Q1) Burak Yildirim
1901042609

a) $2^n + n^3 \leq c \cdot 4^n$
$c = 3$
$n_0 = 5$
True because formula apply for all $n \geq 5$ while $c = 3$.

b) $c \cdot n \leq \sqrt{10n^2 + 7n + 3}$
$c = 1$
$n_0 = 5$
True because formula apply for all $n \geq 5$ while $c = 1$.

c) $n^2 + n < c \cdot n^2$
$c = 1$
$n_0 = 1$
false because formula must apply for all positive constants but doesn't apply for $c = 1$.

d) $c_1 \cdot \log_2 n^2 \leq 3 \cdot \log_2^2 n \leq c_2 \cdot \log_2 n^2$
$\Rightarrow 2 \cdot c_1 \leq 3 \cdot \log_2 n \leq 2 \cdot c_2$
False because as n goes to infinity middle part will go to infinity and upper bound will not hold no matter what we assign to c2 because it's constant and middle part isn't.

e) $(n^3+1)^6 \leq c \cdot n^3$
$(n^3+1)^6 \cong n^{18}$
$\Rightarrow n^{18} \leq c \cdot n^3 \Rightarrow n^{15} \leq c$
False because as n goes to infinity left-hand side will go to infinity and upper bound will not hold no matter what we assign to c because it's a constant and left-hand side isn't.

Q2)

a) $2n\log(n+2)^2 + (n+2)^2\log\left(\frac{n}{2}\right)$

$\Rightarrow 4n.\log(n+2) + \underbrace{(n^2+4n+4).\log\left(\frac{n}{2}\right)}$

$n^2.\log\left(\frac{n}{2}\right) \Rightarrow$ highest order

$g(n) = n^2.\log n$

b) $\underbrace{0.001n^4}_{\text{highest order}} + 3n^3 + 1$

$g(n) = n^4$

Q3)

a) $\log n$, $n^{\log n}$, $n^{1,5}$

$$\lim_{n\to\infty} \frac{n^{\log n}}{n^{1,5}} \Rightarrow \lim_{n\to\infty} n^{\log n - 1,5} = \infty$$

$$\lim_{n\to\infty} \frac{n^{\log n}}{\log n} \Rightarrow \frac{\frac{2\log n.\, n^{\log n}}{n}}{\frac{1}{n.\log 10}} \Rightarrow \frac{2\log n.\, n^{\log n}}{\log 10} = \infty$$

$$\lim_{n\to\infty} \frac{n^{1,5}}{\log n} = \frac{1,5.\, n^{0,5}}{\frac{1}{n.\log 10}} = 1,5.\log 10.\, n^{1,5} = \infty$$

$n^{\log n} > n^{1,5} > \log n$

b) $n!, 2^n, n^2$

$$\lim_{n\to\infty} \frac{n!}{2^n} = \lim_{n\to\infty} \frac{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n}{2^n} = \lim_{n\to\infty} \sqrt{2\pi n}\,\frac{n^n}{2^n \cdot e^n} = \lim_{n\to\infty} \sqrt{2\pi n}\left(\frac{n}{2e}\right)^n = \infty$$

$$\lim_{n\to\infty} \frac{2^n}{n^2} = \lim_{n\to\infty} \frac{2^n \log 2}{2n} = \frac{2^n \cdot \log^2 2}{2} = \infty$$

$$n! > 2^n > n^2$$

c) $n\log n, \sqrt{n}$

$$\lim_{n\to\infty} \frac{n\log n}{\sqrt{n}} = \lim_{n\to\infty} \frac{n\log n}{n^{1/2}} = \lim_{n\to\infty} n^{1/2}\cdot\log n = \infty$$

$$n\cdot\log n > \sqrt{n}$$

d) $n2^n, 3^n$

$$\lim_{n\to\infty} \frac{n\cdot 2^n}{3^n} = \lim_{n\to\infty} n\cdot\left(\frac{2}{3}\right)^n \Rightarrow \lim_{n\to\infty} \frac{n}{\left(\frac{2}{3}\right)^{-n}}$$

$$\Rightarrow \lim_{n\to\infty} \frac{1}{-\left(\frac{2}{3}\right)^{-n}\log\left(\frac{2}{3}\right)} = 0$$

$$3^n > n.2^n$$

e) $\sqrt{n+10}, n^3$

$$\lim_{n\to\infty} \frac{n^3}{\sqrt{n+10}} = \lim_{n\to\infty} \frac{3n^2}{\frac{1}{2\sqrt{n+10}}} \Rightarrow \lim_{n\to\infty} 3n^2 \cdot 2\sqrt{n+10} = \infty$$

$$n^3 > \sqrt{n+10}$$

Q4)

a) Basic operation : comparison

b) $\sum_{i=0}^{n-2} \cdot \sum_{j=i+1}^{n-1} 1 = \sum_{i=0}^{n-2}(n-i-1) = (n-1)+(n-2)+\dots+1$

$= \frac{(n-1)n}{2}$

c) $W(n) \in \Theta(n^2)$

Q5)

a) Basic operation: increment the entry by the multiplication of two entry

b) $\sum_{i=0}^{n-1} \cdot \sum_{j=0}^{n-1} \cdot \sum_{k=0}^{n-1} 1 = \sum_{i=0}^{n-1} \cdot \sum_{j=0}^{n-1} n = \sum_{i=0}^{n-1} n^2 = n^3$

c) $A(n) \in \Theta(n^3)$

Q6)

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algorithm (A[0.. n-1], x)
    for i=0 to n-2 do
        for j=i+1 to n-1 do
            if A[i] * A[j] == x
                print (A[i], A[j])
```

a) Basic operation : comparison

b) $\sum_{i=0}^{n-2} \cdot \sum_{j=i+1}^{n-1} 1 = \sum_{i=0}^{n-2}(n-i-1) = (n-1)+(n-2)+\dots+1$

$= \frac{(n-1).n}{2}$

c) $A(n) \in \Theta(n^2)$