Exercise:

Many mathematical problems involve solving equations. While linear equations can be solved easily, nonlinear ones cannot. A nonlinear equation can be written as

$$f(x) = 0$$

for a suitably chosen function *f*.

For example, if we want to find *x* so that $\tanh(x) = \frac{x}{3}$, we could instead choose $f(x) = \tanh(x) - \frac{x}{3} = 0$ and solve equation $f(x) = 0$. Finding solutions to the equation is called root-finding (a "root" being a value of x for which the equation is satisfied).

We almost have all the tools we need to build a basic and powerful root-finding algorithm by using an iterative method. This means that there is a basic mechanism for taking an approximation to the root, and finding a better one. After enough iterations of this, one is left with an approximation that can be as good as you like (you are also limited by the accuracy of the computation). Iterative methods entail doing the exact same thing over and over again. This is perfect for a computer.
The actual iteration starts from an approximation x_n at the n_{th} step and defines the next one, x_{n+1}:

$$x_{n+1} = x_n - f(x_n)\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \qquad (1)$$

Notice that we only need to evaluate the function *f* at x_{n+1}. When we calculate the equation multiple times (x_3 is calculated by using x_1 and x_2. In the next iteration, x_4 is calculated by using x_2 and x_3, and so on...), after some iterations, consecutive *x* values will converge to each other.

You are required to:

- ✓ Write a function with the name **CalcTerm_ID1234567** that takes x_n and x_{n-1} as input arguments, calculates equality (1), and returns x_{n+1} as output. Take $x_1 = 2$ and $x_2 = 10$ as initial values.
- ✓ Write a main script for root finding algorithm explained above. Use iterations structure, and call your user-defined function **CalcTerm_ID1234567.**

Solve the equation (2) and find the root of the function (You are not required to plot the function!).

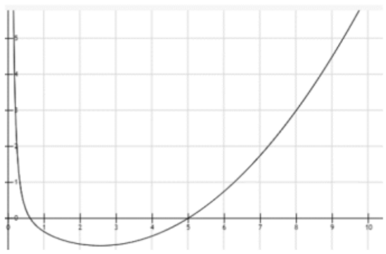

Figure 1 Plot of the given function

$$f(x) = \frac{(x^4 - 5x^3 + 1)}{8x^2} \qquad (2)$$

Use different stopping criteria, 0.0001 and 0.00000001 (which is the difference between two consecutive x values).

- How many iterations do you need until the solution seems to converge? What did it converge to?
- If you try different x_0 and x_1 values such that they are closer to each other, is convergence of the algorithm affected?

You can discuss your results in your script as comment. DO NOT take any inputs from the user. Submit both the main script and the function file.