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Kinematics

ME 2984

“We see in order to move; we move in order to see.” – William Gibson

Project Proposals

- Project Proposals are due in 1 week
- 1 page to describe initial proposal for class project
- Start forming teams and putting together ideas
  - 1 proposal per team, every student must ensure a proposal includes them

Review Week

- Next week
  - Topic Review
  - Q&A
  - Project discussions
- Survey for gathering questions is out
  - Submit 3 - 5 questions
  - Most popular questions will be discussed

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Office Hours

- Open window directly after class
  - 30 minutes
- By appointment only outside that
  - Confirmed appointments only
  - Limiting appointments to 15 minute slots day homework is due

Assignment 2 Review

What are Kinematics?

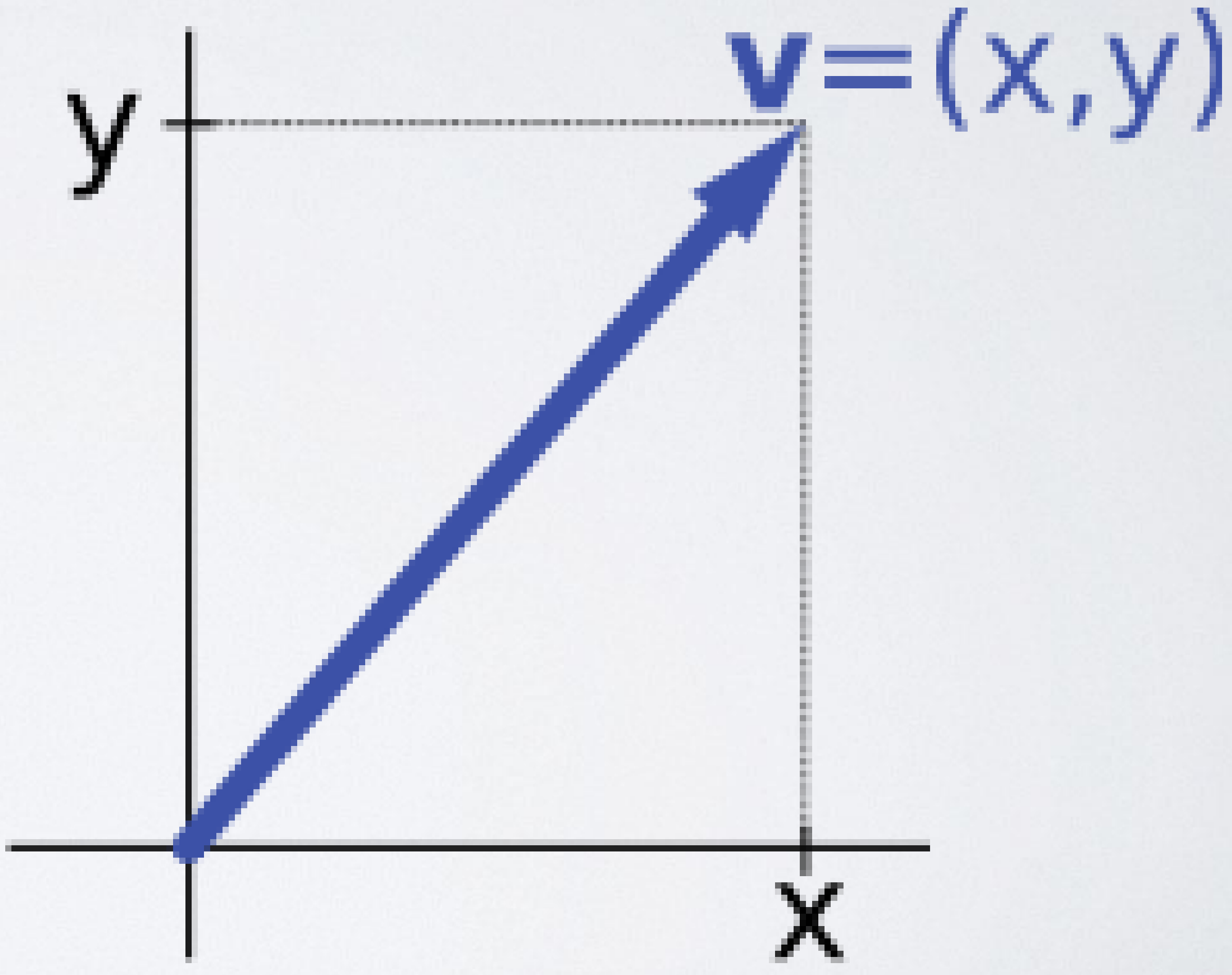
- The study of motion of bodies (kinetics)
- Without taking masses or forces into account (dynamics)
- Many robots tend to require consideration of both kinematics and dynamics

Areas of Kinematics

- Kinematic Analysis
  - Given a linkage, analyze its motion
- Kinematic Synthesis
  - Design a linkage to match a motion profile
- Forward Kinematics
  - How do joint motions translate to motion of whole robot
- Inverse Kinematics
  - How should individual joints move to effect a desired robot motion

Vector Notation

- **P** or $\vec{P}$= $[p_x, p_y]^T$
  - Column or Row
- Unit Vectors
  - $\hat{\imath} = (1,0,0)$
  - $\hat{\jmath} = (0,1,0)$
  - $\hat{k} = (0,0,1)$
- $\vec{P}$= $p_1\hat{\boldsymbol{\imath}} + p_2\hat{\boldsymbol{\jmath}}{}^{+}p_3\hat{\boldsymbol{k}}$



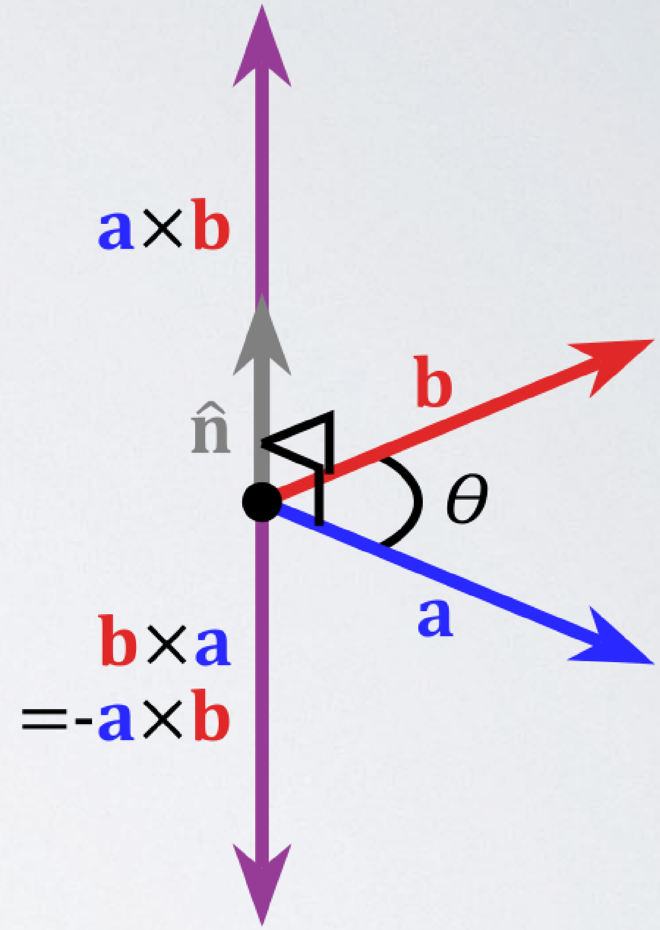
Source: [Wikipedia]

Vector Math

- Magnitude
  - $||a|| = \sqrt{a_1 + a_2 + a_3}$
- Addition and Subtraction
  - $\vec{a} + \vec{b} = (a_1 + b_1)\hat{\boldsymbol{\imath}} + (a_2 + b_2)\hat{\boldsymbol{\jmath}} + (a_3 + b_3)\widehat{\boldsymbol{k}}$
  - $\vec{a} - \vec{b} = (a_1 - b_1)\hat{\boldsymbol{\imath}} + (a_2 - b_2)\hat{\boldsymbol{\jmath}} + (a_3 - b_3)\widehat{\boldsymbol{k}}$

Vector Math

- Multiplication
  - $x\vec{a} = xa_1\hat{\boldsymbol{\imath}} + xa_2\hat{\boldsymbol{\jmath}} + xa_3\widehat{\boldsymbol{k}}$
- Dot Product
  - $\vec{a} \cdot \vec{b} = \|\vec{a}\|\|\vec{b}\|cos\theta =$ $a_1b_1 + a_2b_2 + a_3b_3$
- Cross Product
  - $\vec{a} \cdot \vec{b} = \|\vec{a}\|\|\vec{b}\|cos\theta =$ $(a_2b_3 - a_3b_2)\hat{\boldsymbol{\imath}} + (a_3b_1 - a_1b_3)\hat{\boldsymbol{\jmath}} + (a_1b_2 - a_2b_{1)}\widehat{\boldsymbol{k}}$



Source: Wikipedia

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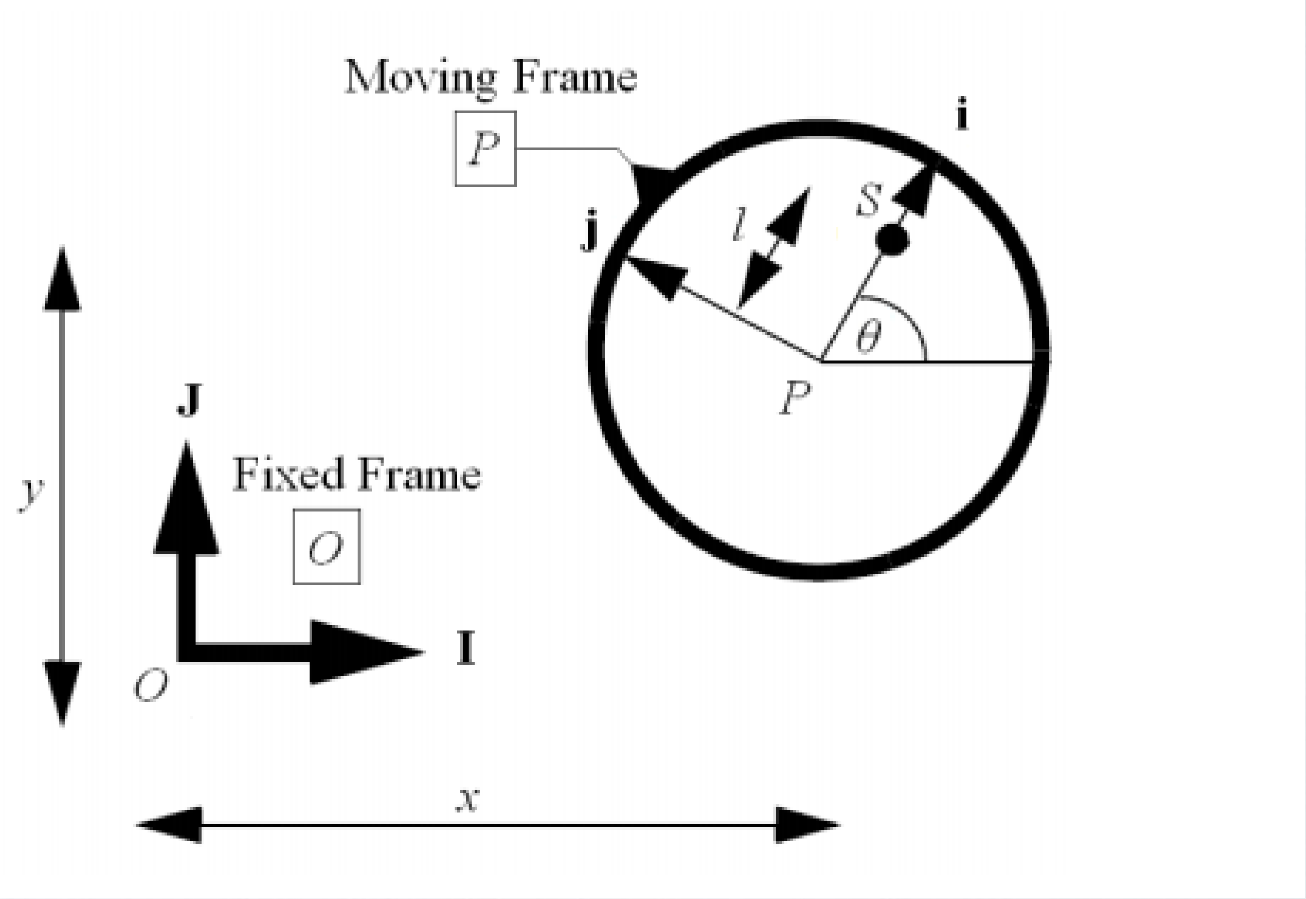
Coordinate Frames

- Fixed Frame vs moving frame
- $\overrightarrow{s_p} = s_1\hat{\imath}$
- $\hat{\imath} = \boldsymbol{I}cos\theta + \boldsymbol{J}sin\theta$
- $\hat{P} = p_1\boldsymbol{I} + p_2\boldsymbol{J} + p_3\mathbf{K}$
- $\widehat{s_o} = (s_1cos\theta + p_1)\boldsymbol{I} + (s_1sin\theta + p_2)\boldsymbol{J} + p_3\boldsymbol{K}$



Source: MIT

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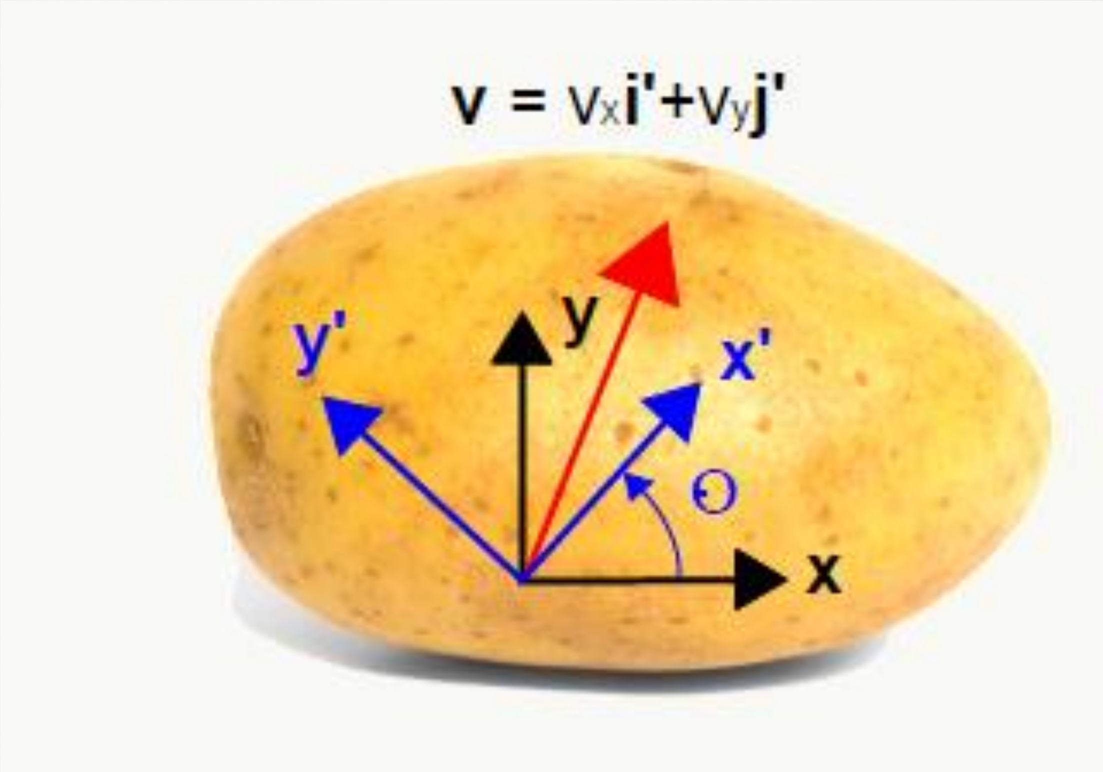
Coordinate Transforms

- $v_x{}' = v_x cos\theta + v_y sin\theta$
- $v_y{}' = -v_x\ cos\theta + v_y sin\theta$

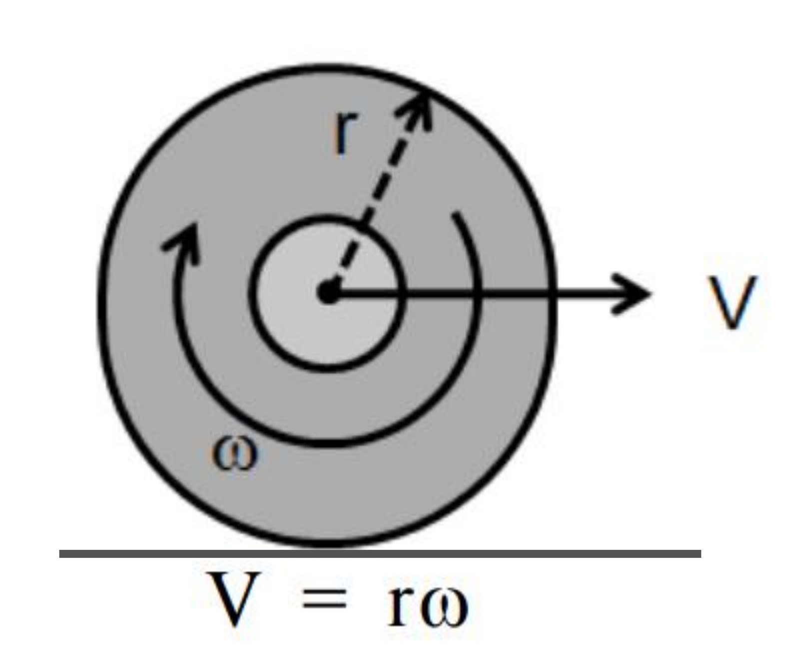


- Transformation Matrix
- $\begin{bmatrix} v'_x \\ v'_y \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix}$

Source: [Continuum Mechanics]

Vehicle Kinematics

- Use encoders to measure wheel velocities
  - Calculate Distance/Tick * total count
  - Robot motion can be described in terms of wheel velocities
  - “No Slip” condition



Source: [Field Robotics]

Wheeled Platforms

- Ackerman Steer
- Bicycle
- Differential Steer
- Holonomic Drive

Ackerman Model

- Standard car model
- Wheels turn different amounts to minimize slip
- Instantaneous Center of Rotation
- Kinematics are hard

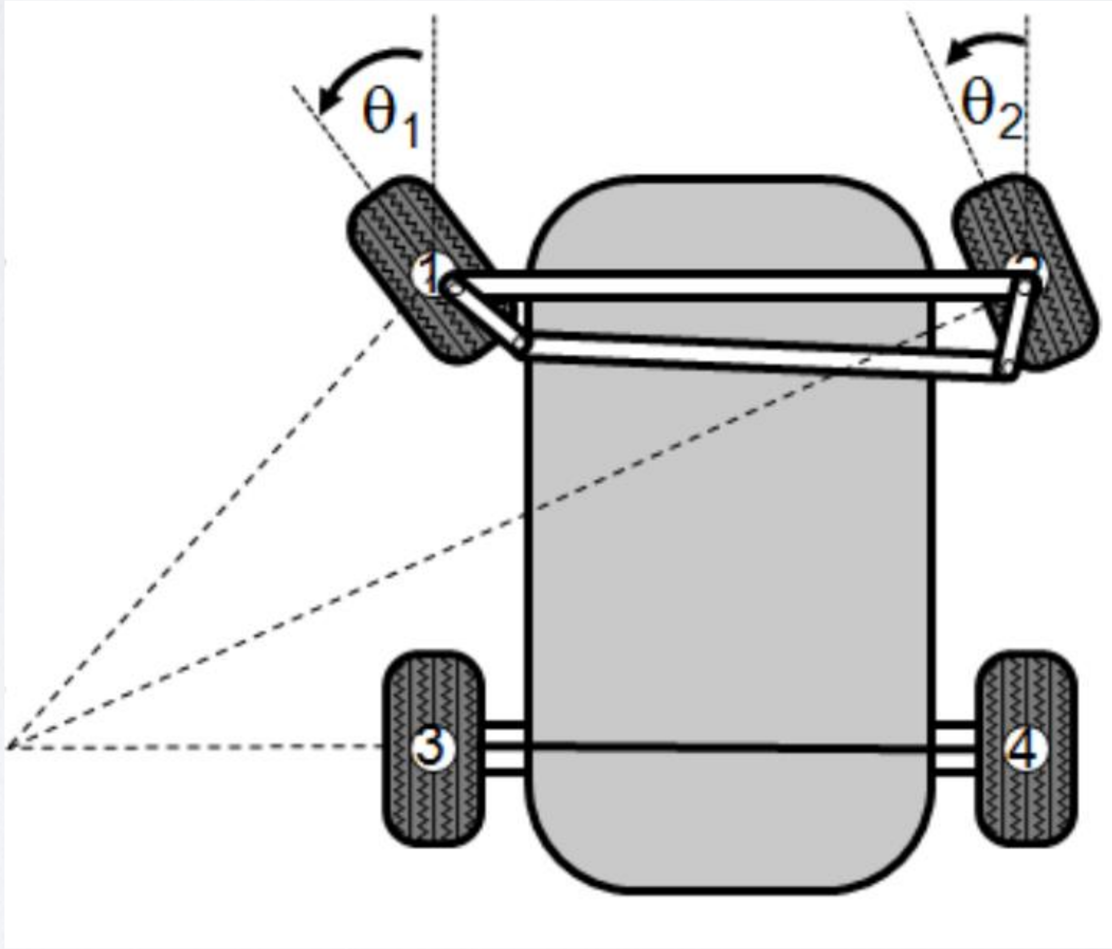
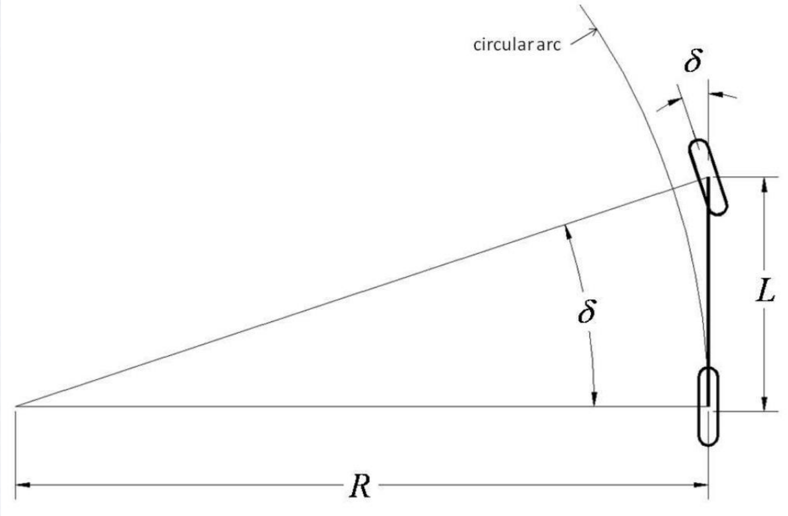


Image Credit: [Alonzo Kelly]

Bicycle Model

- Simple Car Model
  - Similar given no roll
- Collapses down to two wheels
  - Can derive kinematics!



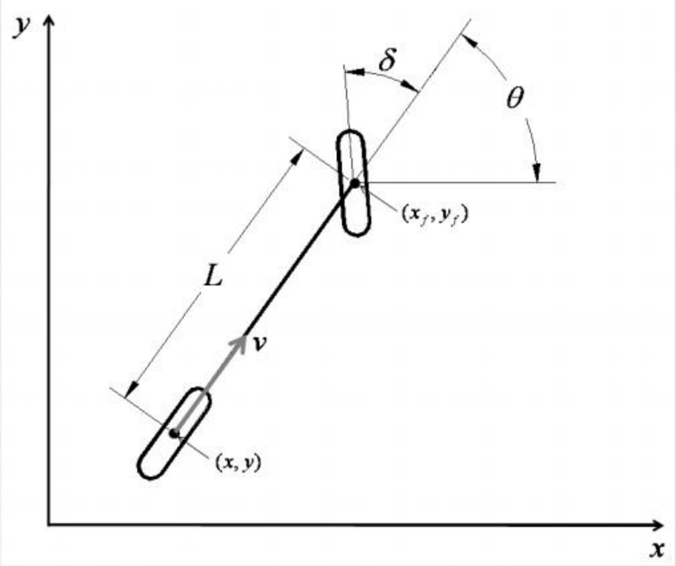


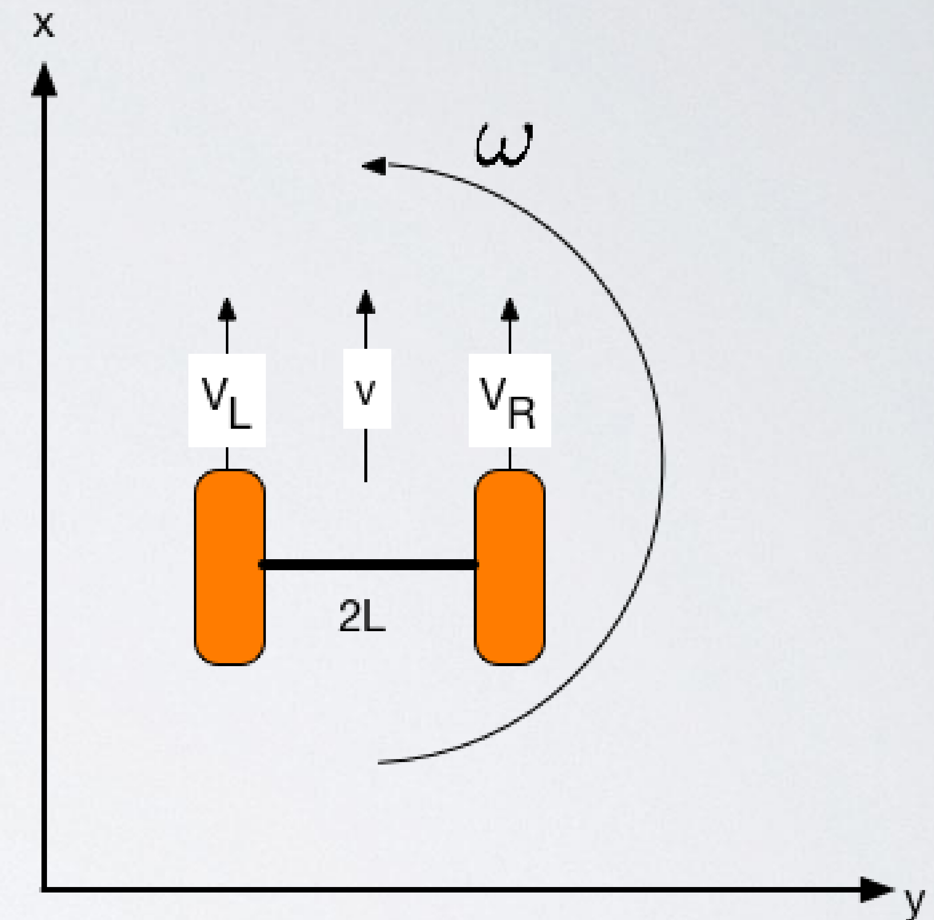
Image Credit: Jarrod Snider

$$tan(\delta) = \frac{L}{R}$$

$$\dot{x} = v * cos(\theta)$$
$$\dot{y} = v * sin(\theta)$$
$$\dot{\theta} = v * \frac{tan(\delta)}{L}$$

Differential Drive

- Two independent wheels
- Can turn in place
- Simplified model of K.H.A.N.



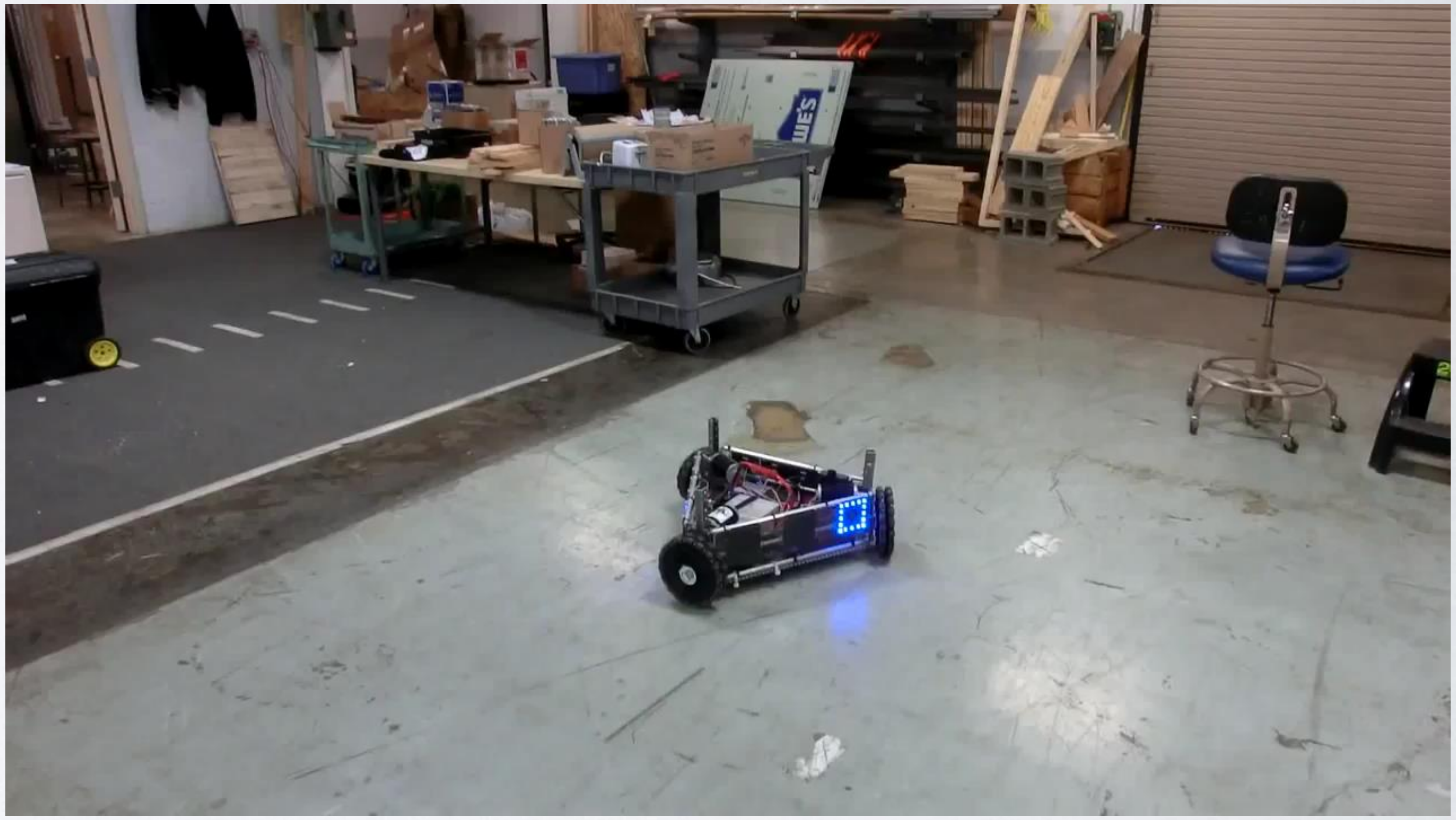
$$v = \frac{V_L + V_R}{2}$$

$$\omega = \frac{V_L - V_R}{2L}$$

$$\dot{x} = v * cos(\theta)$$
$$\dot{y} = v * sin(\theta)$$

Holonomic Drive

- What is holonomic?
  - #Controllable DOF = #Total DOF
  - Trains are holonomic!
- Maneuverable
- Harder to control



Video Credit: [AndyMark]

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QUESTIONS?