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Answer 1

Firstly, let's look at first 3 terms;
$3a_1 + 4a_0 = a_2$
$3a_2 + 4a_1 = a_3$
$3a_3 + 4a_2 = a_4$
To use generating functions, multiply nth row with x^n;
(n = 2) $3a_1.x^2 - 4a_0.x^2 = a_2.x^2$
(n = 3) $3a_2.x^3 - 4a_1.x^3 = a_3.x^3$
(n = 4) $3a_3.x^4 - 4a_2.x^4 = a_4.x^4$
Then taking the same column elements(some coefficients ordering with x^n) :
3a$_1.x^2 + 3a_2.x^3 + 3a_3.x^4 +$ .... $= 3\Sigma_{i\geq 1}^{\infty} = a_i.x^{i+1}$
$4a_0.x^2 + 4a_1.x^3 + 4a_2.x^4 +$ .... $= 4\Sigma_{i\geq 0}^{\infty} = a_i.x^{i+2}$
$a_2.x^2 + a_3.x^3 + a_4.x^4 +$ .... $=\Sigma_{i\geq 2}^{\infty} = a_i.x^i$
let's put these sigma notations in recurrence relation:
$3\Sigma_{i\geq 1}^{\infty} = a_i.x^{i+1} + 4\Sigma_{i\geq 0}^{\infty} = a_i.x^{i+2} = \Sigma_{i\geq 2}^{\infty} = a_i.x^i$
If we assume that f(x) is $\Sigma_{i\geq 0}^{\infty} = a_i.x^i$, we can write the above equation like this:
$3x(f(x) - a_0) + 4x^2 f(x) = f(x) - a_0 - a_1.x$ and substitute a_0 and a_1:
$3x(f(x) - 1) + 4x^2 f(x) = f(x) - 1 - x$
Arranged form of the equation :
$f(x) = \frac{2x-1}{4x^2+3x-1}$ then if we should make partial fraction:
$f(x) = \frac{A}{4x-1} + \frac{B}{x+1} = A.(x+1) + B(4x-1) = 2x - 1$
After solve the above equation for A and B:
$A = \frac{-2}{5}$ and $B = \frac{3}{5}$
$f(x) = \frac{-2/5}{4x-1} + \frac{3/5}{x+1}$
The closed form of the $\frac{-2/5}{4x-1} = -2/5. - (4)^n$
The closed form of the $\frac{3/5}{x+1} = 3/5.(-1)^n$
So $a_n = 2/5.(4)^n + 3/5.(-1)^n$

Answer 2

a)

I can separate the following sequence like this:
$< 2, 5, 11, 29, 83, 245, \ >=< 2, 2, 2, 2, 2, 2, 2, 2, ... > + < 0, 3, 9, 27, 81, ... >$
The power series notation of the sequence $< 2, 2, 2, 2, 2, 2, 2, ... >$is $2\Sigma_{n=0}^{\infty} = x^n$

and closed form of this sigma notation is $\frac{2}{1-x}$
The power series notation of the sequence $< 1, 3, 9, 27, 81, ...>$is $\Sigma_{n=0}^{\infty} = 3^n.x^n$ but for first term to be equal subtract 1 and get $\Sigma_{n=0}^{\infty} = 3^n.x^n - 1$ and closed form of this sigma notation is $\frac{1}{1-3x} - 1$ $= \frac{3x}{1-3x}$

So closed form of the first given sequence is $G(x) = \frac{2}{1-x} + \frac{3x}{1-3x}$

b)

Firstly, to separate 2 distinct fractions, we use partial fractions and get:
$G(x) = \frac{A}{2x-1} + \frac{B}{x-1}$ If we solve for this, A = -5 and B = -2.
$G(x) = \frac{-5}{2x-1} + \frac{-2}{x-1}$
Expansion of the $\frac{-5}{2x-1}$ which is $\frac{5}{1-2x}$ is $5. < 1, 2, 4, 8, 16, 32, ... >$
and expansion of the $\frac{-2}{x-1}$ which is $\frac{2}{1-x}$ is $2. < 1, 1, 1, 1, 1, ... >$.
So if I add them because of the above G(x) function, I get
$G(x) = 5. < 1, 2, 4, 8, 16, 32, ... > +2. < 1, 1, 1, 1, 1, ... >=< 7, 12, 22, 42, 82, 162, ... >$

Answer 3

a)

If we consider R is a equivalence relation, then 3R4 because there is a right triangle which is edges are 3,4 and 5 which 5$\in Z$.

Also if R is equivalence then there should be a pair of 3R3 because of the reflexivity of the relation.
After that there should be a right triangle providing the relation. On the other hand, if the two sides of a right triangle are 3 and 3, then the last side has to be $3\sqrt{2}$ which is $\notin Z$.
These mean that R is not an equivalence relation.

b)

There is a 3 conditions to be equivalence relation. Let's check them:
Let $\forall a, b \in R$
Is the relation Reflexive: 2a+b = 2a+b $\longrightarrow$ (a,b) R (a,b) (YES)

Let $\forall a, b, c, d \in R$
Is the relation Symmetric: 2a+b = 2c+d $\longrightarrow$ (a,b) R (c,d)
2c+d = 2a+b $\longrightarrow$ (c,d) R (a,b) (YES)

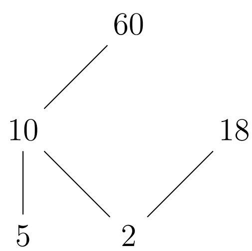
Let $\forall a, b, c, d, e, f \in R$

Is the relation Transitive: 2a+b = 2c+d $\longrightarrow$ (a,b) R (c,d)
2c+d = 2e+f $\longrightarrow$ (c,d) R (e,f)
2a+b = 2e+f $\longrightarrow$ (a,b) R (e,f) (YES)
It provides all conditions so it is a equivalence relation.

For (1,-2), it is 2.(1)+(-2) = 0. Everything which has result of 0 can be equivalence class of this.
The general showing of this equivalence class is $\forall k \in R$ (-k, 2k).
$[(1,2)]_R = \{(-k, 2k)|x \in R\}$ and it represents a line in the cartesian coordinate system which formula of it is y = 2x.

Answer 4

a) Hasse diagram of R is:



b) R = $\{(2,2),(5,5),(10,10),(18,18),(60,60),(2,60),(2,10),(2,18),(5,10),(5,60),(10,60)\}$
Matrix Representation for R is :

$$\begin{vmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

c) R_s is the symmetric closure of R. So we should add symmetric pairs which is not in the R.
All pairs (x,y) where (x, y) $\in R_s \wedge (x,y) \notin R = \{(60,2),(10,2),(18,2),(10,5),(60,10),(60,5)\}$
Matrix Representation for R_s is :

$$\begin{vmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{vmatrix}$$

d) No, it is impossible with 1 removing. We can not do that total ordering because original hasse diagram do not include such a element or do not have such a shape.

On the other hand, with two removing such as 5 and 18, I can get total ordering. Also I can preserve total ordering by adding a number that is a multiple of 60.