Sequence-to-sequence models: Part I

Recurrent Neural Networks, Long short-term memories (LSTMs)

Ryan Chan

17 April 2023

Outline

Language modelling

- Language models provide a probability distribution of sequences of *tokens* in a sentence
- Given a *vocabulary* $V = \{w_1, \dots, w_{|V|}\}$ of words, a language model assigns a probability to a sequence of tokens, $y_1, \dots, y_t$: $p(y_1, \dots, y_t)$
- Common way to write this in terms of conditional probability

$$p(y_1, \dots, y_t) = p(y_1) \cdot p(y_2|y_1) \cdot p(y_3|y_2, y_1) \cdots p(y_t|y_{1:t-1}) \tag{1}$$

$$= \prod_{i=1}^{t} p(y_i|y_{1:i-1}) \tag{2}$$

- Text generation algorithm:
  - For $i = 1, \dots, t$, sample next word $y_i \sim p(y_i|y_{1:i-1})$
  - Note: Some people introduce a *temperature* parameter T to control the randomness of the language model and sample $y_i \sim p(y_i|y_{1:i-1})^{\frac{1}{T}}$
- Question: how do we compute $p(y_i|y_{1:i-1})$?

n-gram models

- Prior to deep learning, n-gram language models [Brown et al., 1992; Brants et al., 2007] were dominant
- Markov assumption: prediction of a token y_t *only* depends on the preceding $(n-1)$ tokens, $y_{t-(n-1):t-1}$, rather than the full history, i.e. we set

$$
\begin{aligned}
p(y_t|y_{1:t-1}) &= p(y_t|y_{t-(n-1)}, \dots, y_{t-1}), \text{ (by assumption)} && (3)\\
&= \frac{p(y_{t-(n-1)}, \dots, y_{t-1}, y_t)}{p(y_{t-(n-1)}, \dots, y_{t-1})}, \text{ (by conditional probability)} && (4)
\end{aligned}
$$

- To compute these n-gram and $n-1$-gram probabilities, we count:

$$
p(y_t|y_{1:t-1}) \approx \frac{\text{count}(y_{t-(n-1)}, \dots, y_{t-1}, y_t)}{\text{count}(y_{t-(n-1)}, \dots, y_{t-1})} \quad (5)
$$

- These models are fixed window models of size n: considers the last $(n-1)$ tokens to predict the next

Problems with n-gram models

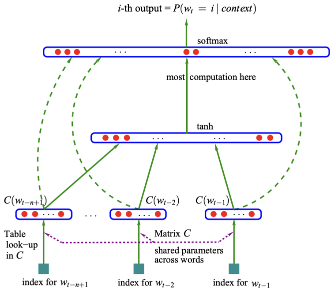
- Sparsity: If n is too large, it's statistically infeasible to get good estimates of the probabilities

$$p(\text{Ryan}|\text{the bug was created by}) = \frac{\text{count(the bug was created by Ryan)}}{\text{count(the bug was created by)}} \tag{6}$$

  - Problem 1: if "the bug was created by Ryan" never occurred, then this is given probability 0 $\rightarrow$ solved by *smoothing* (adding small δ to each word count)
  - Problem 2: if "the bug was created by" never occurred, we have division by 0 $\rightarrow$ solved by *backoff* (condition on $n-2, \ldots$ words)
  - Increasing n makes sparsity worse: typically can only go to $n = 5$
- Storage: you need to store the counts for all n-grams you saw in the corpus (and the $n-1$ grams)
  - Increasing n or the corpus increases model size

Neural language models

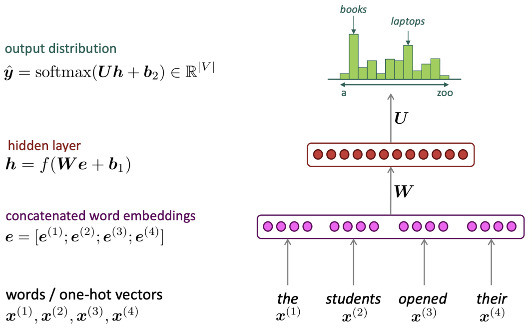
- Recall the language model by Bengio et al. [2003]:



- Model could also learn *distributed representation of words*, i.e. can produce word embeddings
- Context width still bound by n, but statistically feasible to estimate neural language models for much larger values of n
- Main challenge: training neural networks were more computationally expensive
  - Bengio et al. [2003] trained network on 14 million (14×10^6) words
  - Brants et al. [2007] trained 5-gram model on 2 trillion tokens (2×10^{12})

Fixed-window neural language model

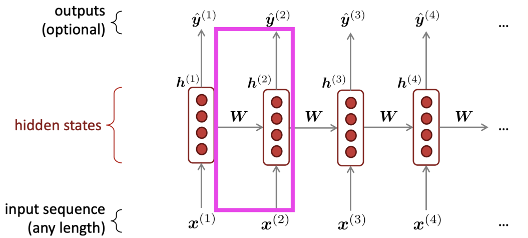
- Slightly simplified version of Bengio et al. [2003] (image taken from Stanford CS244n slides):



  - No more sparsity problems
  - Don't need to store all observed *n*-grams
- But: fixed window is too limiting - enlarging window enlarges W

Recurrent Neural Networks

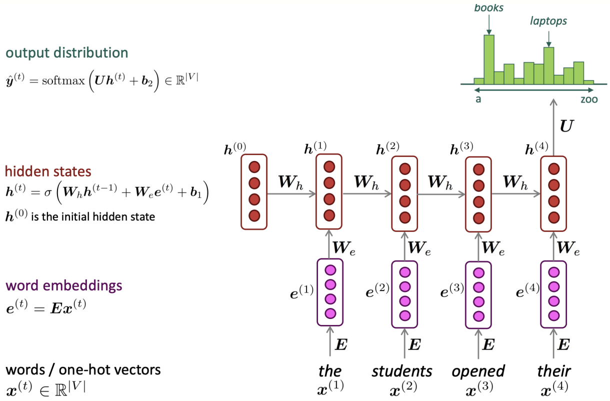
- RNNs [Hopfield, 1982; Rumelhart et al., 1985] are capable of conditioning the model on *all* previous tokens (in theory) (image taken from Stanford CS244n slides)



- Core idea: the hidden layer from previous timestep provides a form of memory or context that informs decisions to be made later in the sequence
    - In theory, context embodied in previous hidden layers can provide information extending to the beginning of the sequence
- The same weights are applied at every timestep
- The model size does not increase for longer input sequence lengths

Recurrent Neural Networks for language modelling

- Instead of a fixed-window approach, sequences are processed by presenting one item/token at a time to the network: (image taken from Stanford CS244n slides)



Training RNN language models: Cross-Entropy loss

- Given a large corpus of text with a sequence of words $\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(T)}$, we feed this into a RNN-LM and compute an output distribution $\hat{y}^{(t)}$ for every time step t
  - $\hat{\boldsymbol{y}}^{(t)}$ is a probability distribution for every word, conditional on all the words prior to t
  - We know the next word: $\boldsymbol{y}^{(t)}$ is a one-hot encoding of $\boldsymbol{x}^{(t+1)}$
- Use the cross-entropy loss between $\hat{\boldsymbol{y}}^{(t)}$ and the *true* word $\boldsymbol{y}^{(t)}$

$$J^{(t)}(\boldsymbol{\theta}) = \mathrm{CE}(\boldsymbol{y}^{(t)}, \hat{\boldsymbol{y}}^{(t)}) = -\sum_{w \in V} \boldsymbol{y}_w^{(t)} \log \hat{\boldsymbol{y}}_w^{(t)} = -\log \hat{\boldsymbol{y}}_{\boldsymbol{x}^{(t+1)}}^{(t)} \tag{7}$$

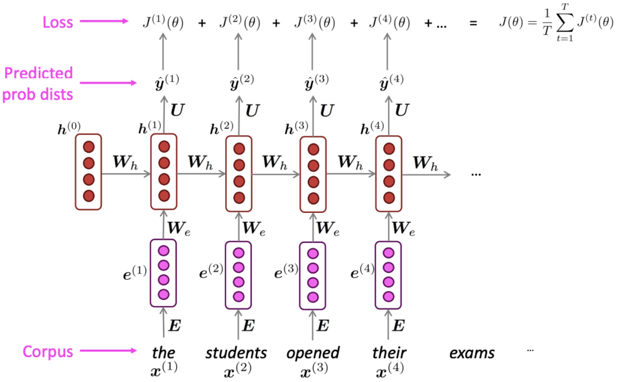
- Average over the loss for the entire corpus:

$$J(\theta) = \frac{1}{T}\sum_{t=1}^{T} J^{(t)}(\theta) = \frac{1}{T}\sum_{t=1}^{T} -\log \hat{\boldsymbol{y}}_{\boldsymbol{x}^{(t+1)}}^{(t)} \tag{8}$$

  - Typically computing loss and gradients for the whole corpus is too expensive
  - In practice, we split the corpus into sentences or documents and use them as *batches* in stochastic gradient descent

Training RNN language models: teacher-forcing

- When making a prediction of the word at time t, $\boldsymbol{y}^{(t)}$, we use the previous words as input to the RNN-LM
  - Rather than using what the model has predicted so far



- But when generating text with RNN-LMs, the sampled output at time t, $\boldsymbol{y}^{(t)}$ does become the next step's input
  - Teacher-forcing is only applied in training

Evaluating Language Model performance

- Standard evaluation metric for language models is perplexity

$$\text{perplexity} = \exp(J(\theta)) \tag{9}$$

  - Is a measure of confusion (or uncertainty) in the model
  - *Lower* perplexity is considered better and implies more confidence in predicting the next word in the sequence (compared to the ground truth)

Problems with RNNs

- Computation is slow
  - The model is inherently sequential and cannot be parallelised easily
- Difficulty in training
  - In practice, it is difficult to access information from many steps back due to vanishing or exploding gradients
    - During backpropagation, hidden layers are subject to repeated multiplications (determined by the length of the sequence)
    - Gradients often get gradually drive to zero (vanishing gradients)
  - The hidden layers/states and weights are asked to do a *lot* of work:
    - Provide information useful for the current decision
    - Update and carry forward information required for future decisions

LSTMs

- Long short-term memory (LSTM) networks [Hochreiter and Schmidhuber, 1997; Gers et al., 2000] are the most commonly used extension to RNNs
- Core idea: divide context management into two problems:
  1. Remove information that is no longer needed from the context
  2. Add information that is likely to be needed later on in decision making
- LSTMs try to *learn* how to manage context by adding in several specialised neural units
  - They add an explicit context layer along with the usual recurrent hidden layer
  - They add additional gates that can add and remove information to the context
- Gated Recurrent Units (GRUs) [Cho et al., 2014] are simpler alternatives to LSTMs which has fewer parameters and are a bit faster than LSTMs

LSTMs: a summary

- Essentially replaces "vanilla" RNN blocks with LSTM blocks that are able to preserve information over longer timesteps
  - In practice, could preserve information to about 100 timesteps rather than 7 in "vanilla" RNNs
- At step t, there is a hidden state vector $\boldsymbol{h}^{(t)}$ and a context state vector $\boldsymbol{c}^{(t)}$ which together store long-term information
- LSTM network can read, erase and write information from the cell
  - The selection of what gets read/erased/written is controlled by gates
- We'll go through the architecture in the next few slides:
  - Let $\circ$ be the Hadamard product, i.e. the element-wise multiplication operation
  - See "Understanding LSTM Networks" [Olah, 2015] for a bit more of a deeper explanation and walkthrough

LSTMs: architecture and gated activation functions I

To compute the current context state $\boldsymbol{c}^{(t)}$:

- New memory generation: $\tilde{\boldsymbol{c}}^{(t)} = \tanh(W^{(c)}\boldsymbol{x}^{(t)} + U^{(c)}\boldsymbol{h}^{(t-1)} + \boldsymbol{b}_c)$
  - Use input $\boldsymbol{x}^{(t)}$ and past hidden state $\boldsymbol{h}^{(t-1)}$ to generate new memory $\tilde{\boldsymbol{c}}^{(t)}$ which includes information about the new word
- Input gate: $\boldsymbol{i}^{(t)} = \sigma(W^{(i)}\boldsymbol{x}^{(t)} + U^{(i)}\boldsymbol{h}^{(t-1)} + \boldsymbol{b}_i)$
- Forget gate: $\boldsymbol{f}^{(t)} = \sigma(W^{(f)}\boldsymbol{x}^{(t)} + U^{(f)}\boldsymbol{h}^{(t-1)} + \boldsymbol{b}_f)$
- Final memory generation / context state: $\boldsymbol{c}^{(t)} = \boldsymbol{f}^{(t)} \circ \boldsymbol{c}^{(t-1)} + \boldsymbol{i}^{(t)} \circ \tilde{\boldsymbol{c}}^{(t)}$
  - $\boldsymbol{f}^{(t)}$ assess if the past memory is useful for the current memory cell - *gates* the past context state $\boldsymbol{c}^{(t-1)}$
  - $\boldsymbol{i}^{(t)}$ assesses if new word is important to the new memory - determines if input word is worth preserving and *gates* the new memory $\tilde{\boldsymbol{c}}^{(t)}$

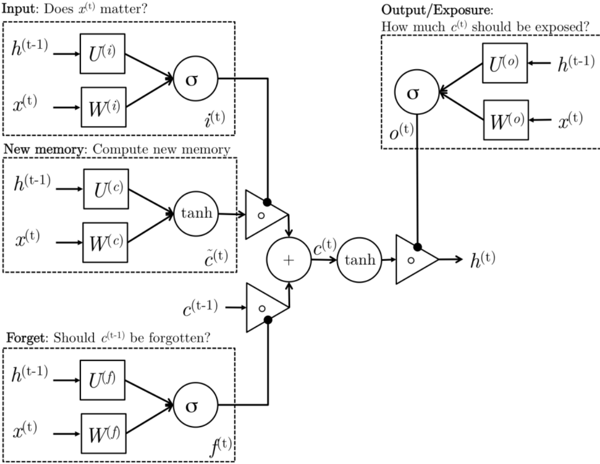
LSTMs: architecture and gated activation functions II

To compute the hidden state $\boldsymbol{h}^{(t)}$ (with the context state $\boldsymbol{c}^{(t)}$):

- Output gate: $\boldsymbol{o}^{(t)} = \sigma(W^{(o)}\boldsymbol{x}^{(t)} + U^{(o)}\boldsymbol{h}^{(t-1)} + \boldsymbol{b}_o)$
- Hidden state: $\boldsymbol{h}^{(t)} = \boldsymbol{o}^{(t)} \circ \tanh(\boldsymbol{c}^{(t)})$
  - $\boldsymbol{o}^{(t)}$ indicates what parts of the memory $\boldsymbol{c}^{(t)}$ needs to be exposed to the hidden state $\boldsymbol{h}^{(t)}$

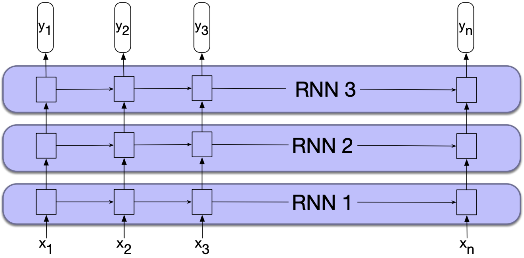
LSTMs: architecture and gated activation functions

- Image taken from Stanford CS244n Lecture Notes (Lecture 5)



Stacked / Deep RNNs

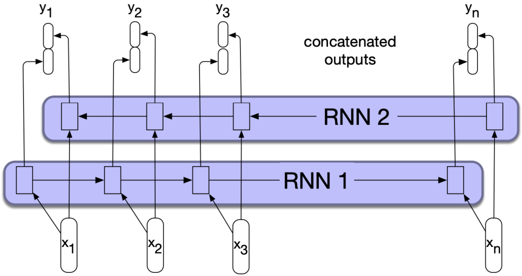
- We can use the entire sequence of outputs from one RNN as the input of another RNN to create a *stack* of RNNs (image taken from Speech and Language Processing)



  - Outputs of one layer serves as input to a subsequent layer
  - Typically outperforms single-layer networks but increases training cost

Bidirectional RNNs

- We can run two separate RNNs: left-to-right (we've seen above) and right-to-left (image taken from Speech and Language Processing)
- We concatentate the hidden states from both RNNs
  - Could combine the two hidden states in anyway you want (e.g. element-wise mean or sum)



  - Not suitable for language modelling as you only have left context
  - Effective if you have access to a full input sentence (to encode a sentence), e.g. sequence classification or sequence labelling
  - "BERT": Bidirectional Encoder Representations from Transformers

References

Bengio, Y., Ducharme, R., and Vincent, P. (2003). A Neural Probabilistic Language Model. *Journal of Machine Learning Research*, 3:1137–1155.

Brants, T., Popat, A. C., Xu, P., Och, F. J., and Dean, J. (2007). Large Language Models in Machine Translation.

Brown, P. F., Della Pietra, V. J., Desouza, P. V., Lai, J. C., and Mercer, R. L. (1992). Class-Based n-gram Models of Natural Language. *Computational Linguistics*, 18(4):467–480.

Cho, K., Van Merriënboer, B., Gulcehre, C., Bahdanau, D., Bougares, F., Schwenk, H., and Bengio, Y. (2014). Learning phrase representations using RNN encoder-decoder for statistical machine translation. *arXiv preprint arXiv:1406.1078*.

Gers, F. A., Schmidhuber, J., and Cummins, F. (2000). Learning to forget: Continual prediction with LSTM. *Neural Computation*, 12(10):2451–2471.

Hochreiter, S. and Schmidhuber, J. (1997). Long Short-Term Memory. *Neural Computation*, 9(8):1735–1780.

Hopfield, J. J. (1982). Neural networks and physical systems with emergent collective computational abilities. *Proceedings of the National Academy of Sciences*, 79(8):2554–2558.

Olah, C. (2015). Understanding LSTM Networks.

Rumelhart, D. E., Hinton, G. E., and Williams, R. J. (1985). Learning internal representations by error propagation. Technical report, California Univ San Diego La Jolla Inst for Cognitive Science.