COM 205 - Digital Logic Design

Boolean Algebra and Logic Gates -IV

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ALKÜ

Last Week

- K-Map method
  - Two-variable
  - Three-variable

m_o	m_1
m_2	m_3

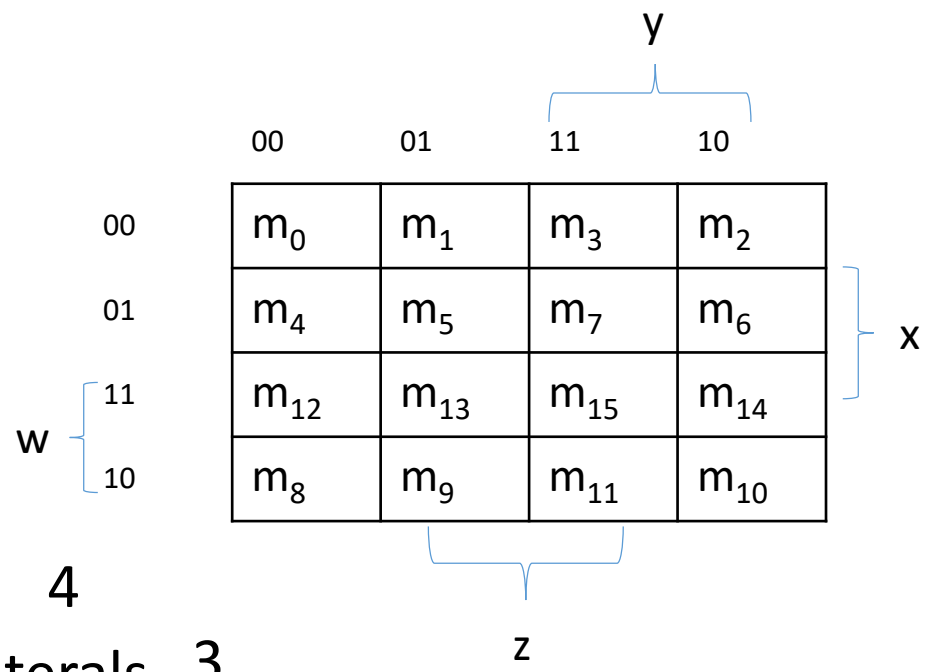
m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6

	00	01	11	10
0	x’y’z’	x’y’z	x’yz	x’yz’
1	xy’z’	xy’z	xyz	xyz’

K-Map Method

- Four-variable Map

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}



- 1 square represents a term with → ? literals 4
- 2 adjacent squares represent a term with ? literals 3
- 4 adjacent squares represent a term with 2 literals
- 8 adjacent squares represent a term with 1 literal
- 16 adjacent squares represent a function that is always 1.

K-Map

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

- Which squares are represented as :
  - x

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

w

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

  - wx

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

K-Map

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

- Which squares are represented as :
  - x’

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

wz

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

  - x’yz

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

K-Map

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

- Which squares are represented as :
  - wy’

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	**m_{13}**	m_{15}	m_{14}
m_8	**m_9**	m_{11}	m_{10}

  - x’yz

m_0	m_1	**m_3**	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	**m_{11}**	m_{10}

- Ex: Simplify the Boolean function $F(w,x,y,z)=\sum(0,1,2,4,5,6,8,9,12,13,14)$

	00	01	11	10
00	1	1		1
01	1	1		1
11	1	1		1
10	1	1		

- F = y’+w’z’+xz’

K-Map Method

- Ex: Simplify the Boolean function F(w,x,y,z)=$\sum(0,1,2,4,5,6,8,9,12,13,14)$

1	1	0	1
1	1	0	1
1	1	0	1
1	1	0	0

- F=y’+w’z’+xz’
- F’= yz+wx’y
- F = (y’+z’)(w’+x+y’)

K-Map Method

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

- Ex: Simplify the Boolean function F=A’B’C’+B’CD’+A’BCD’+AB’C’

1	1		1
			1
1	1		1

- F(A,B,C,D)=B’C’+B’D’+A’CD’

K-Map Method

- Ex: Simplify the Boolean function F=A’B’C’+B’CD’+A’BCD’+AB’C’

1	1		1
			1
1	1		1

- F=B’D’+B’C’+A’CD’

Prime Implicants

- Sometimes there may be two or more expressions that satisfy the simplification criteria.
  - All the minterms of the function must be covered when we combine the squares.
  - Number of terms in the expression is minimized.
  - There are no redundant terms.

Prime Implicants

- A prime implicant is a product term obtained by combining the maximum possible number of adjacent squares in the map.
- If a minterm in a square is covered by only one prime implicant, that prime implicant is said to be essential.
- The prime implicants of a function can be obtained from the map by combining all possible maximum number of squares.
- A single 1 on a map represents a prime implicant if it is not adjacent to any other 1's.
- Two adjacent 1's form a prime implicant, provided they are not within a group of four adjacent squares.
- Four adjacent 1's form a prime implicant, provided they are not within a group of eight adjacent squares.
- The prime implicant is essential if it is the only prime implicant that covers the minterm.

Ex:

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

- $F(A,B,C,D)=\sum(0,2,3,5,7,8,9,10,11,13,15)$

	00	01	11	10
00	1		1	1
01		1	1	
11		1	1	
10	1	1	1	1

- What are the essential prime implicants? BD, B'D'
- What are the prime implicants? B'D', CD, BD, AB', B'C,AD
- What is the simplified function:
- F=BD+B'D'+B'C+AB'
- F=BD+B'D'+CD+AD
- F=BD+B'D'+CD+AB'
- F=BD+B'D'+B'C+AD

m0 → B'D'
m2 → B'D',B'C
m3 → CD,B'C
m5 → BD
m7 → CD,BD
m8 → B'D',AB'
m9 → AB',AD
m10 → B'D'
m11 → AB',AD,CD
m13 → BD,AB
m15 → CD,AD,BD

Ex:

- F(A,B,C,D)=$\sum(0,2,3,5,7,8,9,10,11,13,15)$
  - Due to m0 and m5 essential prime implicants are B'D' and BD

		1	1
	1	1	
	1	1	
1	1	1	1

  - Prime implicants:
    - 2 essential prime implicants cover 8 minterms. Remaining minterm m3,m9,m11
    - M3 → CD or B'C
    - M9 → AD or AB'
    - M11 → can be covered by one of the four prime implicants
  - Simplified function:
    - F=BD+B'D'+CD+AD
    - F=BD+B'D'+CD+AB'
    - F=BD+B'D'+B'C+AD
    - F=BD+B'D'+B'C+AB'

Simplified Expression

- First all essential prime implicants are determined.
- The simplified expression is obtained from the logical sum of all the essential prime implicants, plus other prime implicants that may be needed to cover any remaining minterms not covered by the essential prime implicants.
- There may be more than one way of combining squares and each combination may produce an equally simlified expression.