yETH weighted stableswap

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Invariant derivation

Constant sum: $\sum_i x_i = c_1$

Constant weighted product: $\prod_i x_i^{w_i} = c_2$ with $\sum_i w_i = 1$

Define $\frac{1}{f} := \prod_i w_i^{w_i}$, $v_i := w_i n$

Balanced pool: $x_i = w_i D \Rightarrow \quad c_1 = D \sum_i w_i = D, \quad c_2 = D \prod_i w_i^{w_i} = \frac{D}{f}$

$$\sum_i x_i = D \qquad \prod_i x_i^{v_i} = \left(\frac{D}{f}\right)^n$$

Leveraged invariant:

$$\chi D^{n-1} \sum_i x_i + \prod_i x_i^{v_i} = \chi D^n + \left(\frac{D}{f}\right)^n$$

Dynamic leverage:

$$\chi = A \frac{\prod_i x_i^{v_i}}{\left(\frac{D}{f}\right)^n}$$

Weighted stableswap invariant:

$$A f^n \sum_i x_i + D = A D f^n + \frac{D^{n+1}}{f^n \prod_i x_i^{v_i}}$$

Reduces to original stableswap invariant if we set equal weights $w_i = \frac{1}{n}$.

Define $\sigma := \sum_i x_i$, $\pi := D^n \prod_i \left(\frac{w_i}{x_i}\right)^{v_i}$:

$$A f^n \sigma + D = A D f^n + D\pi \tag{1}$$

Supply calculation

Given a pool with weights $\{w_i\}$ and virtual balances $\{x_i\}$, we can find the equilibrium supply by solving Equation 1 iteratively for D:

$$\begin{aligned} D_{m+1} &= (A f^n \sigma - D_m \pi_m) \,/\, (A f^n - 1) \\ \pi_m &= \left(\frac{D_m}{D_{m-1}}\right)^n \pi_{m-1} \\ \pi_0 &= \prod_i \left(D_0 \frac{w_i}{x_i}\right)^{v_i} \end{aligned} \tag{2}$$

The iterative process is started with a good guess for D_0 (such as σ) and continued until the desired precision is achieved.

Rate update

$$x_i = b_i r_i \to x_i{}' = b_i r_i{}'$$

$$\sigma \to \sigma' = \sigma + b_i({r_i}' - r_i)$$

$$D \to D'$$

$$\pi \to \pi' = \left(\frac{D'}{D}\right)^n \left(\frac{r_i}{r'_i}\right)^{v_i} \pi$$

The iterative process in Equation 2 is used to find both D' and π', starting off with $D'_0 = D$ and $\pi'_0 = \left(\frac{r_i}{{r_i}'}\right)^{v_i} \pi$.

Balance calculation

Given a pool with weights $\{w_i\}$, virtual balances $\{x_i\}_{i \neq j}$ and supply D, we can find the balance of a specific asset j by solving Equation 1 for $y := x_j$.

First, we define intermediary variables $\tilde{\sigma} := \sum_{i \neq j} x_i$ and $\tilde{\pi} := D^n w_j^{v_j} \prod_{i \neq j} \left(\frac{w_i}{x_i}\right)^{v_i}$. This allows us to rewrite Equation 1 to

$$Af^n(\tilde{\sigma} + y) + D = ADf^n + D\frac{\tilde{\pi}}{y^{v_j}}$$

Rearranging gives us

$$y^{v_j+1} + \left(\tilde{\sigma} + \frac{D}{Af^n} - D\right) y^{v_j} - \frac{D}{Af_n}\tilde{\pi} = 0$$

This is equivalent to finding the root of $g(y) = y^{a+1} + by^a - c$, which is something that can be done iteratively using Newtons method: $y_{m+1} = y_m - \frac{g(y_m)}{g'(y_m)}$. Plugging in our function yields

$$\begin{gathered} y_{m+1} = \frac{v_j y_m^2 + b(v_j - 1)y_m + cy_m^{1-v_j}}{(v_j + 1)y_m + v_j b} \\ \text{where} \quad b = \tilde{\sigma} + \frac{D}{Af^n} - D, \quad c = \frac{D}{Af^n}\tilde{\pi} \end{gathered} \tag{3}$$

Swaps

User swaps asset k for asset l ($k \neq l$).

$$x_k \to {x_k}' = x_k + \Delta b_k r_k$$

$$x_l \to {x_l}' = x_l - \Delta b_l r_l$$

Exact input

Our goal is to find Δb_l, given Δb_k, i.e. how much of asset l the user will receive (is taken out of the pool) in exchange for sending a fixed amount of asset k (is added to the pool). To that end, we solve Equation 3 for $y = {x_l}'$, where we set the intermediary variables to

$$\tilde{\sigma} = \sigma + \Delta b_k r_k - x_l \qquad \tilde{\pi} = \left(\frac{x_k}{{x_k}'}\right)^{v_k} x_l^{v_l} \pi$$

From this we obtain the amount to send to the user:

$$\Delta b_l = \frac{x_l - {x_l}'}{r_l}$$

Exact output

Alternatively we can compute Δb_k given Δb_l, i.e. how much of asset k the user will have to send (is added to the pool) in exchange for receiving a fixed amount of asset k (is taken out of the pool). In this scenario the intermediary variables are set to

$$\tilde{\sigma} = \sigma - x_k - \Delta b_l r_l \qquad \tilde{\pi} = x_k^{v_k} \left(\frac{x_l}{x_l{}'} \right)^{v_l} \pi$$

and we use Equation 3 to find $y = x_k$. Finally we obtain the amount to take from the user:

$$\Delta b_k = \frac{x_k{}' - x_k}{r_k}$$