COM 205 - Digital Logic Design
Boolean Algebra and Logic Gates -II

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Last Week

- Boolean Algebra
  - Postulates, proofs etc.

Postulate 2	(a) $x+0=x$	(b)$x \cdot 1 = x$
Postulate 5	(a) $x+x'=1$	(b)$x \cdot x'=0$
Theorem 1	(a) $x+x=x$	(b) $x \cdot x = x$
Theorem 2	(a) $x+1=x$	(b) $X \cdot 0 = 0$
Theorem 3 (involution)	$(x')'=x$	
Postulate 3, commutative	(a) $x+y=y+x$	(b) $x \cdot y = y \cdot x$
Theorem 4 , associative	(a)$x+(y+z)=(x+y)+z$	(b) $x \cdot (y \cdot z)= (x \cdot y) \cdot z$
Postulate 4, distributive	(a) $x \cdot (y+z)=(x \cdot y)+(x \cdot z)$	(b) $x+(y \cdot z)=(x+y) \cdot (x+z)$
Theorem 5, DeMorgan	(a) $(x+y)'=x' \cdot y'$	(b) $(xy)'=x'+y'$
Theorem 6, Absorption	(a) $x+x \cdot y=x$	(b) $x \cdot (x+y)=x$

Boolean Algebra

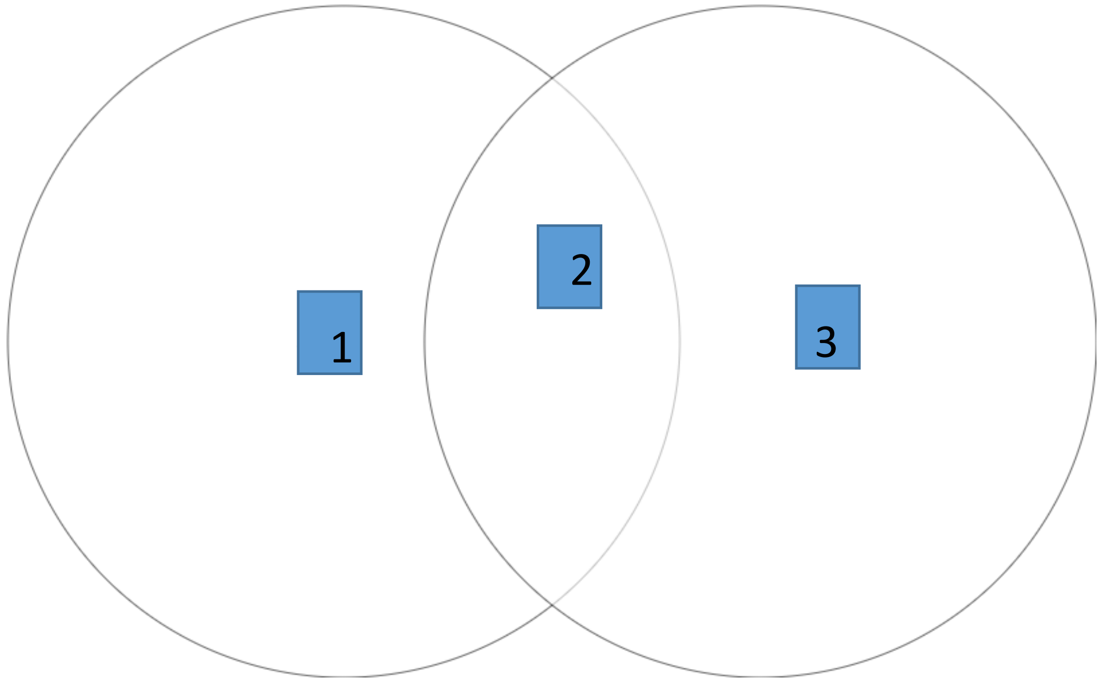
- Operator Presedence

- Can you sort these operators considering their presedence?
  - Not, or, and, paranthesis

  - Parenthesis
  - Not
  - And
  - Or

Boolean Algebra

- Venn Diagram
  - xy’
  - x’y
  - xy



Boolean Functions

- Boolean algebra deals with
  - Binary variables
  - Logic operations

- Boolean functions consist of
  - Binary variables
  - Constants 0 and 1
  - Logic operation symbols

- Boolean function can be in
  - Algebraic form $f = xyz'$
  - Truth table; for n Binary variables, 2^n combinations in truth table

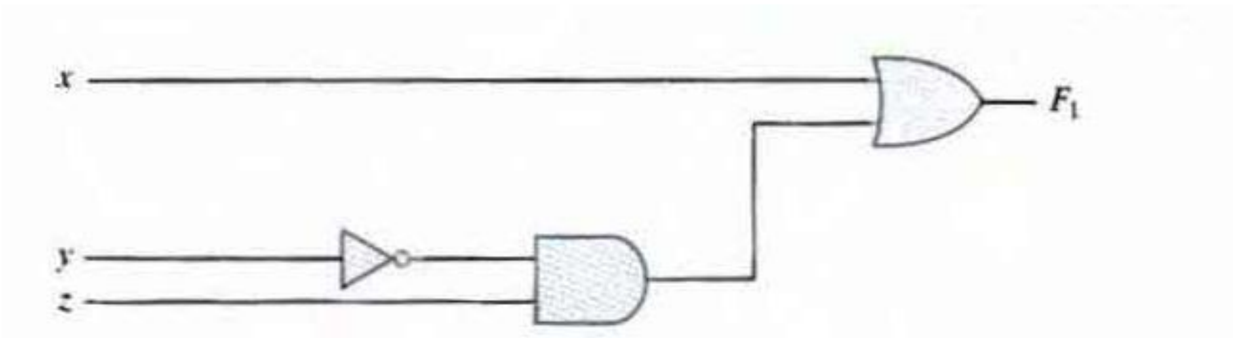
Boolean Functions

- Ex: $F_1 = xyz'$, $F_2 = x+y'z$, $F_3 = x'y'z + x'yz+xy'$, $F_4 =xy'+x'z$

	x	y	z	F_1	F_2	F_3	F_4
1	0	0	0	0	0	0	0
2	0	0	1	0	1	1	1
3	0	1	0	0	0	0	0
4	0	1	1	0	0	1	1
5	1	0	0	0	1	1	1
6	1	0	1	0	1	1	1
7	1	1	0	1	1	0	0
8	1	1	1	0	1	0	0

Boolean Functions

- Gate implementation of the Boolean Function:
  - Ex:



  - Which Boolean function does the given logic gate iimplementation correspond to?

- F_3 and F_4 in the previous example are the same!
- Boolean algebra is used to simplify boolean expressions.

Algebraic Manipulation

- A literal is a single variable with a term, in a complemented or uncomplemented form.
- $F = x'y'z + x'yz + xy'$ → has three terms and eight literals
- By reducing the number of terms, the number of literals, or both in a Boolean expression, it is often possible to obtain a simpler circuit.
- There are various methods for simplifying Boolean functions but there is not an exact way to do that.
- We will learn map method. Designers make use of minimization programs.

Algebraic Manipulation

- Ex: Simplify the Boolean functions to a minimum number of literals.
- $x+x'y$
  $= (x+x')(x+y)$ By distributive law of + over .
  $=1(x+y)$
  $=x+y$
- $x(x'+y)$
  $=xx'+xy$
  $=0+xy$
  $=xy$
- $x'y'z+x'yz+xy'$
  $= x'z(y'+y)+xy'$
  $=x'z+xy'$

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Algebraic Manipulation

- Ex:
- $xy+x'z+yz$

  $=xy+x'z+yz(x+x')$

  $=xy+x'z+xyz+x'yz$

  $=xy(1+z)+x'z(1+y)$

  $=xy+x'z$
- $(x+y)(x'+z)(y+z)$

  $=(x+y)(x'+z)(y+z+(xx)')$

  $=(x+y)(x'+z)(x+y+z)(x'+y+z)$

  $=(x+y)(1+z)(x'+z)(1+y)$

  $=(x+y)(x'+z)$

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Complement of a Function

- The complement of a function F is F' and is obtained from an interchange of 0's for 1's and 1's for 0's in the value of F.
- The complement of a function may be derived algebraically through DeMorgan's theorems:
  - For two variables:
    - $(x+y)'=x'y'$
    - $(xy)'=x'+y'$
  - For three variables:
    - $(A+B+C)'=(A+x)'$ — $(B+C)=x$
      - $=A'x'$ — DeMorgan theorem
      - $=A'(B+C)'$ — $(B+C)=x$
      - $=A'(B'C')$
      - $=A'B'C'$ — Associative Rule

Complement of a Function

- Generalization of DeMorgan's Theorem:
  - $(A+B+C+\ldots+F)'=A'B'C'\ldots F'$
  - $(ABCD..F)'=A'+B'+C'+D'+\ldots+F'$

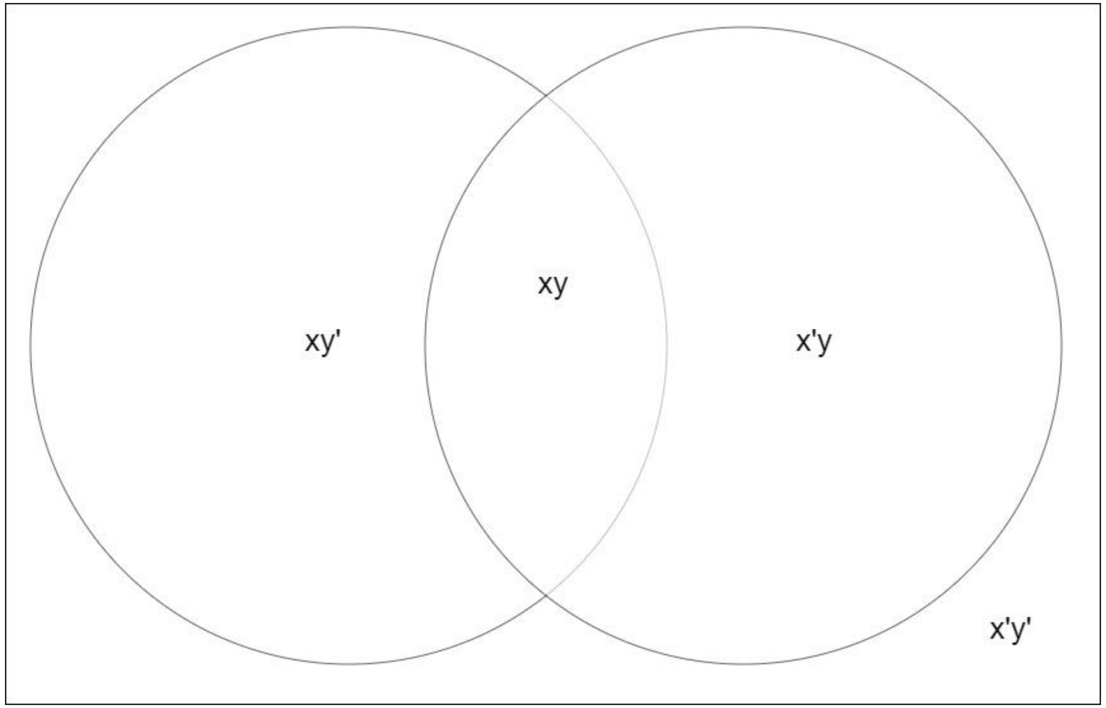
- The complement of a function can be obtained by interchanging AND and OR operators (taking dual) and complementing each literal.

  - Ex: $F1'=(x'yz'+x'y'z)'$
    - $=(x'yz')'(x'y'z)'$
    - $=(x+y'+z)(x+y+z')$

$$
\begin{aligned}
F2' &= [x(y'z'+yz)]' \\
&= x'+(y'z'+yz)' \\
&= x'+(y'z')'(yz)' \\
&= x'+(y+z)(y'+z')
\end{aligned}
$$

Canonical and Standard Forms

- A Binary variable may appear either in its normal form (x) or in its complement form x’.
- Venn diagram for two variables:



Canonical and Standard Forms

3 terms			Minterms		Maxterms	
x	y	z	term	Designation	term	designation
0	0	0	x’y’z’	m0	x+y+z	M0
0	0	1	x’y’z	m1	x+y+z’	M1
0	1	0	x’yz’	m2	x+y’+z	M2
0	1	1	x’yz	m3	x+y’+z’	M3
1	0	0	xy’z’	m4	x’+y+z	M4
1	0	1	xy’z	m5	x’+y+z’	M5
1	1	0	xyz’	m6	x’+y’+z	M6
1	1	1	xyz	m7	x’+y’+z’	M7

Canonical and Standard Forms

- Minterm: standard product
- Maxterm: standard sum

- Each maxterm is the complement of its corresponding minterm.

- A Boolean Function can be expressed algebraically from a given truth table by forming a minterm for each combination of the variables that produces a 1 in the function and then taking the OR of all those terms.

Canonical and Standard Forms

x	y	z	f_1	f_2	f_2'
0	0	0	0	0	1
0	0	1	1	0	1
0	1	0	0	0	1
0	1	1	0	1	0
1	0	0	1	0	1
1	0	1	0	1	0
1	1	0	0	1	0
1	1	1	1	1	0

- Ex:
- f_1=?
  - $=x'y'z+xy'z'+xyz$
  - $=m_1+m_4+m_7$
- f_2=?
  - $=x'yz+xy'z+xyz'+xyz$
  - $=m_3+m_5+m_6+m_7$

Canonical and Standard Forms

x	y	z	f_2
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

- Any Boolean function can be expressed as a sum of minterms (sum meaning OR)
- How to find the complement of a function?
- Complement of a function can be obtained by forming a minterm for each combination that produces 0 in the function and then OR ing these terms.

  Ex: $f_2' = ?$

  $f_2'=(x'y'z'+x'yz'+x'yz+xy'z+xyz')'$

  $f_2 = ?$

  $f_2=(x+y+z)(x+y'+z)(x+y'+z')(x'+y+z')(x'+y'+z)$

  $=M_0M_2M_3M_5M_6$

- Any Boolean Function can be expressed as a product of maxterms (product meaning AND)
  - For each term that produces 0 in the function find the corresponding maxterm
  - AND those terms.

Canonical and Standard Forms

- Expressing Boole functions in the form of sums of minterms or product of maxterms is referred as the canonical forms.
- Sum of Minterms:
  - In order to express a function as the sum of minterm variables
    - Expand the expression into a sum of AND terms
    - Inspect terms to see if it contains all variables. If it misses one or more variables, it is ANDed with an expression such as x+x’

Canonical and Standard Forms

- Ex: F=A+B'C Express as a sum of minterms

  - A=A(B+B')=AB+AB'
  - A=AB(C+C')+AB'(C+C')
  - =ABC+ABC'+AB'C+A'B'C

  - B'C = A'B'C+AB'C

  - F=A+B'C
  - =ABC+ABC'+AB'C+AB'C'+AB'C+A'B'C
  - =A'B'C+AB'C'+AB'C+ABC'+ABC

  - =m1+m4+m5+m6+m7
  - F(A,B,C)=$\sum(1,4,5,6,7)$

Canonical and Standard Forms

- An alternative procedure for deriving minterms of a Boolean function is to obtain the truth table of the function directly from the algebraic expression and then read the minterms from the truth table.
- Ex: F = A+B'C

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1