$$(\boldsymbol{Hw\,01\ \ Question-2})$$

$$\epsilon<10^{-2}\quad p_0=1\quad on[1,2]$$
$$f(x)=x^4-3x^2-3=0$$
$$x^4=3x^2+3$$
$$g(x_{(n+1)})=(3x_{(n)}^2+3)^{(1/4)}$$
$$g'(x)=\frac{(3x)}{(2(3x^2+3)^{(3/4)})}\quad |(g'(x))|\leq k\leq 1$$

$\rightarrow i=0 \qquad p_0=1$

$\rightarrow i=1 \qquad P_1=g(p_0)=1.5651 \quad |(p_1-p_0)|>\epsilon \quad then\ \ continue$

$\rightarrow i=2 \qquad p_2=g(p_1)=1.7936 \quad |(p_2-p_1)|>\epsilon \quad then\ \ continue$

$\rightarrow i=3 \qquad p_3=g(p_2)=1.8860 \quad |(p_3-p_2)|>\epsilon \quad then\,continue$

$\rightarrow i=4 \qquad p_4=g(p_3)=1.9229 \quad |(p_4-p_3)|>\epsilon \quad then\,continue$

$\rightarrow i=5 \qquad p_5=g(p_4)=1.9375 \quad |(p_5-p_4)|>\epsilon \quad then\,continue$

$\rightarrow i=6 \qquad p_6=g(p_5)=1.9433 \quad |(p_6-p_5)|<\epsilon \quad so\,the\,root\,is\,p_6=1.9433$

$$(\boldsymbol{The\,theorical\,number\,of\,itereations:})$$

$$|(p_n-p)|\quad is\,10^{-2}\,that\,is\,given\,so$$
$$|(p_n-p)|\leq\frac{k^n}{(1-k)}|(p_1-p_0)|\quad for\,all\,n\geq 1$$
$$g'(1)=0.391\quad \text{and}\quad g'(2)=0.393\quad let's\quad say\quad k=0.393\leq 1$$

$\rightarrow 10^{-2}\leq\frac{(0.393)^n}{(1-0.393)}(|(1.565-1)|)$

$\rightarrow 10^{-2}\leq(0.393)^n(0.931)$

$\rightarrow \frac{10^{-2}}{0.931}\leq(0.393)^n$

$\rightarrow \log(0.0107)\leq\log((0.393)^n)$

$\rightarrow -1.969\leq n.(-0.406)$

$\rightarrow n\geq 4.854\quad so\quad the\quad theroical\quad number\quad of\quad iterations\quad is\quad 5.$