Summary of Equity Valuation Formulas[1]

Consider a company that pays expected dividends D_1, D_2, D_3 going out forever. The price of this company would then be:

$$P_0 = \frac{D_1}{1+r} + \frac{D_2}{(1+r)^2} + \frac{D_3}{(1+r)^3} + \cdots .$$

where r is the discount rate for investments of similar risk. Its hard to forecast dividends for every period going forward. So we begin with some simplifying assumptions.

Constant Dividends

Suppose that the stock pays <u>the same</u> dividend every year forever:

$$P_0 = \frac{D}{1+r} + \frac{D}{(1+r)^2} + \cdots$$

What would the price be in this case?

$$P_0 = \frac{D}{r}$$

This formula can be related to the price-earnings ratio. To do this, say that the relationship between earnings and dividends is approximated by

$$D = (1-b)E$$

where b denote the *plowback ratio* – the portion of earnings that is kept in the firm. $1-b$ is paid out as dividends. Substituting in

$$P_0 = \frac{(1-b)E}{r}$$

Then, the price-earnings ratio equals:

$$\frac{P_0}{E} = \frac{1-b}{r}$$

[1] Notes for Finance 604 & 612 prepared by Jessica A. Wachter.

Constant Dividend Growth

In reality, some companies grow faster than others. A model of firm value based on level dividends alone will fail to take this into account. Suppose D_0 is the dividend paid today and g is the dividend growth rate. Then

$$D_1 = D_0(1+g)$$

$$D_2 = D_0(1+g)^2$$

and so forth. Therefore:

$$P_0 = \frac{D_0(1+g)}{1+r} + \frac{D_0(1+g)^2}{(1+r)^2} + \cdots$$

from the formula for the sum of a geometric progression:

$$P_0 = \frac{D_0(1+g)}{r-g}$$

so

$$P_0 = \frac{D_1}{r-g}.$$

When there is dividend growth, the price equals to the dividend next year divided by the difference between the market cap rate and the growth rate. Note that this sum converges only if $r > g$.

We can also write this in terms of earnings:

$$P_0 = \frac{E_1(1-b)}{r-g}$$

The price-earnings ratio equals:

$$\frac{P_0}{E_1} = \frac{(1-b)}{r-g}$$

We can also rewrite this equation so that it tells us what the market expects the growth rate is. Suppose $P/E = 18$, $r = 11.6$, $b = 1/3$. What is the expectations for growth embedded in this price-earnings ratio?

$$g = r - \frac{E(1-b)}{P} = .116 - \frac{1}{18}\frac{2}{3} = .078.$$

We can use this formula in one of too ways. We can take an estimate of growth and it will tell us a P/E, which may be different from the P/E that we see when we look at the actual price and the actual earnings.

Or we can use the actual P/E, the one from the data, and use the formula to tell us when the market expects growth to be.

Where does growth come from?

Growth g can be estimated directly, but sometimes it is useful to talk about it in terms of another measure: *Return on Equity.* The advantage of ROE is it is directly related to profitability of the firm's assets. Growth and ROE are related by:

$$g = b\text{ROE}$$

Growth in the firm is equal to the fraction of earnings plowed back, multiplied by the profitability of those earnings. That Now we've decomposed growth:

$$P_0 = \frac{D_1}{r - \text{ROE}b}$$

or

$$P_0 = \frac{(1-b)E_1}{r - \text{ROE}b}$$

Also have a new equation for the price-earnings ratio

$$\frac{P_0}{E_1} = \frac{1-b}{r - \text{ROE}b}$$

When $r = \text{ROE}$, $P/E = 1/r$, the same equation we got assuming no growth.

$$\frac{P_0}{E_1} = \frac{1-b}{r - rb} = \frac{1}{r}$$

ROE is the return inside the firm, r is the return outside the firm. When they are equal, it doesn't matter for the price whether earnings are kept in the firm or not.

Why does this happen? There are two effects of b on the price-earnings ratio. It lowers the price-earnings ratio because you are getting less cash paid out (which you could invest in other sources), but it raises it because earnings are growing more.

Suppose $r = .12$ and ROE $= .10$, $b = .6$. Then

$$\frac{P_0}{E_1} = \frac{1 - .6}{.12 - .6(.1)} = \frac{.4}{.06}$$

As a manager, how can you raise the price of your stock? Set $b = 0$ so that nothing is retained in the firm. Then:

$$\frac{P_0}{E_1} = \frac{1}{.12} = 8.33$$

You are returning the cash to the shareholders so they can put it to more productive use. This firm is also a takeover target. A raider says, I can raise the price just by changing the amount that I pay out.

This discussion suggests that, at least in this constant growth framework, whether reinvestment of earnings raises the value depends on whether the discount rate r is below or above k. We can prove this by taking derivatives:

$$\begin{aligned}
\frac{\partial P_0}{\partial b} &= \frac{-E_1(r - b\text{ROE}) + (\text{ROE})E_1(1-b)}{(r - b\text{ROE})^2} \\
&= E_1 \frac{b\text{ROE} - r - (\text{ROE})b + \text{ROE}}{(r - b\text{ROE})^2} \\
&= E_1 \frac{\text{ROE} - r}{(r - b\text{ROE})^2}
\end{aligned}$$

Because the denominator is always positive, if ROE $> r$, increasing the plowback increases value. If ROE $= r$, the plowback has no effect on value. If ROE $< r$ value can be increased by decreasing the plowback.