

Student Information

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Answer 1

a) $\deg(a)=3$

$\deg(b)=3$

$\deg(c)=3$

$\deg(d)=2$

$\deg(e)=3$

The sum of degrees of all nodes of G is 14.

b)

	a	b	c	d	e
a	0	1	1	0	1
b	1	0	1	0	1
c	1	1	0	1	0
d	0	0	1	0	1
e	1	1	0	1	0

The number of non-zero entries in the adjacency matrix representation of G is 14.

c)

	e_1	e_2	e_3	e_4	e_5	e_6	e_7
a	0	1	0	1	1	0	0
b	1	1	1	0	0	0	0
c	1	0	0	1	0	0	1
d	0	0	0	0	0	1	1
e	0	0	1	0	1	1	0

The number of zero entries in the incidence matrix representation of G is 21.

- d) No, there is no such complete graph with at least 4 vertices as a subgraph because if we choose any four vertices, not every vertex has an edge with each other.
- e) No, it is not, because there are no 2 separate sets that do not have a connection between their elements. To be, bipartite, there should be 2 separate sets with no connection between them.
- f) There are 2^7 directed graphs that has G as their underlying undirected graph because there is 7 edge and there are 2 possibilities for each edge. So there are $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^7$ possible directed graphs.
- g) There are many possibilities for the longest path with length 4, but my choice is b-a-c-d-e.

- h) This graph is a connected graph so there is a path between every vertex. If you follow the path by passing every vertex only 1 time, there will not be a remaining vertex. This means that there is 1 connected component.
- i) No, there is no Euler circuit in G because there are more than 0 vertexes with odd degrees.
- j) No, there is no Euler path in G because there are more than 0 or 2 vertexes with odd degrees.
- k) Yes, G has a Hamilton circuit which is b-a-c-d-e-b. It returns the same vertex by passing by every vertex exactly one time.
- l) Yes, G has a Hamilton path which is b-a-c-d-e. It passes by every vertex exactly one time.

Answer 2

$G(V_1, E_1)$ and $H(V_2, E_2)$ are isomorphic because \exists a bijection (1-to-1 onto) $f: V_1 \implies V_2$ such that $\forall \{x_1, x_2\} \in E_1 \iff \{f(x), f(y)\} \in E_2$

If we make it more clear,

$V_1 = \{a, b, c, d, e\}$ and $V_2 = \{a', b', c', d', e'\}$. So, there is the same number of vertexes, and their appearance in the function is equivalent. For instance,

$$f(a, b) = f(a', b')$$

$$f(a, e) = f(a', e')$$

$$f(d, c) = f(d', c')$$

$$f(d, e) = f(d', e')$$

$$f(b, c) = f(b', c')$$

So, they are isomorphic.

Answer 3

Let's begin with vertex s, and its distance is zero. All remaining vertex distances are infinity.

	s	u	v	w	x	y	z	t
	①	∞	∞	∞	∞	∞	∞	∞
previous	-	-	-	-	-	-	-	-

If the lengths of the vertexes adjacent to s are less than the previous ones, they are changed and the next vertex with the smallest value is selected.

	s	u	v	w	x	y	z	t
	①	4	5	③	∞	∞	∞	∞
previous	-	-	-	s	-	-	-	-

Let's continue with vertex w and then the vertex with less length.

	s	u	v	w	x	y	z	t
	①	④	5	③	11	∞	15	∞
previous	-	s	-	s	-	-	-	-

Let's continue with vertex u and then the vertex with less length.

	s	u	v	w	x	y	z	t
	①	④	⑤	③	11	15	15	∞
previous	-	s	s	s	-	-	-	-

Let's continue with vertex v and then the vertex with less length.

	s	u	v	w	x	y	z	t
	①	④	⑤	③	⑦	11	15	∞
previous	-	s	s	s	v	-	-	-

Let's continue with vertex x and then the vertex with less length.

	s	u	v	w	x	y	z	t
	①	④	⑤	③	⑦	⑧	13	∞
previous	-	s	s	s	v	x	-	-

Let's continue with vertex y and then the vertex with less length.

	s	u	v	w	x	y	z	t
	①	④	⑤	③	⑦	⑧	⑫	17
previous	-	s	s	s	v	x	y	-

Let's continue with vertex z and then the vertex with less length.

	s	u	v	w	x	y	z	t
	①	④	⑤	③	⑦	⑧	⑫	⑮
previous	-	s	s	s	v	x	y	z

If we backtrack the path, our path is s-v-x-y-z-t and its length is 15.

Answer 4

I choose Kruskal's algorithm.

a) When we select it, we should not select the edges that make a circle. First, added edges with length 2,

(b,c),(c,f),(h,i),(d,k),

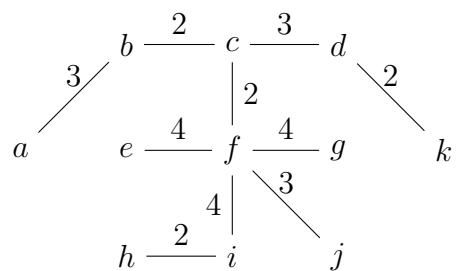
Secondly, added edges with length 3,

(a,b),(c,d),(f,j),

Then, added edges with length 4,

(e,f),(f,g),(f,i).

b)



c)

No, it is not unique because it is dependent on which order I choose the edges. For example, if I choose one edge with weight 3, I can not choose another edge with weight 3 because of the avoiding loop and this means I choose 1 possibility out of 2.