COM 205 - Digital Logic Design
Digital Systems and Binary Numbers

Assist. Prof. Özge ÖZTİMUR KARADAĞ

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Academic Staff

- Instructor: Özge Öztimur Karadağ
  - Email: ozge.karadag@alanya.edu.tr
  - Ofis: 217

- Assistant: Gürkan Çelik
  - Email: gurkan.celik@alanya.edu.tr

Book

- Digital Design, Morris Mano, Michael D. Ciletti

Weekly Plan

Week	Content
1	Introoduction to Digital Logic Design
2	Binary Numbers – Number-Base Conversions, Complements
3	Binary Numbers – Signed binary numbers, binary codes, binary logic
4	Boolean Algebra and Logic Gates – Axiomatic Definitions, basic theorems and properties, Boolean Functions
5	Boolean Algebra and Logic Gates – Canonical and Standard Forms, digital logic Gates, integrated circuits
6	Gate-Level Minimization – The Map Method, Product-of-Sums Simplification
7	Gate-Level Minimization – Don’t-Care Conditions, NAND and NOR Implementation
8	Midterm
9	Combinational Logic – Adders,Subtractors, Toplayıcılar, Çıkarıcılar, Kod Dönüştürme, Analysis Procedure
10	Combinational Logic – Other Two Level Implementations, Exclusive-OR Function
11	MSI Elements – Binary Adder-Subtractor, Decimal Adder, Magnitude Comparator
12	MSI Elements – Decoders, Encoders, Multiplexers
13	Problem Solving on Combinational Logic
14	Problem Solving on MSI elements
15	Review

Grading

• Midterm	40%
• Final	60%

Lab Work

- Schedule and details will be announced soon.

Digital Systems

- Represent and manipulate discrete elements of information.
  - Examples of discrete sets:
    - 10 decimal digits
    - 26 letters of the alphabet
    - 52 playing cards

- Today, electronic digital systems use two discrete value; 0 and 1 →binary

- Digital system is a system that manipulates discrete elements of information represented internally in binary form .

Binary Numbers

- Powers of two:

n	2^n	n	2^n	n	2^n
0	1	8	256	16	65,536
1	2	9	512	17	131,072
2	4	10	1,024 (1K)	18	262,144
3	8	11	2,048	19	524,288
4	16	12	4,096 (4K)	20	1,048,576 (1M)
5	32	13	8,192	21	2,097,152
6	64	14	16,384	22	4,194,304
7	128	15	32,768	23	8,388,608

Binary Numbers

- Base conversion
  - Binary number 11010.11 is equivalent to:

$$1 \text{ x } 2^4 + \text{I X } 2^3 + 0 \text{ X } 2^2 + \text{I X } 2^1 + 0 \text{ x } 2^{\circ} + \text{I X } 2^{-1} + \text{I X } 2^{-2} = 26.75$$

Examples to Base Conversions

- Ex: Convert decimal 41 to binary.
  - $(41)_{10}=(101001)_2$

Integer	Remainder	
41		
20	1	
10	0	
5	0	
2	1	
1	0	
0	1	101001 = answer

- Convert decimal 153 to octal.

153	
19	1
2	3
0	$2 = (231)_8$

- Convert 0.6875 to binary.
  - $(0.1011)_2$

	Integer		Fraction	Coefficient
$0.6875 \times 2 =$	1	+	0.3750	$a_{-1} = 1$
$0.3750 \times 2 =$	0	+	0.7500	$a_{-2} = 0$
$0.7500 \times 2 =$	1	+	0.5000	$a_{-3} = 1$
$0.5000 \times 2 =$	1	+	0.0000	$a_{-4} = 1$

Numbers with Different Bases

Decimal (base 10)	Binary (base 2)	Octal (base 8)	Hexadecimal (base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Octal and Hexadecimal Numbers

- Conversion from Binary to octal:
  - Partition Binary number into groups of three digits each
  - Start from Binary point proceed to left and right
  - Ex:

$$\begin{array}{cccccccccc} (10 & 110 & 001 & 101 & 011 & \cdot & 111 & 100 & 000 & 110)_2 = (26153.7406)_8 \\ 2 & 6 & 1 & 5 & 3 & & 7 & 4 & 0 & 6 \end{array}$$

Octal and Hexadecimal Numbers

- Conversion from octal to Binary:
  - Do the reverse:
  - Ex:

$$(673.124)_8 = (\underset{6}{110}\quad \underset{7}{111}\quad \underset{3}{011}\quad \cdot\quad \underset{1}{001}\quad \underset{2}{010}\quad \underset{4}{100})_2$$

Complements

- Used to simplify subtraction and for logical manipulation
- Diminished Radix Complement ((r-1)'s complement)
  - For a number N (with n digits) in base r, its (r-1)'s complement is:
    - $(r^n-1)-N$

    Ex: 9's complement of 546700 is 999999-546700 = 453299

    1's complement of 1011000 is 0100111.

- Radix Complement (r's complement)
  - Obtained by adding 1 to (r-1)'s complement
    - r^n-N

    Ex: 10's complement of 012398 → 987601+1= 987602

    2's complement of 1101100 is 0010100

Subtraction with Complements

- Subtraction of two n-digit unsigned numbers M-N in base r:

1. Add the minuend M to the r's complement of the subtrahend N. $M+(r^n-N)=M-N+r^n$
2. if M>=N the sum will produce an end carry r^n which can be discarded, what is left is the result M-N
3. if M<N, the sum does not produce an end carry and is equal to $r^n – (N-M)$, which is the r's complement of (N-M). To obtain the answer in familiar form, take the r's complement of the sum and place a negative sign in front.

Subtraction with Complements

- Ex:Using 10’s complement, subtract 72532-3250

M = 72532

10’s complement of 03250 → 96749+1 = 96750

sum = 169282

discard end carry 10^5=-100000

Answer = 69282

Subtraction with Complements

- Ex: Using 10’s complement subtract 3250 – 72532

M=3250

10’s complement of 72532 =27468

Sum=30718

No end carry! Answer is –(10’s complement of 30718)=-69282

Signed Binary Numbers

- Signed magnitude representation: number consists of a magnitude and a symbol(+ or -) or bit (0 or 1)
- Signed 1’s complement representation
- Signed 2’s complement representation

- Ex: -9   9=$(00001001)_2$
  - Signed magnitude: 10001001
  - Signed 1’s complement: 11110110
  - Signed 2’s complement: 11110111

Signed Binary Numbers

Decimal	Signed-2's Complement	Signed-1's Complement	Signed Magnitude
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
−0	—	1111	1000
−1	1111	1110	1001
−2	1110	1101	1010
−3	1101	1100	1011
−4	1100	1011	1100
−5	1011	1010	1101
−6	1010	1001	1110
−7	1001	1000	1111
−8	1000	—	—

References

- Digital Design, Morris Mano, Michael D. Ciletti