COM 205 - Digital Logic Design COMBINATIONAL LOGIC - I

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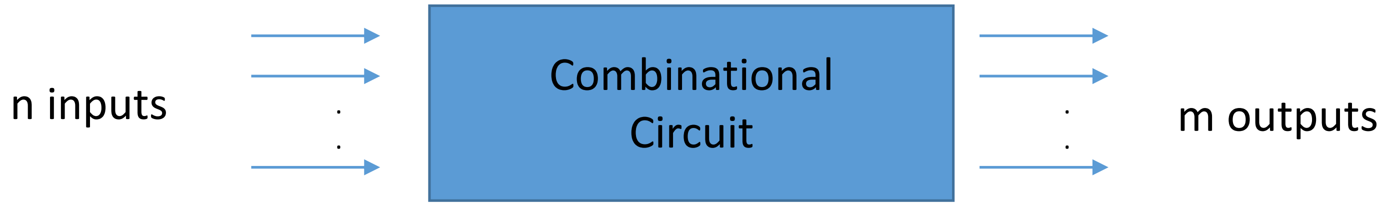
Circuits in Digital Systems

- Two kinds:
  - Combinational: Outputs are determined from only the present combination of inputs. Its operations can be specified by a set of Boolean functions.
  - Sequential: In addition to the logic gates, employ storage elements. Outputs are a function of the inputs and the state of the storage elements. State of the storage elements is a function of previous inputs.

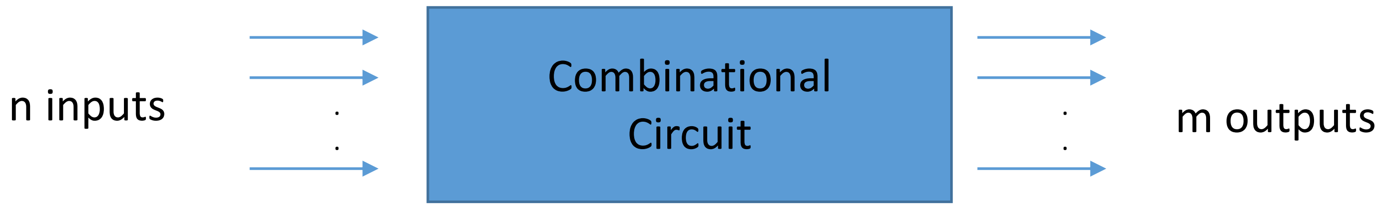
Combinational Circuits

- Consists of:
  - Input variables
  - Logic gates
  - Output variables

  - Block diagram of combinational circuit:



Combinational Circuit



- For n input variables there are 2^n possible input combinations.
- For each possible input combination, there is one possible output value.
- A combinational circuit can be specified with a **truth table** that lists the output values for each combination of input variables.
- A combinational circuit also can be described by m **Boolean functions**, one for each output variable. Each output function is expressed in terms of the n input variables.

Combinational Ciruits

- Two tasks:

  - **Analysis:** a logic circuit is given, and corresponding Boole functions, truth table or explanation of the circuit in words is expected.

  - **Design:** function is expressed in words and corresponding Boole function or logic circuit is expected.

Analysis Procedure

To obtain a **truth table** directly from the logic diagram:

1. Determine the number of input variables in the circuit. For n inputs, form the 2^n possible input combinations and list the binary numbers from 0 to 2^n-1 in a table.
2. Label the outputs of selected gates with arbitrary tables.
3. Obtain the truth table for the outputs of those gates which are a function of the input variables only.
4. Proceed to obtain the truth table for the outputs of those gates which are a function of previously defined values until the columns for all outputs are determined.

Analysis Procedure

- The **truth table** of a combinational circuit consists of columns for input and output variables. Columns for input variables represent 2^n binary combinations for n variables.
- Binary values for output variables are determined by examining the problem definition. For each valid input combination the output variables can be either 0 or 1. For some problems there might be undefined input combinations which are referred as don't care conditions.
- Boolean function obtained from the truth table is simplified (algebraic operations, Karnaugh map) among various simplified expression, one is selected depending on the problem definition:
  - Ex:
    - minimum number of gates
    - Min. Number of gate input
    - Min. Latency of the signal through the circuit
    - Number of min. Mid-connections.

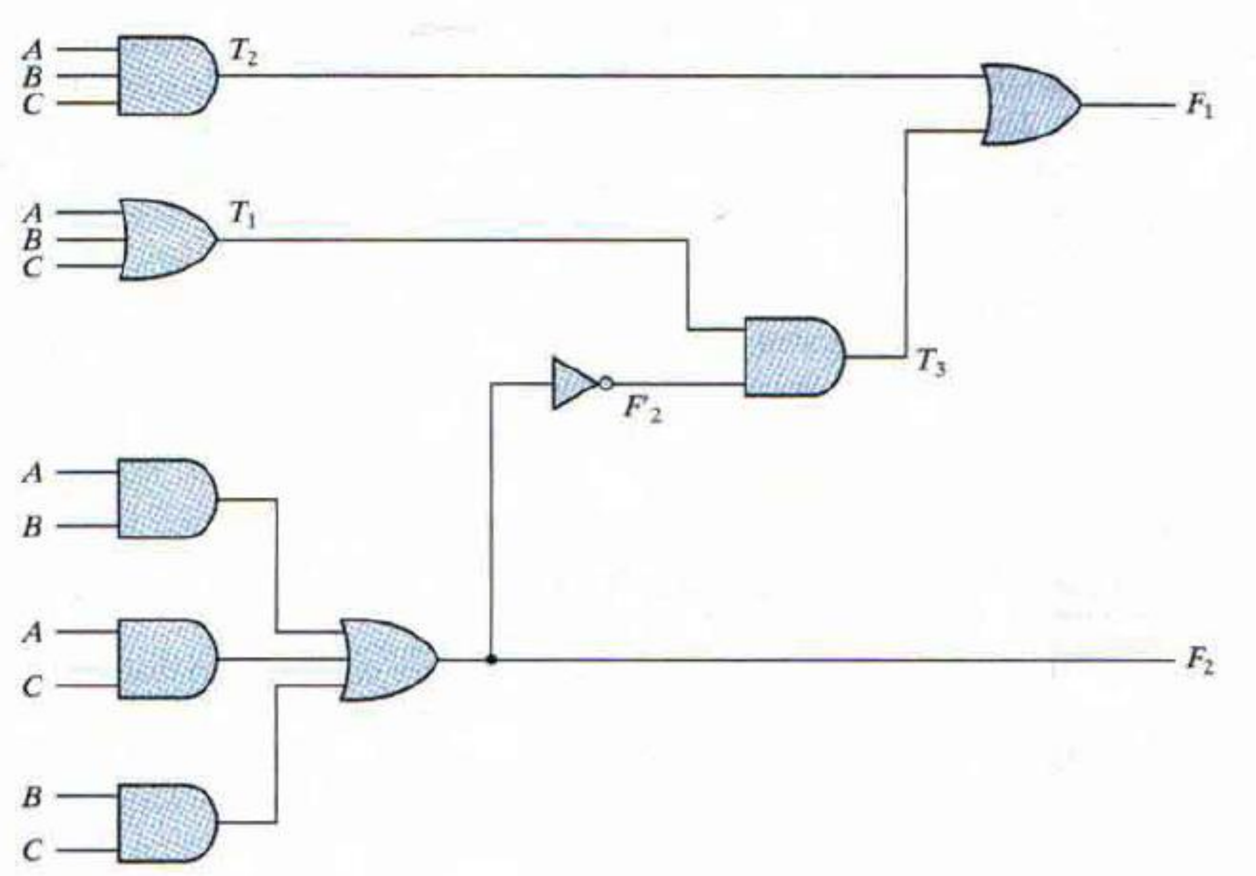
Analysis Procedure

- To obtain the output Boolean Function:
  1. Label all gate outputs that are a function of input variables with arbitrary symbols-but with meaningful names. Determine the Boolean functions for each gate output.
  2. Label the gates that are a function of input variables and previously labeled gates with other arbitrary symbols. Find the Boolean functions for these gates.
  3. Repeat the process outlined in step 2 until the outputs of the circuit are obtained.
  4. By repeated substitution of previously defined functions. obtain the output Boolean functions in term s of input variables

Analysis Example

Obtain the output Boolean functions.

- Given the circuit below:



Inputs: A, B, C

Outputs: F_1, F_2

$F_2 = AB+AC+BC$

$T_1 = (A+B+C)$

$T_2 = ABC$

$T_3 = F_2'T_1$

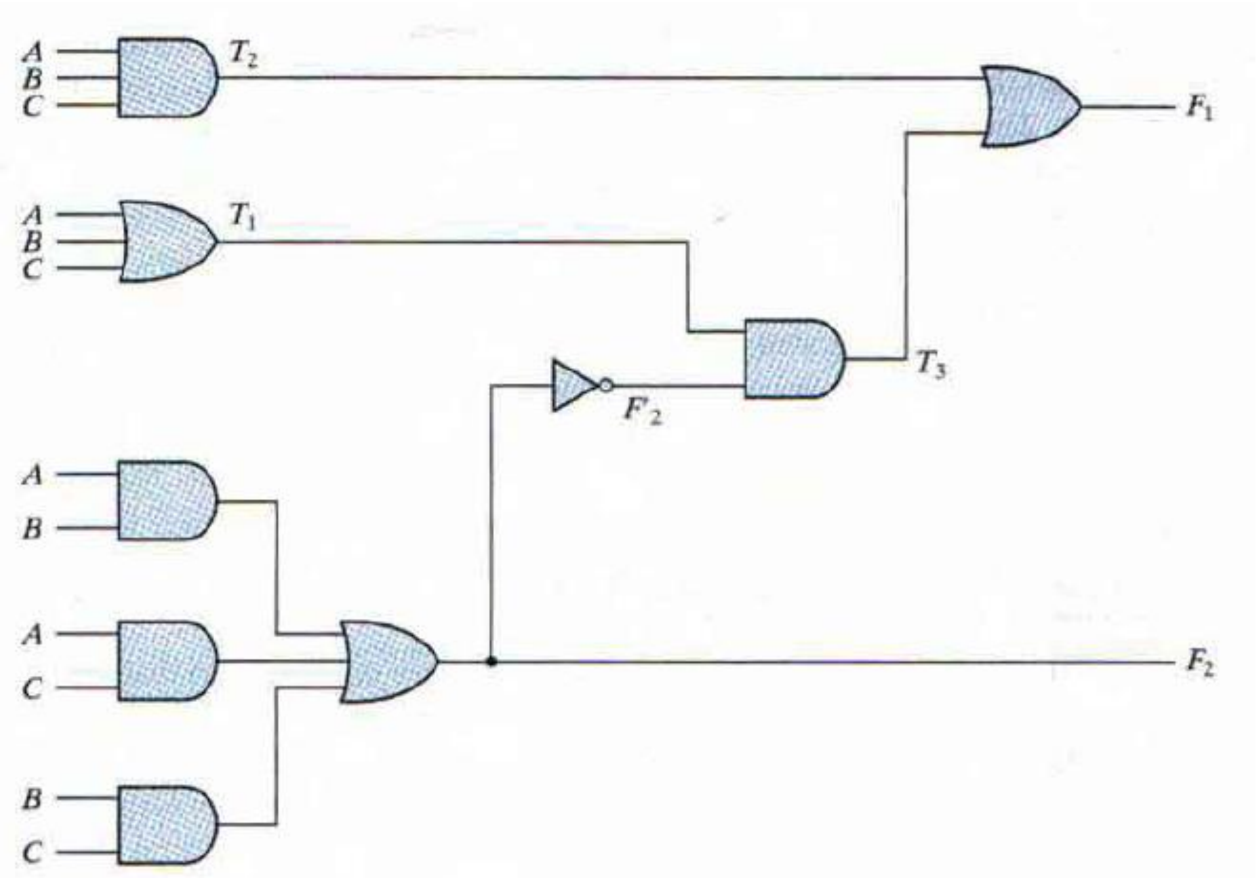
$F_1 = T_2+T_3$

$F_1 = ABC+F_2'T_1$
$= ABC+(AB+AC+BC)'(A+B+C)$
$= ABC+(A'+B')(A'+C')(B'+C')(A+B+C)$
$= ABC+(A'+A'C'+A'B'+B'C')(B'+C')(A+B+C)$

Analysis Example

Obtain the output Boolean functions.

- Given the circuit below:



Inputs: A,B,C

Outputs: F_1, F_2

$F_2 = AB+AC+BC$

$T_1 = A + B + C$

$T_2 = ABC$

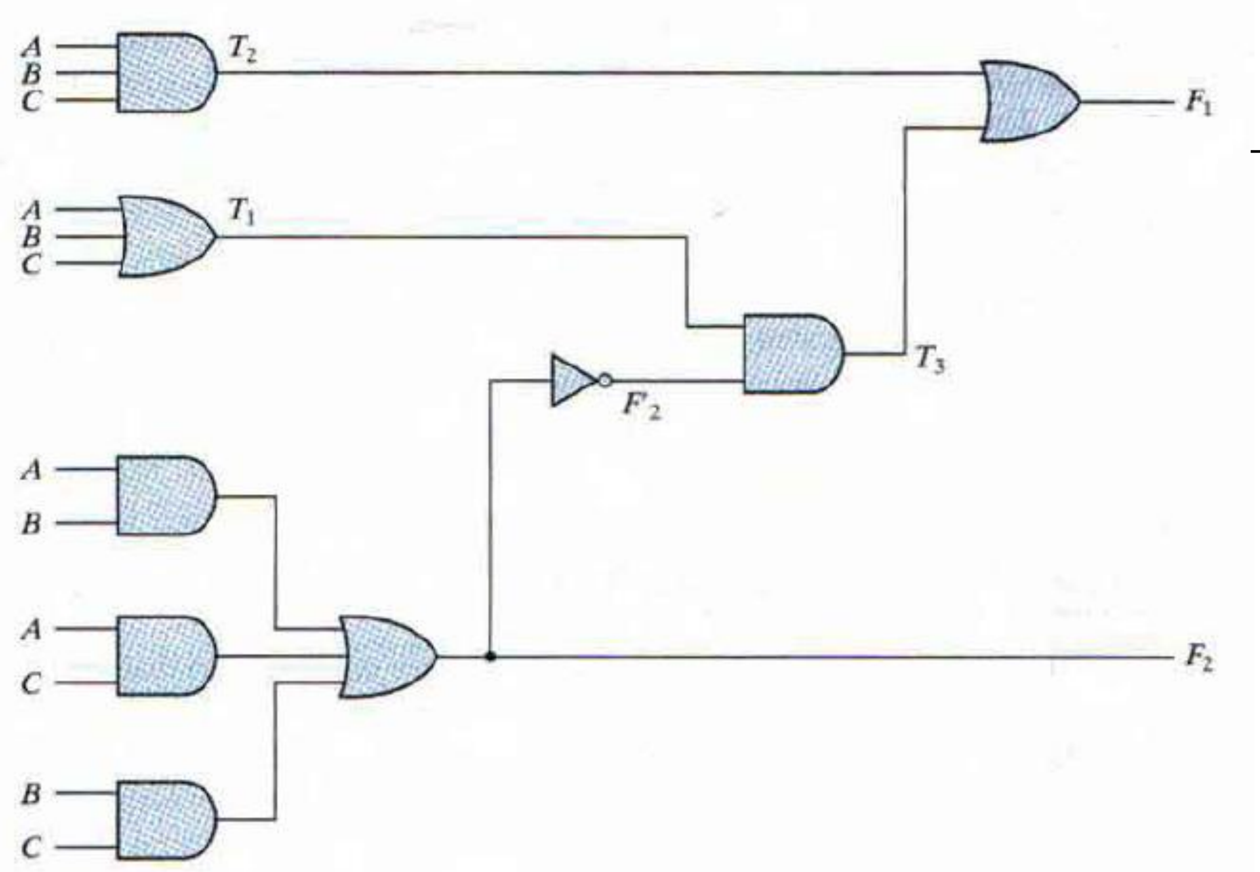
$T_3 = F_2'T_1$

$F_1 = T_3 + T_2$

$F1 = T3 + T2 = F_2'T_1 + ABC$
$=(AB+AC+BC)'(A+B+C)+ ABC$
$=(A'+B')(A'+C')(B'+C')(A+B+C)+ABC$
$=A'BC'+A'B'C+AB'C'+ABC$

Analysis Example

- Given the circuit below:



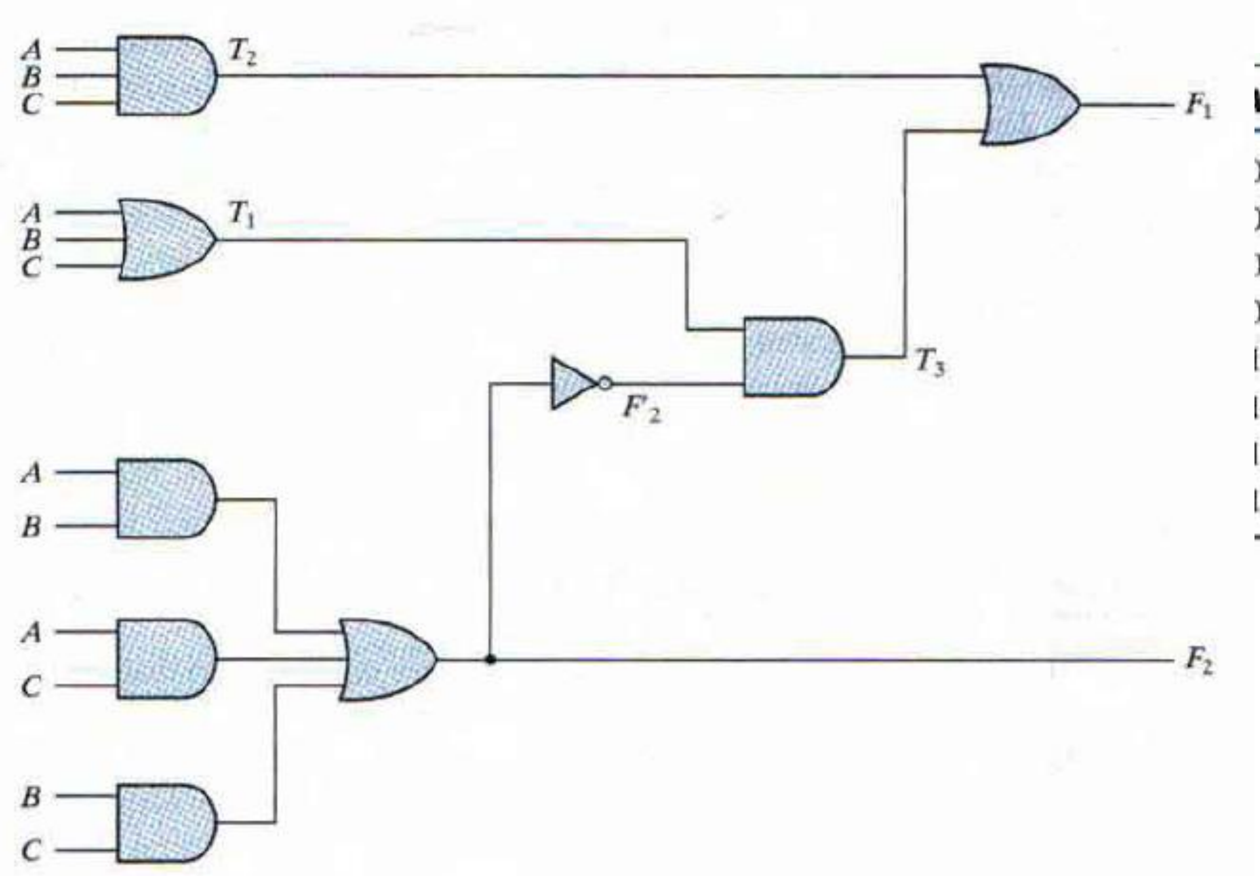
Obtain the truth table

A	B	C	T_1	T_2	F_2	T_3	F_1
0	0	0	0	0	0	0	0
0	0	1	1	0	0	1	1
0	1	0	1	0	0	1	1
0	1	1	1	0	1	0	0
1	0	0	1	0	0	1	1
1	0	1	1	0	1	0	0
1	1	0	1	0	1	0	0
1	1	1	1	1	1	0	1

Analysis Example

- Given the circuit below:

Obtain the truth table



A	B	C	F_2	F'_2	T_1	T_2	T_3	F_1
0	0	0	0	1	0	0	0	0
0	0	1	0	1	1	0	1	1
0	1	0	0	1	1	0	1	1
0	1	1	1	0	1	0	0	0
1	0	0	0	1	1	0	1	1
1	0	1	1	0	1	0	0	0
1	1	0	1	0	1	0	0	0
1	1	1	1	0	1	1	0	1

Design Procedure

- Given the specifications of the design objective, which defines the problem in words:
    1. Number of input and output variables are determined.
    2. Alpabetical symbols are assigned to input and output variables.
    3. Truth table is constructed to show the relation btw. input output
    4. For each output variable a simplified Boolean function is obtained.
    5. Logic circuit is drawn.

Design Example

- Design a a circuit that will convert Binary Coded Decimal (BCD) to Excess-3 code for decimal digits:

Truth table?

İnputs (BCD representation)				Outputs (Excess-3 representation)			
A	B	C	D	w	x	y	z
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	0
0	0	1	0	0	1	0	1
0	0	1	1	0	1	1	0
0	1	0	0	0	1	1	1
0	1	0	1	1	0	0	0
0	1	1	0	1	0	0	1
0	1	1	1	1	0	1	0
1	0	0	0	1	0	1	1
1	0	0	1	1	1	0	0

Design Example

- Design a circuit that will convert Binary Coded Decimal to Excess-3 code for decimal digits:
- Truth table?

Input BCD				Output Excess-3 Code			
A	*B*	*C*	*D*	*w*	*x*	*y*	*z*
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	0
0	0	1	0	0	1	0	1
0	0	1	1	0	1	1	0
0	1	0	0	0	1	1	1
0	1	0	1	1	0	0	0
0	1	1	0	1	0	0	1
0	1	1	1	1	0	1	0
1	0	0	0	1	0	1	1
1	0	0	1	1	1	0	0

K-Maps for outputs?

	00	01	11	10
00	0	0	0	0
01	0	1	1	1
11	X	X	X	X
10	1	1	X	X

w= A + BC + BD

	00	01	11	10
00	0	1	1	1
01	1	0	0	0
11	X	X	X	X
10	0	1	X	X

x= BC’D’ + B’C + B’D

	00	01	11	10
00	1	0	1	0
01	1	0	1	0
11	X	X	X	X
10	1	0	X	X

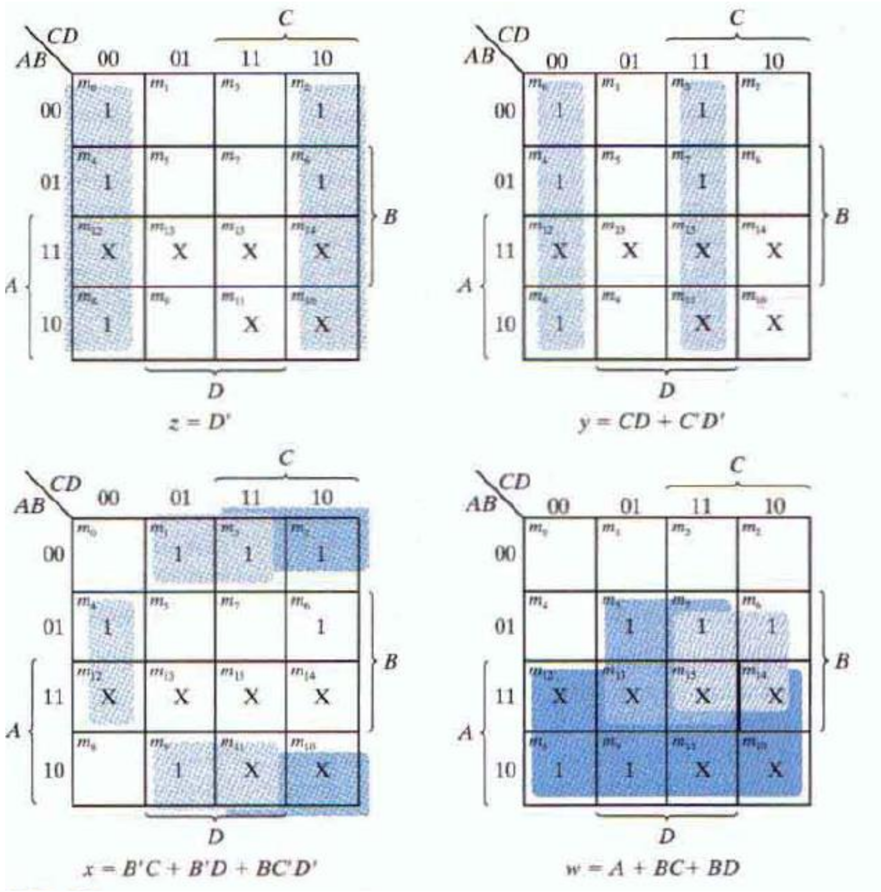
y= CD + C’D’

	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	X	X	X	X
10	1	0	X	X

z= D’

Design Example

- Design a a circuit that will convert Binary Coded Decimal to Excess-3 code for decimal digits:
- Truth table?



Input BCD				Output Excess-3 Code			
A	B	C	D	w	x	y	z
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	0
0	0	1	0	0	1	0	1
0	0	1	1	0	1	1	0
0	1	0	0	0	1	1	1
0	1	0	1	1	0	0	0
0	1	1	0	1	0	0	1
0	1	1	1	1	0	1	0
1	0	0	0	1	0	1	1
1	0	0	1	1	1	0	0

Design Example

- Obtain the logic circuit using the Boole functions:

$$z = D'$$
$$y = CD + C'D' = CD + (C + D)'$$
$$x = B'C + B'D + BC'D' = B'(C + D) + BC'D'$$
$$= B'(C + D) + B(C + D)'$$
$$w = A + BC + BD = A + B(C + D)$$