Holding Period Return and Yield to Maturity for Zero-Coupon Bonds[1]

Notation:

F Face value of a bond

P Price of a bond

V Value of a security

T Years to maturity

t Years until you sell the bond

1. The yield to maturity (assuming annual compounding) is defined as

$$\text{YTM} = \left(\frac{F}{P_0}\right)^{1/T} - 1$$

Because the final payment F is known with certainty (the U.S. government will not default), the yield to maturity is known with certainty. P_0 is the price "today".

2. Recall that for any security, the holding period return is

$$\text{HPR} = \left(\frac{V_t}{V_0}\right)^{1/t} - 1.$$

where V is the value at which you bought the security, and t is the number of years you held the security.

For a zero-coupon bond, the value equals the price (because there are no intermediate cash flows). From the definition above,

$$\text{HPR} = \left(\frac{P_t}{P_0}\right)^{1/t} - 1$$

How are holding period returns and yield to maturities related? It matters whether the bond is held to maturity, or sold before it reaches maturity.

[1] Notes for Finance 604 & 612 prepared by Jessica A. Wachter.

3. Consider the case of a bond that matures in 10 years ($T = 10$), with a price of \$450.11 per \$1000 face value. Then

$$\begin{aligned} \text{YTM} &= \left(\frac{\$1000}{\$450.11}\right)^{\frac{1}{10}} - 1. \\ &= 0.0831 \end{aligned}$$

Suppose the bond is held until maturity. Then the HPR and the YTM are the same. We can see this by directly applying the formula for HPR. When it matures, the bond is worth F, its face value. Assume the bond matures in t years, the HPR equals

$$\text{HPR} = \left(\frac{F}{P_0}\right)^{1/t} - 1.$$

Looking at the definition of the YTM, we see that this is exactly the YTM for a bond with a face value of F, price P, maturing in t years.

4. Now suppose that, instead of waiting for 10 years, we want to sell the bond after 1 year. The bond is now a 9-year bond.

If the YTM does not change, the price of the bond now equals

$$\begin{aligned} P_t &= \frac{\$1000}{(1+0.0831)^9} \\ &= \$487.51. \end{aligned}$$

To compute the HPR, substitute into the definition:

$$\begin{aligned} \text{HPR} &= \frac{\$487.51}{\$450.11} - 1 \\ &= 0.0831. \end{aligned}$$

So, when the yield to maturity stays the same, the HPR equals the YTM. Note that the price of the bond has risen even though the YTM stays the same: the bond's price is "pulled" to the par value of \$1000.

5. However, *the yield to maturity may change between this year and next year*, because investors may be more or less willing to buy bonds next year. Suppose first that the yield to maturity falls to 8%. Now the price of the bond is

$$\begin{aligned} P_t &= \frac{\$1000}{(1+0.08)^9} \\ &= \$500.25. \end{aligned}$$

In this scenario, investors are more willing to hold bonds and they have pushed up the price.

The HPR equals:

$$\begin{aligned} \text{HPR} &= \frac{\$500.25}{\$450.11} - 1 \\ &= 0.1114. \end{aligned}$$

So the HPR (11.14%) we receive from holding the bond for one year is greater than the YTM (8.31%) we would have received if we held the bond to maturity.

Now suppose that the yield to maturity rises to 8.6%. The price of the bond is

$$\begin{aligned} P &= \frac{\$1000}{(1.086)^9} \\ &= \$475.92. \end{aligned}$$

In this scenario, investors are less willing to buy bonds and they have pushed down the price.

The HPR equals:

$$\begin{aligned} \text{HPR} &= \frac{\$475.92}{\$450.11} - 1 \\ &= 0.057 \end{aligned}$$

So the HPR (5.7%) is now lower than the YTM (8.31%) that we would have received if we held the bond to maturity.

6. What should we conclude? Unlike the yield to maturity, the holding period return is *uncertain*. When we buy a bond, we do not know what price we will be able to sell if for next period. This happens even though these bonds are a liability of the U.S. government. It is the cash flows on these bonds that are known with certainty, not the holding period return.