CSE 211: Discrete Mathematics **(Due: 17/01/21)**

Homework #4

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- The homeworks (both latex and pdf files in a zip file) will be submitted into the course page of Moodle.
- The latex, pdf and zip files of the homeworks should be saved as "Name_Surname_StudentId".{tex, pdf, zip}.
- If the answers of the homeworks have only calculations without any formula or any explanation -when needed- will get zero.
- Writing the homeworks on Latex is strongly suggested. However, hand-written paper is still accepted **IFF** hand writing of the student is clear and understandable to read, and the paper is well-organized. Otherwise, the assistant cannot grade the student's homework.

Problem 1 (15+15=30 points)

Consider the nonhomogeneous linear recurrence relation $a_n = 3a_{n-1} + 2^n$.

(a) Show that whether $a_n = -2^{n+1}$ is a solution of the given recurrence relation or not. Show your work step by step.
(Solution)
1) $a_n = a_n^{(h)} + a_n^{(p)}$

2)$a_n - 3a_{n-1} = 0$

3)$r - 3 = 0$

4)$r = 3$

5)$a_n^{(h)} = \alpha(3)^n$

6)$a_n - 3a_{n-1} = 2^n$ Guess $a_n = A2^n$

7)$A2^n - 3A2^{n-1} = 2^n$

8)$A - \frac{3}{2}A = 1 \Rightarrow A = -2$

9)$a_n^{(p)} = -2(2^n) = -2^{n+1}$

10) $a_n = \alpha(3)^n$ - 2^{n+1}
(b) Find the solution with $a_0 = 1$.
(Solution)
$a_0 = \alpha(-3)^0$ - 2^{0+1}
$1 = \alpha$ - 2
$\alpha = 3$
$a_n = 3(3)^n$ - $2^{n+1} = 3^{n+1}$ - 2^{n+1}

Problem 2 (35 points)

Solve the recurrence relation f(n) = 4f(n-1) - 4f(n-2) + n^2 for f(0) = 2 and f(1) = 5.
(Solution)
$f(n) = f(n)^{(h)} + f(n)^{(p)}$
$f(n)$ - $4f(n-1)$ + $4f(n-2) = 0$
r^2 - 4r + 4 = 0
$r_1 = 2 \quad r_2 = 2$
$f(n)^{(h)} = \alpha 2^n + \beta n 2^n$
$f(n)$ - $4f(n-1)$ + $4f(n-2) = n^2$ guess $An^2 + Bn + C$
$A(n^2 - 8n + 12) + B(n-4) + C = n^2$
$n = 0; \quad 12A - 4B + C = 0$
$n = 1; \quad 5A - 3B + C = 1$
$n = 2; \quad 0A - 4B + C = 4$

$$\begin{bmatrix} 12 & -4 & 1 \\ 5 & -3 & 1 \\ 0 & -2 & 1 \end{bmatrix}^{-1} \times \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ 20 \end{bmatrix} \Rightarrow \begin{matrix} A = 1 \\ B = 8 \\ C = 20 \end{matrix}$$

$f(n)^{(p)} = n^2 + 8n + 20$
$f(n) = f(n)^{(h)} + f(n)^{(p)} \Longrightarrow f(n) = \alpha 2^n + \beta n 2^n + n^2 + 8n + 20$
$f(0) = 2 = \alpha + 20 \Rightarrow \alpha = -18$
$f(1) = 5 = 2\beta - 7 \Rightarrow \beta = 6$
$f(n) = (-18)2^n + 6n2^n + n^2 + 8n + 20 \Longrightarrow f(n) = 2^n(6n - 18) + n^2 + 8n + 20$

Problem 3 (20+15 = 35 points)

Consider the linear homogeneous recurrence relation $a_n = 2a_{n-1}$ - $2a_{n-2}$.
(a) Find the characteristic roots of the recurrence relation.
(Solution)
a_n - $2a_{n-1}$ + $2a_{n-2} = 0$
r^2 - 2r + 2 = 0
$\Delta = (-2)^2$ - $4 \times 1 \times 2 = -4$
$r_1 = \frac{-(-2)+\sqrt{-4}}{2} = 1 + i$
$r_2 = \frac{-(-2)-\sqrt{-4}}{2} = 1 - i$
(b) Find the solution of the recurrence relation with $a_0 = 1$ and $a_1 = 2$.
(Solution)
$a_n = \alpha(1+i)^n + \beta(1-i)^n$
$a_0 = 1 = \alpha + \beta$
$a_1 = 2 = \alpha(1+i) + \beta(1-i)$
$\alpha = \frac{1-i}{2} \quad \beta = \frac{1+i}{2}$
$a_n = \frac{1-i}{2}(1+i)^n + \frac{1+i}{2}(1-i)^n$