Week 3

Ex: Suppose that a pair of dice is cast. Let A and B be two events. A is defined as a summation greater than 9 and B is defined as a summation greater than 7.

A={(5,5),(6,4),(4,6),(5,6),(6,5),(6,6)}

B={(4,4),(2,6),(6,2),(3,5),(5,3),(3,6),(6,3),(5,4),(4,5), (4,6), (6,4),(5,5),(5,6),(6,5),(6,6)}

$$A \subset B \Leftrightarrow P(A) < P(B)$$

Theorem: Let A be any event denoted by A ⊂S, $P(A) \leq 1$

Ex: Suppose that a pair of dice is cast. Let A be an event. A is defined as a summation greater than 9.

A={(5,5),(6,4),(4,6),(5,6),(6,5),(6,6)}

$$P(A) = \frac{6}{36} \leq 1$$

Theorem: Let A and A^C be events in S. A^C is called the complement of A. $P(A^C) = 1 - P(A)$

Ex: Suppose that a pair of dice is cast. Let A be an event. A is defined as a summation greater than 7. What is the probability of $P(A^C)$?

A={(4,4),(2,6),(6,2),(3,5),(5,3),(3,6),(6,3),(5,4),(4,5), (4,6), (6,4),(5,5),(5,6),(6,5),(6,6)}

$$P(A^C) = 1 - P(A) = 1 - \frac{15}{36} = \frac{21}{36}$$

Theorem: $P(\emptyset) = 0 \; and \; P(S) = 1$

Ex:$(\emptyset)^C = S$

P(S)=1

$$P(\emptyset) = 1 - P(\emptyset^C) = 1 - P(S) = 1 - 1 = 0$$

Theorem: Let A and B be two events in sample space S.

$$P(A) = P(A \cap B) + P(A \cap B^C)$$

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B={(4,4),(2,6),(6,2),(3,5),(5,3),(3,6),(6,3),(5,4),(4,5), (4,6), (6,4),(5,5),(5,6),(6,5),(6,6)}

$$B^C = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$$

$$(2,1), (2,2), (2,3), (2,4), (2,5), (3,1), (3,2), (3,3), (3,4),$$

(4,1), (4,2), (4,3), (5,1),(5,2),(6,1)}

$$P(A) = \frac{6}{36}$$

$$P(A \cap B) = \frac{6}{36}$$

$$P(A \cap B^C) = P(\emptyset) = 0$$

$$P(A) = P(A \cap B) + P(A \cap B^C)$$

$$\frac{6}{36} = \frac{6}{36} + 0$$

Theorem: Let A and B be two events in S.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Ex: Suppose that a pair dice is cast. What is the probability of getting 7 or 10?

A= {(1,6), (6,1), (2,5),(5,2),(3,4),(4,3)}

B= {(4,6), (64), (5,5)}

Since A and B are disjoint $A \cap B = \emptyset$

$$P(A \cup B) = P(A) + P(B) = \frac{6}{36} + \frac{3}{36} = \frac{9}{36}$$

Ex: Suppose that a pair of dice is cast. Let A and B be two events. A is defined as a summation of 4. B is defined as observing same numbers in a pair. What is the probability of getting summation of 4 or observing the same number in pairs?

A={(1,3),(3,1),(2,2)}

B={(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)}

$$A \cap B = \{(2,2)\}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{3}{36} + \frac{6}{36} - \frac{1}{36} = \frac{8}{36}$$

Theorem: Let A_is be events in S where i=1,2,…,n.

$$P\left(\coprod_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$$

Remark: When three events exist, the probability of union of three events can be written as follows:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) +$$

$$P(A \cap B \cap C)$$

Conditional Probability

Definition: Let A and B be two events in S. Assuming that the occurrence of event A is affected by an event B or vice versa so are the probabilities too. Given that event B is occurred, the conditional probability of A is defined as follows:

$P(A|B) = \frac{P(A \cap B)}{P(B)},\ P(B) > 0$

Similarly,

$P(B|A) = \frac{P(A \cap B)}{P(A)},\ P(A) > 0$

Since P(A∩B)=P(B∩A)

$$P(A \cap B) = P(B)P(A|B) = P(A)P(B|A)$$

Theorem: Let A_i s be events in S.

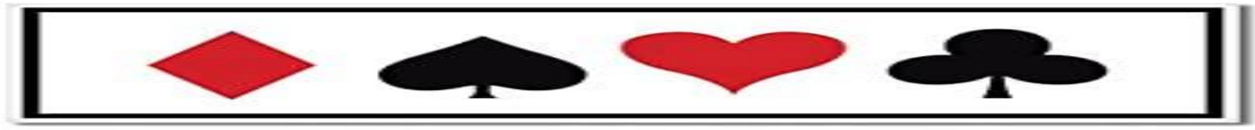
$$P(A_1 \cap A_2 \cap \ldots \cap A_n) = P(A_1)P(A_2|A_1)P(A_3|A_1 \cap A_2) \ldots P(A_n|A_1 \cap A_2 \cap \ldots \cap A_{n-1})$$

Ex: A factory produces some tools that are classified as either defective or undefective. The production of 50 tools are stored in a box where 40 of them is indefective and 10 of them is defectives. Two tools are randomly chosen from this box without putting the selected one into box. What is the probability of getting two defective ones from this box.

A_1: the first tool defective and A_2: the second tool defective

$$P(Choosing\ two\ defective) = P(A_1)P(A_2|A_1) = \frac{10}{50}\frac{9}{49}$$

Ex: Three cards are chosen to form a deck of 52 cards without putting the selected one into deck. What is the probability of getting 3 aces (number 1) (Hint: four types of cards (diamond, club, heart, spade ranging from 1 to 10 and queen, king and jacks



$$P(Choosing\ three\ aces) = P(A_1)P(A_2|A_1)P(A_3|A_1 \cap A_2) = \frac{4}{52}\frac{3}{51}\frac{2}{50}$$

Ex: Ex: Suppose that a pair of dice is cast. Let A and B be two events. A is defined as a summation of 4. B is defined as summation is even number. Given that summation is even, what is the probability of getting 4 with odd number pairs

A={(1,3),(3,1),(2,2)}

B={(1,1),(2,2),(3,3),(4,4),(5,5),(6,6),(1,3),(1,5),(2,4),(2,6),(3,1),(3,5),(4,2),(2,4),(6,2)

(4,6),(5,1),(5,3),(6,4),(6,6)}

Then,

C={(1,3),(3,1)}

$$P(C|B) = \frac{P(B \cap C)}{P(B)} = \frac{2/36}{20/36} = \frac{2}{20}$$