Applications of Lasers

Burak Kara
Boğaziçi University • Physics Department

***Abstract*—A laser is a device that emits coherent monochromatic light. This feature of the laser makes it one of the essential tools for precise measurements. In this experiment, we conducted 4 different experiments, which are: double slit, single slit, measuring index of refraction using Michelson Interferometer, and measuring index of refraction using Pfund's Method. We have found the wavelength of a diode laser using a double slit experiment to be 660 $\pm$ 7.4 nm. Which is 1.4 σ away from the true value 650 nm. We have found the separation of the single slit using a known wavelength to be 0.16 $\pm$ 0.0036 mm. Which is 0.0 σ away from the true value 0.16 mm. Using the Michelson Interferometer, the index of refraction of a transparent solid is measured to be 1.4 $\pm$ 0.017, Which is 6.8 σ away from the true value 1.515 mm. Using Pfund's Method we have found the index of refraction of a transparent solid to be 1.5 $\pm$ 0.083, which is 0.18 σ away from the true value 1.515.**

I. Introduction

A laser is a device that emits light. The word laser stands for Light Amplification by Stimulated Emission of Radiation. As the name implies, a laser produces light through optical amplification with stimulation. Different than other sources of light, lasers are capable of producing coherent and monochromatic light beams. This property of lasers makes them a crucial tool for precise measurements.

In 1917, Einstein laid the theoretical foundation of the lasers in his paper "Zur Quantentheorie der Strahlung" ("On the Quantum Theory of Radiation") [1]. After Einstein, several scientists contributed to the development of lasers through both experimental and theoretical works. Eventually, the first laser was built in 1960 by Theodore Maiman, based on theoretical work by Charles H. Townes and Arthur Leonard Schawlow [2].

In this experiment, we have used HeNe laser to find the index of refraction of a glass. Also, we have used a diode laser to find the separation of a single slit. Moreover, we have found the wavelength of the diode laser using double slits. These experiments aim to both understand the behavior of lasers, their benefits in making precise measurements, and the quantum mechanical phenomena.

II. Theory

In this section, we will give the theoretical background of the measurements we have done. We will give the classical wave-optics formulation.

A. Double Slit Interference

As can be seen in the Figure 1, there exists a path difference due to the separation between slits. The path difference is $\Delta l = dsin\theta$ (assuming the light beam comes perpendicular to the slits.). Therefore, if the path difference is a multiple of the wavelength, we will see constructive interference.

$$\Delta l = dsin\theta_m = m\lambda, \text{: Constructive} \qquad \text{(Eq.1)}$$

$$\Delta l = dsin\theta_m = (m+\frac{1}{2})\lambda, \text{: Destructive} \qquad \text{(Eq.2)}$$

,where m = 0, $\pm$ 1, $\pm$ 2, ...

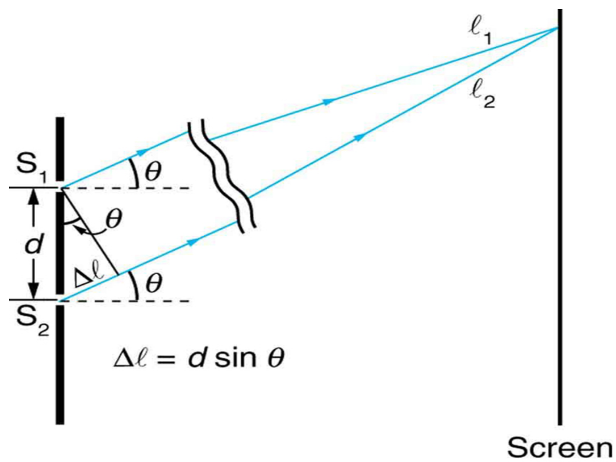

Fig. 1. Double Slit [3]

B. Single Slit Diffraction

The single-slit diffraction phenomenon occurs due to the Huygens–Fresnel Principle. The principle states that every point on a wavefront is itself a source. If the path difference of the sources is $\frac{\lambda}{2}$, then it will cause a destructive interference. Therefore we may derive the formula by dividing the slit into two repeatedly, effectively obtaining infinitely many double slits.

$$\text{Divide by 2: } \Delta l = \frac{d}{2}sin\theta_1 = \frac{\lambda}{2}$$

$$\text{Divide by 4: } \Delta l = \frac{d}{4}sin\theta_2 = \frac{\lambda}{2}$$

$$\text{Divide by 2n: } \Delta l = \frac{d}{2n}sin\theta_n = \frac{\lambda}{2}$$

$$dsin\theta_n = n\lambda \qquad \text{(Eq.3)}$$

, where n = $\pm$ 1, $\pm$ 2, ...

C. Index of Refraction of a Transparent Solid Rotated Inside the Michelson Interferometer

The Michelson Interferometer is a tool to produce optical interference. Using the interferometer, the light beam is split into two arms. Then, the lights reflect back from mirrors on the end of the arms. By superposition principle, they combine. Therefore, one can investigate the path difference between the two arms using the superposed light.

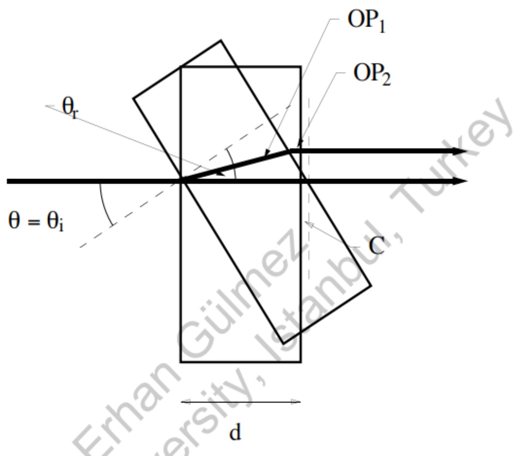

Fig. 2. Measuring the index of refraction [4]

The derivation of the theory can be found in the "Modern Physics Lab Book - Erhan Gülmez" [4].

Optical Path Length:

$$L = nd + \frac{d}{\cos\theta_i} - d$$

Calculating OP1, OP2. (See Fig.2):

$$OP_1 = \frac{nd}{\cos\theta_r}$$

$$OP_2 = \frac{d\tan\theta_i \sin\theta_i - \theta_r}{\cos\theta_r}$$

$$\Delta L = L - OP_1 - OP_2$$

Destructive interference:

$$\frac{\lambda}{2} = m\Delta L$$

Putting the Equations and Simplifying:

$$n = \frac{(d - \frac{m\lambda}{2})(1-\cos(\theta))}{d(1-\cos(\theta)) - \frac{m\lambda}{2}} \qquad \text{(Eq.4)}$$

where n is the index of refraction,

m is the number of shifted fringes,

λ is the wavelength of the light,

θ is the angle and d is the thickness.

D. Index of refraction of a transparent solid found by Pfund's method

According to Snell's Law, when light travels from a medium with a higher refractive index to a lower refractive index inner reflection occurs after a specific angle. Namely, critical angle θ_c.

If one sends a light beam to a transparent solid with a barrier at the bottom. The reflection from the barrier will lead to an inner reflection after the critical angle. (See Figure 3)

Snells Law:

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

Critical Angle:

$$\sin\theta_c = \frac{n_2}{n_1}\sin\theta_2$$

$$\theta_2 = 90^o, n_2 = 1 \implies n_1 = \frac{1}{\sin\theta_c}$$

Calculating The $\sin\theta_c$:

$$\sin\theta_c = \sqrt{\frac{(\frac{r}{2})^2}{(\frac{r}{2})^2 + d^2}}$$

Putting Together and Simplifying:

$$n = \sqrt{1 + \left(\frac{2d}{r}\right)^2} \qquad \text{(Eq.5)}$$

where n is the index of refraction,

d is thickness, and r is the radius of the ring.

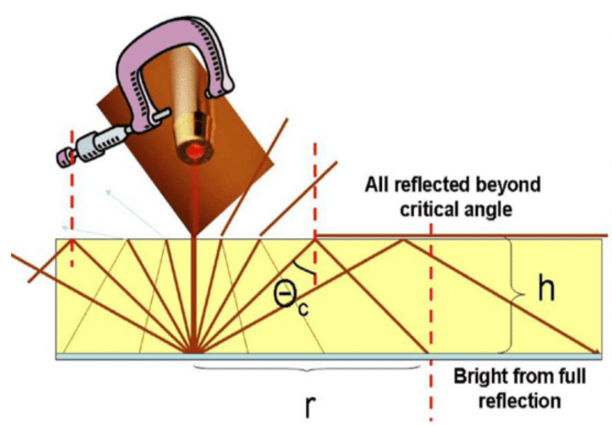

Fig. 3. Visualization of the Pfund's Method [5]

III. The Experimental Setup

A. Measuring the Wavelength of Light with Double Slits

- Diode Laser with 650nm Wavelength.
- Double Slit with 0.50 and 0.25mm separations.

- Screen.
- Ruler.

B. Measuring the Width of a Single Slit

- Diode Laser with 650nm wavelength.
- Single Slit with 0.16mm separation.
- Screen.
- Ruler.

Fig. 4. Apparatus - Double and Single Slit.

C. Index of Refraction of a Transparent Solid Rotated Inside the Michelson Interferometer

- HeNe Laser with 632.8 nm wavelength.
- Transparent solid with 1.515 index of refraction.
- Michelson Interferometer.
- Reflecting Mirrors.

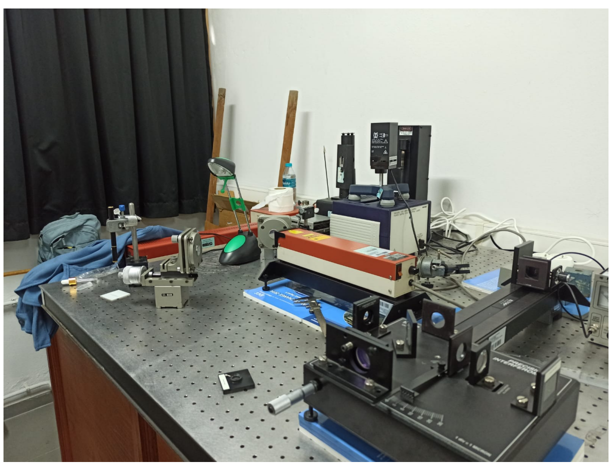

Fig. 5. Apparatus - Michelson Interferometer.

D. Index of refraction of a Transparent Solid Found by Pfund's Method

- Diode Laser with 650nm wavelength.
- Transparent solid with 1.515 index of refraction.
- Ruler.

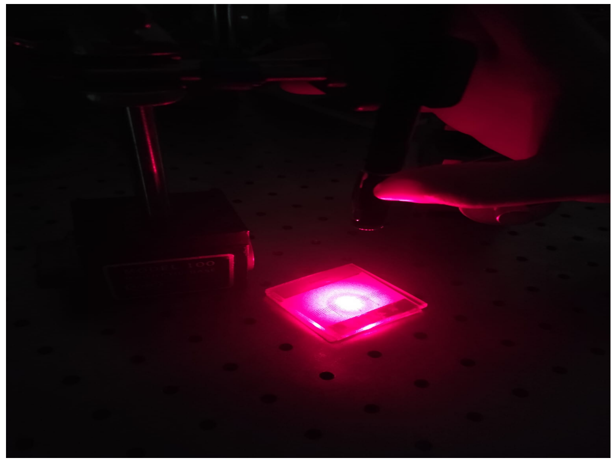

Fig. 6. Apparatus - Pfund's Method.

IV. PROCEDURE

A. Measuring the Wavelength of Light with Double Slits

1) The slit is placed.
2) Marked the maxima of the interference pattern.
3) measured the distance between maxima $\pm1, \pm2, \pm3, \pm4$.

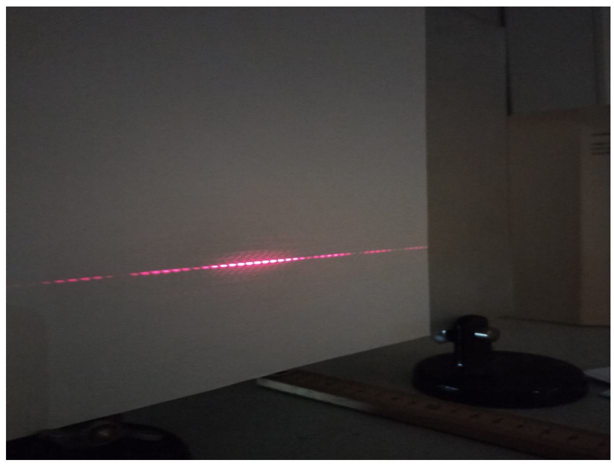

Fig. 7. Double Slit with 0.25mm Separation

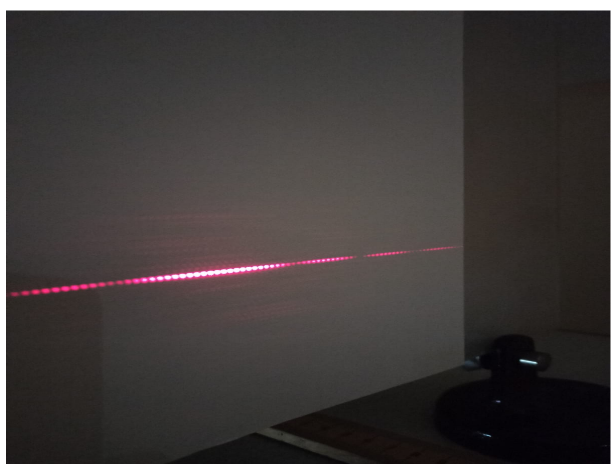

Fig. 8. Double Slit with 0.50mm Separation

B. Measuring the Width of a Single Slit

1) The slit is placed.
2) Marked the minima of the interference pattern.
3) Measured the distance between minima $\pm 1, \pm 2, \pm 3, \pm 4$.

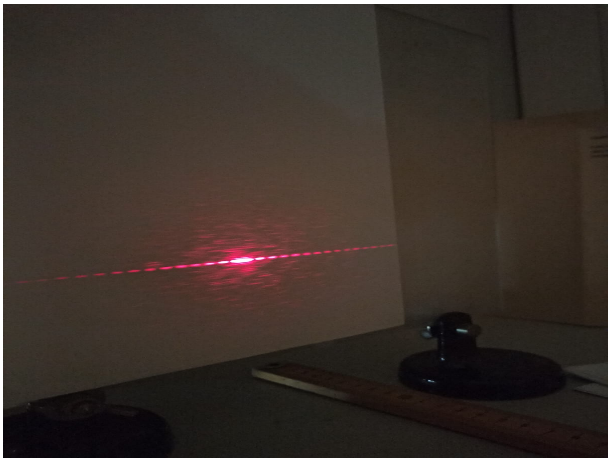

Fig. 9. Single Slit with 0.16mm Separation

C. Index of Refraction of a Transparent Solid Rotated Inside the Michelson Interferometer

1) The Michelson Interferometer is adjusted to obtain a perpendicular incident light.
2) The transparent solid is placed on one of the arms.
3) The reference angle i.e. the angle at which the fringes stop to change is recorded. (0.2 Deg)
4) Rotated the solid while counting the number of fringe changes.
5) The angle of rotations are recorded for the first 20 fringe change, and then every 10 fringe change.
6) The recorded angles are diminished by reference angle.

D. Index of refraction of a Transparent Solid Found by Pfund's Method

1) The solid is placed above a wet, white plate.
2) The diameter of the circle is measured. (See Fig.6)

V. The Data

We have obtained the distance between $\pm n^{th}$ maxima and minima for double slits and single slit respectively. The Double Slit-1 has a known separation $0.25 \pm 0.01mm$ and Double Slit-2 has a known separation $0.50 \pm 0.01mm$. The separation of the single slit is 0.16 ± 0.01.

TABLE I
Difference Between Minima/Maxima

n	Single Slit (cm)	Double Slit - 1 (cm)	Double Slit - 2 (cm)
1	1.3 ± 0.1	0.8 ± 0.1	0.4 ± 0.1
2	2.5 ± 0.1	1.6 ± 0.1	0.8 ± 0.1
3	3.8 ± 0.1	2.5 ± 0.1	1.2 ± 0.1
4	4.9 ± 0.1	3.2 ± 0.1	1.6 ± 0.1

We measured the thickness of the transparent solid as $0.55 \pm 0.01cm$. Also, using Pfund's Method, we measured the diameter of the ring (See Fig.6) as $2.0 \pm 0.1cm$.

Using the Michelson interferometer, we have obtained the number of fringes and the corresponding angles.

TABLE II
Count and Corresponding Angle

Count	Angle (Deg)
0	0.2
20	4.9 ± 0.1
30	6.6 ± 0.1
40	7.5 ± 0.1
50	8.4 ± 0.1
60	8.9 ± 0.1
70	9.6 ± 0.1
80	10.0 ± 0.1
90	10.7 ± 0.1
100	10.9 ± 0.1
110	11.3

VI. The Analysis and Results

We considered the following equation for error propagation, assuming there is not any correlation variables.

$$\sigma_f = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2 + \left(\frac{\partial f}{\partial z}\right)^2 \sigma_z^2 + \cdots}$$

A. Measuring the Wavelength of Light with Double Slits

According to the Eq.1, $\lambda = \frac{d sin\theta_n}{n}$. We have calculated the $sin\theta_n$ using trigonometry, which yields to:

$$sin\theta_n = sin(tan^{-1}\frac{y_n}{2L}) = \frac{y_n}{\sqrt{4L^2 + y_n^2}}$$

, where y is the difference between $\pm n^{th}$ maxima and L is the distance from the slit to the screen. Putting $\sin\theta_n$, we have arrived at the equation:

$$\lambda = \frac{d \cdot y_n}{n\sqrt{4L^2 + y_n^2}}$$

Here, only the n variable is exact and the others are independent of each other. Therefore, we can use the error propagation formula which yields to:

$$\frac{\partial \lambda}{\partial d} = \frac{y_n}{n\sqrt{4L^2 + y_n^2}}$$

$$\frac{\partial \lambda}{\partial y_n} = \frac{d y_n}{n\sqrt{4L^2 + y_n^2}}$$

$$\frac{\partial \lambda}{\partial L} = -\frac{4dLy_n}{n(4L^2 + y_n^2)^{3/2}}$$

Putting Together:

$$\sigma_\lambda = \sqrt{(\frac{\partial \lambda}{\partial d})^2 \cdot (\sigma_d)^2 + (\frac{\partial \lambda}{\partial y_n})^2 \cdot (\sigma_y)^2 + (\frac{\partial \lambda}{\partial L})^2 \cdot (\sigma_L)^2}$$

where, $\sigma_d = 0.01mm, \sigma_y = 0.1cm, \sigma_L = 1cm.$

Considering Table 1, Equation 1, and the error propagation formula, we have found the wavelengths as:

TABLE III
DOUBLE SLIT WITH 0.25MM SEPARATION

n	λ (nm)	σ	Relative Error (%)
1	660 $\pm$ 27	0.37	1.5%
2	660 $\pm$ 27	0.37	1.5%
3	690 $\pm$ 28	1.4	6.2%
4	660 $\pm$ 27	0.37	1.5%
Average	660 $\pm$ 13	0.77	1.5%

TABLE IV
DOUBLE SLIT WITH 0.50MM SEPARATION

n	λ (nm)	σ	Relative Error (%)
1	660 $\pm$ 14	0.71	1.5%
2	660 $\pm$ 14	0.71	1.5%
3	660 $\pm$ 14	0.71	1.5%
4	660 $\pm$ 14	0.71	1.5%
Average	660 $\pm$ 6.9	1.4	1.5%

Taking the average of the two double slits and using the error propagation yields the desired result. We have found the wavelength of the laser beam to be: 660 $\pm$ 7.4 nm. Which is 1.4 σ away from the true value 650 nm, with 1.5% relative error.

B. Measuring the Width of a Single Slit

Similar to the double slit experiment, we have found the $sin\theta_n$ to be:

$$sin\theta_n = sin(tan^{-1}\frac{y_n}{2L}) = \frac{y_n}{\sqrt{4L^2 + y_n^2}}$$

Therefore, according to equation 3, the separation of the slit formula becomes:

$$d = \frac{n\lambda\sqrt{4L^2 + y_n^2}}{y_n}$$

Using the general error propagation formula, we have obtained the desired uncertainty formula for the slit:

$$\frac{\partial d}{\partial \lambda} = \frac{n\sqrt{4L^2 + y_n^2}}{y_n}$$

$$\frac{\partial d}{\partial y_n} = -\frac{4\lambda L^2 n}{y_n^2\sqrt{4L^2 + y_n^2}}$$

$$\frac{\partial d}{\partial L} = \frac{4\lambda L n}{y_n\sqrt{4L^2 + y_N^2}}$$

Putting Together:

$$\sigma_d = \sqrt{(\frac{\partial d}{\partial \lambda})^2 \cdot (\sigma_\lambda)^2 + (\frac{\partial d}{\partial y_n})^2 \cdot (\sigma_y)^2 + (\frac{\partial d}{\partial L})^2 \cdot (\sigma_L)^2}$$

where, $\sigma_\lambda = 0.1nm, \sigma_y = 0.1cm, \sigma_L = 1cm.$

Considering Table 1, Equation 3, and the error propagation formula, we have found the separation of the single slit as:

TABLE V
SINGLE SLIT WITH 0.16MM SEPARATION

n	λ (nm)	σ	Relative Error (%)
1	0.15 $\pm$ 0.012	0.83	6.3%
2	0.16 $\pm$ 0.0064	0	0%
3	0.16 $\pm$ 0.0042	0	0%
4	0.16 $\pm$ 0.0035	0	0%
Average	0.16 $\pm$ 0.0036	0	0%

The average value is the desired result. We have found it to be: 0.16 $\pm$ 0.0036 mm. Which is 0.0 σ away from the true value 0.16 mm, with 0.0% relative error.

C. Index of Refraction of a Transparent Solid Rotated Inside the Michelson Interferometer

We have found the uncertainty formula for the index of refraction, according to the general error propagation formula. However, since the equations break the integrity of the paper, we have added them to the appendix.

TABLE VI
INDEX OF REFRACTION - MICHELSON INTERFEROMETER

Count	Value $\pm$	Sigma	Relative Error (%)
20	1.5 $\pm$ 0.13	0.12	0.99%
30	1.4 $\pm$ 0.059	1.9	7.6%
40	1.4 $\pm$ 0.047	2.4	7.6%
50	1.4 $\pm$ 0.038	3	7.6%
60	1.4 $\pm$ 0.037	3.1	7.6%
70	1.4 $\pm$ 0.032	3.6	7.6%
80	1.5 $\pm$ 0.032	0.47	0.99%
90	1.4 $\pm$ 0.028	4.1	7.6%
100	1.5 $\pm$ 0.03	0.5	0.99%
110	1.5 $\pm$ 0.029	0.52	0.99%
Average	1.4 $\pm$ 0.017	6.8	7.6%

The average value is the desired result. We have found it to be: 1.4 $\pm$ 0.017, Which is 6.8 σ away from the true value 1.515 mm, with 7.6% relative error.

D. Index of refraction of a transparent solid found by Pfund's method

We have Equation 5, and the general error propagation formula which yields to:

$$\frac{\partial n}{\partial d} = \frac{d \cdot 8.0}{2.0\sqrt{\left(\frac{2d}{r}\right)^2 + 1.0}} \cdot \frac{1}{r^2}$$

$$\frac{\partial n}{\partial r} = -\frac{d^2 \cdot 8.0}{2.0\sqrt{\left(\frac{2d}{r}\right)^2 + 1.0}} \cdot \frac{1}{r^3}$$

Putting together:

$$\sigma_n = \sqrt{\left(\frac{\partial n}{\partial d}\sigma_d\right)^2 + \left(\frac{\partial n}{\partial r}\sigma_r\right)^2}$$

where, $r = R/2, \sigma_r = \sigma_R/2 = 0.05cm, \sigma_d = 0.01cm$

According to Equation 5, and the data we have obtained, the index of refraction is calculated to be: 1.5 $\pm$ 0.083, which is 0.18 σ away from the true value 1.515, with 0.99% relative error.

VII. THE CONCLUSION

We have conducted 4 different experiments to understand the quantum phenomena and benefits of lasers. The Experiments are successful enough since the sigma values are close to 1. However, the Michelson Interferometer experiment was a failure. The reasons for this will be more explicitly discussed in the subsection.

In all experiments, we have observed a common problem which is the precision of the measurement devices we have used e.g. ruler. Since they are not precise, we could only get 2 significant figures. This yields lower sigma values; however, less satisfactory results. Therefore, the device precisions can be improved.

A. Measuring the Wavelength of Light with Double Slits

We have found the wavelength of the laser beam to be: 660 $\pm$ 7.4 nm. Which is 1.4 σ away from the true value 650 nm, with 1.5% relative error. Therefore, we have concluded that the experiment was partly successful, but needs precise measurements.
The main problem was that we marked the maxima by hand. Therefore, we could not mark them with high precision. A more sophisticated experiment would be using a detector to obtain the length between maxima.
As can be seen from Fig.8 and Fig.9, the distance from maxima is more spread for the double slit with 0.25mm Separation. Therefore, the uncertainty of the wavelength is higher. One can see this from both comparing Table III and Table IV or directly from the error propagation formula.

B. Measuring the Width of a Single Slit

We have found the separation of the single slit to be: 0.16 $\pm$ 0.0036 mm. Which is 0.0 σ away from the true value 0.16 mm, with 0.0% relative error. Therefore, we have concluded that the experiment was a success.
Even if this looks like a perfect result the lack of precision in measurement devices leads to this "perfect" result.

C. Index of Refraction of a Transparent Solid Rotated Inside the Michelson Interferometer

We have found the index of refraction to be: 1.4 $\pm$ 0.017, Which is 6.8 σ away from the true value 1.515 mm, with 7.6% relative error. Therefore, we have concluded that the experiment was a failure.
The main problem of the failure comes from the adjustment of the Michelson Interferometer. Most probably, we could not adjust the interferometer perfectly. Since the interferometer is highly sensitive, the lack of perfect adjustment led to the high sigma value. Moreover, the rotator of the interferometer stuck frequently; therefore, we missed or overcounted some of the fringes. Even if we added uncertainty for the count number, it seems that the uncertainty was higher than we expected.

D. Index of refraction of a Transparent Solid Found by Pfund's Method

We have found the index of refraction to be: 1.5 $\pm$ 0.083, which is 0.18 σ away from the true value 1.515, with 0.99% relative error. Therefore, we have concluded that the experiment was a success.
The main advantage of the Pfund's Method was the simplicity of the procedure. The procedure does not need any perfect adjustments. Therefore, the result was more reasonable than the interferometer result.

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[5] *Basic concept of Pfund's method for measuring the refractive index of a transparent plate*. URL: https://www.researchgate.net/figure/Basic-concept-of-Pfunds-method-for-measuring-the-refractive-index-of-a-transparent_fig4_232005217 (visited on 11/11/2023).

VIII. APPENDIX

We have used ROOT Version: 6.28/04 on Ubuntu 22.04.2 LTS machine for the following macros.
The macro to calculate the results of the Double Slit and Single Slit experiment:

```
#include <math.h>
#include <iostream>
#include <vector>
using namespace std;

double significant_round(double value, int
    digits)
{
  if (value == 0.0)
      return 0.0;
   double factor = pow(10.0, digits -
       ceil(log10(fabs(value))));
   return round(value * factor) / factor;
}

void print_results(double true_value, double
    value, double error){
     double relative_error = abs(true_value -
         value) * 100 / true_value;
     double sigma = abs(true_value - value) /
         error;
     cout << value << " +- " << error;
     cout << ", Sigma = " <<
         significant_round(sigma,2);
     cout << ", Relative Error = " <<
         significant_round(relative_error,2)
         << "%" << endl;
}

double calculateLambdaError(double n, double
    L, double y_n, double d, double sigma_d,
    double sigma_y, double sigma_L) {
    // Partial derivatives
```

```
    double partial_lambda_d = y_n / (n * sqrt(4
        * pow(L, 2) + pow(y_n, 2)));
    double partial_lambda_y = y_n * d / (n *
        sqrt(4 * pow(L, 2) + pow(y_n, 2)));
    double partial_lambda_L = -4 * d * L * y_n
        / (n * pow(4 * pow(L, 2) + pow(y_n, 2),
        1.5));

    // Error calculation
    double sigma_lambda =
        sqrt(pow(partial_lambda_d * sigma_d, 2)
        + pow(partial_lambda_y * sigma_y, 2) +
        pow(partial_lambda_L * sigma_L, 2));

    return sigma_lambda;
}

double calculateSeparationError(double n,
    double L, double y_n, double lambda,
    double sigma_lambda, double sigma_y,
    double sigma_L) {
    // Partial derivatives
    double partial_d_lambda = n * sqrt(4 *
        pow(L, 2) + pow(y_n, 2)) / y_n;
    double partial_d_y = -4 * lambda * pow(L,
        2) * n / (pow(y_n, 2) * sqrt(4 * pow(L,
        2) + pow(y_n, 2)));
    double partial_d_L = 4 * lambda * L * n /
        (y_n * sqrt(4 * pow(L, 2) + pow(y_n,
        2)));

    // Error calculation
    double sigma_d = sqrt(pow(partial_d_lambda
        * sigma_lambda, 2) + pow(partial_d_y *
        sigma_y, 2) + pow(partial_d_L *
        sigma_L, 2));

    return sigma_d;
}

void find_lambda(vector<double> measure,
    double L, double d,
            double measure_error, double
                L_error, double d_error)
{
  double average;
  double average_error_squared;
  int n = measure.size();
  for (int i = 0; i < n; i++)
  {
    double y_n = measure[i];
    double sin_theta = y_n / sqrt(4 * pow(L,
        2) + pow(y_n, 2));
    double lambda = d * sin_theta / (i + 1);

    double error =
        calculateLambdaError((i+1), L, y_n,
        d, d_error, measure_error, L_error);
    cout << "n = " << i+1 << ", Value = ";
    print_results(650,
        significant_round(lambda*1e9,2) ,
        significant_round(error*1e9,2));
    average += (lambda / n);
    average_error_squared += pow(error / n,
        2);
  }
  double error = sqrt(average_error_squared);
  cout << "Average = ";
  print_results(650,
      significant_round(average*1e9,2),
      significant_round(error*1e9,2));
}

void find_separation(vector<double> measure,
    double L, double lambda,
            double measure_error, double
                L_error, double
                lambda_error)
{
  double average;
  double average_error_squared;
  int n = measure.size();
  for (int i = 0; i < n; i++)
  {
    double y_n = measure[i];
    double sin_theta = y_n / sqrt(4 * pow(L,
        2) + pow(y_n, 2));
    double separation = (i + 1) * lambda /
        sin_theta;

    double error =
        calculateSeparationError((i+1),L,y_n,
    lambda,lambda_error,
    measure_error,L_error);
    cout << "n = " << i+1 << ", Value = ";
    print_results(0.16,
    significant_round(separation*1e3,2) ,
    significant_round(error*1e3,2));

    average += (separation / n);
    average_error_squared += pow(error/n, 2);
  }

  double error = sqrt(average_error_squared);
  cout << "Average = ";
  print_results(0.16,
      significant_round(average*1e3,2),
      significant_round(error*1e3,2));
}

void slit()
{
  double distance = 152 * 1e-2;
  double distance_error = 1 * 1e-2;
  double lambda = 650 * 1e-9;
  double lambda_error = 1 * 1e-9;
  double measure_error = 1 * 1e-3;
  double slit_error = 0.01 * 1e-3;
  vector<double> single_slit = {0.013, 0.025,
      0.038, 0.049};
  vector<double> double_slit_25 = {0.008,
      0.016, 0.025, 0.032};
  vector<double> double_slit_50 = {0.004,
      0.008, 0.012, 0.016};

  cout << "Double Slit with 0.25mm
      Separation" << endl;
  find_lambda(double_slit_25, distance, 0.25
      * 1e-3, measure_error, distance_error,
      slit_error);
  cout << "\nDouble Slit with 0.50mm
      Separation" << endl;
  find_lambda(double_slit_50, distance, 0.50
      * 1e-3, measure_error, distance_error,
      slit_error);
  cout << "\nSingle Slit with 0.16mm
      Separation" << endl;
  find_separation(single_slit, distance,
```

```
    lambda, measure_error, distance_error,
    lambda_error);
}
```

The Macro to calculate the index of refraction of a transparent solid using the Michelson Interferometer:

```
  #include <math.h>
#include <iostream>
#include <vector>
using namespace std;

double refraction(double d, double lambda,
    double m, double theta)
{
   double up = (d - (m * lambda / 2)) * (1 -
       cos(theta));
   double down = d * (1 - cos(theta)) - (m *
       lambda / 2);
   return up / down;
}

double find_error(double d, double l, double
    m, double t,
          double d_error, double lambda_error,
              double m_error, double
              theta_error)
{
   double partial_R_d = ((1 - cos(t)) * (d *
       (1 - cos(t)) + m * -1 / 2 * l) - pow(1
       - cos(t), 2) * (d + m * -1 / 2 * l)) /
       pow(d * (1 - cos(t)) + m * -1 / 2 * l,
       2);
   double partial_R_m = (l * -1 / 2 * (1 -
       cos(t)) * (d * (1 - cos(t)) + m * -1 /
       2 * l) + (d + m * -1 / 2 * l) / 2 * (1
       - cos(t)) * l) / pow(d * (1 - cos(t)) +
       m * -1 / 2 * l, 2);
   double partial_R_l = (m * -1 / 2 * (1 -
       cos(t)) * (d * (1 - cos(t)) + m * -1 /
       2 * l) + (d + m * -1 / 2 * l) / 2 * (1
       - cos(t)) * m) / pow(d * (1 - cos(t)) +
       m * -1 / 2 * l, 2);
   double partial_R_t = ((d + m * -1 / 2 * l)
       * sin(t) * (d * (1 - cos(t)) + m * -1 /
       2 * l) - (d + m * -1 / 2 * l) * (1 -
       cos(t)) * d * sin(t)) / pow(d * (1 -
       cos(t)) + m * -1 / 2 * l, 2);

   double delta_R = sqrt(pow(partial_R_d *
       d_error, 2) + pow(partial_R_m *
       m_error, 2) + pow(partial_R_l *
       lambda_error, 2) + pow(partial_R_t *
       theta_error, 2));
   return delta_R;
}

double significant_round(double value, int
    digits)
{
  if (value == 0.0)
      return 0.0;
   double factor = pow(10.0, digits -
       ceil(log10(fabs(value))));
   return round(value * factor) / factor;
}

void print_results(double true_value, double
    value, double error){
    double relative_error = abs(true_value -
        value) * 100 / true_value;
    double sigma = abs(true_value - value) /
        error;
    cout << value << " +- " << error;
    cout << ", Sigma = " <<
        significant_round(sigma,2);
    cout << ", Relative Error = " <<
        significant_round(relative_error,2)
        << "%" << endl;
}

void interferometer()
{
   double d = 0.55 * 1e-2;
   double d_error = 0.01 * 1e-2;
   double theta_error = 0.1;
   double lambda = 632.8 * 1e-9;
   double lambda_error = 0.1 * 1e-9;
   double m_error = sqrt(10);
   vector<int> m_vector = {20, 30, 40, 50, 60,
       70, 80, 90, 100, 110};
   vector<double> theta_vector = {4.7, 6.4,
       7.3, 8.2, 8.7, 9.4, 9.8, 10.5, 10.7,
       11.1};

   double average = 0;
   double average_error_squared = 0;
   int num = m_vector.size();
   for (size_t i = 0; i < num; i++)
   {
      double n = refraction(d, lambda,
          m_vector[i], theta_vector[i] * M_PI
          / 180);
      double error = find_error(d, lambda,
          m_vector[i], theta_vector[i] * M_PI
          / 180,
      d_error, lambda_error, m_error,
          theta_error * M_PI / 180);
      cout << "Count = " << m_vector[i] <<", n
          = ";
      print_results(1.515,significant_round(n,2),
      significant_round(error,2));

      average += n / num;
      average_error_squared += pow(error/num,
          2);
   }

   double error = sqrt(average_error_squared);

   cout << "Average, n = ";
   print_results(1.515,
       significant_round(average,2),
   significant_round(error,2));
}
```

The macro to calculate the result of Pfund's Method:

```
#include <math.h>
#include <iostream>
#include <vector>
using namespace std;

double significant_round(double value, int
    digits)
{
```

```
  if (value == 0.0)
      return 0.0;
   double factor = pow(10.0, digits -
       ceil(log10(fabs(value))));
   return round(value * factor) / factor;
}

double find_error(double d, double r, double
    delta_d, double delta_r){
   double partial_R_d = (d * 8.0 / (2.0 *
       sqrt(pow(d * 2.0 / r, 2) + 1.0)) /
       pow(r, 2));
   double partial_R_r = -(pow(d, 2) * 8.0 /
       (2.0 * sqrt(pow(d * 2.0 / r, 2) + 1.0))
       / pow(r, 3));
   double delta_R = sqrt(pow(partial_R_d *
       delta_d, 2) + pow(partial_R_r *
       delta_r, 2));

   return delta_R;
}

void print_results(double true_value, double
    value, double error){
     double relative_error = abs(true_value -
         value) * 100 / true_value;
     double sigma = abs(true_value - value) /
         error;
     cout << value << " +- " << error;
     cout << ", Sigma = " <<
         significant_round(sigma,2);
     cout << ", Relative Error = " <<
         significant_round(relative_error,2)
         << "%" << endl;
}

void pfund(){
   double d = 0.55 * 1e-2;
   double d_error = 0.01 * 1e-2;
   double r = 2.0 * 1e-2 / 2;
   double r_error = 0.2 * 1e-2 / 2;

   double n = sqrt (1+ pow(2*d/r,2));
   double error =
       find_error(d,r,d_error,r_error);
   cout << "n = ";
   print_results(1.515,
       significant_round(n,2),
       significant_round(error,2));
}
```

Index of Refraction of a Transparent Solid Rotated Inside the Michelson Interferometer, Error Formula:

$$\frac{\partial n}{\partial d} = \frac{(1-\cos(\theta))(d(1-\cos(\theta)) + m\left(-\frac{1}{2}\right)\lambda) - (1-\cos(\theta))^2(d + m\left(-\frac{1}{2}\right)\lambda)}{(d(1-\cos(\theta)) + m\left(-\frac{1}{2}\right)\lambda)^2}$$

$$\frac{\partial n}{\partial m} = \frac{\frac{\lambda}{-2}(1-\cos(\theta))(d(1-\cos(\theta)) + m\left(-\frac{1}{2}\right)\lambda) + \frac{(d+m\left(-\frac{1}{2}\right)\lambda)}{2}(1-\cos(\theta))\lambda}{\left(d(1-\cos(\theta)) + m\left(-\frac{1}{2}\right)\lambda\right)^2}$$

$$\frac{\partial n}{\partial \lambda} = \frac{\frac{m}{-2}(1-\cos(\theta))(d(1-\cos(\theta)) + m\left(-\frac{1}{2}\right)\lambda) + \frac{(d+m\left(-\frac{1}{2}\right)\lambda)}{2}(1-\cos(\theta))m}{\left(d(1-\cos(\theta)) + m\left(-\frac{1}{2}\right)\lambda\right)^2}$$

$$\frac{\partial n}{\partial \theta} = \frac{(d + m\left(-\frac{1}{2}\right)\lambda)\sin(\theta)(d(1-\cos(\theta)) + m\left(-\frac{1}{2}\right)\lambda) - (d + m\left(-\frac{1}{2}\right)\lambda)(1-\cos(\theta))d\sin(\theta)}{\left(d(1-\cos(\theta)) + m\left(-\frac{1}{2}\right)\lambda\right)^2}$$

Putting Together:

$$\sigma_n = \sqrt{\left(\frac{\partial n}{\partial d}\sigma_d\right)^2 + \left(\frac{\partial n}{\partial m}\sigma_m\right)^2 + \left(\frac{\partial n}{\partial \lambda}\sigma_\lambda\right)^2 + \left(\frac{\partial n}{\partial \theta}\sigma_\theta\right)^2}$$

where, $\sigma_d = 0.01cm, \sigma_m = \sqrt{10}, \sigma_\lambda = 0.1nm.$