

$$X = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 0 & 1 & 2 \end{bmatrix}$$

$$X^T = \begin{bmatrix} 1 & 5 & 9 \\ 2 & 6 & 0 \\ 3 & 7 & 1 \\ 4 & 8 & 2 \end{bmatrix}$$

$$XX^T = \begin{bmatrix} 30 & 70 & 20 \\ 70 & 174 & 68 \\ 20 & 68 & 86 \end{bmatrix}$$

$$X^T X = \begin{bmatrix} 104 & 32 & 47 & 62 \\ 32 & 40 & 48 & 56 \\ 47 & 48 & 59 & 70 \\ 62 & 56 & 70 & 84 \end{bmatrix}$$

$$\det(XX^T - \lambda I) = 0$$

$$\begin{bmatrix} 30-\lambda & 70 & 20 \\ 70 & 174-\lambda & 68 \\ 20 & 68 & 86-\lambda \end{bmatrix} = 0$$

$$(30-\lambda) \begin{bmatrix} 174-\lambda & 68 \\ 68 & 86-\lambda \end{bmatrix} - 70 \begin{bmatrix} 40 & 68 \\ 20 & 86-\lambda \end{bmatrix} + 20 \begin{bmatrix} 40 & 174-\lambda \\ 20 & 68 \end{bmatrix} =$$

$$= (30-\lambda) ((174-\lambda)(86-\lambda) - 68^2) - 70(40(86-\lambda) - 1360)$$

$$+ 20(70 \times 68 - 3480 + 20\lambda) = (30-\lambda)(10340 - 260\lambda + \lambda^2)$$

$$- 70(4660 - 40\lambda) + 20(1280 + 20\lambda) = 310200 - 18140\lambda$$

$$+ 280\lambda^2 - \lambda^3 - 300600 + 5300\lambda = 9600 - 12840\lambda$$

$$+ 280\lambda^2 - \lambda^3 = 0$$

$$\lambda^3 - 280\lambda^2 + 12840\lambda - 9600 = 0$$

$$\lambda_0 = 0.7607 \quad \lambda_1 = 53.5435 \quad \lambda_2 = 235.6958$$

$$\lambda_4 = 0$$

$$A = 235.6958$$

Eigenvalues are rounded
up to 4 decimal numbers.

$$b_1 = \sqrt{235.6958} = 15.3524$$

$$b_2 = \sqrt{53.5435} = 7.3173$$

$$b_3 = \sqrt{0.7607} = 0.8722$$

$$\Sigma = \begin{bmatrix} 15.3524 & 0 & 0 & 0 \\ 0 & 7.3173 & 0 & 0 \\ 0 & 0 & 0.8722 & 0 \end{bmatrix}$$