Q/M Ratio of Electron Experiment

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***Abstract*—The experiment aims to calculate the ratio of mass and charge of the electron. The experiment is done by accelerating a beam of electrons with an electric field. The accelerated beams are affected by a magnetic field which is created by a Helmholtz Coil. A circular motion of an electron beam is created by changing the electric and magnetic fields. So, the Q/m ratio of the electron is calculated. In this experiment, we have found that the Q/m ratio of electron is $-1.77 \times 10^{11} \pm 4.03 \times 10^{8}$ with an error of $\%0.57$ relative to the recommended value -1.76×10^{11} [1].**

I. Theory

The ratio was first calculated by J.J. Thomson in 1897[2]. He showed that the electron has a charge and mass with a certain ratio. Since the mass of the electron is too small to calculate, it is significant to find the ratio.

To calculate the ratio, we have tested the following theory:

$$\text{Energy of accelerated beam: } E_k = V.q = \frac{m_e.v_e^2}{2} \tag{1}$$

$$\text{Lorentz Force: } \overrightarrow{\mathrm{F}} = Q(\overrightarrow{\mathrm{E}} + \overrightarrow{\mathrm{v_e}} \times \overrightarrow{\mathrm{B}}) \tag{2}$$

$$\text{Force on uniform circular motion: } m_e.\frac{v_e^2}{r} \tag{3}$$

Since $\overrightarrow{\mathrm{v_e}}$ and $\overrightarrow{\mathrm{B}}$ are perpendicular,

$$Q.v_e.B = m_e.\frac{v_e^2}{r} \Rightarrow v_e = \frac{Q.B.r}{m_e} \tag{4}$$

Combining eq.4 and eq.1,

$$\frac{Q}{m_e} = \frac{2V}{(r.B)^2} \tag{5}$$

Equation for Magnetic Field

$$B = (\frac{4}{5})^{\frac{3}{2}}.\mu_0.N_{coil}.\frac{I}{R_{coil}} \tag{6}$$

where, $\mu_0 = 1.257 \times 10^{-6}\,\frac{Vs}{Am}$, $N = 154$, $R_{coil} = 0.2m$

Combining eq.4 and eq.6,

$$\frac{Q}{m_e} = \frac{2V.R_{coil}^2.5^3}{I^2.r^2.\mu_0^2.N_{coil}^2.4^3} \tag{7}$$

Here, we have assumed that the magnetic field is uniform and perpendicular to the velocity of the electron beam, also the beam makes a uniform circular motion. However, the magnetic field is not uniform, but it is lower at the edges(see appendix). As can be seen from eq.7, depending on V, I, r charge - mass ratio can be obtained. We expect to see that this ratio is constant.

We use low pressure Neon Gas to observe the emitted electrons. Exited Neon Gas emits light so that we can observe the electron beam. Since the Neon Gas is low pressure, it does not have much molecules, so the effect is negligible.

II. Method

To create an electron beam, the filament is heated. The created beam is accelerated with the electric field. An Electron beam is observed in the Fine-beam tube. A magnetic field is created with a Helmholtz coil to achieve circular motion. Depending on the I, V the circular motion is created with the same radius.

1) Filament is heated until an electron beam can be seen easily.
2) The emitted electrons are accelerated with the Electric Field by increasing the voltage.
3) A straight line is observed.
4) The Magnetic Field is created by increasing the current.
5) The circular motion is observed. If the beam makes spiral instead of circular motion, the axis of fine-beam tube is fixed.(See Fig.2,3)
6) Volt and current data are taken.

III. The Experimental Setup

- Helmholtz Coil.
- Fine-Beam Tube.
- Ammeter for Magnetic Field.
- Voltmeter for Electric Field.
- Power Supply for Magnetic Field.
- Power Supply for Electric Field and Filament

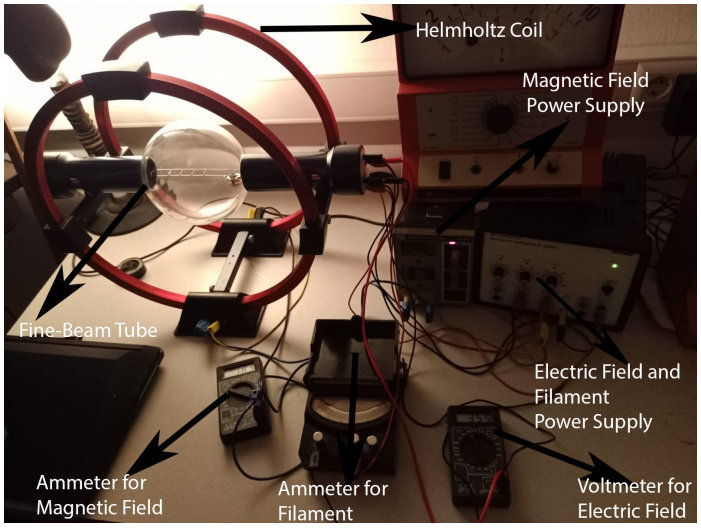

Fig. 1. Apparatus

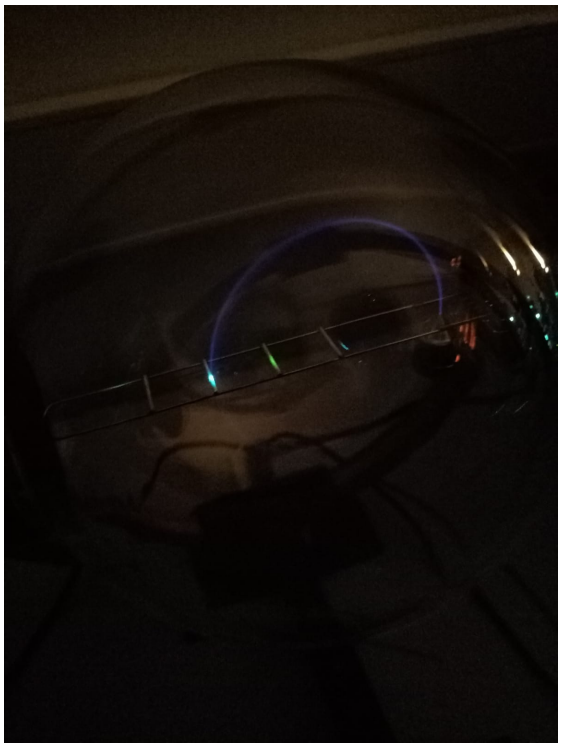

Fig. 2. Circular motion in the fine-beam tube

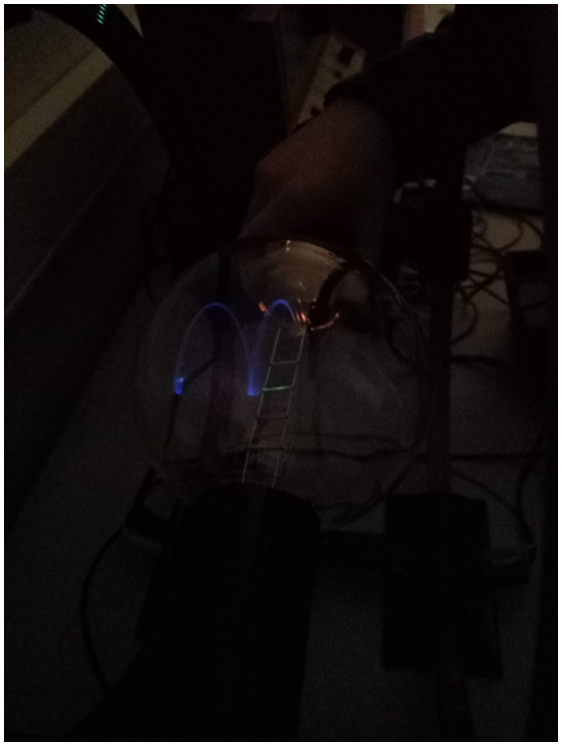

Fig. 3. Arranging position to obtain perpendicular magnetic field and velocity

IV. The Data

Values which create a circular motion are listed.

TABLE I
VOLT AND CURRENT FOR CORRESPONDING RADIUS

Radius (cm)	Volt(V)	Current(A)
2 ± 0.2	151 ± 1	3.09 ± 0.01
2 ± 0.2	135 ± 1	2.89 ± 0.01
2 ± 0.2	91 ± 1	2.15 ± 0.01
3 ± 0.2	151 ± 1	1.97 ± 0.01
3 ± 0.2	305 ± 1	2.88 ± 0.01
3 ± 0.2	135 ± 1	1.85 ± 0.01
3 ± 0.2	132 ± 1	1.82 ± 0.01
3 ± 0.2	173 ± 1	2.15 ± 0.01
3 ± 0.2	103 ± 1	1.48 ± 0.01
4 ± 0.2	151 ± 1	1.43 ± 0.01
4 ± 0.2	135 ± 1	1.36 ± 0.01
4 ± 0.2	305 ± 1	2.13 ± 0.01
4 ± 0.2	308 ± 1	2.15 ± 0.01
4 ± 0.2	224 ± 1	1.82 ± 0.01
4 ± 0.2	158 ± 1	1.48 ± 0.01
5 ± 0.2	151 ± 1	1.12 ± 0.01
5 ± 0.2	135 ± 1	1.02 ± 0.01
5 ± 0.2	305 ± 1	1.71 ± 0.01
5 ± 0.2	346 ± 1	1.82 ± 0.01
5 ± 0.2	233 ± 1	1.48 ± 0.01

V. The Analysis

We have investigated different volt and current values for the same radius. So, there are 4 sets of data. As the eq.7 suggests, there should be a linear relationship between $\frac{C_B}{I^2}$ and $\frac{r^2}{2V}$. Where C_B is the constant for the magnetic field. To find uncertainties of $\frac{C_B}{I^2}$ and $\frac{r^2}{2V}$ uncertainty propagation is done according to the formulas:

$$C_B = (\frac{5}{4})^3.(\frac{R_{coil}}{\mu_0.N_{coil}})^2 \tag{8}$$

$$f = aA \ \Rightarrow \sigma_f = |a|\sigma_A \tag{9}$$

$$f = aA^b \Rightarrow \sigma_f \approx \left|abA^{b-1}\sigma_A\right| = \left|\frac{fb\sigma_A}{A}\right| \tag{10}$$

$$f = \frac{A}{B} \ \Rightarrow \sigma_f \approx |f| \sqrt{\left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2 - 2\frac{\sigma_{AB}}{AB}}[3] \tag{11}$$

Since A and B values are independent, $\sigma_{AB} = 0$.

$$f = \frac{A}{B} \ \Rightarrow \sigma_f \approx |f| \sqrt{\left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2} \tag{12}$$

where, f is the function
A and B are corresponding variables
σ_A and σ_B are corresponding uncertainties
a and b are corresponding constants.

After calculating the uncertainties of the $\frac{C_B}{I^2}$ and $\frac{r^2}{2V}$, weights of data points are calculated and the line is fitted accordingly. The line fit is both done by Root's built-in function and by weighted average (See Appendix). From Fig-4-8 we can see that, the weighted average method obtains closer value to the CODATA recommended value for Q/m (-1.76×10^{11}).

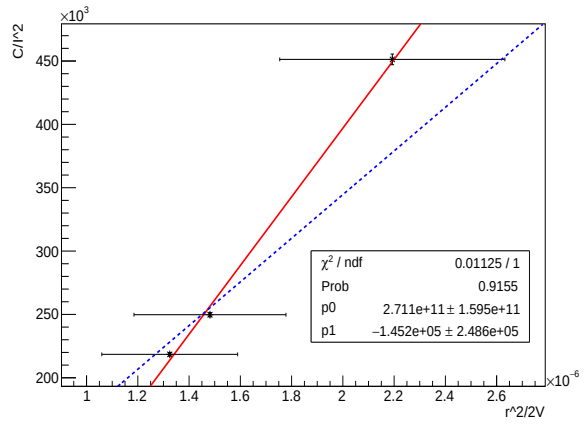

Fig. 4. Graph for r=2cm.
Blue Line: Weighted Average, Red Line: Root
Slope of Blue Line: 1.72283e+11 +- 7.28398e+08
Intercept of Blue Line: -0.0025167 +- 1

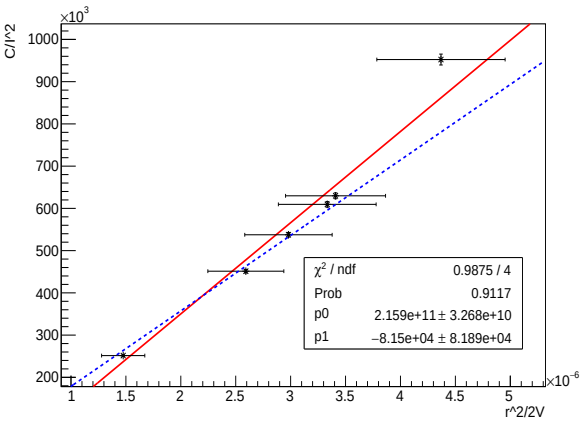

Fig. 5. Graph for r=3cm.
Blue Line: Weighted Average, Red Line: Root
Slope of Blue Line: 1.7866e+11 +- 7.05888e+08
Intercept of Blue Line: -0.00267291 +- 1

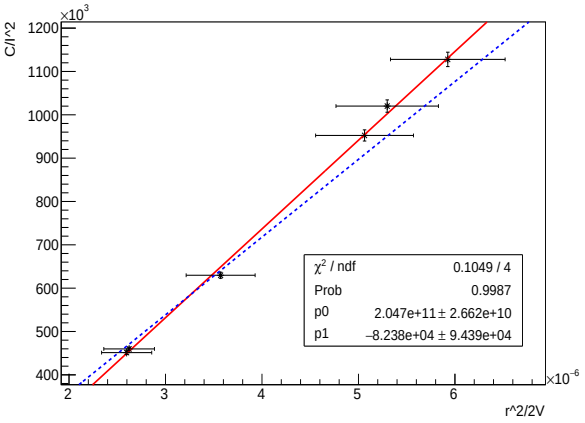

Fig. 6. Graph for r=4cm.
Blue Line: Weighted Average, Red Line: Root
Slope of Blue Line: 1.79441e+11 +- 8.33753e+08
Intercept of Blue Line: -0.000813723 +- 1

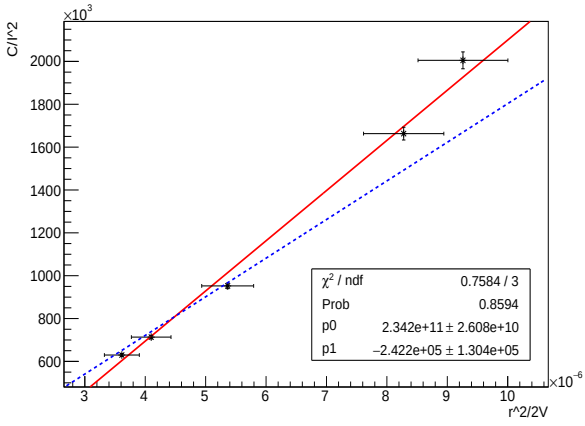

Fig. 7. Graph for r=5cm.
Blue Line: Weighted Average, Red Line: Root
Slope of Blue Line: 1.80295e+11 +- 1.10384e+09
Intercept of Blue Line: -0.000496268 +- 1

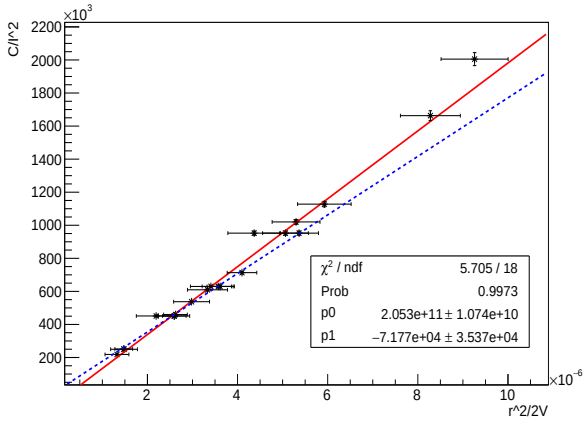

Fig. 8. Graph for all radius.
Blue Line: Weighted Average, Red Line: Root
Slope of Blue Line: 1.77107e+11 +- 4.03208e+08
Intercept of Blue Line: -0.00967139 +- 1

VI. The Result

We have found the magnitude of Q/m ratio to be 1.77×10^{11} with the uncertainty of 4.03×10^{8}. Since the electron beam is attracted to the positive direction, we conclude that the electron has a negative charge, and therefore, the ratio is $-1.77 \times 10^{11} \pm 4.03 \times 10^{8}$. The CODATA recommended value for Q/m of electron is -1.76×10^{11}. The relative error is $\%0.57$.

VII. The Conclusion

The result is consistent with the theory. The $\%0.57$ relative error is acceptable with the apparatus we have used. So, we have concluded that the experiment was a success. However, the uncertainty could be much less. The problem is due to the measurement devices which are not adequately precise for such an experiment.

The power supplies' capabilities are limited, so that, it is impossible to experiment with much higher or lower volt and current values. Therefore, we could not observe the behavior of electrons at higher magnetic and electric fields.

The experiment could be conducted with fewer internal and external impurities. The magnetic and electric field around the apparatus could be eliminated by shielding the environment. Even if it is assumed to get a uniform magnetic field from Helmholtz Coil, this is not the case. Instead of using low-pressure gas to detect electron movement, we could only use phosphor where electron hits the ladder in the tube, so that the electrons are not affected by the gas.

References

[1] *CODATA Value: electron charge to mass quotient*. URL: https://physics.nist.gov/cgi-bin/cuu/Value?esme (visited on 03/12/2023).

[2] *J. J. Thomson 1897*. URL: https://web.lemoyne.edu/~giunta/thomson1897.html (visited on 03/12/2023).

[3] *Strategies for Variance Estimation*. URL: https://www.sagepub.com/sites/default/files/upm-binaries/6427_Chapter_4__Lee_(Analyzing)_I_PDF_6.pdf % 20html (visited on 03/12/2023).

[4] *Magnetic field of two Helmholtz coils*. URL: https://virtuelle-experimente.de/en/b-feld/b-feld/helmholtzspulenpaar.php (visited on 03/12/2023).

VIII. Appendix

[4]

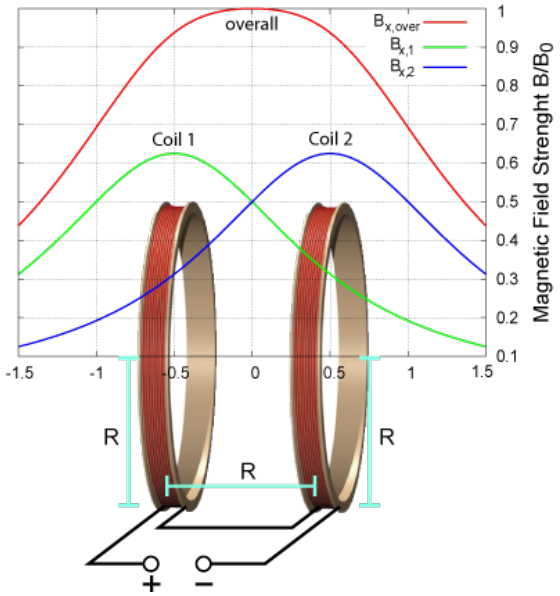

Fig. 9. Magnetic field created by Helmholtz Coil

The fit has been done with both Root's built-in function and with the line fit macro. Root's function is used for additional information about the fit (See Fig4-8). We have used root release 6.28/00 for Ubuntu22.

```
{
 TTree *t = new TTree("t", "t"); //Creating
     Tree and Branches
 t->ReadFile("data.csv");
 float a,v,r;
 t->SetBranchAddress("a", &a);
 t->SetBranchAddress("v", &v);
 t->SetBranchAddress("r", &r);

float * amp, * volt, * rad, * x, * y, * sx,
    * sy ; // Creating Arrays
int n = t->GetEntries();
amp = new float[n];
volt = new float[n];
rad = new float[n];
x= new float[n];
y = new float[n];
sx = new float[n];
sy = new float[n];


for (int i = 0; t->LoadTree(i) >= 0; i++){
    //Filling Arrays
 t->GetEntry(i);
 amp[i] = a;
 volt[i] = v;
 rad[i] = r;
}

float del_r = 0.002;
float del_v = 1;
float del_a = 0.01;

float cons = 2086069.919;

for (int i=0; i<n; ++i){ // Error Propagation
  float r2 = rad[i]*rad[i];
  float a2 = amp[i]*amp[i];
  float del_r2 = r2 * 2 * del_r / rad[i];

  x[i] = r2 / (2 * volt[i]);
  y[i] = cons / a2;
  sx[i] = x[i] * sqrt( (del_v/volt[i]) *
      (del_v/volt[i]) + (del_r2 / r2) *
      (del_r2 / r2));
  sy[i] = (cons/a2) * 2 * del_a / amp[i];
}

float weight = 0;
float totw = 0; // Total weight
float xybar = 0, xbar = 0, ybar = 0, x2bar =
    0, y2bar = 0; // weighted averages

for (int i=0; i<n; ++i) {
  weight = 1./((sy[i]*sy[i]) + (sx[i]*sx[i]));
  totw += weight;
  xybar += (x[i]*y[i]*weight);
  xbar += (x[i]*weight);
  ybar += (y[i]*weight);
  x2bar += (x[i]*x[i]*weight);
  y2bar += (y[i]*y[i]*weight);
}

float sy2bar = n; // weighted average error
    squared
float slope = (xybar - xbar*ybar) / (x2bar -
    xbar*xbar);
float itcpt = ybar - slope * xbar;
float slopeerr = sqrt ( sy2bar / (n * (x2bar
    - xbar*xbar) ) );
float itcpterr = sqrt ( x2bar ) * slopeerr;
cout << slopeerr << endl;
cout << "slope of fit line = " << slope << "
    +- " << slopeerr << endl;
cout << "intercept of fit line = " << itcpt
    << " +- " << itcpterr << endl;
```

```
 TGraphErrors *mygraph = new
     TGraphErrors(n,x,y,sx,sy);
 mygraph->Draw("A*");

 gStyle->SetOptFit(1111);

 TF1 *ffitline = new
     TF1("ffitline","[0]*x+[1]",0,6);
 ffitline->SetParameter(0,slope);
 ffitline->SetParameter(1,itcpt);
 ffitline->SetLineColor(kBlue); // draw in
     blue color
 ffitline->SetLineStyle(2); // draw dotted
     line
 ffitline->Draw("same");

 TF1 *fnew = new TF1("fnew","[0]*x+[1]",0,6);
 fnew->SetParameters(1200,7); // arbitrary
     starting parameters
 mygraph->Fit(fnew);
}
```