MATH 2425 **Midterm 1(Version A)** **Spring 2017**

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Part I (40 Points)	Your Score:
#9 (10 points)	
#10 (10 points)	
#11 (10 points)	
#12 (10 points)	
#13 (10 points)	
#14 (10 points)	
Part II Total (60 Points)	Your Score:
Midterm One Total:	

INSTRUCTIONS FOR PART I: Write your answers for these questions on a scantron form SC882-E and mark only one answer per question. You may use an approved calculator. You may write on this exam or request scratch paper if needed. **Scantrons will not be returned so mark your answers on your exam paper; however, your score in Part I will be determined solely by what you mark on your scantron.**

1. Assume $f'(x)$ is continuous on the interval $[0, \ln 5]$. Evaluate $\displaystyle\int_0^{\ln 5} e^x f'(x)\,dx$ given that $f(0) = 3$, $f(\ln 5) = 2$, and $\displaystyle\int_0^{\ln 5} e^x f(x)\,dx = 4$

   (a) $2\ln 5 - 7$

   (b) 7

   (c) 6

   (d) 3

   (e) Not enough information to evaluate the integral.

$$\int_0^{\ln(5)} e^x f'(x)dx = f(x)e^x\Big|_0^{\ln(5)} - \int_0^{\ln(5)} f(x)e^x dx$$

$u = e^x \quad dv = f'(x)dx$

$du = e^x dx \quad v = f(x)$

$= [2(5) - 3(1)] - 4$

$[10 - 3] - 4 = 3$

2. When computing the integral

$$\int \csc^4 x \cot^{3/2} x\,dx$$

via a u-substitution, which one of the following integrals could result from the appropriate substitution?

CAREFUL

   (a) $-\displaystyle\int (1+u^2)^2 u^{3/2}\,du$

   (b) $-\displaystyle\int u^{5/2}\,du$

   (c) $-\displaystyle\int (u^{3/2} + u^{7/2})\,du$

   (d) $-\displaystyle\int u^{11/2}du$

   (e) $\displaystyle\int (u^{5/2} - u^{7/2})\,du$

$$\int \csc^4(x) \cot^{3/2}(x)dx$$

$$= \int \csc^2(x)\cot^{7/2}(x)\csc^2(x)dx$$

$$= -\int [\cot^2(x)+1]\cot^{3/2}(x)(-\csc^2(x)dx)$$

$u = \cot(x)$

$du = -\csc^2(x)dx$

$$= -\int [u^2+1]u^{7/2} du$$

$$= -\int (u^{7/2} + u^{3/2})du$$

3. Evaluate $\int_0^{\pi/2} \cos^3\theta\,(\sin\theta)^{5/2}\,d\theta$.

$$= \int_0^{\pi/2} \cos^2(\theta)\sin^{5/2}(\theta)\,\cos(\theta)d\theta =$$

LET $u = \sin(\theta)$

$du = \cos(\theta)d\theta$

$$= \int_0^{\pi/2} (1-\sin^2(\theta))\sin^{5/2}(\theta)\,\cos(\theta)d\theta$$

$$= \int_0^1 (1-u^2)u^{5/2}\,du = \int_0^1 (u^{5/2} - u^{9/2})\,du$$

$$= \left(\frac{2}{7}u^{7/2} - \frac{2}{11}u^{11/2}\right)\Big|_0^1 = \left(\frac{2}{7} - \frac{2}{11}\right) - 0 = \frac{8}{77}$$

(a) 0

(b) $\frac{36}{77}$

(c) $\frac{8}{45}$

(d) $\frac{28}{45}$

(e) $\frac{8}{77}$

4. The trigonometric substitution $x - 7 = 2\tan\theta$ could be useful in integrating a function (i.e., an integrand) involving...

$$[(x-7)^2 + 4] = [x^2 - 14x + 49 + 4] =$$

$$[x^2 - 14x + 53]$$

(a) $\sqrt{x^2 - 14x + 45}$

(b) $\sqrt{x^2 - 14x + 53}$

(c) $\sqrt{(x-7)^2 - 2}$

(d) $\sqrt{2 + (x-7)^2}$

(e) $\sqrt{x^2 - 7x - 2}$

5. The correct partial fraction decomposition of $\dfrac{3x^2 - x + 3}{(x^2-9)(x^2+3)^2}$ is

$0 - 4(1)(3) =$

$-12 < 0$

$$= \frac{3x^2 - x + 3}{(x+3)(x-3)(x^2+3)^2}$$

$$= \frac{A}{(x+3)} + \frac{B}{(x-3)} + \frac{(x+D)}{(x^2+3)} + \frac{Ex+F}{(x^2+3)^2}$$

(a) $\dfrac{A}{x+3} + \dfrac{B}{(x+3)^2} + \dfrac{C}{x-3} + \dfrac{D}{(x-3)^2}$

(b) $\dfrac{A}{x+3} + \dfrac{B}{(x-3)} + \dfrac{Cx+D}{(x^2+3)^2}$

(c) $\dfrac{A}{x-3} + \dfrac{B}{(x+3)} + \dfrac{Cx+D}{x^2+3} + \dfrac{Ex+F}{(x^2+3)^2}$

(d) $\dfrac{Ax+B}{x^2-9} + \dfrac{Cx+D}{x^2+3} + \dfrac{Ex+F}{(x^2+3)^2}$

(e) $\dfrac{A}{x-3} + \dfrac{B}{(x-3)^2} + \dfrac{Cx+D}{x^2+3} + \dfrac{Ex+F}{(x^2+3)^2}$

6. Evalute $\int_{-1}^{2} \frac{1}{x^5}\,dx$. $= \lim_{a\to 0} \int_{a}^{2} \frac{1}{x^5}\,dx + \lim_{b\to 0} \int_{-1}^{b} \frac{1}{x^5}\,dx$

$= \lim_{a\to 0} -\frac{x^{-4}}{4}\Big|_a^2 + \lim_{b\to 0} -\frac{x^{-4}}{4}\Big|_{-1}^{b}$

$-\frac{2^{-4}}{4} + \lim_{a\to 0} \frac{a^{-4}}{4} - \lim_{b\to 0} \frac{b^{-4}}{4} + \frac{1}{4}$

(a) 0
(b) $\frac{15}{64}$
(c) $\frac{21}{128}$
(d) $2 \ln 4$
(e) The integral diverges.

7. The Monotone Convergence Theorem cannot be applied to the sequence $\left\{2 + \frac{\cos n}{n}\right\}$ because

(a) The sequence is not monotonic.
~~(b) The sequence does not converge.~~
~~(c) The sequence is not bounded.~~
~~(d) The sequence is not positive.~~
~~(e) All of the above.~~

$\frac{-1}{n} \le \frac{\cos(n)}{n} \le \frac{1}{n}$

$\to 2 - \frac{1}{n} \le 2 + \frac{\cos(n)}{n} \le 2 + \frac{1}{n}$

$1 < 2 - \frac{1}{n} \le 2 + \frac{\cos(n)}{n} \le 2 + \frac{1}{n} < 3$

BOUNDED

8. What is the largest set of values for r such that the sequence $\lim_{n\to\infty} r^n$ converges?

(a) $-1 < r \le 1$
(b) $0 \le r \le 1$
(c) $-1 < r < 1$
(d) $-1 \le r < 1$
(e) $-1 \le r \le 1$

GEOMETRIC SEQUENCE

Ex: $\{2^n\} = \{1, 2, 4, 8, 16, \ldots\}$

NON-CONVERGENT

Ex: $\{1^n\} = \{1, 1, 1, 1, \ldots\}$

Ex: $\{(\frac{1}{2})^n\} = \{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\}$

INSTRUCTIONS FOR PART II: For these questions, you must write down work, including the justification of any limits, to support your answer; answers without supporting work will receive no credit. Write legibly and carefully label any graphs or pictures. Presentation is a component of your grade. Draw a box around your final answer. Partial credit will be given for those parts of your solution that are correct. Each of the questions in this section counts 10 points, for a total possible score of 60 points. SHOW DETAILED WORK OF INTEGRATION TECHNIQUES. FOR THESE PROBLEMS< DO NOT USE TABULAR INTEGRATION TO JUSTIFY WORK FOR THE INTEGRATION BY PARTS METHOD.

9. In great detail, evaluate the following integral. (Do NOT use a reduction formula.) Draw a box around your final answer. Work shown is part of your grade.

$$\int \sec x \tan^2 x \, dx$$

$$= \int \sec(x)[\sec^2(x) - 1]\,dx = \int [\sec^3(x) - \sec(x)]\,dx$$

$$= \underbrace{\int \sec^3(x)\,dx}_{(**)} - \underbrace{\int \sec(x)\,dx}_{(*)} = \frac{\sec(x)\tan(x)}{2} - \frac{1}{2}\int \sec(x)\,dx$$

$$= \boxed{\frac{\sec(x)\tan(x)}{2} - \frac{1}{2}\ln|\sec(x)+\tan(x)| + C}$$

$(*)$ $\int \sec(x)\,dx = \ln|\sec(x)+\tan(x)| + C$

$(**)$ $\int \sec^3(x)\,dx = \sec(x)\tan(x) - \int \sec(x)\tan^2(x)\,dx = \sec(x)\tan(x) - \int \sec^3(x)\,dx + \int \sec(x)\,dx$

$u = \sec(x)$ $\quad dv = \sec^2(x)\,dx$

$du = \sec(x)\tan(x)$ $\quad v = \tan(x)$

$$\Rightarrow \cancel{2}\int \sec^3(x)\,dx = \frac{\sec(x)\tan(x)}{2} + \int \frac{\sec(x)\,dx}{2}$$

10. In great detail, use <u>**TRIGONOMETRIC SUBSTITUTION**</u> to evaluate the following integral. Draw a box around your final answer.

$$\int \frac{dx}{(9+x^2)^{3/2}}$$

$a^2+x^2 \rightarrow a\,\mathrm{TAN}(\theta)$

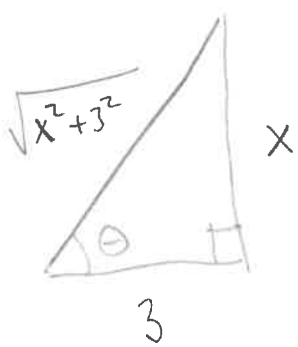
$$\int \frac{dx}{(3^2+x^2)^{3/2}} \rightarrow \int \frac{3\sec^2(\theta)\,d\theta}{(9+9\tan^2(\theta))^{3/2}} = \int \frac{3\sec^2(\theta)\,d\theta}{27(1+\tan^2(\theta))^{3/2}}$$

LET* $x = 3\tan(\theta)$

$dx = 3\sec^2(\theta)$

$$= \frac{1}{9}\int \frac{\sec^2(\theta)\,d\theta}{\sec^3(\theta)} = \frac{1}{9}\int \frac{1}{\sec(\theta)}\,d\theta$$

$$= \frac{1}{9}\int \cos(\theta)\,d\theta = \frac{1}{9}\left[\sin(\theta)\right] + C = \underbrace{\frac{1}{9}\sin(\theta) + C}$$



$$\sin(\theta) = \frac{x}{\sqrt{x^2+3^2}}$$

$$\boxed{\frac{1}{9}\left[\frac{x}{\sqrt{x^2+9}}\right] + C}$$

11. In great detail, evaluate the following integral. Draw a box around your final answer. Work shown is part of your grade.

$$\int_0^{\pi/6} \sin^3 x \cos^4 x \, dx$$

$$\int_0^{\pi/6} \sin^2(x) \underbrace{\cos^4(x)}_{u} \underbrace{\sin(x)\,dx}_{du} = \int_0^{\pi/6} [1-\cos^2(x)] \cos^4(x) \sin(x)\,dx$$

LET $u = \cos(x)$

$du = -\sin(x)dx$

$$= -\int_1^{\sqrt{3}/2} [1-u^2] u^4 \, du$$

$$= \int_{\sqrt{3}/2}^1 [u^4 - u^6] \, du = \left(\frac{1}{5}u^5 - \frac{1}{7}u^7\right)\Big|_{\sqrt{3}/2}^1$$

$$= \left(\frac{1}{5} - \frac{1}{7}\right) - \left(\frac{1}{5}(\sqrt{3}/2)^5 - \frac{1}{7}(\sqrt{3}/2)^7\right)$$

$$\frac{2}{35} - \left[\frac{1}{5}\left(\frac{9\sqrt{3}}{32}\right) - \frac{1}{7}\left(\frac{27\sqrt{3}}{128}\right)\right]$$

$$\boxed{\frac{2}{35} - \frac{9\sqrt{3}}{160} + \frac{27\sqrt{3}}{896}}$$

$\sim .0119$

12. In great detail, use the method of **PARTIAL FRACTIONS** to evaluate the following integral. Draw a box around your final answer. Work shown is part of your grade.

$$\int \frac{x+9}{(x-1)(x^2+9)}\,dx = (*)$$

$$\frac{x+9}{(x-1)(x^2+9)} = \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+9)} = \frac{1}{(x-1)} - \frac{(x+9)}{x^2+9}$$

$$x+9 = A(x^2+9) + (Bx+C)(x-1)$$

$A=1$

$B=-1$

$C=0$

$$x+9 = Ax^2 + 9A + Bx^2 - Bx + Cx - C$$

$$x+9 = (A+B)x^2 + (9A-B+C)x - C$$

$A+B=0$

$9A-B+C=1$

$-C=9$

$\boxed{\cancel{C=-9}} \to 0$

$9A-B-9=1$

$9A-B=10$

$A = \frac{10+B}{9}$

$\boxed{A=1}$

$\frac{10+B}{9} + B = 0$

$10+B+9B=0$

$10+10B=0$

$\frac{10B}{10} = \frac{-10}{10}$

$\boxed{B=-1}$

(*)

$\shortparallel$

$$\int \frac{1}{(x-1)}dx - \int \frac{x \cancel{+9}}{x^2+9}dx = \int \frac{1}{(x-1)}dx - \int \frac{x}{x^2+9}dx \cancel{- \int \frac{9}{x^2+9}dx}$$

$$= \int \frac{1}{(x-1)}dx - \frac{1}{2}\int \frac{1}{u}du \cancel{- 9\int \frac{3\sec^2(\theta)}{9\tan^2(\theta)+9}d\theta} = \ln|x-1| - \frac{1}{2}\ln|x^2+9| - \cancel{3\arctan\left(\frac{x}{3}\right) + C}$$

$u = x^2+9$

$\cancel{x = 3\tan(\theta)}$

13. In great detail, evaluate the following **IMPROPER INTEGRAL.** Draw a box around your final answer. Work shown is part of your grade. Show all steps in evaluating the integral. Show detailed use of L'Hopital's rule when used. . (For this problem do not use the growth theorem as justification for your limit reasoning.)

$$\int_0^1 \ln x dx$$

$$\int_0^1 \ln(x)dx = \lim_{b\to 0} \int_b^1 \ln(x)dx = \lim_{b\to 0} \left[x\ln(x) - x \right]_b^1$$

$$\left[(1)\ln(1) - 1\right] - \lim_{b\to 0}\left[(b)\ln(b) - b\right] \to 0$$

$$\lim_{b\to 0} (b)(\ln(b)) = \lim_{b\to 0} \frac{\ln(b)}{1/b} \overset{L'H}{=} \lim_{b\to 0} \frac{\frac{1}{b}}{\frac{-1}{b^2}} = \lim_{b\to 0} -b = 0.$$

$$= \left[(1)(0) - 1 - 0\right] = \underline{\underline{-1}}$$

$$\int_0^1 \ln(x)dx = \underline{\underline{-1}}$$

14. In great detail, evaluate the following sequences for convergence or divergence. Show detailed use of L'Hopital's rule when used. . (For this problem do not use the growth theorem as justification for your limit reasoning.)

   (a) $\{\cos(n\pi)\}$

   (b) $\left\{\frac{\ln(2^n)}{e^n}\right\}$

a) $\cos(n\pi) = \begin{cases} 1 & \text{IF } n \text{ IS EVEN} \\ -1 & \text{IF } n \text{ IS ODD} \end{cases}$

$\{\cos(n\pi)\} = \{1, -1, 1, -1, 1, -1, \ldots\}$

HERE THE SEQUENCE WILL ALTERNATE BACK AND FORTH AND NEVER CONVERGE. i.e. $\lim_{n\to\infty} \cos(n\pi) \to \underline{\text{DNE}}$

b) $\left\{\frac{\ln(2^n)}{e^n}\right\}$

$$\lim_{n\to\infty} \frac{\ln(2^n)}{e^n} = \lim_{n\to\infty} \frac{n\ln(2)}{e^n} \to \frac{\infty}{\infty} \quad \text{L'H}$$

$$= \lim_{n\to\infty} \frac{\ln(2)}{e^n} = 0$$

THUS, THE SEQUENCE CONVERGES!