COM 205 - Digital Logic Design
Digital Systems and Binary Numbers - II

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Last Week

- Digital Systems
- Binary numbers
- Base conversions
- Complements

This Week

- Arithmetic
- Binary codes
- Registers

Arithmetic Addition

- Sign-magnitude representation:
  - The rules of ordinary arithmetic is applied
  - If signs are the same $\rightarrow$ add two numbers and give sum the common sign
  - If signs are different $\rightarrow$ subtract the smaller magnitude from the larger and give the difference the sign of the larger magnitude.
  - Ex: (25)+(-37)

    =-(37-25)=-12

Arithmetic Addition

- 2's complement representation:
  - Add two numbers including the sign bits.
  - A carry out of the sign bit position is discarded.
  - Ex:

+6	00000110
+13	00001101
+19	00010011
-6	
+13	

+6	00000110	
-13	11110011	
-7	11111001	→ (-)00000111
-6	11111010	
-13	11110011	
-19	11101101	→(-) 00010011

Negative results are automatically in 2's complement form. Negative numbers are unfamiliar. Take 2's complement and see its value ;)

Arithmetic Subtraction

- Subtraction of two signed Binary numbers when negative numbers are in 2’s complement form:
  - Take the 2’s complement of the subtrahend (including the sign bit) and add it to the minued (including the sign bit).
  - A carry out of the sign bit position is discarded.
    - A-(+B)=A+(-B)
    - A-(-B)=A+B

    Changing a positive number to a negative number is easily done by taking the 2’s complement of the positive number.

    Ex: (-6)-(-13)=7

6=00000110	11111010		11111010
13=00001101	11110011	Take 2’s complement	+00001101
			00000111

Arithmetic Operations

- Binary numbers in the signed-complement system are added and subtracted by the same basic addion and subtraction rules ad unsigned numbers.
- Therefore, computers need only one common hardware circuit to handle both types of arithmetic.

Binary Codes

- An n-bit Binary code is a group of n bits that assumes up to 2^n distinct combinations of 1's and 0's.
- Ex: 2bit Binary code can represent: 00, 01,10,11
- A set of eight elements requires a three-bit code.
- Minimum number of bits required to code 2^n distinct quantities is n.

Binary Codes

- Decimal Codes
  - Binary codes for representing decimal numbers require at least 4 bits.
  - It is possible to design various combinations for decimal numbers using 4 or more bits.

Binary Codes

- Binary Coded Decimal

Decimal Symbol	BCD Digit
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

Binary Codes

- Four Different Binary Codes for Decimal Digits

Decimal Digit	BCD 8421	2421	Excess-3	8, 4, −2, −1
0	0000	0000	0011	0000
1	0001	0001	0100	0111
2	0010	0010	0101	0110
3	0011	0011	0110	0101
4	0100	0100	0111	0100
5	0101	1011	1000	1011
6	0110	1100	1001	1010
7	0111	1101	1010	1001
8	1000	1110	1011	1000
9	1001	1111	1100	1111
	1010	0101	0000	0001
Unused bit combi-nations	1011	0110	0001	0010
	1100	0111	0010	0011
	1101	1000	1101	1100
	1110	1001	1110	1101
	1111	1010	1111	1110

Binary Codes

- BCD and 2421 codes are examples of weighted codes
- BCD code has weights 8,4,2,1
  - Ex: 0110 is interpreted as 8x0+4x1+2x1+1x0=6

- 2421 and Excess-3 are examples of self-complementing codes.
  - 9’s complement of a decimal is obtained directly by changing 1’s to 0’s and 0’s to 1’s.
  - Ex: 395 is represented in Excess-3 code as:
    - 011011001000
    - 9’s complement of 395 is:
    - 604, which can be obtained by 1’s complement of 011011001000 → 100100110111

Binary Codes

- Error Detecting Code
  - The aim of the error detecting code is to detect bit changes from 0 to 1 and 1 to 0 in data communication. The most common method is the parity bit.
  - A parity bit is an extra bit included with a message to make the total number of 1's either even or odd.
  - 4-bit messages and corresponding parity bits:
  - Can detect one, three or any odd combination of errors in each character that is transmitted.
  - Additional error detection codes are required to detect even combination of errors.

Odd parity		Even Parity	
Message	P	Message	P
0000	1	0000	0
0001	0	0001	1
0010	0	0010	1
0011	1	0011	0
0100	0	0100	1
0101	1	0101	0
0110	1	0110	0
0111	0	0111	1
1000	0	1000	1
1001	1	1001	0
1010	1	1010	0
1100	1	1100	0
1101	0	1101	1
1110	0	1110	1
1111	1	1111	0

Binary Codes

- Gray-code
  - Only one bit in the code group changes in going from one number to the next.
  - Ex: 7→8 0100 →1100

  - Gray code is especially useful in cases where a transition from one number to the next may produce error or ambiguity.

Gray Code	Decimal Equivalent
0000	0
0001	1
0011	2
0010	3
0110	4
0111	5
0101	6
0100	7
1100	8
1101	9
1111	10
1110	11
1010	12
1011	13
1001	14
1000	15

ASCII Code

- American Standard Code for Information Interchange
- ASCII is 7 bits long code, representing 128 characters.
- Standard Binary code for alphanumeric characters.

	$b_7b_6b_5$							
$b_4b_3b_2b_1$	000	001	010	011	100	101	110	111
0000	NUL	DLE	SP	0	@	P	‘	p
0001	SOH	DC1	!	1	A	Q	a	q
0010	STX	DC2	“	2	B	R	b	r
0011	ETX	DC3	#	3	C	S	c	s
0100	EOT	DC4	$	4	D	T	d	t
0101	ENQ	NAK	%	5	E	U	e	u
0110	ACK	SYN	&	6	F	V	f	v
0111	BEL	ETB	‘	7	G	W	g	w
1000	BS	CAN	(	8	H	X	h	x
1001	HT	EM	)	9	I	Y	i	y
1010	LF	SUB	*	:	J	Z	j	z
1011	VT	ESC	+	;	K	[	k	{
1100	FF	FS	,	<	L		l	|
1101	CR	GS	–	=	M	]	m	}
1110	SO	RS	.	>	N	∧	n	~
1111	SI	US	/	?	O	–	o	DEL

Control Characters

NUL	Null	DLE	Data-link escape
SOH	Start of heading	DC1	Device control 1
STX	Start of text	DC2	Device control 2
ETX	End of text	DC3	Device control 3
EOT	End of transmission	DC4	Device control 4
ENQ	Enquiry	NAK	Negative acknowledge
ACK	Acknowledge	SYN	Synchronous idle
BEL	Bell	ETB	End-of-transmission block
BS	Backspace	CAN	Cancel
HT	Horizontal tab	EM	End of medium
LF	Line feed	SUB	Substitute
VT	Vertical tab	ESC	Escape
FF	Form feed	FS	File separator
CR	Carriage return	GS	Group separator
SO	Shift out	RS	Record separator
SI	Shift in	US	Unit separator
SP	Space	DEL	Delete

Binary Storage and Registers

- A Binary cell is a device that possesses two stable states and is capable of storing one bit (0 or 1) of information.
- Registers: A register is a group of Binary cells. A register with n cells can store any discrete quantity of information that contains n bits.

Binary Storage and Registers

- Ex: 16 bit register

1	1	0	0	0	0	1	1	1	1	0	0	1	0	0	1
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

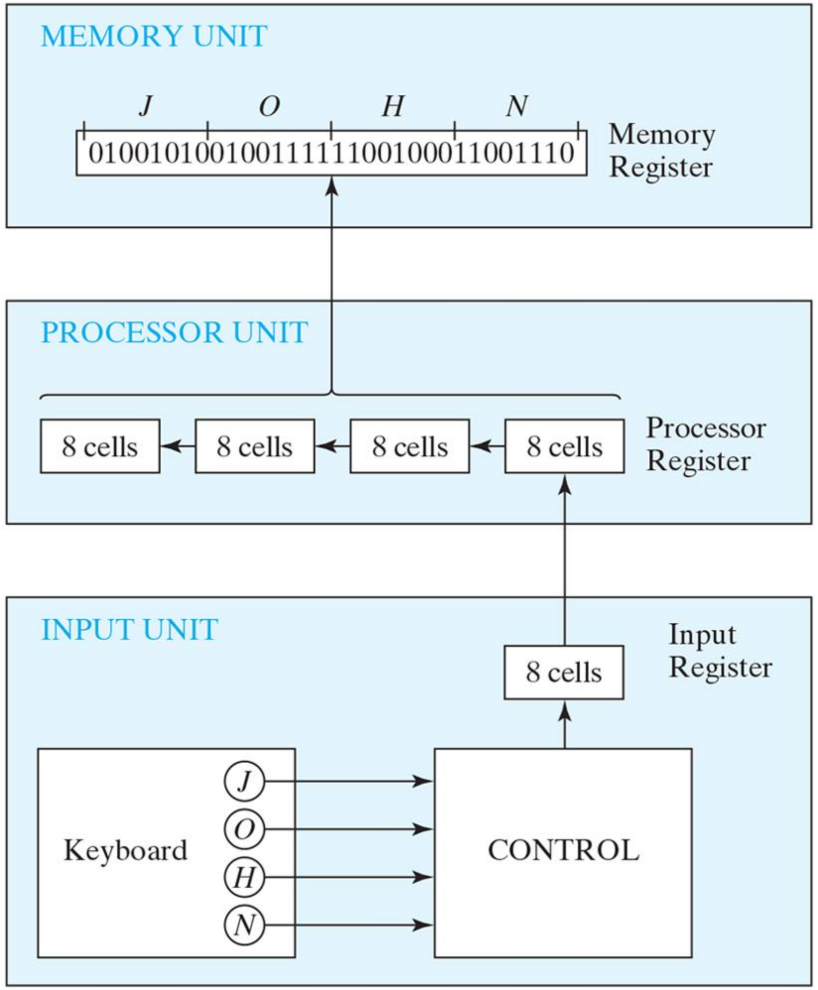
  - A register with 16 cells can be in one of 2^{16} possible states.
  - Register can store any Binary number from 0 to 2^{16}-1.
  - If the register stores alphanumeric characters of an 8 bit code, then the content of the register is any two meaningful characters. The most significant bit can be considered as the parity bit.
    - Ex: (assuming even parity) the register in the example above contains CI
  - If the register contains four decimal digits represented by a four-bit code, the content of the register is a four-digit decimal number.
    - in excess-3 code: 9096
    - BCD: Meaningless as 1100 is not assigned to any decimal digit

Binary Storage and Registers

- A register can store discrete elements of information and the same bit configuration may be interpreted differently for different types of data.

Register Transfer

- Data transfer between various parts of digital computer (control unit, arithmetic unit, input/output units, memory units) is accomplished via registers.
- Transfer of information among registers:
  - It is assumed that ASCII code with odd-parity is used.
  - Each time a key is stuck, the control circuit enters an equivalent eight-bit alphanumeric character code into the input register.
  - The information from the input register is transferred into the eight least significant cells of processor register.
  - After every transfer, the input register is cleated to enable the control to insert a new eight bit code when the keyboard is stuck again.
  - Each eight-bit character transferred to the processor register is preceded by a shift of the previous character to the next eight cells on its left.
  - When a transfer of four characters is completed, the processor register is full, and its contents are transferred into the memory register.



Binary Information Processing in Registers

- To process discrete quantities of information in Binary form, a computer must be provided with devices
  - That hold the data to be processed
  - With circuit elements that manipulate individual bits of information
- Process of adding two 10-bit Binary numbers: