Summary of NPV Rule[1]

Before defining the NPV rule, we need several present value concepts:

- Future Value:

  Suppose you deposit \$1000 in a bank account, paying 10% interest.

  What is the value of the deposit in 1 year

$$
\begin{aligned}
\text{FV} &= \text{Principal} + \text{Interest} \\
&= \$1000 + \$1000(.10) \\
&= \$1100
\end{aligned}
$$

  In general, suppose r is the interest rate:

$$
\begin{aligned}
&= \$1000 + \$1000r \\
&= \$1000(1 + r)
\end{aligned}
$$

  The future value of your investment at the interest rate r is $\$1000(1 + r)$.

  Definition *The future value of C_0 at interest rate r in 1 year is*

$$\text{FV} = C_0(1 + r)$$

- Present Value:

  Suppose you need \$1000 in one year. What amount of money do you need to put aside today?

$$\$1000 = \text{PV} \times (1 + r)$$

[1] Notes for Finance 604 & 612 prepared by Jessica A. Wachter.

Rearranging,

$$\text{PV} = \frac{\$1000}{1+r}$$

when $r = 10\%$, $\text{PV} = \$1000/(1.10) = \$909.90$. \$909.90 is the *Present value* of \$1000, at 10%. Note that we are bringing the \$1000 backward in time.

In general, suppose you will have an *cash flow* of C_1 in one year.

Definition *The Present value of C_1 at interest rate r is*

$$\text{PV} = \frac{C_1}{1+r}$$

- Net Present Value:

  Consider an investment that pays a cash flow of C_1 in one year and costs $-C_0$ today. The interest rate is r.

  Definition *The NPV of the investment is*

  $$\text{NPV} = C_0 + \frac{C_1}{1+r}$$

  Basically NPV is really just present value. The "net" emphasizes that we have a negative cash flow at time zero.

We are now ready to define the NPV rule:

Definition *NPV rule: Accept projects with* NPV > 0 *and reject projects* NPV < 0.

We are going to establish:

Result. *Following the NPV rule maximizes the value of the corporation.*

To give some intuition for this result, we will look at a very simple corporation, consisting of a single person (Suzy), who has access to 1M and can borrow and lend from a bank at 20%.

Suzy allocates wealth between youth and old age. Assuming this bank represents the only opportunity for Suzy, what should she do?

1. She could go on a trip around the world, and then live in poverty in old age
2. She could go on a smaller trip have a moderate lifestyle in her youth (spending .5M) and still have $.5(1.2) = .6M$ for her old age.
3. She could put all her money in the bank and go for an even better trip around the world in old age. ($1.2M)

Note: all these possibilities are equally "correct".

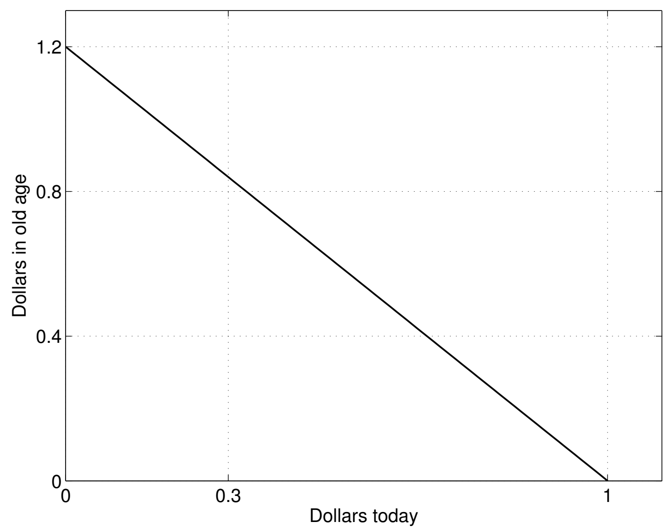
- Derivation of line: Let $y = \$$ in old age, $x = \$$ today. Then FV reasoning tells us

  $$y = (1 - x)(1 + r)$$

  So the relation between consumption today and consumption in old age is a straight line with a negative slope of $-(1 + r)$. For example, if she consumes everything today (1 M) she will have nothing in old age. If she saves everything, she will have 1.2M in old age.

- Suzy's preferences between now and old age determine where she is on the line.

Figure 1: Suzy's possibilities provided by the bank



- This figure represents the trade-off between spending in now and spending in old age. Its slope is negative, and equal to $-(1+r)$. For every \$1 you give up in spending now, you get $\$1(1+r)$ in old age.

- Now suppose Suzy considers opening a restaurant. The start-up costs would be .70M. She could invest in this restaurant and have .80M in her old age. Should she invest? *No.*

- Why? If she puts the .70M in the bank, she has $.70(1.2) = .84$M. Don't need to know anything about Suzy's preferences to make this decision.

- Now suppose Suzy spies a plot of land for a vineyard that costs .70M and will yield .91 in old age. Should she make the investment? It seems like the answer should be yes.

- But what if she is planning to spend money on college and an MBA? This will cost her $.2M$. Now she only has $.1M$ left. Should she still make the investment?

Do we need more information to determine whether Suzy should invest?