COM 205 - Digital Logic Design
Boolean Algebra and Logic Gates -III

Assist. Prof. Özge ÖZTİMUR KARADAĞ

ALKÜ

Last Week

- Conversion among canonical forms
- Digital Logic Gates

Example

- Simplify the functions:
  - $f = x'y+xy'+xy+x'y'$
    - $f = y(x'+x) + y'(x+x')$ (distributive law)
    - $= y + y'$
    - $= 1$
  - $F=(BC'+A'D)(AB'+CD')$
    - $=AB'BC'+BC'CD'+AA'B'D+A'CDD'$ (distributive law)
    - $=0$

Example

- Given the truth table of F
  - Find its minterms
  - Find the minterms for F’
  - Represent F in sum of products form.
  - Represent the function with minimum number of terms.

x	y	z	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Example

- Given the truth table of F
    - Find its minterms
    - Find the minterms for F'
    - Represent F in sum of products form.
    - Represent the function with minimum number of terms.

x	y	z	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

- $F(x,y,z)=\sum(2,3,6,7)$
- $F(x,y,z)=\sum(0,1,4,5)$
- $F(x,y,z)=x'yz'+x'yz+xyz'+xyz$
- $F(x,y,z)=x'y(z'+z)+xy(z'+z)=x'y+xy = y(x'+x)=y$

Gate-Level Minimization

- Task of finding an optimal gate-level implementation of the Boolean Functions
- The Map Method: Simple, straightforward procedure for minimizing Boolean functions. Karnaugh map (K-map)
  - K-map is a diagram made-up of squares, each representing one minterm of the function to be minimized.
  - The simplified expressions produces by the map are in one of the two standard forms: sum of products or product of sums.
  - It is assumed that the simplest algebraic expression is an algraic expression with a minimum number of terms and with the smallest possible number of literals in each term.
  - Simplest expression is not unique.

K-Map Method

- Two-variable Map

m_o	m_1
m_2	m_3

	0	1
0	x’y’	x’y
1	xy’	xy

- Representation of functions in the map:

	1

F=xy

	1
1	1

F=x+y

$F=m_1+m_2+m_3=x'y+xy'+xy=x+y$

K-Map Method

- Three-variable map:

m_0	m_1	m_2	m_3
m_4	m_5	m_7	m_6

	00	01	11	10
0	x’y’z’	x’y’z	x’yz	x’yz’
1	xy’z’	xy’z	xyz	xyz’

- Which squares corresspond to x?

	00	01	11	10
0				
1				

K-Map Method

- Any two adjacent squares in the map differ by only one variable, which is primed in one square and unprimed in the other.
  - For example: m_5 and m_7 in two adjacent squares.
  - $m_5+m_7 = xy'z+xyz = xz(y+y')=xz$

m_0	m_1	m_2	m_3
m_4	m_5	m_7	m_6

	00	01	11	10
0	x’y’z’	x’y’z	x’yz	x’yz’
1	xy’z’	xy’z	xyz	xyz’

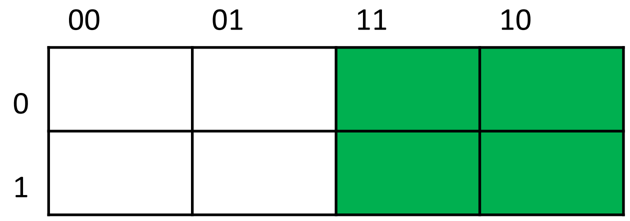
K-Map Method

- Three-variable map:

m_0	m_1	m_2	m_3
m_4	m_5	m_7	m_6

	00	01	11	10
0	x'y'z'	x'y'z	x'yz	x'yz'
1	xy'z'	xy'z	xyz	xyz'

- Which squares corresspond to y?



K-Map Method

- Three-variable map:

m_0	m_1	m_2	m_3
m_4	m_5	m_7	m_6

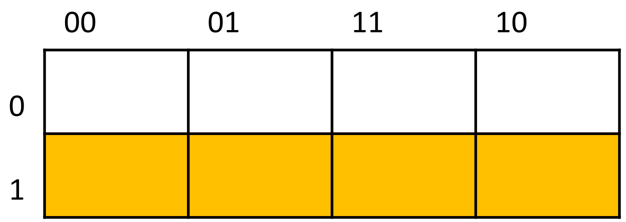
	00	01	11	10
0	x'y'z'	x'y'z	x'yz	x'yz'
1	xy'z'	xy'z	xyz	xyz'

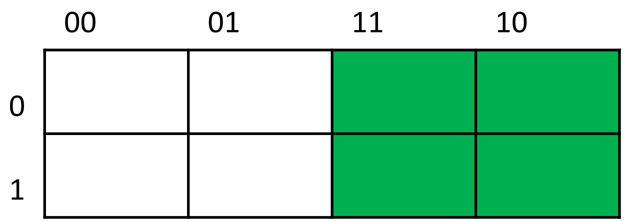
- Which squares corresspond to z?

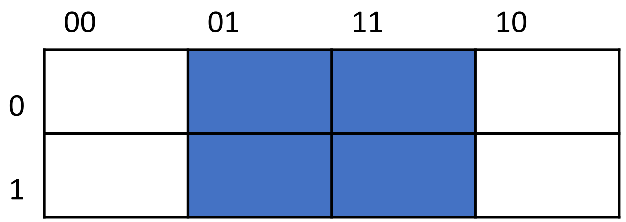
	00	01	11	10
0				
1				

K-Map Method

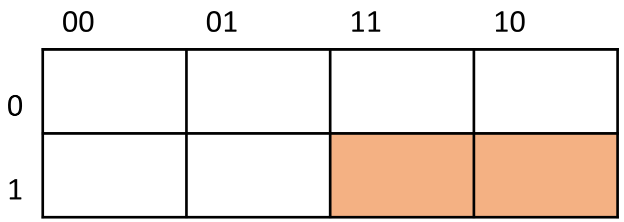
m_0	m_1	m_2	m_3
m_4	m_5	m_7	m_6





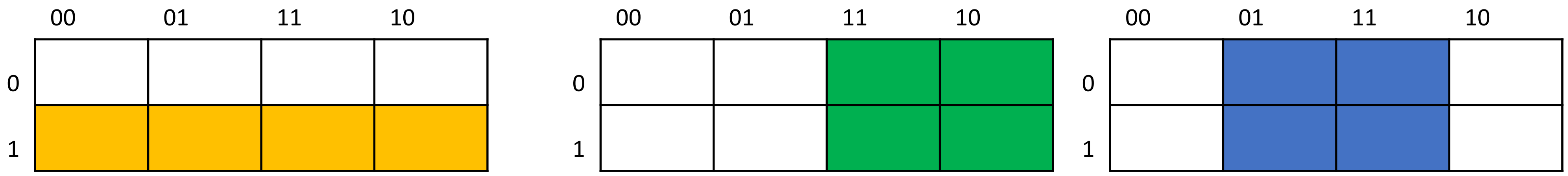


- Which squares correspond to xy?

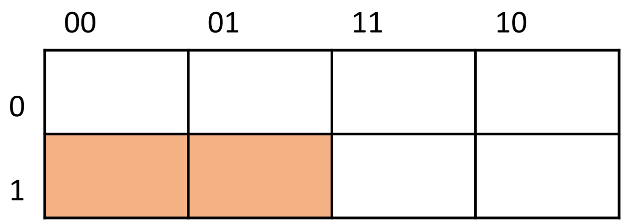


m_0	m_1	m_2	m_3
m_4	m_5	m_7	m_6

K-Map Method



- Which squares correspond to xy'?



K-Map Method

- Ex: Simplify the Boolean Function
- F(x,y,z)=$\sum(2,3,4,5)$

	00	01	11	10
0			1	1
1	1	1		

- F=xy’+

K-Map Method

- Ex: Simplify the Boolean Function
- F(x,y,z)=$\sum(2,3,4,5)$

	00	01	11	10
0			1	1
1	1	1		

- F = xy'+x'y

K-Map Method

- In certain cases, two squares in the map are considered to be adjacent even though they do not touch eachother.
  - m_0 and m_2 are adjacent $m_0+m_2 = x'y'z'+x'yz'=x'z'(y'+y)=x'z'$
  - m_4 and m_6 are adjacent $m_4+m_6 = xy'z' + xyz' = xz'(y'+y)=xz'$

m_0	m_1	m_2	m_3
m_4	m_5	m_7	m_6

	00	01	11	10
0	x'y'z'	x'y'z	x'yz	x'yz'
1	xy'z'	xy'z	xyz	xyz'

K-Map

- Ex: Simplify the Boolean Function
- F(x,y,z)=$\sum(3,4,6,7)$

	00	01	11	10
0			1	
1	1		1	1

- F=yz+xz’

K-Map

- Ex: Simplify the Boolean Function
- $F(x,y,z)=\sum(3,4,6,7)$

	00	01	11	10
0			1	
1	1		1	1

- $F=xz'+ yz$

K-Map

- The number of adjacent squares that may be combined must always represent a number that is a power of two, such as 1,2,4,8.
- As more squares are combined, we obtain a product term with fewer literals.

- One square represents one minterm → three literals
- Two squares represent a term with two literals
- Four squares represent a term with one literal
- Eight squares produce a function that is always equal to 1.

K-Map Method

- Ex: Simplify the following Boolean Function
- F(x,y,z) = $\sum(0,2,4,5,6)$

	00	01	11	10
0	1			1
1	1	1		1

F= z'y+xy‘ ???

F= z’+xy’

K-Map Method

- Ex: Simplify the following Boolean Function
- F(x,y,z) = $\sum(0,2,4,5,6)$

	00	01	11	10
0	1			1
1	1	1		1

- F = z'+xy'

K-Map Method

- Let the Boolean Function F=A’C+A’B+AB’C+BC
- a) Express this function as a sum of minterms
- b) Find the minimal sum-of-products expression

- F(A,B,C)=∑
- Obtain the K-map,
- Find the function.

	00	01	11	10
0		1	1	1
1		1	1	

- BASRİ: m1,m2,m3,m5,m6
- İrem: m2 m3 m4 m5 m6 m7

K-Map Method

- Let the Boolean Function F=A’C+A’B+AB’C+BC
- a) Express this function as a sum of minterms
- b) Find the minimal sum-of-products expression

- $F(A,B,C)=\sum(1,2,3,5,7)$

	00	01	11	10
0		1	1	1
1		1	1	

- F= C+A’B