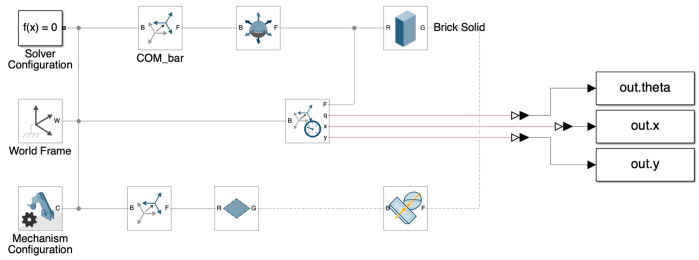
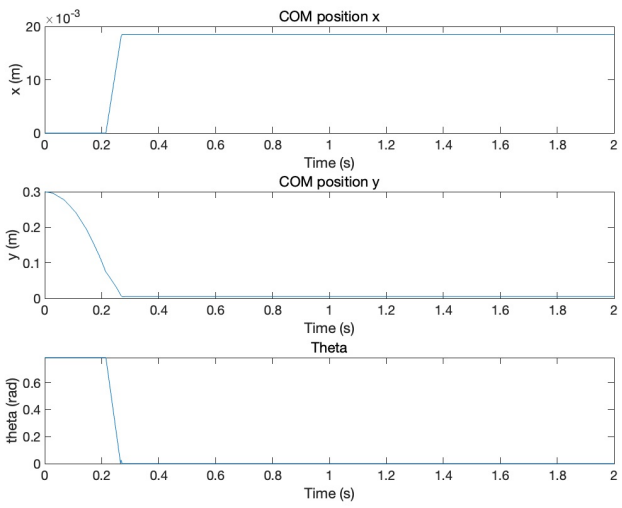
AME 556 HW3 Shuai Zhao

Problem 1

a.



b.



c.

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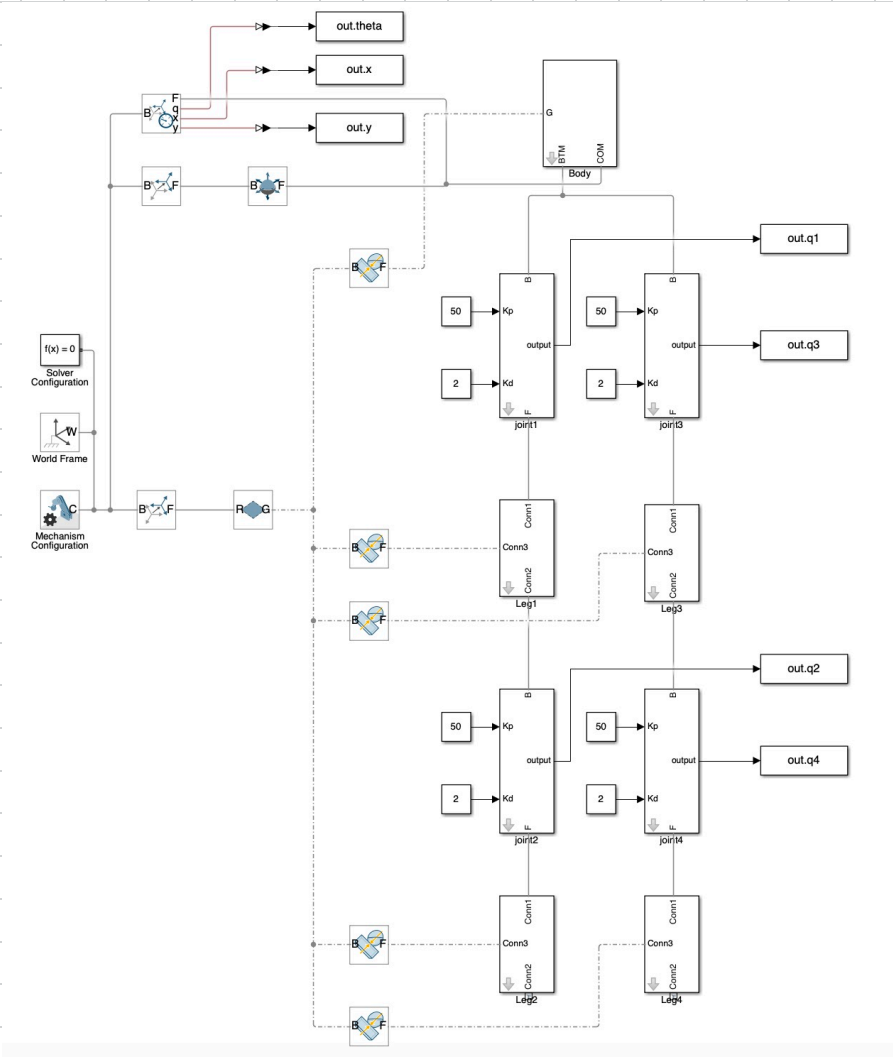
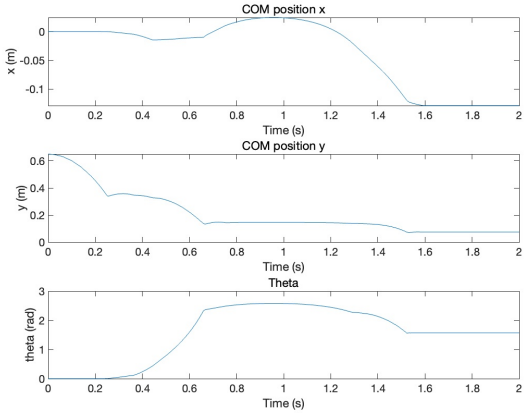
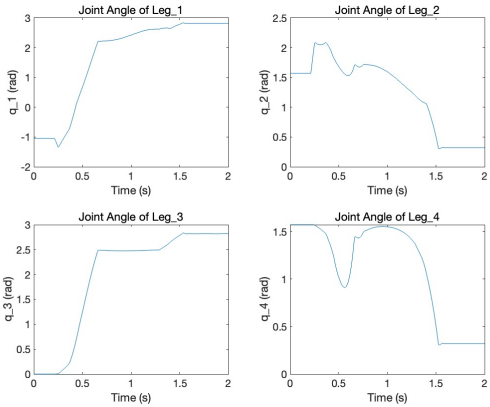
Problem 2


out.theta
out.x
out.y
F
q
x
y
G
BTM
COM
Body
out.q1
out.q3
out.q2
out.q4
50
Kp
2
Kd
output
joint1
joint3
joint2
joint4
f(x) = 0
Solver Configuration
World Frame
Mechanism Configuration
R
Conn1
Conn3
Conn2
Leg1
Leg3
Leg2
Leg4

a. unforced system





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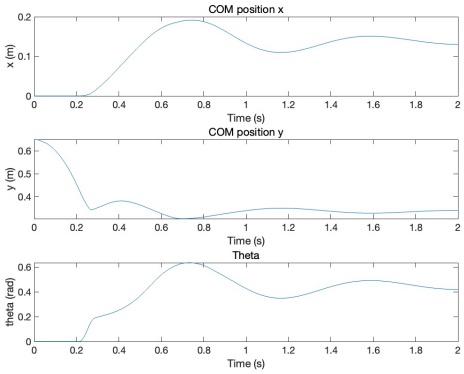
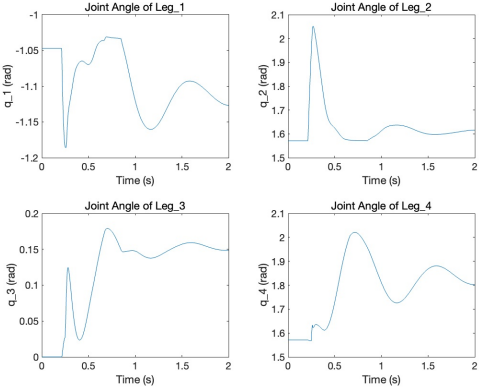
b. controlled system





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Problem 3

a. Linearization :

$$\begin{cases} (M+m)\ddot{x} + mL\sin(\theta)\cdot\dot{\theta}^2 - mL\cos(\theta)\cdot\ddot{\theta} = u \\ mL^2\ddot{\theta} - mL\cos(\theta)\cdot\ddot{x} - mgL\sin(\theta) = 0 \end{cases}$$

Sub $M=1kg$, $m=0.2kg$ $L=0.3kg$ $g=9.81 m/s^2$ and Solve it for $\ddot{x}$ and $\ddot{\theta}$:

$$\ddot{x} = \frac{0.3\sin(\theta)\cdot\dot{\theta}^2 - 9.81\sin(\theta)\cos(\theta)}{\cos^2(\theta)-6} + \frac{5}{\sin^2(\theta)+5}u$$

$$\ddot{\theta} = \frac{\sin(\theta)\cos(\theta)\cdot\dot{\theta}^2 - 196.2\sin(\theta)}{\cos^2(\theta)-6} - \frac{50\cos(\theta)}{3(\cos^2(\theta)-6)}\cdot u$$

Let $X=[x,\theta,\dot{x},\dot{\theta}]^T$, $\dot{X}=[\dot{x},\dot{\theta},\ddot{x},\ddot{\theta}]^T$

$$\dot{q} = \begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \frac{0.3\sin(\theta)\cdot\dot{\theta}^2 - 9.81\sin(\theta)\cos(\theta)}{\cos^2(\theta)-6} + \frac{5}{\sin^2(\theta)+5}u \\ \frac{\sin(\theta)\cos(\theta)\cdot\dot{\theta}^2 - 196.2\sin(\theta)}{\cos^2(\theta)-6} - \frac{50\cos(\theta)}{3(\cos^2(\theta)-6)}\cdot u \end{bmatrix} = f(q,u)$$

$$A = \left.\frac{\partial f}{\partial q}\right|_{\bar{q}=(0,0,0,0)^T} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 981/500 & 0 & 0 \\ 0 & 981/25 & 0 & 0 \end{bmatrix}, \quad B = \left.\frac{\partial q}{\partial u}\right|_{(0,0,0,0)^T} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 10/3 \end{bmatrix}$$

the linearized system is $\delta\dot{X} = A\delta X + B\delta u$

b. Design

Using the formula of 2nd-order system :

settling time $[0.5\ 1] \Rightarrow \zeta\omega_n \in [5\ \ 10]$

overshoot $[10\%\ \ 20\%] \Rightarrow \zeta \in [0.46\ \ 0.59]$

so we could guess the poles should located at around $[-5+5j, -5-5j, -50, -51]$

After a few choice, I finally choose desired pole at $[-6+3j, -6-3j, -60, -55]$ which associated with $K = [-45541\ \ 2792\ \ -1369\ \ 449]$

i.

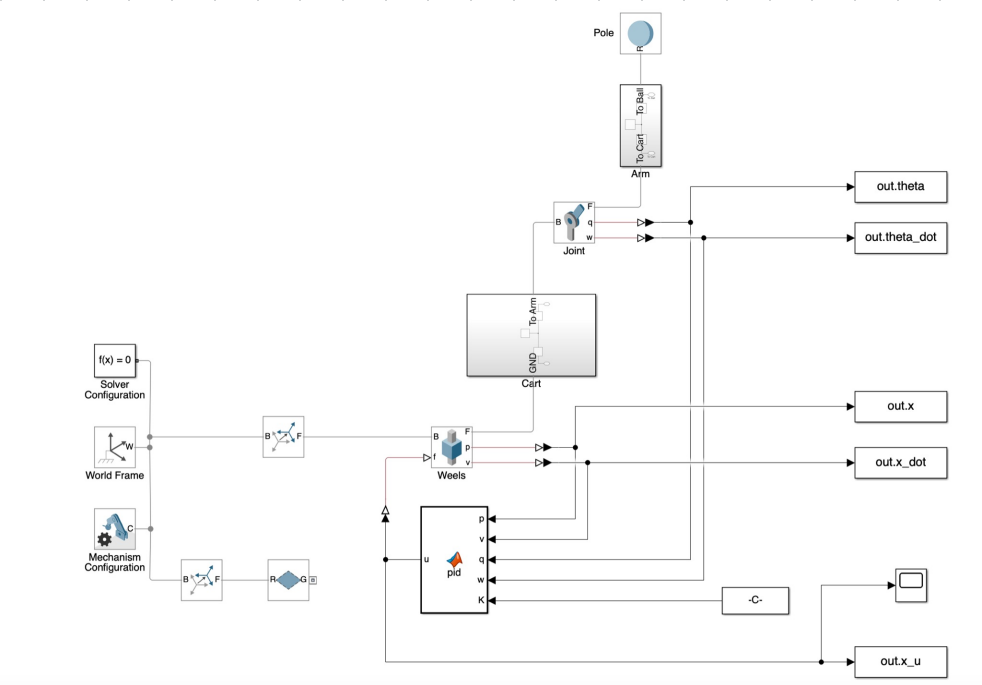
$$A_{cl} = A - BK = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 4541 & -2892 & 1369 & -448 \\ 15137 & -9266 & 4560 & -1496 \end{bmatrix}$$

whose eigenvalues are:

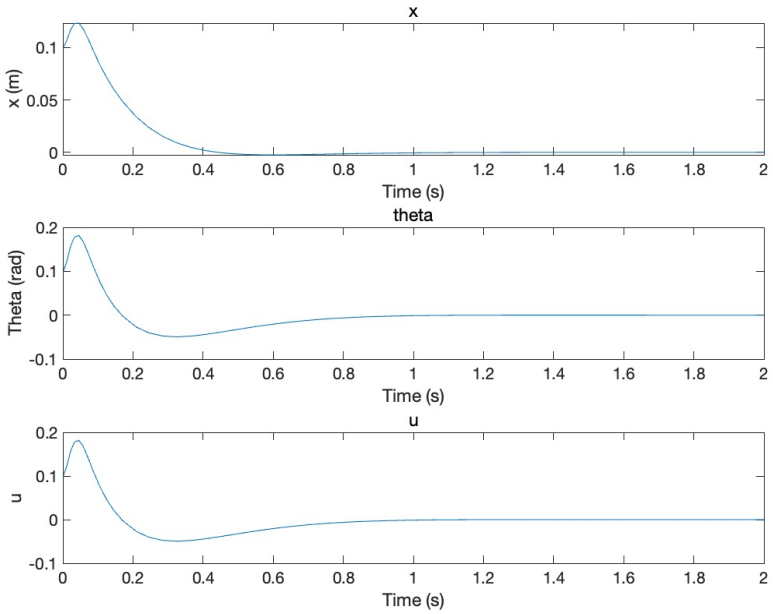
$$\lambda_1 = -60, \quad \lambda_2 = -55, \quad \lambda_{3,4} = -6 \pm 3i$$

All of the $Re[\lambda_i]$ are negative, so the CL system is exponentially stable.

ii.



iii.



iv.

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