Exercises 6

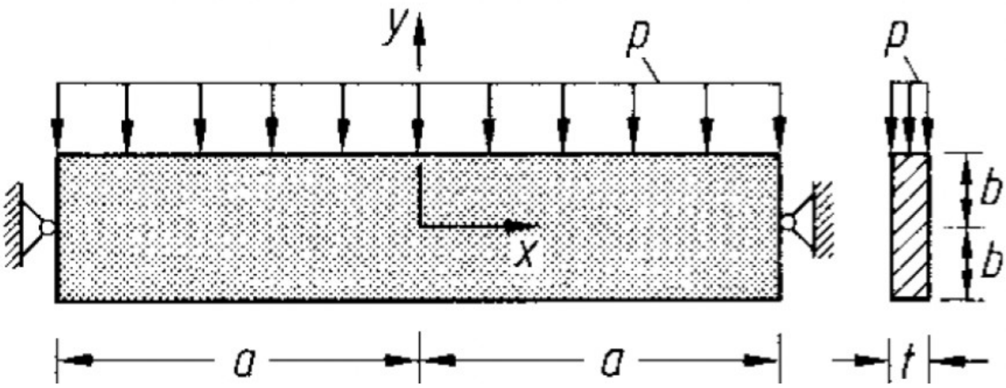
(Learning content: *Functions, numpy, -.meshgrid, matplotlib, -.contour*)

Calculation of principal and maximum shear stresses

task

The aim of the task is to determine the principal stresses σ_1 and σ_2 as well as the maximum shear stress τ_{Max} to be calculated and represented in isoline diagrams.

Given a bar of the kind



In the example, sides a and b have a ratio of 5: 1; however, it should be possible to carry out the calculations for different aspect ratios.

calculation

Within the framework of the disk theory, the stresses under constant line load can be approximated using the following formulas.

$$\sigma_x = -\frac{3pa^2y}{4b^3}\left(1 - \frac{x^2}{a^2} + \frac{2}{3}\frac{y^2}{a^2} - \frac{2}{5}\frac{b^2}{a^2}\right) \tag{1}$$

$$\sigma_y = -p\left(\frac{1}{2} + \frac{3}{4}\frac{y}{b} - \frac{1}{4}\frac{y^3}{b^3}\right) \tag{2}$$

$$\tau_{xy} = \frac{3px}{4b}\left(1 - \frac{y^2}{b^2}\right) \tag{3}$$

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{4}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \tau_{max} \tag{5}$$

In the following, however, the values of x, y, σ_x, σ_y and τ_{Max} considered, but the *relative* Sizes x / a, y / a, σ_x/ p, σ_y/ p and τ_{Max}/ p. The coordinates are therefore on the length a *normalized* and the stresses on the load p. The stress results are therefore dimensionless. Use the identities to reformulate the above equations to the relative values

x / b = x / a (a / b)
y / b = y / a (a / b)

programming

Proceed as follows to create the diagrams:

1. Define a constant A_B that contains the selected aspect ratio a: b (eg 5). This constant is not passed as a parameter to the following functions, but is used as a "global" constant in their formulas.
2. Define a function sigma_x_p (x_a, y_a). This function should be σ_xCalculate / p for given values of x_a = x / a, y_a = y / a and a / b. For the necessary conversion to the *relative* You replace variables in the above equation (1), for example, terms such as x / a with the function parameter x_a.
3. Define sigma_y_p (x_a, y_a) and tau_xy_p (x_a, y_a) accordingly.
4. Define functions tau_max_p, sigma_1_p and sigma_2_p, the τ_{Max}/ p, σ_1/ p and $\sigma_2$compute / p. The parameters of the functions are again x / a, y / a.
5. Place by means of np.linspace () Vectors for the value ranges of x / a and y / a in such a way that the normalized x covers the interval from -1 to +1 and the normalized y accordingly covers the interval from -1 / A_B to + 1 / A_B. As the number of subdivisions, you can use, for example, 200 for the longer side, for the shorter a number that is smaller according to the aspect ratio a: b.
6. Place means np.meshgrid () from the 1D vectors generated in step 5, 2D coordinate matrices X and Y of the evaluation points / support points in the plane.
7. Use ("automatically" vectorized) calls to the functions from step 4 to calculate the matrices of the relative stress results Sigma1 and Sigma2 from X and Y.
8. Generate using plt.subplots () a plot with 3 subplots arranged one above the other.
9. Set Sigma1 and Sigma2 in separate subplots as an isoline diagram using plt.contour () Use the predefined *jet*-colormap for mapping the values to the colors:
   ```
   ctr1 = ax1.contour (..., cmap = 'jet')
   ```
   but note the selected "reverse" color assignment for Sigma2 in the sample plot. Supplement the diagrams with meaningful titles and axis labels, using the options of the extended text formatting[1].
10. Add using plt.colorbar () Color legends for the isolines. Make sure that the respectivecolorbar through the 1st parameter of the corresponding contour and about the named parameter ax must be assigned to the correct subplot, outlines:
    ```
    ctr2 = ax2.contour (...)
    cb2 = plt.colorbar (ctr2, ax = ax2, shrink = 0.75)
    ```
11. Proceed accordingly to create an isoline diagram for τ in a third subplot$_{Max}$to generate / p.

[1] Matplotlib text tutorial: [Writing mathematical expressions]

For an aspect ratio of a: b = 5: 1, the result should be an image of the following type (shown here on a reduced scale), set the corresponding value ranges for your results in the *colorbars* and the axles safely.