Sequence-to-sequence models: Part I

Recurrent Neural Networks, Long short-term memories (LSTMs)

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Outline

Language modelling

- Language models provide a probability distribution of sequences of *tokens* in a sentence
- Given a *vocabulary* $V = \{w_1, \ldots, w_{|V|}\}$ of words, a language model assigns a probability to a sequence of tokens, $y_1, \ldots, y_t$: $p(y_1, \ldots, y_t)$
- Common way to write this in terms of conditional probability

$$p(y_1, \ldots, y_t) = p(y_1) \cdot p(y_2|y_1) \cdot p(y_3|y_2, y_1) \cdots p(y_t|y_{1:t-1}) \tag{1}$$

$$= \prod_{i=1}^{t} p(y_i|y_{1:i-1}) \tag{2}$$

- Text generation algorithm:
  - For $i = 1, \ldots, t$, sample next word $y_i \sim p(y_i|y_{1:i-1})$
  - Note: Some people introduce a *temperature* parameter T to control the randomness of the language model and sample $y_i \sim p(y_i|y_{1:i-1})^{\frac{1}{T}}$
- Question: how do we compute $p(y_i|y_{1:i-1})$?

n-gram models

- Prior to deep learning, n-gram language models [Brown et al., 1992; Brants et al., 2007] were dominant
- Markov assumption: prediction of a token y_t *only* depends on the preceding $(n-1)$ tokens, $y_{t-(n-1):t-1}$, rather than the full history, i.e. we set

$$
\begin{aligned}
p(y_t | y_{1:t-1}) &= p(y_t | y_{t-(n-1)}, \dots, y_{t-1}), \text{ (by assumption)} \quad (3)\\
&= \frac{p(y_{t-(n-1)}, \dots, y_{t-1}, y_t)}{p(y_{t-(n-1)}, \dots, y_{t-1})}, \text{ (by conditional probability)} \quad (4)
\end{aligned}
$$

- To compute these n-gram and $n-1$-gram probabilities, we count:

$$
p(y_t | y_{1:t-1}) \approx \frac{\text{count}(y_{t-(n-1)}, \dots, y_{t-1}, y_t)}{\text{count}(y_{t-(n-1)}, \dots, y_{t-1})} \quad (5)
$$

- These models are fixed window models of size n: considers the last $(n-1)$ tokens to predict the next

Problems with n-gram models

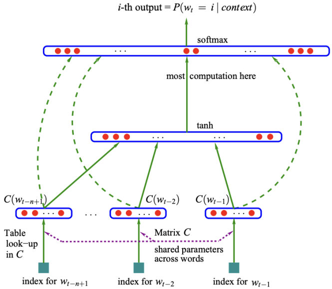
- Sparsity: If n is too large, it's statistically infeasible to get good estimates of the probabilities

$$p(\text{Ryan}|\text{the bug was created by}) = \frac{\text{count(the bug was created by Ryan)}}{\text{count(the bug was created by)}} \tag{6}$$

  - Problem 1: if "the bug was created by Ryan" never occurred, then this is given probability 0 $\rightarrow$ solved by *smoothing* (adding small δ to each word count)
  - Problem 2: if "the bug was created by" never occurred, we have division by 0 $\rightarrow$ solved by *backoff* (condition on $n-2, \ldots$ words)
  - Increasing n makes sparsity worse: typically can only go to $n = 5$
- Storage: you need to store the counts for all n-grams you saw in the corpus (and the $n-1$ grams)
  - Increasing n or the corpus increases model size

Neural language models

- Recall the language model by Bengio et al. [2003]:



- Model could also learn *distributed representation of words*, i.e. can produce word embeddings
- Context width still bound by n, but statistically feasible to estimate neural language models for much larger values of n
- Main challenge: training neural networks were more computationally expensive
  - Bengio et al. [2003] trained network on 14 million (14×10^6) words
  - Brants et al. [2007] trained 5-gram model on 2 trillion tokens (2×10^{12})

Fixed-window neural language model

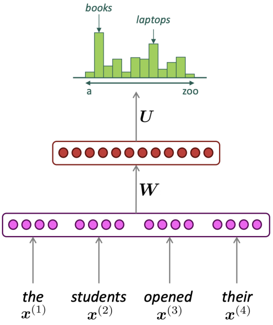
- Slightly simplified version of Bengio et al. [2003]:

output distribution
$\hat{\boldsymbol{y}} = \mathrm{softmax}(\boldsymbol{U}\boldsymbol{h} + \boldsymbol{b}_2) \in \mathbb{R}^{|V|}$

hidden layer
$\boldsymbol{h} = f(\boldsymbol{W}\boldsymbol{e} + \boldsymbol{b}_1)$

concatenated word embeddings
$\boldsymbol{e} = [\boldsymbol{e}^{(1)}; \boldsymbol{e}^{(2)}; \boldsymbol{e}^{(3)}; \boldsymbol{e}^{(4)}]$

words / one-hot vectors
$\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}, \boldsymbol{x}^{(3)}, \boldsymbol{x}^{(4)}$



[1]

  - No more sparsity problems
  - Don't need to store all observed *n*-grams
- But: fixed window is too limiting - enlarging window enlarges *W*

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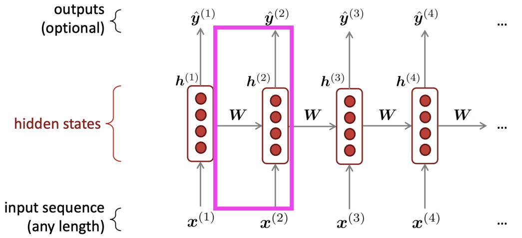
[1] Manning et al. [2017]

Recurrent Neural Networks

- RNNs [Hopfield, 1982; Rumelhart et al., 1985] are capable of conditioning the model on *all* previous tokens (in theory)



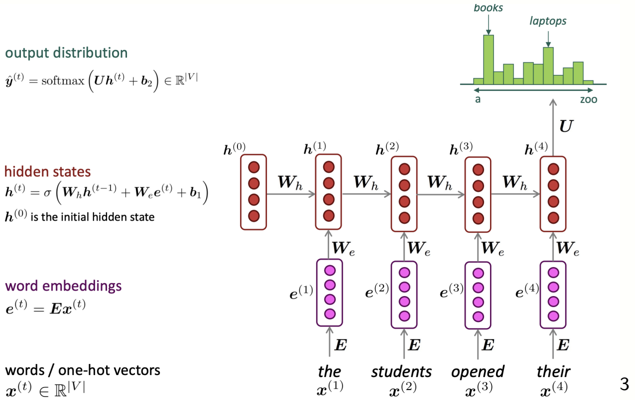
[2]

- Core idea: the hidden layer from previous timestep provides a form of memory or context that informs decisions to be made later in the sequence
  - In theory, context embodied in previous hidden layers can provide information extending to the beginning of the sequence
- The same weights are applied at every timestep
- The model size does not increase for longer input sequence lengths

[2] Manning et al. [2017]

Recurrent Neural Networks for language modelling

- Instead of a fixed-window approach, sequences are processed by presenting one item/token at a time to the network: (image taken from Stanford CS244n slides)



[3] Manning et al. [2017]

Training RNN language models: Cross-Entropy loss

- Given a large corpus of text with a sequence of words $\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(T)}$, we feed this into a RNN-LM and compute an output distribution $\hat{y}^{(t)}$ for every time step t
  - $\hat{\boldsymbol{y}}^{(t)}$ is a probability distribution for every word, conditional on all the words prior to t
  - We know the next word: $\boldsymbol{y}^{(t)}$ is a one-hot encoding of $\boldsymbol{x}^{(t+1)}$
- Use the cross-entropy loss between $\hat{\boldsymbol{y}}^{(t)}$ and the *true* word $\boldsymbol{y}^{(t)}$

$$J^{(t)}(\boldsymbol{\theta}) = \mathrm{CE}(\boldsymbol{y}^{(t)}, \hat{\boldsymbol{y}}^{(t)}) = -\sum_{w \in V} \boldsymbol{y}_w^{(t)} \log \hat{\boldsymbol{y}}_w^{(t)} = -\log \hat{\boldsymbol{y}}_{\boldsymbol{x}^{(t+1)}}^{(t)} \tag{7}$$

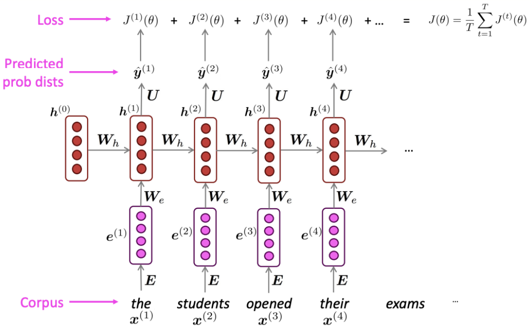
- Average over the loss for the entire corpus:

$$J(\theta) = \frac{1}{T} \sum_{t=1}^{T} J^{(t)}(\theta) = \frac{1}{T} \sum_{t=1}^{T} -\log \hat{\boldsymbol{y}}_{\boldsymbol{x}^{(t+1)}}^{(t)} \tag{8}$$

  - Typically computing loss and gradients for the whole corpus is too expensive
  - In practice, we split the corpus into sentences or documents and use them as *batches* in stochastic gradient descent

Training RNN language models: teacher-forcing

- Teacher-forcing: when making a prediction of the word at time t, $\boldsymbol{y}^{(t)}$, we use the previous words as input to the RNN-LM
  - Rather than using what the model has predicted so far



[4]

- But when generating text with RNN-LMs, the sampled output at time t, $\boldsymbol{y}^{(t)}$ does become the next step's input

[4] Manning et al. [2017]

Evaluating Language Model performance

- Standard evaluation metric for language models is perplexity

$$\text{perplexity} = \exp(J(\theta)) \tag{9}$$

  - Is a measure of confusion (or uncertainty) in the model
  - *Lower* perplexity is considered better and implies more confidence in predicting the next word in the sequence (compared to the ground truth)

Problems with RNNs

- Computation is slow
  - The model is inherently sequential and cannot be parallelised easily
- Difficulty in training
  - In practice, it is difficult to access information from many steps back due to vanishing or exploding gradients
    - During backpropagation, hidden layers are subject to repeated multiplications (determined by the length of the sequence)
    - Gradients often get gradually drive to zero (vanishing gradients)
  - The hidden layers/states and weights are asked to do a *lot* of work:
    - Provide information useful for the current decision
    - Update and carry forward information required for future decisions

LSTMs

- Long short-term memory (LSTM) networks [Hochreiter and Schmidhuber, 1997; Gers et al., 2000] are the most commonly used extension to RNNs
- Core idea: divide context management into two problems:
  1. Remove information that is no longer needed from the context
  2. Add information that is likely to be needed later on in decision making
- LSTMs try to *learn* how to manage context by adding in several specialised neural units
  - They add an explicit context layer along with the usual recurrent hidden layer
  - They add additional gates that can add and remove information to the context
- Gated Recurrent Units (GRUs) [Cho et al., 2014] are simpler alternatives to LSTMs which has fewer parameters and are a bit faster than LSTMs

LSTMs: a summary

- Essentially replaces "vanilla" RNN blocks with LSTM blocks that are able to preserve information over longer timesteps
  - In practice, could preserve information to about 100 timesteps rather than 7 in "vanilla" RNNs
- At step t, there is a hidden state vector $\boldsymbol{h}^{(t)}$ and a context state vector $\boldsymbol{c}^{(t)}$ which together store long-term information
- LSTM network can read, erase and write information from the cell
  - The selection of what gets read/erased/written is controlled by gates
- We'll go through the architecture in the next few slides:
  - Let $\circ$ be the Hadamard product, i.e. the element-wise multiplication operation
  - See "Understanding LSTM Networks" [Olah, 2015] for a bit more of a deeper explanation and walkthrough

LSTMs: architecture and gated activation functions I

To compute the current context state $\boldsymbol{c}^{(t)}$:

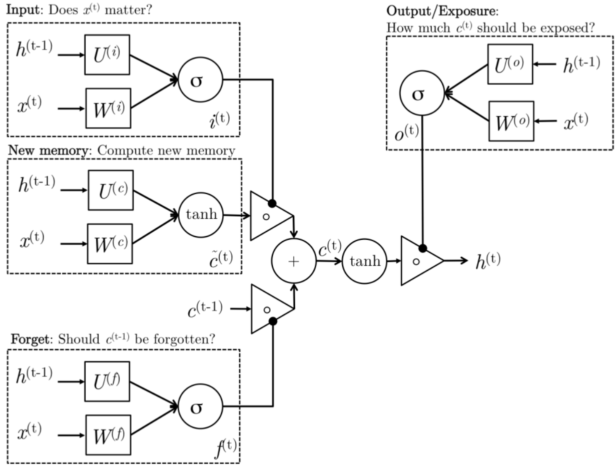
- New memory generation: $\tilde{\boldsymbol{c}}^{(t)} = \tanh(W^{(c)}\boldsymbol{x}^{(t)} + U^{(c)}\boldsymbol{h}^{(t-1)} + \boldsymbol{b}_c)$
  - Use input $\boldsymbol{x}^{(t)}$ and past hidden state $\boldsymbol{h}^{(t-1)}$ to generate new memory $\tilde{\boldsymbol{c}}^{(t)}$ which includes information about the new word
- Input gate: $\boldsymbol{i}^{(t)} = \sigma(W^{(i)}\boldsymbol{x}^{(t)} + U^{(i)}\boldsymbol{h}^{(t-1)} + \boldsymbol{b}_i)$
- Forget gate: $\boldsymbol{f}^{(t)} = \sigma(W^{(f)}\boldsymbol{x}^{(t)} + U^{(f)}\boldsymbol{h}^{(t-1)} + \boldsymbol{b}_f)$
- Final memory generation / context state: $\boldsymbol{c}^{(t)} = \boldsymbol{f}^{(t)} \circ \boldsymbol{c}^{(t-1)} + \boldsymbol{i}^{(t)} \circ \tilde{\boldsymbol{c}}^{(t)}$
  - $\boldsymbol{f}^{(t)}$ assess if the past memory is useful for the current memory cell - *gates* the past context state $\boldsymbol{c}^{(t-1)}$
  - $\boldsymbol{i}^{(t)}$ assesses if new word is important to the new memory - determines if input word is worth preserving and *gates* the new memory $\tilde{\boldsymbol{c}}^{(t)}$

LSTMs: architecture and gated activation functions II

To compute the hidden state $\boldsymbol{h}^{(t)}$ (with the context state $\boldsymbol{c}^{(t)}$):

- Output gate: $\boldsymbol{o}^{(t)} = \sigma(W^{(o)}\boldsymbol{x}^{(t)} + U^{(o)}\boldsymbol{h}^{(t-1)} + \boldsymbol{b}_o)$
- Hidden state: $\boldsymbol{h}^{(t)} = \boldsymbol{o}^{(t)} \circ \tanh(\boldsymbol{c}^{(t)})$
    - $\boldsymbol{o}^{(t)}$ indicates what parts of the memory $\boldsymbol{c}^{(t)}$ needs to be exposed to the hidden state $\boldsymbol{h}^{(t)}$

LSTMs: architecture and gated activation functions

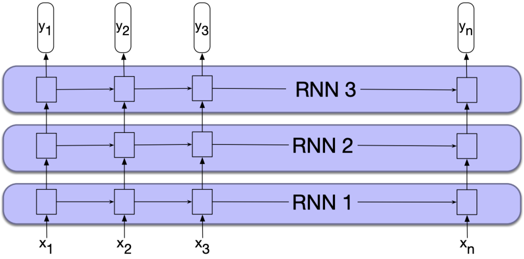


[5]

[5] Manning et al. [2017]

Stacked / Deep RNNs

- We can use the entire sequence of outputs from one RNN as the input of another RNN to create a *stack* of RNNs



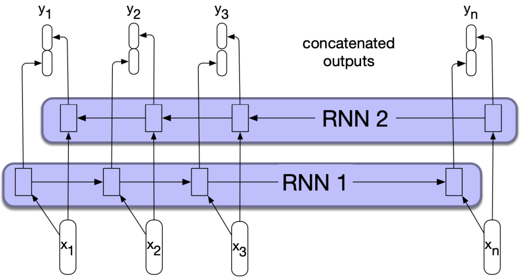
[6]

- Outputs of one layer serves as input to a subsequent layer
- Typically outperforms single-layer networks but increases training cost

[6] Jurafsky and Martin [2019, Chapter 9]

Bidirectional RNNs

- Run two separate RNNs: left-to-right (we've seen above) and right-to-left
- Then concatentate the hidden states from both RNNs
  - Alternatives: element-wise mean or sum



[7]

- Effective if you have access to a full input sentence (to encode a sentence), e.g. sequence classification or sequence labelling
- "BERT": Bidirectional Encoder Representations from Transformers

[7] Jurafsky and Martin [2019, Chapter 9]

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