



# BSM307

## İşaretler ve Sistemler

Dr. Seçkin Arı

Fourier Seri Açılım Özellikleri

- Doğrusallık
- Zamanda Öteleme
- Zamanda
  - ♦ Ters Çevirme
  - ♦ Ölçekleme
  - ♦ Çarpma
  - ♦ Türev
  - ♦ İntegral

# Doğrusallık

- $\mathcal{FS}\{x(t)\} \rightarrow a_k$  ve  $\mathcal{FS}\{y(t)\} \rightarrow b_k$  biliniyorsa
  - ♦ Aynı T periyodu

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- $\mathcal{FS}\{x(t)\} \rightarrow a_k$  ve  $\mathcal{FS}\{y(t)\} \rightarrow b_k$  biliniyorsa
  - ♦ Aynı T periyodu
- $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$
- $y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\frac{2\pi}{T}t}$
- $z(t) = Ax(t) + By(t)$  ise
- $\mathcal{FS}\{z(t)\} \rightarrow$

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- $\mathcal{FS}\{z(t)\} \rightarrow c_k = Aa_k + Bb_k$  olur.
- $z(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\frac{2\pi}{T}t}$



# Örnek 1

- $\mathcal{FS}\{x(t)\} \rightarrow c_k = ?$ 
  - ♦  $x(t) = 2 \cos(2t) - \sin(2t)$

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- $\mathcal{FS}\{x(t)\} \rightarrow c_k = ?$ 
  - ♦  $x(t) = 2 \underbrace{\cos(2t)}_{a_k=?} - \underbrace{\sin(2t)}_{b_k=?}$
  - ♦  $\omega_0 =$



# Örnek 1

- $\mathcal{FS}\{x(t)\} \rightarrow c_k = ?$

- ♦  $x(t) = 2 \underbrace{\cos(2t)}_{a_{\pm 1} = \frac{1}{2}} - \underbrace{\sin(2t)}_{\substack{b_1 = \frac{1}{2j} \\ b_{-1} = -\frac{1}{2j}}}$

- ♦  $\omega_0 = 2 \text{ rad/s}$

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- ♦  $\omega_0 = 2 \text{ rad/s}$

- $c_k = 2a_k - b_k$

- ♦  $c_1 =$

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- ♦  $c_{-1} =$

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  - ♦  $\omega_0 = 2 \text{ rad/sn}$
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  - ♦  $c_{-1} = 2a_{-1} - b_{-1} = 1 + \frac{1}{2j}$
  - ♦  $k \neq \pm 1$  iken  $c_k = 0$

# Zamanda Öteleme

- $\mathcal{FS}\{x(t)\} \rightarrow a_k$  biliniyorsa
- $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$
- $\mathcal{FS}\{x(t - t_0)\} \rightarrow b_k = e^{-jk\frac{2\pi}{T}t_0} a_k$  olur.
- $x(t - t_0) = \sum_{k=-\infty}^{\infty} b_k e^{jk\frac{2\pi}{T}t}$



## Örnek 2

- $x(t) = \cos(2t)$ 
  - ♦  $a_{\pm 1} = \frac{1}{2}$
  - ♦  $k \neq \pm 1$  iken  $a_k = 0$
- $x_1(t) = \cos\left(2t - \frac{\pi}{4}\right)$  ise  $b_k = ?$

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- $x_1(t) = x(\quad)$

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- $x_1(t) = \cos\left(2t - \frac{\pi}{4}\right)$  ise  $b_k = ?$
- $x_1(t) = x\left(t - \frac{\pi}{8}\right)$ 
  - ♦  $b_k =$

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  - ♦  $b_k = e^{-jk2\frac{\pi}{8}} a_k$
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  - ♦  $b_1 = e^{-j\frac{\pi}{4}} a_1 = \frac{e^{-j\frac{\pi}{4}}}{2}$
  - ♦  $b_{-1} =$

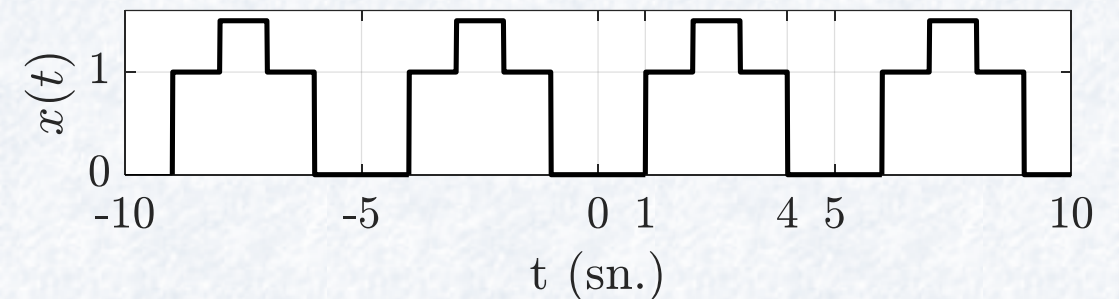
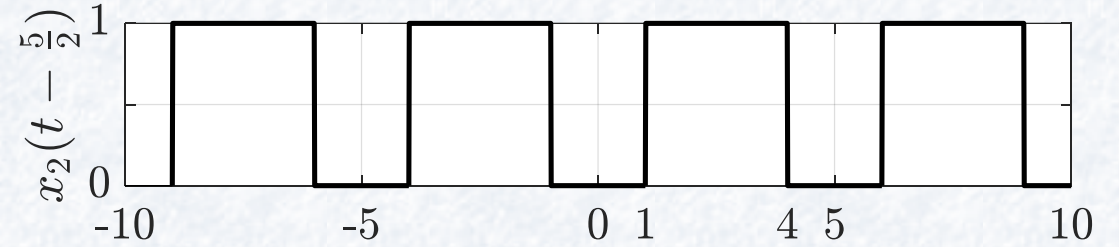
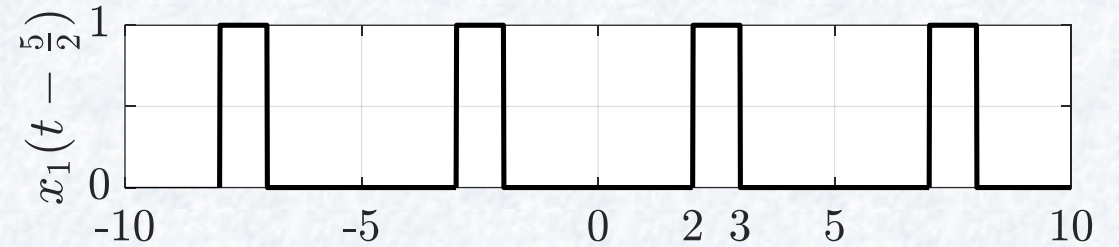
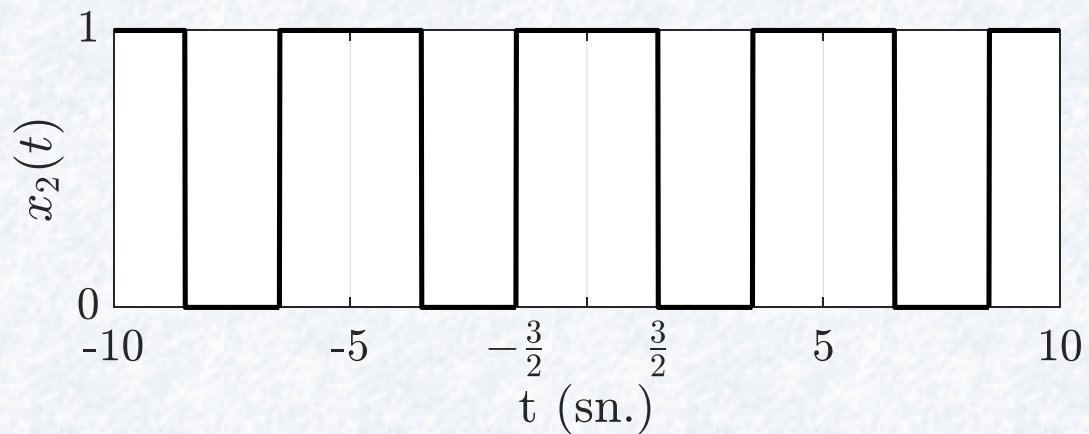
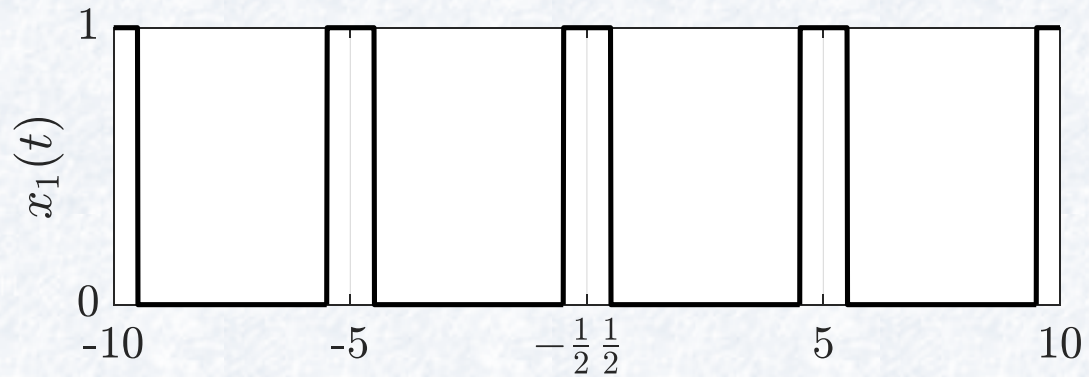
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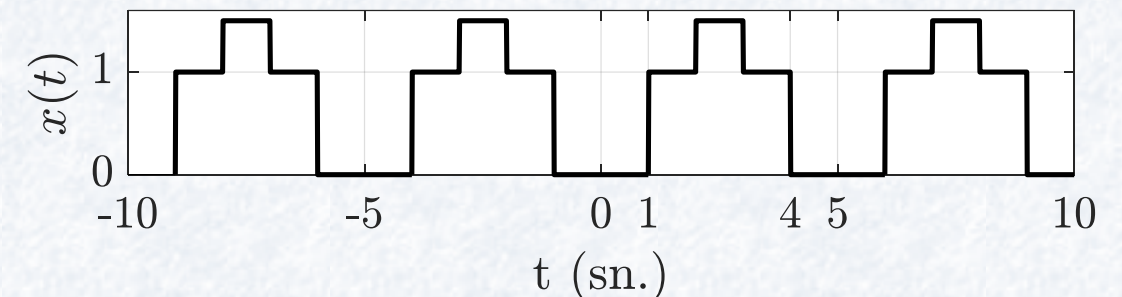
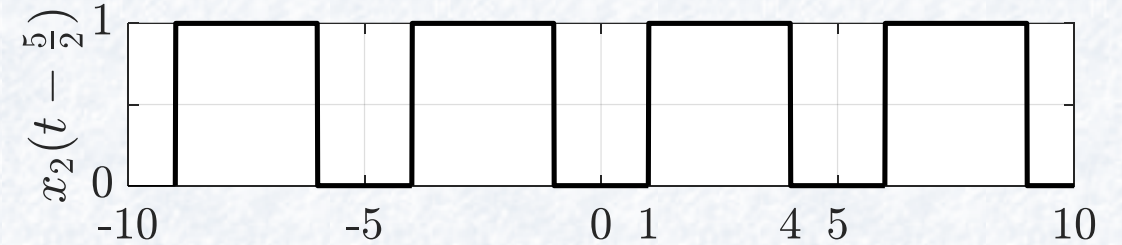
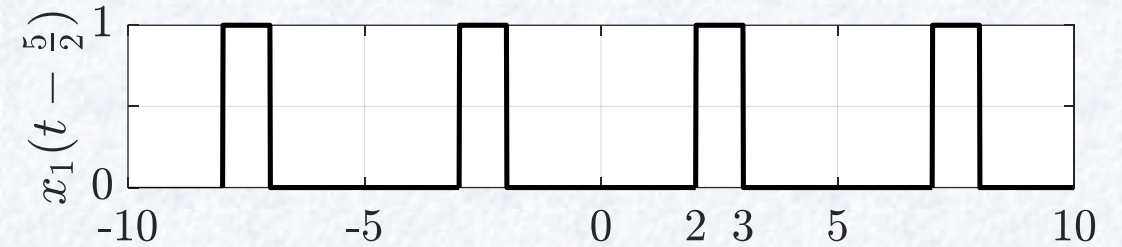
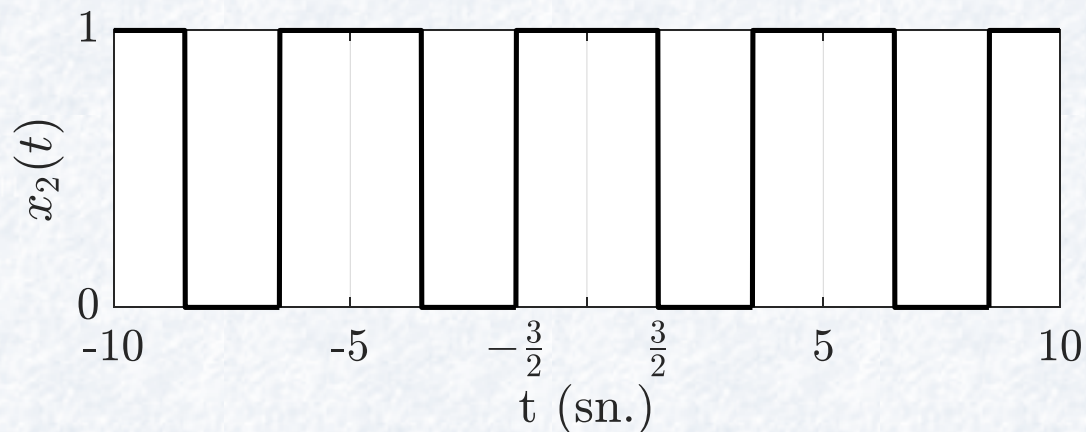
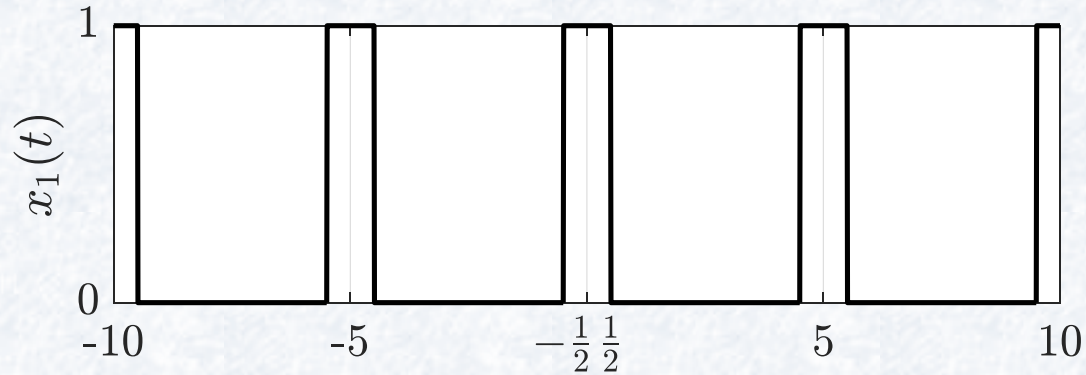
# Örnek 3

- $\mathcal{FS}\{x(t)\} \rightarrow e_k = ?$

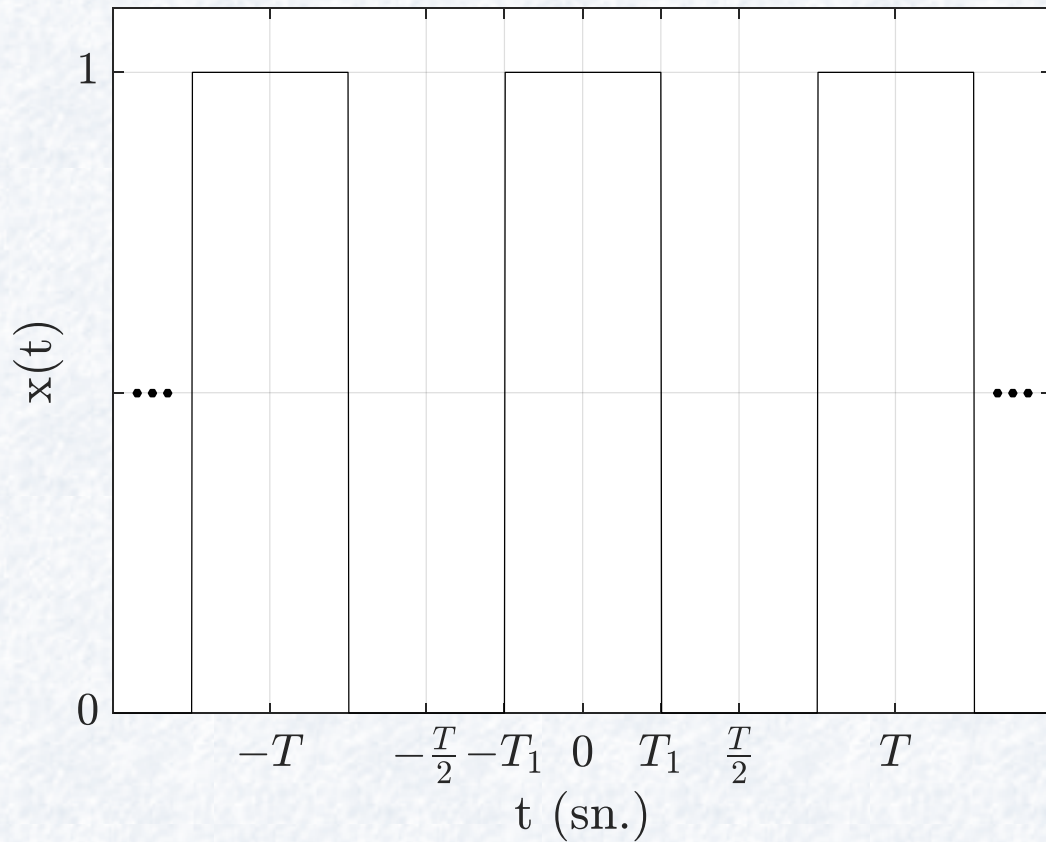


# Örnek 3

- $\mathcal{FS}\{x(t)\} \rightarrow e_k = ? \quad x(t) = \frac{1}{2}x_1\left(t - \frac{5}{2}\right) + x_2\left(t - \frac{5}{2}\right)$



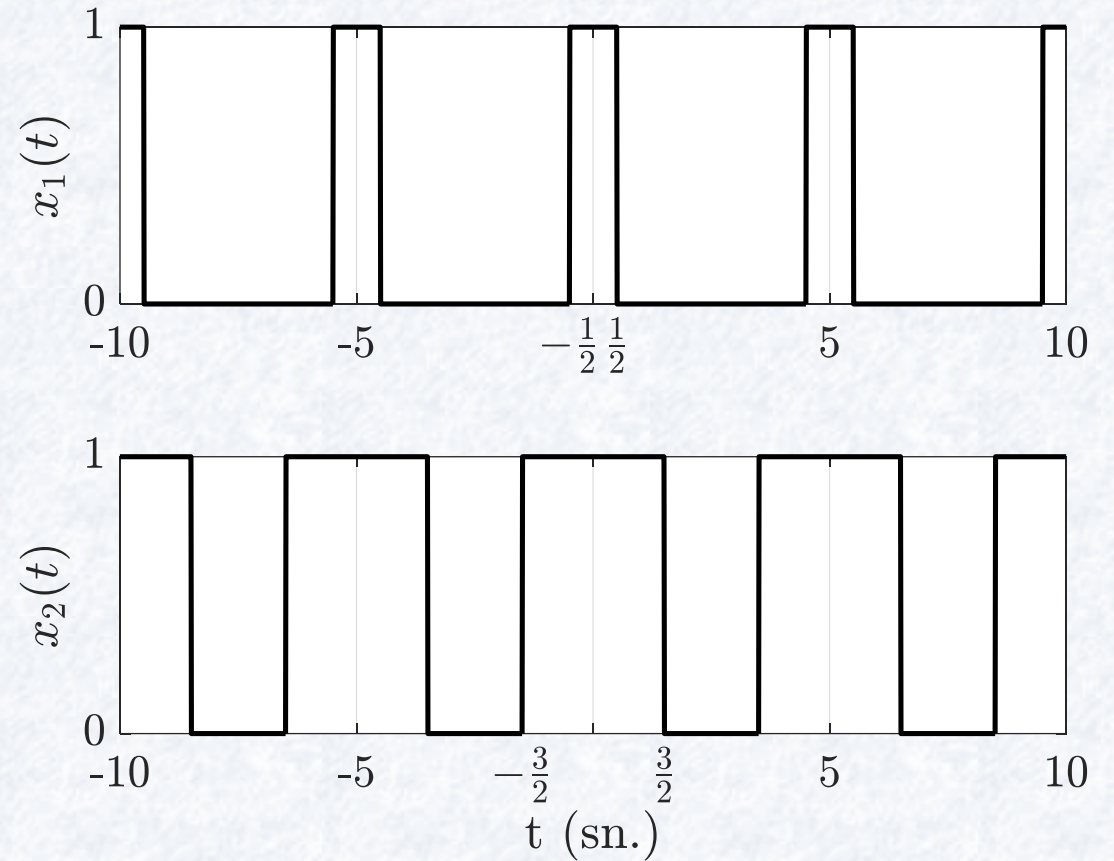
# Örnek 3



- $\omega_0 = \frac{2\pi}{T} \text{ rad/sn}$
- $a_k = \frac{\sin\left(k\frac{2\pi T_1}{T}\right)}{\pi k}$
- $a_0 = \frac{1}{T} \int_{-T_1}^{T_1} dt = \frac{1}{T} t \Big|_{-T_1}^{T_1} = \frac{2T_1}{T}$

# Örnek 3

- $\mathcal{FS}\{x_1(t)\} \rightarrow a_k = ?$
- $\mathcal{FS}\{x_2(t)\} \rightarrow b_k = ?$



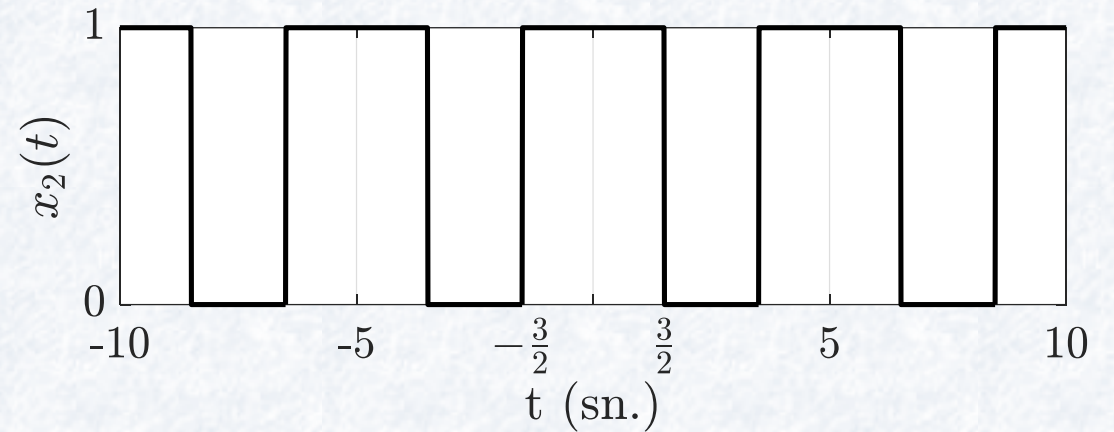
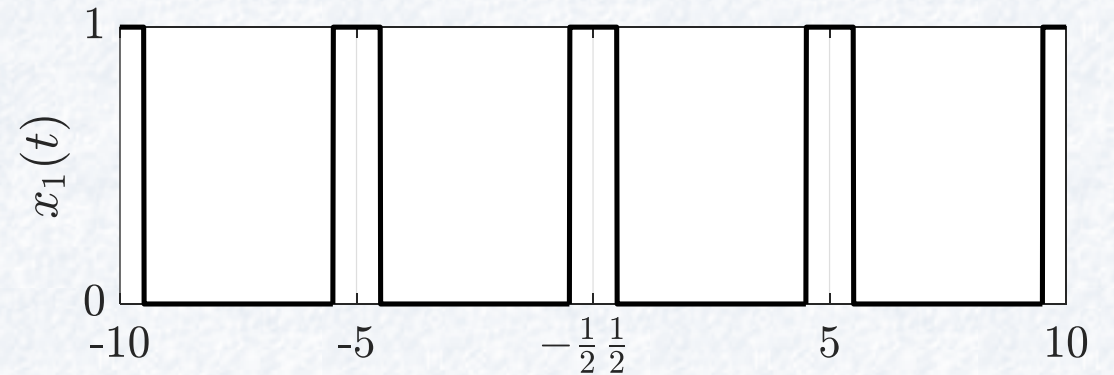
# Örnek 3

- $\mathcal{FS}\{x_1(t)\} \rightarrow a_k = \frac{\sin\left(k\frac{2\pi T_1}{T}\right)}{\pi k}$   
 $a_0 = \frac{2T_1}{T}$

♦  $T = \square, T_1 = \square$

- $\mathcal{FS}\{x_2(t)\} \rightarrow b_k = \frac{\sin\left(k\frac{2\pi T_1}{T}\right)}{\pi k}$   
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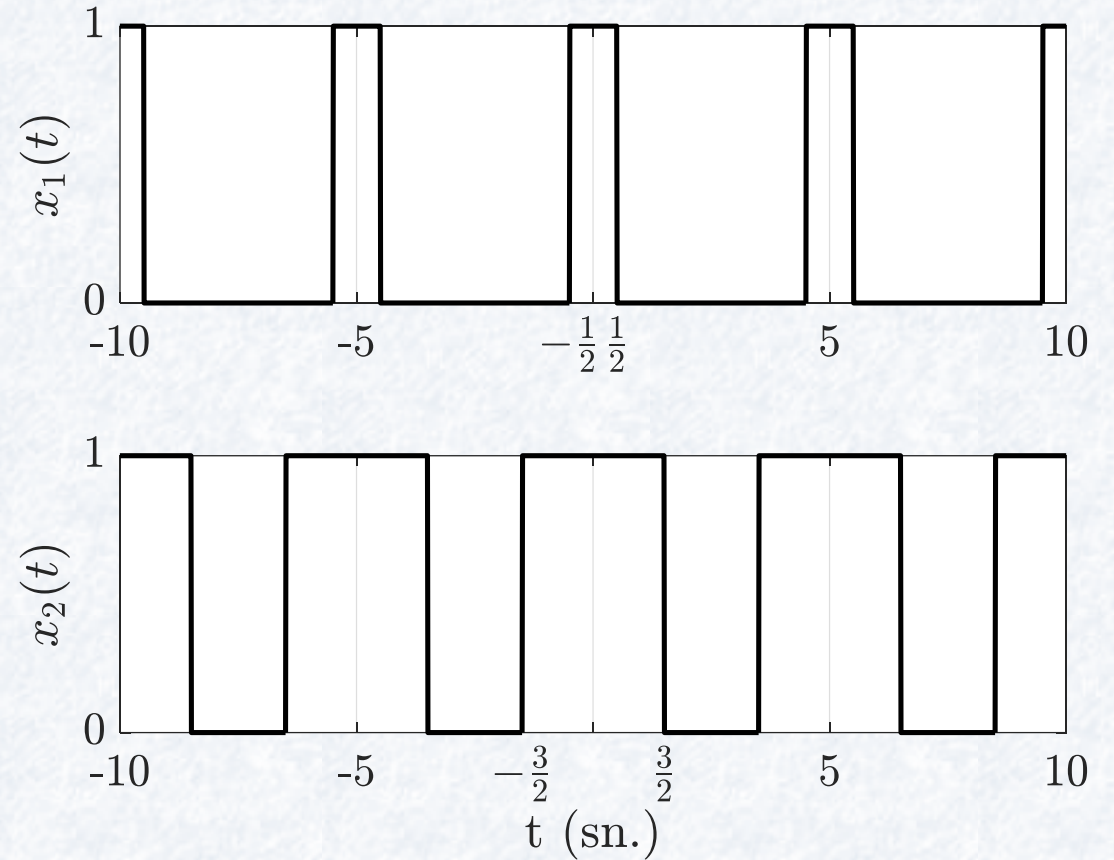
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- ♦  $T = 5, T_1 = \frac{1}{2}$

- $\mathcal{FS}\{x_2(t)\} \rightarrow b_k = \frac{\sin\left(k\frac{2\pi T_1}{T}\right)}{\pi k}$   
 $b_0 = \frac{2T_1}{T}$

- ♦  $T = 5, T_1 = \frac{3}{2}$





# Örnek 3

- $\mathcal{FS}\{x_1(t)\} \rightarrow a_k = \frac{\sin\left(k\frac{\pi}{5}\right)}{\pi k}$

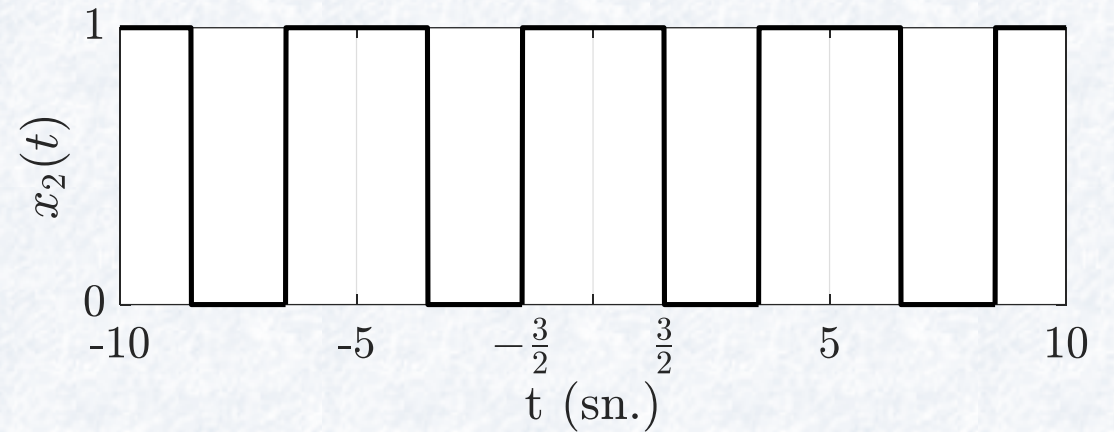
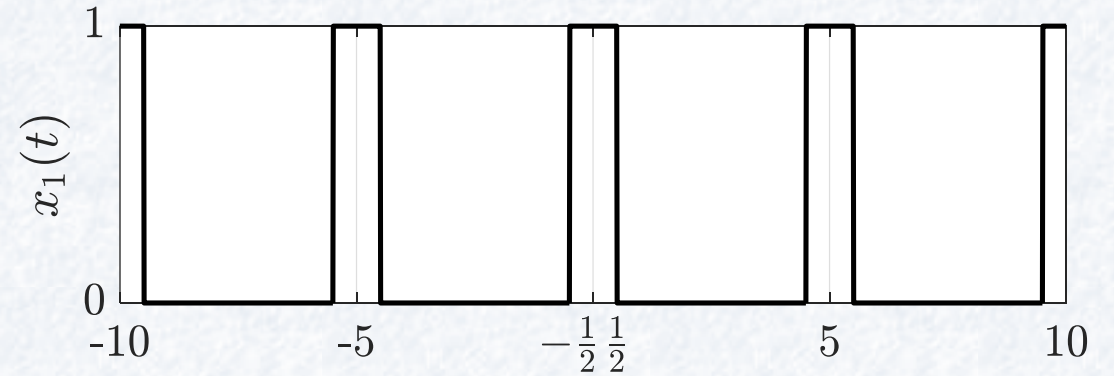
$$a_0 = \frac{1}{5}$$

- ♦  $T = 5, T_1 = \frac{1}{2}$

- $\mathcal{FS}\{x_2(t)\} \rightarrow b_k = \frac{\sin\left(k\frac{3\pi}{5}\right)}{\pi k}$

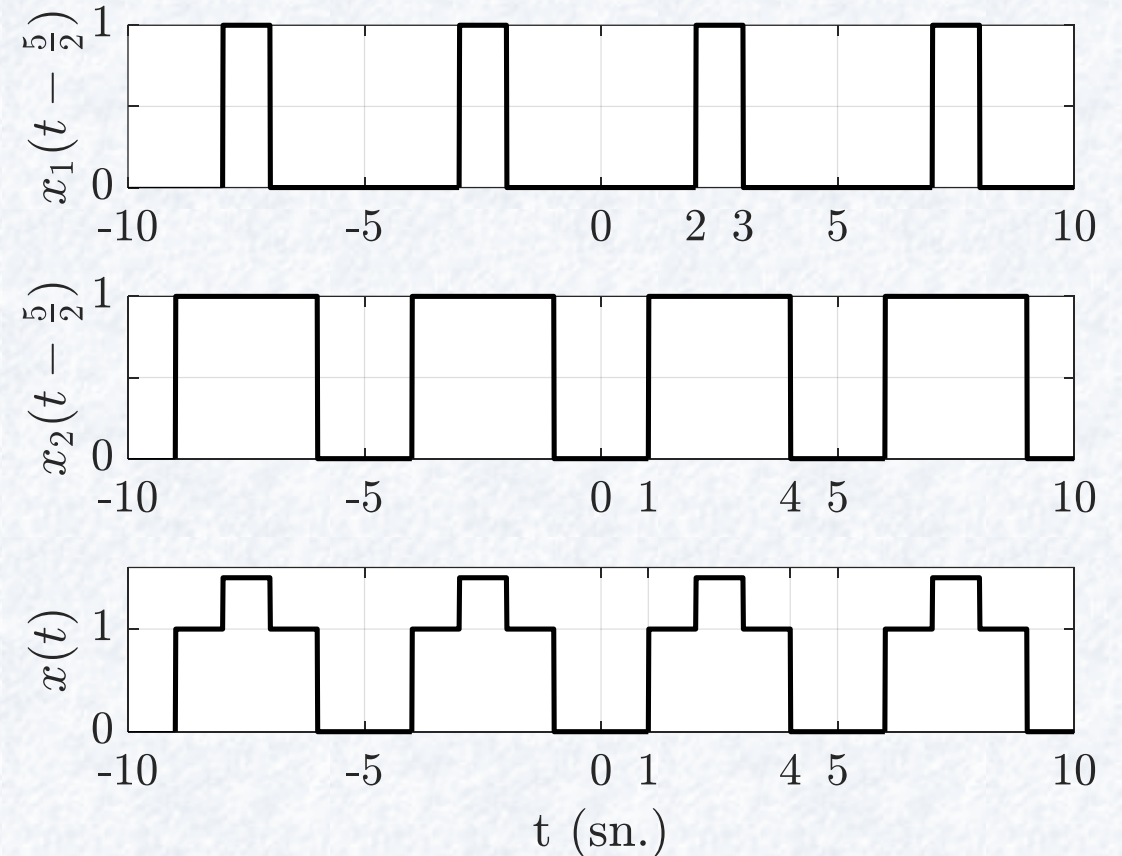
$$b_0 = \frac{3}{5}$$

- ♦  $T = 5, T_1 = \frac{3}{2}$



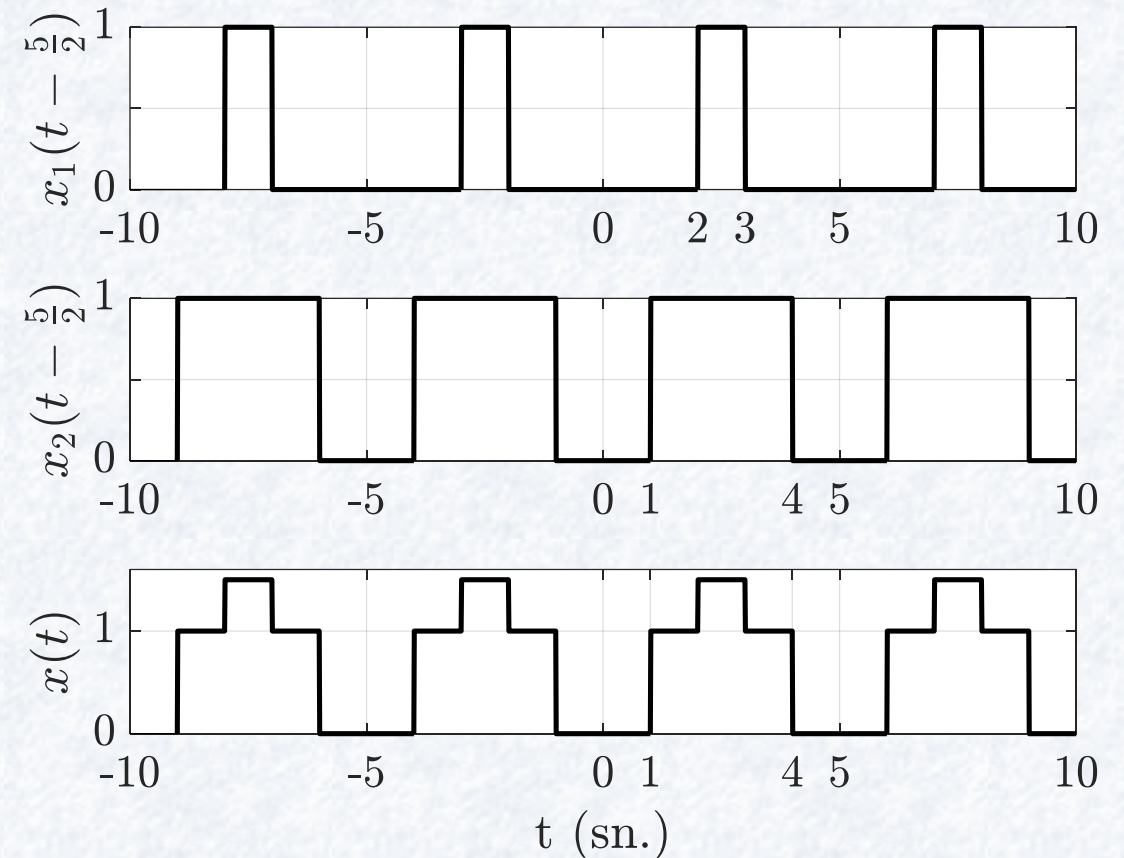
# Örnek 3

- $\mathcal{FS}\{x_1(t)\} \rightarrow a_k = \frac{\sin(k\frac{\pi}{5})}{\pi k}$   
 $a_0 = \frac{1}{5}$
- $\mathcal{FS}\left\{x_1\left(t - \frac{5}{2}\right)\right\} \rightarrow c_k = e^{-jk\pi} a_k$   
 $c_0 = a_0$
- $\mathcal{FS}\{x_2(t)\} \rightarrow b_k = \frac{\sin(k\frac{3\pi}{5})}{\pi k}$   
 $b_0 = \frac{3}{5}$
- $\mathcal{FS}\left\{x_2\left(t - \frac{5}{2}\right)\right\} \rightarrow d_k = e^{-jk\pi} b_k$   
 $d_0 = b_0$



# Örnek 3

- $\mathcal{FS}\{x_1(t)\} \rightarrow a_k = \frac{\sin(k\frac{\pi}{5})}{\pi k}$   
 $a_0 = \frac{1}{5}$
- $\mathcal{FS}\left\{x_1\left(t - \frac{5}{2}\right)\right\} \rightarrow c_k = (-1)^k a_k$   
 $c_0 = a_0$
- $\mathcal{FS}\{x_2(t)\} \rightarrow b_k = \frac{\sin(k\frac{3\pi}{5})}{\pi k}$   
 $b_0 = \frac{3}{5}$
- $\mathcal{FS}\left\{x_2\left(t - \frac{5}{2}\right)\right\} \rightarrow d_k = (-1)^k b_k$   
 $d_0 = b_0$



# Örnek 3

- $\mathcal{FS} \left\{ x_1 \left( t - \frac{5}{2} \right) \right\}$ 
  - ♦  $c_k = (-1)^k \frac{\sin\left(k\frac{\pi}{5}\right)}{\pi k}$
  - ♦  $c_0 = \frac{1}{5}$
- $\mathcal{FS} \left\{ x_2 \left( t - \frac{5}{2} \right) \right\}$ 
  - ♦  $d_k = (-1)^k \frac{\sin\left(k\frac{3\pi}{5}\right)}{\pi k}$
  - ♦  $d_0 = \frac{3}{5}$
- $x(t) = \frac{1}{2} x_1 \left( t - \frac{5}{2} \right) + x_2 \left( t - \frac{5}{2} \right)$
- $e_k =$
- $e_0 =$

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- $x(t) = \frac{1}{2} x_1 \left( t - \frac{5}{2} \right) + x_2 \left( t - \frac{5}{2} \right)$
- $e_k = \frac{1}{2} c_k + d_k$
- $e_0 = \frac{1}{2} c_0 + d_0$

# Örnek 3

- $\mathcal{FS} \left\{ x_1 \left( t - \frac{5}{2} \right) \right\}$

- ♦  $c_k = (-1)^k \frac{\sin\left(k\frac{\pi}{5}\right)}{\pi k}$

- ♦  $c_0 = \frac{1}{5}$

- $\mathcal{FS} \left\{ x_2 \left( t - \frac{5}{2} \right) \right\}$

- ♦  $d_k = (-1)^k \frac{\sin\left(k\frac{3\pi}{5}\right)}{\pi k}$

- ♦  $d_0 = \frac{3}{5}$

- $x(t) = \frac{1}{2} x_1 \left( t - \frac{5}{2} \right) + x_2 \left( t - \frac{5}{2} \right)$

- $e_k = \frac{1}{2} c_k + d_k$

- $e_k = (-1)^k \left( \frac{1}{2} \frac{\sin\left(k\frac{\pi}{5}\right)}{\pi k} + \frac{\sin\left(k\frac{3\pi}{5}\right)}{\pi k} \right)$

- $e_0 = \frac{1}{2} c_0 + d_0$

- $e_0 = \frac{1}{2} \frac{1}{5} + \frac{3}{5} = \frac{7}{10}$



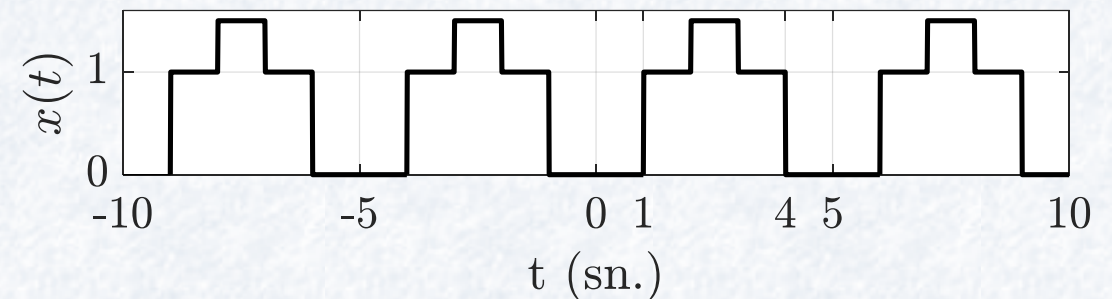
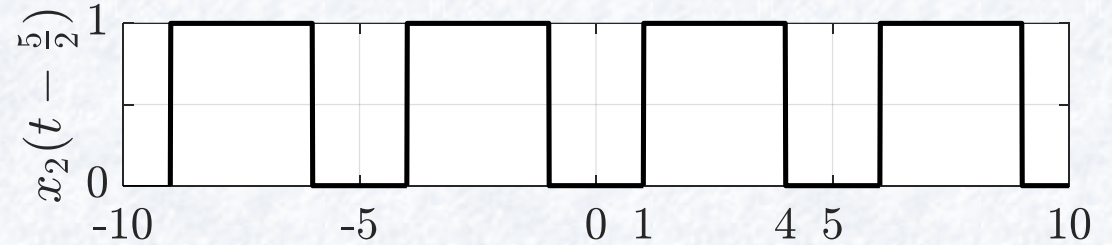
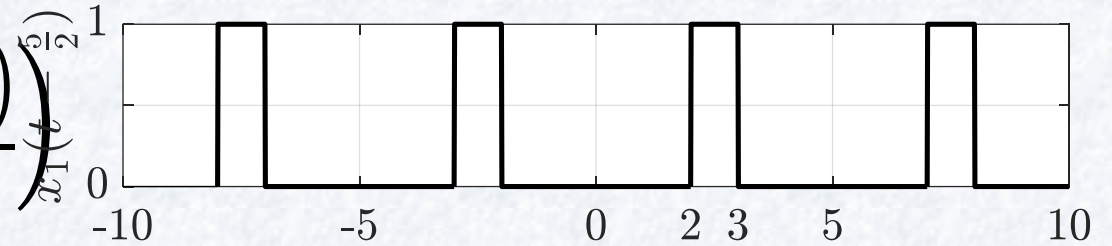
# Örnek 3

- $x(t) = \frac{1}{2}x_1\left(t - \frac{5}{2}\right) + x_2\left(t - \frac{5}{2}\right)$

- $e_k = (-1)^k \left( \frac{1}{2} \frac{\sin\left(k\frac{\pi}{5}\right)}{\pi k} + \frac{\sin\left(k\frac{3\pi}{5}\right)}{\pi k} \right)$

- $e_0 = \frac{7}{10}$

- $x(t) = \sum_{k=-\infty}^{\infty} e_k e^{jk\frac{2\pi}{5}t}$



# Zamanda Ters Çevirme

- $\mathcal{FS}\{x(t)\} \rightarrow a_k$  biliniyorsa
- $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$
- $\mathcal{FS}\{x(-t)\} \rightarrow b_k = a_{-k}$  olur.
- $x(-t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\frac{2\pi}{T}t}$

# Örnek 4

- $x(t) = \cos(2t)$ 
  - ♦  $a_{\pm 1} = \frac{1}{2}$
  - ♦  $k \neq \pm 1$  iken  $a_k = 0$
- $x_1(t) = \cos\left(2t - \frac{\pi}{4}\right)$ 
  - ♦  $b_1 = e^{-j\frac{\pi}{4}}a_1 = \frac{e^{-j\frac{\pi}{4}}}{2}$
  - ♦  $b_{-1} = e^{j\frac{\pi}{4}}a_{-1} = \frac{e^{j\frac{\pi}{4}}}{2}$
  - ♦  $k \neq \pm 1$  iken  $b_k = 0$
- $x_2(t) = \cos\left(-2t - \frac{\pi}{4}\right)$  ise  $c_k = ?$

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  - ♦  $k \neq \pm 1$  iken  $b_k = 0$
- $x_2(t) = \cos\left(-2t - \frac{\pi}{4}\right)$  ise  $c_k = ?$
- $x_2(t) = x_1(-t)$ 
  - ♦  $c_k =$

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  - ♦  $c_k = b_{-k}$
  - ♦  $c_1 =$

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- $x_1(t) = \cos\left(2t - \frac{\pi}{4}\right)$ 
  - ♦  $b_1 = e^{-j\frac{\pi}{4}}a_1 = \frac{e^{-j\frac{\pi}{4}}}{2}$
  - ♦  $b_{-1} = e^{j\frac{\pi}{4}}a_{-1} = \frac{e^{j\frac{\pi}{4}}}{2}$
  - ♦  $k \neq \pm 1$  iken  $b_k = 0$
- $x_2(t) = \cos\left(-2t - \frac{\pi}{4}\right)$  ise  $c_k = ?$
- $x_2(t) = x_1(-t)$ 
  - ♦  $c_k = b_{-k}$
  - ♦  $c_1 = b_{-1} = \frac{e^{j\frac{\pi}{4}}}{2}$
  - ♦  $c_{-1} =$



# Örnek 4

- $x(t) = \cos(2t)$ 
  - ♦  $a_{\pm 1} = \frac{1}{2}$
  - ♦  $k \neq \pm 1$  iken  $a_k = 0$
- $x_1(t) = \cos\left(2t - \frac{\pi}{4}\right)$ 
  - ♦  $b_1 = e^{-j\frac{\pi}{4}}a_1 = \frac{e^{-j\frac{\pi}{4}}}{2}$
  - ♦  $b_{-1} = e^{j\frac{\pi}{4}}a_{-1} = \frac{e^{j\frac{\pi}{4}}}{2}$
  - ♦  $k \neq \pm 1$  iken  $b_k = 0$
- $x_2(t) = \cos\left(-2t - \frac{\pi}{4}\right)$  ise  $c_k = ?$
- $x_2(t) = x_1(-t)$ 
  - ♦  $c_k = b_{-k}$
  - ♦  $c_1 = b_{-1} = \frac{e^{j\frac{\pi}{4}}}{2}$
  - ♦  $c_{-1} = b_1 = \frac{e^{-j\frac{\pi}{4}}}{2}$
  - ♦  $k \neq \pm 1$  iken  $c_k = 0$

# Zamanda Ölçekleme

- $\mathcal{FS}\{x(t)\} \rightarrow a_k$  biliniyorsa
- $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$
- $\mathcal{FS}\{x(\beta t)\} \rightarrow b_k = a_k$  olur.
  - ♦  $T_{yeni} = \frac{T}{\beta}$
  - ♦  $\omega_{0,yeni} = \beta \omega_0$
- $x(\beta t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\beta\frac{2\pi}{T}t}$

## Örnek 5

- $x_2(t) = \cos\left(-2t - \frac{\pi}{4}\right)$ 
  - ♦  $c_1 = b_{-1} = \frac{e^{j\frac{\pi}{4}}}{2}$
  - ♦  $c_{-1} = b_1 = \frac{e^{-j\frac{\pi}{4}}}{2}$
  - ♦  $k \neq \pm 1$  iken  $c_k = 0$
- $x_3(t) = \cos\left(-t - \frac{\pi}{4}\right)$  ise  $d_k = ?$

## Örnek 5

- $x_2(t) = \cos\left(-2t - \frac{\pi}{4}\right)$ 
  - ♦  $c_1 = b_{-1} = \frac{e^{j\frac{\pi}{4}}}{2}$
  - ♦  $c_{-1} = b_1 = \frac{e^{-j\frac{\pi}{4}}}{2}$
  - ♦  $k \neq \pm 1$  iken  $c_k = 0$
- $x_3(t) = \cos\left(-t - \frac{\pi}{4}\right)$  ise  $d_k = ?$
- $x_3(t) = x_2(\quad)$

## Örnek 5

- $x_2(t) = \cos\left(-2t - \frac{\pi}{4}\right)$ 
  - ♦  $c_1 = b_{-1} = \frac{e^{j\frac{\pi}{4}}}{2}$
  - ♦  $c_{-1} = b_1 = \frac{e^{-j\frac{\pi}{4}}}{2}$
  - ♦  $k \neq \pm 1$  iken  $c_k = 0$
- $x_3(t) = \cos\left(-t - \frac{\pi}{4}\right)$  ise  $d_k = ?$
- $x_3(t) = x_2\left(\frac{t}{2}\right)$ 
  - ♦  $d_k = c_k$
  - ♦  $T_3 =$

## Örnek 5

- $x_2(t) = \cos\left(-2t - \frac{\pi}{4}\right)$ 
  - ♦  $c_1 = b_{-1} = \frac{e^{j\frac{\pi}{4}}}{2}$
  - ♦  $c_{-1} = b_1 = \frac{e^{-j\frac{\pi}{4}}}{2}$
  - ♦  $k \neq \pm 1$  iken  $c_k = 0$
- $x_3(t) = \cos\left(-t - \frac{\pi}{4}\right)$  ise  $d_k = ?$
- $x_3(t) = x_2\left(\frac{t}{2}\right)$ 
  - ♦  $d_k = c_k$
  - ♦  $T_3 = 2T = 2\pi$  sn
  - ♦  $\omega_{0_3} = 1$  rad/sn



# Zamanda Çarpma

- $\mathcal{FS}\{x(t)\} \rightarrow a_k$  ve  $\mathcal{FS}\{y(t)\} \rightarrow b_k$  biliniyorsa
  - ♦ Aynı T periyodu
- $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$
- $y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\frac{2\pi}{T}t}$
- $z(t) = x(t)y(t)$  ise
- $\mathcal{FS}\{z(t)\} \rightarrow c_k = \sum_{l=-\infty}^{\infty} a_l b_{k-l}$  olur.
- $z(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\frac{2\pi}{T}t}$

## Örnek 6

- $x(t) = \cos(2t)$ 
  - ♦  $a_{\pm 1} = \frac{1}{2}$
- $y(t) = \sin(2t)$ 
  - ♦  $b_{\pm 1} = \pm \frac{1}{2j}$
- $\mathcal{FS}\{z(t) = x(t)y(t)\} \rightarrow c_k = ?$

## Örnek 6

- $x(t) = \cos(2t)$ 
  - ♦  $a_{\pm 1} = \frac{1}{2}$
- $y(t) = \sin(2t)$ 
  - ♦  $b_{\pm 1} = \pm \frac{1}{2j}$
- $\mathcal{FS}\{z(t) = x(t)y(t)\} \rightarrow c_k = \sum_{l=-\infty}^{\infty} a_l b_{k-l}$

## Örnek 6

- $x(t) = \cos(2t)$ 
  - ♦  $a_{\pm 1} = \frac{1}{2}$
- $y(t) = \sin(2t)$ 
  - ♦  $b_{\pm 1} = \pm \frac{1}{2j}$
- $\mathcal{FS}\{z(t) = x(t)y(t)\} \rightarrow c_k = \sum_{l=-\infty}^{\infty} a_l b_{k-l}$ 
  - ♦  $c_{-2} =$

## Örnek 6

- $x(t) = \cos(2t)$ 
  - ♦  $a_{\pm 1} = \frac{1}{2}$
- $y(t) = \sin(2t)$ 
  - ♦  $b_{\pm 1} = \pm \frac{1}{2j}$
- $\mathcal{FS}\{z(t) = x(t)y(t)\} \rightarrow c_k = \sum_{l=-\infty}^{\infty} a_l b_{k-l}$ 
  - ♦  $c_{-2} = a_{-1}b_{-1} + a_1b_{-3} = -\frac{1}{4j}$
  - ♦  $c_0 =$

# Örnek 6

- $x(t) = \cos(2t)$ 
  - ♦  $a_{\pm 1} = \frac{1}{2}$
- $y(t) = \sin(2t)$ 
  - ♦  $b_{\pm 1} = \pm \frac{1}{2j}$
- $\mathcal{FS}\{z(t) = x(t)y(t)\} \rightarrow c_k = \sum_{l=-\infty}^{\infty} a_l b_{k-l}$ 
  - ♦  $c_{-2} = a_{-1}b_{-1} + a_1b_{-3} = -\frac{1}{4j}$
  - ♦  $c_0 = a_{-1}b_1 + a_1b_{-1} = 0$
  - ♦  $c_2 =$



# Örnek 6

- $x(t) = \cos(2t)$ 
  - ♦  $a_{\pm 1} = \frac{1}{2}$
- $y(t) = \sin(2t)$ 
  - ♦  $b_{\pm 1} = \pm \frac{1}{2j}$
- $\mathcal{FS}\{z(t) = x(t)y(t)\} \rightarrow c_k = \sum_{l=-\infty}^{\infty} a_l b_{k-l}$ 
  - ♦  $c_{-2} = a_{-1}b_{-1} + a_1b_{-3} = -\frac{1}{4j}$
  - ♦  $c_0 = a_{-1}b_1 + a_1b_{-1} = 0$
  - ♦  $c_2 = a_{-1}b_3 + a_1b_1 = \frac{1}{4j}$

# Zamanda Türev

- $\mathcal{FS}\{x(t)\} \rightarrow a_k$  biliniyorsa
- $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$
- $\mathcal{FS}\left\{\frac{\partial x(t)}{\partial t}\right\} \rightarrow b_k = jk\omega_0 a_k$  olur.
- $\frac{\partial x(t)}{\partial t} = \sum_{k=-\infty}^{\infty} b_k e^{jk\frac{2\pi}{T}t}$

# Örnek 7

- $x(t) = \cos(2t)$ 
  - ♦  $a_{\pm 1} = \frac{1}{2}$
- $\mathcal{FS}\{x_4(t) = \sin(2t)\} \rightarrow f_k =$

# Örnek 7

- $x(t) = \cos(2t)$ 
  - ♦  $a_{\pm 1} = \frac{1}{2}$
- $\mathcal{FS}\{x_4(t) = \sin(2t)\} \rightarrow f_k =$
- $\mathcal{FS}\left\{\frac{\partial x(t)}{\partial t} = -2 \sin(2t)\right\} \rightarrow$

# Örnek 7

- $x(t) = \cos(2t)$ 
  - ♦  $a_{\pm 1} = \frac{1}{2}$
- $\mathcal{FS}\{x_4(t) = \sin(2t)\} \rightarrow f_k =$
- $\mathcal{FS}\left\{\frac{\partial x(t)}{\partial t} = -2 \sin(2t)\right\} \rightarrow jk2a_k$

# Örnek 7

- $x(t) = \cos(2t)$ 
  - ♦  $a_{\pm 1} = \frac{1}{2}$
- $\mathcal{FS}\{x_4(t) = \sin(2t)\} \rightarrow f_k =$
- $\mathcal{FS}\left\{\frac{\partial x(t)}{\partial t} = -2 \sin(2t)\right\} \rightarrow jk2a_k$
- $\mathcal{FS}\left\{\frac{\partial x(t)}{-2\partial t} = \sin(2t)\right\} \rightarrow f_k =$



# Örnek 7

- $x(t) = \cos(2t)$ 
  - ♦  $a_{\pm 1} = \frac{1}{2}$
- $\mathcal{FS}\{x_4(t) = \sin(2t)\} \rightarrow f_k =$
- $\mathcal{FS}\left\{\frac{\partial x(t)}{\partial t} = -2 \sin(2t)\right\} \rightarrow jk2a_k$
- $\mathcal{FS}\left\{\frac{\partial x(t)}{-2\partial t} = \sin(2t)\right\} \rightarrow f_k = \frac{jk2a_k}{-2}$ 
  - ♦  $f_1 =$

# Örnek 7

- $x(t) = \cos(2t)$ 
  - ♦  $a_{\pm 1} = \frac{1}{2}$
- $\mathcal{FS}\{x_4(t) = \sin(2t)\} \rightarrow f_k =$
- $\mathcal{FS}\left\{\frac{\partial x(t)}{\partial t} = -2 \sin(2t)\right\} \rightarrow jk2a_k$
- $\mathcal{FS}\left\{\frac{\partial x(t)}{-2\partial t} = \sin(2t)\right\} \rightarrow f_k = \frac{jk2a_k}{-2}$ 
  - ♦  $f_1 = -ja_1 = -\frac{j}{2}$
  - ♦  $f_{-1} =$

# Örnek 7

- $x(t) = \cos(2t)$ 
  - ♦  $a_{\pm 1} = \frac{1}{2}$
- $\mathcal{FS}\{x_4(t) = \sin(2t)\} \rightarrow f_k =$
- $\mathcal{FS}\left\{\frac{\partial x(t)}{\partial t} = -2 \sin(2t)\right\} \rightarrow jk2a_k$
- $\mathcal{FS}\left\{\frac{\partial x(t)}{-2\partial t} = \sin(2t)\right\} \rightarrow f_k = \frac{jk2a_k}{-2}$ 
  - ♦  $f_1 = -ja_1 = -\frac{j}{2}$
  - ♦  $f_{-1} = ja_{-1} = \frac{j}{2}$

# Zamanda İntegral

- $\mathcal{FS}\{x(t)\} \rightarrow a_k$  biliniyorsa
- $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$
- $\mathcal{FS}\{\int x(t)\} \rightarrow b_k = \frac{a_k}{jk\omega_0}$  olur.
- $\int x(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\frac{2\pi}{T}t}$

# Örnek 7

- $x(t) = \cos(2t)$ 
  - ♦  $a_{\pm 1} = \frac{1}{2}$
- $\mathcal{FS}\{x_5(t) = \sin(2t)\} \rightarrow g_k =$
- $\mathcal{FS}\left\{\int x(t) = \frac{\sin(2t)}{2}\right\} \rightarrow$

# Örnek 7

- $x(t) = \cos(2t)$ 
  - ♦  $a_{\pm 1} = \frac{1}{2}$
- $\mathcal{FS}\{x_5(t) = \sin(2t)\} \rightarrow g_k =$
- $\mathcal{FS}\left\{\int x(t) = \frac{\sin(2t)}{2}\right\} \rightarrow \frac{a_k}{jk2}$
- $\mathcal{FS}\{2 \int x(t) = \sin(2t)\} \rightarrow g_k =$

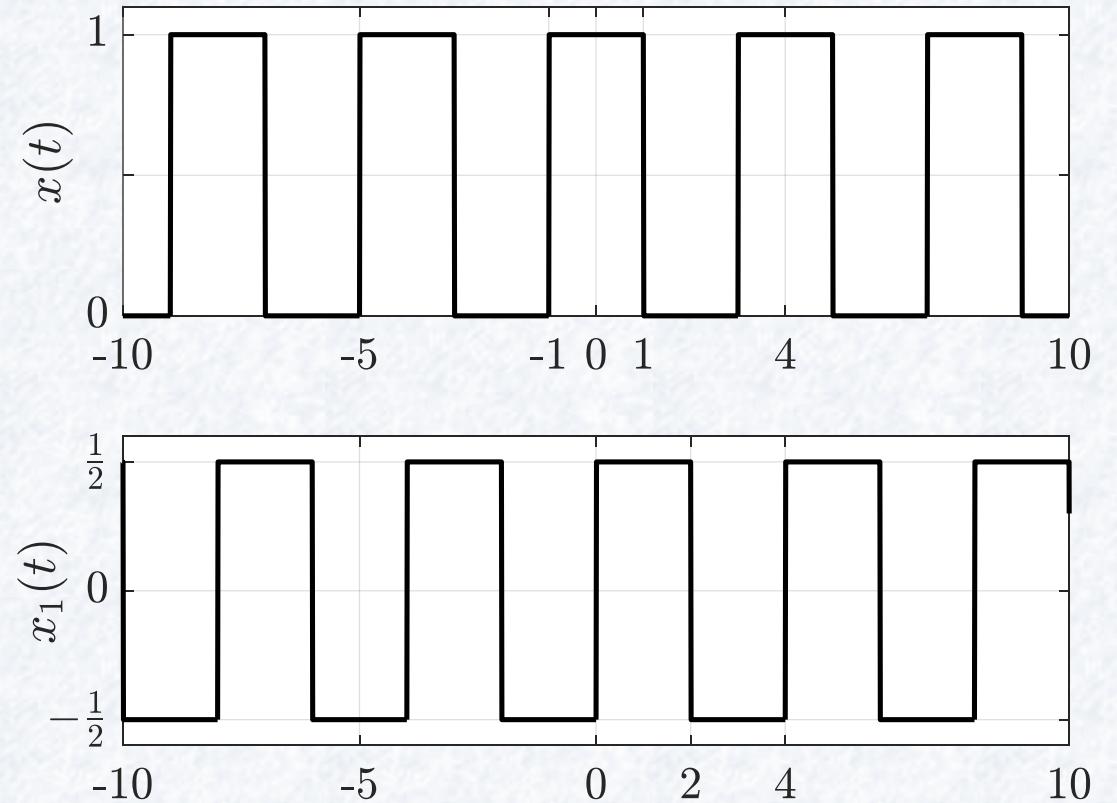


# Örnek 7

- $x(t) = \cos(2t)$ 
  - ♦  $a_{\pm 1} = \frac{1}{2}$
- $\mathcal{FS}\{x_5(t) = \sin(2t)\} \rightarrow g_k =$
- $\mathcal{FS}\left\{\int x(t) = \frac{\sin(2t)}{2}\right\} \rightarrow \frac{a_k}{jk2}$
- $\mathcal{FS}\{2 \int x(t) = \sin(2t)\} \rightarrow g_k = \frac{a_k}{jk}$ 
  - ♦  $g_1 = \frac{a_1}{j} = \frac{1}{2j}$
  - ♦  $g_{-1} = \frac{a_{-1}}{-j} = -\frac{1}{2j}$

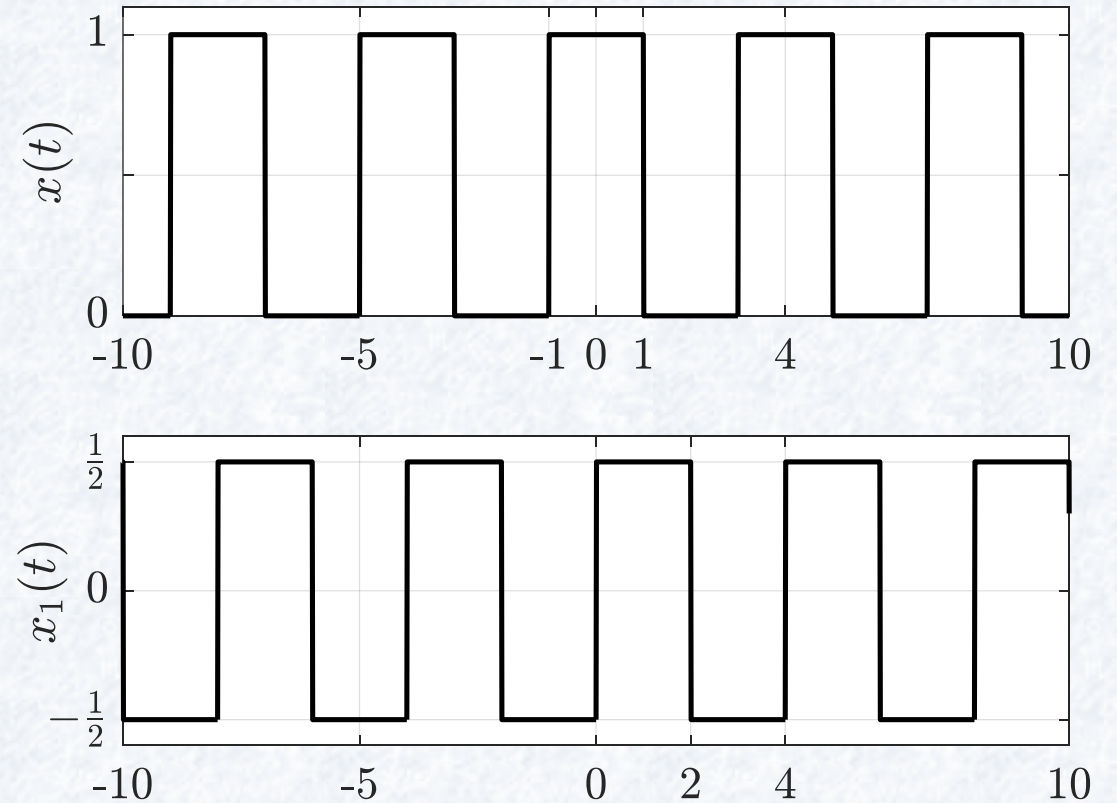
# Örnek 8

- $\mathcal{FS}\{x_1(t)\} \rightarrow c_k = ?$



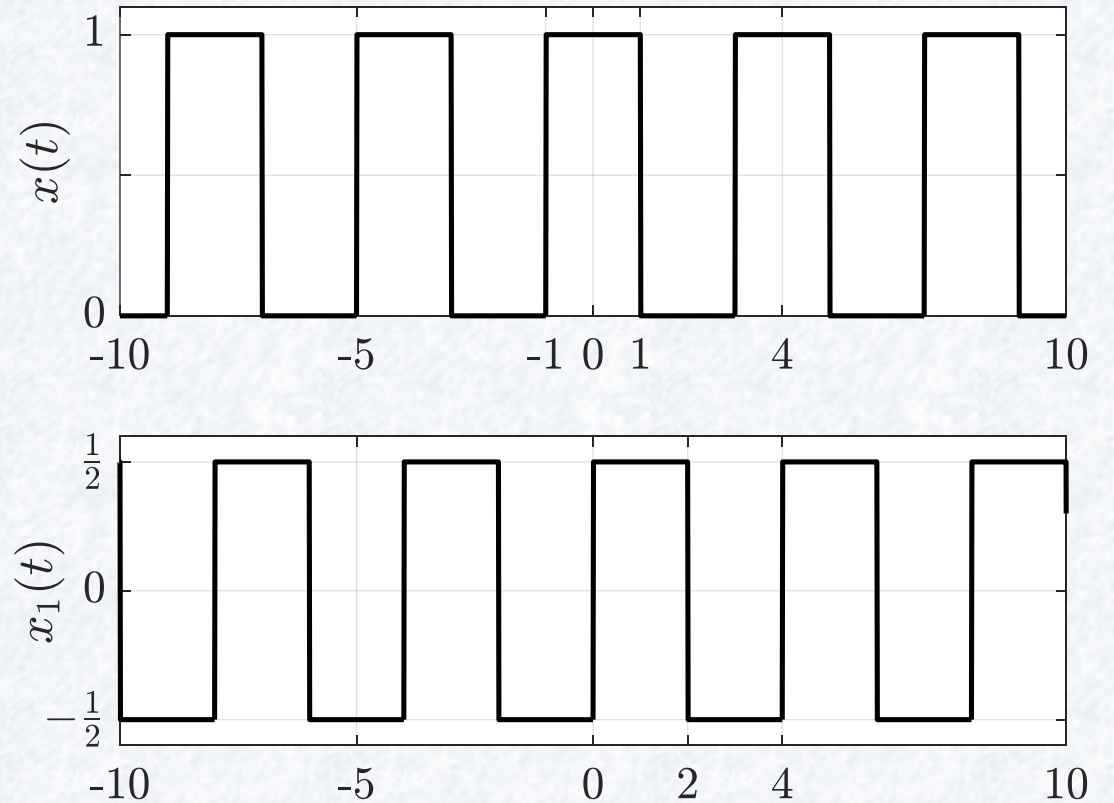
# Örnek 8

- $\mathcal{FS}\{x_1(t)\} \rightarrow c_k = ?$
- $\mathcal{FS}\{x(t)\}$ 
  - ♦  $a_k = \frac{\sin(k\frac{\pi}{2})}{\pi k}$
  - ♦  $a_0 = \frac{1}{2}$
- $x_1(t) = x( \quad )$



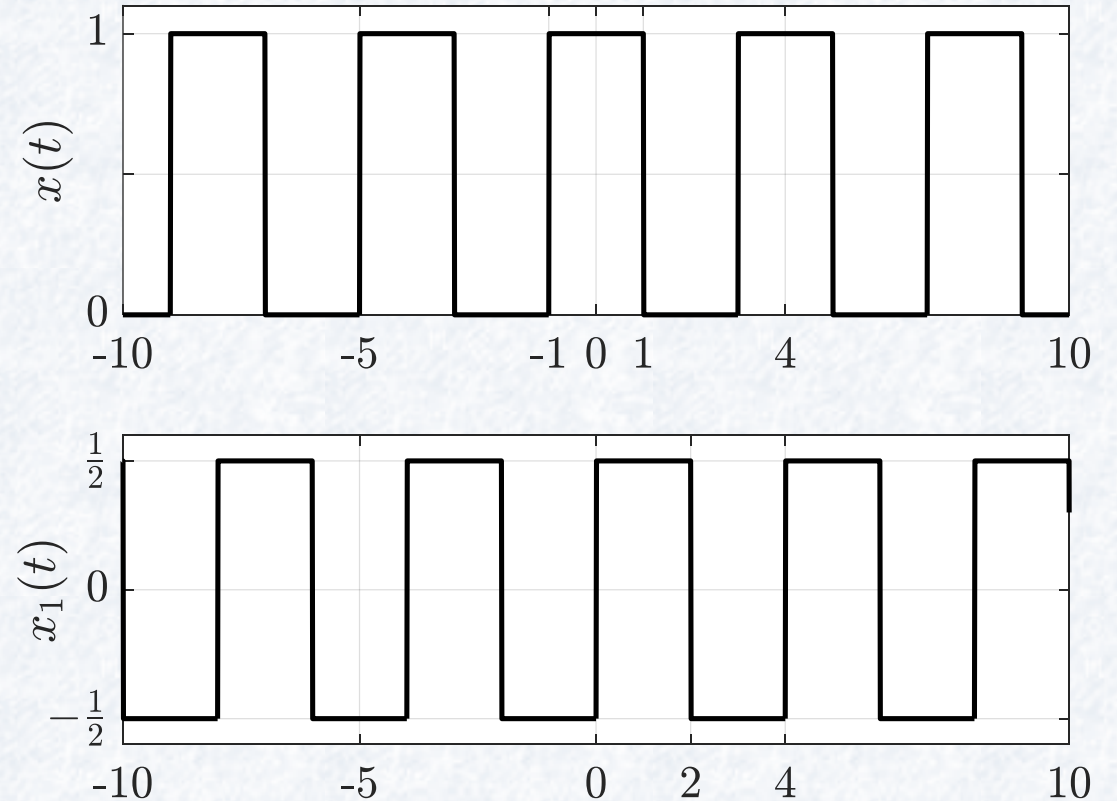
# Örnek 8

- $\mathcal{FS}\{x_1(t)\} \rightarrow c_k = ?$
- $\mathcal{FS}\{x(t)\}$ 
  - ♦  $a_k = \frac{\sin(k\frac{\pi}{2})}{\pi k}$
  - ♦  $a_0 = \frac{1}{2}$
- $x_1(t) = x(t - 1) - \frac{1}{2}$



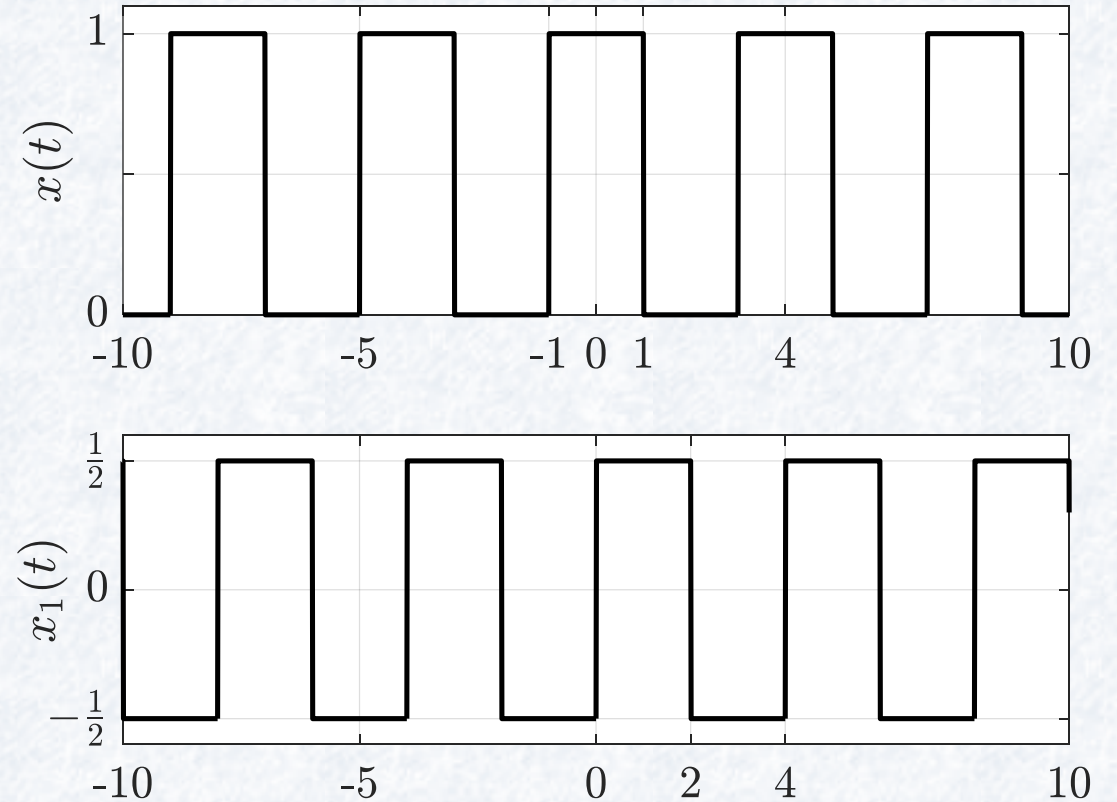
# Örnek 8

- $\mathcal{FS}\{x_1(t)\} \rightarrow c_k = ?$
- $\mathcal{FS}\{x(t)\}$ 
  - ♦  $a_k = \frac{\sin(k\frac{\pi}{2})}{\pi k}$
  - ♦  $a_0 = \frac{1}{2}$
- $x_1(t) = x(t - 1) - \frac{1}{2}$
- $\mathcal{FS}\{x(t - 1)\}$ 
  - ♦  $b_k =$



# Örnek 8

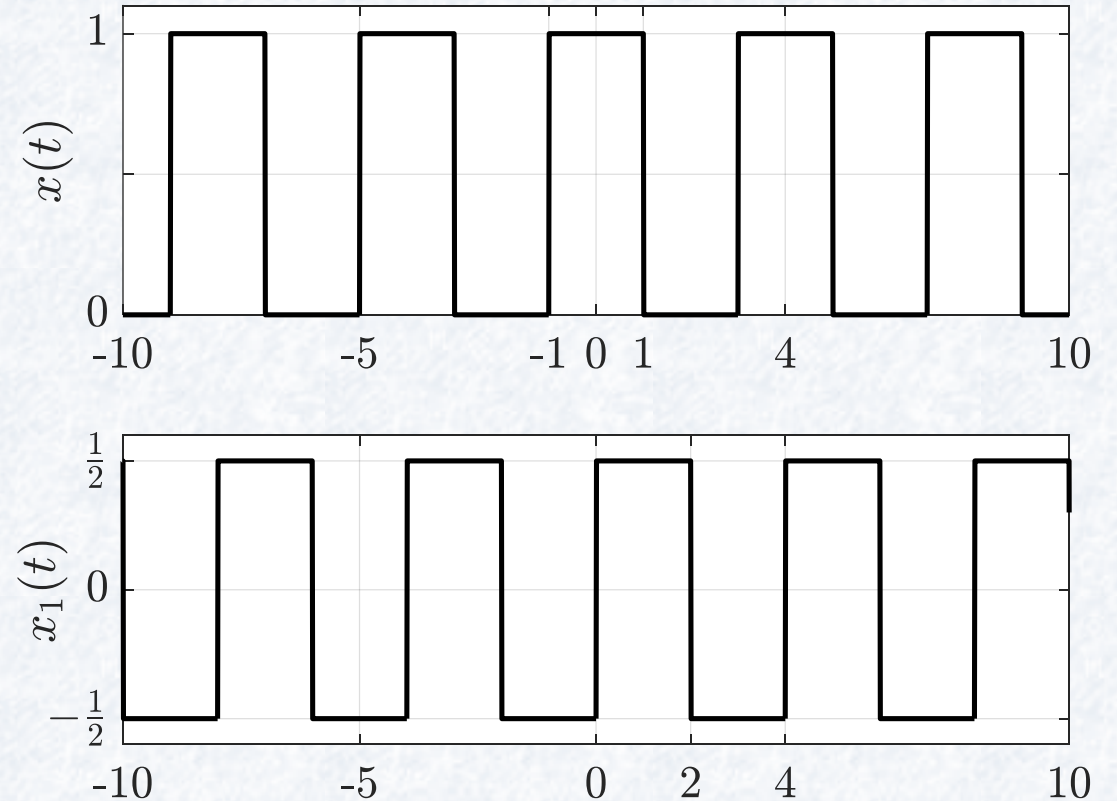
- $\mathcal{FS}\{x_1(t)\} \rightarrow c_k = ?$
- $\mathcal{FS}\{x(t)\}$ 
  - ♦  $a_k = \frac{\sin(k\frac{\pi}{2})}{\pi k}$
  - ♦  $a_0 = \frac{1}{2}$
- $x_1(t) = x(t - 1) - \frac{1}{2}$
- $\mathcal{FS}\{x(t - 1)\}$ 
  - ♦  $b_k = e^{-jk\frac{\pi}{2}} a_k = e^{-jk\frac{\pi}{2}} \frac{\sin(k\frac{\pi}{2})}{\pi k}$
  - ♦  $b_0 =$





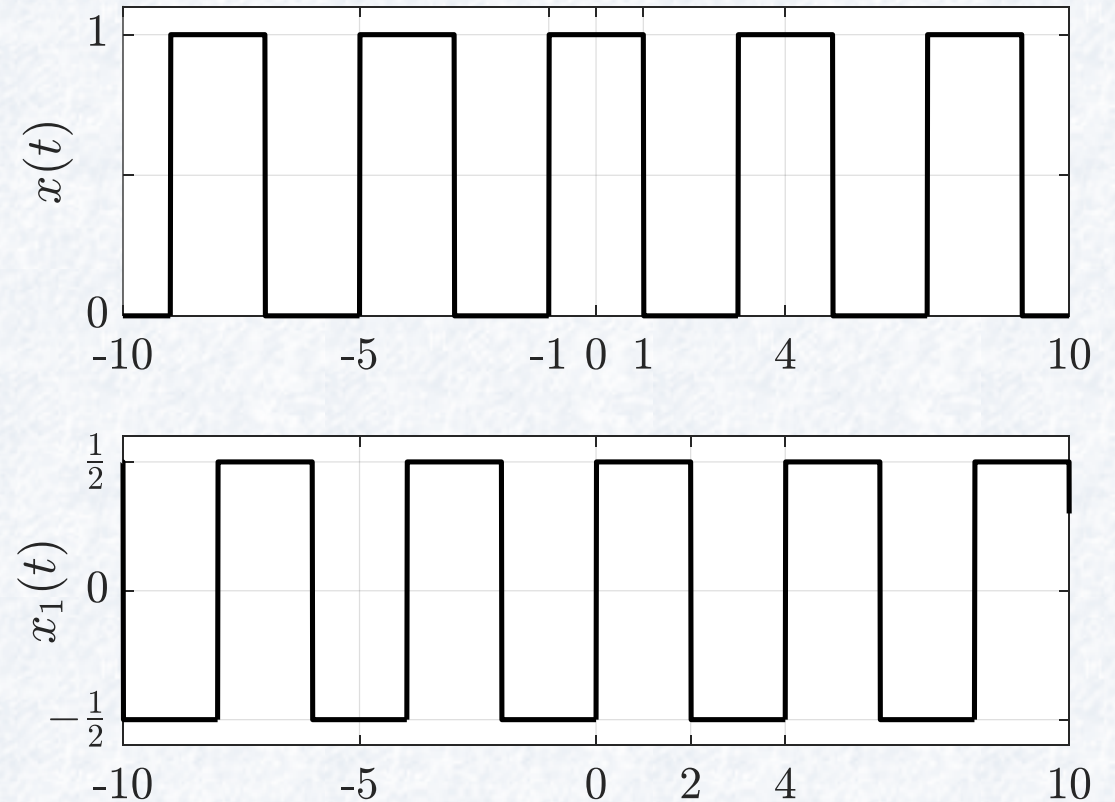
# Örnek 8

- $\mathcal{FS}\{x_1(t)\} \rightarrow c_k = ?$
- $\mathcal{FS}\{x(t)\}$ 
  - ♦  $a_k = \frac{\sin(k\frac{\pi}{2})}{\pi k}$
  - ♦  $a_0 = \frac{1}{2}$
- $x_1(t) = x(t - 1) - \frac{1}{2}$
- $\mathcal{FS}\{x(t - 1)\}$ 
  - ♦  $b_k = e^{-jk\frac{\pi}{2}} a_k = e^{-jk\frac{\pi}{2}} \frac{\sin(k\frac{\pi}{2})}{\pi k}$
  - ♦  $b_0 = a_0 = \frac{1}{2}$



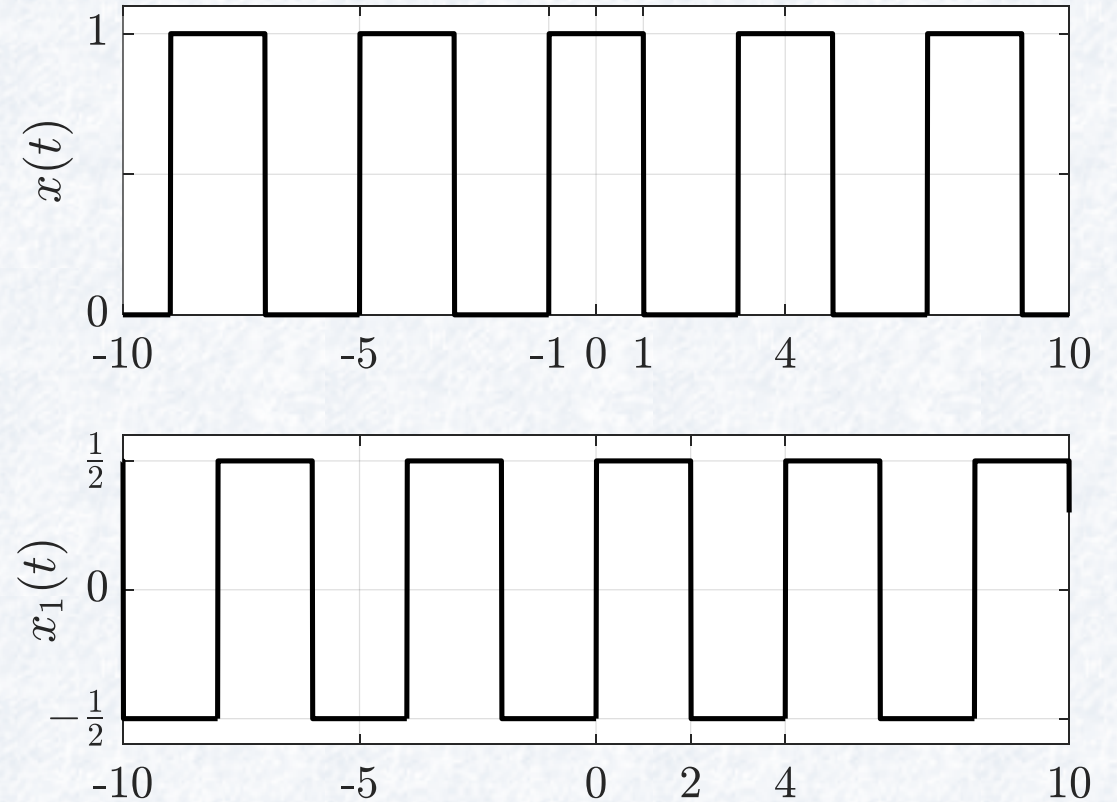
# Örnek 8

- $\mathcal{FS}\{x_1(t)\} \rightarrow c_k = ?$
- $x_1(t) = x(t - 1) - \frac{1}{2}$
- $\mathcal{FS}\{x(t - 1)\}$ 
  - ♦  $b_k = e^{-jk\frac{\pi}{2}} a_k = e^{-jk\frac{\pi}{2}} \frac{\sin(k\frac{\pi}{2})}{\pi k}$
  - ♦  $b_0 = a_0 = \frac{1}{2}$
- $\mathcal{FS}\{x_1(t)\}$ 
  - ♦  $c_k =$



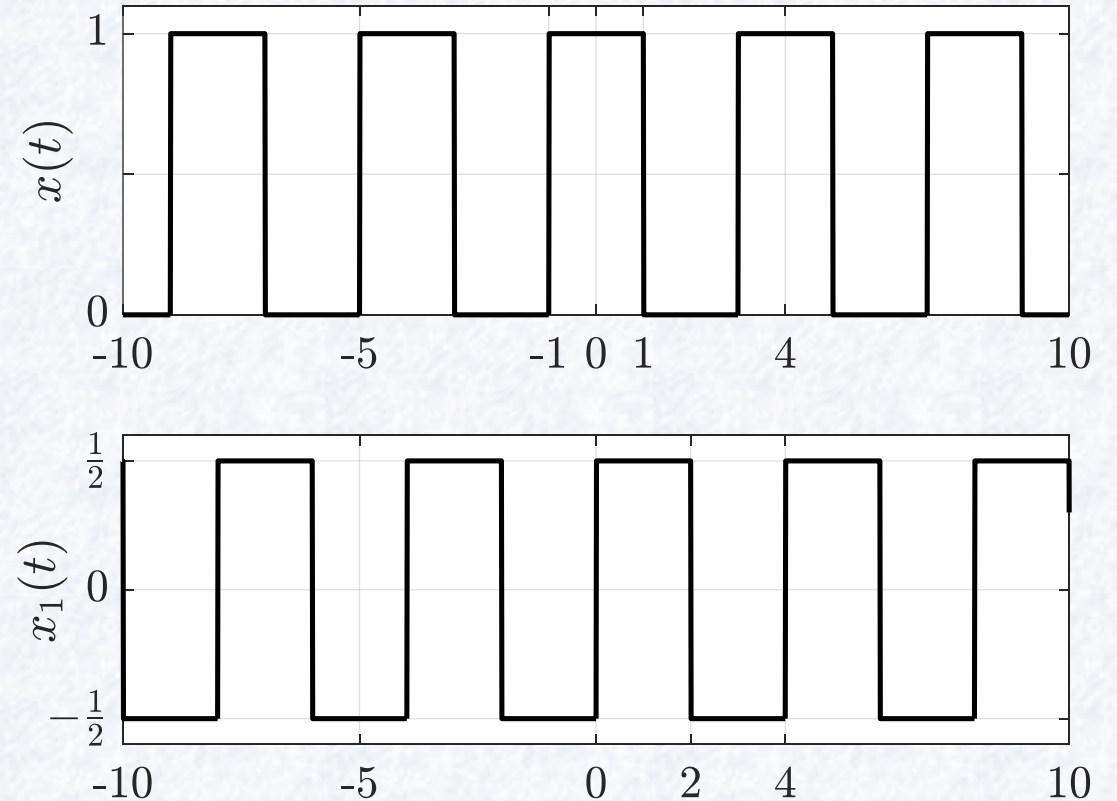
# Örnek 8

- $\mathcal{FS}\{x_1(t)\} \rightarrow c_k = ?$
- $x_1(t) = x(t - 1) - \frac{1}{2}$
- $\mathcal{FS}\{x(t - 1)\}$ 
  - ♦  $b_k = e^{-jk\frac{\pi}{2}} a_k = e^{-jk\frac{\pi}{2}} \frac{\sin(k\frac{\pi}{2})}{\pi k}$
  - ♦  $b_0 = a_0 = \frac{1}{2}$
- $\mathcal{FS}\{x_1(t)\}$ 
  - ♦  $c_k = b_k$
  - ♦  $c_0 =$



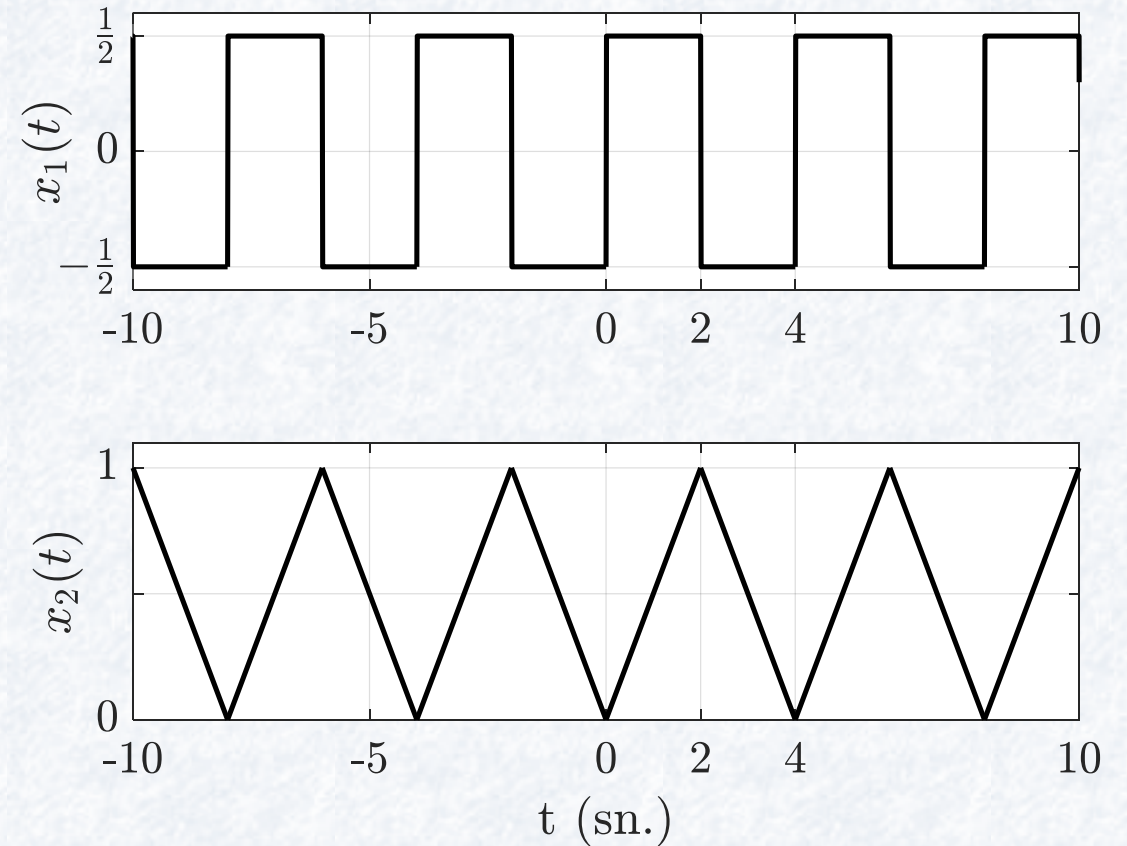
# Örnek 8

- $\mathcal{FS}\{x_1(t)\} \rightarrow c_k = ?$
- $x_1(t) = x(t - 1) - \frac{1}{2}$
- $\mathcal{FS}\{x(t - 1)\}$ 
  - ♦  $b_k = e^{-jk\frac{\pi}{2}} a_k = e^{-jk\frac{\pi}{2}} \frac{\sin(k\frac{\pi}{2})}{\pi k}$
  - ♦  $b_0 = a_0 = \frac{1}{2}$
- $\mathcal{FS}\{x_1(t)\}$ 
  - ♦  $c_k = b_k$
  - ♦  $c_0 = b_0 - \frac{1}{2} = 0$



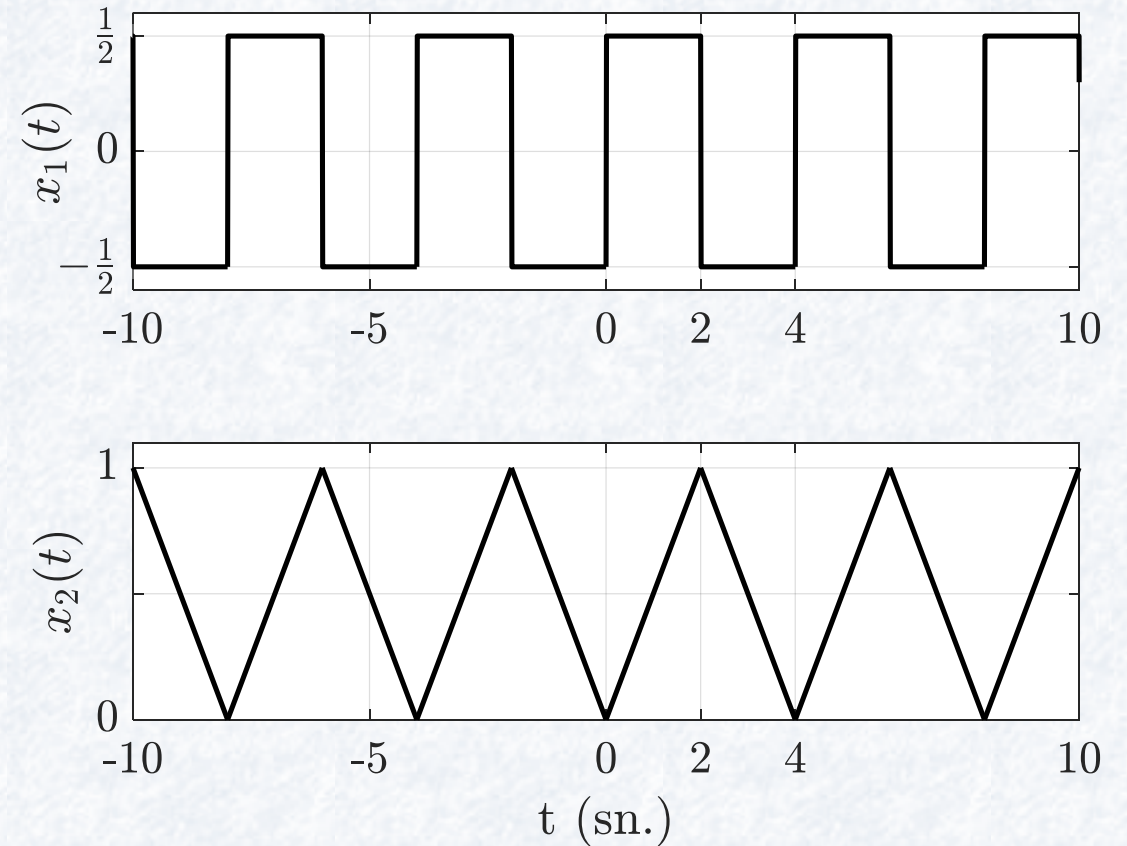
# Örnek 8

- $\mathcal{FS}\{x_2(t)\} \rightarrow d_k = ?$
- $\mathcal{FS}\{x_1(t)\}$ 
  - ♦  $c_k = b_k = e^{-jk\frac{\pi}{2}} \frac{\sin(k\frac{\pi}{2})}{\pi k}$
  - ♦  $c_0 = b_0 - \frac{1}{2} = 0$



# Örnek 8

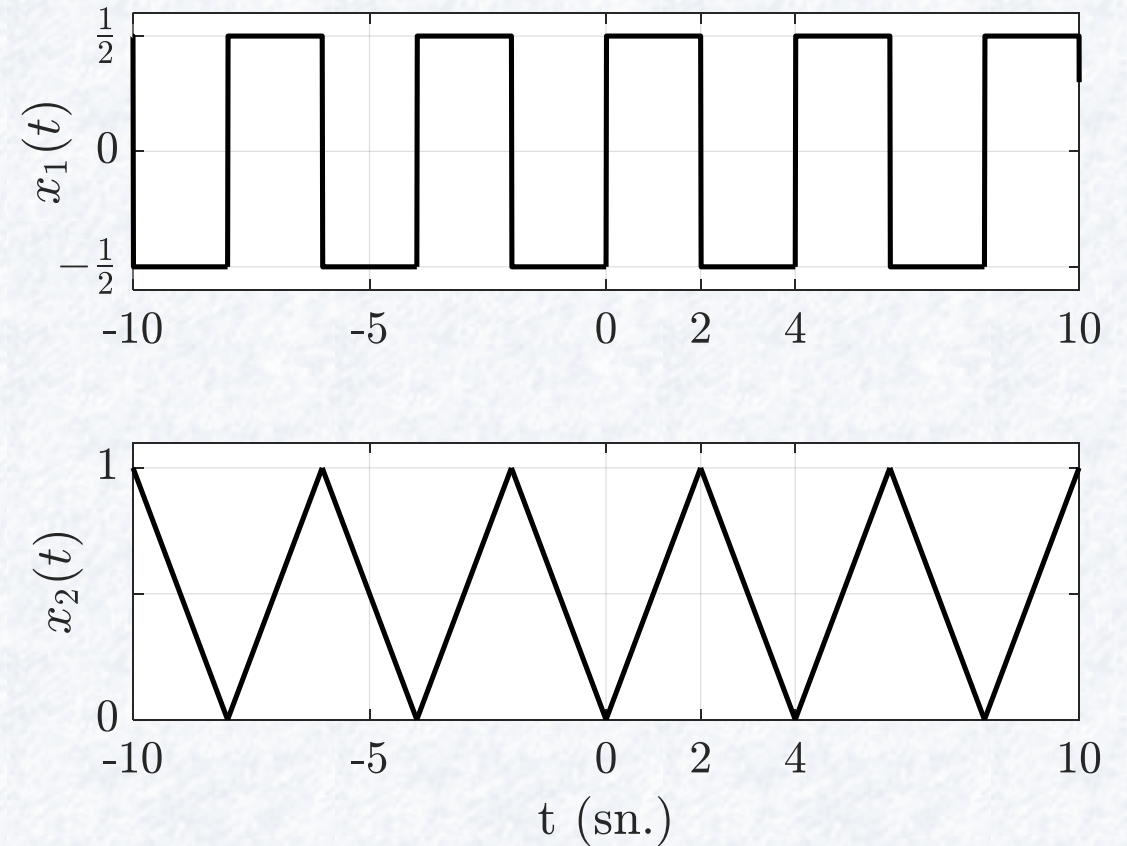
- $\mathcal{FS}\{x_2(t)\} \rightarrow d_k = ?$
- $\mathcal{FS}\{x_1(t)\}$ 
  - ♦  $c_k = b_k = e^{-jk\frac{\pi}{2}} \frac{\sin(k\frac{\pi}{2})}{\pi k}$
  - ♦  $c_0 = b_0 - \frac{1}{2} = 0$
- $x_2(t) = x_1( \quad )$





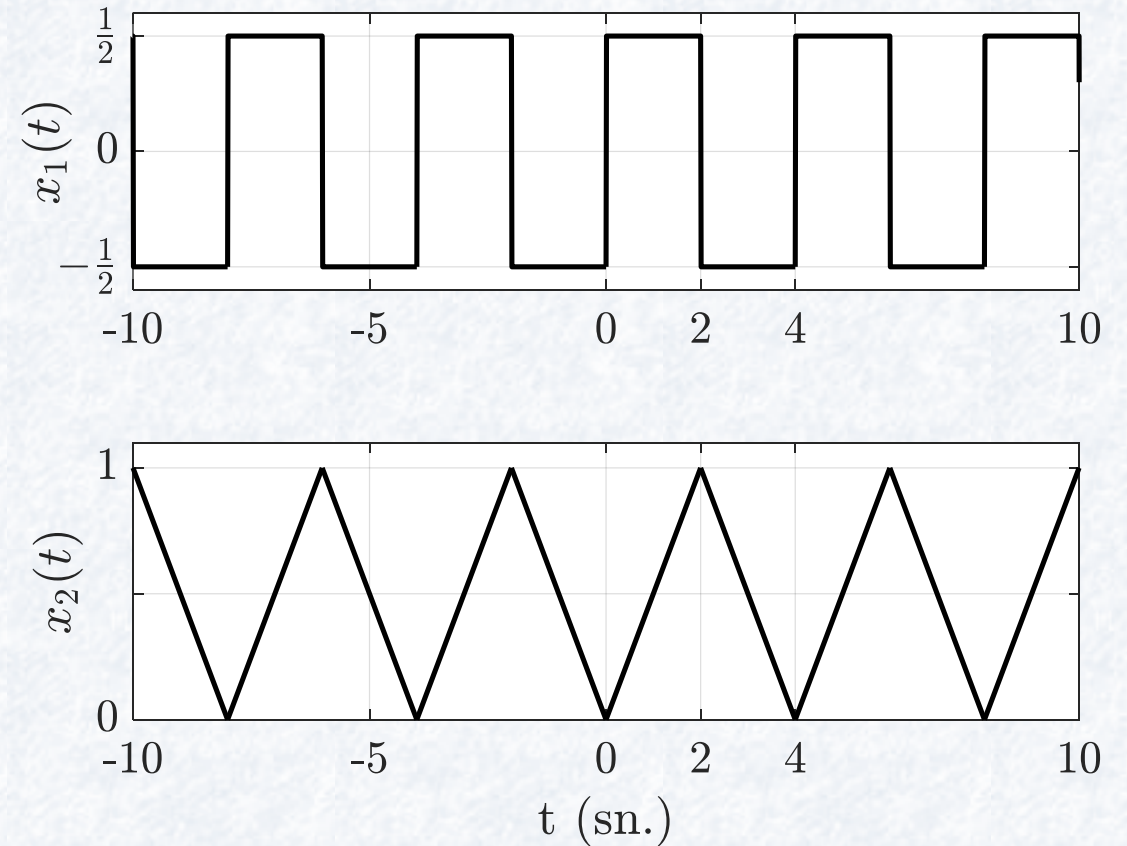
# Örnek 8

- $\mathcal{FS}\{x_2(t)\} \rightarrow d_k = ?$
- $\mathcal{FS}\{x_1(t)\}$ 
  - ♦  $c_k = b_k = e^{-jk\frac{\pi}{2}} \frac{\sin(k\frac{\pi}{2})}{\pi k}$
  - ♦  $c_0 = b_0 - \frac{1}{2} = 0$
- $\frac{\partial x_2(t)}{\partial t} = x_1(t)$



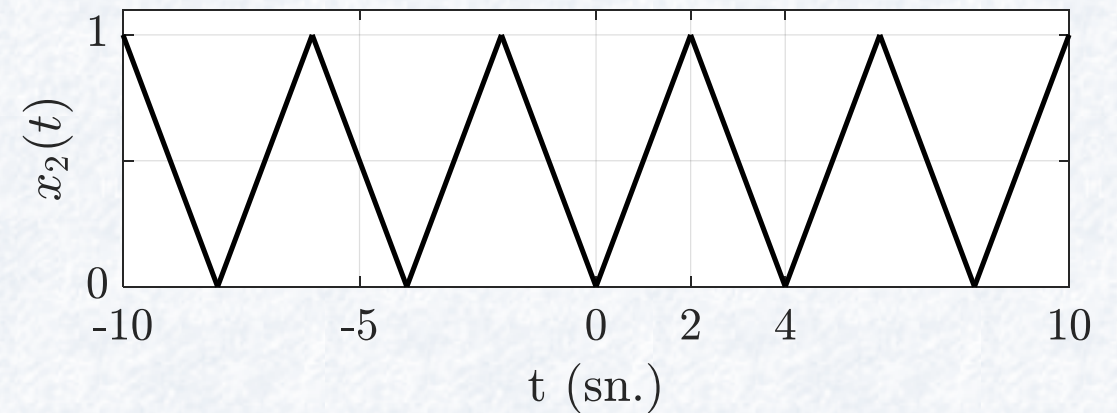
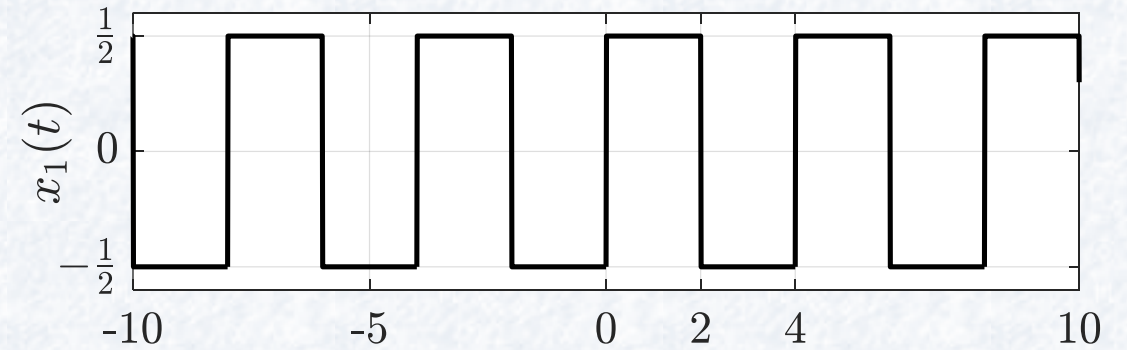
# Örnek 8

- $\mathcal{FS}\{x_2(t)\} \rightarrow d_k = ?$
- $\mathcal{FS}\{x_1(t)\}$ 
  - ♦  $c_k = b_k = e^{-jk\frac{\pi}{2}} \frac{\sin(k\frac{\pi}{2})}{\pi k}$
  - ♦  $c_0 = b_0 - \frac{1}{2} = 0$
- $\frac{\partial x_2(t)}{\partial t} = x_1(t)$
- $jk\frac{\pi}{2} d_k = c_k$



# Örnek 8

- $\mathcal{FS}\{x_2(t)\} \rightarrow d_k = ?$
- $\mathcal{FS}\{x_1(t)\}$ 
  - ♦  $c_k = b_k = e^{-jk\frac{\pi}{2}} \frac{\sin(k\frac{\pi}{2})}{\pi k}$
  - ♦  $c_0 = b_0 - \frac{1}{2} = 0$
- $\frac{\partial x_2(t)}{\partial t} = x_1(t)$
- $jk\frac{\pi}{2}d_k = c_k$
- $d_k = \frac{1}{jk\frac{\pi}{2}} e^{-jk\frac{\pi}{2}} \frac{\sin(k\frac{\pi}{2})}{\pi k}$
- $d_0 =$



# Örnek 8

- $\mathcal{FS}\{x_2(t)\} \rightarrow d_k = ?$
- $\mathcal{FS}\{x_1(t)\}$ 
  - ♦  $c_k = b_k = e^{-jk\frac{\pi}{2}} \frac{\sin(k\frac{\pi}{2})}{\pi k}$
  - ♦  $c_0 = b_0 - \frac{1}{2} = 0$
- $\frac{\partial x_2(t)}{\partial t} = x_1(t)$
- $jk\frac{\pi}{2}d_k = c_k$
- $d_k = \frac{1}{jk\frac{\pi}{2}} e^{-jk\frac{\pi}{2}} \frac{\sin(k\frac{\pi}{2})}{\pi k}$
- $d_0 = \frac{1}{4} \left( \int_0^2 \frac{t}{2} dt + \int_2^4 \left(2 - \frac{t}{2}\right) dt \right) = \frac{1}{2}$

