# HW1

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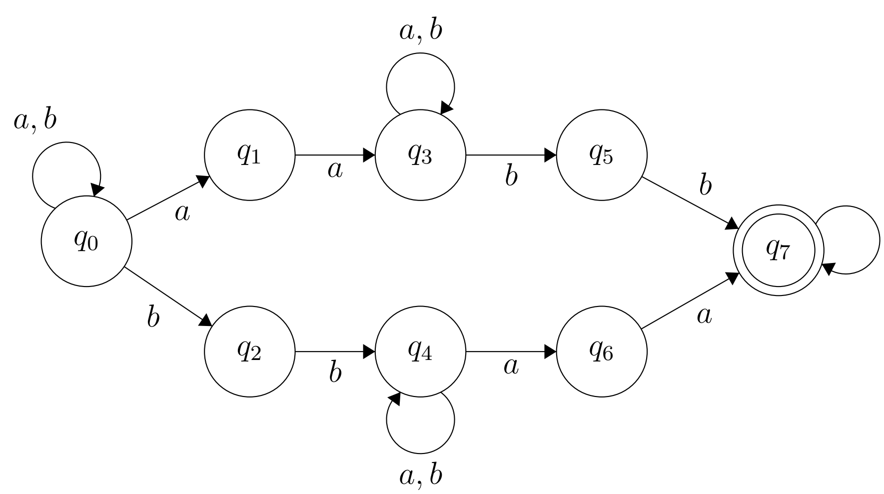
# 1

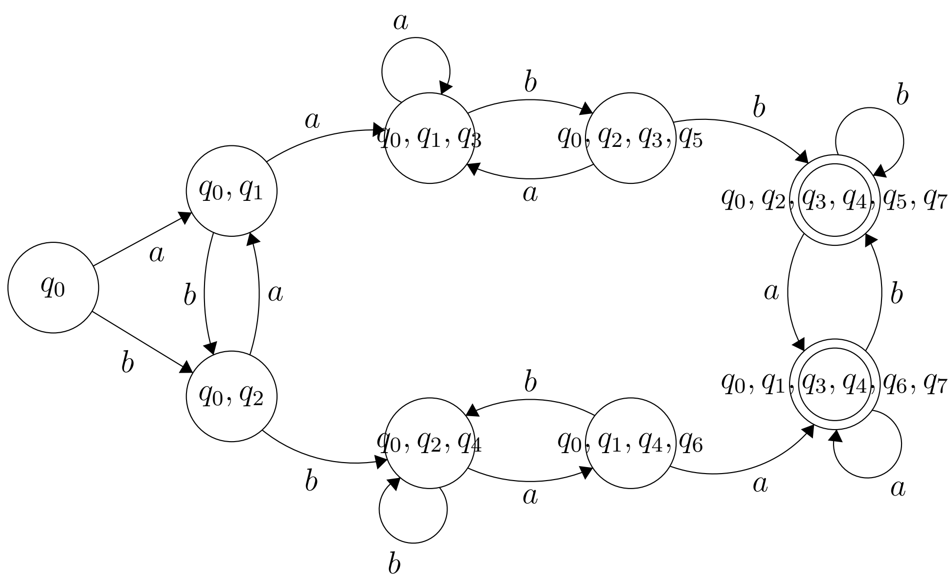
## 1.1 (a ∪ b)*((aa(a ∪ b)*bb )∪ (bb(a ∪ b)*aa))*(a ∪ b)*

## 1.2

$L = \{Q,\ \Sigma,\ \delta,\ s,\ F\}$
$Q = \{\ q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$
$\Sigma = \{a, b\}$
$\delta = \{(q_0, a, q_0), (q_0, b, q_0), (q_0, a, q_1), (q_0, b, q_2), (q_1, a, q_3),$
$(q_2, b, q_4), (q_3, a, q_3), (q_3, b, q_3), (q_3, b, q_5), (q_4, a, q_4),$
$(q_4, b, q_4), (q_4, a, q_6), (q_5, b, q_7), (q_6, a, q_7), (q_7, a, q_7), (q_7, b, q_7)\}$
$s = \{q_0\}$
$F = \{q_7\}$



**1.3** $(\{q_0\}, a) = \{q_0, q_1\})$
$(\{q_0\}, b) = \{q_0, q_2\})$
$(\{q_0, q_1\}, a) = \{q_0, q_1, q_3\})$
$(\{q_0, q_1\}, b) = \{q_0, q_2\})$
$(\{q_0, q_2\}, a) = \{q_0, q_1\})$
$(\{q_0, q_2\}, b) = \{q_0, q_2, q_4\})$
$(\{q_0, q_1, q_3\}, a) = \{q_0, q_1, q_3\})$
$(\{q_0, q_1, q_3\}, b) = \{q_0, q_2, q_3, q_5\})$
$(\{q_0, q_2, q_4\}, a) = \{q_0, q_1, q_4, q_6\})$
$(\{q_0, q_2, q_4\}, b) = \{q_0, q_2, q_4\})$
$(\{q_0, q_2, q_3, q_5\}, a) = \{q_0, q_1, q_3\})$
$(\{q_0, q_2, q_3, q_5\}, b) = \{q_0, q_2, q_3, q_4, q_5, q_7\})$
$(\{q_0, q_1, q_4, q_6\}, a) = \{q_0, q_1, q_3, q_4, q_6, q_7\})$
$(\{q_0, q_1, q_4, q_6\}, b) = \{q_0, q_2, q_4\})$
$(\{q_0, q_2, q_3, q_4, q_5, q_7\}, a) = \{q_0, q_1, q_3, q_4, q_6, q_7\})$
$(\{q_0, q_2, q_3, q_4, q_5, q_7\}, b) = \{q_0, q_2, q_3, q_4, q_5, q_7\})$
$(\{q_0, q_1, q_3, q_4, q_6, q_7\}, a) = \{q_0, q_1, q_3, q_4, q_6, q_7\})$
$(\{q_0, q_1, q_3, q_4, q_6, q_7\}, b) = \{q_0, q_2, q_3, q_4, q_5, q_7\})$



**1.4** **For DFA,**
$(q_0, bbabb) \vdash ((q_0, q_2), babb) \vdash ((q_0, q_2, q_4), abb) \vdash ((q_0, q_1, q_4, q_6), bb) \vdash ((q_0, q_2, q_4), b) \vdash ((q_0, q_2, q_4), e)$ $q_4$ **is not a final state the word is not accepted by the automaton.**

**For NFA we have few possible alternatives,**
$(q_0, bbabb) \vdash (q_2, babb) \vdash (q_4, abb) \vdash (q_4, bb) \vdash (q_4, b) \vdash (q_4, e)$. **Not a final state. Automata does not accept the word.**
$(q_0, bbabb) \vdash (q_2, babb) \vdash (q_4, abb) \vdash (q_6, bb)$. **No way to go, automata does not accept the word.**
$(q_0, bbabb) \vdash (q_0, babb) \vdash (q_2, abb)$. **No way to go, automata does not accept the word.**
$(q_0, bbabb) \vdash (q_0, babb) \vdash (q_0, abb) \vdash (q_1, bb)$. **No way to go, automata does not accept the word.**
$(q_0, bbabb) \vdash (q_0, babb) \vdash (q_0, abb) \vdash (q_0, bb) \vdash (q_2, b) \vdash (q_4, e)$ . **Not a final state. Automata does not accept the word.**
$(q_0, bbabb) \vdash (q_0, babb) \vdash (q_0, abb) \vdash (q_0, bb) \vdash (q_0, b) \vdash (q_2, e)$. **Not a final state. Automata does not accept the word.**
$(q_0, bbabb) \vdash (q_0, babb) \vdash (q_0, abb) \vdash (q_0, bb) \vdash (q_0, b) \vdash (q_0, e)$. **Not a final state. Automata does not accept the word.**

# 2

**2.1** **The complement of** $L_1$ **is** $\{\ a^m b^n \mid m \leq n$ **and** $m, n \in \mathbb{N}\}$
**Let** $a^m = xv$ **and** $b^n = z$ **so that** $m <= n$
**and** $a^t = x, a^h = v, b^n = z$ **such that** $x + h = m$. **We can pump the v enough to have** $t + h * p > n$ **so** $L_2$ **is not regular from the pumping theorem.**

**2.2** $L_4 = \{\ a^n b^n \mid n \in \mathbb{N}^+\}$
**Let** $a^n = xv$ **and** $b^n = z$ **so that** $n <= n$
**and** $a^{n-1} = x, a = v, b^n = z$. **For** $xv^iz$ **and** $i > 0$, **word is not in language.So by pumping theorem** $L_4$ **is not regular.**
$L^5$ **has regular expression** $a^*b^*$ **since m and n can be any natural number, and** $L_5$ **is regular since if a language has regular expression it is a regular language. Same Thing apply to** $L^6$ **so it is regular too.**
**Also** $L_4 \subset L_5$ **for if** $m = n$ **in** $L_5$, $L_5 = L_4$ **so** $L_4 \cup L_5$ **is regular and regular languages are closed under union so** $(L_4 \cup L_5) \cup L_6$ **is regular.**