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Answer 1

Firstly I will show that $6^{(2n)} - 1$ is divisible by 5.

Step 1 Because of the least number in the domain, n = 1 is the first number to check. if we substitute one for n, $6^2 - 1 = 35$ which divisible by 5. So for 1, this claim is true.

Step 2 Let's assume that this is true for any k which is an element of the domain, so when $n = k$, 6^{2k} is divisible by 5.
$6^{2k} - 1 = 5m$ which m is a positive integer. $6^{2k} = 5m + 1$

Step 3 If it is true for any, it should be true for any k+1. We have to prove this. Let's show, when n:k+1, is $6^{2n} - 1$ divisible by 5?
$6^{2(k+1)} - 1 = 6^{2k}.6^2 - 1$ we can use the equation from step2.
$(5m + 1).36 - 1 = 180m + 35$

So, $6^{2n} - 1 = 5.(36m + 7)$ and it is a multiple of 5. It is divisible by 5.

Secondly, Lets do it same thing for 7 and show it is divisible by 7.

Step 1 n=1 and $6^2 - 1 = 35$, 35 is divisible by 7.

Step 2 Let's assume that this is true for any k which is an element of the domain, so when $n = k$, 6^{2k} is divisible by 7.
$6^{2k} - 1 = 7m$ which m is a positive integer. $6^{2k} = 7m + 1$

Step 3 If it is true for any, it should be true for any k+1. We have to prove this. Let's show, when n:k+1, is $6^{2n} - 1$ divisible by 7?
$6^{2(k+1)} - 1 = 6^{2k}.6^2 - 1$ we can use the equation from step2.
$(7m + 1).36 - 1 = 252m + 35$

So, $6^{2n} - 1 = 7.(36m + 5)$ and it is a multiple of 7. It is divisible by 7.

We conclude that $6^{2n} - 1$ is divisible by both 5 and 7.

Answer 2

1-) Base: $H_0 = 1, H_1 = 5, H_2 = 7$, Least element of domain of $n \geq 3$ is 3 so another base case is 3. Let's try it for 3.

P(3) $= H_3 = 8H_2 + 8H_1 + 9H_0$ should be less or equal than 9^n
P(3) $= 8.7 + 8.5 + 9.1 = 115 \leq 9^3$

2-) There is a 'k' which is $0 \leq k \geq n$, Let's assume that this claim is true for any k.
P(k) $= H_{k-1} + H_{k-2} + H_{k-3}$ and $H_k \leq 9^k$
So if this is true, this should satisfy any k+1
P(k+1) $= 8H_k + 8H_{k-1} + 9H_{k-2}$
And I know that from for any k step $H_k \leq 9^k$
$H_{k-1} \leq 9^{k-1}$
$H_{k-2} \leq 9^{k-2}$

Let's substitute these:
P(k+1) $= H_{k+1} \leq 8.9^k + 8.9^{k-1} + 9.9^{k-2}$
If I arrange this I can clearly see that: P(k+1) $= H_{k+1} \leq 9^{k+1}$

So $H^n \leq 9^n$ is true for any element of the domain.

Answer 3

For 8 bit string firstly, I will find for 4 consecutive 1 case:
Let's put these four 1 in the first 4 slot:

1	1	1	1	

We don't know other 4 slot so, there is 2 option for every slot and total permutations are $2.2.2.2 = 2^4 = 16$
Let's move one step to the right each 1: Now we put 0 beginning of the 1's because at the beginning we include the case first slot is 1.

0	1	1	1	1	

Now for the remaining three-step, we have $2^3 = 8$ options total.
Move on one more step: Again we put 0 at the beginning of 1's because 1 case is included above.

	0	1	1	1	1	

Now we don't know the first slot and remaning 2 slot, there are $2^3 = 8$ options.
This continue like this:

		0	1	1	1	1	

There are $2^3 = 8$ options.

			0	1	1	1	1

There are $2^3 = 8$ options.

There are total $16 + 8 + 8 + 8 + 8 = 48$ options for four consecutive 1.

Let's calculate for 0's:
$\begin{array}{|c|c|c|c|c}\hline 0 & 0 & 0 & 0 & \\ \hline\end{array}$ There are $2^4 = 16$ options.

$\begin{array}{|c|c|c|c|c|}\hline 1 & 0 & 0 & 0 & 0 \\ \hline\end{array}$ There are $2^3 = 8$ options.

$\begin{array}{|c|c|c|c|c|c|}\hline & 1 & 0 & 0 & 0 & 0 \\ \hline\end{array}$ There are $2^3 = 8$ options.

$\begin{array}{|c|c|c|c|c|c|c|}\hline & & 1 & 0 & 0 & 0 & 0 \\ \hline\end{array}$ There are $2^3 = 8$ options.

$\begin{array}{|c|c|c|c|c|c|c|c|}\hline & & & 1 & 0 & 0 & 0 & 0 \\ \hline\end{array}$ There are $2^3 = 8$ options.

There are total $16 + 8 + 8 + 8 + 8 = 48$ options for four consecutive 0.
On the other hand, we calculate the case:
$\begin{array}{|c|c|c|c|c|c|c|c|}\hline 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ \hline\end{array}$ or vice versa above. Let's subtract these two case:

$48 + 48 - 2 = 94$ options.

Answer 4

To form a galaxy:
I have to choose 1 Star: C(10,1)
I have to choose 2 habitable planets: C(20,2)
I have to choose 8 unhabitable planets: C(80,8)
So we can choose it by C(10,1).C(20,2).C(80,8)

Let's order these planets with given conditions:
Also, we can change the order of two habitable ones with 2!. Firstly, There are 6 unhabitable planets between two habitable ones. We can choose this 6 planet with C(8,6) and order with 6!. If we think of these 8 planets like a package, then there will be 3 elements to order. So I can order them with 3!. To sum up, there will be C(8,6).6!.3! options for this one.

Secondly, There are 7 unhabitable planets between two habitable ones. We can choose this 7 planets with C(8,7) and order with 7!. If we think this 9 planet as a package, then there will be 2 elements to order. So I can order them with 2!. To sum up, there will be C(8,7).7!.2! options for this one.

Finally, There are 8 unhabitable planets between two habitable ones. We can choose this 8 planet with C(8,8) and order with 8!. So there are no more element to order. So There are C(8,8).8! options.

In addition, I can order two habitable planet with 2! for above three case. In conclusion totally, From begining, ıf I them choose and order for each case, there will be

C(10,1). C(20,2). C(80,8). 2!. (C(8,6).6!,3! + C(8,7).7!.2! + C(8,8).8!)

Answer 5

a) Let's say P(n) is the number of paths for the robot to go to n cells away from the starting position.

In order to move to 1 cell away from its initial location, there is 1 possibility. So P(1) = 1

To move 2 cells, P(2) = 2
P(3)=4

For P(4) there are P(4) = P(3)+P(2)+P(1) because each jump reduces the probability by its own jump amount. So in general, I can say:

P(n) = P(n-1) + P(n-2) + P(n-3)

b) P(1) = 1, P(2) = 2, P(3) = 4 for every n which is $n \geq 4$

c) Let's calculate the P(9)
P(4) = P(3) + P(2) + P(1) = 7
P(5) = 13
P(6) = 24
P(7) = 44
P(8) = 81
P(9) = 149