1) $T_1(n) = 3n^4 + 3n^3 + 1 \in O(n^4)$

$T_2(n) = 3^n \in O(3^n)$

$T_3(n) = (n-2)! \in O(n!)$

$T_4(n) = \ln^2 n = \ln \ln n \in O(\log \log n)$

$T_5(n) = 2^{2n} = (2^2)^n = 4^n \in O(4^n)$

$T_6(n) = n^{1/3} \in O(n^{1/3})$

$\Rightarrow$ According to these asymptotic complexity, the order is like that:

$$\boxed{T_4 < T_6 < T_1 < T_2 < T_5 < T_3}$$ //

Property

$f(n) \in o(g(n)) \Rightarrow f(n) \in O(g(n))$

Then Prove;

$T_4 \stackrel{?}{=} O(T_6)$

$$\lim_{n\to\infty} \frac{T_4}{T_6} = \lim_{n\to\infty} \underbrace{\frac{\ln \ln n}{n^{1/3}}}_{\frac{\infty}{\infty}} = \lim_{n\to\infty} \frac{\frac{1}{n \cdot \ln n}}{\frac{1}{3} \cdot n^{-2/3}} = \lim_{n\to\infty} \frac{3}{n^{1/3} \cdot \ln n} = 0$$ then $T_4 \in o(T_6) \Rightarrow T_4 \in O(T_6)$ ✓

$T_6 \stackrel{?}{=} O(T_1)$

$$\lim_{n\to\infty} \frac{T_6}{T_1} = \lim_{n\to\infty} \underbrace{\frac{n^{1/3}}{n^4}}_{\frac{\infty}{\infty}} = \lim_{n\to\infty} \frac{\frac{1}{3} \cdot n^{-2/3}}{4 \cdot n^3} = \lim_{n\to\infty} \frac{1}{4 \cdot n^3 \cdot n^{2/3}} = 0$$ then $T_6 \in o(T_1) \Rightarrow T_6 \in O(T_1)$ ✓

$T_1 \stackrel{?}{=} O(T_2)$

$$\lim_{n\to\infty} \frac{T_1}{T_2} = \lim_{n\to\infty} \underbrace{\frac{n^4}{3^n}}_{\frac{\infty}{\infty}} = \lim_{n\to\infty} \underbrace{\frac{4 \cdot n^3}{3^n \cdot \ln 3}}_{\frac{\infty}{\infty}} = \lim_{n\to\infty} \underbrace{\frac{4 \cdot 3 \cdot n^2}{(\ln 3)^2 \cdot 3^n}}_{\frac{\infty}{\infty}} = \lim_{n\to\infty} \underbrace{\frac{4 \cdot 3 \cdot 2 \cdot n}{(\ln 3)^3 \cdot 3^n}}_{\frac{\infty}{\infty}} = \lim_{n\to\infty} \frac{4 \cdot 3 \cdot 2 \cdot 1}{(\ln 3)^4 \cdot 3^n} = 0$$ then $T_1 \in o(T_2) \Rightarrow T_1 \in O(T_2)$ ✓

$T_2 \stackrel{?}{=} O(T_5)$

$$\lim_{n\to\infty} \frac{T_2}{T_5} = \lim_{n\to\infty} \frac{3^n}{4^n} = 0$$ then $T_2 \in o(T_5) \Rightarrow T_2 \in O(T_5)$ ✓

↳ Growth rate is grother than 3^n growth rate.

$T_5 \stackrel{?}{=} O(T_3)$

$$\lim_{n\to\infty} \frac{T_5}{T_3} = \lim_{n\to\infty} \underbrace{\frac{4^n}{n!}}_{\frac{\infty}{\infty}} = \lim_{n\to\infty} \frac{4^n}{\sqrt{2\pi n} \cdot \left(\frac{n}{e}\right)^n} = 0$$ then $T_5 \in o(T_3) \Rightarrow T_5 \in O(T_3)$ ✓

$n! \cong \sqrt{2\pi n} \cdot \left(\frac{n}{e}\right)^n$

↳ Stirling Formula

2

a) This algorithm finds the nearest value that in fruits to the result of (max + min) // 2. ⇒ max : The maximum value in fruits
min : The minimum value in fruits.

- The variable called orange holds the return value that I mentioned above.
- The variable called watermelon holds the maximum value in fruits array.
- The variable called plum holds the minimum value in fruits array.
- The variable called orangeTime for controlling the while loop.

b) i ⇒ Worst case scenario

For the worst case the minimum value must be at the end of the fruits array. According to the this scenario the complexity of worst case is $O(n)$.

$W(n) = O(n)$ //

j ⇒ Best case Scenario

For the best case scenario the fruits array must be not shifted. According to this scenario the complexity of best case is $\Omega(n)$.

$B(n) = \Omega(n)$ //

k ⇒ Average Case Scenario

The worst case (upper bound) is $O(n)$.

The best case (lower bound) is $\Omega(n)$

So the Average case complexity is $\Theta(n)$.

$A(n) = \Theta(n)$ //

(3)

3-a) $\sum_{i=0}^{n-1}(i^2+1)^2 \Rightarrow \int_0^{n-1}(i^4+2i^2+1)di \leq f(n) \leq \int_1^n (i^4+2i^2+1)\,di$

$\frac{i^5}{5}+2.\frac{i^3}{3}+i\Big|_0^{n-1} \leq f(n) \leq \frac{i^5}{5}+2.\frac{i^3}{3}+i\Big|_1^n$

$\frac{(n-1)^5}{5}+\frac{2}{3}.(n-1)^3+(n-1) \leq f(n) \leq \frac{n^5}{5}+\frac{2}{3}n^3+n-\left(\frac{1}{5}+\frac{2}{3}+1\right)$

$\boxed{f(n)\in\Theta(n^5)}$

3-b) $\sum_{i=2}^{n-1}\log i^2$

$\Rightarrow 2.\int_1^{n-1}\log i.di \leq f(n) \leq 2.\int_2^n \log i.di$

$2.(i.\log i-i)\Big|_1^{n-1} \leq f(n) \leq 2.(i.\log i-i)\Big|_2^n$

$2.((n-1).\log(n-1)-(n-1))+2.1 \leq f(n) \leq 2.(n.\log n-n)-(2.(2.\log 2-2))$

$\boxed{f(n)\in\Theta(n.\log n)}$

3-c)

c) $\sum_{i=1}^{n}(i+1).2^{i-1}$

$\Rightarrow \int_0^n (i+1).2^{i-1}.di \leq f(n) \leq \int_1^{n+1}(i+1).2^{i-1}.di$

$i+1=u \qquad 2^{i-1}.di=dv$

$di=du \qquad \frac{2^{i-1}}{\ln 2}=v$

$\frac{(i+1).2^{i-1}}{\ln 2}\Big|_0^n-\frac{2^{i-1}}{(\ln 2)^2}\Big|_0^n \leq f(n) \leq \frac{(i+1).2^{i-1}}{\ln 2}\Big|_1^{n+1}-\frac{2^{i-1}}{(\ln 2)^2}\Big|_1^{n+1}$

$\boxed{f(n)\in\Theta(n.2^n)}$

3-d) $\sum_{i=0}^{n-1}\sum_{j=0}^{i-1}(i+j)$

$\sum_{j=0}^{i-1}(i+j)=\underbrace{(i+0)+(i+1)+(i+2)+\cdots+(i+(i-1))}_{(i-1)\text{ times}} \Rightarrow \boxed{\sum_{i=1}^{n-1}\frac{3}{2}.(i-1).i}$

$=(i-1).i+\frac{(i-1).i}{2}=\frac{3}{2}.(i-1).i$

$\frac{3}{2}\int_0^{n-1}(i^2-i).di \leq f(n) \leq \frac{3}{2}\int_1^n(i^2-i).di$

$\frac{3}{2}\left[\frac{i^3}{3}-\frac{i^2}{2}\right]\Big|_0^{n-1} \leq f(n) \leq \frac{3}{2}\left[\frac{i^3}{3}-\frac{i^2}{2}\right]\Big|_1^n$

$\rightarrow \frac{3}{2}\left(\frac{(n-1)^3}{3}-\frac{(n-1)^2}{2}\right) \leq f(n) \leq \frac{3}{2}\left(\frac{n^3}{3}-\frac{n^2}{2}-\frac{1}{3}+\frac{1}{2}\right)$

$\boxed{f(n)\in\Theta(n^3)}$

3-a

Code

```
def Q3-a (n):

    return [i for i in range(n*n*n*n*n)]
```

3-d

Code

```
def Q3-d (n):
    count =0
    for i in range(n*n*n):
        count += 1
    return count
```

4)
```
int fun(int n){
    int count = 0;
    for (int i=n; i>0; i/=2)
        for(int j=0; j<1; j++)
            count +=1;
    return count;
}
```

$\Longrightarrow \underbrace{n + \frac{n}{2} + \frac{n}{4} + \cdots + 1}$

Summation representation of this code is:

$$\boxed{\sum_{i=0}^{\log_2 n} \frac{n}{2^i} = n + \frac{n}{2} + \frac{n}{4} + \cdots + 1}$$

Then find the complexity of this code according to this summation representation by using integral method.

$$\int_0^{\log_2 n} \frac{n}{2^i} \cdot di \leq f(n) \leq \int_1^{\log_2 n+1} \frac{n}{2^i} \cdot di$$

$$n \cdot \int_0^{\log_2 n} 2^{-i} \cdot di \leq f(n) \leq n \cdot \int_1^{\log_2 (n+1)} 2^{-i} \cdot di$$

$$n \cdot \left[-\ln 2 \cdot 2^{-i} \Big|_0^{\log_2 n} \right] \leq f(n) \leq n \cdot \left[-\ln 2 \cdot 2^{-i} \Big|_1^{\log_2 n+1} \right]$$

$$n \cdot \left[-\ln 2 \cdot 2^{-\log_2 n} + \ln 2 \cdot 2^0 \right] \leq f(n) \leq n \cdot \left[-\ln 2 \cdot 2^{-\log_2 n+1} + 2^{-1} \right]$$

$$n \cdot \left[-\ln 2 \cdot \frac{1}{n} + \ln 2 \right] \leq f(n) \leq n \cdot \left[-\ln 2 \cdot \frac{1}{n+1} + \frac{1}{2} \right]$$

$$f(n) \in \Theta(n)$$

5)

5.a) $n^3 \overset{?}{\in} O(3^{2n})$

$f(n) = n^3 \quad , \quad g(n) = (3^2)^n = 9^n$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{n^3}{9^n} = \lim_{n\to\infty} \frac{3n^2}{\ln 9 . 9^n} = \lim_{n\to\infty} \frac{6n}{(\ln 9)^2 . 9^n} = \lim_{n\to\infty} \frac{6}{(\ln 9)^3 . 9^n} = 0$$

$\frac{\infty}{\infty} \quad \frac{\infty}{\infty} \quad \frac{\infty}{\infty}$

Then $f(n) \in o(g(n)) \Rightarrow f(n) \in O(g(n))$

5.b)

$n^3 \in O(3^{2n})$ ✓

$n \overset{?}{\in} o(\log \log n)$

$f(n) = n \quad , g(n) = \log \log n \Rightarrow$ we can assume that $g(n) = \ln(\ln n)$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{n}{\ln(\ln n)} = \lim_{n\to\infty} \frac{1}{\frac{1}{n . \ln(n)}} = n . \ln(n)$$

$\frac{\infty}{\infty}$

Then $f(n) \in \Theta(n.\log n)$

$n \notin o(\log \log n)$ X

5.c) $n^2 . \log^2 n \overset{?}{\in} O(n!)$

$$\lim_{n\to\infty} \frac{n^2 . \log(\log n)}{n!} = \lim_{n\to\infty} \frac{n^2 . \log(\log n)}{\sqrt{2\pi n} . \left(\frac{n}{e}\right)^n} = \ldots\ldots$$

$\frac{\infty}{\infty} \quad \frac{\infty}{\infty}$

5.d) $\sqrt{10n^2 + 7n + 3} \overset{?}{\in} \Theta(n)$

$f(n) = \sqrt{10n^2 + 7n + 3}$

$g(n) = n$

ignore this part

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{\sqrt{10n^2 + 7n + 3}}{n} = \lim_{n\to\infty} \frac{\sqrt{10} . \cancel{n}}{\cancel{n}} = \sqrt{10}$$

constant

$f(n) \in \Theta(g(n))$

$\sqrt{10n^2 + 7n + 3} \in \Theta(n)$ ✓