

Biçimsel Diller ve Soyut Makineler

Matematiksel Temeller

Hafta 1

Mathematical Preliminaries

- Kümeler
- Fonksiyonlar
- Bağıntılar
- Graflar

KÜMELER

Küme: Ortak özellikli elemanlar topluluğu

$$A = \{1, 2, 3\}$$

$$B = \{train, bus, bicycle, airplane\}$$

Burada

$$1 \in A$$

$$ship \notin B$$

Küme Gösterimleri

$$C = \{ a, b, c, d, e, f, g, h, i, j, k \}$$

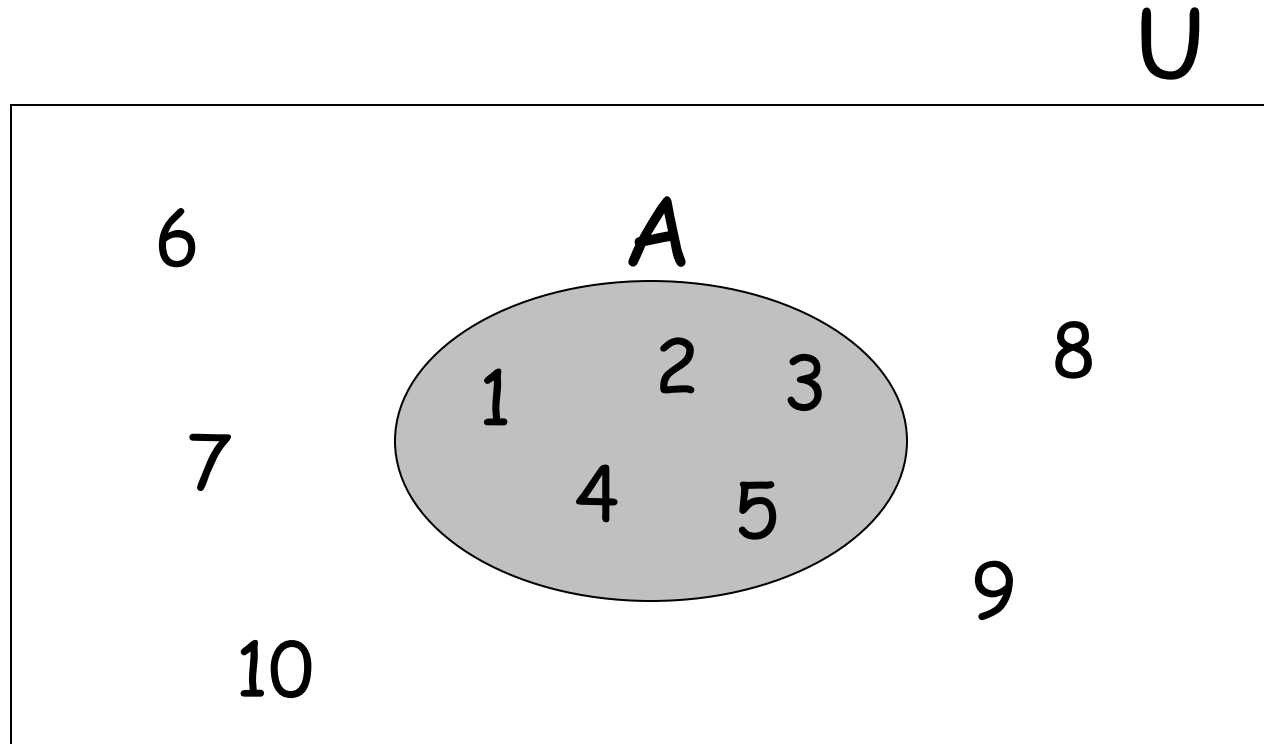
$$C = \{ a, b, \dots, k \} \longrightarrow \text{Sonlu (finite) set}$$

$$S = \{ 2, 4, 6, \dots \} \longrightarrow \text{Sonsuz (infinite) set}$$

$$S = \{ j : j > 0, \text{ ve } j = 2k, k > 0 \}$$

$$S = \{ j : j, \text{ pozitif ve çift sayıdır} \}$$

$$A = \{1, 2, 3, 4, 5\}$$



Evrensel Küme: olası bütün elemanlar

$$U = \{1, \dots, 10\}$$

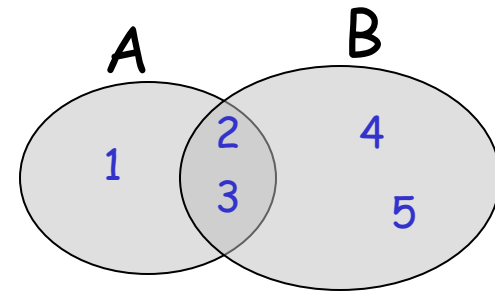
Küme işlemleri

$$A = \{1, 2, 3\}$$

$$B = \{2, 3, 4, 5\}$$

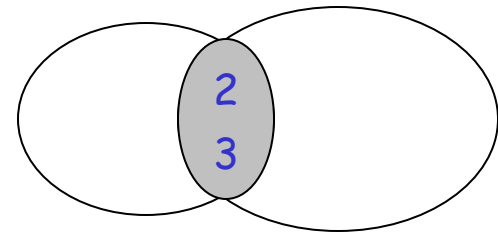
- Birleşim (Union)

$$A \cup B = \{1, 2, 3, 4, 5\}$$



- Kesişim (Intersection)

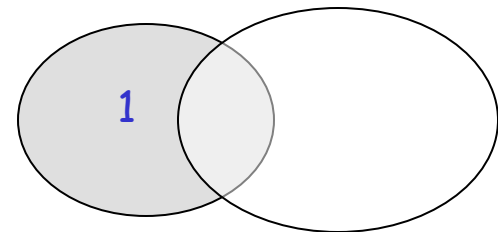
$$A \cap B = \{2, 3\}$$



- Fark (Difference)

$$A - B = \{1\}$$

$$B - A = \{4, 5\}$$

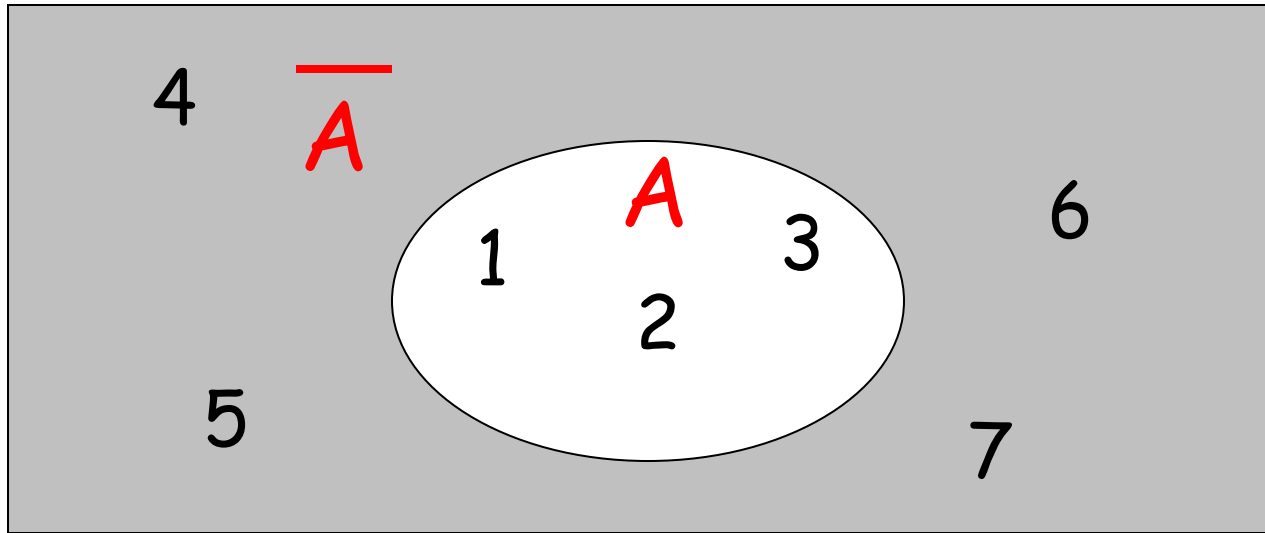


Venn şeması

- Tümleyen

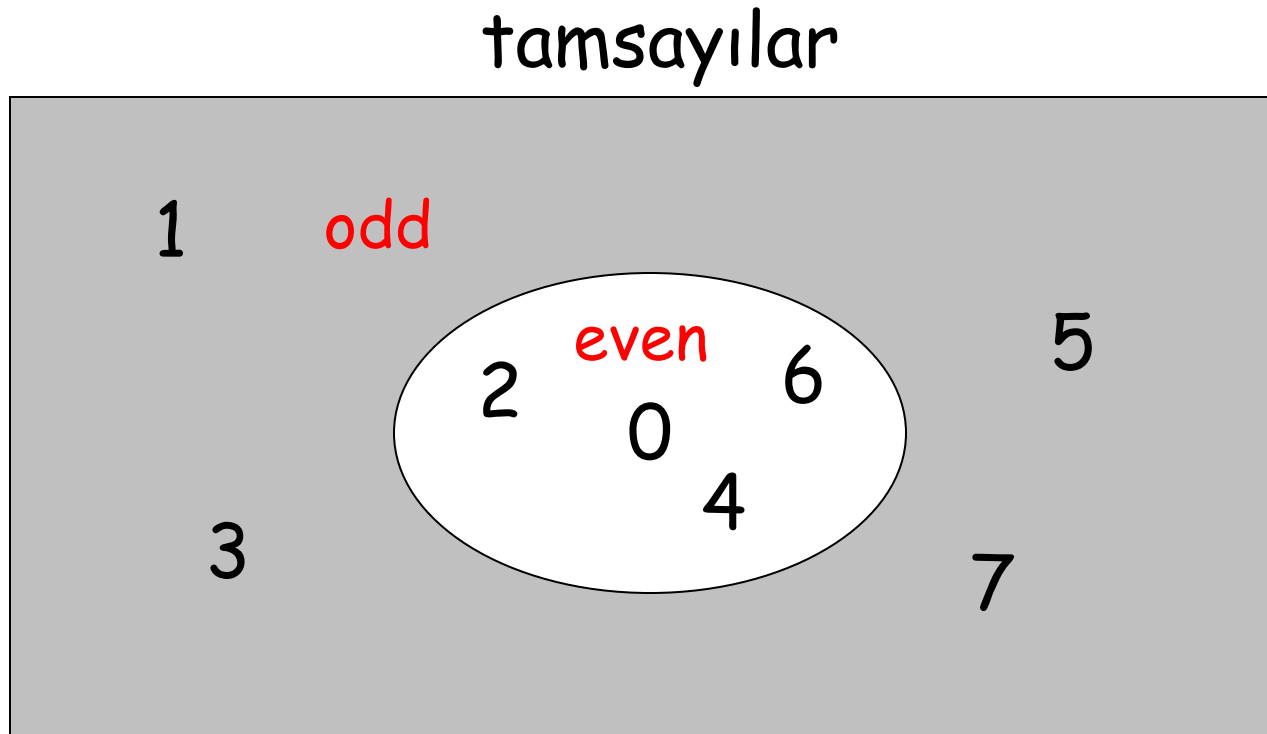
Evrensel Küme= $\{1, \dots, 7\}$

$$A = \{1, 2, 3\} \longrightarrow \overline{A} = \{4, 5, 6, 7\}$$



$$\overline{\overline{A}} = A$$

$$\{\text{çift tamsayılar}\} = \{\text{tek tamsayılar}\}$$



DeMorgan Kuralları

$$\overline{A \cup B} = \bar{A} \cap \bar{B}$$

$$\overline{A \cap B} = \bar{A} \cup \bar{B}$$

Boş küme:

\emptyset

$$\emptyset = \{ \}$$

$$S \cup \emptyset = S$$

$$S \cap \emptyset = \emptyset$$

$$S - \emptyset = S$$

$$\emptyset - S = \emptyset$$

$\overline{\emptyset}$ = Evrensel Küme

Alt küme

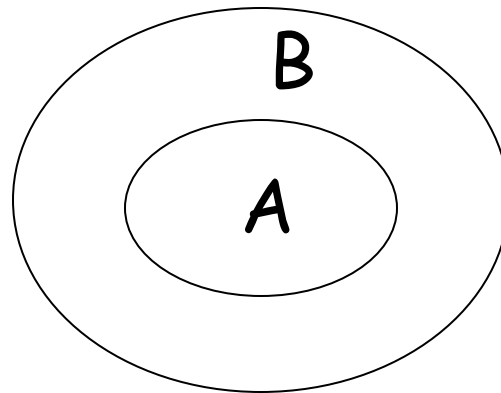
$$A = \{1, 2, 3\}$$

$$B = \{1, 2, 3, 4, 5\}$$

$$A \subseteq B$$

Kapsama:

$$A \subset B$$

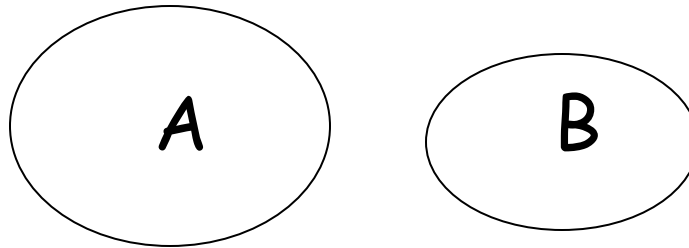


Ayrık Kümeler

$$A = \{ 1, 2, 3 \}$$

$$B = \{ 5, 6 \}$$

$$A \cap B = \emptyset$$



Eleman sayısı

$$A = \{ 12, 15, 17 \}$$

$$|A| = 3$$

Alt kümeler

$$S = \{ a, b, c \}$$

$$2^S = \{ \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\} \}$$

Alt Küme sayısı: $| 2^S | = 2^{|S|} \quad (8 = 2^3)$

Kartezyen Çarpım

$$A = \{ 2, 4 \}$$

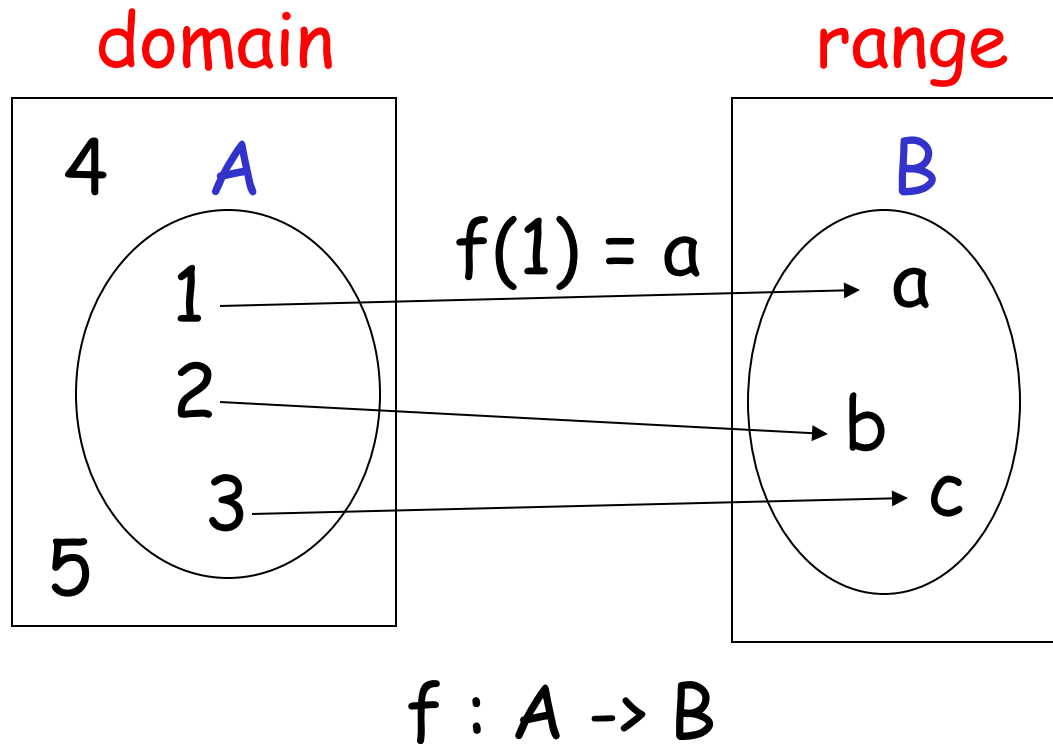
$$B = \{ 2, 3, 5 \}$$

$$A \times B = \{ (2, 2), (2, 3), (2, 5), (4, 2), (4, 3), (4, 5) \}$$

$$|A \times B| = |A| |B|$$

$$A \times B \times \dots \times Z$$

Fonksiyon



Bağıntılar

$$R = \{(x_1, y_1), (x_2, y_2), (x_3, y_3), \dots\}$$

$$x_i R y_i$$

e. g. if $R = '>': 2 > 1, 3 > 2, 3 > 1$

Eşdeğer bağıntılar

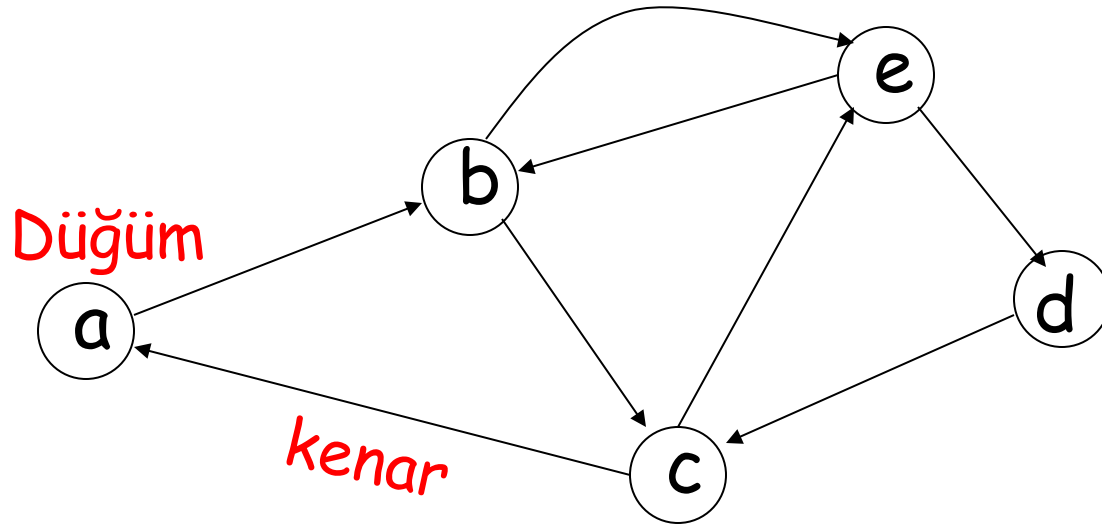
- Reflexive: $x R x$
- Symmetric: $x R y \longrightarrow y R x$
- Transitive: $x R y$ and $y R z \longrightarrow x R z$

Örnek: $R = '='$

- $x = x$
- $x = y \longrightarrow y = x$
- $x = y$ and $y = z \longrightarrow x = z$

GRAFLAR

Yönlü graf



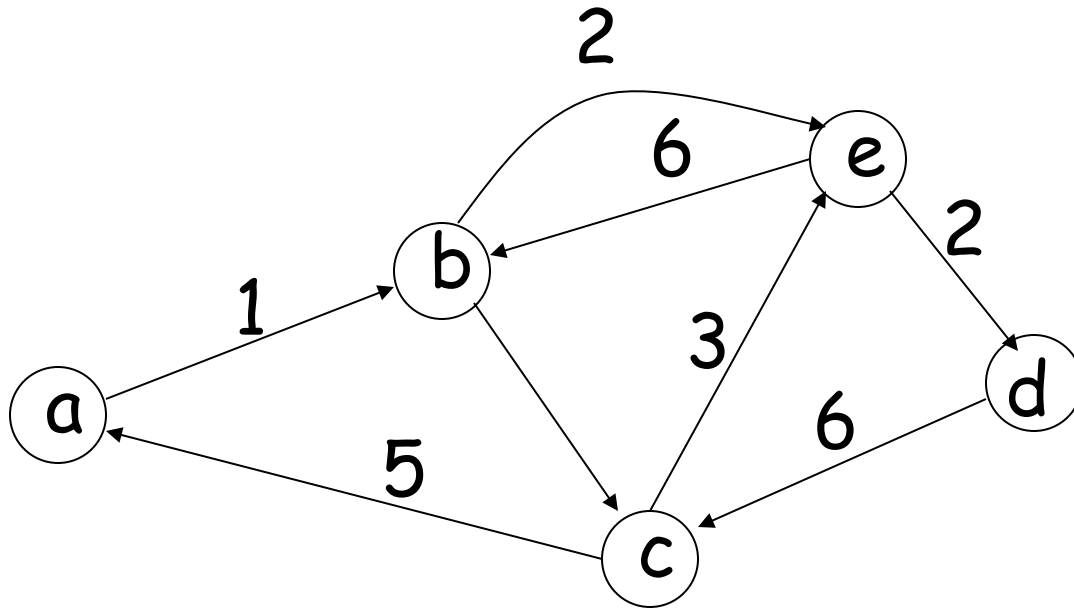
- Düğümler (Vertices)

$$V = \{ a, b, c, d, e \}$$

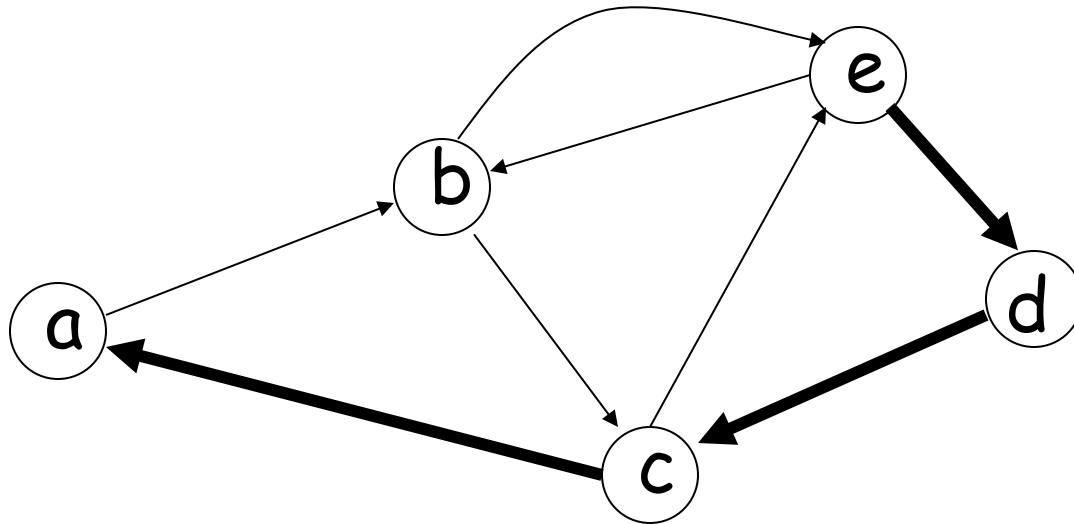
- Kenarlar

$$E = \{ (a,b), (b,c), (b,e), (c,a), (c,e), (d,c), (e,b), (e,d) \}$$

Etiketli graflar



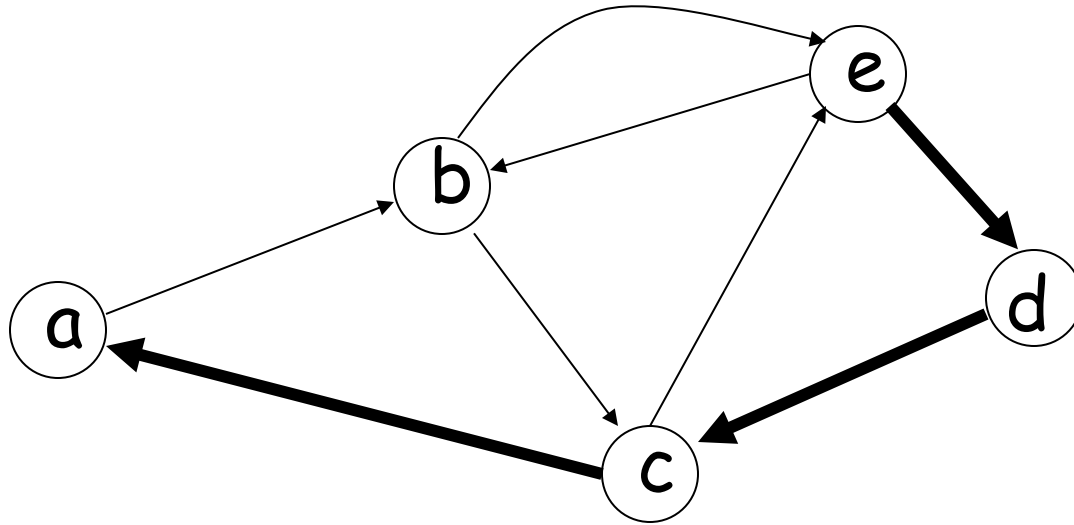
Yol



Yol

$(e, d), (d, c), (c, a)$

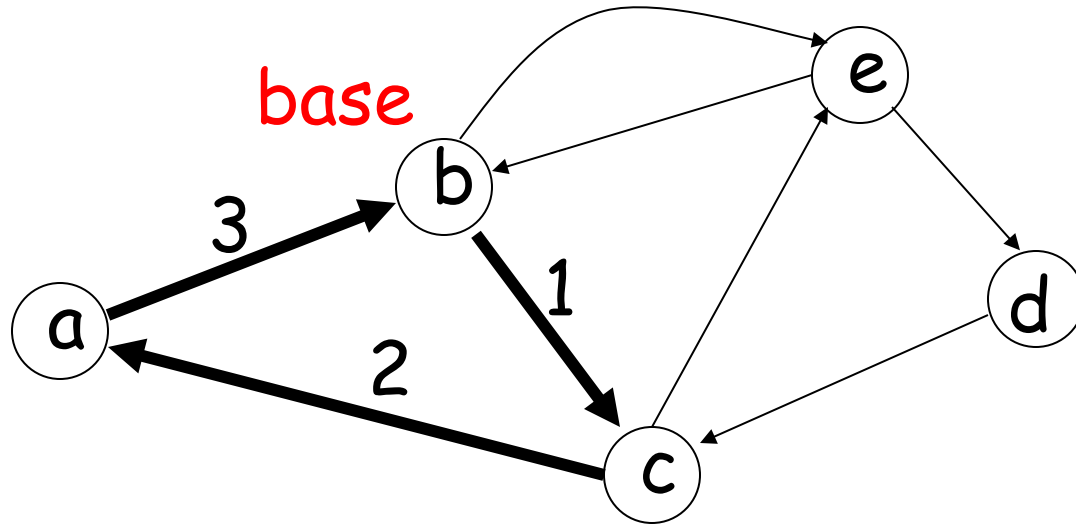
Path



Path: Hiçbir düğümün tekrarlanmadığı alternatif Yol

Simple path: hiçbir düğüm ve kenarın tekrarlanmadığı
paralel kenarların bulunmadığı yol

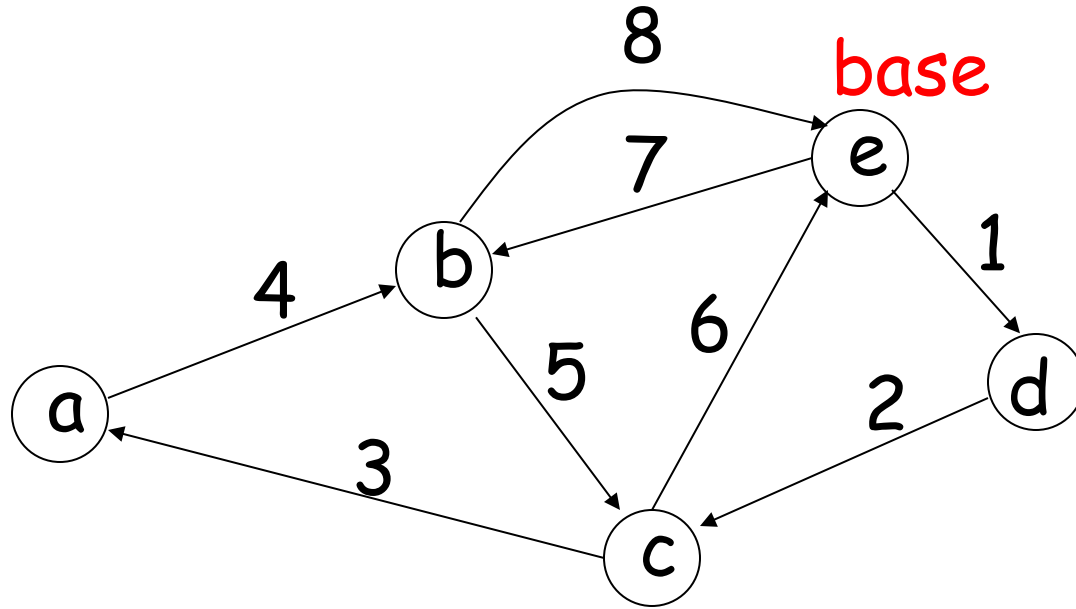
Döngü (Cycle)



Cycle: Bir düğümden çıkıp tekrar kendine dönen kenar

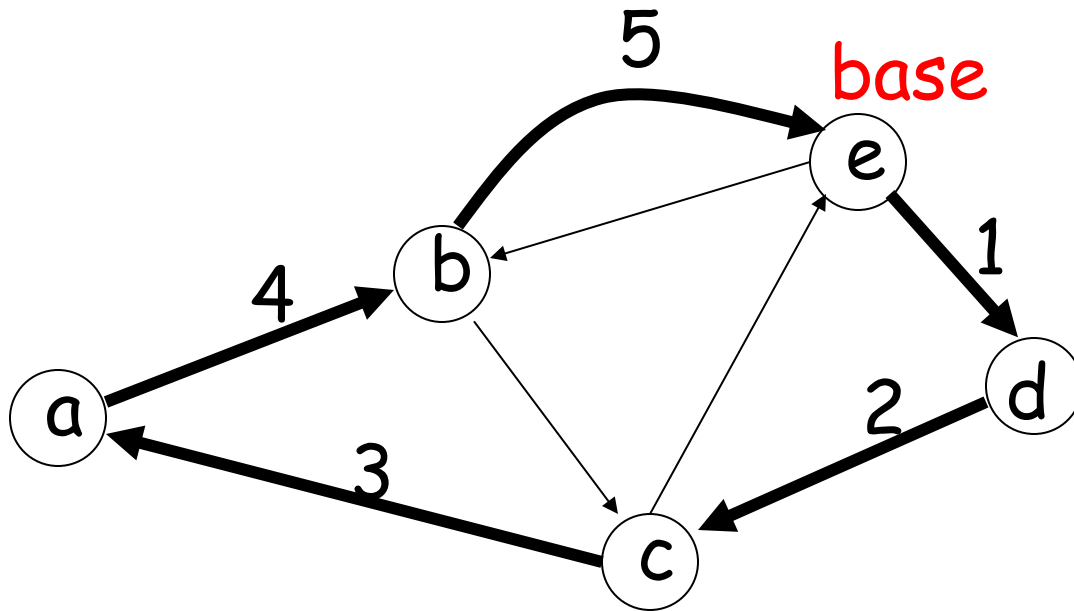
Simple cycle: sadece taban düğüm tekrarlanır

Euler Halkası



Tüm kenarlardan sadece birkez geçilerek oluşan
Başlangıç ve bitiş düğümleri farklı olan
Yola Euler yolu denir. Eğer başlangıç ve bitiş
düğümleri aynı ise buna Euler Halkası denir.

Hamiltonian Cycle



Bütün düğümleri içeren basit bir döngü

İçerik

Otomatlar

Düzgün İfadeler (Regüler Expression)

DFA-NFA-NFA-E ve Dönüşümler,

Grammer(Tür-0,Tür-1, Tür-2, Tür-3)

PDA ve Dönüşümler

Turing Makineleri

Parsing (Ayrıştırma)

LL1

LR1