Summary of NPVGO and the Constant Growth Model[1]

Notation:

P_0	Price of the stock at time 0
D_t	Expected dividend at time t
E_t	Expected earnings at time t
I_t	Expected Investment at time t
b	plowback ratio
ROE	Return on equity
r	Discount rate

Note: P_0, D_t, E_t, I_t are assumed to be a per-share basis.

Assumptions: plowback (b) and return on equity (ROE) are constant over time.

Because dividends are the cash flows to equity:

$$P_0 = \frac{D_1}{1+r} + \frac{D_2}{(1+r)^2} + \frac{D_3}{(1+r)^3} + \cdots \tag{1}$$

Note that growth in earnings is given by

$$g = \mathrm{ROE}b$$

and is constant over time.

Under the assumptions that b and ROE are constant, we have derived a formula for (1). Because

$$D_t = (1-b)E_t = (1-b)(1+g)^{t-1}E_1,$$

the growing perpetuity formula can be applied to (1) to obtain

$$P_0 = \frac{(1-b)E_1}{r - \mathrm{ROE}b}. \tag{2}$$

Claim:

$$P_0 = \frac{E_1}{r} + \mathrm{NPVGO} \tag{3}$$

[1] Notes for Finance 604 & 612 prepared by Jessica A. Wachter.

where NPVGO is the net present value of growth opportunities. Equation 3 is very general; it holds whether b and ROE are constants or not. We will show that (2) and (3) are the same in the constant growth case and derive an intuitive formula for NPVGO.

This firm has an opportunity to make a positive NPV investment each year. In year t, the firm makes investment I_t. This investment pays off ROEI_t in all future years. The NPV of this investment is

$$\begin{aligned} NPV_t &= -I_t + \frac{\text{ROE}I_t}{1+r} + \frac{\text{ROE}I_t}{(1+r)^2} + \cdots \\ &= -I_t + \frac{\text{ROE}I_t}{r} \\ &= I_t\left(\frac{\text{ROE}}{r} - 1\right) \end{aligned}$$

What is investment I_t? Because of our assumption that the plowback is constant

$$I_t = bE_t = bE_1(1+g)^{t-1}$$

So

$$NPV_t = bE_1\left(\frac{\text{ROE}}{r} - 1\right)(1+g)^{t-1}$$

The NPVGO is the sum of the net present value of all these growth opportunities, discounted back to the present:

$$\begin{aligned} \text{NPVGO} &= \frac{NPV_1}{1+r} + \frac{NPV_2}{(1+r)^2} + \cdots \\ &= bE_1\left(\frac{\text{ROE}}{r} - 1\right)\left[\frac{1}{1+r} + \frac{1+g}{(1+r)^2} + \cdots\right] \end{aligned}$$

applying the growing perpetuity formula:

$$\text{NPVGO} = bE_1\left(\frac{\text{ROE}}{r} - 1\right)\frac{1}{r-g}.$$

Is this consistent with equation (2)? In fact it is. Because

$$P_0 = \frac{E_1}{r} + \text{NPVGO},$$

and $g = b\text{ROE}$, we have

$$P_0 = \frac{E_1(r-g) + E_1 g - rbE_1}{r(r-g)}$$

Canceling the terms $E_1 g$ with each other, and canceling r in the numerator and denominator gives us (2).