**CSE 211: Discrete Mathematics** **(Due: 17/11/20)**

# Homework #1

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*Assistant: Gizem Süngü*

**Course Policy**: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- The homeworks (both latex and pdf files in a zip file) will be submitted into the course page of Moodle.
- The latex, pdf and zip files of the homeworks should be saved as "Name_Surname_StudentId".{tex, pdf, zip}.
- If the answers of the homeworks have only calculations without any formula or any explanation -when needed- will get zero.
- Writing the homeworks on Latex is strongly suggested. However, hand-written paper is still accepted **IFF** hand writing of the student is clear and understandable to read, and the paper is well-organized. Otherwise, the assistant cannot grade the student's homework.

**Problem 1: Conditional Statements** (5+5+5=15 points)

State the converse, contrapositive, and inverse of each of these conditional statements.

**(a)** If it snows tonight, then I will stay at home.
***(Solution)***

**Converse:** If I stay at home, then it will snow tonight.

**Contrapositive:** If I don't stay at home, then it won't snow tonight.

**Inverse:** If it doesn't snow tonight, then I won't stay at home.

**(b)** I go to the beach whenever it is a sunny summer day.
***(Solution)***

**Converse:** It is a sunny summer day whenever I go to the beach.

**Contrapositive:** It is not a sunny summer day whenever I don't go to the beach.

**Inverse:** I don't go to the beach whenever it is not a sunny summer day.

**(c)** If I stay up late, then I sleep until noon.
***(Solution)***

**Converse:** If I sleep until noon, then I stay up late.

**Contrapositive:** If I don't sleep until noon, then I don't stay up late.

**Inverse:** If I don't stay up late, then I don't sleep until noon.

**Problem 2: Truth Tables For Logic Operators** (5+5+5=15 points)

Construct a truth table for each of the following compound propositions.
**(a)** $(p \oplus \neg q)$
***(Solution)***

| p | q | ¬ q | p ⊕ ¬ q |
|---|---|---|---|
| 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 |

**(b)** $(p \iff q) \oplus (\neg p \iff \neg r)$
***(Solution)***

| p | q | r | ¬ p | ¬ r | p ⟺ q | ¬ p ⟺ ¬ r | (p ⟺ q) ⊕ ( ¬ p ⟺ ¬ r) |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |

**(c)** $(p \oplus q) \Rightarrow (p \oplus \neg q)$
***(Solution)***

| p | q | ¬ q | p ⊕ q | p ⊕ ¬ q | (p ⊕ q) ⇒ (p ⊕ ¬ q) |
|---|---|---|---|---|---|
| 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 |

**Problem 3: Predicates and Quantifiers** (21 points)

There are three predicate logic statements which represent English sentences as follows.

- P(x): ”x can speak English.”
- Q(x): ”x knows Python.”
- H(x): ”x is happy.”

Express each of the following sentences in terms of P(x), Q(x), H(x), quantifiers, and logical connectives or vice versa. The domain for quantifiers consists of all students at the university.

**(a)** There is a student at the university who can speak English and who knows Python.
***(Solution)***
∃x(P(x) ∧ Q(x)).
**(b)** There is a student at the university who can speak English but who doesn't know Python.
***(Solution)***
∃x(P(x) ∧ ¬ Q(x)).
**(c)** Every student at the university either can speak English or knows Python.
***(Solution)***
∀ x (P(x) ⊕ Q(x)).
**(d)** No student at the university can speak English or knows Python.
***(Solution)***
¬ ∃ x (P(x) ∨ Q(x)).
**(e)** If there is a student at the university who can speak English and know Python, then she/he is happy.
***(Solution)***
(P(x) ∧ Q(x)) $\Longrightarrow$ H(x).
**(f)** At least two students are happy.
***(Solution)***
∃x∃y(x ≠ y ∧ H(x) ∧ H(y)).
**(g)** $\neg\forall x(Q(x) \wedge P(x))$
***(Solution)***
No student at the university knows Python and can speak English.

**Problem 4: Mathematical Induction** (21 points)

Prove that 3 + 3 . 5 + 3 . $5^2$ + . . . + 3 . $5^n$ $=\frac{3(5^{n+1}-1)}{4}$ whenever n is a nonnegative integer.
***(Solution)***
for n = 1;
$3 + 3.5 = \frac{3(5^{1+1}-1)}{4} \qquad 18 = \frac{3.24}{4} \qquad 18 = 18$
for n = k;
$3 + 3 \,.\, 5 + 3 \,.\, 5^2 + \ldots + 3 \,.\, 5^k = \frac{3(5^{k+1}-1)}{4}$
for n = k + 1;
$3 + 3 \,.\, 5 + 3 \,.\, 5^2 + \ldots + 3 \,.\, 5^k + 3.5^{k+1} = \frac{3(5^{k+2}-1)}{4}$

$\frac{3(5^{k+1}-1)}{4} + 3.5^{k+1} = \frac{3(5^{k+2}-1)}{4} \qquad \frac{15.5^{k+1}-3}{4} = \frac{3.5^{k+2}-3}{4}$

$75.5^k$ - 3 = $75.5^k$ - 3

**Problem 5: Mathematical Induction** (20 points)

Prove that $n^2$ - 1 is divisible by 8 whenever n is an odd positive integer.
***(Solution)***
n = 2k + 1 $\qquad n^2$ - 1 = $4k^2$ + 4k
for k = 1;
$4(1^2)$ + 4.1 = 8 is divisible by 8.
for k = 2;

$4(2^2) + 4.2 = 24$ is divisible by 8.
n = 2k $\quad n^2 - 1 = 4k^2 - 1$
for k = 1;
$4(1^2) - 1 = 3$ is not divisible by 8.
for k = 2;
$4(2^2) - 1 = 15$ is not divisible by 8.
This indicates that $4k^2$ + 4k is divisible by 8 for all positive iterations of k and $4k^2$ - 1 is not divisible by 8 for all positive iterations of k so $n^2$ - 1 is divisible by 8 whenever n is an odd positive integers

**Problem 6: Sets** (8 points)

Which of the following sets are equal? Show your work step by step.

**(a)** {t : t is a root of $x^2 - 6x + 8 = 0$}

**(b)** {y : y is a real number in the closed interval [2, 3]}

**(c)** {4, 2, 5, 4}

**(d)** {4, 5, 7, 2} - {5, 7}

**(e)** {q: q is either the number of sides of a rectangle or the number of digits in any integer between 11 and 99}

***(Solution)***

(a) $\Delta = 6^2 - 4.8.1 = 4$. $\quad x_1 = \frac{6+\sqrt{4}}{2.1} = 4$. $\quad x_2 = \frac{6-\sqrt{4}}{2.1} = 2$. $\quad$ a = {2, 4}

(b) b has infinite number of elements because there is infinite number of real numbers between 2 and 3.

(c) {4, 2, 5, 4}

(d) {4, 2}

(e) e = {4, 2} because rectangle has 4 sides and any integer between 11 and 99 has 2 digits.

a, d and e have the same elements so a = d = e.

**Problem Bonus: Logic in Algorithms** (20 points)

Let p and q be the statements as follows.

- **p:** It is sunny.
- **q:** The flowers are blooming.

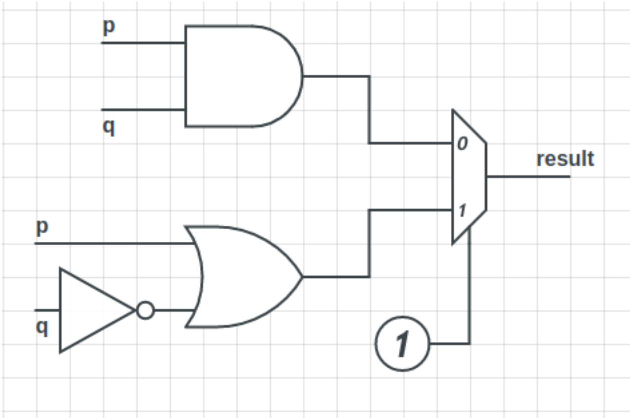


Figure 1: Combinational Circuit

In Figure 1, the two statements are used as input. The circuit has 3 gates as AND, OR and NOT operators. It has also a 2x1 multiplexer[1] which provides to select one of the two options.
**(a)** Write the sentence that "result" output has.
***(Solution)***
It is sunny or the flowers are not blooming.
**(b)** Convert Figure 1 to an algorithm which you can write in any programming language that you prefer (including pseudocode).
***(Solution)***

```
int main(){
int s0 = 1;
if(s0 == 1){
cout << "It is sunny or the flowers are not blooming." << endl;
}
else{
cout << "It is sunny and the flowers are blooming." << endl;
}
}
```

[1]https://www.geeksforgeeks.org/multiplexers-in-digital-logic/