Computation & optimization for Lasso - part 2

Luyang Han & Janosch Ott

ETH Zürich

22 October 2018

Coordinate Descent

A Simulation Study

Least Angle Regression

ADMM

Min-Max Algorithms

Alternating Minimizations

Screening Rules

Overview

1. Coordinate Descent
2. A Simulation Study
3. Least Angle Regression
4. ADMM
5. Min-Max Algorithms
6. Alternating Minimizations
7. Screening Rules

Alternating Direction Method of Multipliers (ADMM)

Problem

$$\underset{\beta \in \mathbb{R}^m, \theta \in \mathbb{R}^n}{\text{minimize}} f(\beta) + g(\theta) \quad \text{subject to } \mathbf{A}\beta + \mathbf{B}\theta = c$$

Augmented Lagrangian

$$L_\rho(\beta, \theta, \mu) := f(\beta) + g(\theta) + \langle \mu, \mathbf{A}\beta + \mathbf{B}\theta - c \rangle + \frac{\rho}{2} ||\mathbf{A}\beta + \mathbf{B}\theta - c||_2^2$$

Dual variable update

$$\beta^{t+1} = \arg\min_{\beta \in \mathbb{R}^m} L_\rho(\beta, \theta^t, \mu^t)$$

$$\theta^{t+1} = \arg\min_{\theta \in \mathbb{R}^m} L_\rho(\beta^{t+1}, \theta, \mu^t)$$

$$\mu^{t+1} = \mu^t + \rho(\mathbf{A}\beta^{t+1} + \mathbf{B}\theta^{t+1} - c)$$

ADMM for the Lasso

Problem in Lagrangian form

$$\underset{\beta\in\mathbb{R}^p,\theta\in\mathbb{R}^p}{\text{minimize}} \left\{ \frac{1}{2}||\mathbf{y}-\mathbf{X}\beta||_2^2 + \lambda||\theta||_1 \right\} \quad \text{such that } \beta-\theta=0$$

Update

$$\begin{aligned}
\beta^{t+1} &= (\mathbf{X}^T\mathbf{X}+\rho\mathbf{I})^{-1}(\mathbf{X}^T\mathbf{y}+\rho\theta^t-\mu^t) \\
\theta^{t+1} &= \mathcal{S}_{\lambda/\rho}(\beta^{t+1}+\mu^t/\rho) \\
\mu^{t+1} &= \mu^t+\rho(\beta^{t+1}-\theta^{t+1})
\end{aligned}$$

where $\mathcal{S}_{\lambda/\rho}(z) = \text{sign}(z)(|z|-\frac{\lambda}{\rho})_+$.

Dual Polytope Projection (DPP)

Suppose we want to calculate a lasso solution at $\lambda < \lambda_{\max}$. The DPP rule discards the j^{th} variable if

$$\left|\mathbf{x}_j^T \mathbf{y}\right| < \lambda_{\max} - ||\mathbf{x}_j||_2 ||\mathbf{y}||_2 \frac{\lambda_{\max} - \lambda}{\lambda}$$

Sequential DPP rule

Suppose we have the lasso solution $\hat{\beta}(\lambda')$ at λ' and want to screen variables for solutions at $\lambda < \lambda'$. We discard the j^{th} variable if

$$\left|\mathbf{x}_j^T (\mathbf{y} - \mathbf{X}\hat{\beta}(\lambda'))\right| < \lambda' - ||\mathbf{x}_j||_2 ||\mathbf{y}||_2 \frac{\lambda_{\max} - \lambda}{\lambda}$$

Global Strong Rule

Suppose we want to calculate a lasso solution at $\lambda < \lambda_{\max}$. The global strong rule discards the j^{th} variable if

$$\left|\mathbf{x}_j^T \mathbf{y}\right| < \lambda - (\lambda_{\max} - \lambda) = 2\lambda - \lambda_{\max}$$

Sequential Strong Rule

Suppose we have the lasso solution $\hat{\beta}(\lambda')$ at λ' and want to screen variables for solutions at $\lambda < \lambda'$. We discard the j^{th} variable if

$$\left|\mathbf{x}_j^T (\mathbf{y} - \mathbf{X}\hat{\beta}(\lambda'))\right| < 2\lambda - \lambda'$$

Paragraphs of Text

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Bullet Points

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- Aliquam blandit faucibus nisi, sit amet dapibus enim tempus eu
- Nulla commodo, erat quis gravida posuere, elit lacus lobortis est, quis porttitor odio mauris at libero
- Nam cursus est eget velit posuere pellentesque
- Vestibulum faucibus velit a augue condimentum quis convallis nulla gravida

Blocks of Highlighted Text

Block 1

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Block 2

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Block 3

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Multiple Columns

Heading

1. Statement
2. Explanation
3. Example

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Table

Treatments	**Response 1**	**Response 2**
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table: Table caption

Theorem

Theorem (Mass–energy equivalence)

$E = mc^2$

Verbatim

Example (Theorem Slide Code)

```
\begin{frame}
\frametitle{Theorem}
\begin{theorem}[Mass--energy equivalence]
$E = mc^2$
\end{theorem}
\end{frame}
```

Figure

Uncomment the code on this slide to include your own image from the same directory as the template .TeX file.

Citation

An example of the `\cite` command to cite within the presentation:

This statement requires citation [Hastie et al., 2015].

References

Trevor Hastie, Robert Tibshirani, and Martin Wainwright (2015)
Statistical learning with sparsity: the Lasso and generalizations
CRC Press; Boca Raton, FL

The End