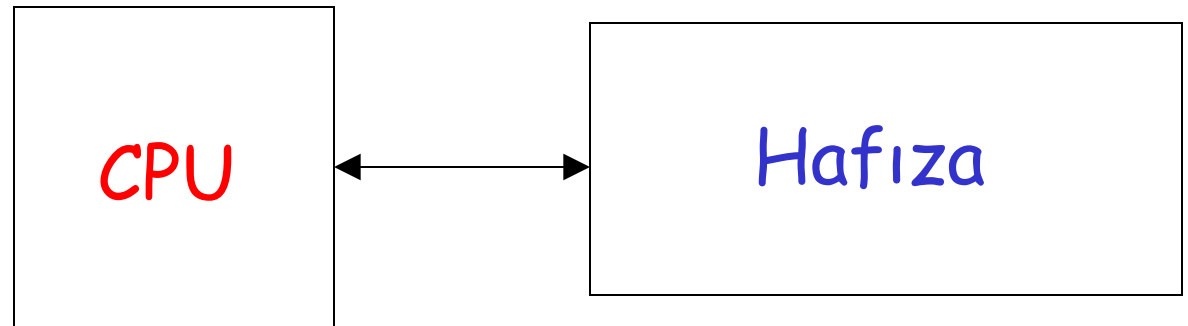
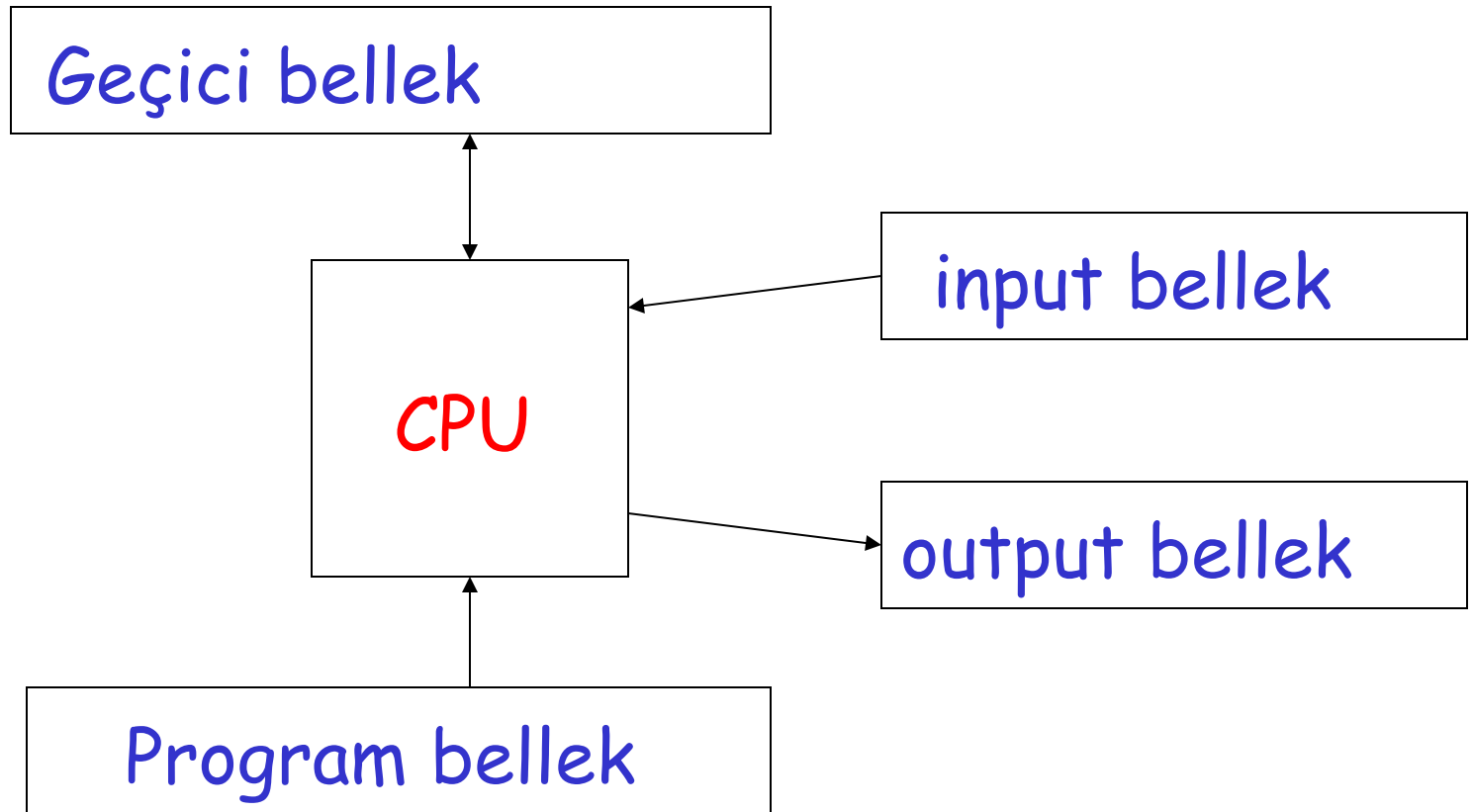


Biçimsel Diller ve Soyut Makineler

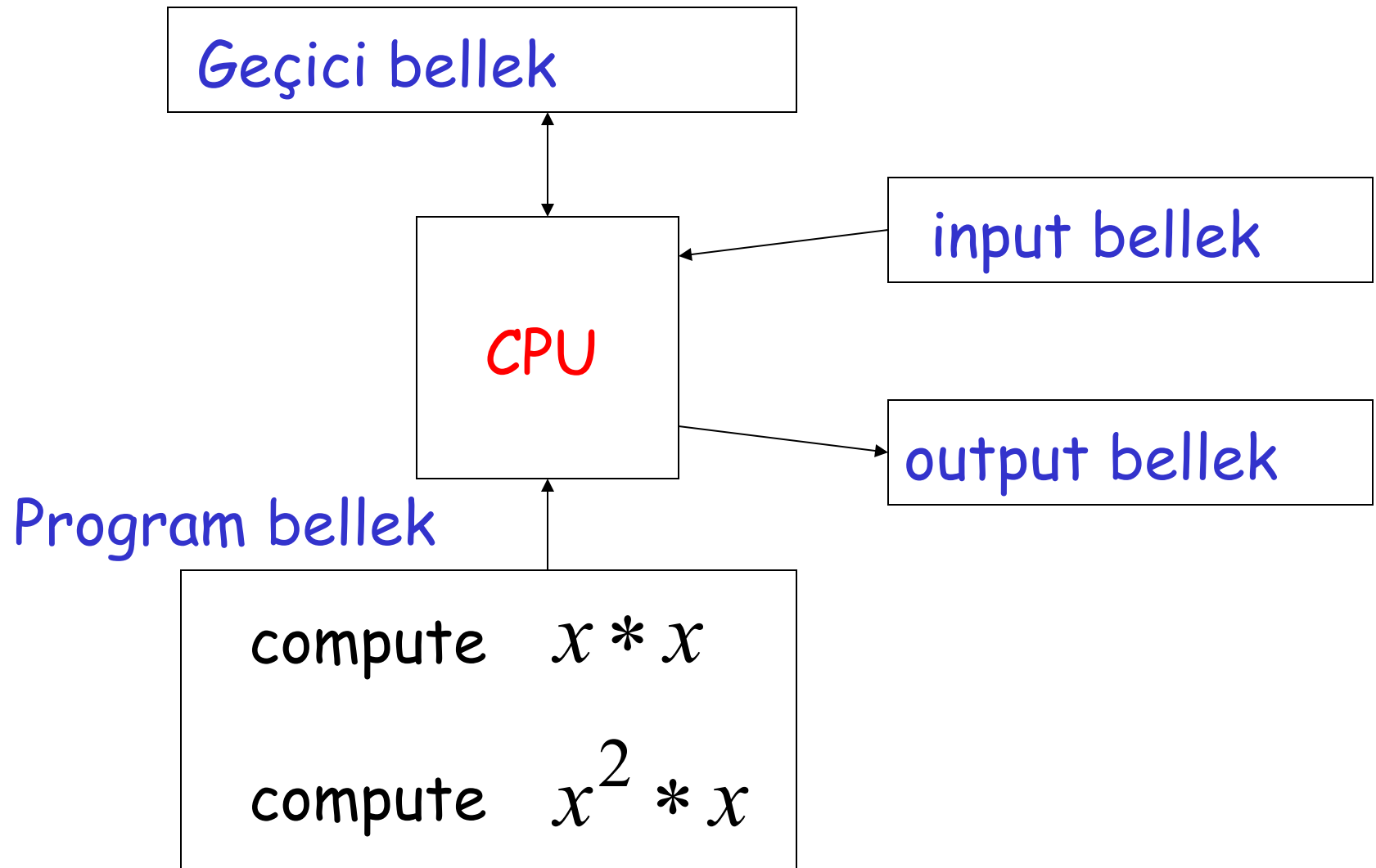
Hafta 1

Hesaplama

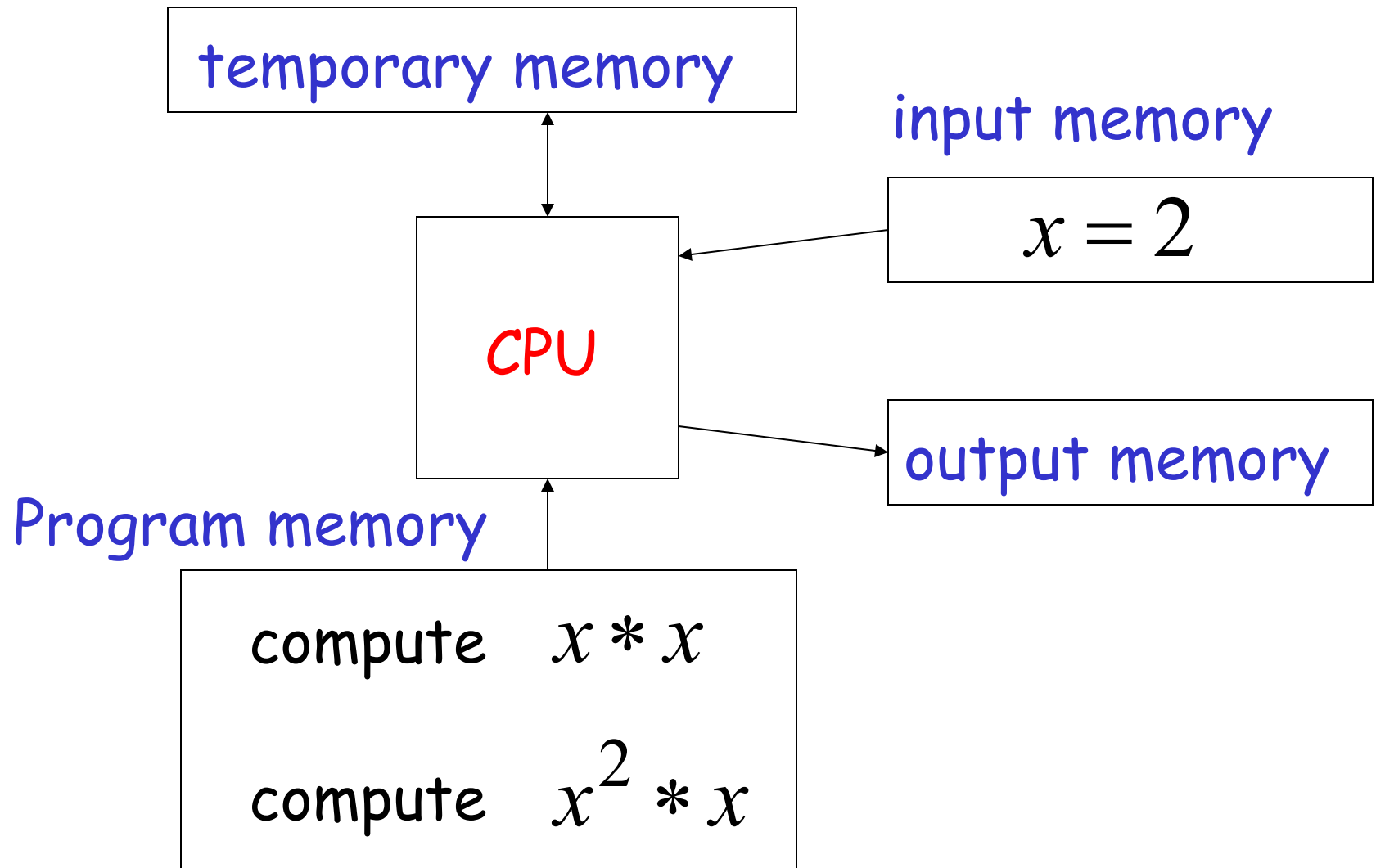




Örnek: $f(x) = x^3$



$$f(x) = x^3$$



temporary memory

$$z = 2 * 2 = 4$$

$$f(x) = z * 2 = 8$$

$$f(x) = x^3$$

input memory

$$x = 2$$

CPU

output memory

Program memory

compute $x * x$

compute $x^2 * x$

temporary memory

$$z = 2 * 2 = 4$$

$$f(x) = z * 2 = 8$$

$$f(x) = x^3$$

input memory

$$x = 2$$

CPU

$$f(x) = 8$$

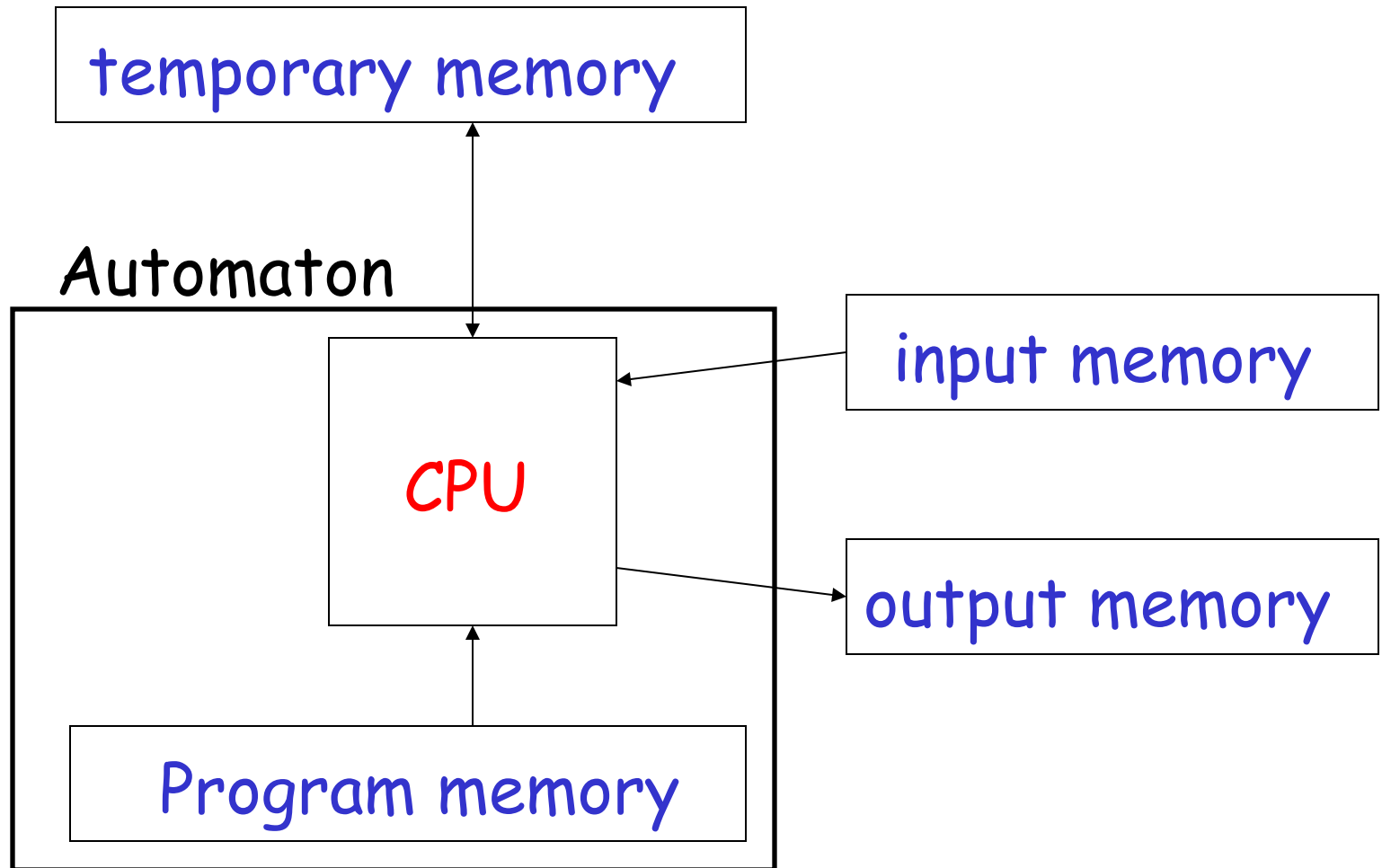
output memory

Program memory

compute $x * x$

compute $x^2 * x$

Otomat

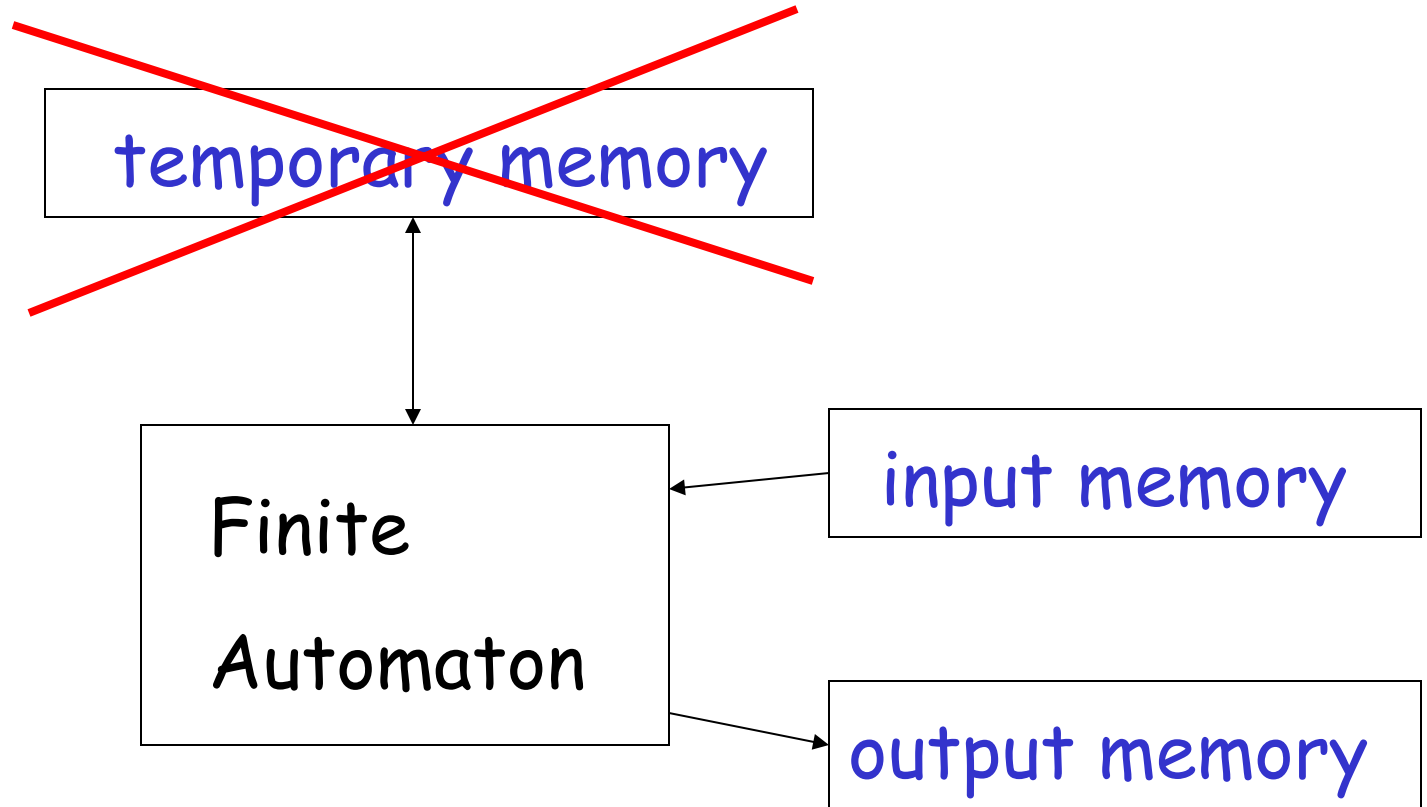


Farklı otomat çeşitleri

Otomatlar geçici bellek kullanımlarına göre ayırdedilir

- **Finite Automata:** bellekleri yoktur.
- **Pushdown Automata:** yığın
- **Turing Machines:** random access memory

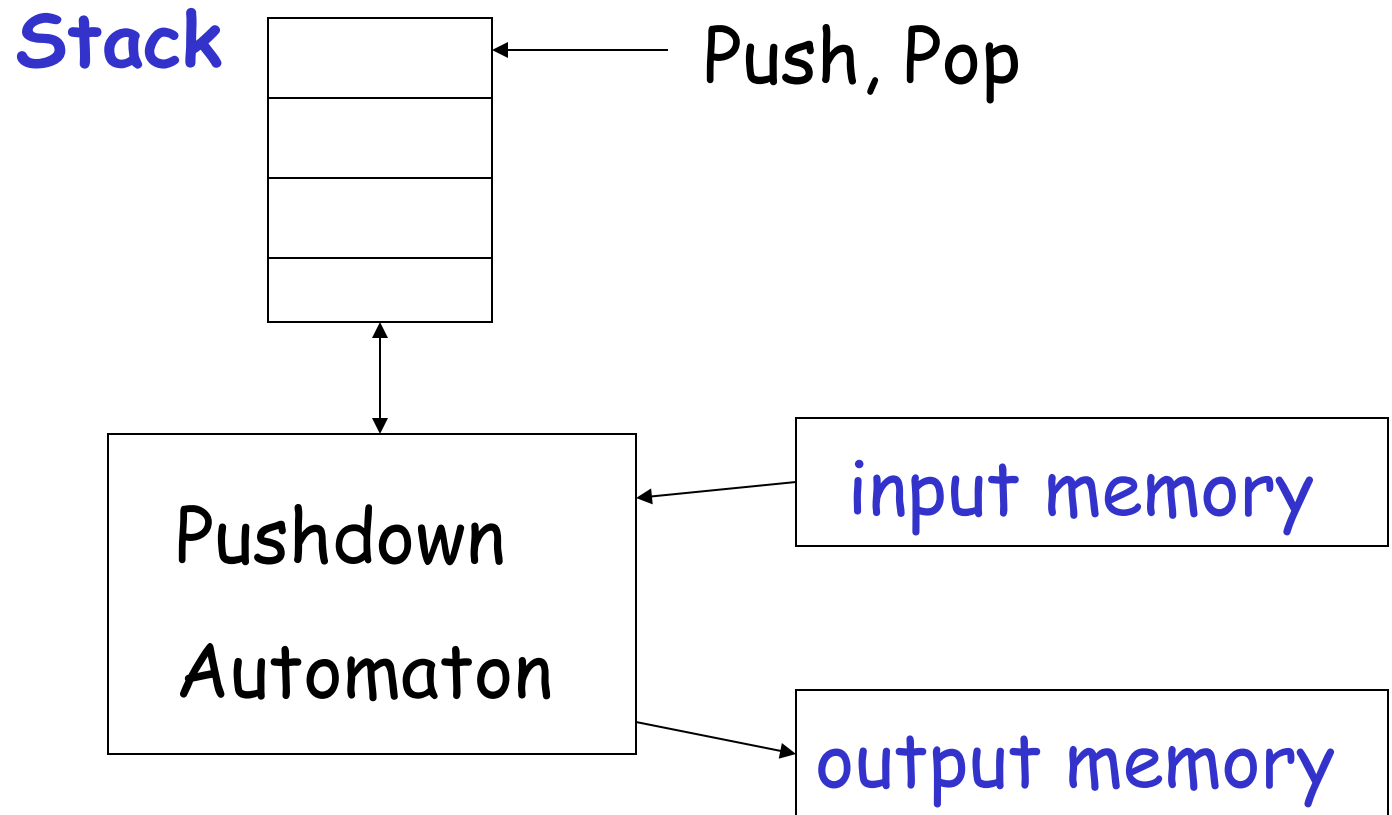
Finite Automaton



Örnek: Vending Machines

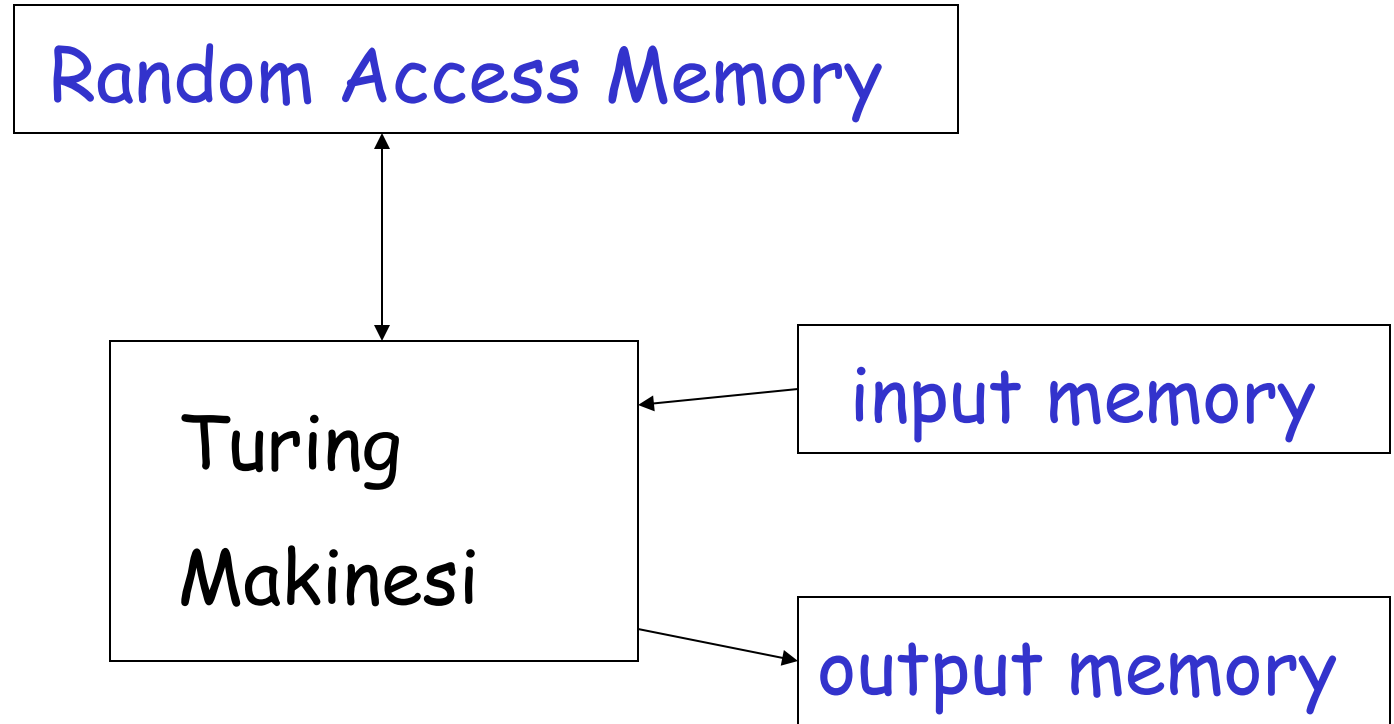
(Hesap gücü düşük)

Pushdown Automaton



Örnek: Programlama Dili derleyicileri
(Hesaplama Gücü orta)

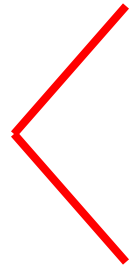
Turing Makinesi



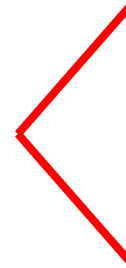
Örnek: Herhangi bir Algoritma
(Hesaplama gücü en yüksek)

Otomatların Güçleri

Finite
Automata



Pushdown
Automata



Turing
Makinesi

Az güç



Yüksek güç

Basit
problemler



Daha karmaşık
problemler

DİLLER

Dil: Karakter katarları kümesidir.

Katar (String): Bir alfabe üzerinde tanımlı Karakter (letter) dizisidir.

$$\Sigma = \{a, b, c, \dots, z\}$$

Examples: "for", "while", "toplam", ...

Alphabets and Strings

Alfabemiz $\Sigma = \{a, b\}$

Strings

a

ab

abba

baba

aaabbbbaabab

u = ab

v = bbbbaaa

w = abba

String işlemleri

$$w = a_1 a_2 \cdots a_n$$

abba

$$v = b_1 b_2 \cdots b_m$$

bbbbaaa

Concatenation

$$wv = a_1 a_2 \cdots a_n b_1 b_2 \cdots b_m$$

abbabbbbaaa

$$w = a_1 a_2 \cdots a_n$$

ababaaaabbb

Reverse

$$w^R = a_n \cdots a_2 a_1$$

bbbbaaababa

Katar uzunluğu

$$w = a_1 a_2 \cdots a_n$$

Uzunluk: $|w| = n$

Örnekler: $|abba| = 4$

$$|aa| = 2$$

$$|a| = 1$$

Bitiřtirme'nin uzunluęu

$$|uv| = |u| + |v|$$

Örnek:

$$u = aab, \quad |u| = 3$$

$$v = abaab, \quad |v| = 5$$

$$|uv| = |aababaab| = 8$$

$$|uv| = |u| + |v| = 3 + 5 = 8$$

Empty String

Sıfır karakterden oluşan katar:

$$\lambda$$

Observations: $|\lambda| = 0$

$$\lambda w = w \lambda = w$$

$$\lambda abba = abba \lambda = abba$$

Alt katar (Substring)

String

Substring

abbab

ab

abbab

abba

abbab

b

abbab

bbab

Prefix and Suffix

abbab

Prefixes

Suffixes

λ

abbab

a

bbab

ab

bab

abb

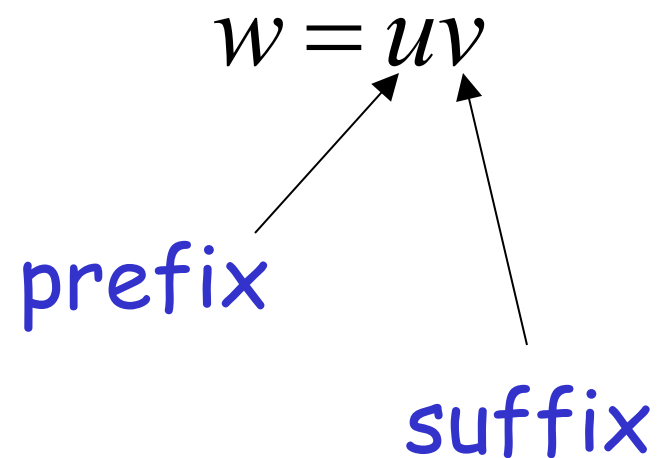
ab

abba

b

abbab

λ



Üs işlemi

$$w^n = \underbrace{ww \cdots w}_n$$

Örnek: $(abba)^2 = abbaabba$

Tanım: $w^0 = \lambda$

$$(abba)^0 = \lambda$$

* (kleene) işlemi

Σ^* : Σ üzerinde tanımlı Olası bütün katarlar kümesi

$$\Sigma = \{a, b\}$$

$$\Sigma^* = \{\lambda, a, b, aa, ab, ba, bb, aaa, aab, \dots\}$$

+ işlemi

Σ^+ : λ dışında, Σ üzerinde tanımlı olası bütün katarlar

$$\Sigma = \{a, b\}$$

$$\Sigma^* = \{\lambda, a, b, aa, ab, ba, bb, aaa, aab, \dots\}$$

$$\Sigma^+ = \Sigma^* - \lambda$$

$$\Sigma^+ = \{a, b, aa, ab, ba, bb, aaa, aab, \dots\}$$

Languages

Dil Σ^* 'in herhangi bir alt kümesi olarak tanımlanabilir.

Örnek:

$$\Sigma = \{a, b\}$$

$$\Sigma^* = \{\lambda, a, b, aa, ab, ba, bb, aaa, \dots\}$$

Diller:

$$\{\lambda\}$$

$$\{a, aa, aab\}$$

$$\{\lambda, abba, baba, aa, ab, aaaaaa\}$$

Dikkat

Sets

$$\emptyset = \{\} \neq \{\lambda\}$$

Set size

$$|\{\}| = |\emptyset| = 0$$

Set size

$$|\{\lambda\}| = 1$$

String length

$$|\lambda| = 0$$

Örnek

$$L = \{a^n b^n : n \geq 0\}$$

$$\left. \begin{array}{l} \lambda \\ ab \\ aabb \\ aaaaaabbbbb \end{array} \right\} \in L \qquad abb \notin L$$

Diller üzerinde işlemler

Genel Küme İşlemleri:

$$\{a, ab, aaaa\} \cup \{bb, ab\} = \{a, ab, bb, aaaa\}$$

$$\{a, ab, aaaa\} \cap \{bb, ab\} = \{ab\}$$

$$\{a, ab, aaaa\} - \{bb, ab\} = \{a, aaaa\}$$

Tümleyen: $\bar{L} = \Sigma^* - L$

$$\overline{\{a, ba\}} = \{\lambda, b, aa, ab, bb, aaaa, \dots\}$$

Reverse

Tanım: $L^R = \{w^R : w \in L\}$

Örnek: $\{ab, aab, baba\}^R = \{ba, baa, abab\}$

$$L = \{a^n b^n : n \geq 0\}$$

$$L^R = ?$$

Reverse

Tanım: $L^R = \{w^R : w \in L\}$

Örnek: $\{ab, aab, baba\}^R = \{ba, baa, abab\}$

$$L = \{a^n b^n : n \geq 0\}$$

$$L^R = \{b^n a^n : n \geq 0\}$$

Bitiştirme (Concatenation)

Tanım: $L_1L_2 = \{xy : x \in L_1, y \in L_2\}$

Örnek: $\{a, ab, ba\}\{b, aa\}$

$$= \{ab, aaa, abb, abaa, bab, baaa\}$$

Üs işlemi

Definition: $L^n = \underbrace{LL \cdots L}_n$

$$\{a,b\}^3 = \{a,b\}\{a,b\}\{a,b\} = \\ \{aaa, aab, aba, abb, baa, bab, bba, bbb\}$$

Özel Durum: $L^0 = \{\lambda\}$

$$\{a, bba, aaa\}^0 = \{\lambda\}$$

Örnek

$$L = \{a^n b^n : n \geq 0\}$$

$$L^2 = \{a^n b^n a^m b^m : n, m \geq 0\}$$

$$aabbbaaabb \in L^2$$

Yıldız Kapanma-Star-Closure (Kleene *)

Tanım: $L^* = L^0 \cup L^1 \cup L^2 \dots$

Örnek:

$$\{a, bb\}^* = \left\{ \begin{array}{l} \lambda, \\ a, bb, \\ aa, abb, bba, bbbb, \\ aaa, aabb, abba, abbbb, \dots \end{array} \right\}$$

Pozitif Kapnma-Positive Closure

Tanım:
$$L^+ = L^1 \cup L^2 \cup \dots$$
$$= L^* - \{\lambda\}$$

$$\{a, bb\}^+ = \left\{ \begin{array}{l} a, bb, \\ aa, abb, bba, bbbb, \\ aaa, aabb, abba, abbbb, \dots \end{array} \right\}$$