AME 556 – Robot Dynamics and Control – HW1

1. Show that the length of a free vector is not changed by rotation:
$$\|\boldsymbol{v}\| = \|R\boldsymbol{v}\|$$
2. Show that the distance between points is not changed by rotations:
$$\|\boldsymbol{p_1} - \boldsymbol{p_2}\| = \|R\boldsymbol{p_1} - R\boldsymbol{p_2}\|$$
3. Consider a rotation R with respect to the Euler angles of
$$[Roll, Pitch, Yaw] = [\frac{\pi}{3}, -\frac{\pi}{4}, \frac{\pi}{2}]$$
around X-Y-Z axis.
    a. Determine the rotation matrix corresponding to this rotation using built-in MATLAB function, report your code and the answer from MATLAB.
    b. Consider a vector $\boldsymbol{p} = \overrightarrow{OP} = [1; 2; 3]$ in a 3D coordinate frame Oxyz. Apply the above rotation to the vector $\boldsymbol{p}$, we get a new vector $\boldsymbol{p_1}$. What is the coordinate of the vector $\boldsymbol{p_1}$. Draw the vectors $\boldsymbol{p}$ and $\boldsymbol{p_1}$ in the same plot using MATLAB.
    Hint:
        - use the function *plot3* in MATLAB
        - use the MATLAB command "*axis equal*" to scale the 3 axes evenly.
4. Consider a triangle ABC with the A=[0;0;0]; B=[2;0;0]; C= [0;1;0] in the world frame $O_0X_0Y_0Z_0$
    a. Draw the triangle in MATLAB in a 3D-view.
    Hint:
        - use the function [patch](*X,Y,Z,C*) in MATLAB
        - use the MATLAB command "*axis equal*" to scale the 3 axes evenly.
    b. Apply a rotation of $\frac{\pi}{6}$ about the X_0 axis. Draw the new triangle in another plot.
    c. Apply a rotation of $-\frac{\pi}{4}$ about the Y_0 axis to the current triangle. Draw the new triangle in another plot.
    d. Apply a rotation of $\frac{2\pi}{3}$ about the Z_0 axis to the current triangle. Draw the new triangle in another plot.

e. Apply a rotation R to the current triangle so that the resulting triangle will be at the same location as the original triangle ABC. Find R and draw the new triangle in another plot.

Please report your code, justification for the code and plots for each part.

5. Consider the following sequence of rotation

   (1) Rotate by ϕ about the world x-axis

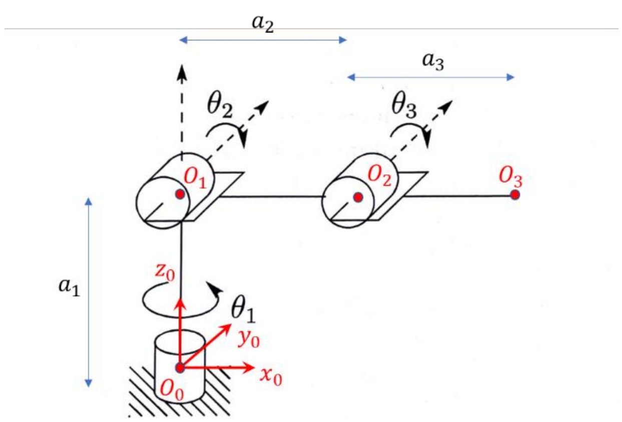
   (2) Rotate by θ about the world z-axis

   (3) Rotate by ψ about the current x-axis

   (4) Rotate by α about the world z-axis

Write the matrix product that will give the resulting rotation matrix (do not perform matrix multiplication).

6. Consider the following 3-DOF robot arm:



$$a_1 = 0.1\ m; a_2 = 0.2\ m; a_3 = 0.2\ m;$$

a. Use homogeneous transformation to derive forward kinematics of the robot.

b. Use MATLAB to draw the robot configuration with the following cases:

   (1) $\theta = [0; 0; 0]$ (corresponding to the configuration in the figure above)

   (2) $\theta = [0; \pi/4\ ; \pi/4\ ]$

   (3) $\theta = [\pi/6; \pi/4\ ; -\pi/2]$

   (4) $\theta = [\pi; \pi/2\ ; \pi/2\ ]$

Hint:

- For each case, plot O_0O_1, O_1O_2, O_2O_3 in 3D on the same plot. You may want to use different color for each link to better represent the robot.

c. Now assume that the robot arm is attached to a moving body instead of a fixed base. Apply the following transformations to the entire arm. Draw the arm in each case with $\theta = [0; \pi/4; \pi/4]$.

   (1) Move forward along the x-axis of the body frame by a distance of 0.5 m and then rotate by $[Roll, Pitch, Yaw] = [\pi/2, 0, \pi/4]$.

   (2) Rotate by $[Roll, Pitch, Yaw] = [\pi/2, 0, \pi/4]$ and then move forward along the x-axis of the body frame by a distance of 0.5 m.

   (3) Rotate by $[Roll, Pitch, Yaw] = [\pi/2, 0, \pi/4]$ and then move forward along the x-axis of the world frame by a distance of 0.5 m.

Please report your code, justification for the code and plots for each part.