Exercise:

For large n,

$$A_n = 1 + \frac{-1/3}{3} + \cdots + \frac{(-3)^{-n}}{2n+1} = \sum_{k=0}^{n} \frac{(-3)^{-k}}{2k+1} \cong \frac{\pi}{\sqrt{12}}$$

$$T_n = 1 + \frac{1}{2^2} + \cdots + \frac{1}{n^2} = \sum_{k=1}^{n} \frac{1}{k^2} \cong \frac{\pi^2}{6}$$

$$R_n = 1 - \frac{1}{3} + \cdots + \frac{(-1)^{n+1}}{2n-1} = \sum_{k=1}^{n} \frac{(-1)^{k+1}}{2k-1} \cong \frac{\pi}{4}$$

gives an estimate for π:

$$\delta_n = A_n\sqrt{12}$$

$$\tau_n = \sqrt{6T_n}$$

$$\rho_n = 4R_n$$

a) Write a script that computes the value of $|\pi - \delta_n|$, $|\pi - \tau_n|$ and $|\pi - \rho_n|$. These values correspond to your errors. In the script, plot these values for $n = 10, 20, \ldots, 1000$ <u>on one figure</u> (x axis: n values, y axis: errors). Format the figure appropriately (axes, titles, legend, etc.). You can use arrays.

b) Now, consider that each element of the sum is rounded up to <u>5 decimals</u> and the summation is performed with the rounded elements. Observe the differences $|\delta'_n - \delta_n|$, $|\tau'_n - \tau_n|$ and $|\rho'_n - \rho_n|$. These values correspond to absolute errors. Using the formulas below,

$$\delta\_RE_n = \frac{|\delta'_n - \pi|}{\pi}$$

$$\tau\_RE_n = \frac{|\tau'_n - \pi|}{\pi}$$

$$\rho\_RE_n = \frac{|\rho'_n - \pi|}{\pi}$$

compute **<u>the relative errors</u>** and **plot** these values as in part (a).