Chapter 5: Algorithms

Computer Science: An Overview
Twelfth Edition

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Chapter 5: Algorithms

- 5.1 The Concept of an Algorithm
- 5.2 Algorithm Representation
- 5.3 Algorithm Discovery
- 5.4 Iterative Structures
- 5.5 Recursive Structures
- 5.6 Efficiency and Correctness

Definition of Algorithm

An algorithm is an **ordered** set of **unambiguous**, **executable** steps that defines a **terminating** process.

Polya's Problem Solving Steps

- 1. Understand the problem.
- 2. Devise a plan for solving the problem.
- 3. Carry out the plan.
- 4. Evaluate the solution for accuracy and its potential as a tool for solving other problems.

Polya's Steps in the Context of Program Development

- 1. Understand the problem.
- 2. Get an idea of how an algorithmic function might solve the problem.
- 3. Formulate the algorithm and represent it as a program.
- 4. Evaluate the solution for accuracy and its potential as a tool for solving other problems.

Getting a Foot in the Door

- Try working the problem backwards
- Solve an easier related problem
  - Relax some of the problem constraints
  - Solve pieces of the problem first (bottom up methodology)
- Stepwise refinement: Divide the problem into smaller problems (top-down methodology)

Ages of Children Problem

- Person A is charged with the task of determining the ages of B's three children.
  - B tells A that the product of the children's ages is 36.
  - A replies that another clue is required.
  - B tells A the sum of the children's ages.
  - A replies that another clue is needed.
  - B tells A that the oldest child plays the piano.
  - A tells B the ages of the three children.
- How old are the three children?

Figure 5.5

a. Triples whose product is 36

(1,1,36)	(1,6,6)
(1,2,18)	(2,2,9)
(1,3,12)	(2,3,6)
(1,4,9)	(3,3,4)

b. Sums of triples from part (a)

1 + 1 + 36 = 38	1 + 6 + 6 = 13
1 + 2 + 18 = 21	2 + 2 + 9 = 13
1 + 3 + 12 = 16	2 + 3 + 6 = 11
1 + 4 + 9 = 14	3 + 3 + 4 = 10

Racing Problem

- Before A, B, C, and D ran a race they made the following predictions:
  - A predicted that B would win.
  - B predicted that D would be last.
  - C predicted that A would be third.
  - D predicted that A's prediction would be correct.
- Only one of these predictions was true, and this was the prediction made by the winner. In what order did A, B, C, and D finish the race?

Algorithm Representation

- Algorithm and its representation are like a story and its book
- Requires well-defined primitives
- A collection of primitives constitutes a programming language.
- Syntax: the primitive's symbolic representation
- Semantics: the meaning of the primitive

Figure 5.2 **Folding a bird from a square piece of paper**

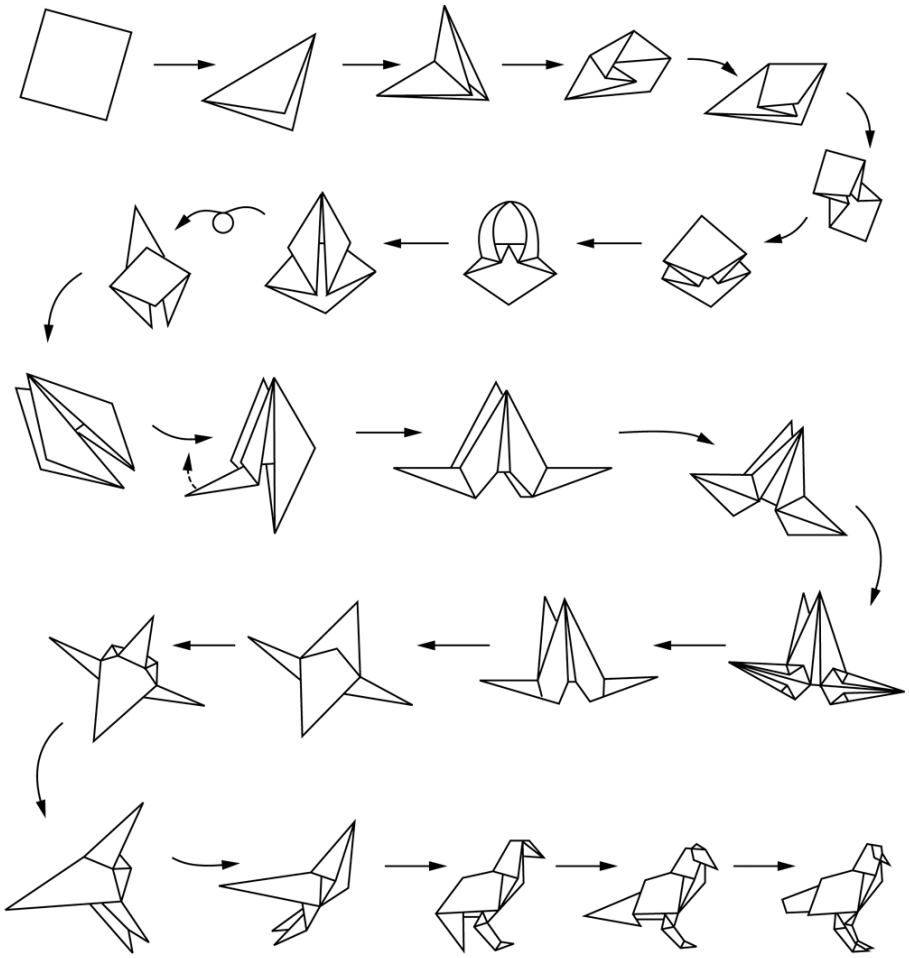
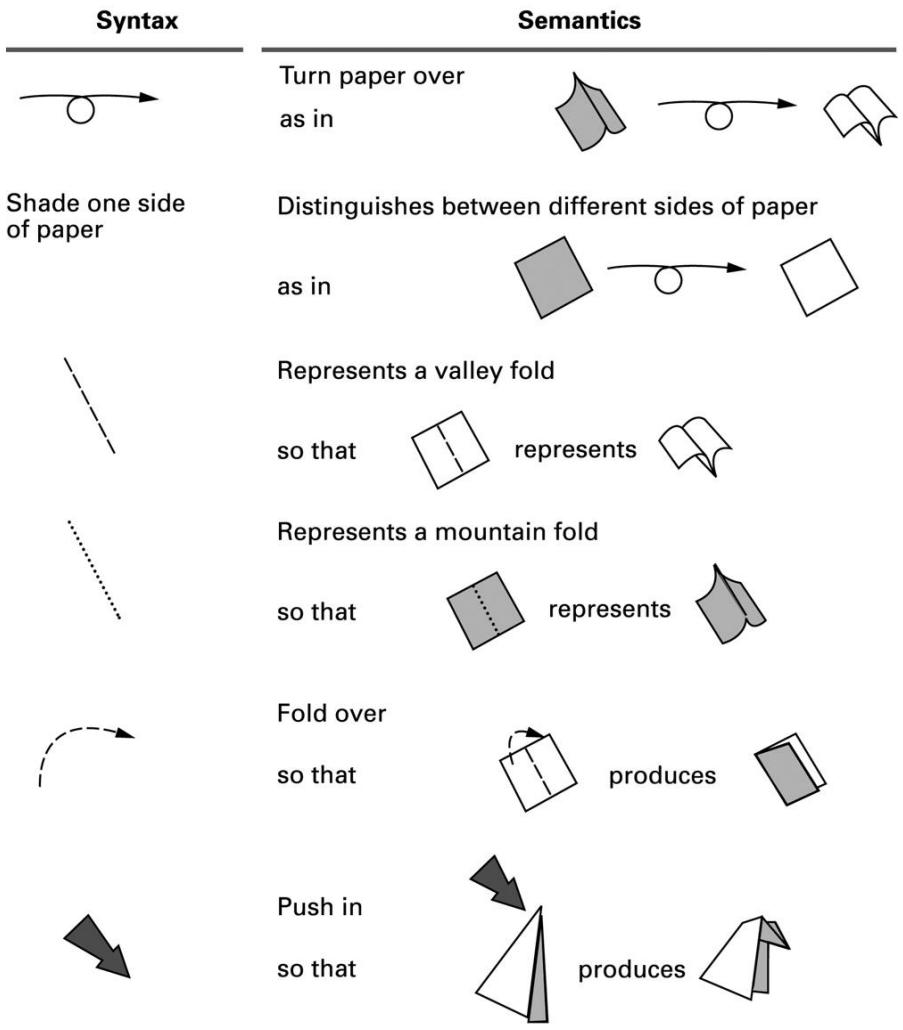


Figure 5.3 **Origami primitives**

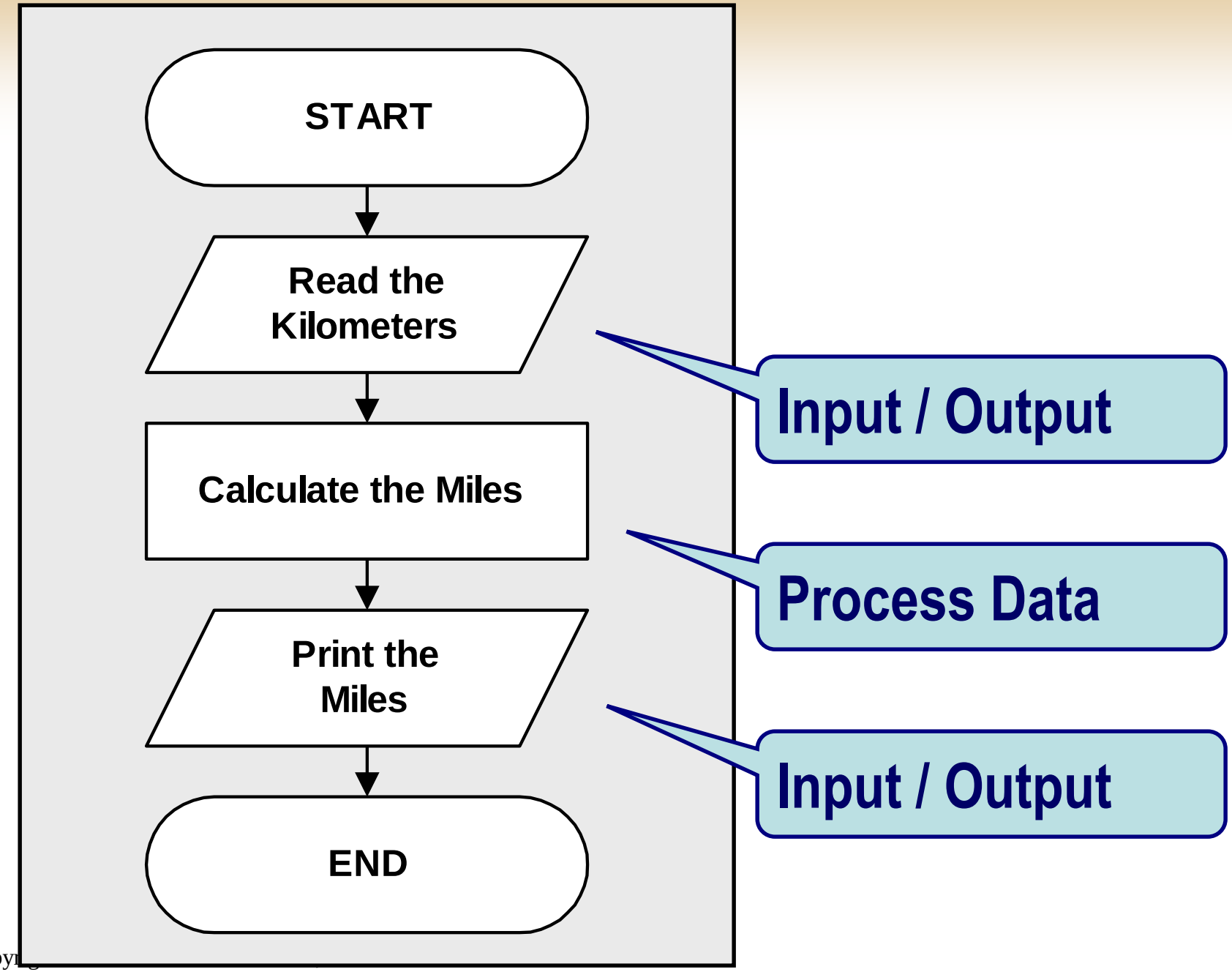


Algorithm Representation

- Two types of representation:
  - Flowchart: Graphical representation
  - Pseudocode: Text based representation

Flow Chart Overview

- Graphical representation
  - each step is a *shape* (box, circle, …)
  - useful for conceptualizing an *algorithm*
  - easy to understand and visualize
- Used to document how an algorithm was designed

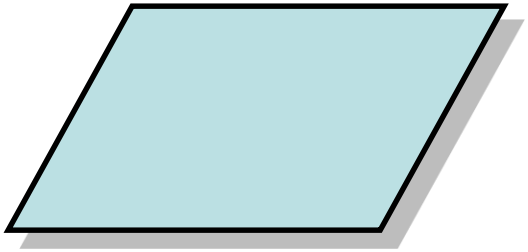

START
Read the Kilometers
Calculate the Miles
Print the Miles
END
Input / Output
Process Data
Input / Output

Start / End

- Indicates the start and end of an algorithm
- Represented by a rectangle with rounded sides
- There are typically two:
  - one to start the flowchart
  - one to end the flowchart

Input / Output

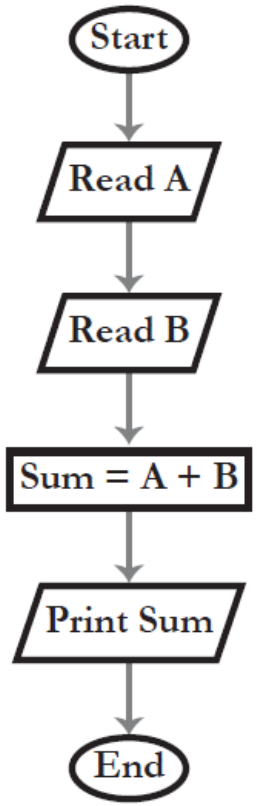
- Indicates data being:
  - inputted into the computer
  - outputted to the user
- Represented by a parallelogram
- Flowcharts can have many of this shape



Processes

- Indicates data:
  - being processed
  - also called "calculations"
- Represented by a rectangle
- The most common shape in a flowchart

Flowchart - How to find sum of two numbers



Finding sum of 845 and 247

Start

|

A= 845

|

B= 247

|

Sum= 845+ 247

|

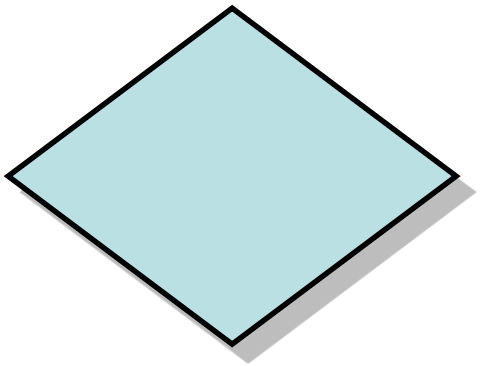
Sum= 1092

|

End

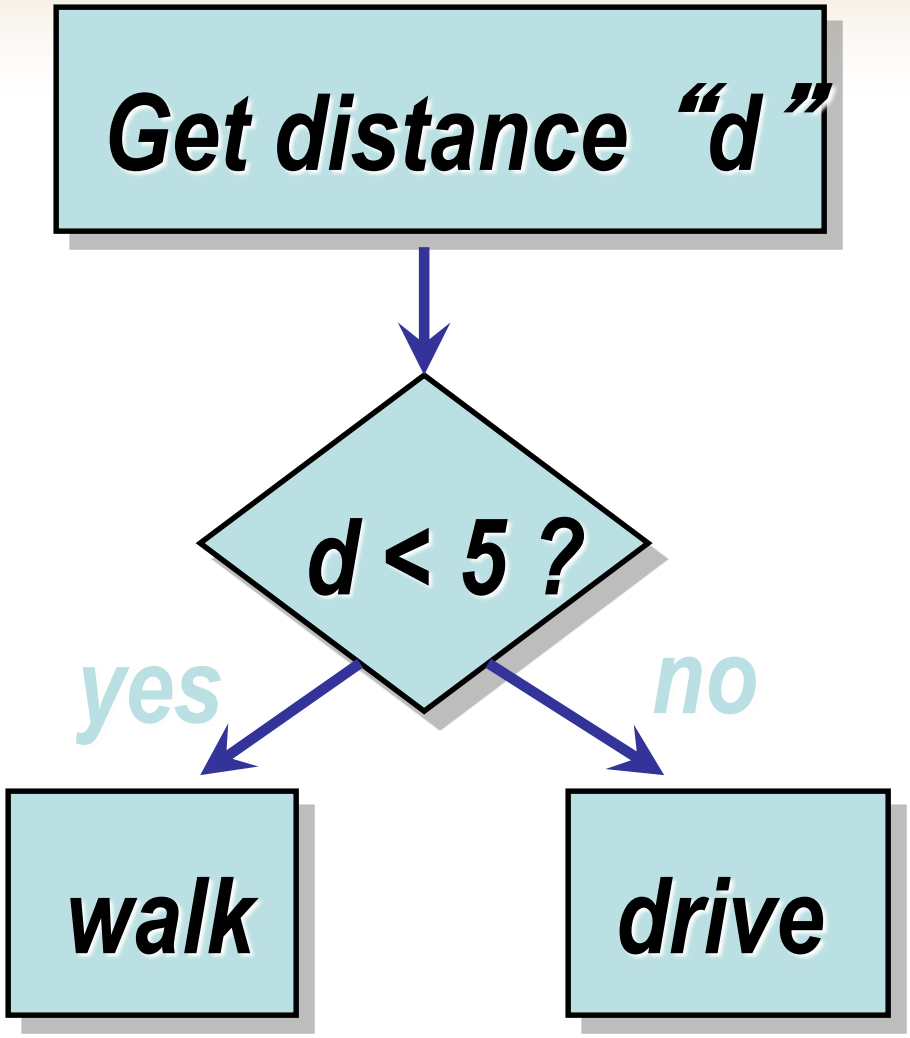
Decisions

- Indicates a conditional branch
  - describe a condition or a question
  - Has more than one outgoing branch, depending on the outcomes to the condition/question
- Represented by a diamond



Comparison Operators

Mathematical	Computer	Explanation
=	=	equal to
≠	<>	not equal to
>	>	greater than
<	<	smaller than
≥	>=	greater or equal to
≤	<=	smaller or equal to



Pseudocode

- It is a system consisting of signs and numbers, describing the algorithm steps in an informal language.
- Algorithm steps is described by daily language similar to a programming language.

Pseudocode Primitives

- Assignment

```
name = expression
```

- Example

```
RemainingFunds = CheckingBalance +
                                 SavingsBalance
```

Pseudocode Primitives (continued)

- Conditional selection

```
if (condition):
    activity
```

- Example

```
if (sales have decreased):
    lower the price by 5%
```

Pseudocode Primitives (continued)

- Conditional selection

```
if (condition):
    activity
else:
    activity
```

- Example

```
if (year is leap year):
    daily total = total / 366
else:
    daily total = total / 365
```

Pseudocode Primitives (continued)

- Indentation shows **nested** conditions

```
if (not raining):
    if (temperature == hot):
        go swimming
    else:
        play golf
else:
    watch television
```

Pseudocode Primitives (continued)

- Repeated execution

```
while (condition):
    body
```

- Example

```
while (tickets remain to be sold):
    sell a ticket
```

Pseudocode Primitives (continued)

- Define a function

```
def name():
```

- Example

```
def ProcessLoan():
```

- Executing a function

```
if (. . .):
    ProcessLoan()
else:
    RejectApplication()
```

Figure 5.4 The procedure Greetings in pseudocode

```
def Greetings():
    Count = 3
    while (Count > 0):
        print('Hello')
        Count = Count - 1
```

Pseudocode Primitives (continued)

- Using parameters

```
def Sort(List):
    .
    .
```

- Executing Sort on different lists

```
Sort(the membership list)
Sort(the wedding guest list)
```

Figure 5.6 The sequential search algorithm in pseudocode

```
def Search (List, TargetValue):
    if (List is empty):
        Declare search a failure
    else:
        Select the first entry in List to be TestEntry
        while (TargetValue > TestEntry and entries remain):
            Select the next entry in List as TestEntry
        if (TargetValue == TestEntry):
            Declare search a success
        else:
            Declare search a failure
```

Figure 5.7 Components of repetitive control

Initialize: Establish an initial state that will be modified toward the termination condition

Test: Compare the current state to the termination condition and terminate the repetition if equal

Modify: Change the state in such a way that it moves toward the termination condition

Loop Example

```
number = 1
while (number != 6)
    number = number + 2
```

- What is wrong with this loop?

Iterative Structures

- Pretest loop:

```
while (condition):
    body
```

- Posttest loop:

```
repeat:
    body
    until(condition)
```

Figure 5.8 The while loop structure

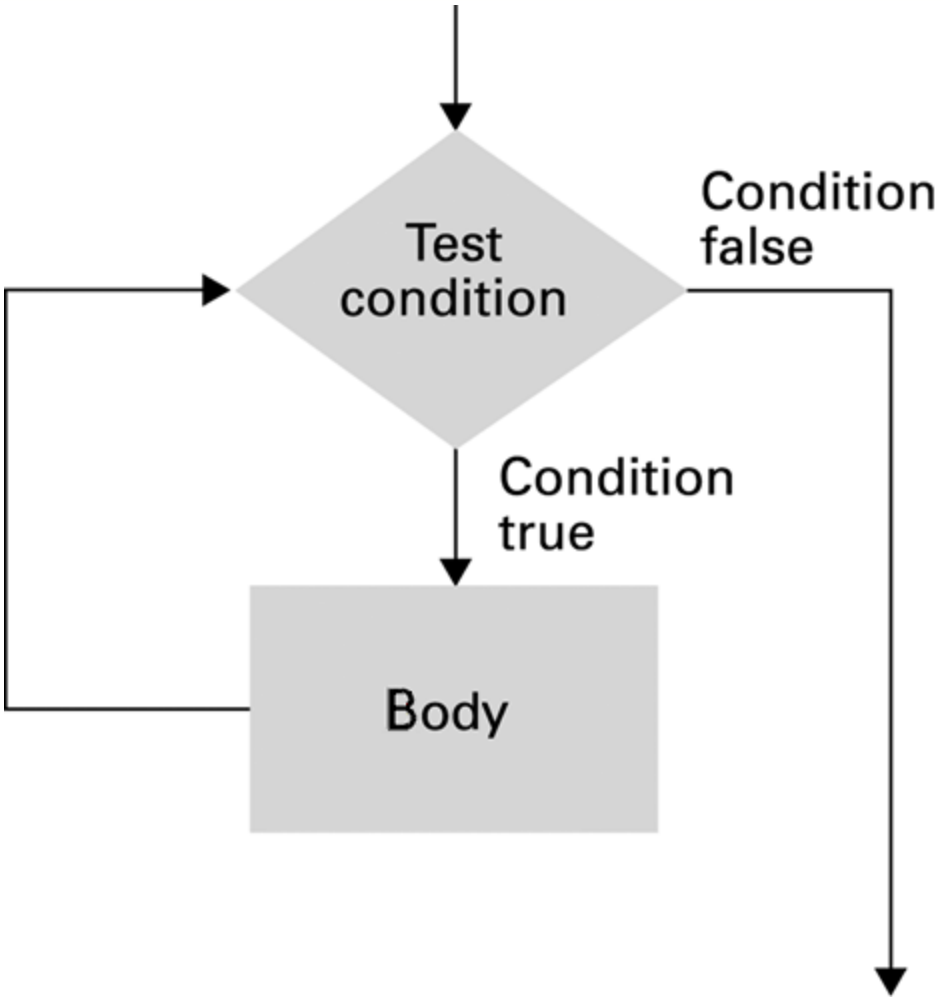


Figure 5.9 The repeat loop structure

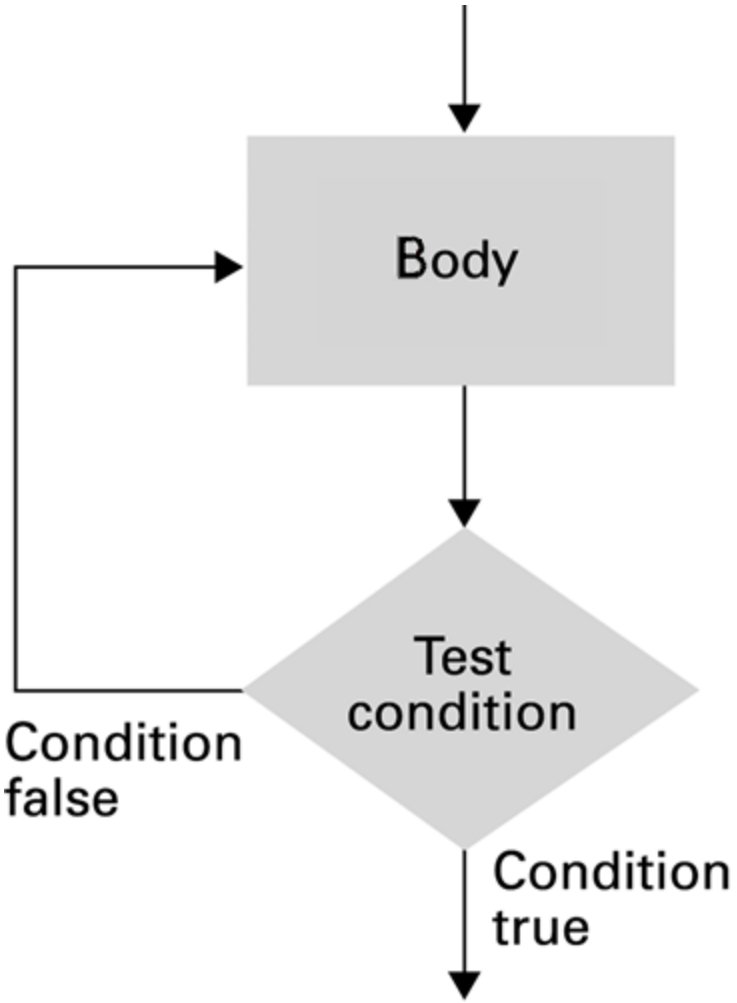
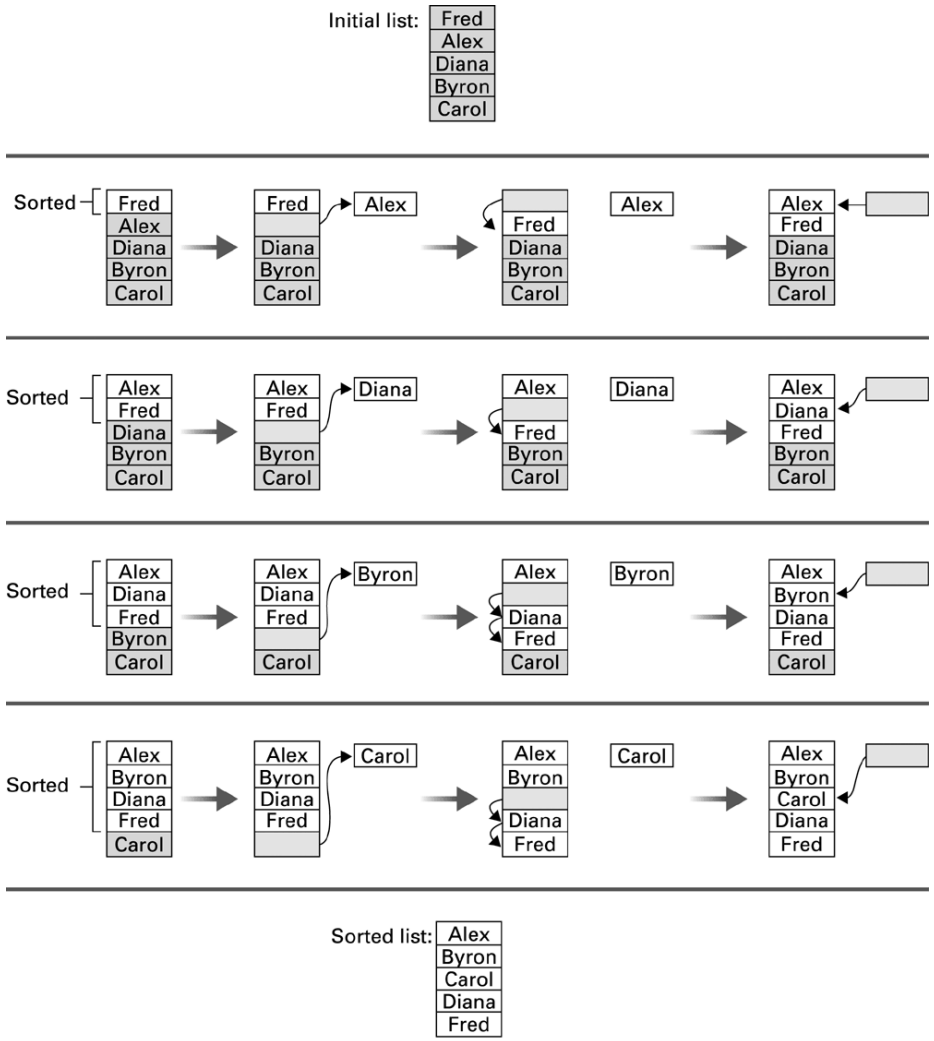


Figure 5.11 The insertion sort algorithm expressed in pseudocode

```
def Sort(List):
    N = 2
    while (N <= length of List):
        Pivot = Nth entry in List
        Remove Nth entry leaving a hole in List
        while (there is an Entry above the
                    hole and Entry > Pivot):
            Move Entry down into the hole leaving
            a hole in the list above the Entry
        Move Pivot into the hole
        N = N + 1
```

Figure 5.10 Sorting the list Fred, Alex, Diana, Byron, and Carol alphabetically



Recursion

- The execution of a procedure leads to another execution of the procedure.
- Multiple activations of the procedure are formed, all but one of which are waiting for other activations to complete.
- Self-calling functions are called recursive functions.
- Recursion is a very powerful programming technique that can replace iterations (loops, repeats).
- To solve small parts of the original problem, a subprogram invokes itself, bringing a different perspective to the solution of repetitive processes.

Recursive Structures

- Factorial Function
- For a given positive integer n, the n factorial, shown as n! and is equal to the multiplication of all integers between n to 1.
- Definition of factorial function:
  - n! = 1 if n==0
  - n! = n * (n-1) * (n-2) * ... * 1 if n>0

Iterative Definition of Factorial

- We can create an algorithm that takes the integer n and returns the n factorial value:

```
prod = 1
x=n
while (x>0)
     prod *= x
     x = x - 1
return prod;
```

- Such an algorithm is an iterative algorithm. Because there is a significant cycle or repetition that lasts until a certain condition is met.

Recursive Definition of Factorial

- The recursive definition of factorial:
  - n! = 1 if n==0
  - n! = n * (n-1)! if n>0
- This is the definition of factorial function in terms of its own kind.
- Such a definition is referred to as recursive identification because it refers to an object as simple as its own.

Figure 5.12 Applying our strategy to search a list for the entry John

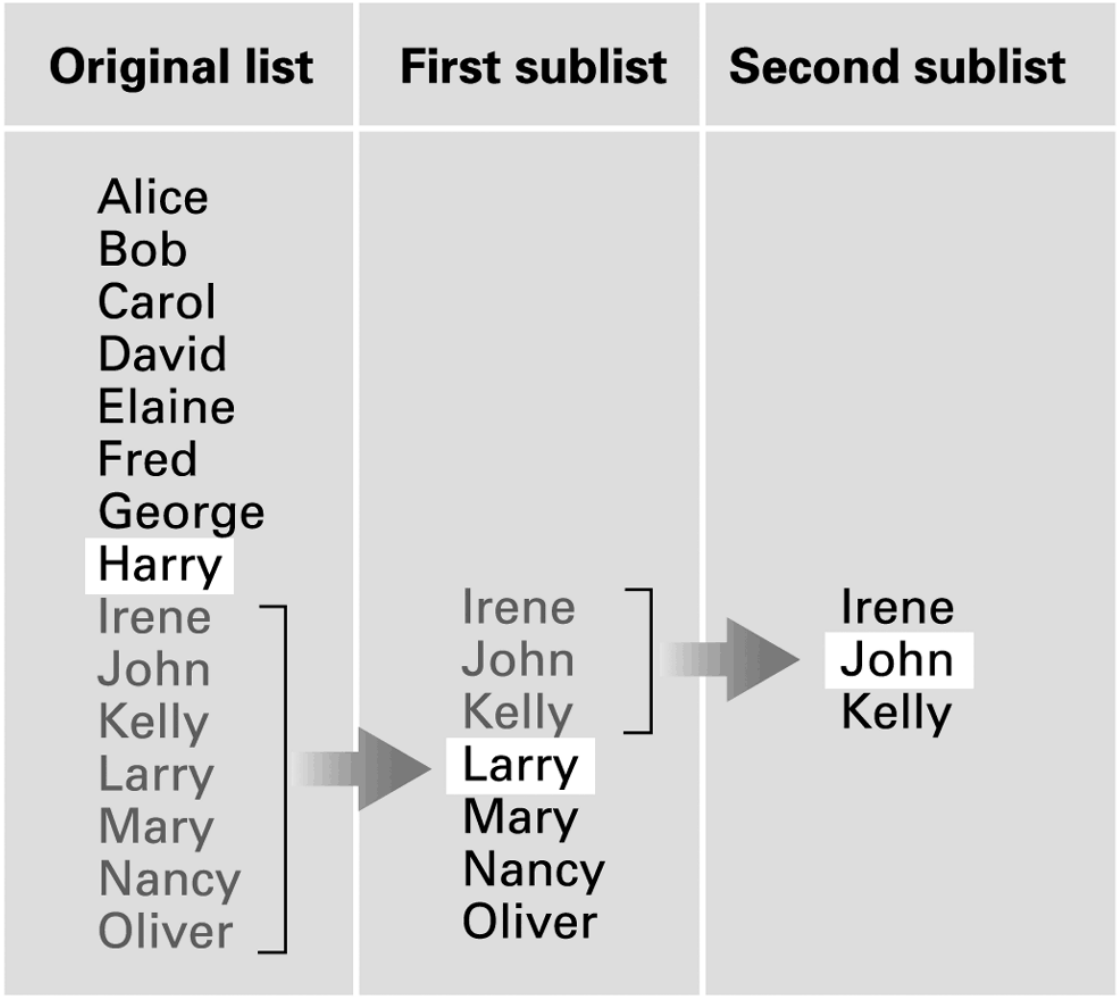


Figure 5.13 A first draft of the binary search technique

```
if (List is empty):
    Report that the search failed
else:
    TestEntry = middle entry in the List
    if (TargetValue == TestEntry):
        Report that the search succeeded
    if (TargetValue < TestEntry):
        Search the portion of List preceding TestEntry for
        TargetValue, and report the result of that search
    if (TargetValue > TestEntry):
        Search the portion of List following TestEntry for
        TargetValue, and report the result of that search
```

Figure 5.14 The binary search algorithm in pseudocode

```
def Search(List, TargetValue):
    if (List is empty):
        Report that the search failed
    else:
        TestEntry = middle entry in the List
        if (TargetValue == TestEntry):
            Report that the search succeeded
        if (TargetValue < TestEntry):
            Sublist = portion of List preceding TestEntry
            Search(Sublist, TargetValue)
        if (TargetValue > TestEntry):
            Sublist = portion of List following TestEntry
            Search(Sublist, TargetValue)
```

Figure 5.15 The First Call

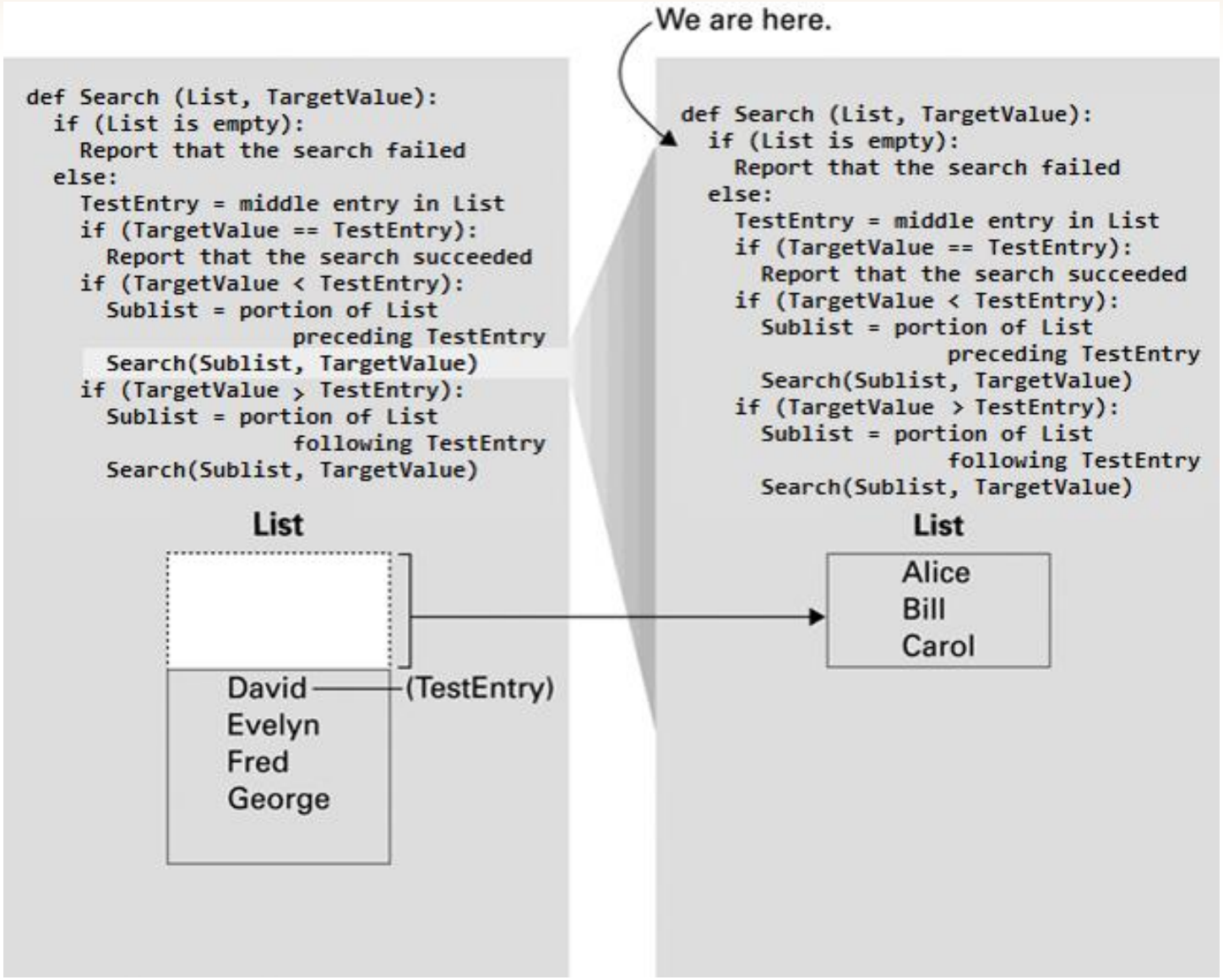


Figure 5.16 The Second Call

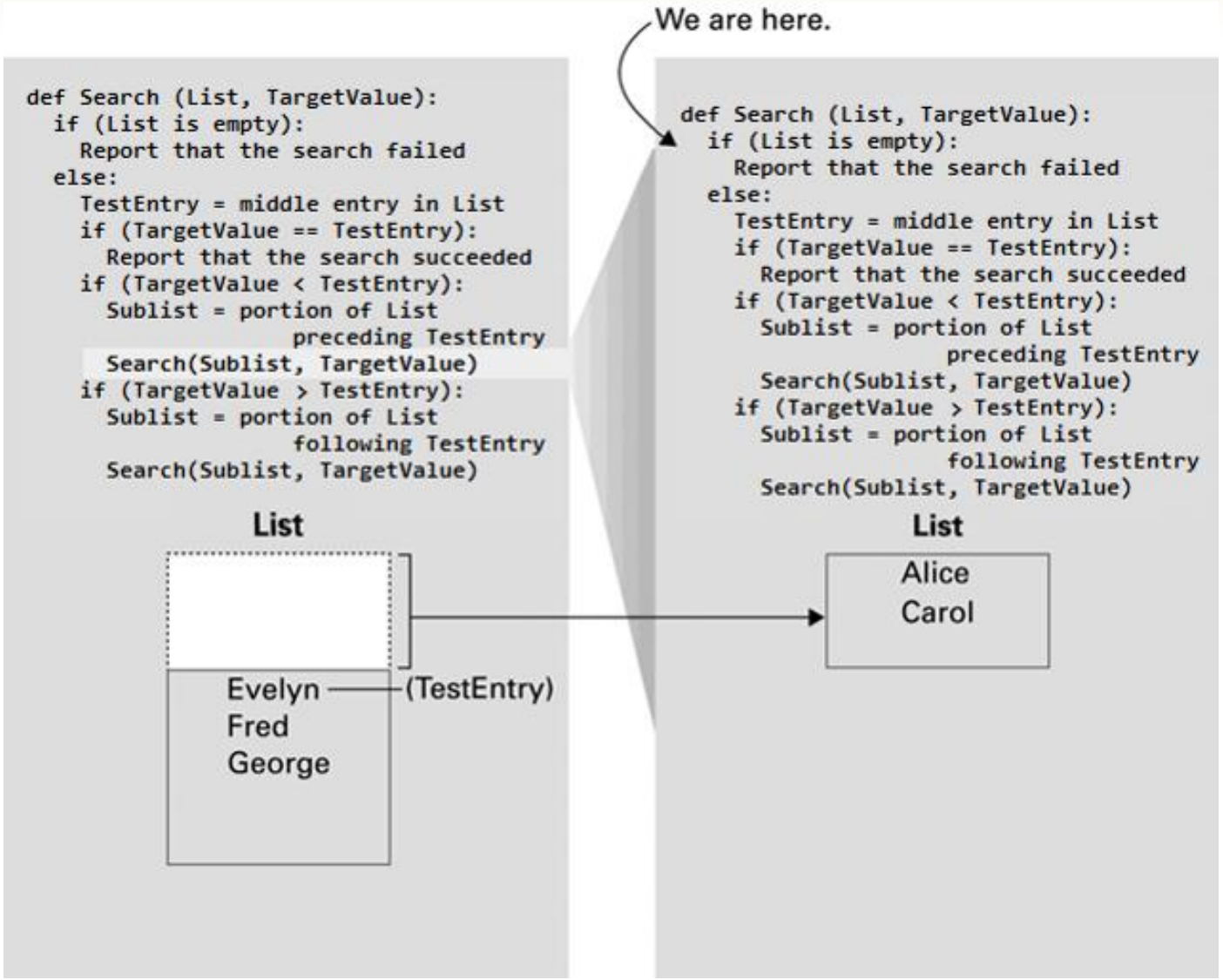
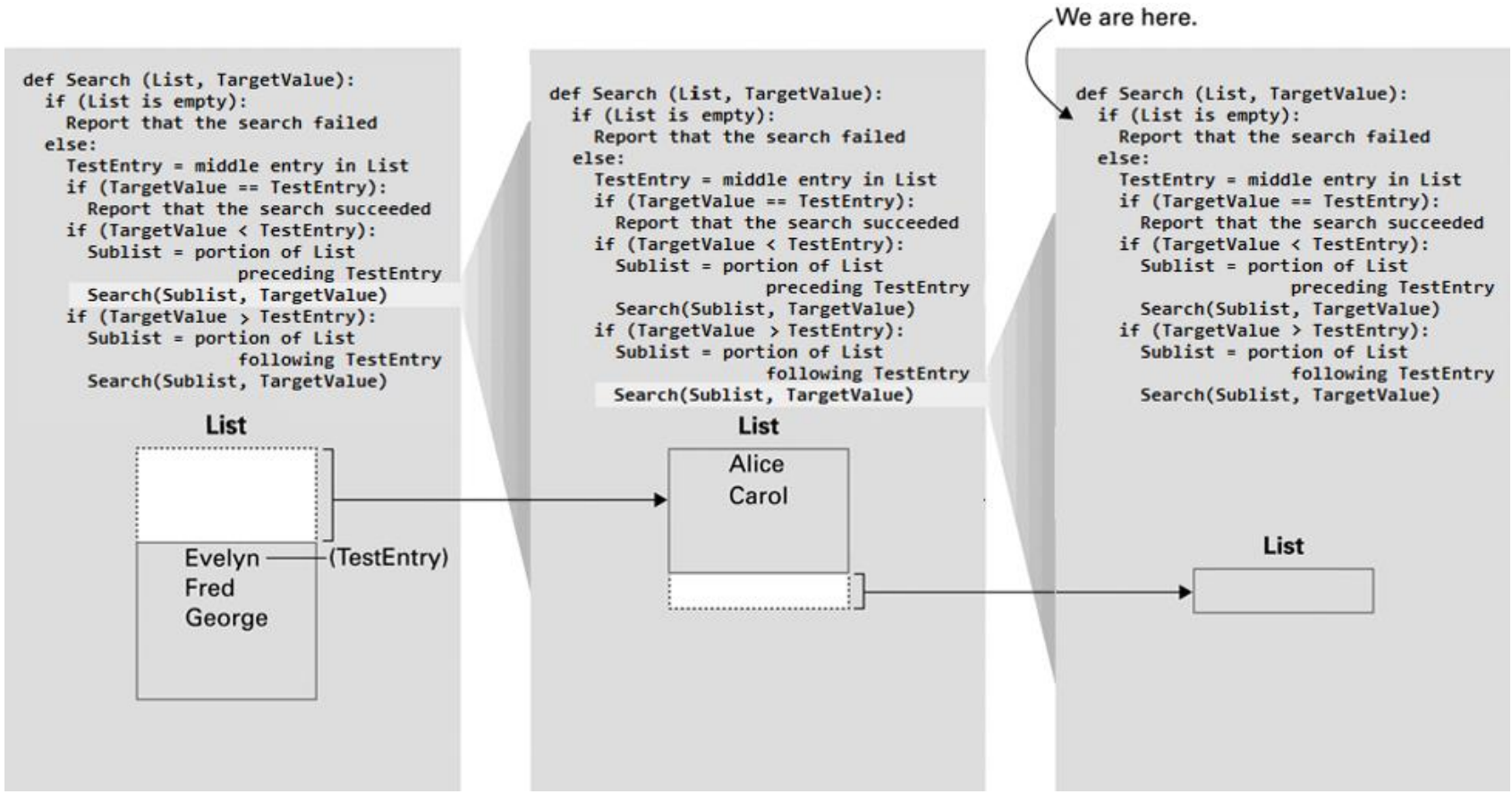


Figure 5.17 The Third Call



Algorithm Efficiency

- Measured as number of instructions executed
- Big theta notation: Used to represent efficiency classes
  - Example: Insertion sort is in $\Theta(n^2)$
- Best, worst, and average case analysis

Figure 5.18 Applying the insertion sort in a worst-case situation

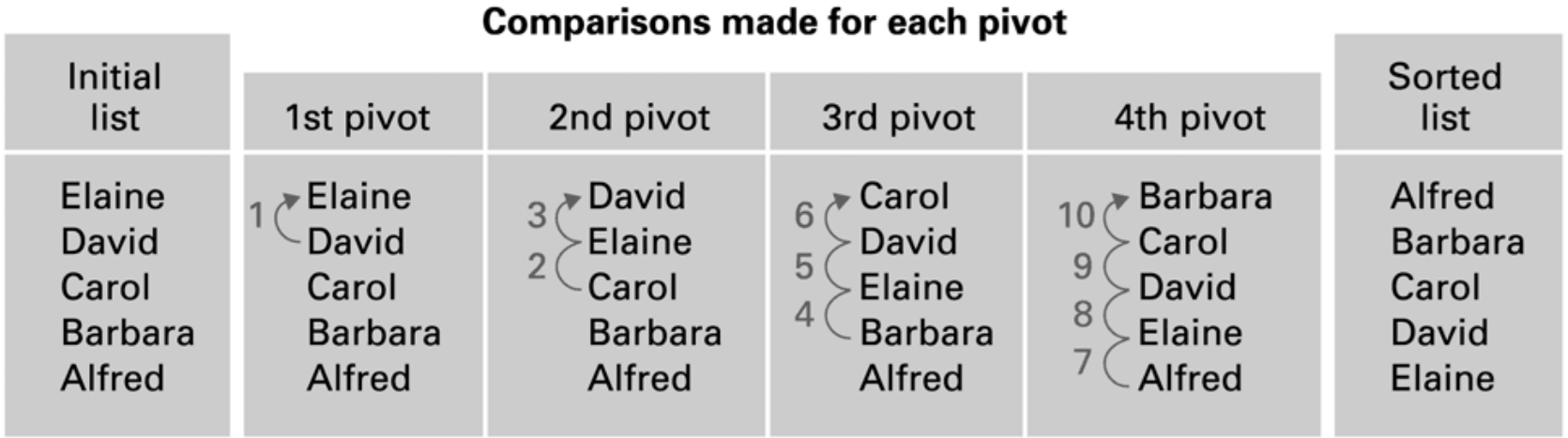


Figure 5.19 Graph of the worst-case analysis of the insertion sort algorithm

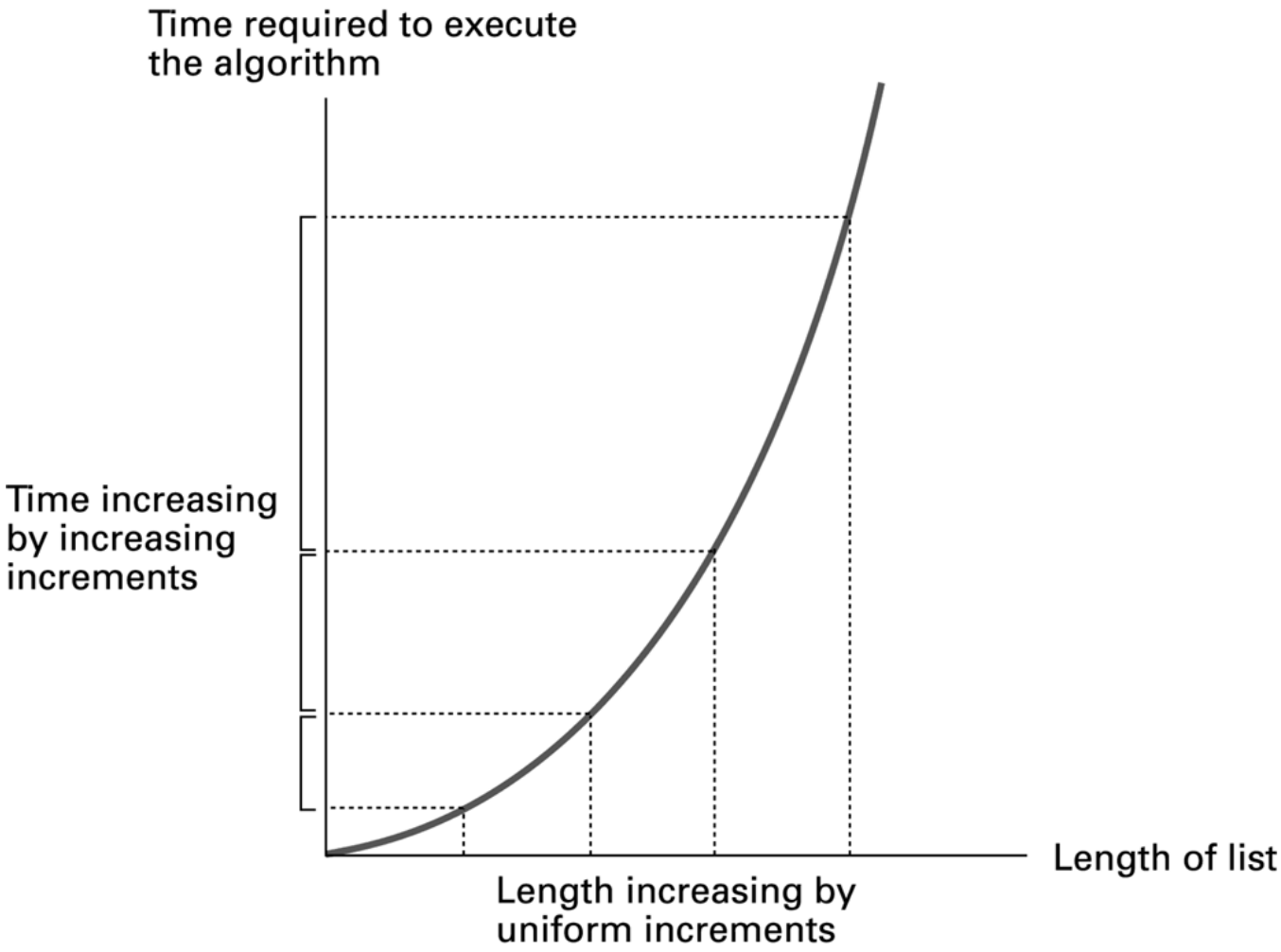
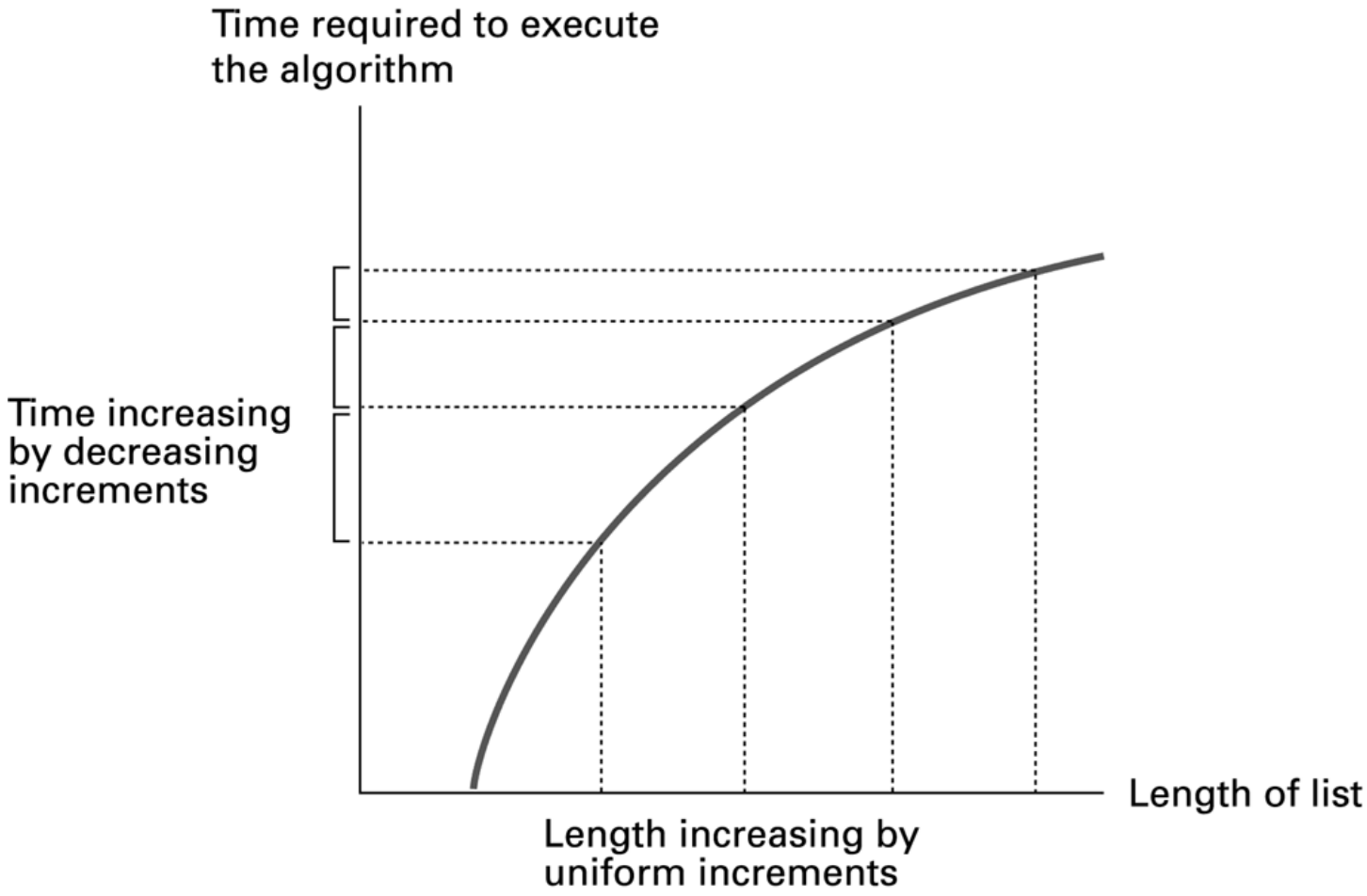


Figure 5.20 Graph of the worst-case analysis of the binary search algorithm



Software Verification

- Proof of correctness
  - Assertions
    - Preconditions
    - Loop invariants
- Test

Chain Separating Problem

- A traveler has a gold chain of seven links.
- He must stay at an isolated hotel for seven nights.
- The rent each night consists of one link from the chain.
- What is the fewest number of links that must be cut so that the traveler can pay the hotel one link of the chain each morning without paying for lodging in advance?

Figure 5.21 **Separating the chain using only three cuts**

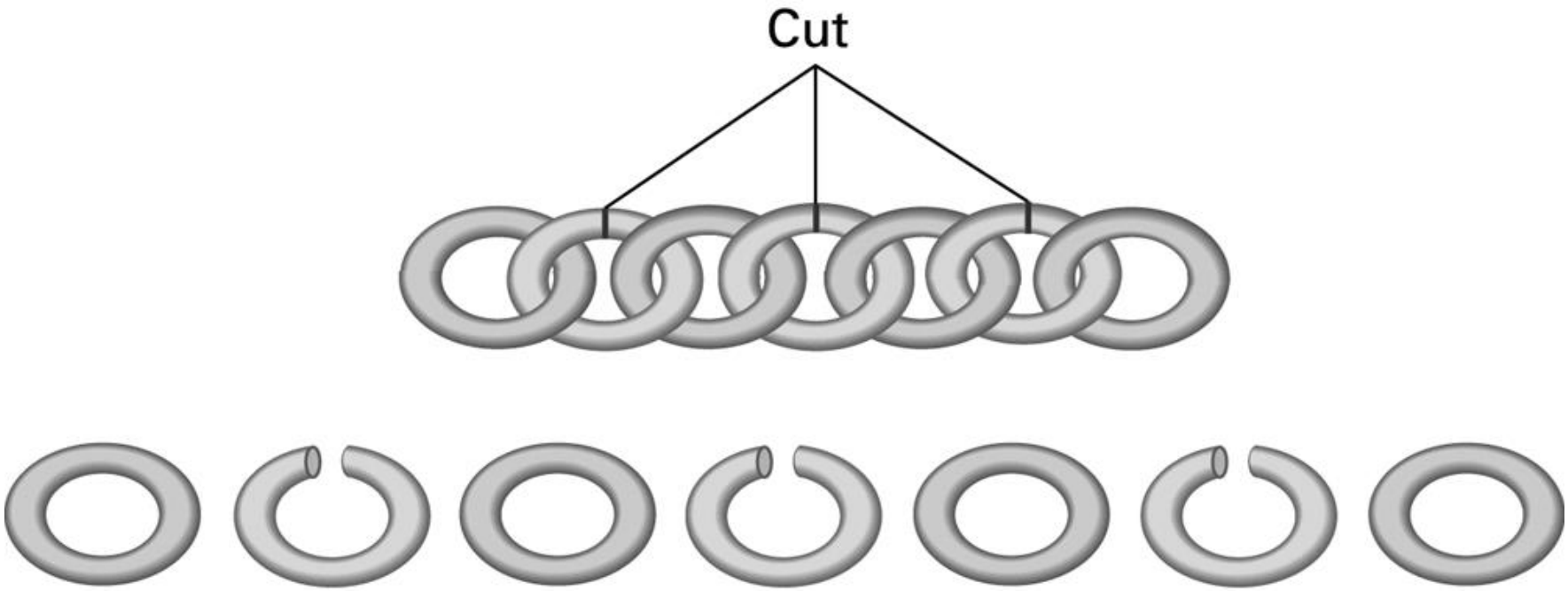


Figure 5.22 **Solving the problem with only one cut**

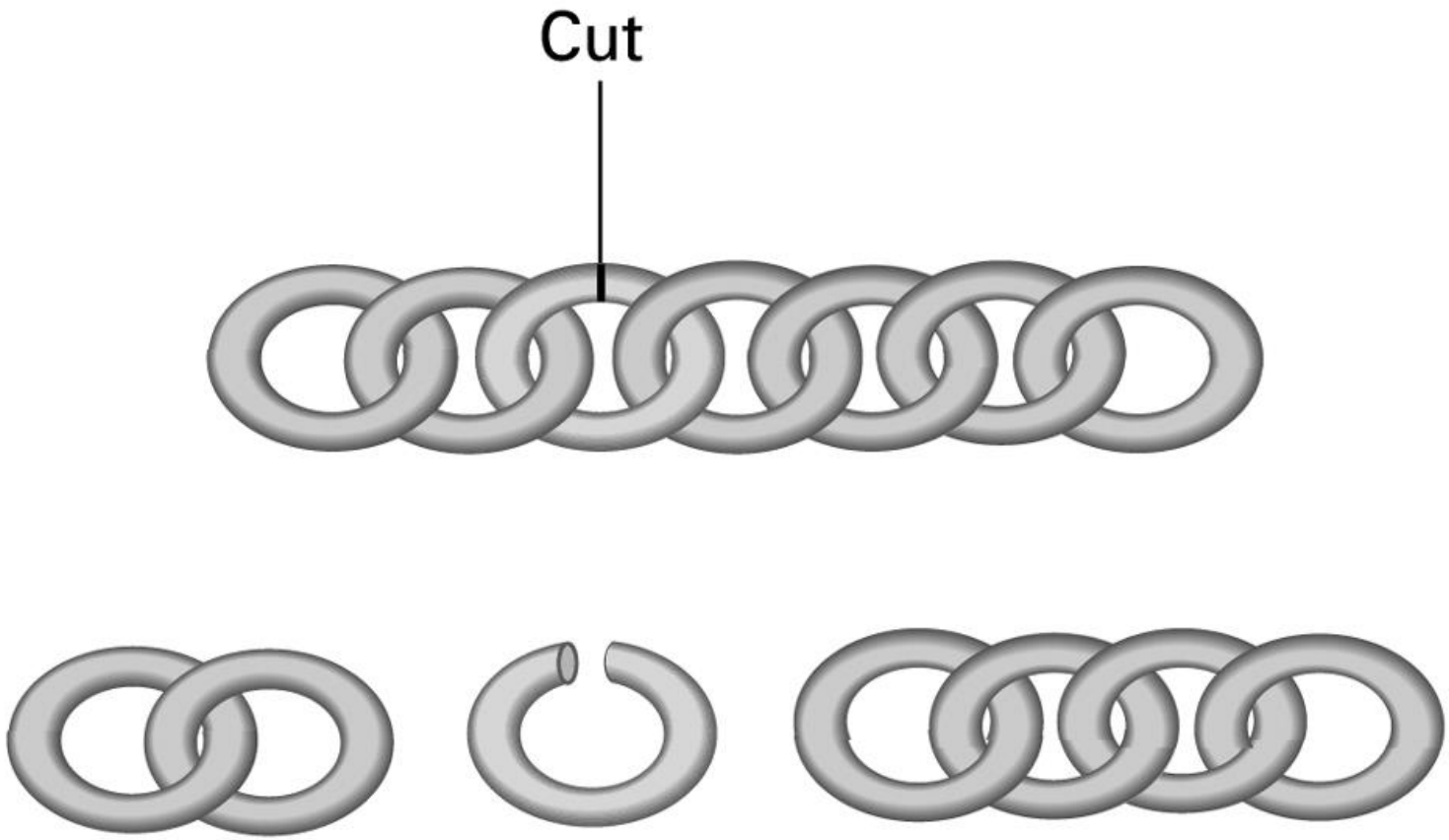


Figure 5.23 The assertions associated with a typical while structure