

Topic 4: Applications to Equity Valuation

(a) Using Present Value Methods to Value Equity

We've determined the value of fixed income securities (bonds) using present value methods. Can we apply the same to stocks?

Definition *Equity: The residual claim on the assets of a corporation.*

Namely, equity is what is left after the bondholders have been paid. If you own equity in a company, you are a partial owner of that company. Another word for equity is common stock.

1. Dividend.
2. Capital Gain (or price appreciation).

Notation:

- P_0 = Current price per share.
- P_1 = Price per share next year.
- D_1 = Dividend per share next year.

Prices are ex-dividend (namely, P_0 is the price just after the current dividend has been paid).

Holding period return (**HPR**):

$$\begin{aligned} r &= \frac{V_1 - V_0}{V_0} \\ &= \frac{D_1 + P_1 - P_0}{P_0} \\ &= \frac{D_1}{P_0} + \frac{P_1 - P_0}{P_0} \end{aligned}$$

The term D_1/P_0 is the **dividend-yield**. The term $(P_1 - P_0)/P_0$ is the percent price appreciation (or percent capital gain).

We can invert the formula for r to write the price today given D_1 and P_1 :

$$1 + r = \frac{D_1 + P_1}{P_0}$$

so that

$$P_0 = \frac{D_1 + P_1}{1 + r}.$$

This also makes sense based on present-value reasoning. Price today is the present discounted value of payments one period from now.

However, this formula is not satisfying - how do we know what P_1 is? We could apply the same formula to P_1 and find:

$$P_1 = \frac{D_2 + P_2}{1 + r}.$$

Substituting this in, we find:

$$P_0 = \frac{D_1}{1 + r} + \frac{D_2}{(1 + r)^2} + \frac{P_2}{(1 + r)^2}.$$

More generally:

$$P_t = \frac{D_1}{1 + r} + \frac{D_2}{(1 + r)^2} + \cdots + \frac{D_t}{(1 + r)^t} + \frac{P_t}{(1 + r)^t}.$$

Under reasonable conditions for P_t (where rate of growth of P is less than r for a sufficiently high period t) we can take limits to find:

$$P_t = \frac{D_1}{1 + r} + \frac{D_2}{(1 + r)^2} + \cdots + \frac{D_t}{(1 + r)^t} + \cdots$$

This is a formula for the price given the (possibly infinite) stream of future dividends.

Note: this is **price per share**. But we can also value the entire firm if given total dividends.

Note: “dividends” can be any cash flow from the corporation to investors. If the company liquidates, the liquidation value is treated as one big dividend. If another company buys the shares for cash, that is also a “dividend” in this analysis. *The bottom line: we value equities the same way we value any other asset – we discount the future stream of cash flows.*

In principle, this sounds simple. However, there is an important difference between bonds and equities: equity cash flows are uncertain. For bonds, remember that cash flows are fixed – bonds are fixed-income securities. However, for equities, while we can estimate dividends, we don’t know what they will be for sure.

Instead, we use *expected dividends* in the numerator. In the denominator, the r we use will in general not be the same r as for bonds. In the second half of the course, we will pay a lot of attention to finding the appropriate r . For now, we take r to be the rate of return on an investment of comparable risk to the firm’s dividend process. Typically (for reasons we will see), this will be higher than the rate available on bonds or at the bank.

We are about to get into the specifics of valuing equity. The approach of valuing equity through its cash flows takes some getting used to, and you may find yourself wondering “do

people really value equity this way?" YES! As we will learn, the principle of using multiples (such as the price-earnings ratio) to value equity all comes down to thinking of equity as a stream of future cash flows. This approach is the standard in the financial industry.

(b) Applying Infinite-Horizon Formulas

1. Constant Dividend Growth

How do we determine expected dividends? We build a model. Our first model will be simple, but nonetheless will capture the important feature that equity is a long-lived asset in which the cash flows (dividends) grow over time.

Let D_1 be expected dividends next year, and g the growth rate in expected dividends. Then the dividend stream is as follows:

$$\begin{aligned} D_2 &= D_1(1+g) \\ D_3 &= D_1(1+g)^2 \\ &\vdots \\ D_t &= D_1(1+g)^{t-1} \\ &\vdots \end{aligned}$$

And the price today is

$$P_0 = \frac{D_1}{1+r} + \frac{D_1(1+g)}{(1+r)^2} + \cdots + \frac{D_1(1+g)^{t-1}}{(1+r)^t} + \cdots$$

By applying the formula for a **growing perpetuity**, we arrive at

$$P_0 = \frac{D_1}{r-g}.$$

Recall that the formula for the growing perpetuity assumes $r > g$.

Example Assume: $D_1=\$3$, $g=10\%$, and $r=15\%$. Then:

$$P_0 = \frac{3}{.15 - .1} = \$60.$$

An aside: you may say – plenty of companies grow at a rate $> 15\%$! Often, you hear forecasts of 50% growth, but not of 50% growth rates that last forever! It may be unrealistic to assume a constant growth rate forever for some companies. Companies typically grow quickly when they are young and more slowly when they get old.

2. OPTIONAL: *Differential Growth*

Example Consider a company whose dividends are expected to grow at 15% for the next five years, and 10% after that. If the dividend next year is expected to be \$2.3/share and the discount rate is 15%, what should price/share be today?

More generally: a company grows for N years at g_1 , then at g_2 . Assume $r > g_2$.

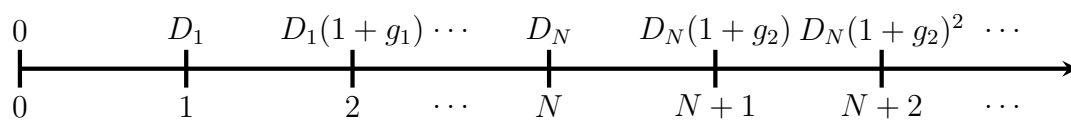
Phase I:

$$\begin{aligned} D_1 &= D_1 \\ D_2 &= D_1(1 + g_1) \\ &\vdots \\ D_N &= D_1(1 + g_1)^{N-1}. \end{aligned}$$

Phase II:

$$\begin{aligned} D_{N+1} &= D_N(1 + g_2) = D_1(1 + g_1)^{N-1}(1 + g_2) \\ D_{N+2} &= D_N(1 + g_2)^2 = D_1(1 + g_1)^{N-1}(1 + g_2)^2 \\ &\vdots \end{aligned}$$

To illustrate the cash flows:



where $D_N = D_1(1 + g_1)^{(N-1)}$.

It is easiest to value each of the phases separately, and then find:

$$P_0 = \text{PV}(\text{Ph I}) + \text{PV}(\text{Ph II}).$$

We will start with phase I:

$$\text{PV, Ph I} = \frac{D_1}{1+r} + \frac{D_1(1+g_1)}{(1+r)^2} + \cdots + \frac{D_1(1+g_1)^{N-1}}{(1+r)^N}.$$

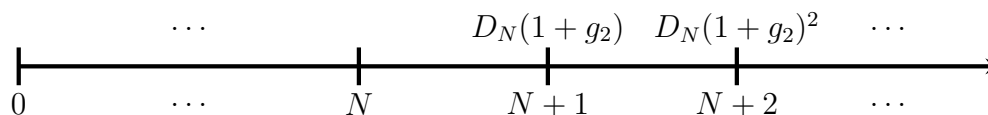
For the present value of the **first phase**, we use the **growing annuity** formula:

$$\text{PV, Ph I} = \begin{cases} D_1 \left(\frac{1}{r-g_1} - \frac{1}{r-g_1} \frac{(1+g_1)^N}{(1+r)^N} \right) & r \neq g_1 \\ \frac{D_1}{1+g_1} N & r = g_1 \end{cases}$$

Now for phase 2:

$$\text{PV, Ph 2} = \frac{D_N(1+g_2)}{(1+r)^{N+1}} + \frac{D_N(1+g_2)^2}{(1+r)^{N+2}} + \dots$$

To find the PV of **Phase II**, we use the **growing perpetuity** formula. However, we need to adjust for the fact that the perpetuity is delayed and be careful about when the growth occurs. Plot just Phase II dividends:



Therefore:

$$\text{PV(Ph II at time } N) = \frac{D_N(1+g_2)}{r-g_2}.$$

Discounting back to time 0:

$$\text{PV(Ph II)} = \frac{1}{(1+r)^N} \frac{D_N(1+g_2)}{r-g_2}.$$

This is the same as our delayed perpetuity formula from Topic 2! Note, however, that we needed to be careful about the form of the time $N+1$ cash flow.

Substituting in for D_N :

$$\text{PV(Ph II)} = \frac{D_1(1+g_1)^{N-1}(1+g_2)}{(1+r)^N(r-g_2)}.$$

Going back to our example: $D_1 = 2.3, g_1 = 15\%, N = 5, g_2 = 10\%, r = 15\%$

$$\begin{aligned} P_0 &= \frac{2.3}{1.15}(5) + \frac{2.3(1.15)^4(1.10)}{(1.15)^5(0.15-0.10)} \\ &= 10 + 2.20/0.05 = \$54. \end{aligned}$$

This is a very useful and flexible method to value stocks. You can see how it would change if, say, there were three periods rather than two. We always have the same three steps:

1. Identify growth stages.
2. Calculate the PV of each stage.
3. Sum together.

(c) Determining Dividend Growth

Where does g come from?

In this subsection, we will relate g to the firm's profitability. We will assume funds for investment are generated internally (rather than from issuing stock or bonds – this assumption is not critical). Under this assumption, we have the following identity:

$$\text{Earnings} = \text{Dividends} + \text{Retained earnings}.$$

Definition *Plowback ratio*: the proportion of earnings plowed back into the company.

We will use the notation b :

$$b = \frac{\text{Retained earnings}}{\text{Earnings}}.$$

RWJ calls this the “retention ratio.” Let

$$E = \text{Earnings per share}$$

Then bE is retained earnings. Because

$$E = D + bE,$$

we can write $D = (1 - b)E$. Dividends (D) are the part of earnings not plowed back. Thus to understand dividends, we need to understand earnings. A firm's earnings will be determined by the amount of investment and how profitable the investment is. A useful measure of profitability is ROE:

Definition *Return on equity (ROE)* is defined as:

$$\text{ROE} = \frac{\text{Earnings}}{\text{Book value of equity}}$$

Assuming ROE doesn't change from one year to the next (specifically, if g_{t+1} is the growth rate between t and $t + 1$, we need $\text{ROE}_t = \text{ROE}_{t+1}$), we can relate ROE to earnings growth:

Claim: $g = \text{ROE}b$

Proof: By definition:

$$\begin{aligned} \text{Equity}_{t+1} &= \text{Equity}_t + (\text{Retained Earnings})_t \\ &= \text{Equity}_t + bE_t. \end{aligned}$$

Note: think of equity as equity/share and retained earnings as RE/share.

Then multiply both sides of this equation by ROE:

$$\begin{aligned}(\text{ROE})(\text{Equity}_{t+1}) &= (\text{ROE})(\text{Equity}_t) + (\text{ROE})bE_t \\ E_{t+1} &= E_t + b\text{ROE } E_t.\end{aligned}$$

This implies:

$$\frac{E_{t+1}}{E_t} = 1 + b\text{ROE},$$

or $g = b\text{ROE}$. Thus, growth in the firm is equal to the fraction of earnings plowed back, multiplied by the profitability of those earnings.

Example Assume $\text{ROE} = 15\%$, \$100 invested (this is equity at time 1), and $b = 0.6$.

At $t = 1$:

$$E_1 = \$100(0.15) = \$15.$$

Now:

$$\text{Retained Earnings} = 0.6(\$15) = \$9.$$

Therefore:

$$\begin{aligned}\text{Equity}_2 &= \$100 + \$9 = \$109. \\ E_2 &= \$109(0.15) = \$16.35. \\ g = \frac{E_2}{E_1} - 1 &= \frac{16.35}{15} - 1 = 9\% = b\text{ROE}.\end{aligned}$$

It is also the case that:

$$\begin{aligned}E_2 &= \$100(0.15) + \$9(0.15) \\ &= E_1 + E_1 b\text{ROE}.\end{aligned}$$

We can use the relation between growth, plowback, and ROE to gain insight into our valuation formulas.

- Assume b and ROE are constant. Then $g = b\text{ROE}$ is also dividend growth!

$$E_{t+1} = (1 + g)E_t.$$

This implies:

$$(1 - b)E_{t+1} = (1 + g)(1 - b)E_t.$$

Which then implies:

$$D_{t+1} = (1 + g)D_t.$$

- Recall the formula under constant growth:

$$P_0 = \frac{D_1}{r - g}.$$

- Now that we've decomposed growth, we can substitute in to arrive at the following:

$$P_0 = \frac{E_1(1 - b)}{r - bROE}.$$

Note: this works as long as $r > bROE$.

I want to step back and discuss this assumption. Suppose you and I saw a business with ROE sufficiently high that $bROE > r$. This could persist for a few years, but eventually everyone would want to go into such a profitable business. Over time, ROE would fall.

We also now have an equation for the price-earnings ratio:

$$\frac{P_0}{E_1} = \frac{(1 - b)}{r - bROE}.$$

Going back to the constant growth model, note that if nothing is plowed back into the company, $b = 0$, $r = ROE$, we have zero growth, and we return to the perpetuity formula:

$$P_0 = \frac{E_1}{r}.$$

This company is known as a *cash cow*. ROE is the firm's internal return, and r is the external return. When $ROE = r$, as in this case, it doesn't matter for price whether earnings are kept in the firm or not. There's no investment – the firm is just being milked for its cash. Why? As we can see from this formula, there are two effects of b on price:

1. $b \uparrow \Rightarrow P_0 \downarrow$ because you are paying less cash out.
2. $b \uparrow \Rightarrow P_0 \uparrow$ because of higher growth in earnings.

Which effect wins? We can figure this out by taking the derivative:

$$\begin{aligned} \frac{\partial P_0}{\partial b} &= \frac{-E_1(r - bROE) + (ROE)E_1(1 - b)}{(r - bROE)^2} \\ &= E_1 \frac{bROE - r - (ROE)b + ROE}{(r - bROE)^2} \\ &= E_1 \frac{ROE - r}{(r - bROE)^2}. \end{aligned}$$

Because the denominator is always positive, if:

- $\text{ROE} > r \Rightarrow$ value increases with increasing plowback (b).
- $\text{ROE} = r \Rightarrow$ increasing plowback (b) has no effect on value.
- $\text{ROE} < r \Rightarrow$ value decreases with increasing plowback (b). Don't grow company!
Pay money out to shareholders!

Intuition: r is the rate of return on \$ invested outside the firm. ROE is the rate of return on \$ invested inside the firm.

Example Suppose: $r = 12\%$, $\text{ROE} = 10\%$, $b = 0.6$, and $E_1 = \$10$. Then:

$$P_0 = \frac{10(1 - 0.6)}{0.12 - 0.6(0.1)} = \frac{4}{0.06} = \$66.67.$$

As a manager, how can you raise your stock price? Set $b = 0$ so nothing is retained. Then:

$$P_0 = \frac{10}{0.12} = 83.33.$$

You are **returning cash to shareholders** so they can use it more productively. This firm is also a **takeover target**. A raider can raise the price just by changing the amount it pays out.

This analysis teaches us not to confuse company growth with higher value. At least in the constant growth framework, whether earnings reinvestment raises value depends on whether the discount rate r is below or above k . So, growth can actually be bad and lower value!

(d) Net Present Value of Growth Opportunities (OPTIONAL)

As we discussed last time, growth doesn't necessarily lead to higher value. Only when growth comes in the form of positive NPV investments does it increase value. There's a formula that makes this explicit. We want to show that:

$$P_0 = \frac{E}{r} + \text{NPVGO},$$

where NPVGO is the net present value of growth opportunities. Note that E/r is the cash cow value. When we use this valuation formula, we get the same answer as if we calculate the PV of dividends:

$$P_0 = \frac{D_1}{1+r} + \frac{D_2}{(1+r)^2} + \dots$$

\Rightarrow That's a surprise.

The NPVGO equation is very general – it holds whether b and ROE are constants or not. We will show that the above two equations are the same in the constant growth case and derive an intuitive formula for NPVGO. We will demonstrate this for two cases:

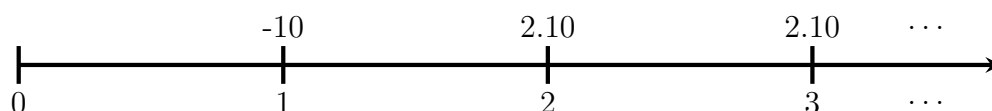
1. Single growth opportunity.
2. Growth opportunities every year, with constant b and ROE.

From this, you will see how to apply the NPVGO formula more generally.

Case 1: Single Growth Opportunity

Example If a firm undertakes no investment, $E = \$10$ (per share) in perpetuity.

Now assume we have a single investment opportunity (say, a marketing campaign) at $t = 1$. Cost = \$10 per share, and earnings are expected to increase by \$2.10 per share in all subsequent periods. Assume $r = 10\%$.



Using the NPVGO formula: at $t = 1$:

$$\text{NPV}_1 = -10 + 2.1/0.1 = \$11.$$

Note that this is a positive NPV opportunity!

At $t = 0$:

$$\text{NPVGO} = \frac{11}{1 + r} = \$10$$

So using the formula for the total price per share:

$$P_0 = \frac{10}{0.1} + 10 = \$110.$$

This is the same answer as we would have gotten by explicitly solving for the present value of future dividends.

To see this, let's calculate the earnings assuming the firm makes the investment:

$$\begin{aligned} E_1 &= \$10, \\ E_2 &= \$10 + \$2.1 = \$12.1 \\ E_3 &= \$10 + \$2.1 = \$12.1 \end{aligned}$$

and so forth. How about dividends? Let

$$I_t = \text{Investment at time } t$$

Investment is another name for retained earnings. Then

$$D_t = E_t - I_t,$$

where

$$\begin{aligned} I_1 &= \$10 \\ I_2 &= 0 \\ I_3 &= 0 \\ &\vdots \end{aligned}$$

It follows that

$$\begin{aligned} D_1 &= 0, \\ D_2 &= \$12.1 \\ D_3 &= \$12.1 \\ &\vdots \end{aligned}$$

Price is the present discounted value of these dividends:

$$\begin{aligned} P_0 &= \frac{0}{1+r} + \frac{12.1}{(1+r)^2} + \frac{12.1}{(1+r)^3} + \dots \\ &= \frac{1}{1+r} \frac{12.1}{r} \\ &= \frac{1}{1.1} 121 \\ &= 110, \end{aligned}$$

so they are the same.

Case 2: Growth Opportunity Every Year – Constant b and ROE

Example We already know that the constant growth model, $g = \text{ROE}b$ and:

$$P_0 = \frac{(1-b)E_1}{r - \text{ROE}b}. \quad (1)$$

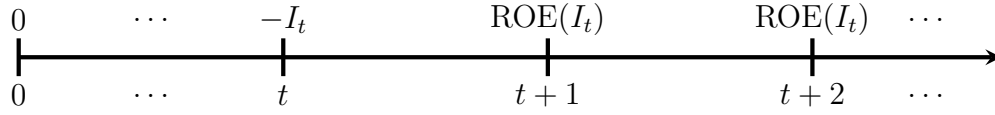
But it is also the case that:

$$P_0 = \frac{E_1}{r} + \text{NPVGO}. \quad (2)$$

How can this be? This firm has a new positive NPV opportunity each year. It invests:

$$I_t = bE_t = bE_1(1+g)^{t-1}.$$

This investment, at time t , pays off $\text{ROE}(I_t)$ every year in perpetuity:



The value of this investment as time t equals

$$\begin{aligned} \text{NPV}_t &= -I_t + \frac{\text{ROE}(I_t)}{1+r} + \frac{\text{ROE}(I_t)}{(1+r)^2} + \dots \\ &= -I_t + \frac{\text{ROE}(I_t)}{r} \\ &= I_t \left(\frac{\text{ROE}}{r} - 1 \right) \end{aligned} \tag{3}$$

$$= bE_1(1+g)^{t-1} \left(\frac{\text{ROE}}{r} - 1 \right) \tag{4}$$

$$= bE_1 \left(\frac{\text{ROE}}{r} - 1 \right) (1+g)^{t-1} \tag{5}$$

NPVGO is the sum of the NPV of all these growth opportunities, discounted to the present:

$$\begin{aligned} \text{NPVGO} &= \frac{\text{NPV}_1}{1+r} + \frac{\text{NPV}_2}{(1+r)^2} + \dots \\ &= bE_1 \left(\frac{\text{ROE}}{r} - 1 \right) \left[\frac{1}{1+r} + \frac{1+g}{(1+r)^2} + \dots \right]. \end{aligned}$$

Applying the growing perpetuity formula:

$$\text{NPVGO} = bE_1 \left(\frac{\text{ROE}}{r} - 1 \right) \frac{1}{r-g}.$$

[While there seems to be a lot going on here, all we are doing is applying the growing perpetuity formula from Section 2 with a cash flow “ C ” that takes a complicated form – $C = bE_1 \left(\frac{\text{ROE}}{r} - 1 \right)$.]

Is this consistent with equation (2)? In fact, it is:

$$P_0 = \frac{E_1}{r} + \text{NPVGO},$$

and because $g = b\text{ROE}$, we have:

$$P_0 = \frac{E_1(r-g) + E_1g - rbE_1}{r(r-g)}.$$

Canceling the terms E_1g with each other, and canceling r in the numerator and denominator gives us (2).

What this reveals is that growth implies an investment every period. As we discussed on the first day, what determines if an investment increases value is whether it is positive NPV.

Using the formula

$$\text{NPVGO} = bE_1 \left(\frac{\text{ROE}}{r} - 1 \right) \frac{1}{r - g}.$$

we see directly that positive NPV projects are those where $\text{ROE} > r$. Note that it is possible to have growth that lowers the value of the company.

Note also that the price-earnings ratio pops out as an interesting measure:

$$\frac{P_0}{E_1} = \frac{1}{r} + \frac{\text{NPVGO}}{E_1}.$$

So, the P/E ratio tells us something about the firm's growth opportunities and discount rate: if we have two firms of roughly the same risk level, if one is priced more highly, P/E tells us this pricing is due to these positive NPV projects the firm will undertake.

We have now completed our discussion of valuation. The next topic is using these techniques for corporate decision making.