X-Ray Experiment

Burak Kara
Boğaziçi University • Physics Department

***Abstract*—The experiment aims to calculate the Planck Constant h. To calculate the Planck Constant, we have scattered X-Rays from NaCl crystal which are emitted from Molybdenum (Mo42). We have used the characteristic radiations of Mo42 to calibrate our data and we have used Bragg's Law and Duane-Hunt Law to calculate the Planck Constant. We have found the Planck Constant to be $h = 6.1 \pm 1.7 \times 10^{-34} JHz^{-}1$. The CODATA recommended value is $6.62607015 \times 10^{-34} JHz^{-}1$ [1]. We are 0.31σ away from the recommended value with $\%7.9$ relative error.**

I. Theory

When an accelerated electron hits an atom whose atomic number is sufficiently big, X-rays are produced. The dominant X-Ray emissions are Bremsstrahlung and de-excitation. [2] "Bremsstrahlung" word is originated from German and it can be translated as "The Breaking Radiation". As the name suggests, the de-acceleration of electron beam creates the Bremsstrahlung. Since the energetic electron beam can lose its energy up to its maximum energy depending on its position and its energy, the produced X-Rays has a continuous spectra. On the other hand, the de-excitation creates a discrete spectra, this discrete spectra depends on excitation energy of the atom. So, it is a Characteristic Radiation. In this experiment, we will use the characteristic radiation to calibrate the data and we will use Bremsstrahlung to calculate the Planck Constant.

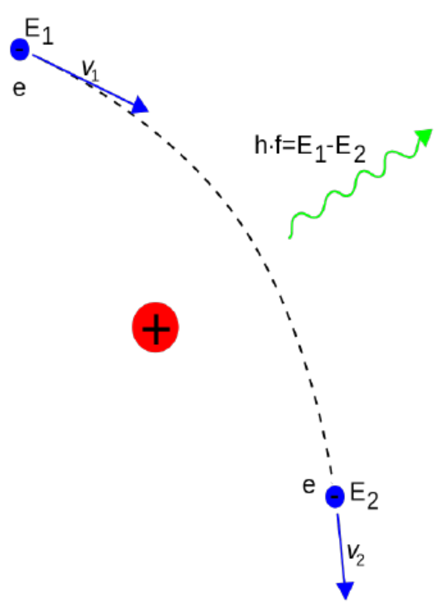

Fig. 1. Bremsstrahlung [3]

We will test the following Theory:

Duane–Hunt law:

$$h\nu_{max} = qV_{acc} \tag{1}$$

where, h is the Planck Constant,
ν_{max} is the maximum frequency of X-Ray,
q is the electron charge,
V_{acc} is the accelerating voltage.

Speed of Light:

$$c = \lambda\nu \tag{2}$$

where, c is the speed of light and λ is the wavelength

Using Eq.1 and Eq.2

$$h = \frac{q\lambda_{min}V_{acc}}{c} \tag{3}$$

We know the electron charge, the speed of light and the accelerating frequency, so, if we can obtain the minimum wavelength of the X-Ray, we can obtain the Planck Constant. To obtain minimum wavelength, one can scatter the emitted X-Rays from a crystal. The scattered X-Rays obeys the Bragg's Law:

Bragg's Law:

$$2dsin(\theta) = m\lambda \tag{4}$$

where, d is the spacing between crystal layers,
θ is the incident angle, λ is wavelength,
and m is the diffraction order.

With the Bragg's Law, we have obtained the wavelengths.

Using Bragg's Law and Eq.3:

$$h = \frac{2dqsin(\theta)V_{acc}}{mc} \tag{5}$$

With the equation 5, we know every parameter at the right-hand side, so we have obtained a reasonable theory to extract the Planck Constant.

II. Method

To extract the Planck Constant, we will use the scattering method mentioned in the theory. We have used the Mo42 as the

X-Ray source and NaCl to scatter the emitted X-Rays. We have used the characteristic X-Ray radiations of Mo42 to calibrate our data and used the Duane-Hunt Law and Bragg's Law to extract the Planck Constant.

1) Electron beams are created with heating the filament.
2) Created electron beams are accelerated with various high voltages. (15kV, 18kV, 21kV, 24kV, 27kV, 30kV, 35kV)
3) The emitted X-Rays are gathered with a diaphragm tube.
4) The gathered X-Rays from Mo42 are scattered from NaCl.
5) The succesive scatterings are counted with Geiger-Muller Counter Tube.
6) The collected data is calibrated according to the characteristic X-Rays of Mo42. For calibration, we have used 35kV.
7) Using Eq.5, the Planck Constant is extracted.

III. The Experimental Setup

- Mo42 atom as the X-Ray source.
- Diaphragm Tube for gathering X-Rays.
- NaCl Crystal for scattering.
- Geiger-Muller Counter Tube.
- Goniometer for angle measurement.
- Computer for getting data.

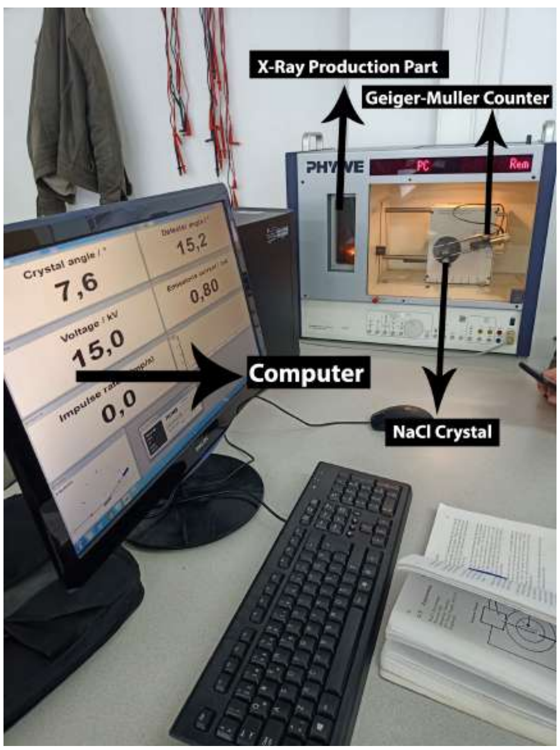

Fig. 2. Apparatus.

IV. The Data

The error on angles is 0.1 Deg and on Counts are $\sqrt{N/5}$.

TABLE I: 15kV

Angle(Deg)	Count per Second
3.0	25.0
3.1	29.0
3.2	20.0
3.3	11.0
3.4	9.0
3.5	7.0
3.6	2.0
3.7	2.0
3.8	0.0
3.9	1.0
4.0	1.0
4.1	1.0
4.2	1.0
4.3	1.0
4.4	0.0
4.5	0.0
4.6	2.0
4.7	0.0
4.8	2.0
4.9	1.0
5.0	1.0
5.1	1.0
5.2	1.0
5.3	1.0
5.4	0.0
5.5	2.0
5.6	0.0
5.7	1.0
5.8	1.0
5.9	1.0
6.0	0.0
6.1	0.0
6.2	0.0
6.3	1.0
6.4	1.0
6.5	0.0
6.6	1.0
6.7	0.0
6.8	0.0

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TABLE I: 15kV (Continued)

Angle(Deg)	Count per Second
6.9	0.0
7.0	1.0
7.1	1.0
7.2	0.0
7.3	0.0
7.4	0.0
7.5	0.0
7.6	0.0
7.7	0.0
7.8	1.0
7.9	1.0
8.0	0.0
8.1	0.0
8.2	0.0
8.3	1.0
8.4	1.0
8.5	1.0
8.6	2.0
8.7	2.0
8.8	5.0
8.9	6.0
9.0	9.0
9.1	11.0
9.2	14.0
9.3	16.0
9.4	20.0
9.5	18.0
9.6	25.0
9.7	27.0
9.8	21.0
9.9	22.0
10.0	26.0
10.1	23.0
10.2	25.0
10.3	21.0
10.4	25.0
10.5	27.0
10.6	27.0
10.7	27.0

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TABLE I: 15kV (Continued)

Angle(Deg)	Count per Second
10.8	27.0
10.9	30.0
11.0	24.0
11.1	30.0
11.2	28.0
11.3	26.0
11.4	28.0
11.5	26.0
11.6	25.0
11.7	27.0
11.8	26.0
11.9	29.0
12.0	25.0
12.1	24.0
12.2	29.0
12.3	25.0
12.4	21.0

TABLE II: 18kV

Angle(Deg)	Count per Second
3.0	57.0
3.1	59.0
3.2	42.0
3.3	26.0
3.4	21.0
3.5	16.0
3.6	8.0
3.7	4.0
3.8	3.0
3.9	2.0
4.0	3.0
4.1	2.0
4.2	3.0
4.3	3.0
4.4	2.0
4.5	2.0
4.6	2.0
4.7	2.0

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TABLE II: 18kV (Continued)

Angle(Deg)	Count per Second
4.8	2.0
4.9	2.0
5.0	1.0
5.1	2.0
5.2	2.0
5.3	2.0
5.4	3.0
5.5	2.0
5.6	1.0
5.7	2.0
5.8	2.0
5.9	1.0
6.0	1.0
6.1	2.0
6.2	0.0
6.3	1.0
6.4	1.0
6.5	2.0
6.6	2.0
6.7	1.0
6.8	0.0
6.9	3.0
7.0	2.0
7.1	4.0
7.2	4.0
7.3	5.0
7.4	6.0
7.5	10.0
7.6	15.0
7.7	15.0
7.8	20.0
7.9	29.0
8.0	27.0
8.1	26.0
8.2	32.0
8.3	32.0
8.4	40.0
8.5	41.0
8.6	45.0

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TABLE II: 18kV (Continued)

Angle(Deg)	Count per Second
8.7	44.0
8.8	46.0
8.9	50.0
9.0	48.0
9.1	57.0
9.2	56.0
9.3	54.0
9.4	54.0
9.5	57.0
9.6	56.0
9.7	60.0
9.8	54.0
9.9	50.0
10.0	44.0
10.1	57.0
10.2	48.0
10.3	48.0
10.4	48.0
10.5	52.0
10.6	44.0
10.7	50.0
10.8	54.0
10.9	50.0
11.0	41.0
11.1	48.0
11.2	51.0
11.3	43.0
11.4	50.0

TABLE III: 21kV

Angle(Deg)	Count per Second
3.0	122.0
3.1	106.0
3.2	86.0
3.3	61.0
3.4	41.0
3.5	25.0
3.6	15.0

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TABLE III: 21kV (Continued)

Angle(Deg)	Count per Second
3.7	6.0
3.8	4.0
3.9	5.0
4.0	5.0
4.1	4.0
4.2	5.0
4.3	6.0
4.4	4.0
4.5	2.0
4.6	4.0
4.7	3.0
4.8	4.0
4.9	5.0
5.0	4.0
5.1	4.0
5.2	4.0
5.3	2.0
5.4	2.0
5.5	3.0
5.6	3.0
5.7	2.0
5.8	1.0
5.9	2.0
6.0	2.0
6.1	5.0
6.2	5.0
6.3	6.0
6.4	9.0
6.5	12.0
6.6	21.0
6.7	21.0
6.8	28.0
6.9	32.0
7.0	34.0
7.1	33.0
7.2	41.0
7.3	38.0
7.4	50.0
7.5	48.0

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TABLE III: 21kV (Continued)

Angle(Deg)	Count per Second
7.6	57.0
7.7	64.0
7.8	64.0
7.9	68.0
8.0	63.0
8.1	66.0
8.2	72.0
8.3	75.0
8.4	81.0
8.5	79.0
8.6	80.0
8.7	85.0
8.8	78.0
8.9	86.0
9.0	83.0
9.1	84.0
9.2	83.0
9.3	82.0
9.4	82.0
9.5	81.0
9.6	89.0
9.7	91.0
9.8	84.0
9.9	82.0
10.0	78.0
10.1	78.0
10.2	65.0
10.3	79.0

TABLE IV: 24kV

Angle(Deg)	Count per Second
3.0	188.0
3.1	179.0
3.2	121.0
3.3	114.0
3.4	70.0
3.5	44.0
3.6	25.0

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TABLE IV: 24kV (Continued)

Angle(Deg)	Count per Second
3.7	15.0
3.8	12.0
3.9	12.0
4.0	0.0
4.1	10.0
4.2	9.0
4.3	8.0
4.4	8.0
4.5	8.0
4.6	8.0
4.7	8.0
4.8	7.0
4.9	5.0
5.0	6.0
5.1	7.0
5.2	5.0
5.3	10.0
5.4	8.0
5.5	9.0
5.6	9.0
5.7	11.0
5.8	17.0
5.9	22.0
6.0	22.0
6.1	29.0
6.2	27.0
6.3	39.0
6.4	47.0
6.5	49.0
6.6	42.0
6.7	50.0
6.8	60.0
6.9	62.0
7.0	62.0
7.1	80.0
7.2	95.0
7.3	109.0
7.4	114.0
7.5	110.0

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TABLE IV: 24kV (Continued)

Angle(Deg)	Count per Second
7.6	115.0
7.7	158.0
7.8	109.0
7.9	86.0
8.0	87.0
8.1	92.0
8.2	92.0
8.3	89.0
8.4	94.0
8.5	91.0
8.6	102.0
8.7	92.0
8.8	96.0
8.9	99.0
9.0	98.0
9.1	94.0
9.2	99.0
9.3	105.0
9.4	107.0
9.5	102.0
9.6	99.0
9.7	98.0
9.8	99.0
9.9	91.0
10.0	94.0
10.1	86.0
10.2	85.0
10.3	85.0
10.4	77.0
10.5	85.0
10.6	79.0
10.7	80.0
10.8	78.0

TABLE V: 27kV

Angle(Deg)	Count per Second
3.0	294.0
3.1	273.0

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TABLE V: 27kV (Continued)

Angle(Deg)	Count per Second
3.2	188.0
3.3	159.0
3.4	106.0
3.5	66.0
3.6	42.0
3.7	32.0
3.8	19.0
3.9	22.0
4.0	21.0
4.1	21.0
4.2	15.0
4.3	14.0
4.4	14.0
4.5	12.0
4.6	15.0
4.7	15.0
4.8	15.0
4.9	15.0
5.0	22.0
5.1	18.0
5.2	26.0
5.3	26.0
5.4	30.0
5.5	34.0
5.6	39.0
5.7	39.0
5.8	48.0
5.9	51.0
6.0	52.0
6.1	64.0
6.2	66.0
6.3	66.0
6.4	76.0
6.5	85.0
6.6	89.0
6.7	87.0
6.8	95.0
6.9	114.0
7.0	115.0

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TABLE V: 27kV (Continued)

Angle(Deg)	Count per Second
7.1	142.0
7.2	159.0
7.3	194.0
7.4	210.0
7.5	218.0
7.6	236.0
7.7	304.0
7.8	202.0
7.9	114.0
8.0	118.0
8.1	121.0
8.2	119.0
8.3	123.0
8.4	123.0
8.5	120.0
8.6	131.0
8.7	130.0
8.8	135.0
8.9	122.0
9.0	132.0
9.1	126.0
9.2	123.0
9.3	121.0
9.4	129.0
9.5	135.0
9.6	133.0
9.7	127.0
9.8	131.0
9.9	121.0
10.0	117.0
10.1	116.0
10.2	109.0
10.3	117.0
10.4	114.0
10.5	108.0
10.6	110.0
10.7	114.0
10.8	119.0
10.9	119.0

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TABLE V: 27kV (Continued)

Angle(Deg)	Count per Second
11.0	117.0
11.1	109.0
11.2	110.0
11.3	116.0
11.4	110.0
11.5	98.0
11.6	103.0
11.7	108.0

TABLE VI: 30kV

Angle(Deg)	Count per Second
3.0	403.0
3.1	386.0
3.2	272.0
3.3	196.0
3.4	144.0
3.5	103.0
3.6	69.0
3.7	55.0
3.8	40.0
3.9	39.0
4.0	37.0
4.1	33.0
4.2	29.0
4.3	29.0
4.4	29.0
4.5	34.0
4.6	31.0
4.7	35.0
4.8	34.0
4.9	44.0
5.0	48.0
5.1	0.0
5.2	61.0
5.3	70.0
5.4	75.0
5.5	74.0
5.6	88.0

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TABLE VI: 30kV (Continued)

Angle(Deg)	Count per Second
5.7	83.0
5.8	77.0
5.9	85.0
6.0	88.0
6.1	86.0
6.2	89.0
6.3	111.0
6.4	117.0
6.5	129.0
6.6	125.0
6.7	130.0
6.8	144.0
6.9	166.0
7.0	163.0
7.1	218.0
7.2	274.0
7.3	297.0
7.4	339.0
7.5	311.0
7.6	375.0
7.7	499.0
7.8	316.0
7.9	147.0
8.0	130.0
8.1	141.0
8.2	142.0
8.3	146.0
8.4	148.0
8.5	152.0
8.6	169.0
8.7	151.0
8.8	151.0
8.9	152.0
9.0	148.0
9.1	169.0
9.2	159.0
9.3	161.0
9.4	164.0
9.5	161.0

Continued on next page

TABLE VI: 30kV (Continued)

Angle(Deg)	Count per Second
9.6	157.0
9.7	168.0
9.8	164.0
9.9	152.0
10.0	165.0
10.1	152.0
10.2	151.0
10.3	145.0
10.4	141.0
10.5	153.0
10.6	140.0
10.7	146.0
10.8	144.0
10.9	144.0
11.0	144.0
11.1	150.0
11.2	138.0
11.3	137.0
11.4	142.0
11.5	135.0
11.6	125.0
11.7	133.0
11.8	130.0
11.9	133.0
12.0	126.0
12.1	122.0
12.2	128.0
12.3	113.0
12.4	126.0
12.5	123.0
12.6	121.0
12.7	123.0
12.8	121.0
12.9	121.0
13.0	122.0
13.1	133.0
13.2	138.0
13.3	178.0
13.4	199.0

Continued on next page

TABLE VI: 30kV (Continued)

Angle(Deg)	Count per Second
13.5	171.0
13.6	122.0
13.7	116.0
13.8	130.0
13.9	104.0
14.0	120.0
14.1	114.0
14.2	119.0
14.3	121.0
14.4	120.0
14.5	121.0
14.6	106.0
14.7	127.0
14.8	143.0
14.9	277.0
15.0	480.0

TABLE VII: 30kV - Calibration

Angle(Deg)	Count per Second
3	923
3.1	768
3.2	489
3.3	343
3.4	257
3.5	187
3.6	135
3.7	124
3.8	117
3.9	105
4	107
4.1	98
4.2	100
4.3	95
4.4	94
4.5	123
4.6	138
4.7	135
4.8	156

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TABLE VII: 30kV - Calibration (Continued)

Angle(Deg)	Count per Second
4.9	150
5	165
5.1	154
5.2	157
5.3	162
5.4	159
5.5	142
5.6	155
5.7	144
5.8	144
5.9	133
6	141
6.1	139
6.2	143
6.3	173
6.4	184
6.5	201
6.6	205
6.7	202
6.8	234
6.9	284
7	269
7.1	355
7.2	428
7.3	494
7.4	600
7.5	596
7.6	712
7.7	930
7.8	631
7.9	217
8	221
8.1	219
8.2	211
8.3	213
8.4	222
8.5	226
8.6	239

Continued on next page

TABLE VII: 30kV - Calibration (Continued)

Angle(Deg)	Count per Second
8.7	238
8.8	239
8.9	226
9	239
9.1	229
9.2	239
9.3	248
9.4	237
9.5	236
9.6	244
9.7	240
9.8	236
9.9	243
10	224
10.1	217
10.2	210
10.3	208
10.4	203
10.5	205
10.6	203
10.7	204
10.8	200
10.9	187
11	197
11.1	199
11.2	192
11.3	187
11.4	182
11.5	184
11.6	182
11.7	192
11.8	186
11.9	173
12	171
12.1	173
12.2	170
12.3	176
12.4	168

Continued on next page

TABLE VII: 30kV - Calibration (Continued)

Angle(Deg)	Count per Second
12.5	174
12.6	181
12.7	170
12.8	162
12.9	170
13	171
13.1	193
13.2	215
13.3	263
13.4	337
13.5	271
13.6	184
13.7	169
13.8	166
13.9	165
14	166
14.1	172
14.2	156
14.3	153
14.4	159
14.5	171
14.6	177
14.7	199
14.8	222
14.9	429
15	810
15.1	810
15.2	442
15.3	226
15.4	167
15.5	131
15.6	130
15.7	123
15.8	129
15.9	110
16	114
16.1	116
16.2	116

Continued on next page

TABLE VII: 30kV - Calibration (Continued)

Angle(Deg)	Count per Second
16.3	105
16.4	112
16.5	102
16.6	103
16.7	108
16.8	107
16.9	89
17	95
17.1	93
17.2	86
17.3	84
17.4	80
17.5	86
17.6	84
17.7	80
17.8	74
17.9	82
18	72
18.1	78
18.2	75
18.3	74
18.4	74
18.5	77
18.6	75
18.7	75
18.8	69
18.9	68
19	75
19.1	71
19.2	79
19.3	68
19.4	69
19.5	68
19.6	62
19.7	71
19.8	69
19.9	66
20	92

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TABLE VII: 30kV - Calibration (Continued)

Angle(Deg)	Count per Second
20.1	96
20.2	67
20.3	65
20.4	54
20.5	53
20.6	52
20.7	47
20.8	49
20.9	48
21	51
21.1	48
21.2	48
21.3	46
21.4	51
21.5	47
21.6	46
21.7	49
21.8	47
21.9	54
22	50
22.1	57
22.2	58
22.3	56
22.4	64
22.5	125
22.6	191
22.7	190
22.8	151
22.9	73
23	52
23.1	48
23.2	52
23.3	41
23.4	43
23.5	39
23.6	41
23.7	47
23.8	36

Continued on next page

TABLE VII: 30kV - Calibration (Continued)

Angle(Deg)	Count per Second
23.9	38
24	40
24.1	35
24.2	32
24.3	36
24.4	40
24.5	35
24.6	39
24.7	37
24.8	37
24.9	31
25	35
25.1	32
25.2	33
25.3	29
25.4	33
25.5	33
25.6	32
25.7	32
25.8	31
25.9	27
26	30
26.1	31
26.2	28
26.3	28
26.4	29
26.5	32
26.6	31
26.7	28
26.8	25
26.9	30
27	38
27.1	35
27.2	34
27.3	28
27.4	29
27.5	32
27.6	31

Continued on next page

TABLE VII: 30kV - Calibration (Continued)

Angle(Deg)	Count per Second
27.7	27
27.8	25
27.9	33
28	27
28.1	26
28.2	25
28.3	28
28.4	28
28.5	26
28.6	25
28.7	20
28.8	25
28.9	27
29	23
29.1	23
29.2	27
29.3	25
29.4	22
29.5	26
29.6	27
29.7	29
29.8	24
29.9	28
30	25
30.1	20
30.2	25
30.3	28
30.4	27
30.5	39
30.6	54
30.7	55
30.8	42
30.9	35
31	27
31.1	26
31.2	23
31.3	21
31.4	21

Continued on next page

TABLE VII: 30kV - Calibration (Continued)

Angle(Deg)	Count per Second
31.5	23
31.6	18
31.7	19
31.8	20
31.9	22
32	24
32.1	19
32.2	19
32.3	17
32.4	18
32.5	18
32.6	20
32.7	17
32.8	21
32.9	18
33	15
33.1	18
33.2	15
33.3	19
33.4	17
33.5	17
33.6	17
33.7	16
33.8	20
33.9	18
34	17
34.1	16
34.2	18
34.3	16
34.4	14
34.5	14
34.6	18
34.7	17
34.8	15
34.9	15
35	19
35.1	17
35.2	13

Continued on next page

TABLE VII: 30kV - Calibration (Continued)

Angle(Deg)	Count per Second
35.3	13
35.4	17
35.5	13
35.6	13
35.7	13
35.8	12
35.9	17
36	15
36.1	16
36.2	17
36.3	13
36.4	16
36.5	17
36.6	15
36.7	12
36.8	13
36.9	12
37	12
37.1	13
37.2	16
37.3	16
37.4	14
37.5	14
37.6	13
37.7	14
37.8	13
37.9	15
38	12
38.1	14
38.2	12
38.3	14
38.4	14
38.5	11
38.6	12
38.7	15
38.8	14
38.9	13
39.0	11

Continued on next page

TABLE VII: 30kV - Calibration (Continued)

Angle(Deg)	Count per Second
39.1	12
39.2	15
39.3	13
39.4	18
39.5	16
39.6	17
39.7	15
39.8	10
39.9	13
40.0	12
40.1	13
40.2	8
40.3	13
40.4	12
40.5	11
40.6	13
40.7	11
40.8	9
40.9	11
41.0	11
41.1	11
41.2	12
41.3	11
41.4	9
41.5	13
41.6	9
41.7	11
41.8	10
41.9	11
42.0	11
42.1	11
42.2	9
42.3	8
42.4	12
42.5	11
42.6	11
42.7	10
42.8	9

Continued on next page

TABLE VII: 30kV - Calibration (Continued)

Angle(Deg)	Count per Second
42.9	10
43.0	8
43.1	11
43.2	8
43.3	9
43.4	10
43.5	10
43.6	10
43.7	12
43.8	9
43.9	9
44.0	11
44.1	8
44.2	8
44.3	9
44.4	10
44.5	8
44.6	11
44.7	10
44.8	9
44.9	10
45.0	11

V. The Analysis

We have used 35kV upto 45 degree to calibrate the proceeding data. As can be seen at Fig.3, we have counted very high X-Rays at the very first angles. This is due to that the Geiger-Muller Tube looks straight to the diaphragm, so the tube also counts non-scattered X-Rays. Therefore, the data at very first angles/wavelengths of figures are meaningless.

We have obtained the theoretical angles corresponds to K_α and K_β according to the Eq.5. The characteristic energies are $K_{\alpha_1} = 17,479.34eV$, $K_{\alpha_2} = 17,374.3eV$, $K_{\beta_1} = 19,608.3eV$ [4]. Since the ratio of intensities of K_{α_1} and K_{α_2} is approximately 2:1, we have taken the weighted average of them to obtain K_{alpha}[5].

We have Gauss Fitted the characteristic emissions as can be seen at Fig.3. The emissions occur as a spike in the continues spectra. The first spike is the merging of K_α and K_β. Since their energy is similar for m=1, we have used the average value of K_α and K_β for the first spike.

We have taken the uncertainty on the Count as $\sqrt{N/5}$ and on the angle as 0.1.

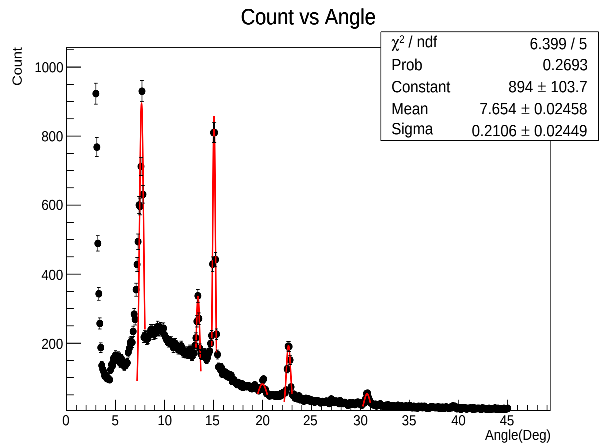

Fig. 3. Experimental Characteristic X-Rays

We have obtained the theoretical and experimental angles to calibrate the data. As can be seen from Fig.4, the calibration obeys the linear equation with Interception = 0.1594 +- 0.03085 and Slope = 0.922 +- 0.001688.

TABLE VIII
THEORETICAL AND EXPERIMENTAL ANGLES

Spike Number	Theoretical	Experimental
1	6.83828 Deg	7.65 +- 0.02 Deg
2	12.9571 Deg	13.41 +- 0.05 Deg
3	14.598 Deg	15.05 +- 0.05 Deg
4	19.6536 Deg	19.98 +- 0.06 Deg
5	22.2132 Deg	22.64 +- 0.03 Deg
6	26.6438 Deg	30.65 +- 0.04 Deg

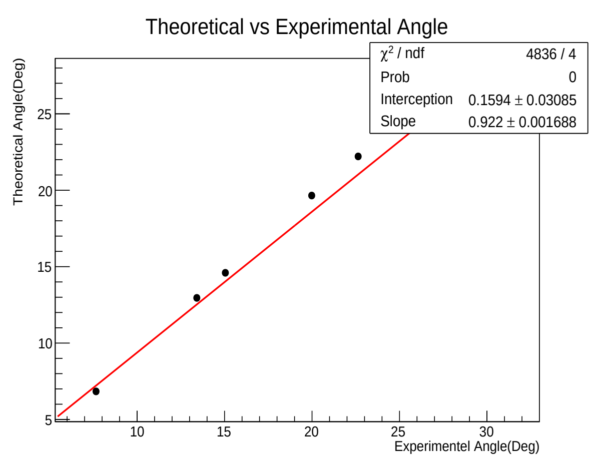

Fig. 4. Calibration - Linear Fit

The χ^2 value of Fig.4 is not close to 1 because of that there is no error on theoretical value.
We have used the uncertainties given by the Root's fit. To calculate the uncertainties of angle and wavelengths, we have used the general formula:

General Case:

$$\sigma_y^2 = \sum_i^m \left(\frac{\partial f}{\partial x_i}\right)^2 \sigma_i^2 + \sum_{j>i}^m \left(\frac{\partial f}{\partial x_i}\right)\left(\frac{\partial f}{\partial x_j}\right)\sigma_{ij}^2 \qquad \text{(C.14)}$$

In the following special cases, it was assumed that there is no correlation between the variables, $\sigma_{ij} = 0$.

Fig. 5. Error Propogation. [2]

To calculate the calibrated angles and the uncertainties, we have used the following equation:

Calculating Calibrated Angles:

$$\theta_{calibrated} = \theta slope + intercept$$

Calculating Uncertainty of Calibrated Angle:

$$\sigma_{\theta_{calibrated}} = \sqrt{(slope)^2\sigma_\theta^2 + \theta^2\sigma_{slope}^2 + \sigma_{intercept}^2}$$

To calculate the wavelength corresponding to the angles, we have used the Bragg's Law and to calculate the uncertainty of the wavelengths, we have used the following equation:

Calculating Uncertainty of Wavelength:

$$\sigma_\lambda = \sqrt{(2Sin(\theta_{calibrated}))^2\sigma_d^2 + (2dCos(\theta_{calibrated}))^2\sigma_{\theta_{calibrated}}^2}$$

where, d = 282.0 +- 0.1 pm

The following plots are created with the raw data and the calibration equation. As can be seen at Fig.5-10, after a certain wavelength, the accumulation rate increases. We have fitted a line to obtain the minimum wavelength.

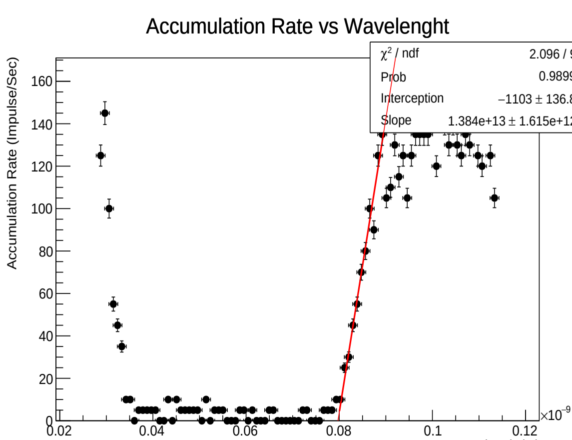

Fig. 6. 15kV

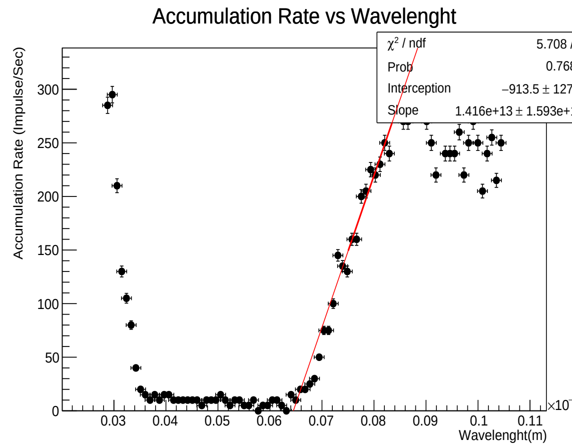

Fig. 7. 18kV

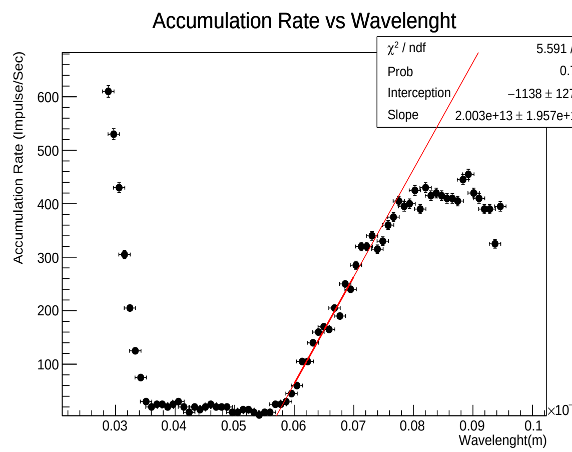

Fig. 8. 21kV

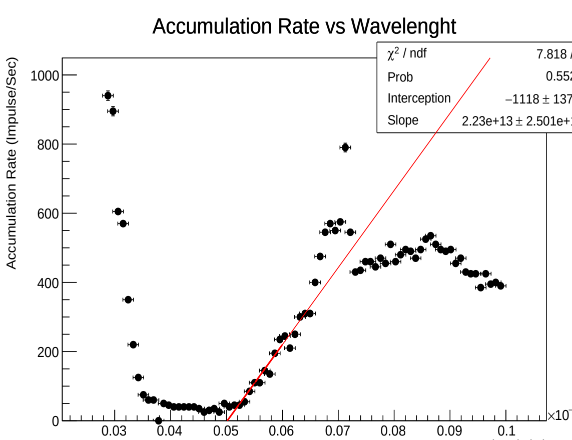

Fig. 9. 24kV

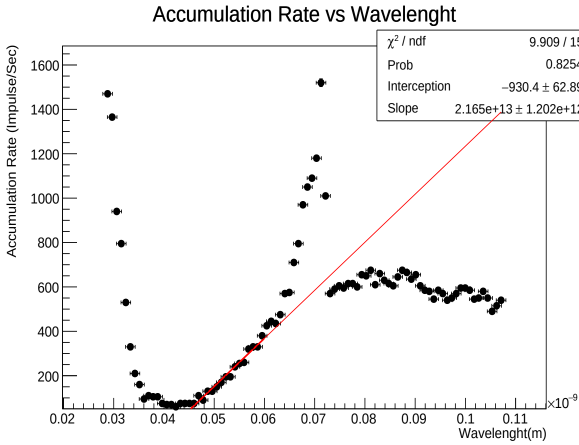

Fig. 10. 27kV

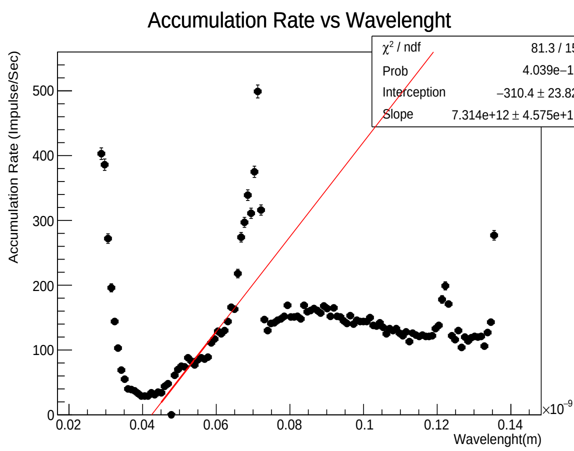

Fig. 11. 30kV

We have used the acceleration voltage to obtain the energy and the minimum wavelength to obtain the maximum frequency using Eq.2.

TABLE IX
FREQUENCY AND WAVELENGTH FOR APPLIED VOLTS

Applied Volt	λ_{min}	ν_{max}
15 ± 1 kV	8.0e-11 +- 1.4e-11 m	3.8e+18 +- 0.6e+18 Hz
18 ± 1 kV	6.4e-11 +- 1.2e-11 m	4.6e+18 +- 0.8e+18 Hz
21 ± 1 kV	5.7e-11 +- 0.8e-11 m	5.3e+18 +- 0.8e+18 Hz
24 ± 1 kV	5.0e-11 +- 0.8e-11 m	6.0e+18 +- 1.0e+18 Hz
27 ± 1 kV	4.3e-11 +- 0.4e-11 m	7.0e+18 +- 0.6e18 Hz
30 ± 1 kV	4.2e-11 +- 0.4e-11 m	7.1e+18 +- 0.7e+18 Hz

As Duane-Hunt Law states, there is a linear relation between the energy and the corresponding maximum frequency. The slope of the linear relation is exactly the Planck Constant.

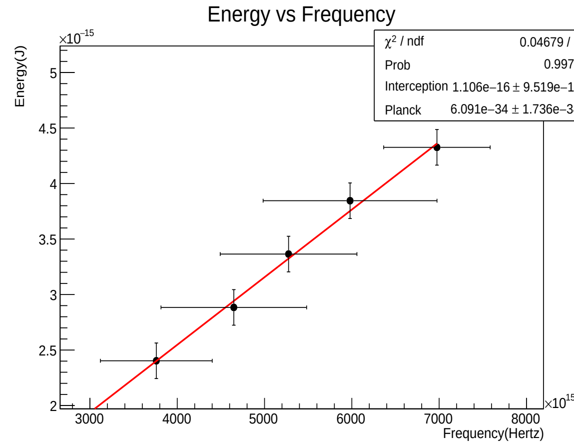

Fig. 12. Planck Constant

We have obtained the Planck Constant with the linear fit.

VI. The Result

We have found the Planck Constant to be $h = 6.1 \pm 1.7 \times 10^{-34} JHz^-1$. The CODATA recommended value is $6.62607015 \times 10^{-34} JHz^-1$ [1]. We are 0.31σ away with $\%7.9$ relative error.

VII. The Conclusion

We have found the Planck Constant to be $h = 6.1 \pm 1.7 \times 10^{-34} JHz^-1$. The CODATA recommended value is $6.62607015 \times 10^{-34} JHz^-1$ [1]. We are 0.31σ away with $\%7.9$ relative error. So, we have concluded that the experiment was a success. However, the uncertainty on the Planck Constant is not sufficiently small, this is due to the apparatus we have used is not adequate for calculating with a high precision.

Moreover, we have observed a high accumulation at the low wavelengths, this is because of that the Geiger-Muller counter look straight to the diaphragm, so it counts non-scattered X-Rays too. So, for a better data the diaphragm could be made smaller.

Another problem we have observed is that, the Geiger-Muller Counter counts several radiations even if we do not apply voltage. This is due to the background radiation. A better shielded environment could lead to a better calculation.

References

[1] *CODATA Value: Planck constant*. URL: https://physics.nist.gov/cgi-bin/cuu/Value?h (visited on 05/10/2023).

[2] E. Gülmez. *Advanced Physics Experiments*. 1st. Boğaziçi University Publications, 1999.

[3] *Bremsstrahlung*. URL: https://commons.wikimedia.org/w/index.php?curid=1531092 (visited on 05/10/2023).

[4] *Molybdenum*. URL: https://xdb.lbl.gov/Section1/Periodic_Table/Mo_Web_data.htm (visited on 05/10/2023).

[5] L. E Klug H. P. Alexander. *X-Ray diffraction procedures: for polycrystalline and amorphous materials*. 2nd. John Wiley and Sons, Inc., 1974.

VIII. Appendix

The fit has been done with Root's built-in function (See Fig.3-11). We have used root release 6.28/00 for Ubuntu22.

Macro for Calibration:

```
{
 TTree *tree = new TTree("tree", "tree");
 tree->ReadFile("XRAY_data/calibration.csv");

 float c, a;
 tree->SetBranchAddress("c", &c);
 tree->SetBranchAddress("a", &a);


 double *count, *angle, *sd_count, *sd_angle;
 double *experimental, *sd_experimental;
 double *theoretical;
 int n_angle = 6;
 int n = tree->GetEntries();
 count = new double[n];
 angle = new double[n];
 sd_count = new double[n];
 sd_angle = new double[n];
 theoretical = new double[n_angle];
 experimental = new double[n_angle];
 sd_experimental = new double[n_angle];


 for (int i = 0; tree->LoadTree(i) >= 0; i++){
  tree->GetEntry(i);
  count[i] = c;
   angle[i] = a;
   sd_count[i] = sqrt(count[i]/5);
   sd_angle[i] = 0.1;
 }


 double h = 6.62607015e-34;
 double light = 299792458.;
 double d = 282.0e-12;
 double sd_d = 0.1e-12;
 double eV = 1.60217663e-19;
 double K_a1 = 17479.34*eV;
 double K_a2 = 17374.3*eV;
 double K_a = (2*K_a1 + 1*K_a2)/3;
 double K_b = 19608.3*eV;
 double rad2deg = 180/TMath::Pi();


 for(int m = 0; m < 4; m++){

 double theta_a =
     rad2deg*TMath::ASin((m+1)*h*light/(2*K_a*d));
 double theta_b =
     rad2deg*TMath::ASin((m+1)*h*light/(2*K_b*d));

 if(m == 0){
   theoretical[m] = (theta_a+theta_b)/2;
 }
 else{
   if(m == 3){
     theoretical[2*m-1] = theta_b;
   }
   else{
   theoretical[2*m - 1] = theta_b;
   theoretical[2*m] = theta_a;
   }
}
cout << "Theoretical K alpha for m = " <<
    m+1 << ": "<< theta_a << endl;
cout << "Theoretical K beta for m = " << m+1
    << ": "<< theta_b << endl;
}


TCanvas *c1 = new TCanvas();
TGraphErrors *graph = new
    TGraphErrors(n,angle,count,sd_angle,sd_count);
graph->SetTitle("Count vs Angle;
    Angle(Deg);Count");


TF1 *f1 = new TF1("f1","gaus",7.2,8);
TF1 *f2 = new TF1("f2","gaus",13.2,13.7);
TF1 *f3 = new TF1("f3","gaus",14.8,15.3);
TF1 *f4 = new TF1("f4","gaus",19.5,20.5);
TF1 *f5 = new TF1("f5","gaus",22.2,23);
TF1 *f6 = new TF1("f6","gaus",30.2,31);

graph->Fit(f1,"R");
graph->Fit(f2,"R+");
graph->Fit(f3,"R+");
graph->Fit(f4,"R+");
graph->Fit(f5,"R+");
graph->Fit(f6,"R+");


experimental[0] = f1->GetParameter(1);
sd_experimental[0] = f1->GetParError(1);

experimental[1] = f2->GetParameter(1);
sd_experimental[1] = f2->GetParError(1);

experimental[2] = f3->GetParameter(1);
sd_experimental[2] = f3->GetParError(1);

experimental[3] = f4->GetParameter(1);
sd_experimental[3] = f4->GetParError(1);

experimental[4] = f5->GetParameter(1);
sd_experimental[4] = f5->GetParError(1);

experimental[5] = f6->GetParameter(1);
sd_experimental[5] = f6->GetParError(1);

graph->SetMarkerStyle(20);
graph->Draw("AP");

TCanvas *c2 = new TCanvas();
TGraphErrors *graph_c = new
    TGraphErrors(n_angle,experimental,theoretical,
sd_experimental,0);
TF1 *f_linear = new TF1("f_linear","[0] +
    [1]*x",0,35);
f_linear->SetParameters(0,1);
f_linear->SetParNames("Interception","Slope");
graph_c->Fit("f_linear","RE");
graph_c->SetTitle("Theoretical vs
    Experimental Angle;Experimentel
    Angle(Deg);Theoretical Angle(Deg)");
graph_c->SetMarkerStyle(20);
graph_c->Draw("AP");
gStyle->SetOptFit(1111);

double slope = f_linear->GetParameter(1);
double intercept = f_linear->GetParameter(0);
```

```
double sd_slope = f_linear->GetParError(1);
double sd_intercept =
    f_linear->GetParError(0);
cout << "Slope of the calibration = " <<
    slope << " +- " << sd_slope << endl;
cout << "Intercept of the calibration = " <<
    intercept << " +- " << sd_intercept <<
    endl;

for(int i = 0; i < n_angle; i++){
  cout << "Experimental Angle For Spike-" <<
      i+1 << " " << experimental[i] << " +- "
      << sd_experimental[i] << endl;;
  cout << "Theoretical Angle For Spike-" <<
      i+1 <<" " << theoretical[i] << endl;
}
c1->Print("xray_gauss.pdf");
 c2->Print("xray_calibration.pdf");
}
```

Macro For Calculating Minimum Wavelengths:

```
  {
TTree *tree = new TTree("tree", "tree");
tree->ReadFile("XRAY_data/30.csv");
double volt = 30*1e3;
double x_min = 0.045e-9;
double x_max = 0.06e-9;
float a, I;
tree->SetBranchAddress("a", &a);
tree->SetBranchAddress("I", &I);

double *count, *angle, *sd_count, *sd_angle;
double *wave, *sd_wave, *extend;
int n = tree->GetEntries();
wave = new double[n];
count = new double[n];
angle = new double[n];
sd_count = new double[n];
sd_angle = new double[n];
sd_wave = new double[n];
extend = new double[n];

double h = 6.62607015e-34;
double light = 299792458.;
double eV = 1.60217663e-19;
double K_a1 = 17479.34*eV;
double K_a2 = 17374.3*eV;
double K_a = (2*K_a1 + 1*K_a2)/3;
double K_b = 19608.3*eV;
double deg2rad = TMath::Pi()/180;
double slope = 0.92199;
double sd_slope = 0.001688;
double intercept = 0.15944;
double sd_intercept = 0.0308526;
double d = 282.0e-12;
double sd_d = 0.1e-12;

for (int i = 0; tree->LoadTree(i) >= 0; i++){
 tree->GetEntry(i);
 count[i] = I;
  angle[i] = a*slope + intercept;
  wave[i] = 2*d*TMath::Sin(deg2rad*angle[i]);
  sd_count[i] = sqrt(count[i]/5);
  sd_angle[i] = sqrt(pow(slope,2)*pow(0.1,2) +
  pow(a,2)*pow(sd_slope,2) +
  pow(1,2)*pow(sd_intercept,2));
  sd_wave[i] =
     sqrt(pow(2*TMath::Sin(deg2rad*angle[i]),2)
  *pow(sd_d,2) +
  pow(2*d*TMath::Cos(deg2rad*angle[i])*deg2rad,2)
  *pow(sd_angle[i],2));
 }


gStyle->SetOptFit(1111);
TCanvas *c1 = new TCanvas();
TGraphErrors *graph = new
    TGraphErrors(n,wave,count,sd_wave,sd_count);
TF1 *f_linear = new TF1("f_linear","[0] +
    [1]*x",x_min,x_max);
double par_1 = 1e12;
double par_0 = -x_min * par_1;
f_linear->SetParameters(par_0,par_1);
f_linear->SetParNames("Interception","Slope");
graph->Fit("f_linear","R");
double new_slope = f_linear->GetParameter(1);
double new_intercept =
    f_linear->GetParameter(0);
double sd_new_slope =
    f_linear->GetParError(1);
double sd_new_intercept =
    f_linear->GetParError(0);
for(int i = 0; i < n; i++){
 extend[i] = new_slope*wave[i] +
     new_intercept;
}
TGraph *graph2 = new TGraph(n,wave,extend);
graph->SetTitle("Accumulation Rate vs
    Wavelenght; Wavelenght(m); Accumulation
    Rate (Impulse/Sec)");
graph->SetMarkerStyle(20);
graph->Draw("AP");
graph2->SetLineColor(2);

graph2->Draw("Same");


double lambda = -new_intercept/new_slope;
double sd_lambda =
    sqrt(pow(-1/new_slope,2)*pow(sd_new_intercept,2)
    +
pow(new_intercept/pow(new_slope,2),2)*pow(sd_new_slope
double freq = light/lambda;
double sd_freq =
    sqrt(pow(light/(lambda*lambda),2)*pow(sd_lambda,2))
cout << "Lambda_Min: " << lambda << " +- "
    << sd_lambda << endl;
cout << "Freq_Max: " << freq << " +- " <<
    sd_freq << endl;
cout << "Volt: " << volt << " +- " << 1e3 <<
    endl;
c1->Print("30.pdf");
}
```

Macro for Calculating Planck Constant:

```
  {
const int n = 6;
double q = 1.602176634e-19;
double freq[n] =
    {3.76064e+18,4.64811e+18,5.27529e+18,
5.97956e+18,6.97476e+18,7.07684e+18};
double sd_freq[n] = {6.40308e+17, 8.3406e+17
    ,
7.82979e+17, 9.95806e+17 ,6.10088e+17,
```

```
    6.56757e+17};
double energy[n] = {q*15e3, q*18e3, q*21e3,
    q*24e3, q*27e3, q*30e3};
double sd_energy[n] =
    {q*1e3,q*1e3,q*1e3,q*1e3,q*1e3,q*1e3};

TCanvas *c1 = new TCanvas();
TGraphErrors *graph = new
    TGraphErrors(n,freq,energy,sd_freq,sd_energy);
TF1 *f_linear = new TF1("f_linear","[0] +
    [1]*x",1e18,7e18);
f_linear->SetParameters(0,6e-34);
f_linear->SetParNames("Interception","Planck");
graph->Fit("f_linear","R");
graph->SetTitle("Energy vs Frequency;
    Frequency(Hertz); Energy(J)");
graph->SetMarkerStyle(20);
graph->Draw("AP");
gStyle->SetOptFit(1111);

double planck = f_linear->GetParameter(1);
double sd_planck = f_linear->GetParError(1);
double h = 6.62607015e-34;

cout << "Experimental Planck Constant: " <<
    planck << " +- " << sd_planck << endl;
cout << "Relative Error: " <<
    abs(planck-h)*100/h << "%"<< endl;
cout << abs(planck-h)/sd_planck << " Sigma
    Away" << endl;
c1->Print("xray_h.pdf");
}
```