Radioactive Decay Experiment

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***Abstract*—The experiment aims to calculate the half-life of ^{220}Rn. We have conducted the experiment with Thorium Salt ^{232}Th, which produces ^{220}Rn in its decay chain. We have calculated the half-life of ^{220}Rn from the Decay Law. It is calculated to be $t_{1/2} = 72.30 \pm 0.09s$. The true value is 55.6s, so we are $186sigma$ away from the true value with $\%30.0$ relative error.**

I. Theory

Radioactivity is discovered by Henri Becquerel and Pierre Curie, Marie Skłodowska-Curie in 1896. In 1903, they are awarded with the Nobel Prize in Physics [1]. The Radioactive Decay is a phenomenon observed when an unstable atomic nucleus loses energy with a radiation. The radiation may be in a different nature. The most common 3 decays are α (helium nucleus), β (electron + antineutrino or positron + neutrino), γ (photon) decays.[2]

The decays may occur with different ways; however, they obeys the same law. [3]

$$\text{Decay Law: } N(t) = N_0 e^{-\lambda t} \tag{1}$$

where, λ is the decay constant

N_0 is the initial number of unstable isotope of nuclei.

II. Method

Eq.1 suggest that the number of unstable isotopes of nuclei will be decreased with time. We aim to observe this reduction and therefore confirm the theory. ^{220}Rn decays to $^(216)Po$, which decays to ^{212}Pb with alpha particle [4]. The alpha particle has positive charge; therefore, creates a current in the system. To calculate that, we have used Wulf's Electroscope. The Wulf's Electroscope is a device which grounds its current after a certain current value. So that, instead of calculating the current, we can calculate the time passed between to successive discharges.

$$Q = Is \Rightarrow I = \frac{Q}{s} \Rightarrow I \propto \frac{1}{s} \tag{2}$$

The current is proportional to the successive decays

$$\text{Number of Decays} \propto \frac{1}{s} \tag{3}$$

1) High Voltage is applied.
2) The Thorium Salt vessel is squeezed to obtain Radon Gas (^{220}Rn) in the Ionization Chamber.
3) Recorded the time at each instant of electroscope discharge, until no movement can be seen.

III. The Experimental Setup

- Wulf's Electroscope.
- Thorium Salt.
- Ionization Chamber.
- HV Power Supply (0-5kV).
- Stopwatch.

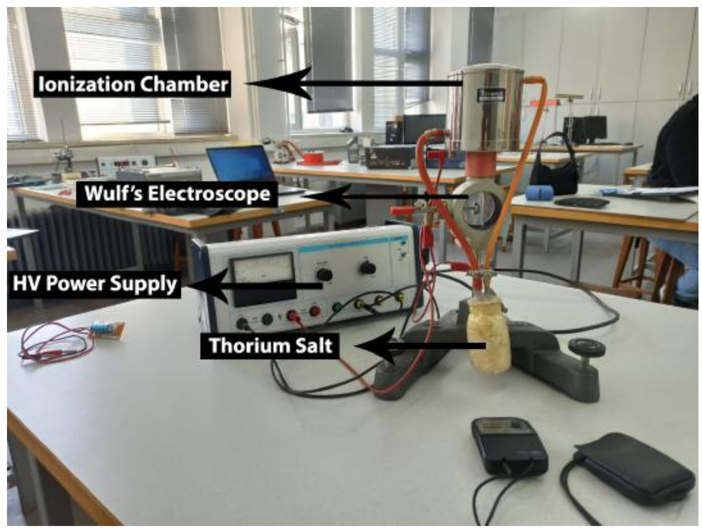

Fig. 1. Apparatus.

IV. The Data

TABLE I
Elapsed Time for 2500V and 2 Squeezes

Elapsed Time(s)
3.573± 0.3
11.564± 0.3
14.675± 0.3
21.111± 0.3
24.965± 0.3
31.399± 0.3
44.585± 0.3
49.321± 0.3
58.405± 0.3
65.581± 0.3
77.634± 0.3
83.147± 0.3
101.671± 0.3
107.114± 0.3
134.121± 0.3
141.615± 0.3
183.623± 0.3
202.609± 0.3
290.544± 0.3

TABLE II
ELAPSED TIME FOR 2500V AND 3 SQUEEZES

Elapsed Time(s)
3.924± 0.3
6.293± 0.3
10.459± 0.3
12.72± 0.3
16.643± 0.3
19.791± 0.3
25.756± 0.3
27.63± 0.3
33.5± 0.3
37.354± 0.3
42.092± 0.3
45.912± 0.3
53.585± 0.3
58.429± 0.3
64.724± 0.3
69.353± 0.3
78.579± 0.3
84.625± 0.3
94.983± 0.3
102.301± 0.3
118.673± 0.3
125.466± 0.3
149.743± 0.3
158.653± 0.3
197.259± 0.3
210.798± 0.3
294.612± 0.3

TABLE III
ELAPSED TIME FOR 2500V AND 4 SQUEEZES

Elapsed Time(s)
9.955± 0.3
2.934± 0.3
6.505± 0.3
8.201± 0.3
12.756± 0.3
14.135± 0.3
18.617± 0.3
21.056± 0.3
24.805± 0.3
28.203± 0.3
31.881± 0.3
35.767± 0.3
39.861± 0.3
43.962± 0.3
48.384± 0.3
53.617± 0.3
58.248± 0.3
63.976± 0.3
69.384± 0.3
76.423± 0.3
88.329± 0.3
90.355± 0.3
97.746± 0.3
107.647± 0.3
114.044± 0.3
130.841± 0.3
139.428± 0.3
156.928± 0.3
175.286± 0.3
200.071± 0.3
231.215± 0.3
285.126± 0.3

TABLE IV
ELAPSED TIME FOR 2500V AND 5 SQUEEZES

Elapsed Time(s)
3.572± 0.3
6.045± 0.3
9.014± 0.3
11.417± 0.3
14.636± 0.3
17.18± 0.3
20.007± 0.3
23.791± 0.3
26.796± 0.3
30.502± 0.3
33.791± 0.3
37.782± 0.3
41.845± 0.3
46.196± 0.3
50.614± 0.3
55.633± 0.3
60.406± 0.3
65.645± 0.3
71.516± 0.3
76.855± 0.3
84.423± 0.3
90.042± 0.3
100.326± 0.3
107.681± 0.3
118.608± 0.3
129.718± 0.3
142.2± 0.3
159.981± 0.3
175.432± 0.3
200.865± 0.3
232.758± 0.3
284.906± 0.3

TABLE V
ELAPSED TIME FOR 3000V AND 2 SQUEEZES

Elapsed Time(s)
2.759± 0.3
9.235± 0.3
12.172± 0.3
24.270± 0.3
27.208± 0.3
32.406± 0.3
41.778± 0.3
45.878± 0.3
50.793± 0.3
56.235± 0.3
62.813± 0.3
68.365± 0.3
76.994± 0.3
84.669± 0.3
93.620± 0.3
115.057± 0.3
123.655± 0.3
141.172± 0.3
155.358± 0.3
183.486± 0.3
207.224± 0.3
278.298± 0.3

TABLE VI
ELAPSED TIME FOR 3000V AND 3 SQUEEZES

Elapsed Time(s)
4.743± 0.3
5.592± 0.3
10.366± 0.3
11.424± 0.3
16.442± 0.3
17.961± 0.3
22.449± 0.3
25.101± 0.3
30.189± 0.3
32.520± 0.3
38.918± 0.3
41.675± 0.3
47.688± 0.3
62.219± 0.3
69.435± 0.3
73.537± 0.3
84.255± 0.3
89.348± 0.3
100.346± 0.3
106.998± 0.3
120.898± 0.3
130.410± 0.3
147.594± 0.3
162.268± 0.3
189.508± 0.3
211.654± 0.3
267.938± 0.3

TABLE VII
ELAPSED TIME FOR 3000V AND 4 SQUEEZES

Elapsed Time(s)
2.124± 0.3
4.314± 0.3
7.424± 0.3
9.934± 0.3
12.867± 0.3
15.447± 0.3
18.805± 0.3
21.564± 0.3
25.454± 0.3
27.823± 0.3
32.411± 0.3
34.531± 0.3
40.537± 0.3
48.770± 0.3
51.028± 0.3
57.382± 0.3
71.625± 0.3
78.273± 0.3
83.543± 0.3
91.952± 0.3
96.512± 0.3
107.935± 0.3
114.020± 0.3
128.486± 0.3
139.347± 0.3
151.585± 0.3
190.421± 0.3
226.412± 0.3
267.679± 0.3
355.011± 0.3

TABLE VIII
ELAPSED TIME FOR 3000V AND 5 SQUEEZES

Elapsed Time(s)
3.783± 0.3
5.516± 0.3
9.894± 0.3
11.554± 0.3
16.432± 0.3
18.235± 0.3
23.151± 0.3
24.917± 0.3
30.894± 0.3
32.626± 0.3
39.414± 0.3
41.147± 0.3
48.503± 0.3
50.693± 0.3
59.181± 0.3
61.691± 0.3
75.448± 0.3
84.995± 0.3
89.379± 0.3
103.134± 0.3
109.217± 0.3
124.427± 0.3
131.642± 0.3
155.692± 0.3
167.463± 0.3
194.874± 0.3
224.972± 0.3
273.439± 0.3
367.433± 0.3

TABLE IX
ELAPSED TIME FOR 3500V AND 2 SQUEEZES

Elapsed Time(s)
1.557± 0.3
6.401± 0.3
9.122± 0.3
14.920± 0.3
17.181± 0.3
22.335± 0.3
26.222± 0.3
30.322± 0.3
36.825± 0.3
40.997± 0.3
46.794± 0.3
53.902± 0.3
59.380± 0.3
67.611± 0.3
74.746± 0.3
84.212± 0.3
94.216± 0.3
104.216± 0.3
119.314± 0.3
130.111± 0.3
155.593± 0.3
175.513± 0.3
208.295± 0.3
251.525± 0.3
336.451± 0.3

TABLE X
ELAPSED TIME FOR 3500V AND 3 SQUEEZES

Elapsed Time(s)
3.855± 0.3
6.540± 0.3
10.676± 0.3
13.539± 0.3
17.284± 0.3
21.170± 0.3
25.129± 0.3
29.762± 0.3
34.112± 0.3
43.197± 0.3
48.183± 0.3
54.018± 0.3
66.285± 0.3
74.425± 0.3
80.892± 0.3
89.690± 0.3
100.056± 0.3
109.604± 0.3
122.871± 0.3
136.765± 0.3
154.263± 0.3
174.275± 0.3
202.245± 0.3
246.732± 0.3
312.195± 0.3

TABLE XI
ELAPSED TIME FOR 3500V AND 4 SQUEEZES

Elapsed Time(s)
3.185± 0.3
6.405± 0.3
8.033± 0.3
16.386± 0.3
18.542± 0.3
20.097± 0.3
25.403± 0.3
26.606± 0.3
32.796± 0.3
41.069± 0.3
43.086± 0.3
49.243± 0.3
52.458± 0.3
59.676± 0.3
63.179± 0.3
70.220± 0.3
75.102± 0.3
84.264± 0.3
89.501± 0.3
99.974± 0.3
107.861± 0.3
118.686± 0.3
130.705± 0.3
143.821± 0.3
162.605± 0.3
176.189± 0.3
207.908± 0.3
228.482± 0.3
302.825± 0.3

TABLE XII
ELAPSED TIME FOR 3500V AND 5 SQUEEZES

Elapsed Time(s)
1.340± 0.3
3.100± 0.3
6.207± 0.3
8.040± 0.3
11.784± 0.3
13.692± 0.3
17.328± 0.3
19.905± 0.3
25.770± 0.3
30.043± 0.3
32.445± 0.3
36.862± 0.3
40.608± 0.3
44.527± 0.3
48.694± 0.3
55.018± 0.3
58.057± 0.3
63.348± 0.3
68.151± 0.3
74.477± 0.3
79.912± 0.3
86.452± 0.3
92.882± 0.3
102.596± 0.3
107.967± 0.3
122.378± 0.3
129.182± 0.3
144.618± 0.3
156.070± 0.3
178.504± 0.3
193.712± 0.3
231.250± 0.3
259.073± 0.3
344.429± 0.3

TABLE XIII
ELAPSED TIME FOR 4000V AND 2 SQUEEZES

Elapsed Time(s)
8.668± 0.3
11.429± 0.3
16.379± 0.3
18.958± 0.3
23.942± 0.3
26.631± 0.3
33.489± 0.3
36.353± 0.3
43.430± 0.3
47.421± 0.3
54.528± 0.3
58.734± 0.3
68.600± 0.3
72.055± 0.3
85.299± 0.3
89.855± 0.3
95.931± 0.3
105.229± 0.3
112.295± 0.3
130.062± 0.3
144.896± 0.3
167.686± 0.3
188.075± 0.3
234.691± 0.3
292.832± 0.3

TABLE XIV
ELAPSED TIME FOR 4000V AND 3 SQUEEZES

Elapsed Time(s)
0.739± 0.3
1.585± 0.3
3.736± 0.3
6.838± 0.3
9.515± 0.3
12.051± 0.3
16.102± 0.3
18.814± 0.3
22.625± 0.3
26.091± 0.3
29.698± 0.3
33.552± 0.3
37.973± 0.3
41.864± 0.3
46.886± 0.3
57.638± 0.3
61.174± 0.3
67.502± 0.3
72.879± 0.3
78.574± 0.3
87.558± 0.3
94.418± 0.3
103.934± 0.3
111.364± 0.3
125.231± 0.3
137.075± 0.3
152.033± 0.3
170.035± 0.3
191.790± 0.3
222.003± 0.3
269.766± 0.3
342.003± 0.3

TABLE XV
ELAPSED TIME FOR 4000V AND 4 SQUEEZES

Elapsed Time(s)
1.028± 0.3
4.319± 0.3
6.016± 0.3
10.297± 0.3
12.170± 0.3
16.061± 0.3
18.924± 0.3
22.318± 0.3
25.925± 0.3
29.675± 0.3
33.634± 0.3
37.101± 0.3
42.052± 0.3
44.843± 0.3
52.022± 0.3
55.982± 0.3
61.676± 0.3
66.132± 0.3
73.337± 0.3
82.497± 0.3
86.067± 0.3
102.517± 0.3
110.722± 0.3
124.096± 0.3
132.972± 0.3
151.510± 0.3
161.128± 0.3
193.888± 0.3
206.294± 0.3
263.138± 0.3

TABLE XVI
ELAPSED TIME FOR 4000V AND 5 SQUEEZES

Elapsed Time(s)
1.239± 0.3
1.804± 0.3
2.863± 0.3
3.675± 0.3
4.804± 0.3
6.181± 0.3
6.817± 0.3
7.807± 0.3
8.266± 0.3
10.953± 0.3
12.933± 0.3
14.948± 0.3
16.893± 0.3
19.226± 0.3
21.806± 0.3
24.494± 0.3
27.04± 0.3
30.294± 0.3
32.664± 0.3
35.952± 0.3
39.133± 0.3
41.289± 0.3
45.994± 0.3
47.795± 0.3
52.852± 0.3
60.169± 0.3
61.651± 0.3
70.103± 0.3
71.692± 0.3
78.128± 0.3
81.839± 0.3
89.725± 0.3
94.003± 0.3
102.025± 0.3
106.623± 0.3
115.638± 0.3
121.438± 0.3
131.022± 0.3
138.946± 0.3
154.967± 0.3
159.034± 0.3
172.084± 0.3
184.323± 0.3
205.266± 0.3
218.069± 0.3
251.041± 0.3
269.256± 0.3
335.827± 0.3

V. THE ANALYSIS

We have investigated the radioactive decay with various volt and squeezes (See. Fig2-17). As can be seen from plots, there is an exponential behaviour of the data points as Eq.1 suggests. We have used the Root's built-in fit function to obtain the decay constant from the exponential fit (See. Appendix).

TABLE XVII
DECAY CONSTANTS FOR VOLT AND SQUEEZES

Decay Constant	Volt(V)	Squeezes
$0.009884 \pm 4.8 \times 10^{-5}$	2500 $\pm$100	2
$0.01179 \pm 6. \times 10^{-5}$	2500 $\pm$100	3
$0.008725 \pm 5.0 \times 10^{-5}$	2500 $\pm$100	4
$0.01114 \pm 6. \times 10^{-5}$	2500 $\pm$100	5
$0.009717 \pm 4.8 \times 10^{-5}$	3000 $\pm$100	2
$0.009570 \pm 5.6 \times 10^{-5}$	3000 $\pm$100	3
$0.007535 \pm 3.9 \times 10^{-5}$	3000 $\pm$100	4
$0.008707 \pm 3.2 \times 10^{-5}$	3000 $\pm$100	5
$0.009630 \pm 4.1 \times 10^{-5}$	3500 $\pm$100	2
$0.009329 \pm 4.8 \times 10^{-5}$	3500 $\pm$100	3
$0.01088 \pm 4. \times 10^{-5}$	3500 $\pm$100	4
$0.01054 \pm 4. \times 10^{-5}$	3500 $\pm$100	5
$0.008875 \pm 5.7 \times 10^{-5}$	4000 $\pm$100	2
$0.009204 \pm 4.4 \times 10^{-5}$	4000 $\pm$100	3
$0.01111 \pm 7. \times 10^{-5}$	4000 $\pm$100	4
$0.01021 \pm 5. \times 10^{-5}$	4000 $\pm$100	5

We have used the data to calculate the weighted-average of the decay constant, which is $\lambda = 0.009587$ +- 1.2e-05. From the decay constant we are able to calculate the half-life of ^{220}Rn according to the formulas:

$$t_{1/2} = \frac{ln2}{\lambda} \tag{4}$$

$$t_{1/2} = \frac{ln2}{0.009587} = 72.30 \tag{5}$$

$$\sigma_t^2 = \frac{1}{\sum_i^N 1/\sigma_i^2}[3] \tag{6}$$

$$\sigma_t = 0.09 \tag{7}$$

So, we have found that the decay constant of the ^{220}Rn is 72.30 +- 0.09 s. We have also investigated the decay constants as a function of volt and squeezes (See. Fig18-19). We could not see any relation between decay constants and various squeezes or volts.

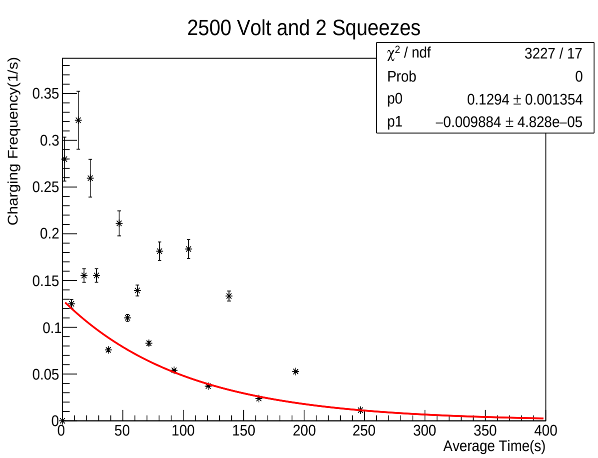

Fig. 2

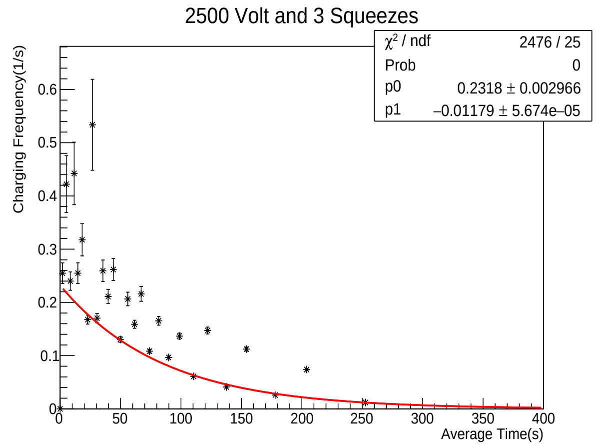

Fig. 3

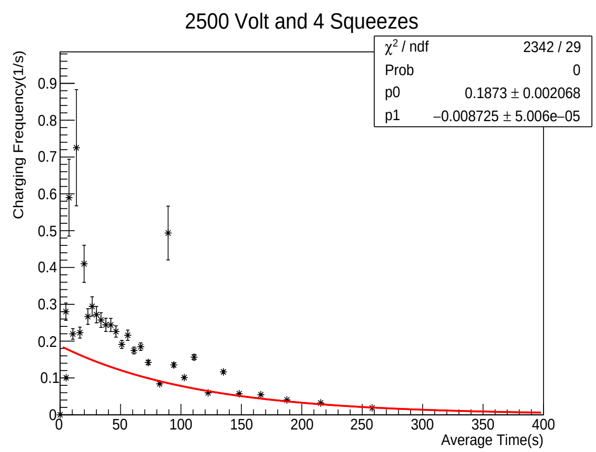

Fig. 4

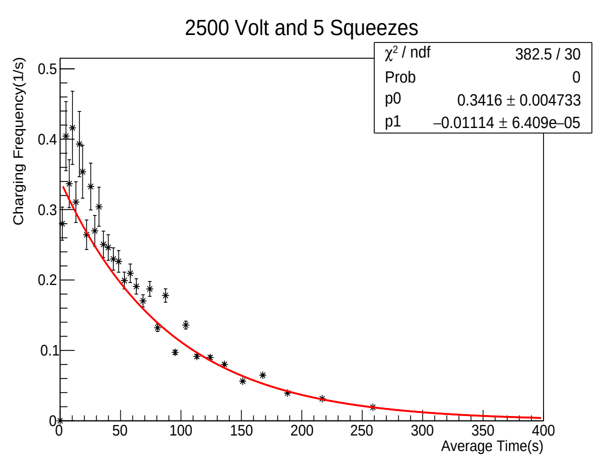

Fig. 5

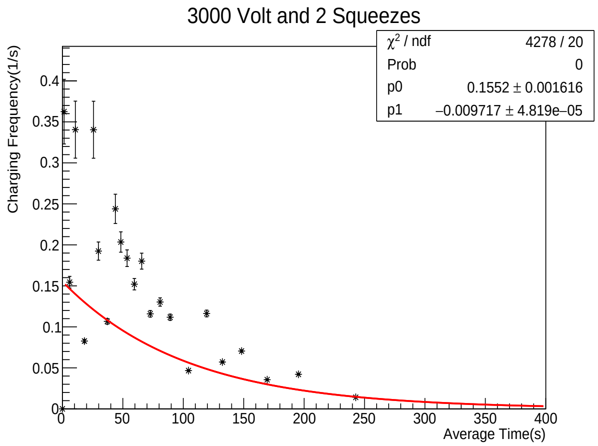

Fig. 6

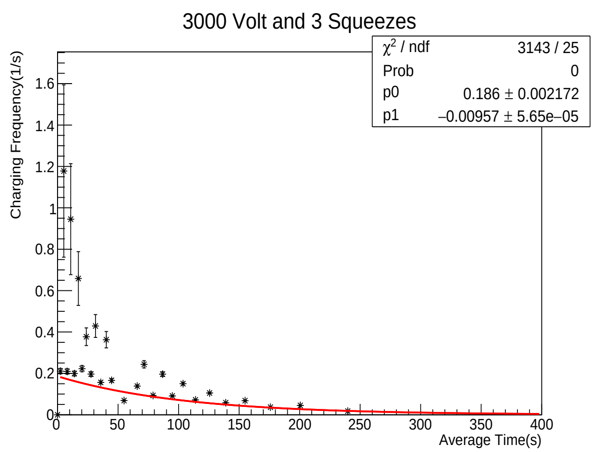

Fig. 7

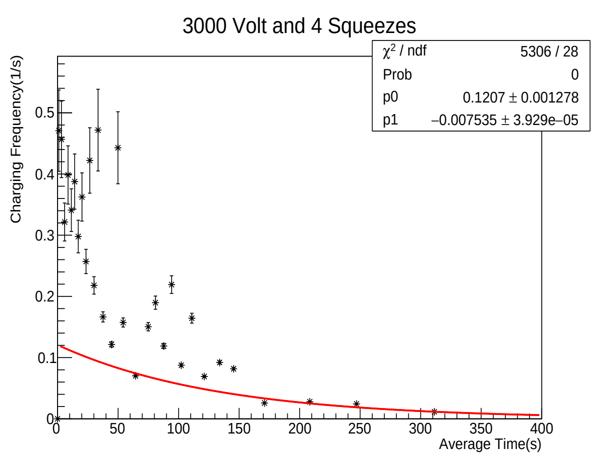

Fig. 8

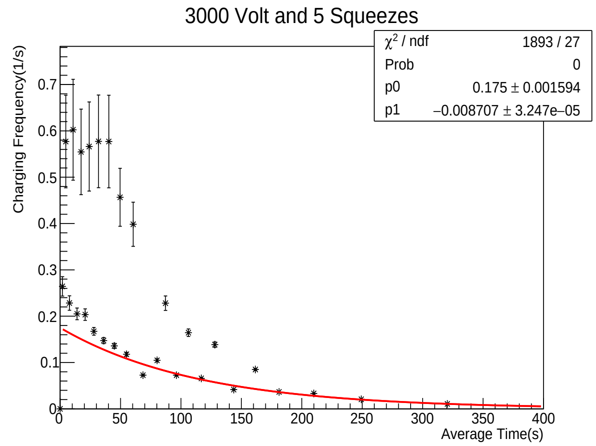

Fig. 9

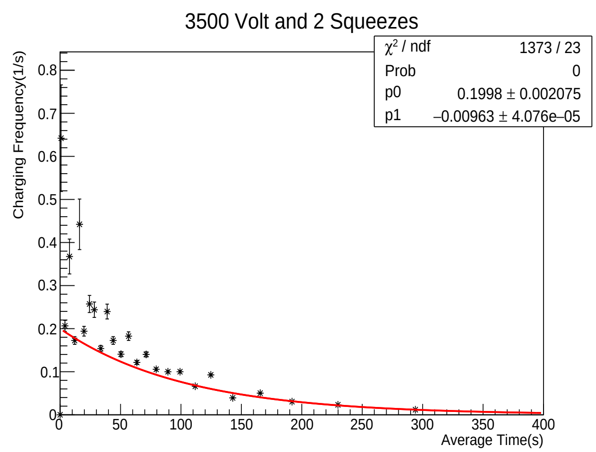

Fig. 10

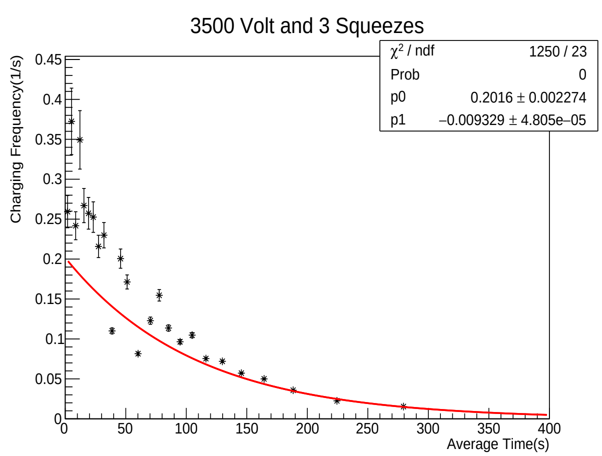

Fig. 11

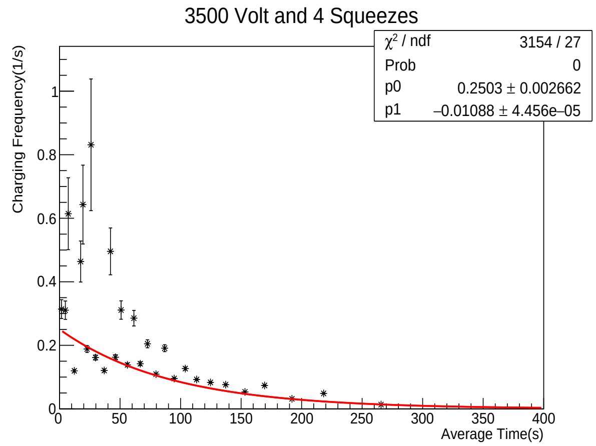

Fig. 12

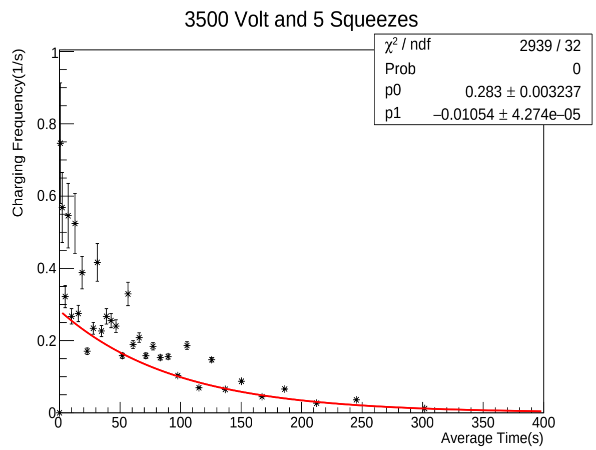

Fig. 13

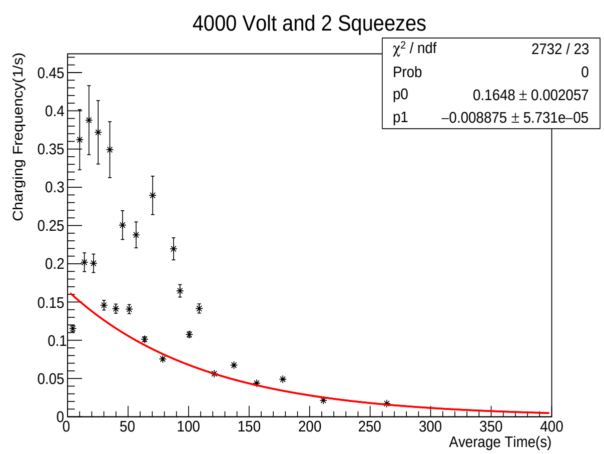

Fig. 14

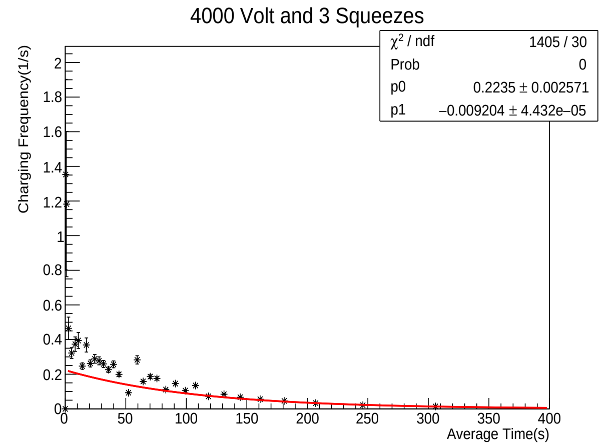

Fig. 15

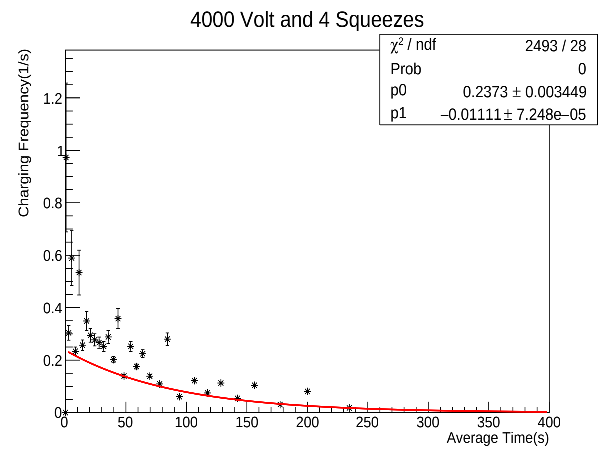

Fig. 16

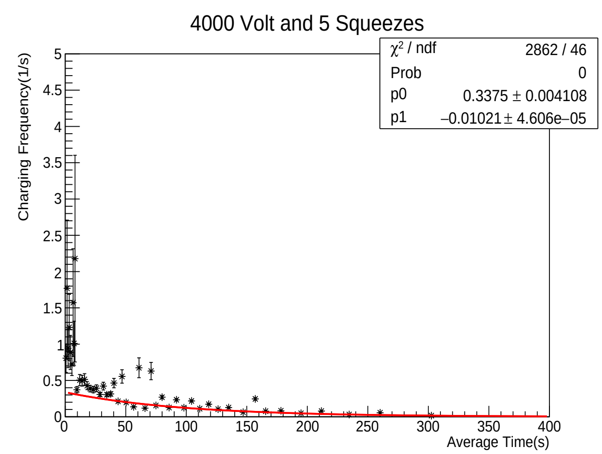

Fig. 17

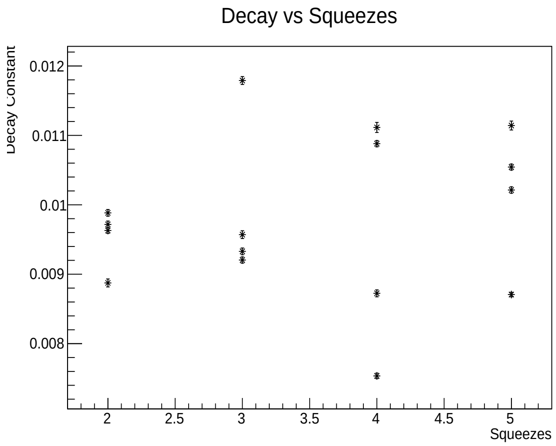

Fig. 18

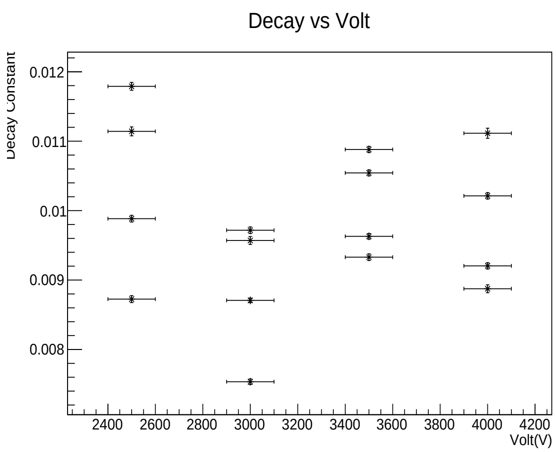

Fig. 19

VI. The Result

We have found the $t_{1/2}$ of ^{220}Rn to be 72.30 +- 0.09 s. The true value for $t_{1/2}$ is 55.6 seconds. So, we are 186 σ away from the true value with a relative error of $\%30.0$.

VII. The Conclusion

The result we have obtained is far away from the recommended value. Therefore, we have concluded that the experiment was a failure. Even if we have tried to obtain the best data we can get, it is obvious that there were some issues. One, and probably the most, important issue we have seen is that we have worked with an inadequate equipment. The Wulf's Electroscope was not fully discharging, so as can be seen in the data, the time between sequential discharges fluctuates, rather than obeying an exponential function.

Another impurity in the experiment is the daughter nuclei ^{216}Po which has 0.15s half-life. Since its half-life is relatively short, its effect is mainly occurs at the first few decays. The effect of daughter nuclei is not sufficient to explain the 186 σ away from the true value

The last thing we should mention is that the time interval that we have taken. We have taken intervals at the mid-points of 2 events. Which in return, we have found a bigger decay constant.

The decay constant can be found with:

$$ln(y_1) = -\lambda T_1$$

$$\lambda = \frac{-ln(y_1)}{T_1}$$

We have taken the midpoint of two time intervals:

$$ln(y_1) = -\lambda * (\frac{2T_1 + \epsilon}{2})$$

$$\lambda = \frac{-ln(y_1)}{T1 + \frac{\epsilon}{2}}$$

However, the error because of the midpoints are not also sufficient to explain the error. Therefore, we have concluded that the main error is due to the apparatus we have used.

References

[1] *The Nobel Prize in Physics 1903*. URL: https://www.nobelprize.org/prizes/physics/1903/summary/ (visited on 03/25/2023).
[2] *Radioactive Decay — US EPA*. URL: https://www.epa.gov/radiation/radioactive-decay (visited on 03/25/2023).
[3] E. Gülmez. *Advanced Physics Experiments*. 1st. Boğaziçi University Publications, 1999.
[4] *Decay Chain of Thorium*. URL: https://pubs.usgs.gov/of/2004/1050/thorium.htm (visited on 03/25/2023).

VIII. Appendix

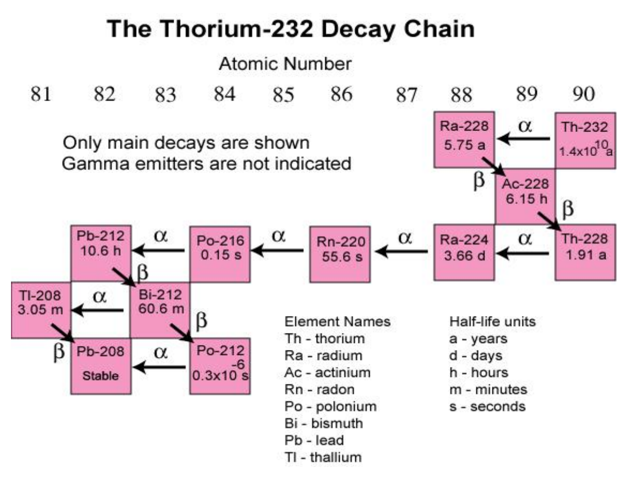

Fig. 20. [4]

The fit has been done with Root's built-in function (See Fig4-9). We have used root release 6.28/00 for Ubuntu22.

Exponential fit and extracting decay constant.

```
{
 TTree *t = new TTree("t", "t");
 TString file("4000-5.csv");
 t->ReadFile(file);
 TString title(file(0,4) + " Volt and " +
     file(5) + " Squeezes");
 float T;
 t->SetBranchAddress("T", &T);
 float * times, * x, * y, * sx, * sy ;
 int n = t->GetEntries();
 times = new float[n];
 x= new float[n-1];
 y = new float[n-1];
 sx = new float[n-1];
 sy = new float[n-1];

 float error = 0.300; //reflex error

 for (int i = 0; n > i; i++){
  t->GetEntry(i);
  times[i] = T;
 }
 for (int i = 0; n-1 > i; i++){
  y[i] = times[i+1] - times[i];
  sy[i] =
      1/y[i]*sqrt((error/y[i])*(error/y[i]));
  y[i] = 1/y[i];
  x[i] = (times[i+1] + times[i])/2;
  sx[i] = error*sqrt(1/2);
 }
 TCanvas *c1 = new TCanvas();

 TGraphErrors *mygraph = new
     TGraphErrors(n,x,y,sx,sy);
 mygraph->Draw("A*");
 mygraph->SetTitle(title);
 mygraph->GetXaxis()->SetTitle("Average
     Time(s)");
 mygraph->GetYaxis()->SetTitle("Charging
     Frequency(1/s)");
 TF1 *expo_fit = new
     TF1("expo_fit","[0]*exp([1]*x)",0.3,400);

 float par_0 = -0.001;
 float par_1 = -TMath::Log(2)/55.6;
 expo_fit->SetParameters(par_0,par_1);
 expo_fit->SetLineColor(kRed);
 expo_fit->SetLineWidth(2);
 mygraph->Fit(expo_fit,"R");
 mygraph->GetXaxis()->SetLimits(0, 400);
 mygraph->SetMinimum(0);
 mygraph->SetMaximum(5);
 expo_fit->Draw("same");
 gStyle->SetOptFit(1111);

 float decay = expo_fit->GetParameter(1);
 float decay_error = expo_fit->GetParError(1);
 cout << "Decay Constant = " << -decay << ","
     << decay_error << endl;
 c1->Print(file(0,6) + ".pdf");
}
```

Plotting Decay Constant vs Volt/Squeezes.
Calculating weighted averages of Decay Constant and Half-Life.

```
{
TTree *tree = new TTree("tree", "tree");
tree->ReadFile("decay_data.csv");
float t,st,v,s;
tree->SetBranchAddress("t", &t);
tree->SetBranchAddress("st", &st);
tree->SetBranchAddress("v", &v);
tree->SetBranchAddress("s", &s);

float * decay, * decay_error, * volt, * sq,
    * volt_error;
int n = tree->GetEntries();
decay = new float[n];
decay_error = new float[n];
volt = new float[n];
squ = new float[n];
volt_error = new float[n];


for (int i = 0; tree->LoadTree(i) >= 0; i++){
 tree->GetEntry(i);
 decay[i] = t;
 decay_error[i] = st;
 volt[i] = v;
 squ[i] = s;
 volt_error[i] = 100;
}
TCanvas *c1 = new TCanvas();
TGraphErrors *mygraph = new
    TGraphErrors(n,volt,decay,volt_error,decay_error);
mygraph->Draw("A*");
mygraph->SetTitle("Decay vs Volt");
mygraph->GetYaxis()->SetTitle("Decay
    Constant");
mygraph->GetXaxis()->SetTitle("Volt(V)");

TCanvas *c2 = new TCanvas();
TGraphErrors *mygraph2 = new
    TGraphErrors(n,squ,decay,0,decay_error);
mygraph2->Draw("A*");
mygraph2->SetTitle("Decay vs Squeezes");
mygraph2->GetYaxis()->SetTitle("Decay
    Constant");
mygraph2->GetXaxis()->SetTitle("Squeezes");

float weight = 0;
float totw = 0;
float decaybar = 0;
float decay_errorbar = 0;

for (int i=0; i<n; ++i) {
  weight = 1./(decay_error[i]*decay_error[i]);
  totw += weight;
  decaybar += (decay[i]*weight);
  //life_errorbar += (life_error[i]*weight);
}

decaybar /= totw;
decay_errorbar = sqrt(1 / totw);

float life = TMath::Log(2)/decaybar;
float life_error =
    life*sqrt((decay_errorbar/decaybar)*(decay_errorbar

cout << "Weighted Average of Decay Constant
    = " << decaybar << " +- " <<
    decay_errorbar << endl;
cout << "Weighted Average of Half-Life = "
    << life << " +- " << life_error << endl;
 }
```