# Noise Models in Qiskit

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https://github.com/BurgerAndreas/qiskit-reduced-noise-model

## 1 Noise Models in Qiskit

Qiskit supplies noise models based on device properties measured during calibration.
https://qiskit.org/documentation/_modules/qiskit/providers/aer/noise/device/models.html

The noise models contain three error sources

1. thermal relaxation (relaxation and dephasing)
2. depolarizing (Pauli) error
3. readout (measurement) error

At every gate, first the thermal relaxation and then the depolarizing error is applied. The strength of the depolarizing error is calculated backwards to reach a target 'gate error' when combined with the thermal relaxation.

**Gate Error** The gate error of a noisy quantum channel $\epsilon$ with a target unitary $U$ is defined as the average infidelity

$$E(\epsilon, U) = 1 - F_{avg}(\epsilon, U) \tag{1}$$

$$F_{avg} = \int d\phi \, \langle\phi| U^\dagger \epsilon(|\phi\rangle \langle\phi|) U |\phi\rangle \, . \tag{2}$$

Gate error: https://qiskit.org/documentation/stubs/qiskit.quantum_info.gate_error.html
Fidelity: https://qiskit.org/documentation/stubs/qiskit.quantum_info.average_gate_fidelity.html#qiskit.quantum_info.average_gate_fidelity

### 1.1 Error sources

**Thermal relaxation** Thermal relaxation is parameterized by the qubit-specific time until relaxation $T1$, qubit-specific time until dephasing $T2$, and the gate-dependent gate time.

In general $T_2 \leq 2T_1$ has to hold. For $T_2 < T_1$ thermal relaxation can be described by (assuming device to be at 0 temperature) [GEZ21]

$$K_{T_0} = \sqrt{p_I}\mathbb{1}, \; K_{T_1} = \sqrt{p_Z}\hat{\sigma}^z, \; K_{T_2} = \sqrt{p_{reset}} \, |0\rangle \langle 0| \tag{3}$$

$$E_T(\rho) = \sum_{i=10}^{2} K_{T_i} \rho K_{T_i}^\dagger \tag{4}$$

It is composed of the probabilities of a phase-flip $p_Z = (1 - p_{reset})(1 - \frac{p_{T_2}}{p_{T_1}})/2$, a reset to the ground state $p_{reset} = 1 - p_{T_1}$, or for nothing to happen $p_I = 1 - p_Z - p_{reset}$.

If $2T_1 \geq T_2 > T_1$ thermal relaxation is described by the Choi matrix $\rho \rightarrow E_T(\rho) = tr_1[C(\rho^T \otimes I)]$

$$\begin{pmatrix} & 1 & 0 & 0 & p_{T_2} \\ & 0 & 0 & 0 & 0 \\ C = & 0 & 0 & p_{reset} & 0 \\ & p_{T_2} & 0 & 0 & 1 - p_{reset} \end{pmatrix} \tag{5}$$

**Depolarizing** The depolarizing noise (or Pauli) channel is composed of either a bit-flip ($\hat{\sigma}^x$), a phase-flip ($\hat{\sigma}^z$) or both at the same time ($\hat{\sigma}^y$) with equal probability [GEZ21].

$$\rho \rightarrow E_P(\rho) = \sum_{i=1}^{3} K_{P_i} \rho K_{P_i}^\dagger \tag{6}$$

$$K_{P_0} = \sqrt{1 - p_p}\mathbb{1}, \ K_{P_1} = \sqrt{\frac{p_p}{3}}\hat{\sigma}^x \tag{7}$$

$$K_{P_2} = \sqrt{\frac{p_p}{3}}\hat{\sigma}^y, \ K_{P_3} = \sqrt{\frac{p_p}{3}}\hat{\sigma}^z \tag{8}$$

In Qiskit the strength of the depolarizing error is calculated from the target gate infidelity $\mathcal{I}_{gate}$, and the infidelity due to thermal relaxation $\mathcal{I}_T$. If we write the depolarizing error in terms of the identity and the complete depolarizing channel $E_P = (1 - p_P) * \mathbb{1} + p_P * D$, we can rewrite the gate fidelity

$$\begin{aligned} \mathcal{F}_{gate} &= 1 - \mathcal{I}_{gate} && (9) \\ &= \mathcal{F}(E_P * E_T) && (10) \\ &= (1 - p_P)\mathcal{F}(\mathbb{1} * E_T) + p_P * \mathcal{F}(D * E_T) && (11) \\ &= (1 - p_P)\mathcal{F}(E_T) + p_P * \mathcal{F}(D) && (12) \\ &= (1 - p_P)\mathcal{F}_T + p_P * \mathcal{F}_P && (13) \\ &= \mathcal{F}_T - p_P * (d * \mathcal{F}_T - 1)/d && (14) \end{aligned}$$

Where $d = 2^{qubits}$ is the dimensionality of the gate. From this the solution for the depolarizing error probability is

$$\begin{aligned} p_P &= d(\mathcal{F}_T - \mathcal{F}_{gate})/(d * \mathcal{F}_T - 1) && (15) \\ &= d * (\mathcal{I}_{gate} - \mathcal{I}_T)/(d * \mathcal{F}_T - 1) && (16) \\ & && (17) \end{aligned}$$

https://qiskit.org/documentation/_modules/qiskit/providers/aer/noise/device/models.html

**Measurement error** The measurement error is equivalent to a bit-flip followed by a noiseless read-out [GEZ21]

$$K_(R_0) = \sqrt{1 - p_R}\mathbb{1}, \ K_(R_1) = \sqrt{p_R}\hat{\sigma}^x \tag{18}$$

In Qiskit the readout error is given by the probability $P(n|m)$ of recording a noisy measurement outcome as n, given the true measurement outcome is m.
https://qiskit.org/documentation/stubs/qiskit.providers.aer.noise.ReadoutError.html

## 1.2 Reduced-Noise Models in Qiskit

For our reduced-noise models we multiply the gate error $\mathcal{I}_{gate}$ and the gate times by a factor $\xi < 1$. I.e. we reduce the average gate infidelity $\mathcal{I}_{gate}$, while keeping the relative contribution of the thermal relaxation and depolarizing error unchanged. In addition, we scale down the false-readout probabilities $P(1|0)$, $P(0|1)$ by the same factor.

$$\mathcal{I}_{gate} \rightarrow \xi * \mathcal{I}_{gate} \tag{19}$$

$$\mathcal{I}_{t} \rightarrow \xi * \mathcal{I}_{t} \tag{20}$$

$$P(1|0),\ P(0|1) \rightarrow \xi * P(1|0),\ \xi * P(0|1) \tag{21}$$

For example, as a basis we use the 27 qubit IBMQ Toronto device with a Falcon r4 processor, V1.7.7. At the time of writing the the average calibration data is: Avg. CNOT Error: $4.936e-2$, Avg. Readout Error: $4.119e-2$, Avg. T1: 113.87 us, Avg. T2: 101.63 us, Avg. Gate time: 454.095 ns, Avg. Qubit Frequency: 5.08 GHz, Avg. Qubit Anharmonicity $-0.329$ GHz.
Implementation: https://github.com/BurgerAndreas/qiskit-reduced-noise-model

## References

[GEZ21] Konstantinos Georgopoulos, Clive Emary, and Paolo Zuliani. Modeling and simulating the noisy behavior of near-term quantum computers. *Physical Review A*, 104(6):062432, dec 2021.