Week 7

Theorem: Let a and b be constants and X be either discrete or a continuous random variable.

$$E(aX+b)=aE(X)+b$$

Results:

1. if a=0, then E(b)=b

2. if a=1, then E(X+b)=E(X)+b

3. if b=0, then E(a.X)=a.E(X)

4. if a=1 and b=−E(X)=−μ then $E\big(X-E(X)\big)=E(X)-E\big(E(X)\big)=\mu-\mu=0$

Definition: Let X be either a discrete or a continuous random variable. Then it has either a probability function or probability density function. $E(X)=\mu$. The variance of X is defined by

$$V(X)=\sigma^2=E((X-\mu)^2)=E(X^2)-(E(X))^2$$

Definition: The positive square root of variance is called standard deviation and denoted by σ.

$$\sigma=\sqrt{\sigma^2}$$

Ex: Let X be a discrete random variable. Find E(X), V(X), and σ.

X=x	P(X=x)	x.P(X=x)	x^2P(X=x)
0	1/8	0.(1/8)=0	0.(1/8)=0
1	3/8	1.(3/8)=3/8	1.(3/8)=3/8
2	3/8	2.(3/8)=6/8	4.(3/8)=12/8
3	1/8	3.(1/8)=3/8	9.(1/8)=9/8

$$E(X)=0+\frac{3}{8}+\frac{6}{8}+\frac{3}{8}=\frac{12}{8}$$

$$E(X^2)=0+\frac{3}{8}+\frac{12}{8}+\frac{9}{8}=\frac{24}{8}$$

$$V(X)=E(X^2)-\big(E(X)\big)^2=\frac{24}{8}-\left(\frac{12}{8}\right)^2=\frac{3}{4}$$

$$\sigma=\sqrt{\frac{3}{4}}$$

Ex. Let X be a continuous random variable whose probability density function is defined by

$$f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & dd \end{cases}$$

Find E(X), V(X) and σ

$$E(X) = \int_{-\infty}^{\infty} x.f(x)dx = \int_{0}^{1} x.1dx = \frac{x^2}{2}|_0^1 = \frac{1}{2} - 0 = \frac{1}{2}$$

$$E(X^2) = \int_{-\infty}^{\infty} x.f(x)dx = \int_{0}^{1} x^2.1dx = \frac{x^3}{3}|_0^1 = \frac{1}{3} - 0 = \frac{1}{3}$$

$$\sigma^2 = E(X) = E(X^2) - \left(E(X)\right)^2 = \frac{1}{3} - \left(\frac{1}{2}\right)^2 = \frac{1}{12}$$

$$\sigma = \sqrt{\frac{1}{12}}$$

The Properties of Variance

Theorem: 1. Let a be constant and X be either a discrete or a continuous random variable.

$$V(aX) = a^2V(X)$$

2. Let b be a constant and X be either a discrete or a continuous random variable.

$$V(X + b) = V(X)$$

Ex: Let X be a discrete random variable and its probability function is defined below.

X=x	0	1	2
P(X=x)	1/5	2/5	2/5

V(3X-3)=?

$$E(X) = 0 + \frac{2}{5} + \frac{4}{5} = \frac{6}{5}$$

$$E(X^2) = 0 + \frac{2}{5} + \frac{8}{5} = \frac{10}{5} = 2$$

$$V(X) = 2 - \frac{36}{25} = \frac{14}{25}$$

$$V(3X-3)=9V(X)=9\frac{14}{25}=\frac{126}{25}$$

Some Special Discrete Probability Distributions

Definition (Bernoulli Random Variable): Let X be a discrete random variable having two different outcomes, then X is called Bernoulli random variable.

Ex:1.Toss a coin. (Tail (T=0) or Head (H=1))

2.A jar contains M black balls and N white balls. A ball is randomly selected from the jar. (Black ball (1) or white ball (0))

3.Defective and undefective tools are stored in a storage. A tool is randomly drawn from the storage. (Defective (1) or Undefective (1))

Definition (Bernoulli Distribution): Let X be Bernoulli random variable either taking 1 or 0. Then its probability function is defined by

$$P(X=1)=p$$

$$P(X=0)=1-p=q$$

or

$$P(X=x)=p^x(1-p)^{1-x}, x=0,1$$

Theorem: Let X be a Bernoulli random variable. Its expected value and variance are defined by

$$E(X)=p$$

$$V(x)=pq$$

Definition (Binomial random Variable): N independent random Bernoulli experiments are run. Let X denote the number of successes out od n trials. P is a probability denoting the success, 1-p=q denotes the probability of failure. Then, X is called a Binomial random variable. The properties of Binomial experiment is defined as follows:

1.Experiment composes of n same trials.

2.Each trial result with 2 different outcomes, which are called success and failure.

3.Each trial has a success probability p and failure probability 1-p=q.

4.Trial are independent of each other.

Ex: Toss a coin 10 times. Let X show the number of Hs.

2.Suppose that a box contains 8 black and 4 white balls. 3 balls are randomly selected from the box. Let X show the number of black balls.

Theorem (Binomial Distribution) Suppose that n indepednet Bernoulli trials are executed. For each trial, p denotes the success probability and 1-p denotes the failure probability. Let X denote the number of successes out of n trial. Then, binomial distribution of X is defined by

$$\mathrm{P}(X = x) = \binom{n}{x} p^x (1-p)^{n-x},\ x = 0,1,2,\dots,n$$

Theorem: Let X be a binomial random variable. The expected value and variance of binomial is defined by

$$E(X) = np$$

$$V(x) = npq$$

Ex. Toss a coin 4 times.

a.2Hs

b.At least one H

c.At most one H

d.E(x) and V(x)

a.$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x},\ x = 0,1,2,\dots,n$

$$P(X = 2) = \binom{4}{2} (\frac{1}{2})^2 (1 - \frac{1}{2})^{4-2} = \binom{4}{2} (\frac{1}{2})^2 (\frac{1}{2})^2$$

b.$P(X \geq 1) = P(X = 1) + P(X = 2) + P(x = 3) + P(X = 4)$

or

$$1 - P(X < 1) = 1 - P(X = 0) = 1 - \binom{4}{0} (\frac{1}{2})^0 (\frac{1}{2})^4$$

c.$P(X \leq 1) = P(X = 0) + P(X = 1) = \binom{4}{0} (\frac{1}{2})^0 (\frac{1}{2})^4 + \binom{4}{1} (\frac{1}{2})^1 (\frac{1}{2})^3$

d.$E(X) = np = 4\frac{1}{2} = 2$ ve $V(X == npq = 4\frac{1}{2}\frac{1}{2} = 1$