ML in Fundamental Physics

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Exercise Sheet 5

Submission by Monday 17$^{\text{th}}$ of May 18:00 via moodle.

This sheet will entirely be peer corrected. Make sure you write clean code, because there will be points for this, too.

1 Kernel methods (peer correction)

The aim of this exercise is to familiarise yourself with the dual optimisation problem.

- We obtain the solution to a linear regression with respect to our weights w as previously discussed in the lectures.

$$C(w) = \frac{1}{2}\sum_{n=1}^{N}\left(\mathbf{w}^T x_n - y_n\right)^2 + \frac{\lambda}{2}\mathbf{w}^T\mathbf{w}$$

  To implement this, consider the underlying model $y = 2x$. Generate data with some Gaussian noise (100 datapoints). Solve the optimisation problem for $\lambda = 0, 0.1, 1$ as described in the lectures.

- We now introduce a kernel $\phi(x) = (x, x^2)$. Solve again the problem for the underlying model $y = 2x$. Note that this time you need to recalculate the target values t_n appropriately.

- Calculate the kernel function $k(x_n, x_m) = \phi(x_n)^T\phi(x_m)$.

- Using this kernel function solve the optimisation problem for the dual variables $\mathbf{a}$.

- Show that both solutions agree by obtaining $\mathbf{w}$ from your solution $\mathbf{a}$.

2 KL divergence, information content (peer correction)

- For independent random variables ($P(X,Y) = P(X)P(Y)$) show that Shannon entropy is additive, i.e. $H(X,Y) = H(X) + H(Y)$.

- A source produces a character x from the alphabet $\mathcal{A} = \{0, 1, \ldots, 9, a, b, \ldots, z\}$; with probability $1/3$, x is a digit $(0, \ldots, 9)$, with probabiblity $1/3$ it is a vowel, and with probability $1/3$ it is one of the 21 consonants. All numerals are equiprobable, and the same goes for the vowels and consonants. Calculate the entropy of X.

Write your solution in a jupyter notebook as markdown.