Week 4

Question: A box contains 12 balls that consist of 5 black, 4 white and 3 red. In how many ways could 6 balls be chosen with a constraint that at least one ball for each color should be in. (Hint: 2 possible solutions exist)

Question: Given a set A={1,2,3,4,5}

a. How many subsets do A have (Hint: all possible subsets)

b. How many subsets do not contain both 1 and 2

c. How many subsets have at least 1 or 2.

$$\binom{12}{6}-\binom{9}{6}-\binom{7}{6}-\binom{8}{6}$$

Or

$$\binom{5}{1}\left\{\binom{4}{4}\binom{3}{1}+\binom{4}{3}\binom{3}{2}+\binom{4}{2}\binom{3}{3}\right\}+\binom{4}{1}\left\{\binom{5}{2}\binom{3}{3}+\binom{5}{3}\binom{3}{2}+\binom{5}{4}\binom{3}{1}\right\}$$
$$+\binom{3}{1}\left\{\binom{5}{3}\binom{4}{2}+\binom{5}{2}\binom{4}{3}\right\}+\left\{\binom{5}{2}\binom{4}{2}\binom{3}{2}\right\}=805$$

a. $\binom{5}{0}+\binom{5}{1}+\binom{5}{2}+\binom{5}{3}+\binom{5}{4}+\binom{5}{5}=1+5+10+10+5+1=32=2^5$

b. 1-2-3-4-5

12-13-14-15-23-24-25-34-35-45

123-124-125-134-135

1234-1235-1245-1345-2345

12345

Answer=32-8=24

c.**1-2**-3-4-5

12-13-14-15-23-24-25-34-35-45

123-124-125-134-135

1234-1235-1245-1345-2345

12345

Answer=20

Ex: Two a's and two b's are arranged in order. All arrangements are equally likely. Given that the last letter, in order, is b, find the probability that two a's are together.

$$S=\{aabb, abab, abba, baab, baba, bbaa\}$$

$$B = \{aabb, abab, baab\}$$

A={aabb, baab, bbaa}

$$A \cap B = \{aabb, baab\}$$

$$P(B) = \frac{3}{6}$$

$$P(A \cap B) = \frac{2}{6}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{2/6}{3/6} = \frac{2}{3}$$

Ex: In the two-dice experiment. The first dice is denoted by r and the second die is denoted by c. Given that r+c=9, what is the probability that the second die is 4?

$$B = \{(3,6), (6,3), (4,5), (5,4)\}$$

$$A = \{(5,4)\}$$

$$P(A \cap B) = \frac{1}{36}$$

$$P(A|B) = \frac{1/36}{4/36} = \frac{1}{4}$$

Ex: In a class, 25 percent students fails physics and 15 percent fails chemistry and 10 percent fail both courses. If one student is randomly selected

a. What is the probability of failing chemistry given that he or she fails physics?

b. What is the probability of failing physics given that he or she fails chemistry?

c. What is the probability of failing either chemistry or physics?

a.P(Phys)=0.25, P(Chem)=0.15, P(Phys∩Chem)=0.10

$$P(Chem|Phys) = \frac{P(Phys \cap Chem)}{P(Phys)} = \frac{0.10}{0.25}$$

b.

$$P(Phys|Chem) = \frac{P(Phys \cap Chem)}{P(Chem)} = \frac{0.10}{0.15}$$

c.$P(Phys \cup Chem) = P(Phys) + P(Chem) - P(Phys \cap Chem) = 0.25 + 0.15 - 0.10 = 0.30$

Ex: Two dice are tossed. If the first de shows 5, what is the probability that the second die shows even?

B= {(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)}

A={(5,2),(5,4),(5,6)}

A∩B={(5,2,(5,4),(5,6)}

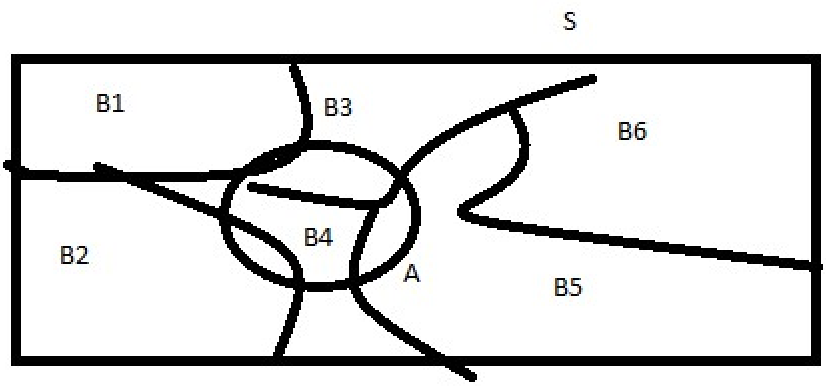
$$P(A|B) = \frac{3/36}{6/36} = \frac{1}{2}$$

Definition (Partition of Sample Space):

a. $B_i \cap B_j = \emptyset \; for \; all \; i \neq j$

b. $\coprod_{i=1}^{n} B_i = S$

c. $P(B_i) > 0 \; for \; all \; i$



If a sample space S is partitioned into five mutually exclusive events, namely pairwise disjoint events, and they satisfy the all three conditions above, it is a called a partition of S

Theorem: S is partitioned into $B_1, B_2, \dots, B_k$ then

$$P(B_1) + P(B_2) + \dots + P(B_k) = 1$$

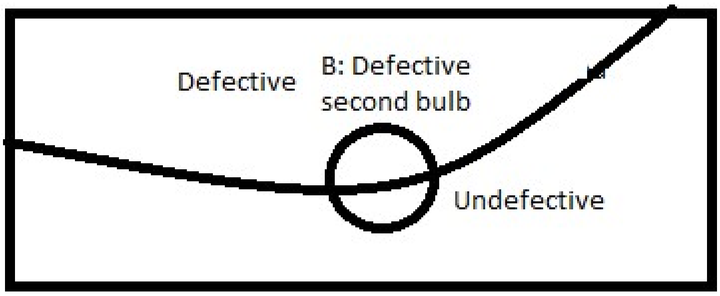
Total probability Formula

Theorem: S is partitioned into $B_1, B_2, \dots, B_k$. If an event A in S is defined then its probability is expressed as

$$P(A) = \sum_{i=1}^{k} P(B_i)P(A|B_i)$$

Ex: A storage has 20 defective and 80 undefective electric bulbs. Two electric bulbs are chosen without putting it back. What is the probability of getting a second bulb that is defective?

S



$$P(B) = P(second\ bulb\ defective).P(defective|first\ bulb\ defective) + P(second\ bulb\ defective)P(defective|first\ bulb\ undefective)$$

$$P(B) = \frac{19}{99}\frac{20}{100} + \frac{20}{99}\frac{80}{100}$$

Independent Events

Definition: Events A and B are called independent if and only if (iff)

$$P(A \cap B) = P(A)P(B)$$

Theorem: If events A and B ($P(A) \neq 0$ and $P(B) \neq 0$) are called independent and there exist at least one common point of both A and B, namely, $A \cap B \neq \emptyset$.

Ex: Suppose that two dice are tossed. What is the probability of getting (1,1) when tossed once?

A:first die is 1

B:second die 1

$$P(A \cap B) = P(A)P(B)$$

$$\frac{1}{36} = \frac{1}{6}\frac{1}{6}$$

Hence, two events are independent

Ex: Suppose two dice are tossed. While the first die is represented by r ,the second die is represented by s. A is defined as r+s=11 and B is defined as $r \neq 5$. Are these two events independent?

A={(5,6),(6,5)}

B={(1,1),(1,2),(1,3),(1,4,(1,5),(1,6), (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), (3,1),(3,2),(3,3),(3,4),(3,5),(3,6), (4,1),(4,2),(4,3),(4,4),(4,5),(4,6), (6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

$$A \cap B = \{(6{,}5)\}$$

$$P(A \cap B) = \frac{1}{36}$$

$$P(A) = \frac{2}{36} \quad P(B) = \frac{30}{36}$$

$$\frac{1}{36} \neq \frac{2}{36}\frac{30}{36}$$

Events A and B are not independent but dependent