

(Hw01 Question-3) for Question 5 on book

Use the Newton's method to find solutions accurate within 10^{-4} for following problems

a. $x^3 - 2x^2 - 5 = 0$ on interval $[1, 4]$ $f(x) = x^3 - 2x^2 - 5$ $f'(x) = 3x^2 - 4x$ $\epsilon = 10^{-4}$ assume $p_0 = 2$

$$\rightarrow p_1 = p_0 - \frac{f(p_0)}{f'(p_0)} = 2 - \frac{(-5)}{4} = 3.25 \quad |(p_1 - p_0)| > \epsilon$$

$$\rightarrow p_2 = 3.25 - \frac{(8.2031)}{(18.6875)} = 2.811 \quad |(p_2 - p_1)| > \epsilon$$

$$\rightarrow p_3 = 2.811 - \frac{(1.4083)}{(12.4612)} = 2.698 \quad |(p_3 - p_2)| > \epsilon$$

$$\rightarrow p_4 = 2.698 - \frac{(0.0809)}{(11.0456)} = 2.69068 \quad |(p_4 - p_3)| > \epsilon$$

$$\rightarrow p_5 = 2.691 - \frac{(0.0039)}{(8.7228)} = 2.69055 \quad |(p_5 - p_4)| < \epsilon \text{ so the root is } p_5 = 2.69055$$

b. $x^3 + 3x^2 - 1 = 0$ on interval $[-3, -2]$ $f(x) = x^3 + 3x^2 - 1$ $f'(x) = 3x^2 + 6x$ $\epsilon = 10^{-4}$ $p_0 = -3$

$$\rightarrow p_1 = p_0 - \frac{f(p_0)}{f'(p_0)} = -2.888889 \quad |(p_1 - p_0)| > \epsilon$$

$$\rightarrow p_2 = p_1 - \frac{f(p_1)}{f'(p_1)} = -2.87945 \quad |(p_2 - p_1)| > \epsilon$$

$$\rightarrow p_3 = p_2 - \frac{f(p_2)}{f'(p_2)} = -2.87939 \quad |(p_3 - p_2)| \leq \epsilon \text{ so the root is } p_3 = -2.87939$$

c. $x - \cos(x)$ on interval $[0, \frac{\pi}{2}]$ $f(x) = x - \cos(x)$ $f'(x) = 1 + \sin(x)$ $\epsilon = 10^{-4}$ let's say $p_0 = 0$

$$\rightarrow p_1 = p_0 - \frac{f(p_0)}{f'(p_0)} = 1.00 \quad |(p_1 - p_0)| > \epsilon$$

$$\rightarrow p_2 = p_1 - \frac{f(p_1)}{f'(p_1)} = 0.24964 \quad |(p_2 - p_1)| > \epsilon$$

$$\rightarrow p_3 = p_2 - \frac{f(p_2)}{f'(p_2)} = 0.01125 \quad |(p_3 - p_2)| > \epsilon$$

$$\rightarrow p_4 = p_3 - \frac{f(p_3)}{f'(p_3)} = 0.73909 \quad |(p_1 - p_0)| \leq \epsilon \text{ so the root is } p_4 = 0.73909$$

d. $x - 0.8 - 0.2 \sin(x)$ on interval $[0, \frac{\pi}{2}]$ $f(x) = x - 0.8 - 0.2 \sin(x)$ $f'(x) = 1 - 0.2 \cos(x)$ $\epsilon = 10^{-4}$ $p_0 = 0$

$$\rightarrow p_1 = p_0 - \frac{f(p_0)}{f'(p_0)} = 1.00 \quad |(p_1 - p_0)| > \epsilon$$

$$\rightarrow p_2 = p_1 - \frac{f(p_1)}{f'(p_1)} = 0.9645 \quad |(p_2 - p_1)| > \epsilon$$

$$\rightarrow p_3 = p_2 - \frac{f(p_2)}{f'(p_2)} = 0.9643 \quad |(p_3 - p_2)| > \epsilon$$

$$\rightarrow p_4 = p_3 - \frac{f(p_3)}{f'(p_3)} = 0.9643 \quad |(p_1 - p_0)| \leq \epsilon \text{ so the root is } p_4 = 0.9643$$