Impacts of Total Factor Productivity and Investment Shocks on General Equilibrium in a Real Business Cycle Dynamic Stochastic General Equilibrium Model

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1. Introduction

Macroeconomic models are used to study the performance of an entire economy by focusing on aggregate units such as overall output, inflation, and investment. The models examine how these factors respond to different policies and shocks and what trends the economy exhibits over time. Dynamic Stochastic General Equilibrium (DSGE) models are an example of macroeconomic models that are frequently used.

When considering the causes and effects of short-term contractions and expansions in an economy, business cycles are typically used to provide a framework to analyse these fluctuations. The Neoclassical Real Business Cycle (RBC) model is a DSGE model focusing on the impact of real (non-monetary) exogenous shocks.

"Investment shocks account for between 50 and 60 % of the variance of output and hours … and for more than 80 % of that of investment" (Justiniano, Primiceri, Tambalotti 2009). Additionally, in the 1997 paper by Goodfriend and King, "The New Neoclassical Synthesis and the Role of Monetary Policy", a total-factor-augmenting productivity (TFP) shock was found to have an "analogous" effect on the economy as the "variations in this composite measure of relative prices". Therefore, in this paper, I will be considering the impact of an investment shock and a TFP shock on an economy in equilibrium with the RBC model.

2. Model RBC Framework

In Real Business Cycle models, fluctuations are caused by "exogenous shocks to the production function" (Greenwood, Hercowitz, and Huffman 1988). The key agents in the model are the households and the firms (the policymakers are considered exogenous). It is a discrete-time model which makes the following assumptions: the agents live indefinitely, there is perfect competition with flexible prices in the markets (i.e., firms are price takers), and the economic agents are rational and make optimal decisions based on perfect information.

2.1 The Household

The representative **household's optimisation problem** is to maximise lifetime utility. To formulate this problem, start with the lifetime utility function and create an objective function:

$$U(C_t, N_t) = E_0 \sum_{t=0}^{\infty} \beta^t [\ln(C_t) - \frac{bN_t^2}{2}] \quad \rightarrow \quad \max_{\{C_t,\, N_t, K_{t+1}, \lambda_t\}} E_0 \sum_{t=0}^{\infty} \beta^t U_t(C_t, N_t)$$

Then, define the budget constraint with the substitution for investment:

$$C_t + \frac{K_{t+1} - (1-\delta)K_t}{\mu_t} \leq W_t N_t + r_t K_t$$

And non-negativity constraints:

$$C_t,\ N_t, K_{t+1} \geq 0$$

To find the **first-order conditions**, start by defining the Lagrangian. The Lagrangian is written with λ_t as the Lagrange multiplier:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \{\mathrm{U}(\mathrm{C_t}, \mathrm{N_t}) + \lambda_t \left[W_t N_t + r_t K_t - C_t - \frac{K_{t+1} - (1-\delta)K_t}{\mu_t} \right] \}$$

Then, differentiate with respect to Consumption, Labour, and Capital variables. The derivatives are expressed both with and without the substitutions of the utility function. After finding the derivatives, equate them to 0 to maximise them:

$\frac{\partial \mathcal{L}}{\partial C_t}$:

$$\beta^t U'_{C,t} - \beta^t \lambda_t = 0 \quad \rightarrow \quad \beta^t \left(\frac{1}{C_t} - \lambda_t \right) = 0 \quad \rightarrow \quad \frac{1}{C_t} = \lambda_t$$

$\frac{\partial \mathcal{L}}{\partial N_t}$:

$$\beta^t U'_{N,t} + \beta^t \lambda_t W_t = \quad \rightarrow \quad \beta^t(-bN_t - \lambda_t W_t) = 0$$

$\frac{\partial \mathcal{L}}{\partial K_{t+1}}$:

$$-\beta^t \left(\frac{\lambda_t}{\mu_t} \right) + \beta^{t+1} \left[r_{t+1} \lambda_{t+1} + \frac{\lambda_{t+1}(1-\delta)}{\mu_{t+1}} \right] = 0$$

Once all the first-order conditions have been obtained, the Euler equation can be obtained by combining the first-order conditions with respect to consumption and Capital, then dividing the β^t out and making the marginal utility of consumption the subject of the resulting expression:

$$U'_{C,t} = \beta \mu_t E_t U'_{C,t+1} \left[r_{t+1} \ + \frac{(1-\delta)}{\mu_{t+1}} \right]$$

The Euler Equation with respect to the physical capital stock represents the household's exchange between present consumption and future consumption. The two most relevant variables to this are the marginal utility of consumption for this period ($U'_{C,t}$) and the next period $U'_{C,t+1}$. The other variables act as a coefficient, showing how much of the marginal utility of future consumption is equal to the marginal utility of present consumption.

The Euler equation can also be used to explain what the marginal utility of consumption is and its connection to the interest rate and substitution shock. The marginal utility of consumption at time t is the additional utility gained by consuming 1 additional unit in time t, and it is inversely proportional to the consumption at time t ($\mathrm{C_t}$). By considering the future marginal utility of consumption $U'_{C,t+1}$ with the future interest rate $\mathrm{r_{t+1}}$ and future investment shock μ_{t+1}, you can see that the marginal utility of consumption is positively related to investment shocks within the same period, and negatively related to interest rates within the same period. This shows that an increase in the current interest rate disincentivises agents from consuming in the current period, but a positive investment shock encourages more consumption.

2.2 The Firm

The objective of all firms in the RBC model is to maximise profit. A firm's profit is equal to its revenue minus the cost of production:

$$\max_{N_t, K_t} \Pi_t \ = A_t K_t^{\alpha} N_t^{1-\alpha} - W_t N_t - r_t K_t$$

Revenue is calculated using a Cobb-Douglas production function, which thus has constant returns to scale. It is the product of the Total Factor of Productivity (TFP) in that period and the capital and labour inputs. Production costs are the total cost of paying wages for labour and the total rental payments for capital. The company aims to maximise profit.

The firm's first-order conditions with respect to N_t and K_t are found by differentiating the profit function with respect to each of the variables and equating them to 0 to maximise profit. The resulting expressions can be simplified to equate the marginal product of labour to the wage rate and the marginal product of capital to the rental rate.

$\frac{\partial \Pi_t}{\partial N_t}$:

$$MPN_t = F'_K(A_t, K_t, N_t) \quad \rightarrow \quad A_t(1-\alpha)K_t^{\alpha} N_t^{-\alpha} - W_t = 0 \quad \rightarrow \quad W_t = (1-\alpha)A_t \left(\frac{K_t}{N_t}\right)^{\alpha}$$

$\frac{\partial \Pi_t}{\partial K_t}$:

$$A_t \alpha K_t^{\alpha-1} N_t^{1-\alpha} - r_t = 0 \quad \rightarrow \quad MPK_t = F'_K(A_t, K_t, N_t) \quad \rightarrow \quad r_t = \alpha A_t \left(\frac{N_t}{K_t}\right)^{1-\alpha}$$

Given the constant returns scale nature of the production function, the rental rate and wage rate can be combined to create an expression equating the cost of production to the production function.

$$r_tK_t + W_tN_t \ = \left[\alpha A_t \left(\frac{N_t}{K_t}\right)^{1-\alpha}\right] K_t + \left[(1-\alpha)A_t\left(\frac{K_t}{N_t}\right)^{\alpha}\right] N_t$$

The expression can then be simplified to:

$$r_tK_t + W_tN_t \ = \alpha A_t N_t^{1-\alpha}K_t^{\alpha} + (1-\alpha)A_tK_t^{\alpha}N_t^{1-\alpha}$$

Because of the simplification, the TFP as well as the labour and capital inputs can be substituted as they make up the firm's production function. The resulting expression can then be simplified to equate the cost of production to the firm's output:

$$r_tK_t + W_tN_t \ = \alpha Y_t + (1-\alpha)Y_t$$

$$r_tK_t + W_tN_t \ = Y_t$$

The marginal product of capital and labour is the benefit from 1 additional unit of capital or labour. The profit function is maximised when the marginal product of labour is equal to the wage rate, and the marginal product of capital is equal to the rental rate. This means that it is profitable for the firm to hire labour until the additional productivity generated by one more unit of labour equals the cost of that labour. This is expressed as hiring more labour while $W_t > MPK_t$. The same is true for every additional unit of capital.

Under equilibrium, the firm's total costs of production are equal to its total production. If this expression is substituted into the profit function, the resulting profit is 0:

$$\Pi_t = A_tK_t^{\alpha}N_t^{1-\alpha} - W_tN_t - r_tK_t$$

$$\Pi_t = Y_t - \ W_tN_t - r_tK_t$$

$$\Pi_t = Y_t - \ Y_t \quad \rightarrow \quad \Pi_t = 0$$

2.3 Market Clearing Condition

We first look at the relationship between firms and households to determine the market clearing condition. The households supply the firms with labour N_t and receive wages W_tN_t. The households then spend that income either on consumption C_t or savings and investments I_t . Additionally, the pre-existing capital the household owns is K_t and the earnings they get from renting it out is r_tK_t. Therefore, the following equations can be formed:

$$Household\ Income = W_tN_t + r_tK_t$$

$$Household\ Expenditure = C_t + I_t$$

The household's budget is its income, which restricts expenditure. Therefore, the household's budget constraint is:

$$C_t + I_t \leq W_t N_t + r_t K_t$$

However, when considering the zero-profit condition, you can substitute the firms' production function into the equation and connect it to the household's consumption:

$$C_t + \ I_t \leq Y_t$$

In this model, there are no exports and no taxes. Therefore, all the income generated in the economy ends up back with the households. Consequently, you can change the inequality to an equality. This is the market clearing condition, which states that the total output produced in an economy is exactly equal to the consumption and investment made by households:

$$C_t + \ I_t = Y_t$$

This can be taken further with the substitution of the production function and the growth of capital equations:

$$C_t + \frac{K_{t+1} - (1-\delta)K_t}{\mu_t} = A_t K_t^{\alpha} N_t^{1-\alpha}$$

2.4 General Equilibrium Conditions

General equilibrium conditions in the RBC model emerge from the combination of the households and the firms' optimised decisions. They are the conditions for all markets in the economy to clear simultaneously, meaning supply equals demand across all markets.

On the household side, equilibrium requires optimising the amount of consumption and leisure in the current period while also balancing their intertemporal trade-offs between present and future consumption. The first-order conditions for the households are used for the general equilibrium conditions:

$$\frac{1}{C_t} = \lambda_t$$

$$bN_t = \lambda_t W_t$$

$$U'_{C,t} = \beta \mu_t E_t U'_{C,t+1} \left[r_{t+1} \ + \frac{(1-\delta)}{\mu_{t+1}} \right]$$

For firms, equilibrium is derived from their combination of labour and capital inputs that seek to maximise profit by equating the marginal products to corresponding factor prices:

$$\max_{N_t, K_t} \Pi_t = A_t K_t^{\alpha} N_t^{1-\alpha} - W_t N_t - r_t K_t$$

$$W_t = (1-\alpha)\left(\frac{Y_t}{N_t}\right)$$

$$r_t = \alpha\left(\frac{Y_t}{K_t}\right)$$

The market clearing condition equates the total output of the firms to the consumption and investment of the households, which acts as the equilibrium condition for aggregate supply and demand for the whole economy:

$$C_t + I_t = Y_t$$

The law of motion for capital illustrates how investments account for depreciation and augment the existing capital stock for the next period:

$$K_{t+1} = (1-\delta)K_t + \mu_t I_t$$

Finally, the production function represents the combination of inputs used by the firm to produce outputs. Because a Cobb-Douglas is used, it captures the constant return to scales while also exhibiting a diminishing marginal rate of substitution for each of the inputs:

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$$

The term A is meant to represent a TFP Shock, which follows an auto-regressive AR(1) process. The Investment Shock μ_t also follows an auto-regressive AR(1) process. The persistence (autocorrelation) parameters are bound between 0 and 1, and the serially uncorrelated noise shocks ε are normally distributed with a constant standard deviation and mean zero:

$$\mu_t = (\mu_{t-1})^{\rho_\mu} \exp\,(sd_\mu \cdot \varepsilon_t^{\mu}) \quad ; \quad 0 < \rho_\mu < 1$$

$$\ln\,(\mu_t) = \rho_\mu \ln(\mu_{t-1}) + sd_\mu \varepsilon_t^{\mu} \quad ; \quad \varepsilon_t^{A} \sim N(0, sd_\mu^2)$$

$$A_t = (A)^{1-\rho_A}(A_{t-1})^{\rho_A} \exp(\varepsilon_t^{A}) \quad ; \quad 0 < \rho_A < 1$$

$$\ln\,(A_t) = \rho_A \ln(A_{t-1}) + \varepsilon_t^{A} \quad ; \quad \varepsilon_t^{A} \sim N(0, sd_A^2)$$

2.5 Steady State Equilibrium

In the RBC model, the steady state is the long-run equilibrium. At a steady state, all variables are constant. Therefore, all the subscripts indicating the period are removed. Furthermore, because all conditions remain constant in a steady state, the investment shock and TFP shock are equal to 1, effectively removing them from consideration. This results in the following steady-state equilibrium conditions:

The first condition relates the rental rate in a steady state economy to the households' time preference. If rental rates were too high, all the money would be invested for future consumption. And if they were too low, all the households would only consume in the present. This relationship is expressed as the rental rate after factoring in depreciation must equal the inverse of the discount factor. This means that households are indifferent between consuming today versus saving and consuming in the future for the long-run equilibrium.

$$R = r + (1 - \delta) = \beta^{-1}$$

In the steady state, the equilibrium capital stock per worker is where the marginal product of capital is equal to the cost of investment. This means the cost of an additional unit per worker is equal to the benefit of that. The ratio also considers the depreciation rate of that capital and time preference for future consumption:

$$\frac{K}{N} = \left[\frac{\alpha}{\beta^{-1} - 1 + \delta}\right]^{\frac{1}{1-\alpha}}$$

The capital stock changes over time with decreases through depreciation and increases through investment, but in a steady state, the investment is the same and perfectly offsets the depreciation. Consequently, the capital stock remains the same relative to the output:

$$K_{t+1} = (1 - \delta)K_t + \mu_t I_t \quad \rightarrow \quad K = (1 - \delta)K + I \quad \rightarrow \quad I = \delta K$$

The steady-state level of consumption is dependent on the discount rate, the growth rate, and the depreciation rate. It reflects how households balance immediate consumption against the maintenance and growth of the economy's productive capacity:

$$C = \left[\frac{\alpha}{\beta^{-1} - 1 + \delta}\right]^{\frac{\alpha}{1-\alpha}} \left[(1 - g) - \frac{\alpha\delta}{\beta^{-1} - 1 + \delta}\right] N$$

In a steady state, The labour supply is at equilibrium when the trade-off between the marginal benefit of labour and the marginal benefit of leisure is equal, i.e., the benefit from one additional unit of work is the same as the benefit from one additional unit of leisure.

$$bN^{1+\varphi} = \frac{1 - \alpha}{\left[(1 - g) - \delta\left(\frac{K}{N}\right)^{1-\alpha}\right]}$$

$$bN^{1+\varphi} = C^{-1}A(1-\alpha)\left(\frac{K}{N}\right)^{\alpha}$$

The market clearing condition remains unchanged from the general equilibrium conditions, except the variables are not specific to any time anymore:

$$C + I = Y$$

The production function also remains unchanged with the only changes being the $A = 1$ and removing any notation that specifies time.

$$Y = K^{\alpha}N^{1-\alpha}$$

3. Impulse Response Functions

To model a negative TFP and investment shock in a stable state economy, I used Dynare for MATLAB to plot the impulse response functions of 9 variables (Output, Consumption, Investment, Capital, Labour, MPK, Interest Rate, Wage Rate, and the Investment / TFP Shock) for 100 periods after a 1% decline in investment and a 1% TFP drop.

3.1 Calibration

The model was calibrated with the following parameters: the capital share in production (α) is 0.35, the discount factor (β) is 0.995, and the depreciation rate (δ) is 0.025. Labour disutility is set with $b_{ss} = 3.67$, and the inverse Frisch elasticity (φ) is 1. Steady-state values, including capital-to-labour ratio $\left(\frac{K}{N}\right)$, output (Y_{ss}), consumption (C_{ss}), and wages (W_{ss}), are derived from these parameters. Both shocks were assumed to be persistent, with ρ=0.95 and have a standard deviation of 0.01.

To properly simulate the shocks, we use Dynare to solve the DSGE models in MATLAB. However, the model is non-linear, which can be hard to solve numerically. So, we use a technique called log-linearization. "Log linearization converts a non-linear equation into a linear in terms of log-deviations of the associated variables from steady state." (Zilberman 2024).

3.2 Simulation Results

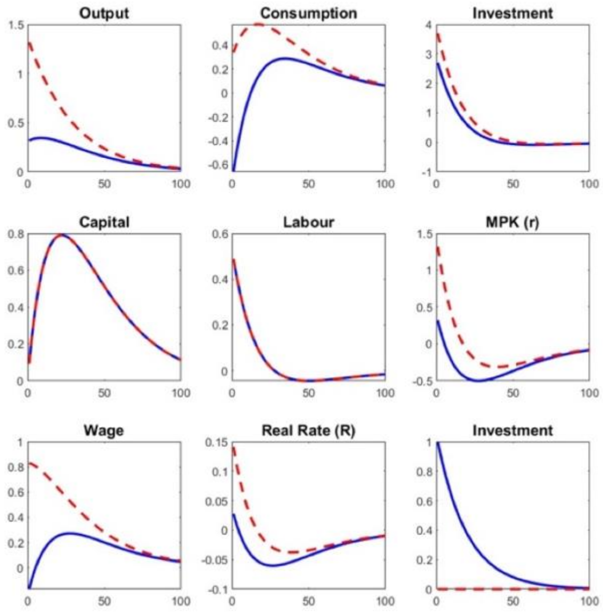

Figure 1 – TFP (red) and Investment (blue) Shock in a Steady State

3.2.1 TFP Shock – Red Dotted Line

Following a negative TFP shock, the output immediately faces a sharp decline. This is because of the decrease in productivity, which lowers the marginal product of both labour and capital, meaning each additional unit is less productive. This discourages the use of more labour and capital, decreasing the level of capital and labour. Because of the decrease in marginal productivity, the wage rate and rental rate also decrease.

The income effect is the change in consumption of goods and labour supply due to a change in income. Because households experience a decline in income, consumption also drops in the short run. Furthermore, because of the decrease in income, there is also a decrease in investment. However, as the real interest rate rises, households adjust their consumption-savings trade-offs, smoothing their consumption over time

Initially, the workers need to work more to compensate for the decrease in wages. However, in the long term, as the shock dissipates and the level of investment returns to normal, they begin to substitute labour for leisure once again, per the substitution effect.

3.2.2 Investment Shock – Solid Blue Line

The first and most obvious effect of a negative investment shock is a decrease in investment. An investment shock reduces output more gradually. This is primarily due to a reduction in investment into productive capital by firms, which slows down capital accumulation.

Because there is a decrease in investment, houses take advantage of the. lower real interest rates and prioritise present consumption over future consumption. This adjustment diminishes the immediate effects of the reduced investment. This is the reason that the decreases seen in wages, consumption, output, and rental rates are a lot smaller than the decreases seen with the TFP shock.

Because of the reduced capital accumulation, the economy also has less capital in the long run to produce goods. Instead of a large change in capital or labour, this means the economy gradually adjusts to the diminished investment, with lower and less persistent effects.

3.3 Conclusion

TFP shocks have broader and more persistent impacts, reducing productivity and triggering declines across all key variables. While investment shocks are much smaller in scope, they disrupt capital formation and indirectly affect other variables. Both highlight the interplay of direct transmission mechanisms and general equilibrium adjustments, providing insights into the economy's response to different disturbances.

4. Historical U.S. Data

Once I obtained the results from the Dynare simulations, I compared them to some historical data to check the validity and accuracy of the models. I used the Federal Reserve Economic Data (F.R.E.D.) on Output, Investment and Consumption. Specifically, I looked at the Real Gross Domestic Product (GDP), the Real Gross Private Domestic Investment and the Personal Consumption Expenditures, all from the beginning of Quarter 1 in 1970 to the beginning of Quarter 1 in 2020.

The data already came quarterly (i.e., the GDP, investment, and consumption were given on the first day of January, April, July, and October). However, to make the data more comparable to the TFP and Investment Shock data, I started by finding the logarithm of each variable for each quarter. Then, I computed the quarterly change of each variable, by looking at the difference between consecutive quarters. Then I multiplied the difference by 100 to make it a percentage change as opposed to a proportion. Finally, I found the standard deviation of the quarterly percentage changes and put them in a table against the standard deviations from the TFP and Investment shocks. All the calculations were done in Microsoft Excel.

Variable (log)	F.R.E.D. σ	TFP + Investment σ
Output	0.7982	1.7726
Investment	3.8579	5.9764
Consumption	0.9268	1.0276

Figure 2 – TFP and Investment Shock natural log variable standard deviation against F.R.E.D. quarterly percentage standard deviation.

As is visible from the table, the results from the TPF and Investment shock were reasonably close to the F.R.E.D. values. However, the model results tended to overestimate the historical data, with the standard deviation from the Output and investment being around double the F.R.E.D standard deviations. This might have been because the persistence of the shocks was quite high at 0.95 or because both shocks were considered together. Otherwise, the model seems to follow the real data quite closely.

References

Roy Zilberman: Course Lecture Notes (Parts I and II).

Dynare and Matlab - Getting Started File.

Greenwood, Jeremy, Zvi Hercowitz, and Gregory W. Huffman. (1988). "Investment, Capacity Utilization, and the Real Business Cycle." American Economic Review, 78(3), 402-417.

Justiniano, A., Primiceri, G. E., & Tambalotti, A. (2009). "Investment shocks and business cycles." Journal of Monetary Economics, 57(2), 132–145.

Goodfriend, M., & King, R. G. (1997). "The new neoclassical synthesis and the role of monetary policy." *NBER Macroeconomics Annual, 12*, 231–283.

6. Appendix

A: Dynare Code

Below is the Dynare code used to model the equations for the Dynamic Stochastic General Equilibrium Model:

```
// RBC model with investment and TFP shocks


// Endogenous Variables

var c k n I y r w R a mu log_y log_c log_I log_k log_n log_r log_R log_w log_a log_mu;


// Exogenous Variables

varexo eps_a eps_mu;

// Parameter values

parameters beta alpha delta a_ss b_ss varphi g_ss k_n_ss n_ss k_ss I_ss c_ss y_ss mu_ss
w_ss r_ss R_ss I_y_ss c_y_ss k_n_ss_annual k_y_ss_annual c_I_ss rho_a sd_a rho_mu
sd_mu;

// Parameter Calibration

alpha = 0.35;       // Capital share in production
beta = 0.995;       // Discount factor
delta = 0.025;      // Depreciation rate
a_ss = 1;           // TFP Shock
```

```
b_ss = 3.67;         // Labor disutility weight
varphi = 1;          // Inverse of Frisch Elasticity of Labour Supply
mu_ss = 1;           // SS investment shock

g_ss = 0.0;

k_n_ss = ((alpha/((1/beta)-(1-delta)))^(1/(1-alpha)));                       // SS capital to labour ratio
n_ss = ((1-alpha)*b_ss^(-1)/(1-g_ss-delta*(k_n_ss)^(1-alpha)))^(1/(1+varphi));   // SS labour
k_ss = k_n_ss*n_ss;                                                 // SS capital
I_ss = delta*k_ss;                                                  // SS investment
c_ss = n_ss*(a_ss*((k_n_ss)^alpha)-delta*(k_n_ss));                           // SS consumption
y_ss = a_ss*(k_ss^alpha)*(n_ss^(1-alpha));                                // SS output

w_ss=a_ss*(1-alpha)*(k_ss/n_ss)^alpha;                                  // SS MPN
r_ss=a_ss*alpha*(n_ss/k_ss)^(1-alpha);                                  // SS MPK

R_ss=r_ss+(1-delta);                                                // SS Gross Real Return on Capital net of depreciation

I_y_ss=I_ss/y_ss;
c_y_ss=c_ss/y_ss;
k_n_ss_annual=k_ss/4*n_ss;
k_y_ss_annual=k_ss/4*y_ss;
c_I_ss=c_ss/I_ss;



rho_mu = 0.95;       // Persistence of investment shock
sd_mu = 0.01;        // Std dev of investment shock

rho_a = 0.95;        // Persistence of TFP shock
sd_a = 0.01;         // Std dev of TFP shock



model;


// Basic Model

// Euler Equation

(1/c) = beta * mu * (1 / c(+1)) * (r(+1) + (1 - delta) / mu(+1));



// Labour Supply Condition

b_ss*n^varphi=(1/c)*a*(1-alpha)*(k(-1)/n)^alpha;
```

```
// Production Function

y=a*(k(-1)^alpha)*(n^(1-alpha));

// Resource Constraint

y=I+c;

// Capital Accumulation Constraint

k=mu*I+(1-delta)*(k(-1));

// Marginal Product of Capital - Demand for Capital

r=a*alpha*(n/k(-1))^(1-alpha);

// The Real Gross Return on Capital

R=r(1)+(1-delta);

// Marginal Product of Labour - Demand for Labour

w=a*(1-alpha)*(k(-1)/n)^alpha;

// Process for TFP A

a=((a_ss)^(1-rho_a))*((a(-1))^rho_a)*exp(sd_a*eps_a);

// Process for mu

mu=(mu(-1)^(rho_mu))*(exp(sd_mu*eps_mu));


// Annualized Log Variables

log_y=100*(y-y_ss)/y_ss;

log_c=100*(c-c_ss)/c_ss;

log_I=100*(I-I_ss)/I_ss;

log_k=100*(k-k_ss)/k_ss;

log_n=100*(n-n_ss)/n_ss;
```

```
log_r=100*(r-r_ss)/r_ss;

log_R=400*(R-R_ss)/R_ss;

log_w=100*(w-w_ss)/w_ss;

log_a=100*(a-a_ss)/a_ss;

log_mu=100*(mu-mu_ss)/mu_ss;

end;



// Below steady state values derived in a separate MATLAB file

initval;

c=c_ss;
k=k_ss;
n=n_ss;

I=I_ss;
y=y_ss;
r=r_ss;

R=R_ss;
w=w_ss;
a=a_ss;
mu=mu_ss;


end;

steady;

check;

shocks;

var eps_a; stderr -1;
var eps_mu; stderr -1;


end;

stoch_simul(order=1,irf=100,hp_filter=1600) log_y log_c log_I log_k log_n log_r log_R
log_w log_a log_mu;
```

B: Plotting Impulse Reponse Functions

Below is the code for the MATLAB file to plot the impulse response functions of a negative 1% shock in The Total Factors of Productivity and Investment variables.

```
clear; close all;

load mu_RBC.mat %change to your mat file
oo1=oo_;

load A_RBC.mat %change to your mat file
oo2=oo_;

lag = (1:1:100);  %change to number of periods

%% V
F1=figure(1);
set(F1, 'numbertitle','off')

set(F1, 'name', 'Impulse response functions (Investment Shock)') %change to your title
  h1 = area(1:30); %number of periods
  set(h1,'FaceColor',[.9 .9 .9]);
subplot(3,3,1)% first two number are number of graphs on one page (2x2=4) last part is location
plot(lag,oo1.irfs.log_y_eps_mu(:,[1: 100])','b','linewidth',2); % load oo1.yourname of responses (see mat file)
hold on
plot(lag,oo2.irfs.log_y_eps_a(:,[1: 100]),'--r','linewidth',2);


hold off
title('Output','fontsize',12)


subplot(3,3,2)
plot(lag,oo1.irfs.log_c_eps_mu(:,[1: 100])','b','linewidth',2);
hold on
plot(lag,oo2.irfs.log_c_eps_a(:,[1: 100]),'--r','linewidth',2);


hold off
title('Consumption','fontsize',12)


subplot(3,3,3)
plot(lag,oo1.irfs.log_I_eps_mu(:,[1: 100])','b','linewidth',2);
hold on
plot(lag,oo2.irfs.log_I_eps_a(:,[1: 100]),'--r','linewidth',2);
```

```
hold off

title('Investment','fontsize',12)

subplot(3,3,4)
plot(lag,oo1.irfs.log_k_eps_mu(:,[1: 100])','b','linewidth',2);
hold on
plot(lag,oo2.irfs.log_k_eps_a(:,[1: 100]),'--r','linewidth',2);


hold off

title('Capital','fontsize',12)


subplot(3,3,5)
plot(lag,oo1.irfs.log_n_eps_mu(:,[1: 100])','b','linewidth',2);
hold on
plot(lag,oo2.irfs.log_n_eps_a(:,[1: 100]),'--r','linewidth',2);


hold off

title('Labour','fontsize',12)

subplot(3,3,6)
plot(lag,oo1.irfs.log_r_eps_mu(:,[1: 100])','b','linewidth',2);
hold on
plot(lag,oo2.irfs.log_r_eps_a(:,[1: 100]),'--r','linewidth',2);


hold off
title('MPK (r)','fontsize',12)

subplot(3,3,7)
plot(lag,oo1.irfs.log_w_eps_mu(:,[1: 100])','b','linewidth',2);
hold on
plot(lag,oo2.irfs.log_w_eps_a(:,[1: 100]),'--r','linewidth',2);


hold off

title('Wage','fontsize',12)

subplot(3,3,8)
plot(lag,oo1.irfs.log_R_eps_mu(:,[1: 100])','b','linewidth',2);
hold on
plot(lag,oo2.irfs.log_R_eps_a(:,[1: 100]),'--r','linewidth',2);
```

```
hold off

title('Real Rate (R)','fontsize',12)

subplot(3,3,9)
plot(lag,oo1.irfs.log_mu_eps_mu(:,[1: 100])','b','linewidth',2);
hold on
plot(lag,oo2.irfs.log_mu_eps_a(:,[1: 100]),'--r','linewidth',2);


hold off

title('Investment','fontsize',12)
```

C: Dynare Results

Below is the output from the Dynare code in the command window.

```
Starting Dynare (version 6.2).
Calling Dynare with arguments: none
Starting preprocessing of the model file ...
Found 20 equation(s).
Evaluating expressions...
Computing static model derivatives (order 1).
Normalizing the static model...
Finding the optimal block decomposition of the static model...
5 block(s) found:
  4 recursive block(s) and 1 simultaneous block(s).
  the largest simultaneous block has 6 equation(s)
                                 and 6 feedback variable(s).
Computing dynamic model derivatives (order 1).
Normalizing the dynamic model...
Finding the optimal block decomposition of the dynamic model...
3 block(s) found:
  2 recursive block(s) and 1 simultaneous block(s).
  the largest simultaneous block has 6 equation(s)
                                 and 6 feedback variable(s).
Preprocessing completed.
Preprocessing time: 0h00m01s.

STEADY-STATE RESULTS:

c                1.32933
k                21.8689
n                0.499954
I                0.546723
y                1.87605
r                0.0300251
w                2.43909
```

R	1.00503
a	1
mu	1
log_y	0
log_c	0
log_I	0
log_k	0
log_n	0
log_r	0
log_R	0
log_w	0
log_a	0
log_mu	0

EIGENVALUES:

Modulus	Real	Imaginary
0.95	0.95	0
0.95	0.95	0
0.9617	0.9617	0
1.045	1.045	0
Inf	-Inf	0
Inf	-Inf	0

There are 3 eigenvalue(s) larger than 1 in modulus for 3 forward-looking variable(s)
The rank condition is verified.

MODEL SUMMARY

Number of variables: 20
Number of stochastic shocks: 2
Number of state variables: 3
Number of jumpers: 3
Number of static variables: 15

MATRIX OF COVARIANCE OF EXOGENOUS SHOCKS

Variables	eps_a	eps_mu
eps_a	1.000000	0.000000
eps_mu	0.000000	1.000000

POLICY AND TRANSITION FUNCTIONS

	log_y	log_c	log_I	log_k	log_n	log_r	log_R	log_w	log_a	log_mu
k(-1)	1.125293	2.587299	-2.429492	4.397646	-0.731003	-3.447407	-0.396194	1.856296	0	0
a(-1)	125.212489	32.250984	351.243082	8.781077	46.480752	125.212489	13.423640	78.731737	95.000000	0
mu(-1)	30.212489	-62.749016	256.243082	8.781077	46.480752	30.212489	2.638765	-16.268263	0	95.000000

eps_a	1.318026	0.339484	3.697296	0.092432	0.489271	1.318026	0.141301	0.828755	1.000000	0
eps_mu	0.318026	-0.660516	2.697296	0.092432	0.489271	0.318026	0.027776	-0.171245	0	1.000000

THEORETICAL MOMENTS (HP filter, lambda = 1600)

VARIABLE	MEAN	STD. DEV.	VARIANCE
log_y	0.0000	1.7726	3.1420
log_c	0.0000	1.0276	1.0559
log_I	0.0000	5.9764	35.7176
log_k	0.0000	0.6053	0.3664
log_n	0.0000	0.9078	0.8241
log_r	0.0000	1.8281	3.3420
log_R	0.0000	0.1952	0.0381
log_w	0.0000	1.1291	1.2748
log_a	0.0000	1.3034	1.6990
log_mu	0.0000	1.3034	1.6990

VARIANCE DECOMPOSITION (in percent) (HP filter, lambda = 1600)

	eps_a	eps_mu
log_y	94.20	5.80
log_c	23.94	76.06
log_I	65.21	34.79
log_k	50.00	50.00
log_n	50.00	50.00
log_r	91.68	8.32
log_R	92.90	7.10
log_w	93.68	6.32
log_a	100.00	0.00
log_mu	0.00	100.00

MATRIX OF CORRELATIONS (HP filter, lambda = 1600)

Variables	log_y	log_c	log_I	log_k	log_n	log_r	log_R	log_w	log_a	log_mu
log_y	1.0000	0.2475	0.9143	0.3139	0.8362	0.9439	0.9364	0.8976	0.9687	0.2334
log_c	0.2475	1.0000	-0.1662	0.2409	-0.3244	0.1288	0.1587	0.6493	0.4288	-0.8396
log_I	0.9143	-0.1662	1.0000	0.2187	0.9867	0.9068	0.8867	0.6421	0.8067	0.5886
log_k	0.3139	0.2409	0.2187	1.0000	0.1701	-0.0134	-0.0390	0.3560	0.1938	0.1938
log_n	0.8362	-0.3244	0.9867	0.1701	1.0000	0.8486	0.8244	0.5088	0.7031	0.7031
log_r	0.9439	0.1288	0.9068	-0.0134	0.8486	1.0000	0.9990	0.7996	0.9411	0.2281

log_R	0.9364	0.1587	0.8867	-0.0390	0.8244	0.9990	1.0000	0.8073	0.9449	0.1869
log_w	0.8976	0.6493	0.6421	0.3560	0.5088	0.7996	0.8073	1.0000	0.9556	-0.1989
log_a	0.9687	0.4288	0.8067	0.1938	0.7031	0.9411	0.9449	0.9556	1.0000	-0.0000
log_mu	0.2334	-0.8396	0.5886	0.1938	0.7031	0.2281	0.1869	-0.1989	-0.0000	1.0000

COEFFICIENTS OF AUTOCORRELATION (HP filter, lambda = 1600)

Order	1	2	3	4	5
log_y	0.7199	0.4817	0.2833	0.1222	-0.0050
log_c	0.7341	0.5041	0.3096	0.1487	0.0193
log_I	0.7104	0.4665	0.2657	0.1044	-0.0213
log_k	0.9596	0.8619	0.7266	0.5702	0.4056
log_n	0.7088	0.4641	0.2629	0.1015	-0.0239
log_r	0.7145	0.4732	0.2734	0.1122	-0.0141
log_R	0.7164	0.4761	0.2769	0.1157	-0.0110
log_w	0.7329	0.5023	0.3074	0.1466	0.0173
log_a	0.7133	0.4711	0.2711	0.1098	-0.0163
log_mu	0.7133	0.4711	0.2711	0.1098	-0.0163

Total computing time : 0h00m21s