Computation & optimization for Lasso - part 2

Luyang Han & Janosch Ott

ETH Zürich

22 October 2018

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Overview

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1. Coordinate Descent
2. A Simulation Study
3. Least Angle Regression
4. ADMM
5. Min-Max Algorithms
6. Alternating Minimizations
7. Screening Rules

Alternating Direction Method of Multipliers (ADMM)

Problem

$$\underset{\beta\in\mathbb{R}^m,\theta\in\mathbb{R}^n}{\text{minimize}}\, f(\beta)+g(\theta) \quad \text{subject to } \mathbf{A}\beta+\mathbf{B}\theta=c$$

Lagrangian

$$f(\beta)+g(\theta)+\rho||\mathbf{A}\beta+\mathbf{B}\theta-c||_2^2$$

Augmented Lagrangian

$$L_\rho(\beta,\theta,\mu) := f(\beta)+g(\theta)+\langle\mu,\mathbf{A}\beta+\mathbf{B}\theta-c\rangle+\frac{\rho}{2}||\mathbf{A}\beta+\mathbf{B}\theta-c||_2^2$$

Augmented: scalar product with μ gets added

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Dual variable update

$$\beta^{t+1} = \arg\min_{\beta \in \mathbb{R}^m} L_\rho(\beta, \theta^t, \mu^t)$$

$$\theta^{t+1} = \arg\min_{\theta \in \mathbb{R}^m} L_\rho(\beta^{t+1}, \theta, \mu^t)$$

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ADMM - Why?

- convex problems with nondifferentiable constraints
- blockwise computation
  - sample blocks
  - feature blocks

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ADMM for the Lasso

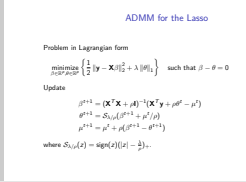
Problem in Lagrangian form

$$\underset{\beta\in\mathbb{R}^p,\theta\in\mathbb{R}^p}{\text{minimize}} \left\{ \frac{1}{2} \|\mathbf{y} - \mathbf{X}\beta\|_2^2 + \lambda \|\theta\|_1 \right\} \quad \text{such that } \beta - \theta = 0$$

Update

$$\begin{aligned}
\beta^{t+1} &= (\mathbf{X}^T\mathbf{X} + \rho\mathbf{I})^{-1}(\mathbf{X}^T\mathbf{y} + \rho\theta^t - \mu^t) \\
\theta^{t+1} &= \mathcal{S}_{\lambda/\rho}(\beta^{t+1} + \mu^t/\rho) \\
\mu^{t+1} &= \mu^t + \rho(\beta^{t+1} - \theta^{t+1})
\end{aligned}$$

where $\mathcal{S}_{\lambda/\rho}(z) = \text{sign}(z)(|z| - \frac{\lambda}{\rho})_+$.



Dual Polytope Projection (DPP)

Suppose we want to calculate a lasso solution at $\lambda < \lambda_{\max}$. The DPP rule discards the j^{th} variable if

$$\left|\mathbf{x}_j^T \mathbf{y}\right| < \lambda_{\max} - \|\mathbf{x}_j\|_2 \|\mathbf{y}\|_2 \frac{\lambda_{\max} - \lambda}{\lambda}$$

Sequential DPP rule

Suppose we have the lasso solution $\hat{\beta}(\lambda')$ at λ' and want to screen variables for solutions at $\lambda < \lambda'$. We discard the j^{th} variable if

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Global Strong Rule

Suppose we want to calculate a lasso solution at $\lambda < \lambda_{\max}$. The global strong rule discards the j^{th} variable if

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Paragraphs of Text

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Bullet Points

- Lorem ipsum dolor sit amet, consectetur adipiscing elit
- Aliquam blandit faucibus nisi, sit amet dapibus enim tempus eu
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Blocks of Highlighted Text

Block 1

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Block 2

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Block 3

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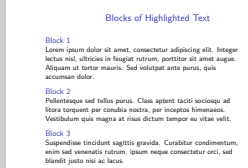

Multiple Columns

Heading

1. Statement
2. Explanation
3. Example

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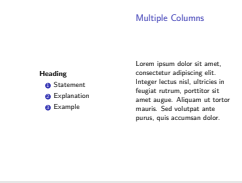

Table

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table: Table caption

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Theorem

Theorem (Mass–energy equivalence)

$E = mc^2$

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Verbatim

Example (Theorem Slide Code)

```
\begin{frame}
\frametitle{Theorem}
\begin{theorem}[Mass--energy equivalence]
$E = mc^2$
\end{theorem}
\end{frame}
```


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Figure

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An example of the `\cite` command to cite within the presentation:

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References

Trevor Hastie, Robert Tibshirani, and Martin Wainwright (2015)
Statistical learning with sparsity: the Lasso and generalizations
CRC Press; Boca Raton, FL

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The End