

- b. To say that a pit causes all adjacent squares to be breezy:

$$\forall s \text{ Pit}(s) \Rightarrow [\forall r \text{ Adjacent}(r, s) \Rightarrow \text{Breezy}(r)] .$$

This axiom allows for breezes to occur spontaneously with no adjacent pits. It would be incorrect to say that a non-pit causes all adjacent squares to be non-breezy, since there might be pits in other squares causing one of the adjacent squares to be breezy. But if *all* adjacent squares have no pits, a square is non-breezy:

$$\forall s [\forall r \text{ Adjacent}(r, s) \Rightarrow \neg \text{Pit}(r)] \Rightarrow \neg \text{Breezy}(s) .$$

8.14 Make sure you write definitions with \Leftrightarrow . If you use \Rightarrow , you are only imposing constraints, not writing a real definition. Note that for aunts and uncles, we include the relations whom the OED says are more strictly defined as aunts-in-law and uncles-in-law, since the latter terms are not in common use.

$$\begin{aligned} \text{Grandchild}(c, a) &\Leftrightarrow \exists b \text{ Child}(c, b) \wedge \text{Child}(b, a) \\ \text{Greatgrandparent}(a, d) &\Leftrightarrow \exists b, c \text{ Child}(d, c) \wedge \text{Child}(c, b) \wedge \text{Child}(b, a) \\ \text{Ancestor}(a, x) &\Leftrightarrow \text{Child}(x, a) \vee \exists b \text{ Child}(b, a) \wedge \text{Ancestor}(b, x) \\ \text{Brother}(x, y) &\Leftrightarrow \text{Male}(x) \wedge \text{Sibling}(x, y) \\ \text{Sister}(x, y) &\Leftrightarrow \text{Female}(x) \wedge \text{Sibling}(x, y) \\ \text{Daughter}(d, p) &\Leftrightarrow \text{Female}(d) \wedge \text{Child}(d, p) \\ \text{Son}(s, p) &\Leftrightarrow \text{Male}(s) \wedge \text{Child}(s, p) \\ \text{FirstCousin}(c, d) &\Leftrightarrow \exists p_1, p_2 \text{ Child}(c, p_1) \wedge \text{Child}(d, p_2) \wedge \text{Sibling}(p_1, p_2) \\ \text{BrotherInLaw}(b, x) &\Leftrightarrow \exists m \text{ Spouse}(x, m) \wedge \text{Brother}(b, m) \\ \text{SisterInLaw}(s, x) &\Leftrightarrow \exists m \text{ Spouse}(x, m) \wedge \text{Sister}(s, m) \\ \text{Aunt}(a, c) &\Leftrightarrow \exists p \text{ Child}(c, p) \wedge [\text{Sister}(a, p) \vee \text{SisterInLaw}(a, p)] \\ \text{Uncle}(u, c) &\Leftrightarrow \exists p \text{ Child}(c, p) \wedge [\text{Brother}(a, p) \vee \text{BrotherInLaw}(a, p)] \end{aligned}$$

There are several equivalent ways to define an m th cousin n times removed. One way is to look at the distance of each person to the nearest common ancestor. Define $\text{Distance}(c, a)$ as follows:

$$\begin{aligned} \text{Distance}(c, c) &= 0 \\ \text{Child}(c, b) \wedge \text{Distance}(b, a) = k &\Rightarrow \text{Distance}(c, a) = k + 1 . \end{aligned}$$

Thus, the distance to one's grandparent is 2, great-great-grandparent is 4, and so on. Now we have

$$\begin{aligned} \text{MthCousinNTimesRemoved}(c, d, m, n) &\Leftrightarrow \\ \exists a \text{ Distance}(c, a) = m + 1 \wedge \text{Distance}(d, a) = m + n + 1 . \end{aligned}$$

The facts in the family tree are simple: each arrow represents two instances of *Child* (e.g., *Child(William, Diana)* and *Child(William, Charles)*), each name represents a sex proposition (e.g., *Male(William)* or *Female(Diana)*), each “bowtie” symbol indicates a *Spouse* proposition (e.g., *Spouse(Charles, Diana)*). Making the queries of the logical reasoning system is just a way of debugging the definitions.

8.15 Although these axioms are sufficient to prove set membership when x is in fact a member of a given set, they have nothing to say about cases where x is not a member. For