Topic 5: Capital Budgeting: NPV vs. Internal rate of return (IRR)

A well-known survey of CFOs finds that 75% of CFOs use the NPV technique. RWJ chapter 5 discusses alternatives to NPV and says thy they are wrong.

Here we are going to focus on the alternative to NPV which is a little bit right – the IRR.

NPV rule $\Longrightarrow$ Accept if NPV > 0. Otherwise, reject.

IRR rule $\Longrightarrow$ Accept if IRR $> r$ (discount rate). Otherwise, reject.

(a) Definition of IRR

Definition *The Internal Rate of Return is the rate of return r such that the* NPV $= 0$*:*

$$0 = C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \cdots$$

Sometimes, the IRR rule works very well.

Example Assume $C_0 = -\$100$ and $C_1 = \$110$:

$$\text{NPV} = -100 + \frac{110}{1+r}.$$

So:

$$-100 + \frac{110}{1+r} = 0 \text{ implies}$$

$$\text{IRR} = \frac{110}{100} - 1 = 10\%.$$

If $r = 8\%$:

- IRR rule says accept.
- NPV rule also says accept because $r = 8\% \Rightarrow \text{NPV} = 1.85 > 0$.

Remarks:

- We've already come across an IRR in this course: The YTM is the IRR for buying a coupon bond!

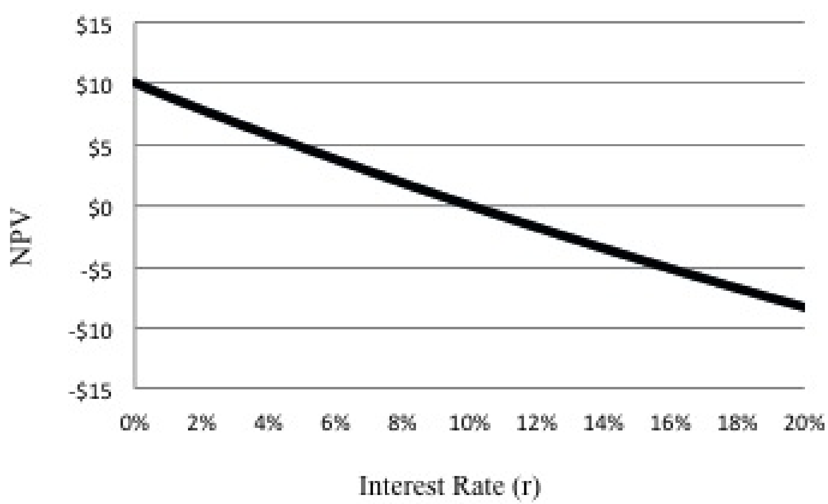
  Remember that the YTM was somewhat of a problematic notion. We wanted to use it as a yardstick for comparing bonds as investments. However, it was a flawed yardstick, as you only receive the YTM if you reinvest at the YTM – which may not happen!

- Similarly, people like to think of IRR as the rate of return on their investment. However, this holds true only if you can invest the intermediate cash flows at the IRR.

  However, this assumption is even less realistic when dealing with a project than when dealing with a bond. A bond is an instrument traded on the market. If rates stay approximately fixed, you might be able to reinvest your cash flows at the same rate. However, we are thinking of IRR as specific to a project in a firm. There may be no wa to reinvest cash flows at the IRR.

Thus thinking about IRR as a rate of return does not make much sense. but does it at least give us the same answers as the NPV rule?

To understand the answer to this question, let's plot the NPV from example 1 as a function of r:



In this example, just as in example 1, NPV is a decreasing function of the discount rate r. This is because r is in the denominator, and all these numbers are positive. This graph gives a visual representation of why IRR and NPV are the same.

When NPV is a declining function of the interest rate, then the NPV and IRR rule will always give us the same answer when we are deciding whether or not to reject projects.

So we know that IRR isn't really a measure of return. But if it yields the same answers as NPV, what's the harm? We will see that there are several pitfalls of IRR.

(b) Comparing NPV and IRR: Accept or Reject Decision

As we've just shown, when a project is considered in isolation, the IRR rule and the NPV rule agree. What happens when NPV is not a decreasing function of r?

Example *Financing project*: Assume $C_0 = \$100$ and $C_1 = -\$130$. IRR solves:

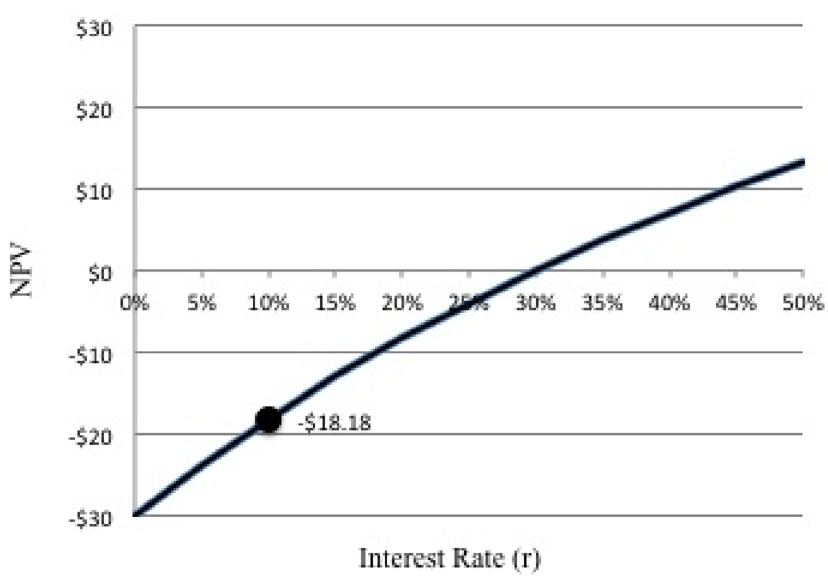
$$0 = -\$100 + \frac{\$130}{1 + \text{IRR}}.$$

This equation is solved by IRR = 30%.
However, suppose $r = 10\%$: IRR rule says accept. However:

$$\text{NPV@10\%} = \$100 - \frac{130}{1.10} = -\$18.2$$

NPV rule says reject!

What's going on? The bigger the r, the *higher* the NPV since the \$130 payment costs less.



When the negative payments are in the future, IRR gives the wrong answer, since NPV is not decreasing in r.

Intuitively, the idea behind the IRR is that it measures the rate of return from a project. But when you borrow, you want a low rate, not a high rate.

Sometimes, you can't use the IRR rule at all, as we can see from the following examples:

Example *Multiple IRRs*: Assume $C_0 = -\$100$, $C_1 = \$230$, and $C_2 = -\$132$:

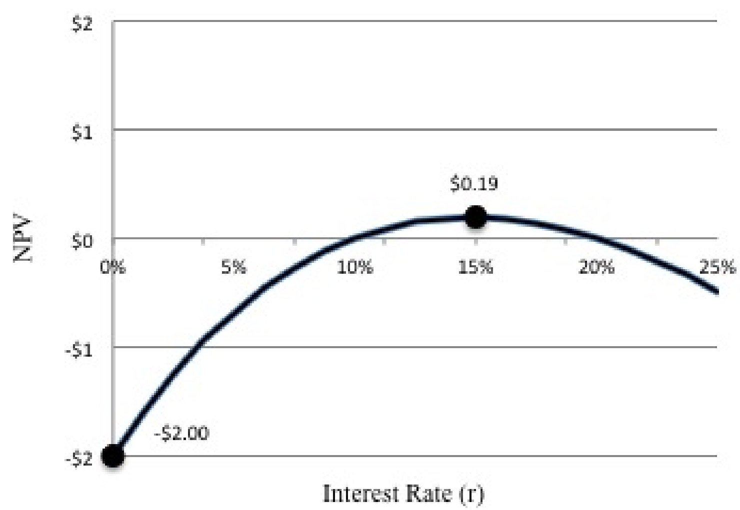
$$\text{NPV@10\%} = -\$100 + \frac{\$230}{1.1} - \frac{\$132}{1.1^2} = 0.$$

so IRR = 10%. However,

$$\text{NPV@20\%} = -\$100 + \frac{\$230}{1.2} - \frac{\$132}{1.2^2} = 0.$$

so IRR = 20%! There is no way to choose.

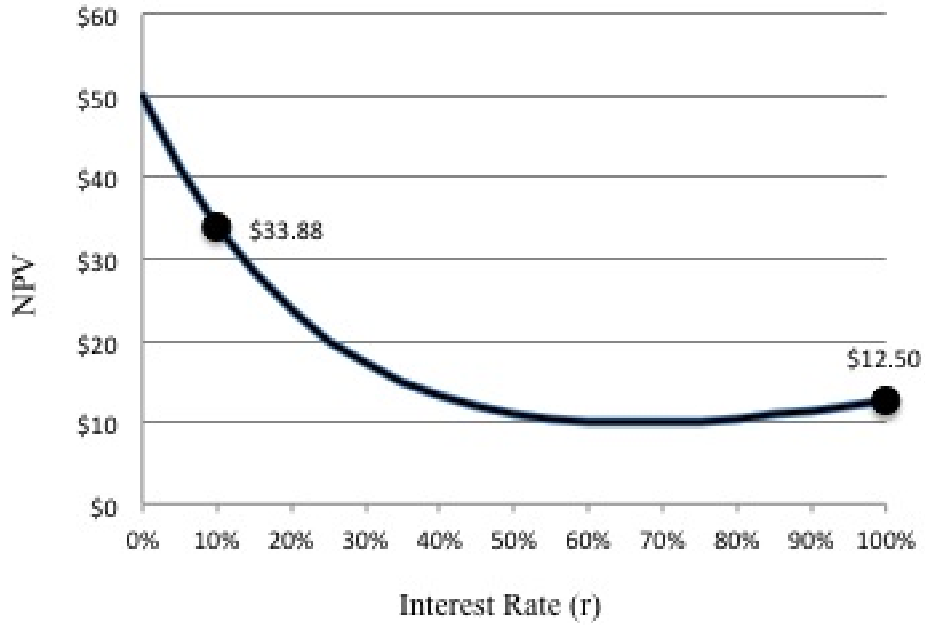
Note that NPV@15% = \$0.19 while NPV@0 = −\$2.0.



To interpret IRRs in this example, we need to draw this graph. In other words, we will need to calculate NPV anyway. Note also that this problem has investing and financing. This is not the only problem we could have.

Example *No IRR*: Assume $C_0 = \$100$, $C_1 = -\$300$, and $C_2 = \$250$.

We can calculate: NPV@10% = \$33.88 and NPV@100% = \$12.50.



When do NPV and IRR give the same answers?

- When $C_0 < 0$ and $C_t > 0$ for all t.
- More generally, when cash flows start out negative and switch to positive.
- When the NPV is a decreasing function of the interest rate.

It may seem that IRR is fine in the case of typical investment projects (which encompass a lot of ground). However, there are an additional set of problems to be aware of.

(c) Comparing NPV and IRR: Mutually Exclusive Projects

Example *Scale* $\Rightarrow$ assume $r = 10\%$:

	C_0	C_1	NPV	IRR
A	-100	400	264	300%
B	-250	650	341	160%

When applied as an accept/reject decision, NPV and IRR agree: you should take both.

But what if you can only choose one? Which will lead to a greater increase in firm value?

- NPV $\Rightarrow$ Pick B.
- IRR $\Rightarrow$ Pick A.

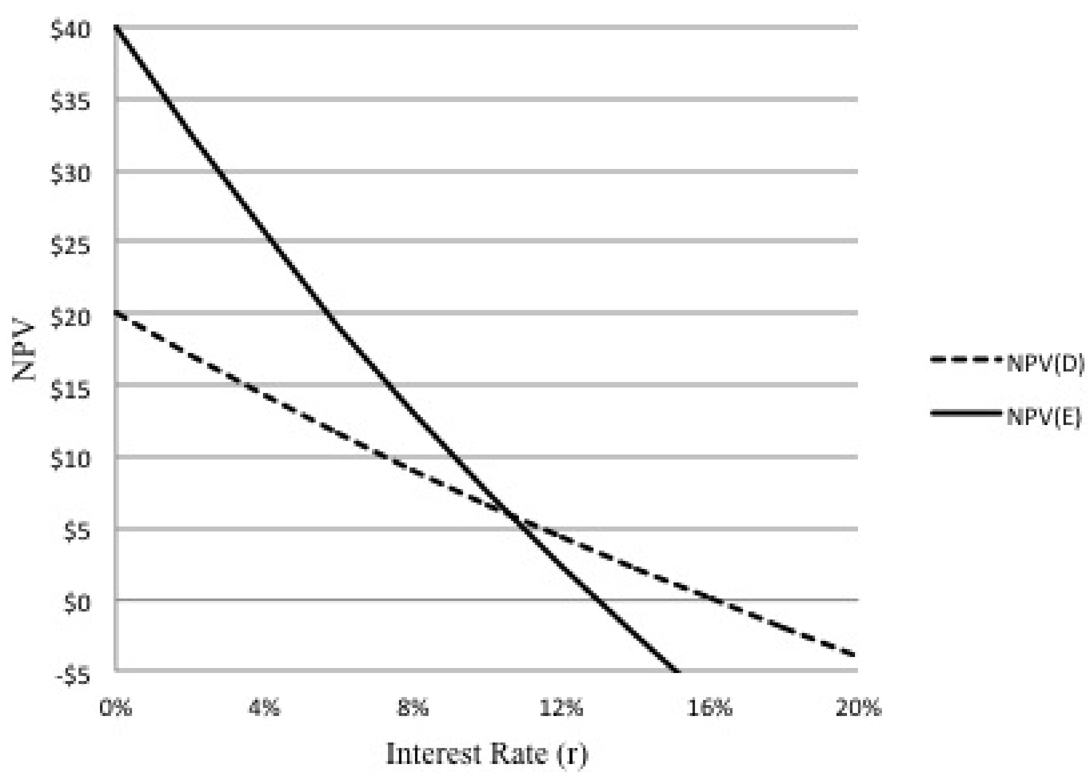
Note that NPV gives you the right answer. B leads to a greater increase in firm value.

What's going on here? IRR is a return measure and therefore ignores the question of scale. What you would really like is to scale A up. However, that is not an option here.

Example *Timing* $\Rightarrow$ again, assume $r = 10\%$:

	C_0	C_1	C_2	C_3	NPV	IRR
D	-100	100	10	10	6.69	16.04%
E	-100	10	10	120	7.51	12.94%

- NPV $\Rightarrow$ Pick E.
- IRR $\Rightarrow$ Pick D.



$$\text{NPV}_D = -100 + \frac{100}{1+r} + \frac{10}{(1+r)^2} + \frac{10}{(1+r)^3}.$$
$$\text{NPV}_E = -100 + \frac{10}{1+r} + \frac{10}{(1+r)^2} + \frac{120}{(1+r)^3}.$$

So, NPV_E falls faster as r rises.

While these problems are ubiquitous, it is sometimes possible to correct for even these. Part of the reason we are getting strange answers is that we are asking the wrong question.

Example *Return to 1st example:*

If you really can't do both A and B, then the decision to choose B has to account for the side effect of not doing A. In fact, the cash flows of interest are really those of $B - A$:

	C_0	C_1	NPV	IRR
$B - A$	-150	250	77	67%

	C_0	C_1	C_2	C_3	NPV	IRR
$E - D$	0	-90	0	110	.82	10.55%