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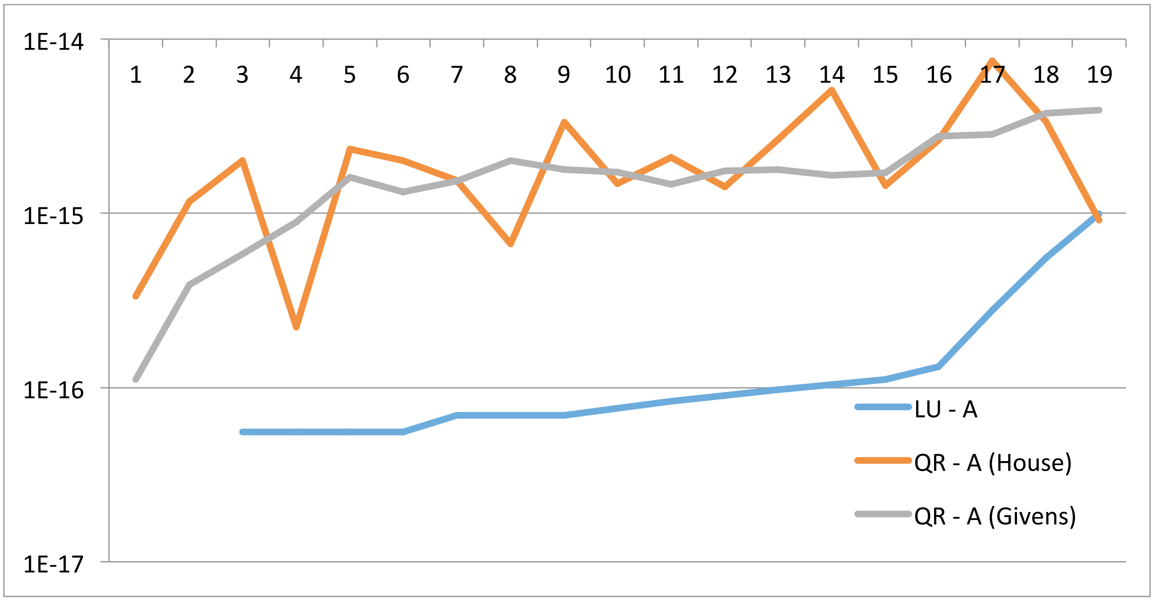
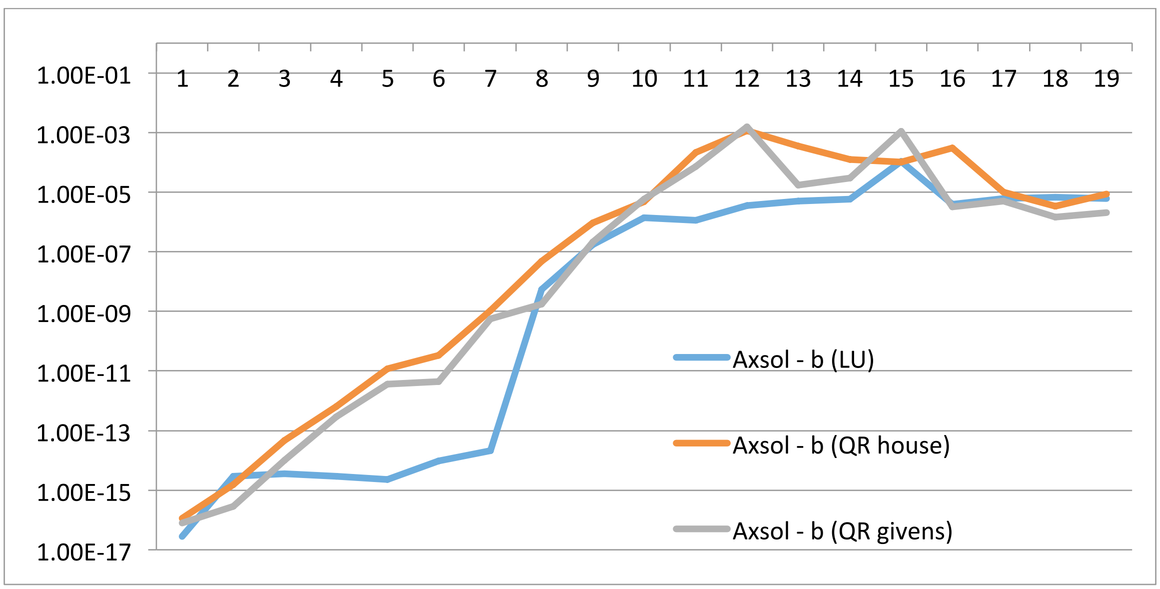
Part #1





It is justified to use the LU or QR-factorizations as opposed to directly calculating an inverse matrix because LU and QR calculate the solutions much more efficiently. For example, for an nxn matrix A, computing A^{-1} requires $2n^3$ "flops" (or floating point operations) whereas computing LU only takes about $2n^3/3$ flops, meaning that it is much faster in comparison. Also, although QR decomposition is slower than LU, it is still much faster than doing the inverse (for instance, a householder reflection takes about $4n^3/3$ floating point operations).

The benefit of using LU or QR-factorizations this way is that it is more numerically stable than a direct matrix inverse. Both LU and QR factorizations produce smaller condition numbers than doing an inverse. They minimize the norm of errors produced during calculation.

Part #2

N, the number of elements in the vector/size of the system, has an incredibly large impact on the number of iterations needed to reach convergence (achieve error tolerance). Jacobi iterations have a far slower runtime than Gauss Seidel. As n increases, the number of Jacobi iterations needed to approach convergence becomes an order of magnitude larger than the number of Gauss Seidel iterations.

Part #3

Urban Population Dynamics

$$A = \begin{bmatrix} 0 & 1.2 & 1.1 & 0.9 & 0.1 & 0 & 0 & 0 & 0 \\ 0.7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.85 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.9 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.88 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.77 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.40 & 0 \end{bmatrix}$$

$\overline{x}(0)$ = (2.1, 1.9, 1.8, 2.1, 2.0, 1.7, 1.2, 0.9, 0.5)($\times 10^5$)

1. Each column of matrix A is a representation of an age group divided into ten years. The first column is for age group 0-9, the second for 10-19, and continues until the last column with represents the age group 80-89. The top of number represents birth rates and the numbers below the top row represent survival rates. This means that the age group of 10-19 produces the most births with an average of 1.2 births per mother, followed by 20-29 with 1.1, 30-39 with 0.9, and 40-49 with 0.1. Few to no births result from any younger or older age group. This information coincides generally with when a female human is most fertile, and likely incorporates societal pressures such as marriage and the need to work. The societal drive for marriage. The need to support oneself with jobs that could interfere with the creation and raising of children likely reduces birth rates all across the board. The advent of contraception and family planning also likely decreases birth rates, especially in the sexually active portion of the 10-19 and 20-29 ranges.

The survival rates reveal that 70% of humans age 0-9 survive to 10 years of age, 85% of humans age 10-19 survive to 20 years of age, 90% of humans age 20-29 survive to 30 years of age, 90% of humans age 30-39 survive to 40 years of age, 88% of humans age 40-49 survive to 50 years of age, 80% of humans 50-59 survive to 60 years of age, 77% of humans age 60-69 survive to 70 years of age, and 40% of humans age 70-79 survive to 80 years of age. These numbers show a clear pattern—survival rates increase from 70% until they peak at 90% for the age range 20-39, then decrease to the 40% survival rates of the 70-79 age range. This indicates a number of things; for one, humans age 0-9 have a relatively low survival rate. This is likely due to the fact that children of that age cannot take care of themselves effectively and that young children are often susceptible to illness and disease. Survival rates increasing in the next age band represents the humans being better able to take care of themselves and an improved immune system. The age bands of 20-29 and 30-39 share the highest survival ratings of 90%. This represents not only the near complete self-sufficiency that characterizes these ages, but also the human body and how it is of peak physical condition in these age ranges. The 40-49 age range has a slightly lower survival rating of 88%, followed by 80% for 50-59, 77% for 60-69 and 40% for 70-79. This is likely indicative of the number of health issues that onset at older ages as well as the lack of self-sufficiency that comes with such health issues.

2. The population distributions total populations, and percent change for 2010, 2020, 2030, 2040 and 2050 are as followed:

Total population for 2000: 1,420,000

2010 = (6.35, 1.47, 1.615, 1.62, 1.89, 1.76, 1.36, 0.924, 0.36)(x10^5)

Total population: 1,734,900

Percent change: 22.1761%

2020 = (5.1875, 4.445, 1.2495, 1.4535, 1.458, 1.6632, 1.408, 1.0472, 0.3996) (x10^5)

Total population: 1,828,150

Percent change: 5.3749%

2030 = (8.1624, 3.63125, 3.77825, 1.12455, 1.30815, 1.28304, 1.33056, 1.08416, 0.41888) (x10^5)

Total population: 2,212,124

Percent change: 21.0034%

2040 = (9.656485, 5.71368, 3.0865625, 3.400425, 1.012095, 1.151172, 1.026432, 1.024531, 0.433664) (x10^5)

Total population: 2,650,505

Percent change: 16.5395%

2050 = (13.41323, 6.75954, 4.85663, 2.77791, 3.06038, 0.89064, 0.920938, 0.79035, 0.40981) (x10^5)

Total population: 3,387,943

Percent change: 27.8225%

3. Using the power method and to eight digits of accuracy, the largest eigenvalue of A is 1.28865624. The corresponding eigenvector is

(1, 0.5432015, 0.35829667, 0.2502351, 0.17476468, 0.11934363, 0.07408873, 0.04426962, 0.01374133)

This means that the population of the city will stabilize to the distribution represented by the eigenvector, and increase exponentially by the dominant eigenvalue. The reason this relationship exists is due to the fact that the iteration used by the Leslie matrix to determine population distributions is nearly identical to the way in which the iteration for the power method is done. The only difference, in fact, is that after each iteration in the power method the new vector x(n) is divided by its absolute maximum element before being passed back into the iteration each time. This absolute maximum element, in fact, converges to the dominant eigenvalue. Because this value is not taken out of the Leslie model calculation after each iteration, the Leslie matrix increases exponentially by this number with each iteration after stabilizing roughly to the population distributions modelled by the eigenvector. Note that the percent change yields the second or "changed" value when one multiplies the first value by $1 + \frac{percent\ change}{100}$. Examining the percent change between 2040 and 2050 reveals that the change in the total population, 27.8225% or 1.278225 by the aforementioned formula is approaching the eigenvalue of 1.28865624, and affirms that as the population distribution stabilizes the total population begins to increase exponentially by the dominant eigenvalue.

4. The new predictions for 2030, 2040, and 2050 are as followed:

2030 = (5.4954, 3.63125, 3.77825, 1.12455, 1.30815, 1.28304, 1.33056, 1.08416, 0.41888) ($x10^5$)

Total population = 1,945,424

Percent change: 6.4149%

2040 = (7.477735, 3.84678, 3.0865625, 3.400425, 1.012095, 1.151172, 1.026432, 1.024531, 0.433664) ($x10^5$)

Total population = 2,245,940

Percent change: 15.4473%

2050 = (8.864879, 5.2344145, 3.29763, 2.77791, 3.06038, 0.89064, 0.920938, 0.79035, 0.40981) ($x10^5$)

Total population = 2,621,909

Percent change: 16.7399%

These values increase at a significantly decreased pace than the original iteration through 2030-2050. One thing of notes is that most of the population distribution is the same as the numbers calculated prior to the change in birth rate. This is due to the altered birth rate taking time to have an effect on the population distribution.

The maximum eigenvalue calculated by the power method to eight degrees of accuracy is 1.16790283. The power method still converges, and converges to an eigenvalue greater than one. This means that the iteration applied to the Leslie matrix will increase exponentially by the eigenvalue, 1.16790283, after stabilizing to the population distributions in the eigenvector, which in this case is the following:

(1, 0.599365, 0.436218, 0.336154, 0.259045, 0.195187, 0.133701, 0.0881494, 0.0301906)

The population will stabilize and increase exponentially, just like in question 3, but it will stabilize to a different distribution and increase at a slower rate. Note that just as in the last question the percent change stabilized to a number close to the eigenvalue, as does this one. This percent change is equivalent to multiplying the first quantity by 1.167399 in the last iteration, and the eigenvalue is 1.16790283. These two numbers are extremely close in value. This further demonstrates and reaffirms that as the iterations continues, the population will increase exponentially by the dominant eigenvalue.