Analysis of Price Volatility in Natural Gas and Crude Oil Markets

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Abstract

The present study aims to examine the return volatility of futures market prices for natural gas and crude oil during the time period of 2010-2023. It provides a comprehensive overview of the global natural gas and crude oil markets, as well as the European Union markets, highlighting the factors that contribute to the high volatility observed in these markets, such as supply and demand issues, political instability, and governance influences. To achieve the objectives, three different types of GARCH models, namely classic GARCH, GJR-GARCH, and EGARCH models, are employed for analysis and prediction. The primary aim of employing these models is to identify the best fit for the selected commodities using the maximum likelihood function and MSE statistic. The analysis indicates that the crude oil market is best fitted with a EGARCH model, while the GARCH model is the best fit for the natural gas market. Furthermore, the paper offers a brief introduction to risk management, and based on the best fitting models for each time series, the Value at Risk (VaR) statistic is calculated and estimated.

Keywords

GARCH models, natural gas price, crude oil price, volatility prediction

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Introduction

Recent global events, including the COVID-19 pandemic and the subsequent restrictions, as well as the Russian invasion in Ukraine, have caused instability and uncertainties in the natural gas and crude oil markets, highlighting the need to analyze these two commodities markets. Natural gas and crude oil are significant commodities used in various human activities such as transportation, chemical industry, and heating, making their prices capable of impacting other economic sectors and the overall economy of a particular country and the world. The objective of this study is to examine the markets of crude oil and natural gas, including their price formation and the factors that impact their prices. Additionally, this study aims to predict volatility in these markets and explore potential methods for measuring and managing risks that are associated with them.

The work of Satlik[1] was taken as an inspiration for this work, where the author refers to natural gas and crude oil market as markets with high volatility and uses different type of GARCH models to analyse volatility and determine which model is the best fit for the data. In this paper the amount of GARCH models is reduced to GARCH, GJR-GARCH and EGARCH model. Their construction, including their strengths and weaknesses, are explained, and their performance is compared using the maximum likelihood function and MSE statistic to select the best fitting model.

The study is structured as follows. Section 1 provides an introduction to the global crude oil and natural gas market, including possible reasons for periods of high volatility. This section also focuses on an overview of these two commodities in the European market and their unique characteristics and statistics. Section 2 explains the concept on which the chosen GARCH models are based, provides definitions, and discusses their implementation. Additionally, this section briefly explains the concept of risk management and VaR calculation, and introduces the RUGARCH package in R. Section 3 provides an overview of the data. Section 4 focuses on empirical results, such as descriptive statistics, model parameters, their evaluation, and VaR calculation. The final section presents the conclusions drawn from the study.

1. Background

Volatility, in the context of finance and economics, refers to the degree of variation of a financial instrument's price or the rate of return of an investment. In simple terms, volatility is the level of risk associated with an investment. A higher level of volatility implies a higher level of risk.

1.1 Volatility in crude oil and natural gas markets

Volatility analysis is particularly important in the case of commodities such as crude oil and natural gas. According to numerous studies[2, 3, 4] the impact of global financial and stock market crises has been shown to affect volatility, with these periods being associated with higher volatility and less volatility persistence. That's why crude oil and natural gas prices are important indicators that are relevant both at the micro level, for investment trading strategies, diversification, and risk management and at the macro level, as volatility in financial and commodity markets has a global economic impact (affecting growth rates, inflation, and unemployment rates due to production cost channels[1]).

According to Serletis and Herbert[5] or Perifanis and Dagoumas[6] crude oil and natural gas prices times series have the tendency to be correlated. Based on this, the study focuses on exploring the assets of these two commodities simultaneously.

1.2 Crude oil overview

Crude oil is an essential part of today's economy as a lot of sectors, such as transportation, the chemical industry, and electricity generation are heavily dependent on it. Although renewable energy sources are gaining popularity, crude oil still holds a leading position in the energy mix due to its abundance, versatility, and affordability. According to bp Statistical Review of World Energy[7] oil remains the leading fuel, accounting for almost 30% of primary energy consumption worldwide in 2021.

As mentioned above, the crude oil market is associated with high volatility. For a better understanding of the nature of that phenomenon, it is important to understand what factors determine global crude oil prices.

1.2.1 Factors that influence prices in the crude oil market

Based on the work of Gyagri[8] the main factors that determine prices of crude oil are:

- **Supply and Demand**

  Supply - crude oil is primarily extracted from underground reserves, with the largest reserves located in the Middle East, followed by Russia, the United States, and China. The production of crude oil is largely controlled by the Organization of the Petroleum Exporting Countries (OPEC), which accounts for about 44% of the world's crude oil production. Non-OPEC countries such as the United States, Canada, and Russia account for the remaining 56%[7]. These shares of the market allow countries to significantly influence prices. Currently, during OPEC meetings, members collectively decide if they want to increase or decrease oil production, which continues to influence the supply side.

  Demand - the demand for crude oil is primarily driven by the energy needs of the transportation and industrial sectors. As a result, the demand for crude oil is heavily influenced by economic growth and development, as countries with growing economies tend to have increased energy needs. On the other hand, during periods of economic crisis, production decreases and growth slow downs which leads to lower energy needs and, as a consequence, lower oil demand.

- **Political instability**

  Political instability can have a significant impact on crude oil prices. This is because many of the world's largest oil-producing nations are located in politically unstable regions, where the risk of supply disruptions is high. For example, conflict in the Middle East or unrest in Venezuela can lead to a reduction in oil production. In addition to that, political statements of major oil-producing countries can cause concerns of investors and lead to overall uncertainties in the market which may cause periods of high volatility.

- **Governance influence**

  Governments can also play a significant role in determining crude oil prices through various policies and regulations. Mainly by controlling the supply though production quotas or export restrictions and demand by taxes or subsidies on crude oil products.

- **Type of crude oil**

  Based on many factors and conditions responsible for its formation crude oil can have different physical and chemical properties. There are numerous types of crude oil. To make it easier investors, traders, and analysts use benchmarks for crude oil. Benchmark crude oil is the primary type of petroleum used as a reference point to determine the prices of other types of oil and oil-based securities. The most common ones are: West Texas Intermediate (extracted from fields located in Texas, North Dakota, and Louisiana of the United States), Brent Blend (North Sea, Europe), OPEC (consists of the basket made up of 13 blends across OPEC member countries). Due to the high correlation between these benchmarks[8] and high demand from Europe this work will focus solely on the prices of Brent oil.

- **Other factors (exchange value of the dollar, speculators and brokers, natural factors, other energy sources etc.)**

1.2.2 Crude oil in the European Union

Crude oil is a fundamental source of energy for the European Union. According to Eurostat's latest data, the final available consumption of oil and petroleum products for energy and non-energy purposes in 2021 for the EU Member States is 495 (Mtoe). This shows a significant growth from the previous year by almost 30%. Although for the last 15 years oil consumption has been experiencing a declining trend, the consumption level in 2021 came back to that of the early 2000s[9].

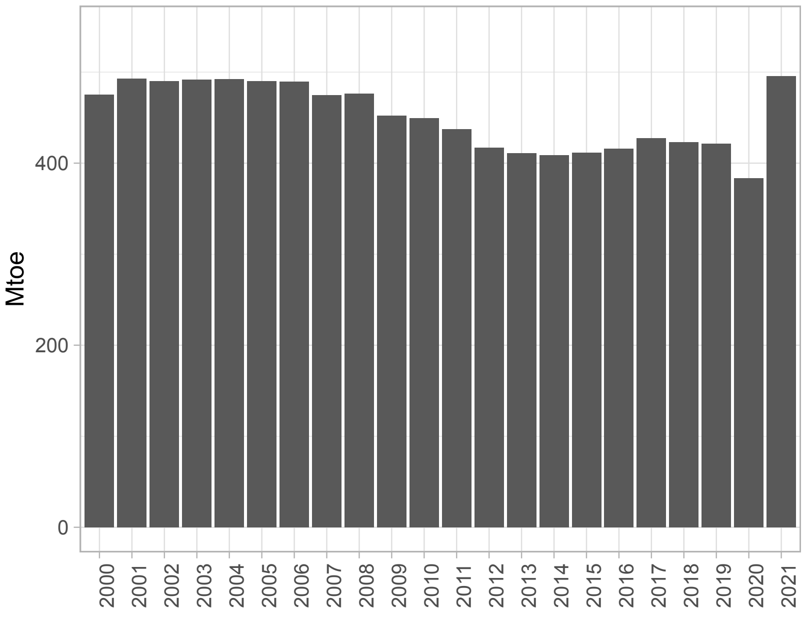


Figure 1.1: The final available consumption of oil and petroleum products in EU-27, 2000-2021

Although the production of crude oil has been declining for several decades, with the region has become increasingly reliant on oil imports to meet its energy needs. Eurostat measures the level of dependency by energy dependency rate which shows the proportion of energy that an economy must import.

It is calculated as net energy imports divided by gross available energy, expressed as a percentage[10]. *The term "gross available energy" refers to the total amount of energy supply available for all activities within a country's territory. This includes energy required for energy transformation processes, such as generating electricity from combustible fuels, as well as support operations within the energy sector, transmission and distribution losses, and the final energy consumption in various sectors, such as industry, transport, households, services, and agriculture. It also encompasses the use of fossil fuel products for non-energy purposes, such as in the chemical industry, and fuel purchased within the country that is used for activities outside the country, such as international aviation and maritime bunkers, and for road transport-related "fuel tourism"*[11]. And calculated as,

$$\begin{aligned}\text{Gross available energy} = \text{Primary production} + \text{Recovered \& Recycled products} + \\ + \text{Imports} - \text{Export} + \text{Stock changes}\end{aligned} \tag{1.1}$$

Since 2000 the important dependency rate has been showing a growing trend reaching its peak in 2019 when the EU relied on net imports for 96.96% of its energy availability[12].

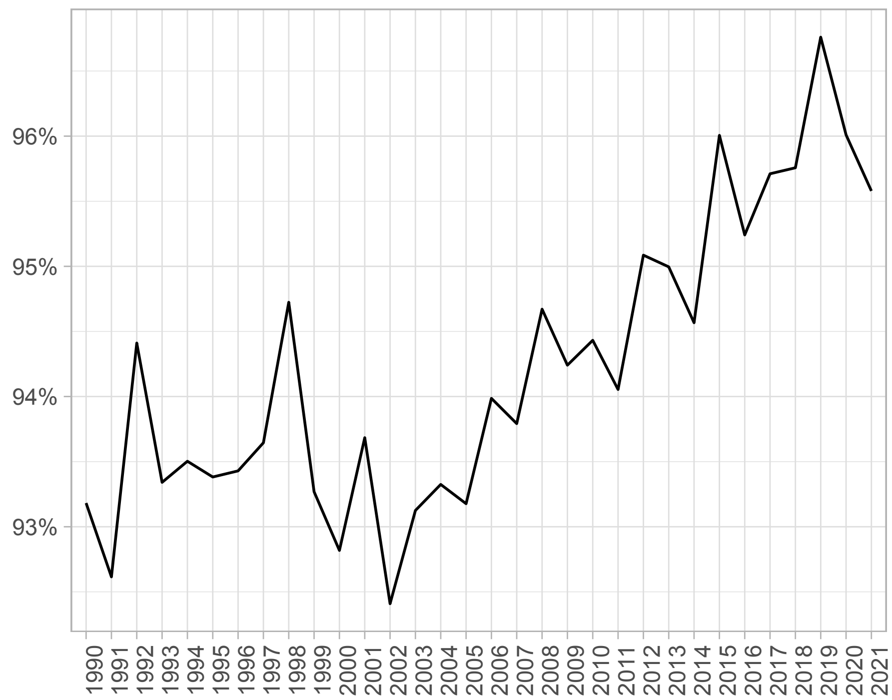


Figure 1.2: Oil import dependency in EU-27 in percentages, 1990-2021

The top EU countries with the highest dependency on crude oil imports are Estonia, Cyprus, Greece, and Italy. These countries have an import dependency of over 90%, which means that they rely heavily on crude oil imports to meet their energy needs. On the other hands, Denmark, Romania, and the Netherlands have the lowest dependency on crude oil imports. These countries have relatively low import dependencies. These countries may have more diverse energy sources or may have significant domestic production of crude oil to meet their energy demands.

The EU's crude oil imports come from a diverse range of sources. Russia, Norway, and Iraq were the top three suppliers for EU member states in 2020[13]. Other significant sources include Saudi Arabia, Nigeria, and Kazakhstan. However, in recent years, the EU has been seeking to reduce its reliance on Russian crude oil imports due to geopolitical concerns and its goal to diversify its energy sources. After the Russian invasion of Ukraine, the EU imposed a ban on crude oil imports from Russia.

This ban could have potential consequences for the EU's energy security. Along with the shift to alternative sources of crude oil and the EU's high import dependency rate on it, this may lead to vulnerability to supply disruptions and price volatility in the global oil market, as the EU adjusts its supply chains to meet its energy needs.

1.3 Natural gas overview

Along with crude oil, natural gas is one of the most important and widely traded commodities on the world markets. Both the bp Statistical Review of World Energy[7] and the International Energy Agency (IEA)[14] estimate that natural gas accounts for about 25% of primary energy consumption worldwide in 2021. Although burning natural gas is associated with greenhouse gas emissions, its amount is relatively low compared to other fossil fuels (such as crude oil and coal) which leads to the natural gas market being the fastest-growing out of all fossil fuels.

1.3.1 Factors that influence prices in the natural gas market

The natural gas market and price formation are similar to the one with crude oil. Possible factors influencing the price are following:

- **Supply and Demand**

  Supply - The supply of natural gas comes from both conventional and unconventional sources, with the vast majority of global production coming from conventional sources. The largest natural gas producers in the world are the United States, Russia, and Iran, followed by Canada, Qatar, and China. Natural gas production in the United States has increased significantly in recent years due to the development of shale gas resources through fracking[7].

  Demand - The demand for natural gas is driven primarily by its use in power generation, industrial processes, and residential and commercial heating. The largest consumers of natural gas are the United States, Russia, China, and Iran[7]. As well as for crude oil during periods of economic growth, demand for natural gas tends to increase as industries and households require more energy and decrease when growth slow downs.

- **Weather conditions**

  One specific characteristic of natural gas usage is shown in Fig.1.3. The residential sector uses natural gas to heat buildings and water. It leads to demand dependency on the weather. For example, colder-than-normal winters can lead to an increase in demand for natural gas for heating, while warmer-than-normal summers can lead to an increase in demand for electricity generation.

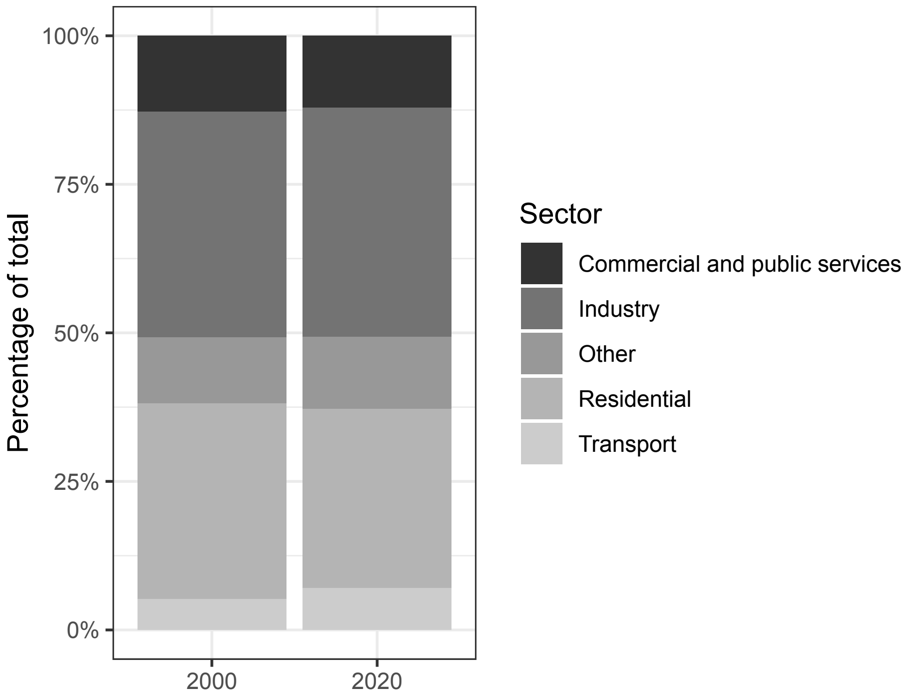


Figure 1.3: Natural gas final consumption worldwide by sector in 2000 and 2020[14]

- **Regionalization and type of natural gas**

  One of the unique characteristics of the natural gas market is that it is highly regionalized, with supply and demand often constrained by infrastructure limitations. This means that the price of natural gas can vary significantly depending on the region, with different pricing mechanisms and benchmarks used in different markets. For instance, in North America, natural gas prices are mainly determined by the Henry Hub benchmark, while in Europe, the Dutch TTF benchmark is commonly used.

- **Governance influence**

  Like oil, natural gas prices are influenced by the government. Government can use different regulation policies, taxes, export restrictions and other price controls mechanisms.

- **Other factors (speculators and brokers, natural factors, inventory levels, other energy sources etc.)**

1.3.2 Natural gas in the European Union

Natural gas remains to be one of the most significant energy sources for European union. According to Eurostat the inland consumption of natural gas in the EU was 15 834 900 terajoules in 2021. It shows an increase in demand by around 4% from the previous year. Historical consumption of natural gas was showing a growing trend until the year 2008 after which it dropped. In current years demand could have potentially reached its previous peak values, but the COVID-19 pandemic, its possible long-lasting effect on demand and the EU's policy on emissions have slowed down consumption[15].

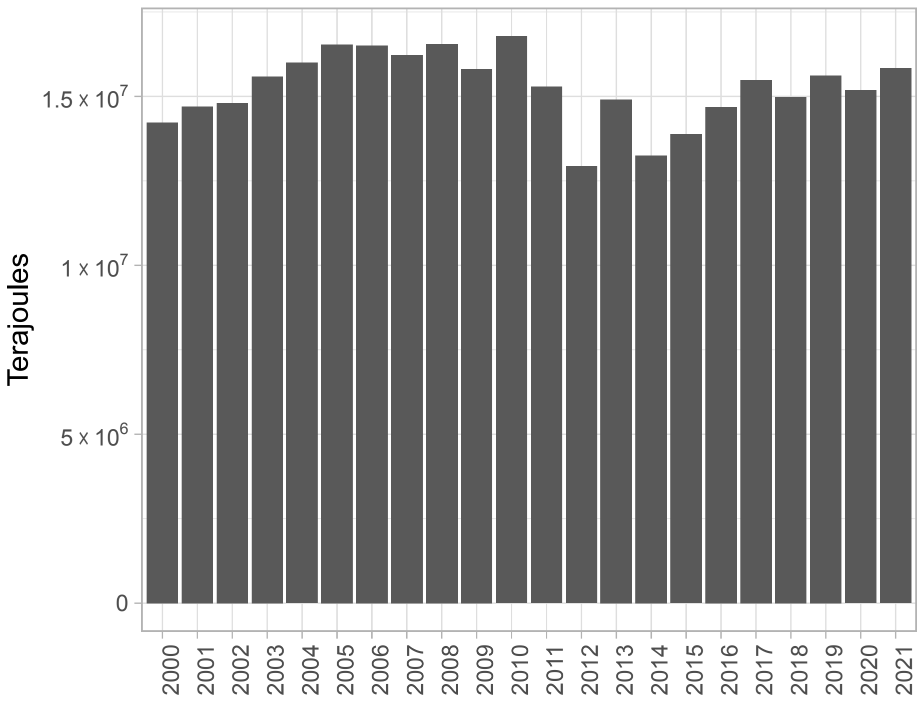


Figure 1.4: Inland consumption of natural gas EU-27, 2000-2021(monthly cumulated data)

Similarly to crude oil natural gas production has been showing a decreasing trend in the EU. The top countries that produce natural gas are the Netherlands(41% of total EU production in 2021), Romania(20%), Germany(10%) and Poland(9%). Along with it, for the last 25 years natural gas dependency rate (defined the same as for crude oil in 1.2.2 sub-chapter) has been showing a growing trend[12].

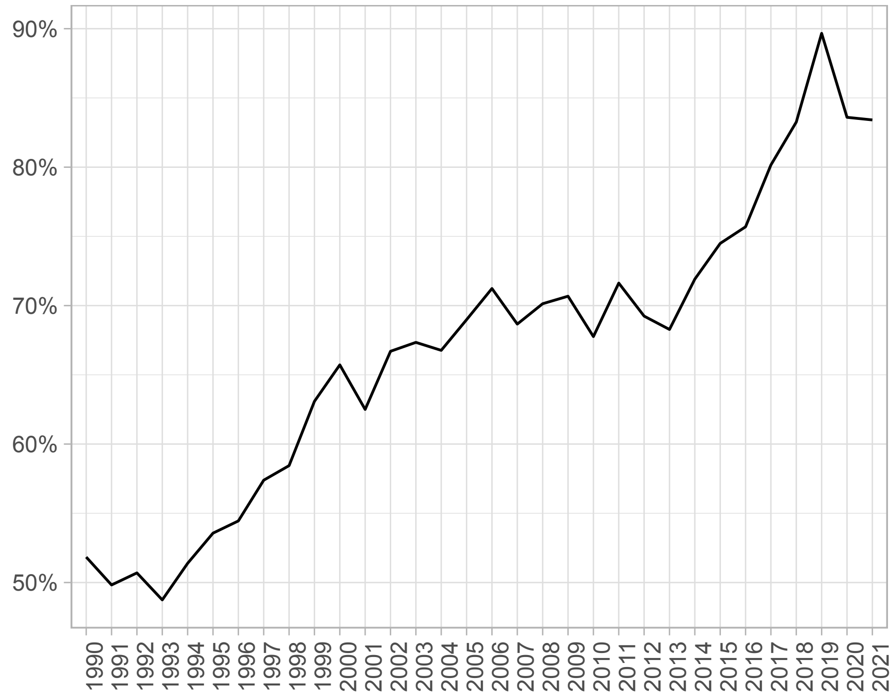


Figure 1.5: Natural gas import dependency in EU-27 in percentages, 1990-2021

15 Member States natural gas dependency was higher than 90% in 2021. The top EU countries with the highest dependency on natural gas imports are Malta, Lithuania, Spain and Portugal (which has a value higher than 100%). On the other hands, Denmark, Romania, and the Netherlands have the lowest dependency on natural gas imports, all bellow 50%.

Regarding the origin of imports, Russia has been the biggest exporter of natural gas for EU. In 2021 Russia was the source of 23% of natural gas entering the EU followed by Norway(22%), Ukraine(10%) and Belarus(9%)[16]. Although the European Union (EU) imports natural gas from various countries, a significant amount of the natural gas imported from Ukraine and Belarus comes from Russia. This means that the EU's reliance on natural gas imports from Russia, in terms of the original source, is effectively greater than that from Norway. In simpler terms, the EU gets a lot of gas from Russia through Ukraine and Belarus, making its reliance on Russia higher than on Norway.

The recent Russian invasion of Ukraine has created instability in the natural gas market leading to high prices of this commodity and its higher-than-usual volatility. Moreover, by adjusting their energy balances after restrictions some countries can face moderate or even significant problems. The most vulnerable countries are located in Central and Eastern Europe (Czech Republic, Slovak Republic and Hungary). Austria, Germany and Italy face this problem as well but the consequence is not that severe. For other countries the effect will be almost insignificant with an estimated impact on GDP under 1%[17].

2. Methodology

Volatility can be measured using a variety of statistics, such as standard deviation, variance, or range. In financial analysis, volatility is typically measured as the standard deviation of asset prices at closing time.

When analysing financial markets instead of using volatility of initial prices assets, volatility of the return rate is used. If P_t is the value of an asset at time t, then a relative measure of the gain/loss (return rate) of the asset at time t is defined as,

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{2.1}$$

Then the corresponding formula for standard deviation computed using T daily returns as,

$$S_r = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(r_t - \bar{r})^2} \tag{2.2}$$

where $\bar{r}$ is the mean of returns.

Although numerous econometric models assume constant variance over time (or homoscedasticity), in financial time series data the variance does not happen to be constant and in most assets heteroscedasticity is often present. One of the key features of volatility is that it tends to cluster in time, which means that high volatility periods are often followed by other high volatility periods, and low volatility periods are followed by other low volatility periods. This clustering effect is due to the fact that market conditions described in previous section tend to persist over time. As a result, volatility levels tend to exhibit a certain degree of persistence, which means that the current level of volatility is influenced by its past level.

2.1 ARCH model

The model ARCH (Autoregressive Conditional Heteroscedastic Model) represents the first systematic approach for modeling conditional variability[18]. The idea of the ARCH model has two assumptions:

(i) the dependence of conditional volatility can be described by a simple quadratic function of its lagged innovations.

(ii) the shock to an asset return is serially uncorrelated, but dependent.

The assumption of serially uncorrelated shocks implies that there is no correlation or pattern in the shock term over time. In other words, the error term is random and not influenced by its previous value. This assumption is necessary for the model to be well-specified and accurate. However, the ARCH model relaxes this assumption by allowing the variance of the error term to vary over time, thereby acknowledging the presence of heteroscedasticity in the data.

In other words, an ARCH(m) is defined as,

$$\begin{aligned} \sigma_t^2 &= \alpha_0 + \alpha_1 e_{t-1}^2 + ... + \alpha_m e_{t-m}^2 \\ e_t &= \sigma_t \epsilon_t \\ r_t &= \mu_t + e_t \end{aligned} \tag{2.3}$$

where σ_t is a conditional variance, ϵ_t is a sequence of independent and identically distributed i.i.d. random variables with mean 0 and variance 1 (normal distribution is the most used case), μ_t is a conditional mean of stochastic process (that is considered to be constant for the purpose of this work) and r_t is a return at time t.

Generally for positive values of σ_t^2 the following regularity conditions are required

$$\alpha_0 > 0, \alpha_1 > 0,...,\alpha_m \geq 0$$

2.2 GARCH model

Due to some disadvantages of ARCH model in practice (such as relatively large number of lags, i. e. large number of parameter and requirements for their positive values), Bollerslev[19] introduced a useful extension known as GARCH model. GARCH(p, q) has form,

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \beta_j \sigma_{t-i}^2 + \sum_{i=1}^{q} \alpha_i e_{t-i}^2 \tag{2.4}$$

General required condition for positive values of σ_t^2 :

$$\alpha_0 > 0, \alpha_i > 0,...,\beta_i \geq 0$$

Compared with the ARCH model, the GARCH model represents linear combinations of lagged values of innovations and its own lagged values.

The parameter α determines the immediate impact of the shock on the conditional variance. Specifically, α represents the weight given to the past squared error in the conditional variance equation. The larger α indicates that the model places more weight on the most recent squared error, resulting in a more significant impact on the current conditional variance and vice versa.

The parameter β, on the other hand, determines the persistence or duration of the impact of the shock on the conditional variance of an asset's return. β represents the weight given to the past conditional variance in the current conditional variance equation. The larger the value of β, the longer the impact of a shock on the current variance of the asset's return, and vice versa.

Furthermore, GARCH(1,1) model is often sufficient for modeling volatility in financial time series as it captures the basic dynamics of volatility while minimizing the risk of overfitting. Based on that 2.4 model can be rewritten as,

$$\sigma_t^2 = \alpha_0 + \beta\sigma_{t-1}^2 + \alpha e_{t-1}^2 \tag{2.5}$$

A crucial factor in analyzing GARCH models is determining whether the conditional variance σ_t^2 is stationary. The GARCH(1,1) process is stationary with,

$$\sigma_e^2 = \frac{\alpha_0}{1 - \alpha - \beta}$$

This requires one more restriction for parameters α and β, that is $\alpha + \beta < 1$

2.3 GJR-GARCH and EGARCH models

The GARCH model assumes that positive and negative shocks have the same impact on volatility. For financial data this assumption is not always true, where usually bad news is followed by a higher increase in volatility than after good news, also known as leverage effect.

Some extensions of the GARCH model were created to capture this feature of financial time series. One of them is the GJR-GARCH model[20]. Specifically, the model assumes that the conditional variance of the current period is a function of past conditional variances, past squared innovations, and a threshold variable that depends on the sign of the past intonations. If the past intonation is negative, the threshold variable has a larger impact on the conditional variance, which captures the idea of greater sensitivity of volatility to negative shocks.

The GJR-GARCH(1,1) model can be expressed as follows:

$$\sigma_t^2 = \alpha_0 + \beta\sigma_{t-1}^2 + (\alpha + \gamma I_{t-1})e_{t-1}^2 \tag{2.6}$$

where

$$I_{t-1} := \begin{cases} 0, \text{ if } e_{t-1} \geq 0 \\ 1, \text{ if } e_{t-1} < 0 \end{cases}$$

with a restriction for parameters α_0,β,$\alpha > 0$. Meanwhile parameter γ can be negative if $(\alpha + \gamma) > 0$.

Similarly to the GARCH model, GJR-GARCH is stationary, if $\alpha + \frac{\gamma}{2} + \beta < 1$, which leads to constant unconditional variance:

$$\sigma_e^2 = \frac{\alpha_0}{1 - \alpha - \frac{\gamma}{2} - \beta}$$

As seen in the definitions above, the GARCH model is in fact a restricted version of the GJR-GARCH with $\gamma = 0$.

Another model that can feature the leverage effect is the EGARCH model introduced by Nelson[21] allows asymmetric responses to positive and negative shocks to volatility. Specifically, it allows for the conditional variance of asset returns to respond differently to positive and negative shocks to the returns.

EGARCH(1,1) is formulated as,

$$\ln \sigma_t^2 = \alpha_0 + \beta \ln \sigma_{t-1}^2 + \alpha \left| \frac{e_{t-1}}{\sigma_{t-1}} \right| + \gamma \frac{e_{t-1}}{\sigma_{t-1}} \tag{2.7}$$

Due to the fact that EGARCH model takes the log of the variance, the positivity of the parameters is guaranteed. However, to maintain stationarity, β must be positive and less than 1. Other parameters do not have any restrictions. The leverage effect is indicated by the value of γ.

While both models might capture leverage effect in financial time series, the EGARCH model is slightly better due to the fact it does not require non-negative constraints on most of the parameters, therefore algorithm for fitting the parameter works faster compared to the GJR-GARCH model.

2.4 Evaluating the models

The Akaike Information Criterion (AIC) is a statistical tool that extends the maximum likelihood principle to combine estimation, as well as structural and dimensional determination into a single procedure. Its primary objective is to select the most appropriate model from a set of multiple models for a given dataset. The AIC works by assessing the model's goodness-of-fit on the training data and incorporating a penalty term for the model's complexity. It yields a single score that indicates the model with the lowest possible AIC value is the most suitable for the dataset.

AIC is usually computed as,

$$AIC = 2K - 2 \ln L \tag{2.8}$$

where K is the number of model parameters and L is the likelihood function of the model.

Package 'rugarch' in R that will be introduced in 2.7 section, calculates AIC as,

$$AIC = \frac{1}{N}(2K - 2\ln L) \tag{2.9}$$

where N is number of observations.

Another popular and simple way to compare models is mean squared error (MSE) computed as:

$$MSE = \frac{1}{N}\sum_{t=1}^{N}(\sigma_t - \hat{\sigma}_t)^2 \tag{2.10}$$

where $\hat{\sigma}_t$ is a fitted conditional variance.

2.5 Verification of GARCH models assumptions

Selected GARCH models have assumptions for construction. After choosing the model it is important verify it on the real data.

All assumptions will be focused on standardized residuals that can be calculated as,

$$\hat{\epsilon}_t = \frac{\hat{e}_t}{\hat{\sigma}_t} \tag{2.11}$$

where $\hat{\sigma}_t$ is the estimated conditional standard deviation.

These standardized residuals are used to verify:

- Close to 0 mean and close to 1 standard deviation.
- Constant variability
  Lagrange multiplier(LM) is used to check this assumption[18].
- No autocorrelation ($Corr(|Z_{t-k}|,|Z_t|) \approx 0$ for $k > 0$)
  The autocorrelation function (ACF) is used to check this assumption.

2.6 VaR in financial risk management

As this thesis emphasizes the significance of predicting volatility in financial markets the final section of this work will discuss one of the crucial statistical measures in financial risk management for evaluating financial risk.

Financial risk management is the process of identifying, assessing, and mitigating risks in financial markets. In the context of commodity markets, such as crude oil and natural gas, where prices can be highly volatile, effective risk management is crucial to minimize potential losses and maximize profits.

This study demonstrates the use of GARCH models to accurately predict volatility in crude oil and natural gas markets, which can be used to calculate Value at Risk (VaR). The ability to accurately estimate VaR using GARCH models is crucial for setting appropriate risk limits, hedging against potential losses, and making informed investment decisions in these commodity markets.

Value at Risk (VaR) is a widely used measure of financial risk that quantifies the potential loss of an investment portfolio at a certain confidence level over a specified time horizon. VaR has three components: (i) time period, (ii) confidence level, (iii) loss amount(or loss percentage).

- **Delta-normal approach**

  This approach of calculation VaR assumes all stock returns are normally distributed. This method consists of going back in time and computing the variance of returns. VaR can be defined as:

$$VaR(p) = \mu + \sigma \cdot N_x(p) \tag{2.12}$$

  where μ is a mean of stock return, σ is the standard deviation of return and $N_x(p)$ is a quantile of normal distribution for corresponding confidence level p.

  The results of such a simple model is often disappointing and are rarely used in practice today. Specifically, the assumption of constant daily variance is rarely accurate and does not reflect the complex and dynamic nature of financial data.

- **GARCH approach**

  This approach assumes that returns have the normal distibution and existence of conditional variance given by one of GARCH variation models. Value at risk then is defined as:

$$VaR(p) = \mu + \sigma_{t|t-1} \cdot N_x(p) \tag{2.13}$$

  where $\sigma_{t|t-1}$ is the conditional standard deviation given the information at $t-1$ and $N_x(p)$ is a quantile of normal distibution for corresponding confidence level p

2.7 R package

The package "rugarch" implemented by Alexios Galanos[22] is used to construct models described above. This package has all the necessary functions for parameters estimations, models analysis and evaluations.

- First of all, it is necessary to specify the type of GARCH model, its order and distribution using the function **ugarchspec**.

- After that function **ugarchfit** can be used, which will estimate parameters for a given specification. For estimation of its parameters, the function will use the maximum likelihood function. This function also includes the necessary test for parameters significance, and evaluation of model performance.

- Function **ugarchroll** is used for constructing a rolling moving ahead forecast.

3. Data

The data presented in this study is taken from January 2010 to February 2023. It includes two time series:

- Daily closing prices of Brent oil futures (in US dollars for 1 barrel)
- Daily closing prices of natural gas futures (in US dollars for 1 Mmbtu)

All data was collected from the London Stock Exchange website. The Brent oil futures time series includes observations for 3387 days, while the Natural gas futures time series includes 3413 days of observations.

4. Empirical results

4.1 Descriptive statistics

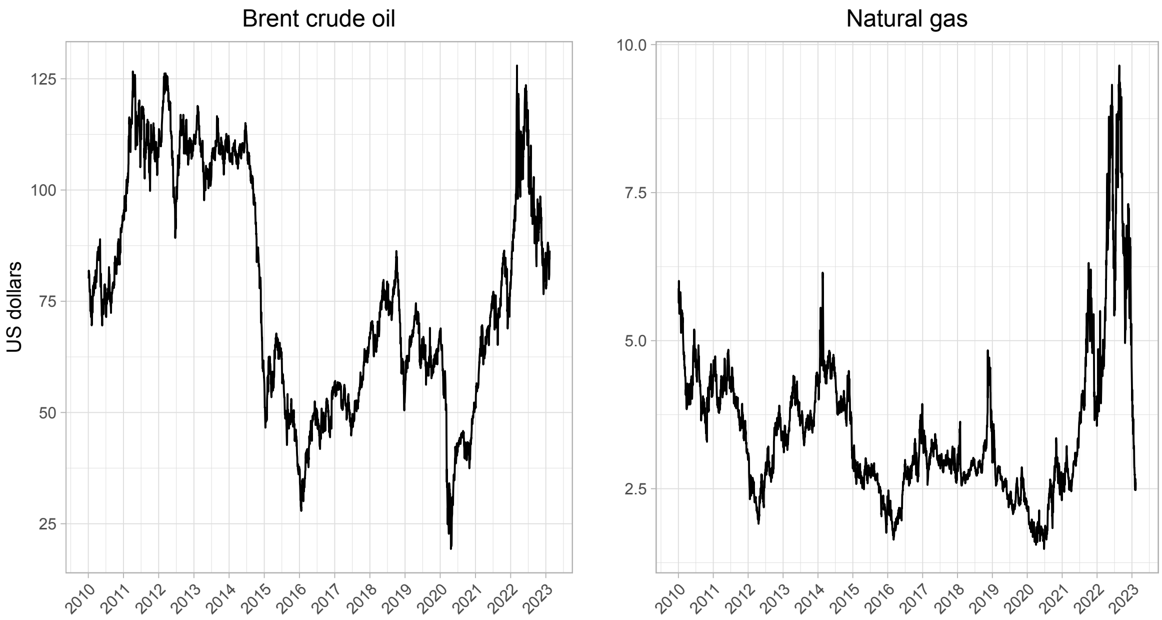


Figure 4.1: Historical prices for Brent crude oil and Natural gas, 2010-2023

Table 4.1: Descriptive statistics for returns

	Mean	**St. deviation**	**Skewness**	**Kurtosis**
Brent crude oil	0.0002911	0.0227557	−0.36040	16.7915
Natural gas	0.0002935	0.0321923	0.305433	6.6872

The mean of both returns is close to 0 and constant overtime. However, in the Fig. 4.2 and Fig. 4.3 of daily return, it can be clearly seen that volatility is changing overtime for both time series. Spikes in both figures indicate about periods with high volatility. High positive values of kurtosis show that both return series are leptokurtic (lepokurtic implies that the distribution has “fat tails”.). It is associated with risk for investors, meaning that extreme events (both positive and negative) are more likely to occur than in a normal distribution. This can be a concern for investors who are looking for stable, predictable returns. Moreover, both return series have non-zero values of skenwenss which indicates about asymmetric distributions. For Brent crude oil time series is left skewed (negative value of skewness) and for natural gas it is right skewed (positive value of skewness).

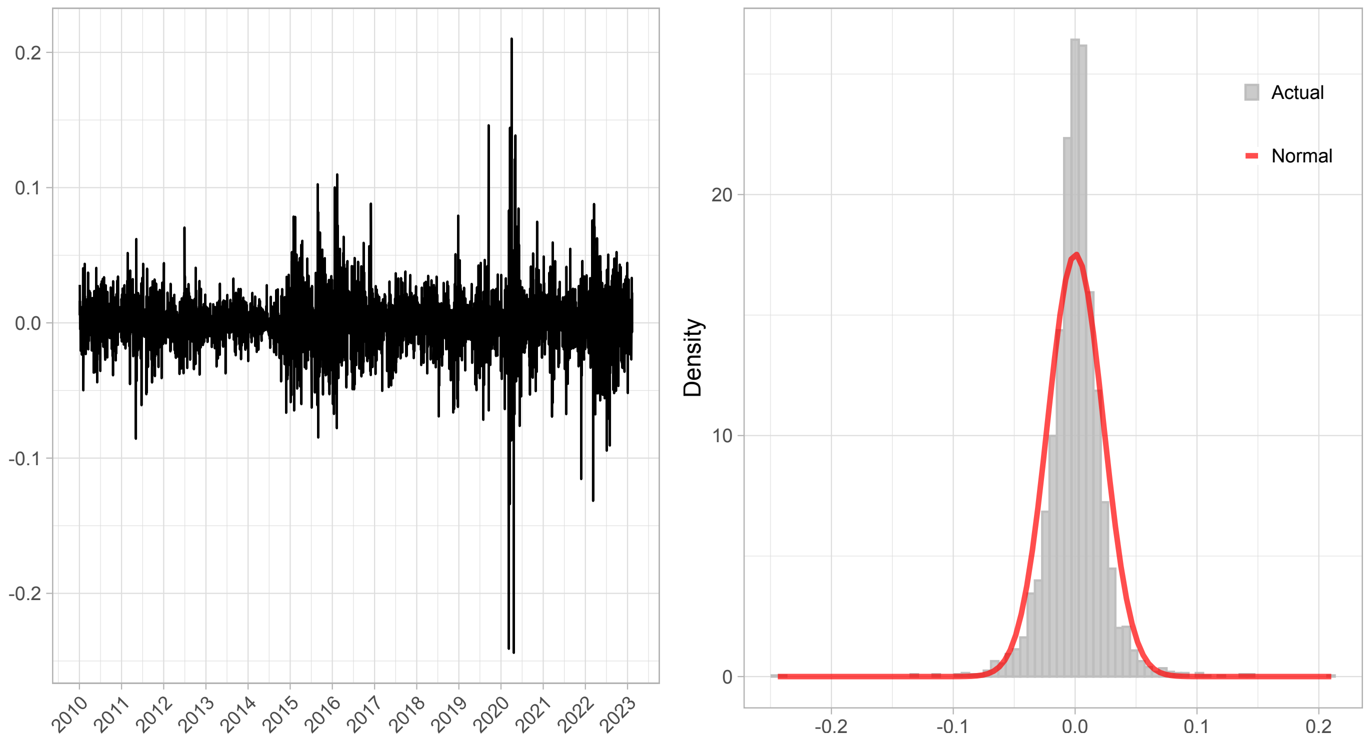


Figure 4.2: Returns and returns density for Brent crude oil, 2010-2023

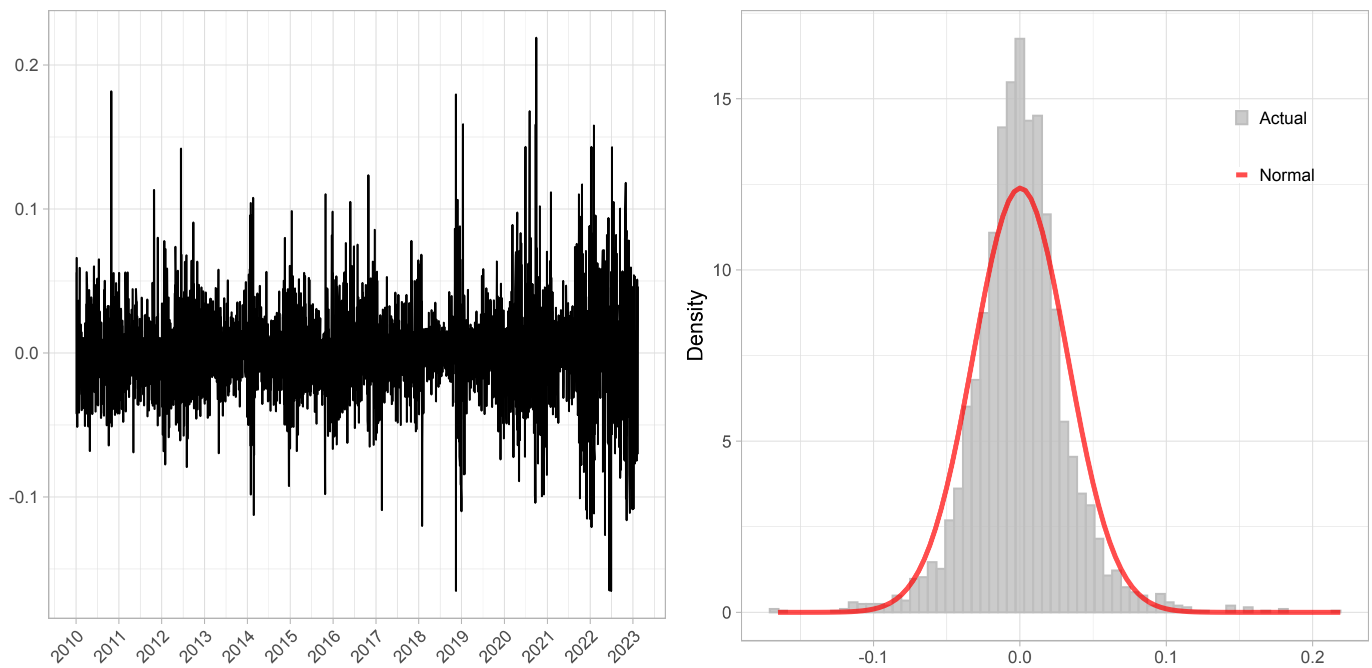


Figure 4.3: Returns and returns density for natural gas, 2010-2023

4.2 Models

In this chapter discussed models are be implemented using data of Brent crude oil and natural gas daily returns. Then the performance of these models is compared using various statistical metrics to assess their predictive power and accuracy in forecasting future volatility. The results of implementing the models are presented in Table 4.2 and 4.3 for crude oil and natural gas respectively.

Table 4.2: Parameters, AIC and MSE criteria for GARCH, GJR-GARCH and EGARCH models for Brent crude oil

	GARCH	**GJR-GARCH**	**EGARCH**
α_0	$5.215 \cdot 10^{-6}$	$4.759 \cdot 10^{-6}$	$9.899 \cdot 10^{-2}$
α	$9.179 \cdot 10^{-2}$	$4.368 \cdot 10^{-2}$	$-6.451 \cdot 10^{-2}$
β	$9.021 \cdot 10^{-1}$	$9.103 \cdot 10^{-1}$	$9.877 \cdot 10^{-1}$
γ		$7.551 \cdot 10^{-2}$	$1.516 \cdot 10^{-1}$
AIC	-5.162	-5.168	-5.173
MSE	$3.793 \cdot 10^{-6}$	$3.753 \cdot 10^{-6}$	$3.706 \cdot 10^{-6}$

Note: All p-values in t-tests for parameters are less than $\alpha = 0.05$

Table 4.3: Parameters, AIC and MSE criteria for GARCH, GJR-GARCH and EGARCH models for natural gas

	GARCH	**GJR-GARCH**	**EGARCH**
α_0	$1.105 \cdot 10^{-5}$	$1.101 \cdot 10^{-5}$	$-8.651 \cdot 10^{-2}$
α	$7.151 \cdot 10^{-2}$	$7.377 \cdot 10^{-2}$	$9.167 \cdot 10^{-3}$
β	$9.187 \cdot 10^{-1}$	$9.194 \cdot 10^{-1}$	$9.879 \cdot 10^{-1}$
γ		$-6.203 \cdot 10^{-3}[*]$	$1.550 \cdot 10^{-1}$
AIC	-4.2946	-4.2941	-4.2945
MSE	$5.641 \cdot 10^{-6}$	$5.640 \cdot 10^{-6}$	$5.622 \cdot 10^{-6}$

Note: All p-values in t-tests for parameters are less than $\alpha = 0.05$ except parameter [*]

Based on the findings presented in 4.2 table, the exponential EGARCH model outperformed the GARCH and GJR-GARCH models for volatility of Brent crude oil returns. The EGARCH model has the lowest AIC and MSE criteria and it is the best fitting model for among the selected ones.

Moreover, for the GJR-GARCH and EGARCH models the parameter γ is significantly different from zero, which indicates about the presence of the leverage effect for Brent crude oil returns.

The analysis of natural gas data revealed that the EGARCH model generated the lowest mean squared error (MSE) among the three models. However, when considering the penalization of model parameters and the best fit for the training data, the GARCH model outperformed the other two models. Moreover, p-value for γ in GJR-GARCH model is 0.395 (greater than significance level $\alpha = 0.05$), which shows that the parameter responsible for the leverage effect is equal to zero. This implies that the GJR-GARCH model for natural gas is equivalent to the simple GARCH model, which explains why the parameters for the GARCH and GJR-GARCH models are almost identical.

However, this raises the question of whether the leverage effect is present in the natural gas time series. Specifically, the inverse leverage effect can be observed in natural gas time series, where energy commodity prices tend to become more volatile as prices increase[23]. While this topic is beyond the scope of this study, it is crucial to test for the presence of this effect in model construction to avoid inefficient estimation or biased outcomes. Therefore, future studies can explore this aspect in detail.

Overall, the best fit for volatility of natural gas returns is GARCH which does not take into account the leverage effect and it is the best choice for volatility modeling, especially when taking into account the complexity and long-range dependence of the data.

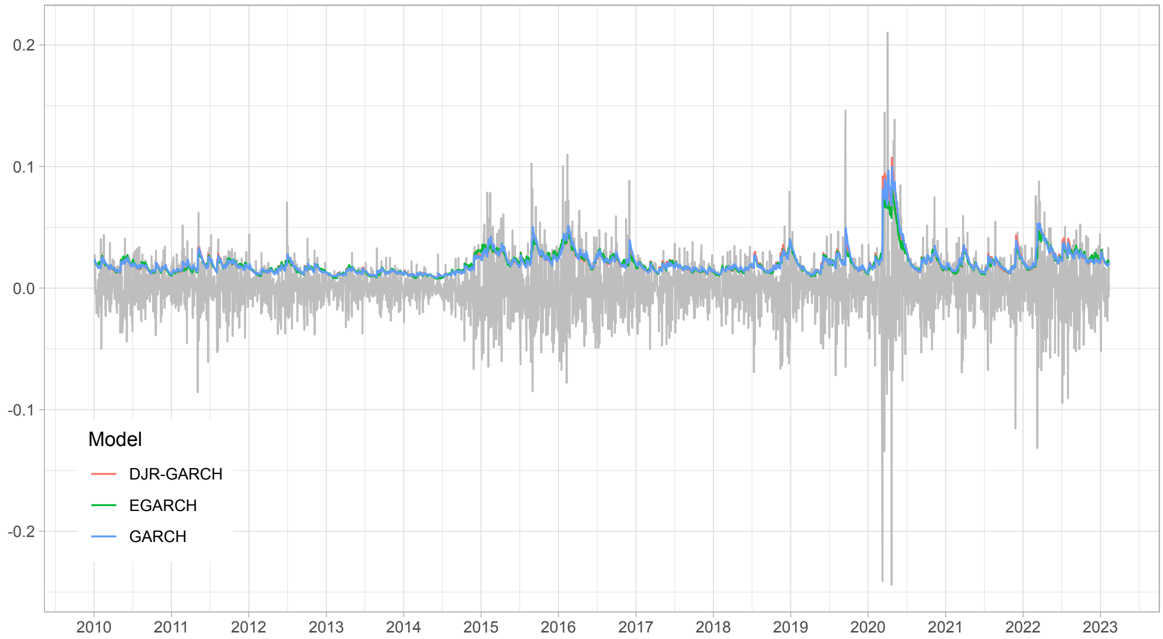
Figure 4.4 shows fitted volatility for both time series by three models. All of them are able to accurately capture the changes in volatility and the clustering of volatility in the original assets. Moreover, the models are able to react appropriately to periods of high volatility, indicating that they are effective in modeling the dynamics of volatility in these energy markets.

4.3 Models verification

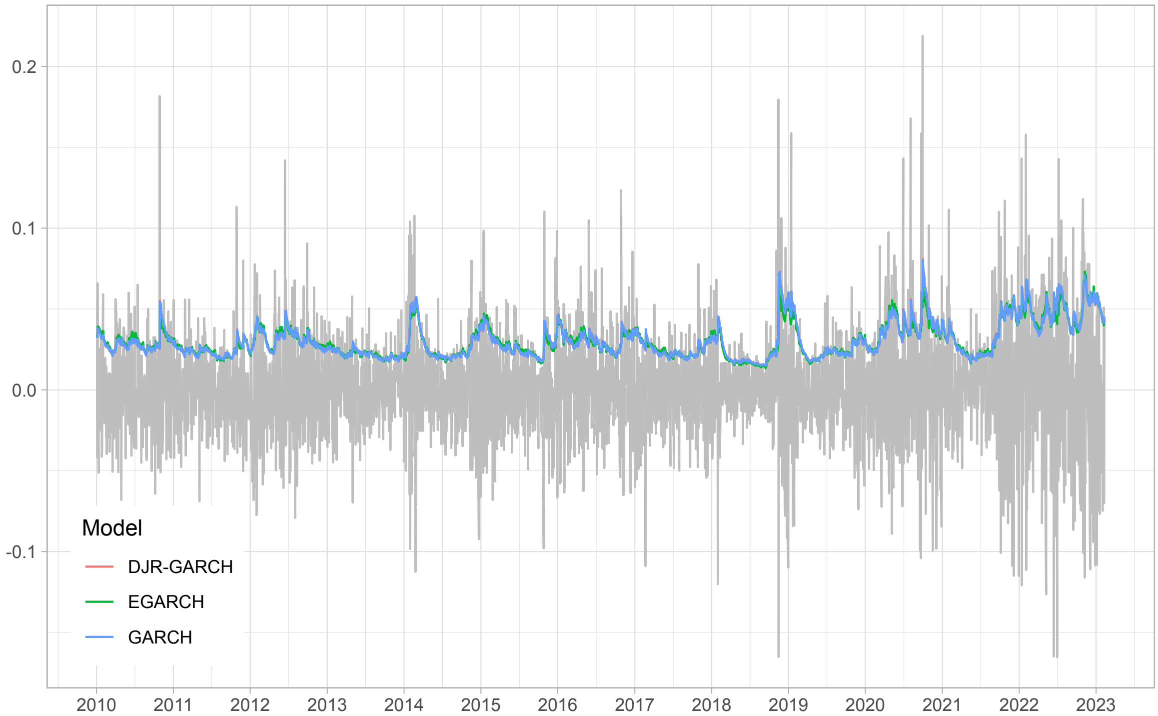
As it previously stated, for model's verification, it should meet three assumptions. According to results presented in table 4.4 standardized residuals for both time series have close to 0 means and standard deviation almost equals to 1.

Although Fig. 4.5 appears to indicate constant variability in the standardized residuals, the p-values for the LM test statistic presented in Table 4.4 are low, leading to the rejection of the null hypothesis about the homoscedasticity of standardized residuals for all confidence levels.

Figures 4.6 depict the absence of autocorrelation for standardized squared residuals with lags bigger than 0 (autocorrelation at point 0 represents the correlation with itself and expectedly equals to 1)



(a) Fitted volatility (GARCH, GJR-GARCH, EGARCH models) with daily return rate for Brent crude oil



(b) Fitted volatility (GARCH, GJR-GARCH, EGARCH models) with daily return rate for natural gas

Figure 4.4: Models fitting with daily return rate for Brent crude oil and natural gas, 2010-2023

Table 4.4: Mean and standard deviation for standardized residuals based on model predictions

	Mean	**Standard dev.**	**LM test statistic[a]**
Brent crude oil(EGARCH)	0.0066	0.9994	7018.8(0.000)
Natural gas(GARCH)	0.0168	1.0051	5403.6(0.000)

Note:[a] $\alpha = 0.05$

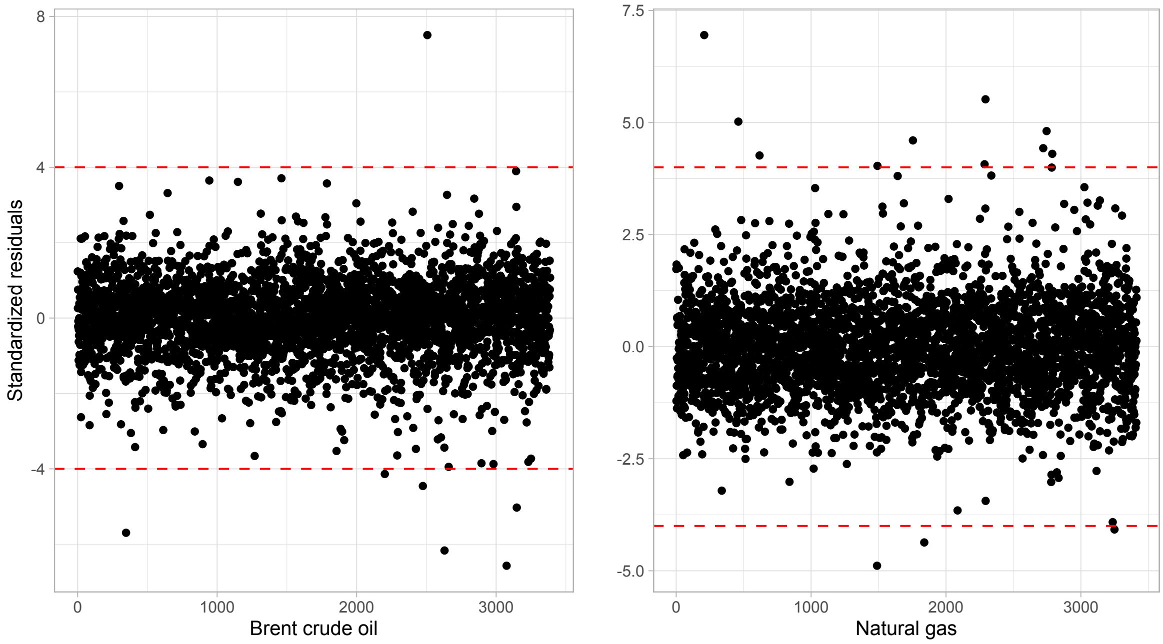


Figure 4.5: Standardized residuals for Brent crude oil and natural gas

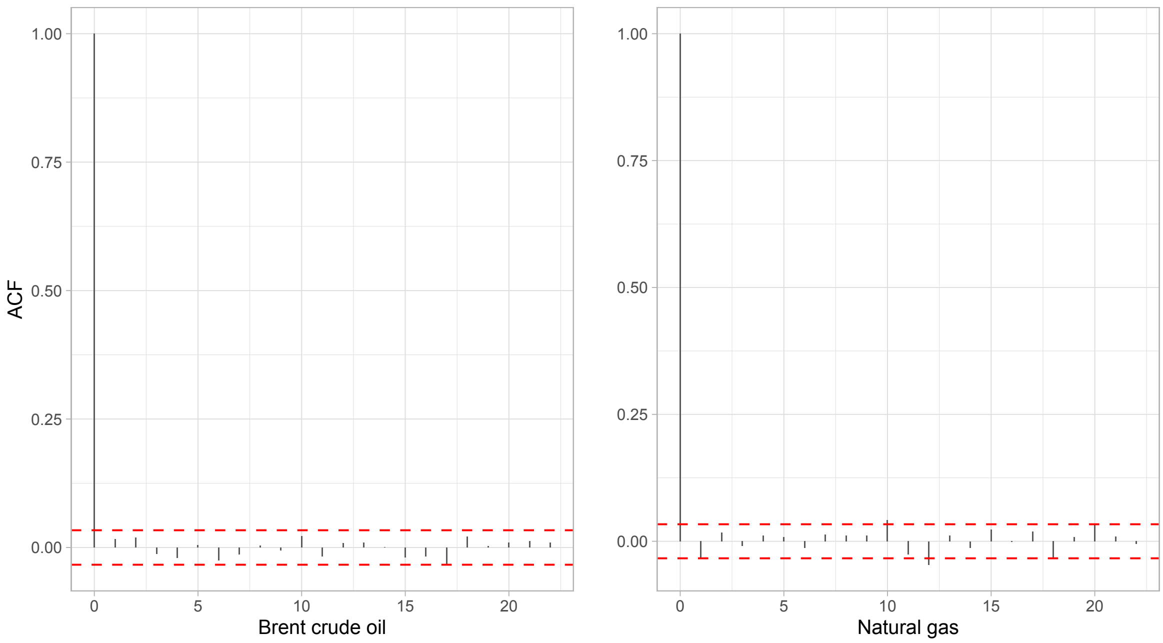


Figure 4.6: ACF functions for standardized squared residuals for Brent crude oil and natural gas

4.4 VaR calculation

In the previous chapter of this work, Value at Risk (VaR) was introduced as a statistical measure for assessing downside risk based on the current position of a financial asset. Using historical data, VaR can be estimated by taking the 5% quantile value. For Brent crude oil this value is -0.03427 and for natural gas this is -0.04834.

For Brent crude oil, this VaR value indicates that there is a 95% probability that the maximum potential loss in the value of Brent crude oil over the priod of 2010-2023 will not exceed -3.43%. In other words, if an investor holds Brent crude oil with a value of $100,000, there is a 95% chance that the maximum expected loss over this period will not exceed $3,427.

Similarly, for natural gas, the VaR value of -0.0483 indicates that there is a 95% probability that the maximum potential loss in the value of natural gas over the same period will not exceed -4.83%. Using the same example as above, if an investor holds natural gas with a value of $100,000, there is a 95% chance that the maximum expected loss in 2010-2023 interval will not exceed $4,830.

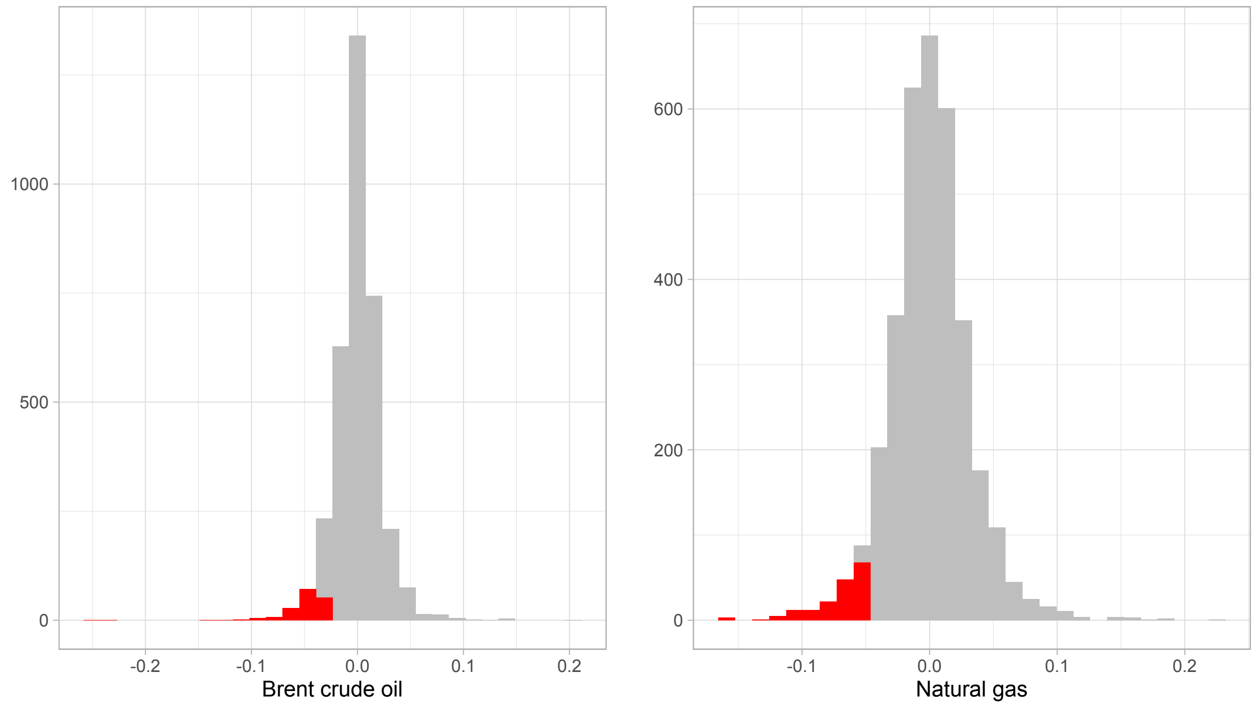


Figure 4.7: Distribution of daily returns for Brent crude oil and natural gas with returns lower than 5% quantile

Using the GARCH models, it is possible to predict conditional volatility and based on that calculate VaR. Fig.4.8 shows VaR comparison between the delta-normal approach and using GARCH modeling for both time series. In this case and later in this work, the EGARCH model for Brent crude oil and the GARCH for natural gas is used as the best fitting models for respective time series.

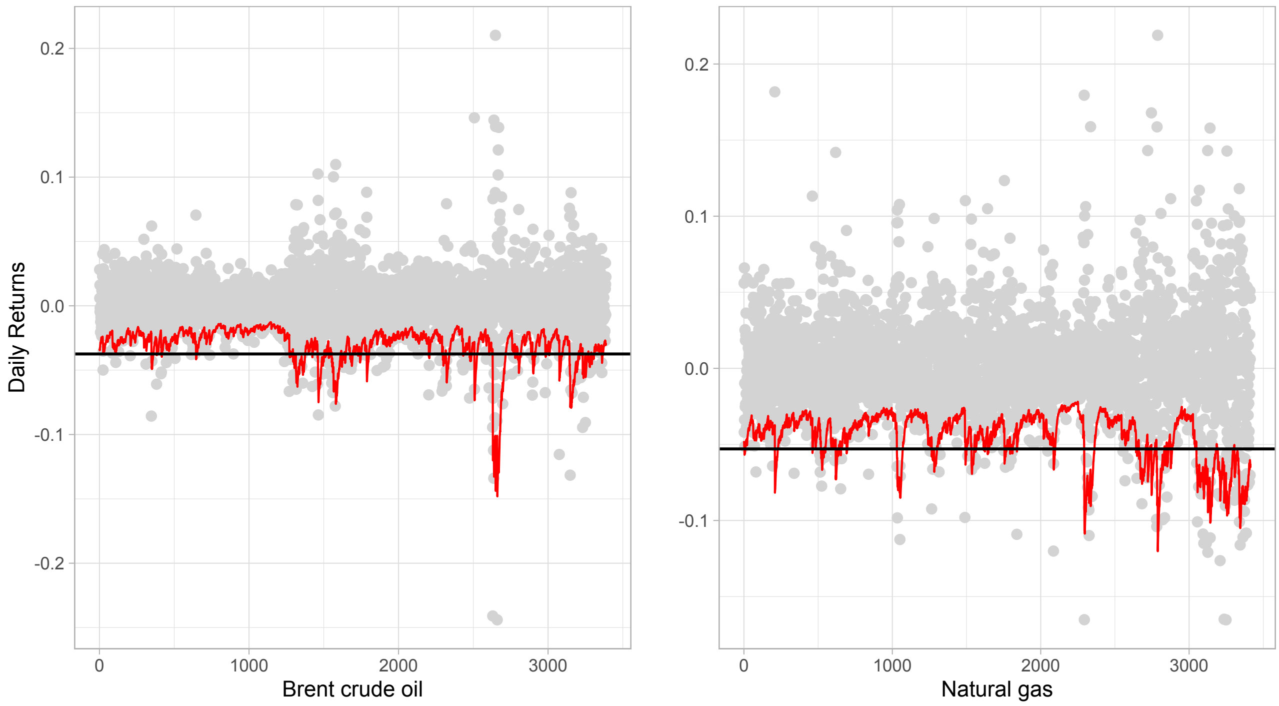


Figure 4.8: Value at risk comparison (Delta-normal vs GARCH) for Brent crude oil and natural gas

One of the properties be derived from the definition of VaR and its values is that value at risk tends to increase during the period of high volatility reflecting the higher potential losses. In the Fig.4.8 models estimation was produced on the whole historical time series, while real-world investors do not have this information but want to predict VaR. For the purpose of this study, the last 500 observations of both time series are taken as a test set and rolling moving 1-step ahead forecast of the conditional standard deviation is performed. Models parameters are re-estimated every 50 observations.

Results of rolling models for value at risk are presented in Fig.4.9, where red dots are actual daily returns that are lower than predicted VaR. For Brent crude oil number of daily returns lower than VaR for 500 last observations is 40, while for natural gas this number is 37.

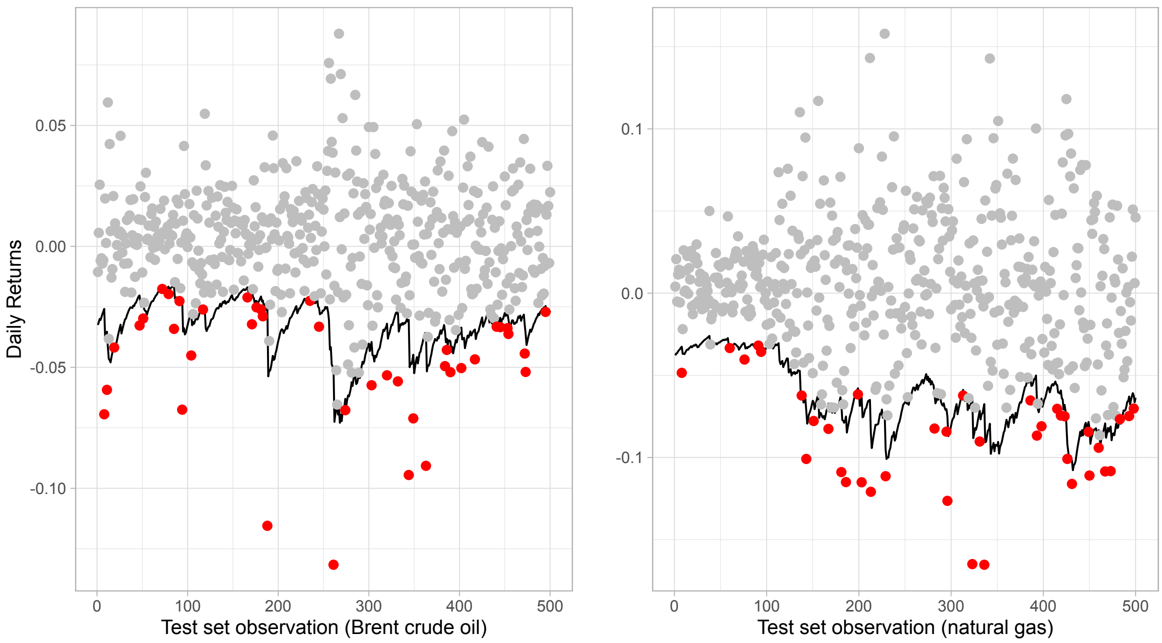


Figure 4.9: Last 500 daily returns for Brent crude oil and natural gas with predicted VaR

Conclusion

The present study investigates the volatility of futures returns of natural gas and crude oil, and the findings reveal that these two markets exhibit high volatility. This is attributed to various political and economic events that directly and indirectly influence the supply and demand of these commodities. Despite the volatility, natural gas and crude oil remain essential commodities that hold a leading position in global and European Union consumption. This highlights the importance of analyzing and predicting the volatility in these markets, particularly for the European Union, where energy dependency on these sources is extremely high.

The GARCH models were found to be effective in capturing the structure of volatility, sudden changes, and clustering of volatility for both time series. All three models, GARCH, GJR-GARCH, and EGARCH, were able to appropriately react to periods of high volatility for Brent crude oil, indicating their effectiveness in modeling the dynamics of this energy market. The EGARCH(1,1) model was found to be the best fitting model for crude oil data. As for natural gas, GARCH(1,1) appears to be the best for training data. However, during models fitting the question of presence of leverage effect has appeared and this can be studies in detail in future works. By utilizing these models and the VaR statistic, it becomes possible to predict the risks associated with these two markets and make adequate adjustments to individual policies.

Furthermore, this study suggests potential avenues for future research, including gaining a deeper understanding of the complex effects in energy markets(such as inverse leverage effect), as well as detailed exploring financial risk management, where discussed properties of crude oil and natural gas market along with GARCH models for volatility prediction can be used to measure risk associated with these commodities and its better prediction.

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