Franck - Hertz Experiment

Burak Kara

Boğaziçi University • Physics Department

Abstract—The experiment aims to study the excitation energy of the Mercury(Hg) vapor and validating the discrete energy levels of the atoms. We have conducted the experiment by shooting energetic electrons into Mercury vapor, and we have observed that the electrons make inelastic collisions at discrete values. These values are multiples of the first excitation energy and we have found it to be 4.96 ± 0.08 eV. The accepted value is 4.9eV; therefore, we are 0.8σ away from the accepted value with 1.2% relative error.

I. THEORY

On April 24, 1914, James Franck and Gustav Hertz presented a paper that was one of the first experiments that clearly shows the experimental evidence for the discrete energy levels, as Niels Bohr suggested a year ago. The experiment has significantly affected the understanding of the nature of atoms and their interactions with electrons [1]. In the experiment, they used mercury vapor which is an inside the tube. Then, they introduced energetic electrons to see if the electrons affect the mercury atom discretely. They have found that the electron can only lose 4.9eV. The results were consistent with Bohr's model[2]. They have awarded The 1925 Nobel Prize in Physics for their success.[1]

Franck-Hertz used the mercury element because of that the mercury vapor is strictly mono-atomic [3]. So that it does not bind to other atoms which guarantees the stability of the system. Noble gases like neon are also usable for this experiment because of the same reason.

In the experiment, we use the same idea as Franck-Hertz's experiment. We shot energetic electrons into the mercury vapor and measure the current at the other side of the tube to see the discreetness of energy levels. We have tested the following theory:

There are two different types of collisions that the electron can make with the mercury vapor. The first one is the elastic collision. The other one is the in-elastic collusion.

In the elastic collusion, if the electron beam hits a mercury atom with the kinetic energy $E_k = \frac{1}{2}mv2$, and since the mercury atoms' velocities are negligible small with respect to the electrons' velocity, the kinetic energy difference is negligibly small.

Conservation of Momentum:
$$mV_1 + MV_2 = mV_3 + MV_4 \tag{1}$$
where, m and M are masses of the electron and mercury,

V_1 and V_2 are the initial velocities,

V_2 and V_3 are final velocities.

Conservation of Kinetic Energy:
$$\frac{1}{2}mV_1{}^2 + \frac{1}{2}MV_2{}^2 = \frac{1}{2}mV_3{}^2 + \frac{1}{2}MV_4{}^2 \tag{2}$$

Solving Eq.1 and Eq.3 for V_3:
$$V_3 = \frac{m-M}{m+M}V_1 + \frac{2M}{m+M}V_2 \tag{3}$$

Solving for $M >> m$
$$V_3 \approx -V_1 + 2V_2 \tag{4}$$

Solving for $V_1 >> V_2$
$$V_3 \approx -V_1 \Rightarrow E_{k_3} \approx E_{k_1} \tag{5}$$

As shown in Eq.5, the kinetic energy of the electron is not changing significantly with the elastic collisions. So we should consider the in-elastic collisions. If one electron makes an inelastic collision, the energy is not stored as kinetic energy, but it is used for the excitation of the atom.

Energy Difference:
$$E_n - E_m = \Delta E = h\nu \tag{7}$$
where E_n and E_m are energy levels of n'th and m'th levels,

h is the Planck Constant ($6.62607015 \times 10^{-34} JHz^{-1}$ [4])

ν is the frequency of the emitted light

Using Franck-Hertz's Conclusion:
$$4.9\text{eV} \approx 7.8 \times 10^{-19}J = h\nu \Rightarrow \nu_{emitted} \approx 1.2 \times 10^{15}Hz \tag{7}$$

Using the Speed of Light:
$$c = \lambda\nu \Rightarrow \lambda_{emitted} \approx 250nm \tag{9}$$

We see from Eq.9 that the light emitted from the mercury atom is ultraviolet, so it is impossible to detect it with the naked eye.

Since the theory suggests that the energy levels are discrete, the energetic electrons can only excite the mercury atom when they have a similar energy of the excitation energy. The excited electrons drop back to the lower energy levels so that they emit light. Therefore, electrons lose all their energy and such a collision occurs as an immediate drop in the current.

In the experiment we have conducted, it is not possible to see higher excitation levels than the first exited state. The reason is that the electrons are gaining energy in the path. So, when they have the first exited state's energy, say at halfway, then they do the inelastic collision. On the remaining way, they gain the first exited state's energy again, so that the electron again does a collision. Therefore, we expect to see this current drop effect at the multiples of the 4.9V.

Ideally, the current would drop to zero; however, there is a possibility of not having successive collisions between the mercury atoms and electrons. Therefore, we should consider the mean free path of the electrons in the tube.

The Mean Free Path:

$$l = \frac{1}{\sigma n} \tag{10}$$

where, l is the mean free path,

σ is the effective cross-sectional area,

n is the number of targets.

We see from Eq.10 that the mean free path is inversely proportional to the number of particles. Since we use the vapor mercury atom, the number of vaporized particles is increased with increasing temperature. Therefore, we expect to see minimal collector current when experimenting with higher temperatures.

II. THE EXPERIMENTAL SETUP

- Oven.
- Digital Thermometer
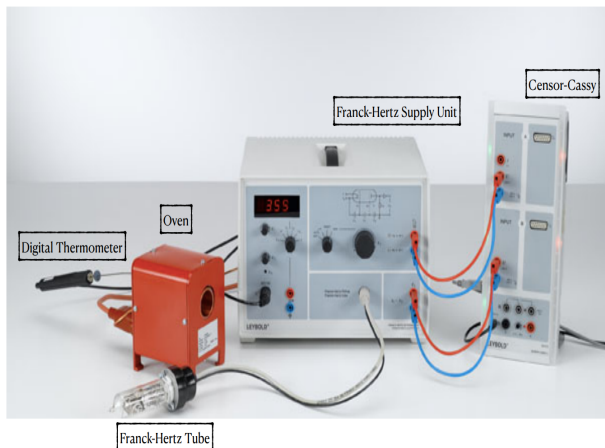- Frank-Hertz Tube.
- Supply Unit.
- Censor-Cassy.

Fig. 1. The Apparatus [5].

On the Supply Unit, there are controllers for temperature and voltage values of the Franck-Hertz Tube. The voltages values are:

- Driving Potential (U1).
- Accelerating Potential (U2). (We can also control from computer to increase it constantly.)
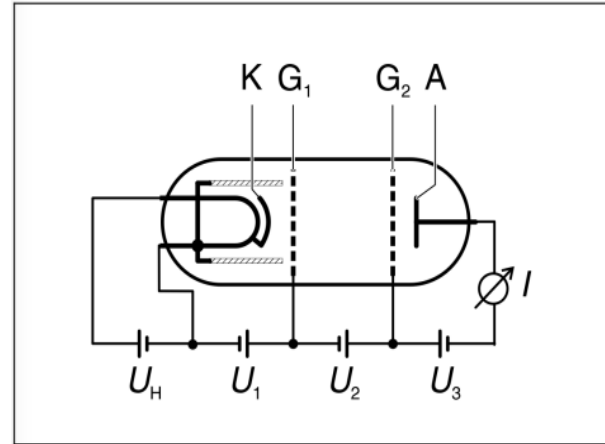- Breaking Potential (U3).

Fig. 2. The Schema of the Frank-Hertz Tube [5]

The current is measured at the collector electrode (A).

III. METHOD

The aim of the experiment is to calculate the first exited state energy of the mercury atom. To do that, we introduced an electron beam which is created by heating a filament and accelerated by applying an acceleration voltage (U2) and passing it from the mercury vapor. Since we wanted to investigate all behaviors of the system, we have changed a single variable while keeping others constant.

1) The oven is heated. (We have used between $180°C - 202°C$.)
2) U1 and U3 is set.
3) Data is taken with constantly increasing accelerating voltage (U2) via computer until seeing at least four peaks.
4) Changed one variable while keeping others constant and repeated the above procedure. (In total we had 10 data sets)

IV. THE DATA

We have plotted the data and fitted a Gaussian on the top of peaks using Root's built-in functions (See. Appendix). The apparatus we have used have uncertainties on voltages (U1, U2, U3) as 0.15 V and the uncertainty on collector current is 0.005 nA.

We have observed behaviours of the system with changing one variable and leaving others constant. The data corresponding to the variables can be seen at figures.

- Changing T: Figures 3-6
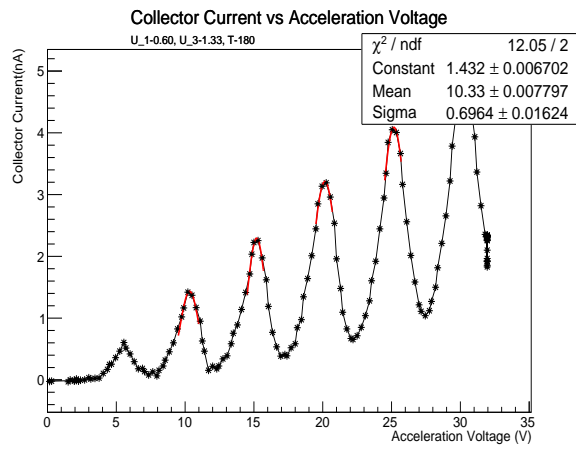- Changing U1: Figures 7-9
- Changing U2: Figures 10-12

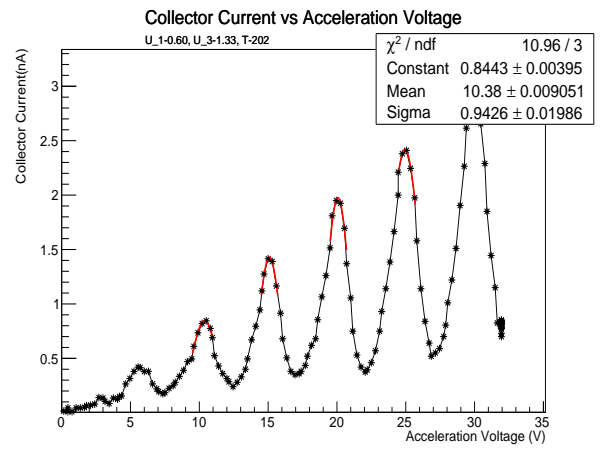Changing T.

Fig. 3. U1:0.60(nA) U3:1.33(nA) T:180($^\circ C$)

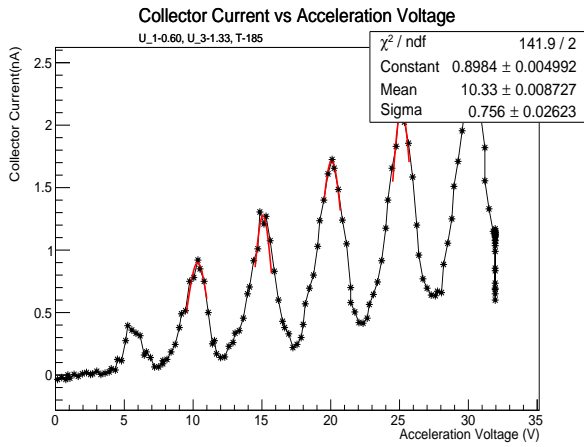Fig. 6. U1:0.60(nA) U3:1.33(nA) T:202($^\circ C$)

Changing U1.

Fig. 4. U1:0.60(nA) U3:1.33(nA) T:185($^\circ C$

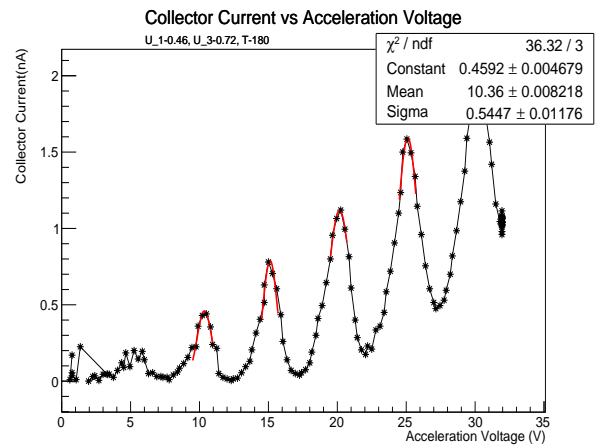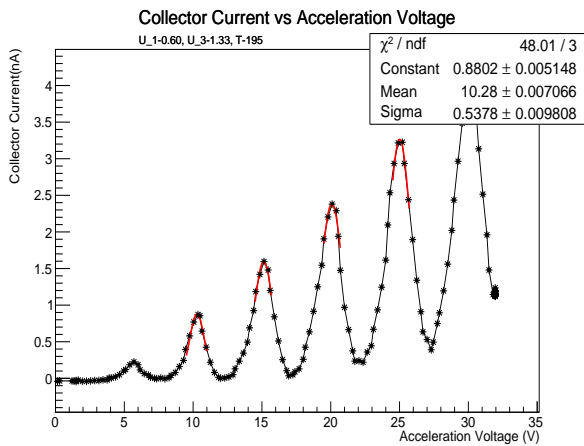

Fig. 7. U1:0.46(nA) U3:1.33(nA) T:180($^\circ C$)
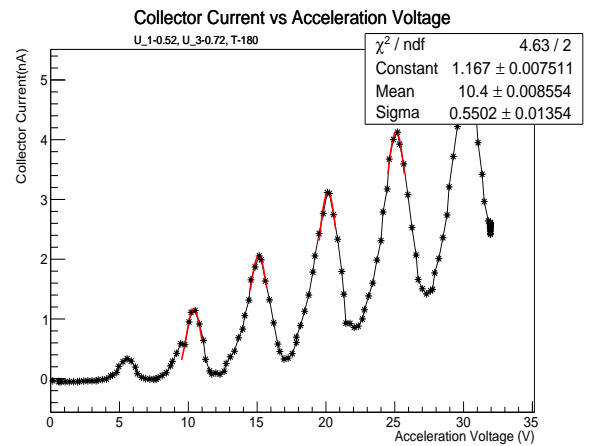

Fig. 5. U1:0.60(nA) U3:1.33(nA) T:195($^\circ C$)

Fig. 8. U1:0.52(nA) U3:1.33(nA) T:180($^\circ C$)
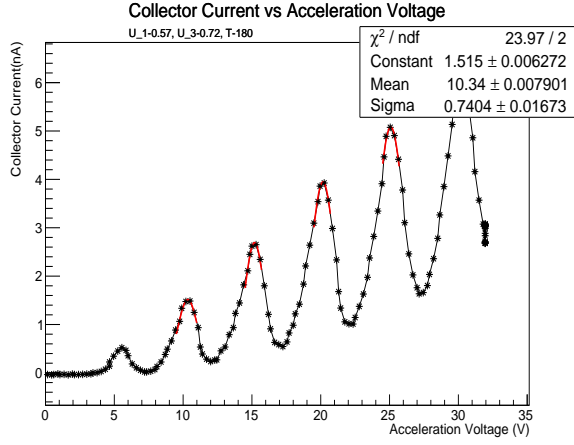
Fig. 9. U1:0.57(nA) U3:1.33(nA) T:180($^{\circ}C$)
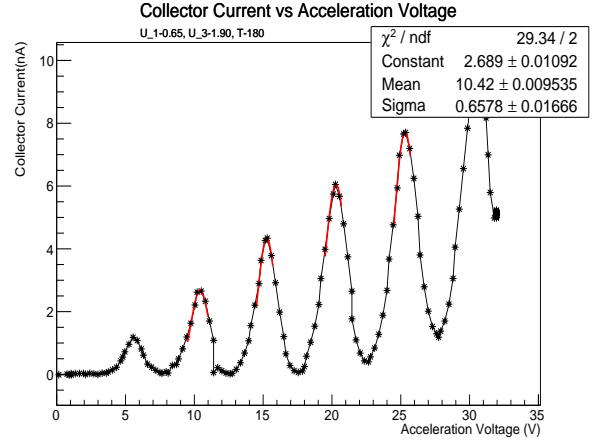
Changing U3.

Fig. 10. U1:0.65(nA) U3:0.28(nA) T:180($^{\circ}C$)

Fig. 11. U1:0.65(nA) U3:0.72(nA) T:180($^{\circ}C$)
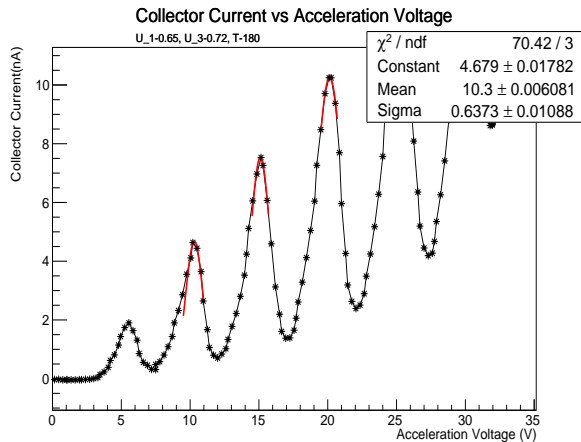

Fig. 12. U1:0.65(nA) U3:1.90(nA) T:180($^{\circ}C$)

V. THE ANALYSIS

We have used Root's built-in functions for plotting histograms and fitting the data. We have calculated the errors in measurement by the general formula:

$$\sigma_f = \sqrt{\sum_i \left(\frac{\partial f}{\partial x_i}\right)^2 \sigma_{x_i}^2} \qquad (11)$$

It is assumed that there is no correlation between the variables.

We have calculated the peak-to-peak voltages using the Gaussian Fit's mean values. As can be seen from the data and tables below, the peak-to-peak voltage values do not change significantly. According to the Eq.11, The peak-to-peak voltages and their uncertainties are calculated as:

Calculating Peak To Peak Voltage:

$$\Delta E_i = \mu_{i+1} - \mu_i \qquad (12)$$

Calculating the Uncertainties Using Eq.11:

$$\sigma_{\Delta E_i} = \sqrt{\sigma_{\mu_{i+1}}^2 + \sigma_{\mu_i}^2} \qquad (13)$$

The corresponding peak-to-peak voltages are listed below:
Changing T.

TABLE I
PEAK TO PEAK VOLTAGES
U1:0.60(V), U3:1.33(V), T:180($^{\circ}C$)

Peak Index	Voltage Difference (V)
1	4.87 ± 0.98 V
2	4.96 ± 1.16 V
3	5.15 ± 1.35 V

TABLE II
PEAK TO PEAK VOLTAGES
U1:0.60(V), U3:1.33(V), T:185($^{\circ}C$)

Peak Index	Voltage Difference (V)
1	4.75 ± 1.00 V
2	4.99 ± 1.08 V
3	5.08 ± 1.18 V

TABLE III
PEAK TO PEAK VOLTAGES
U1:0.60(V), U3:1.33(V), T:195($^\circ C$)

Peak Index	Voltage Difference (V)
1	4.84 ± 0.86 V
2	4.95 ± 1.05 V
3	4.93 ± 1.15 V

TABLE IV
PEAK TO PEAK VOLTAGES
U1:0.60(V), U3:1.33(V), T:202($^\circ C$)

Peak Index	Voltage Difference (V)
1	4.71 ± 1.23 V
2	4.97 ± 1.15 V
3	4.87 ± 1.38 V

Changing U1.

TABLE V
PEAK TO PEAK VOLTAGES
U1:0.46(V), U3:0.72(V), T:180($^\circ C$)

Peak Index	Voltage Difference (V)
1	4.76 ± 0.76 V
2	5.04 ± 0.98 V
3	4.95 ± 1.14 V

TABLE VI
PEAK TO PEAK VOLTAGES
U1:0.52(V), U3:0.72(V), T:180($^\circ C$)

Peak Index	Voltage Difference (V)
1	4.71 ± 0.93 V
2	5.04 ± 1.12 V
3	4.95 ± 1.28 V

TABLE VII
PEAK TO PEAK VOLTAGES
U1:0.57(V), U3:0.72(V), T:180($^\circ C$)

Peak Index	Voltage Difference (V)
1	4.85 ± 1.05 V
2	4.97 ± 1.15 V
3	4.93 ± 1.36 V

Changing U3.

TABLE VIII
PEAK TO PEAK VOLTAGES
U1:0.65(V), U3:0.28(V), T:180($^\circ C$)

Peak Index	Voltage Difference (V)
1	4.80 ± 1.07 V
2	5.05 ± 1.30 V
3	4.96 ± 1.34 V

TABLE IX
PEAK TO PEAK VOLTAGES
U1:0.65(V), U3:0.72(V), T:180($^\circ C$)

Peak Index	Voltage Difference (V)
1	4.80 ± 0.99 V
2	5.04 ± 1.27 V
3	4.96 ± 3.76 V

TABLE X
PEAK TO PEAK VOLTAGES
U1:0.65(V), U3:1.90(V), T:180($^\circ C$)

Peak Index	Voltage Difference (V)
1	4.84 ± 0.92 V
2	5.03 ± 1.03 V
3	5.02 ± 1.16 V

We have obtained the peak-to-peak voltages of the different data sets. We have used the weighted average formula:

Weight of the ΔE_i:

$$w_i = \frac{1}{\sigma_{\Delta E_i}{}^2} \tag{14}$$

Calculating the Weighted Average of ΔE:

$$\bar{\Delta E} = \frac{\sum_{i=1}^{n} w_i E_i}{\sum_{i=1}^{n} w_i} \tag{15}$$

Using weighted averages, we have calculated the peak-to-peak voltages for the corresponding data set:

TABLE XI
PEAK TO PEAK VOLTAGES

Data Set	Voltage (V)
U1:0.46(V), U3:0.72(V), T:180($^\circ C$)	4.92 ± 0.95 V
U1:0.52(V), U3:0.72(V), T:180($^\circ C$)	4.92 ± 0.59 V
U1:0.57(V), U3:0.72(V), T:180($^\circ C$)	4.92 ± 0.40 V
U1:0.60(V), U3:1.33(V), T:180($^\circ C$)	4.94 ± 0.27 V
U1:0.60(V), U3:1.33(V), T:185($^\circ C$)	4.93 ± 0.16 V
U1:0.60(V), U3:1.33(V), T:195($^\circ C$)	4.90 ± 0.20 V
U1:0.60(V), U3:1.33(V), T:202($^\circ C$)	4.86 ± 0.18 V
U1:0.65(V), U3:0.28(V), T:180($^\circ C$)	4.92 ± 0.21 V
U1:0.65(V), U3:0.72(V), T:180($^\circ C$)	4.92 ± 0.25 V
U1:0.65(V), U3:1.90(V), T:180($^\circ C$)	4.98 ± 0.01 V

We have plotted a histogram of average peak-to-peak voltages and fitted a Gaussian. We have used the uncertainties of the data points to calculate weights of the each data point as Eq.14 suggests. The final result is the mean value of the fit. We expect from the theory that the values should be the same. However, we see a deviation, therefore we will take the uncertainty of the final value as the sigma value of the fit. So, we can check if the true value is in our sigma.
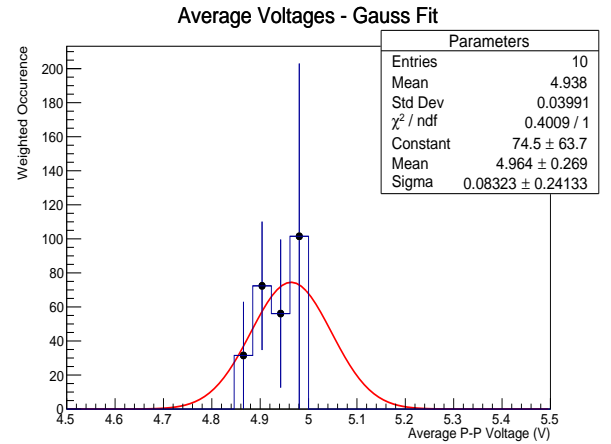

Fig. 13. Histogram - Gauss Fit

From Fig.13 we see that the First Excitation Energy of Mercury is 4.96 ± 0.08 eV.

VI. THE RESULT

We have found the First Excitation Energy of Mercury as 4.96 ± 0.08 eV. The true value is 4.9 eV [2]. We are 0.8σ away from the true value with 1.2% relative error.

VII. The Conclusion

We have found the First Excitation Energy of Mercury as 4.96 ± 0.08 eV. The true value is 4.9 eV [2]. We are 0.8σ away from the true value with 1.2% relative error. Therefore we have concluded that the experiment was a success. With the experiment, we have shown the discreteness of energy levels as Frank and Hertz showed in 1914. While analyzing the data we have picked the maximum current values to determine the excitation energy. This is because the position of the maximum current is uniquely determined by the acceleration voltage (multiples of 4.9V). However, the minimum may depend on the configurations.

We have conducted a controlled experiment that keeps 2 variables constant. We have controlled the temperature of the oven (T), driving potential (U1), and breaking potential (U3).

- Temperature(T): As can be seen from Fig.3-6, increasing temperature leads to a smaller collector current at the minima. As Eq.10 suggests, the mean free path of the electrons decreases with increasing temperature. Therefore successive, collisions occur more frequently.
- Driving Potential(U1): As the name suggests and as can be seen in Fig.7-9, increasing U1 yields greater electron emission. Therefore, greater readings at the collector current.
- Breaking Potential(U3): Increasing U3 leads to finer maximum and minimum values. This effect can be seen from the last three data points' uncertainties shown in Table.XI. Increasing U3 significantly affects the uncertainty. Moreover, it reduces the collector currents.
- Even if we have changed every variable systematically, the peak-to-peak voltages did not change significantly (See.Table.XI).

While taking the data, we observed plots as shown in Fig.14. We have tried to obtain Fig.14-e. In each case, the problem is:

- a) The driving potential is too high. So, the maximum collector current is exceeded.
- b) The breaking potential is not sufficiently high, so the peaks are not fine enough to analyze. Also, the driving potential is not sufficiently high.
- c) The maximum and minimum points are not adequate to analyze. Breaking potential or driving potential is low.
- d)The breaking potential is too high. It should be lowered to obtain the figure e.
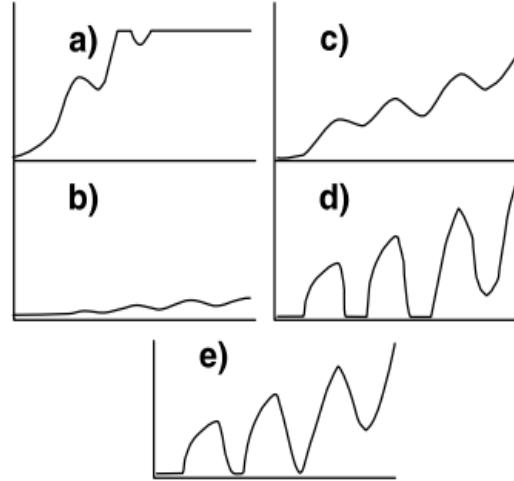

Fig. 14. Observed Plots.

Even if we got a reasonable result, there were some issues we experienced. One of them is because of the electrical isolation of the equipment. While reading data, we have observed that (also can be seen at the very beginning and ending of the plots), there are fluctuations in the readings which corrupts the data.

Another issue we have experienced is the limitations of the apparatus we have used. The apparatus does not get data after a certain volt and also it cannot get the collector current after 10nA. Therefore, we could not observe the behaviors of the system at higher currents and voltages. So the equipment and the software could be designed better. Also, the uncertainties of the voltage and the current could be improved.

References

[1] *Nobel Prize in Physics 1925 - Presentation Speech*. URL: https://www.nobelprize.org/prizes/physics/1925/ceremony-speech/.

[2] Friedel Weinert. "Franck—Hertz Experiment". In: *Compendium of Quantum Physics*. Ed. by Daniel Greenberger, Klaus Hentschel, and Friedel Weinert. Berlin, Heidelberg: Springer Berlin Heidelberg, 2009, pp. 241–243. ISBN: 978-3-540-70626-7. DOI: 10.1007/978-3-540-70626-7_74. URL: https://doi.org/10.1007/978-3-540-70626-7_74.

[3] *The formation of mercury molecules*. URL: https://royalsocietypublishing.org/doi/pdf/10.1098/rspa.1936.0008 (visited on 05/14/2023).

[4] *CODATA Value: Planck constant*. URL: https://physics.nist.gov/cgi-bin/cuu/Value?h (visited on 05/14/2023).

[5] Berare Göktürk. *PHYS 442 - 22 Spring Franck-Hertz Experiment*. 2nd ed. 2023-03-02.

[6] E. Gülmez. *Advanced Physics Experiments*. 1st. Boğaziçi University Publications, 1999.

VIII. Appendix

The fit has been done with Root's built-in function (See Fig.5-8). We have used root release 6.28/00 for Ubuntu22.

```cpp
#include <iostream>
#include <fstream>
#include <string>
{
    gStyle->SetOptFit(1);
    const int n_peak = 10;
    double *delta_E, *sd_delta_E;
    delta_E = new double[n_peak];
    sd_delta_E = new double[n_peak];
    string file_names[] =
        {"u1-0.46_u3-0.72_t-180.csv",
                    "u1-0.52_u3-0.72_t-180.csv",
                    "u1-0.57_u3-0.72_t-180.csv",
                    "u1-0.60_u3-1.33_t-180.csv",
                    "u1-0.60_u3-1.33_t-185.csv",
                    "u1-0.60_u3-1.33_t-195.csv",
                    "u1-0.60_u3-1.33_t-202.csv",
                    "u1-0.65_u3-0.28_t-180.csv",
                    "u1-0.65_u3-0.72_t-180.csv",
                    "u1-0.65_u3-1.90_t-180.csv"};
    const string DELIMITER = "_";
    string u1, u3, t;
    int count = 0;
    for (string file_name : file_names) {
        double gauss_mean[4] = {0,0,0,0};
        double sd_gauss_mean[4] = {0,0,0,0};
        double gauss_sigma[4] = {0,0,0,0};
        double sd_gauss_sigma[4] = {0,0,0,0};
        double delta_peak[3] = {0,0,0};
        double sd_delta_peak[3] = {0,0,0};

        string str1 = "Franck_Hertz-Data/";
        string str2 = str1 + file_name;
        const char *path = str2.c_str();

        int pos1 = file_name.find(DELIMITER);
        int pos2 = file_name.find(DELIMITER,
            pos1 + 1);
        int pos3 = file_name.find(".csv");
        u1 = file_name.substr(2, pos1 - 2);
        u3 = file_name.substr(pos1 + 3, pos2 -
            pos1 - 3);
        t = file_name.substr(pos2 + 2, pos3 -
            pos2 - 2);

        string title_1 =
            "#splitline{#scale[0.8]{Collector
            Current vs Acceleration
            Voltage}}{#scale[0.5]";
        string title_2 = "{U_1" + u1 + ", U_3" +
            u3 + ", T" + t + "}}";
        string title_3 = title_1 + title_2 +
            ";Acceleration Voltage (V);
            Collector Current(nA)";
        string str4 = file_name + ".pdf";

        const char *save = str4.c_str();
        const char *title = title_3.c_str();
        TTree *tree = new TTree("tree", "tree");
        tree->ReadFile(path);
        float I, U;
        tree->SetBranchAddress("U", &U);
        tree->SetBranchAddress("I", &I);

        double *x, *y, *sd_x, *sd_y;
        int n = tree->GetEntries();
        x = new double[n];
        y = new double[n];
        sd_x = new double[n];
        sd_y = new double[n];

        for (int i = 0; tree->LoadTree(i) >= 0;
            i++){
            tree->GetEntry(i);
            x[i] = U;
            y[i] = I;
            sd_y[i] = 0.005;
            sd_x[i] = 0.015;
        }

        TCanvas *c1 = new TCanvas();

        TGraphErrors *graph = new
            TGraphErrors(n,x,y,sd_x,sd_y);
        graph->SetTitle(title);
        graph->SetMarkerStyle(20);
        TF1 *f1 = new TF1("f1","gaus",9.5,11);
        TF1 *f2 = new TF1("f2","gaus",14.5,15.7);
        TF1 *f3 = new TF1("f3","gaus",19.5,20.7);
        TF1 *f4 = new TF1("f4","gaus",24.5,25.7);

        graph->Fit(f1, "QSR");
        graph->Fit(f2, "QSR+");
        graph->Fit(f3, "QSR+");
        graph->Fit(f4, "QSR+");

        graph->Draw("A*");
        graph->Draw();

        gauss_mean[0] = f1->GetParameter(1);
        gauss_mean[1] = f2->GetParameter(1);
        gauss_mean[2] = f3->GetParameter(1);
        gauss_mean[3] = f4->GetParameter(1);

        gauss_sigma[0] = f1->GetParameter(2);
        gauss_sigma[1] = f2->GetParameter(2);
        gauss_sigma[2] = f3->GetParameter(2);
        gauss_sigma[3] = f4->GetParameter(2);

        sd_gauss_mean[0] = f1->GetParError(1);
        sd_gauss_mean[1] = f2->GetParError(1);
        sd_gauss_mean[2] = f3->GetParError(1);
        sd_gauss_mean[3] = f4->GetParError(1);

        sd_gauss_sigma[0] = f1->GetParError(1);
        sd_gauss_sigma[1] = f2->GetParError(1);
        sd_gauss_sigma[2] = f3->GetParError(1);
        sd_gauss_sigma[3] = f4->GetParError(1);

        for(int i = 0; i < 3; i++){
            delta_peak[i] = gauss_mean[i+1] -
                gauss_mean[i];
            sd_delta_peak[i] =
                sqrt(pow(gauss_sigma[i+1],2) +
                pow(gauss_sigma[i],2));
            cout << "Delta_E_" << i+1 << " for "
                << file_name << " : " <<
            delta_peak[i] << " +- " <<
                sd_delta_peak[i] << " V" << endl;
        }

        double weight_peak = 0, weight_sd_peak =
            0;
```

```cpp
        double totw_peak = 0, totw_sd_peak;
        double delta_peak_bar = 0,
            sd_delta_peak_bar = 0;

        for (int i=0; i<3; ++i) {
            weight_peak =
                1./(sd_gauss_mean[i]*sd_gauss_mean[i]);
            weight_sd_peak =
                1./(sd_gauss_sigma[i]*sd_gauss_sigma[i]);
            totw_peak += weight_peak;
            totw_sd_peak += weight_sd_peak;
            delta_peak_bar += delta_peak[i] *
                weight_peak;
            sd_delta_peak_bar += sd_delta_peak[i]
                * weight_sd_peak;
        }

        delta_peak_bar /= totw_peak;
        sd_delta_peak_bar /= totw_sd_peak;

        delta_E[count] = delta_peak_bar;
        sd_delta_E[count] = sd_delta_peak_bar;
        count += 1;
        c1->Print(save);
    }

    TCanvas *c2 = new TCanvas();
    TH1D *h1 = new TH1D("Parameters","Average
        Voltages - Gauss Fit",26,4.5,5.5);
    for (int i=0; i<10; i++)
        h1->Fill(delta_E[i]
        ,1./(pow(sd_delta_E[i],2)));
    h1->SetMarkerStyle(kFullCircle);
    h1->GetXaxis()->SetTitle("Average P-P
        Voltage (V)");
    h1->GetYaxis()->SetTitle("Weighted
        Occurence");
    TF1 *f_gauss = new
        TF1("f_gauss","gaus",4.85,5);
    f_gauss->SetParameters(h1->GetMaximum(),
        h1->GetMean(), h1->GetRMS() );
    h1->Fit("f_gauss", "Q");
    h1->Draw("HIST same E");


    double result = f_gauss->GetParameter(1);
    double sd_result = f_gauss->GetParameter(2);
        for(int i = 0; i < 10; i++){
        cout << delta_E[i] << " +- " <<
            sd_delta_E[i] << endl;
    }

    cout << "First Excitation Energy = " <<
        result << " +- "
    << sd_result << " eV" <<endl;
    c2->Print("Franck-Hertz.pdf");
    return 0;
}
```