work shown is part of your grade.

$$\int \tan^3 x \sec^3 x \, dx$$

$\tan^2 x \sec^2 x$

$(\sec^2 x - 1)\sec^2 x$

$u = \sec x$

$du = \sec x \tan x \, dx$

$$\int (u^2-1)u^2 \, du$$

$$= \int u^4 - u^2 \, du = \frac{1}{5}u^5 - \frac{1}{3}u^3 + C$$

$$\boxed{= \frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C}$$

12. In great detail, **USE TRIGONOMETRIC SUBSTITUTION** to evaluate the following integral. Draw a box around your final answer. Work shown is part of your grade.

$$\int_0^1 x^3\sqrt{1-x^2}\,dx$$

$x = \sin\theta \qquad dx = \cos\theta\, d\theta$

$\theta(x) = \arcsin(x) \qquad \theta(0) = 0 \qquad \theta(1) = \pi/2$

$$\int_0^{\pi/2} \sin^3\theta\,(\cos\theta)\cos\theta\, d\theta = \int_0^{\pi/2} (1-\cos^2\theta)\cos^2\theta\,\sin\theta\, d\theta$$

$u = \cos\theta \qquad du = -\sin\theta\, d\theta$

$u(0) = 1 \qquad u(\pi/2) = 0$

$$-\int_1^0 (1-u^2)\,u^2\,du = \int_0^1 u^2 - u^4\,du = \frac{1}{3}u^3 - \frac{1}{5}u^5 \Big|_0^1$$

$$= \frac{1}{3} - \frac{1}{5} = \frac{5-3}{15} = \frac{2}{15}$$

14. In great detail, **USE PARTIAL FRACTIONS** to evaluate the following integral. Draw a box around your final answer. Work shown is part of your grade.

$$\int \frac{x+4}{x^3+4x}\,dx = \int \frac{x+4}{x(x^2+4)}\,dx$$

We want A, B, C such that $\frac{x+4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$

$$x+4 = A(x^2+4) + Bx^2 + Cx \quad \Rightarrow \quad A+B=0 \quad \text{so } B=-1$$
$$= (A+B)x^2 + Cx + 4A \qquad C = 1$$
$$A = 1$$

$$\int \frac{x+4}{x(x^2+4)}\,dx = \int \frac{1}{x} + \frac{-x+1}{x^2+4}\,dx = \int \frac{1}{x}\,dx - \int \frac{x-1}{x^2+4}\,dx$$
$$= \ln|x| - \int \frac{x}{x^2+4}\,dx + \int \frac{1}{x^2+4}\,dx$$

For $\int \frac{x}{x^2+4}\,dx$, $u = x^2+4$ $\quad du = 2x\,dx$

$$\frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| = \frac{1}{2}\ln|x^2+4|$$

and for $\int \frac{1}{x^2+4}\,dx$, $x = 2\tan\theta$ $\quad dx = 2\sec^2\theta$

$$\int \frac{2\sec^2\theta}{4\sec^2\theta}\,d\theta = \int \frac{1}{2}\,d\theta = \frac{1}{2}\arctan\left(\frac{x}{2}\right)$$

then

$$\boxed{\int \frac{x+4}{x(x^2+4)}\,dx = \ln|x| - \frac{1}{2}\ln|x^2+4| + \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C}$$

15. In great detail, evaluate the following **IMPROPER INTEGRAL**. Draw a box around your final answer. Work shown is part of your grade.

$$\int_0^1 x \ln x dx$$

$u = \ln x$ $\quad dv = x\, dx$

$du = \frac{1}{x} dx$ $\quad v = \frac{1}{2} x^2$

$\frac{1}{2} x^2 \ln(x) - \int \frac{1}{2} x \, dx$

$= \frac{1}{2} x^2 \ln x - \frac{1}{2}\left(\frac{1}{2} x\right)$

$= \frac{1}{2} x^2 \ln x - \frac{1}{4} x$

$\frac{1}{2} x^2 \ln x - \frac{1}{4} x \Big|_0^1 = \left(0 - \frac{1}{4}\right) - (0) = \boxed{-\frac{1}{4}}$