

Lab 9

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11:59PM April 14, 2019

“data wrangling / munging / carpentry” with dplyr.

First load dplyr, tidyr, magrittr and lubridate in one line.

```
rm(list = ls())
```

```
pacman::p_load(dplyr, tidyr, magrittr, lubridate)
```

```
## Installing package into 'C:/Users/burha/Documents/R/win-library/3.6'
## (as 'lib' is unspecified)
```

```
## Warning: unable to access index for repository http://www.stats.ox.ac.uk/pub/RWin/bin/windows/contrib/
## cannot open URL 'http://www.stats.ox.ac.uk/pub/RWin/bin/windows/contrib/3.6/PACKAGES'
```

```
## package 'tidyr' successfully unpacked and MD5 sums checked
##
```

```
## The downloaded binary packages are in
```

```
## C:\Users\burha\AppData\Local\Temp\Rtmpmw9d97\downloaded_packages
```

```
##
```

```
## tidyr installed
```

```
## Installing package into 'C:/Users/burha/Documents/R/win-library/3.6'
## (as 'lib' is unspecified)
```

```
## Warning: unable to access index for repository http://www.stats.ox.ac.uk/pub/RWin/bin/windows/contrib/
## cannot open URL 'http://www.stats.ox.ac.uk/pub/RWin/bin/windows/contrib/3.6/PACKAGES'
```

```
## package 'lubridate' successfully unpacked and MD5 sums checked
##
```

```
## The downloaded binary packages are in
```

```
## C:\Users\burha\AppData\Local\Temp\Rtmpmw9d97\downloaded_packages
```

```
##
```

```
## lubridate installed
```

Load the `storms` dataset from the `dplyr` package and investigate it using `str` and `summary` and `head`. Which two columns should be converted to type factor? Do so below using the `mutate` and the overwrite pipe operator `%<>%`. Verify.

```
data("storms")
```

```
str(storms)
```

```
## Classes 'tbl_df', 'tbl' and 'data.frame': 10010 obs. of 13 variables:
```

```
## $ name : chr "Amy" "Amy" "Amy" "Amy" ...
```

```
## $ year : num 1975 1975 1975 1975 1975 ...
```

```
## $ month : num 6 6 6 6 6 6 6 6 6 6 ...
```

```
## $ day : int 27 27 27 27 28 28 28 28 29 29 ...
```

```
## $ hour : num 0 6 12 18 0 6 12 18 0 6 ...
```

```
## $ lat : num 27.5 28.5 29.5 30.5 31.5 32.4 33.3 34 34.4 34 ...
```

```
## $ long : num -79 -79 -79 -79 -78.8 -78.7 -78 -77 -75.8 -74.8 ...
```

```
## $ status : chr "tropical depression" "tropical depression" "tropical depression" "tropical dep
```

```
## $ category : Ord.factor w/ 7 levels "-1"<"0"<"1"<"2"<...: 1 1 1 1 1 1 1 1 2 2 ...
```

```
## $ wind      : int  25 25 25 25 25 25 25 30 35 40 ...
## $ pressure   : int 1013 1013 1013 1013 1012 1012 1011 1006 1004 1002 ...
## $ ts_diameter: num  NA NA NA NA NA NA NA NA NA NA ...
## $ hu_diameter: num  NA NA NA NA NA NA NA NA NA NA ...
```

```
summary(storms)
```

```
##      name          year      month      day
## Length:10010      Min.   :1975      Min.   : 1.000      Min.   : 1.00
## Class :character  1st Qu.:1990      1st Qu.: 8.000      1st Qu.: 8.00
## Mode  :character  Median :1999      Median : 9.000      Median :16.00
##                      Mean  :1998      Mean  : 8.779      Mean   :15.86
##                      3rd Qu.:2006      3rd Qu.: 9.000      3rd Qu.:24.00
##                      Max.   :2015      Max.   :12.000      Max.   :31.00
##
##      hour          lat          long      status
## Min.   : 0.000      Min.   : 7.20      Min.   : -109.30      Length:10010
## 1st Qu.: 6.000      1st Qu.:17.50      1st Qu.: -80.70      Class :character
## Median :12.000      Median :24.40      Median : -64.50      Mode  :character
## Mean    : 9.114      Mean    :24.76      Mean    : -64.23
## 3rd Qu.:18.000      3rd Qu.:31.30      3rd Qu.: -48.60
## Max.    :23.000      Max.    :51.90      Max.    :  -6.00
##
## category      wind          pressure      ts_diameter
## -1:2545      Min.   : 10.00      Min.   : 882.0      Min.   :  0.00
## 0 :4373      1st Qu.: 30.00      1st Qu.: 985.0      1st Qu.: 69.05
## 1 :1685      Median : 45.00      Median : 999.0      Median :138.09
## 2 : 628      Mean    : 53.49      Mean    : 992.1      Mean    :166.76
## 3 : 363      3rd Qu.: 65.00      3rd Qu.:1006.0      3rd Qu.:241.66
## 4 : 348      Max.    :160.00      Max.    :1022.0      Max.    :1001.18
## 5 : 68
##                      NA's      :6528
## hu_diameter
## Min.   : 0.00
## 1st Qu.: 0.00
## Median : 0.00
## Mean    :21.41
## 3rd Qu.:28.77
## Max.    :345.23
## NA's    :6528
```

```
head(storms)
```

```
## # A tibble: 6 x 13
##   name year month day hour lat long status category wind pressure
##   <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr> <ord> <int> <int>
## 1 Amy  1975  6 27 0 27.5 -79 tropi~ -1 25 1013
## 2 Amy  1975  6 27 6 28.5 -79 tropi~ -1 25 1013
## 3 Amy  1975  6 27 12 29.5 -79 tropi~ -1 25 1013
## 4 Amy  1975  6 27 18 30.5 -79 tropi~ -1 25 1013
## 5 Amy  1975  6 28 0 31.5 -78.8 tropi~ -1 25 1012
## 6 Amy  1975  6 28 6 32.4 -78.7 tropi~ -1 25 1012
## # ... with 2 more variables: ts_diameter <dbl>, hu_diameter <dbl>
```

```
storms %<>%
```

```
  mutate(name = factor(name), status = factor(status))
str(storms)
```

```
## Classes 'tbl_df', 'tbl' and 'data.frame': 10010 obs. of 13 variables:
## $ name : Factor w/ 198 levels "AL011993","AL012000",...: 44 44 44 44 44 44 44 44 44 44 ...
## $ year : num 1975 1975 1975 1975 1975 ...
## $ month : num 6 6 6 6 6 6 6 6 6 ...
## $ day : int 27 27 27 27 28 28 28 28 29 29 ...
## $ hour : num 0 6 12 18 0 6 12 18 0 6 ...
## $ lat : num 27.5 28.5 29.5 30.5 31.5 32.4 33.3 34 34.4 34 ...
## $ long : num -79 -79 -79 -79 -78.8 -78.7 -78 -77 -75.8 -74.8 ...
## $ status : Factor w/ 3 levels "hurricane","tropical depression",...: 2 2 2 2 2 2 2 2 3 3 ...
## $ category : Ord.factor w/ 7 levels "-1"<"0"<"1"<"2"<...: 1 1 1 1 1 1 1 1 2 2 ...
## $ wind : int 25 25 25 25 25 25 25 30 35 40 ...
## $ pressure : int 1013 1013 1013 1013 1012 1012 1011 1006 1004 1002 ...
## $ ts_diameter: num NA NA NA NA NA NA NA NA NA NA ...
## $ hu_diameter: num NA NA NA NA NA NA NA NA NA NA ...
```

Reorder the columns so name is first, status is second, category is third and the rest are the same. Verify.

```
storms %<>%
  select(name, status, category, everything())
storms
```

```
## # A tibble: 10,010 x 13
##   name status category year month day hour lat long wind pressure
##   <fct> <fct> <ord>   <dbl> <dbl> <int> <dbl> <dbl> <dbl> <int>   <int>
## 1 Amy tropi~ -1      1975 6 27 0 27.5 -79 25 1013
## 2 Amy tropi~ -1      1975 6 27 6 28.5 -79 25 1013
## 3 Amy tropi~ -1      1975 6 27 12 29.5 -79 25 1013
## 4 Amy tropi~ -1      1975 6 27 18 30.5 -79 25 1013
## 5 Amy tropi~ -1      1975 6 28 0 31.5 -78.8 25 1012
## 6 Amy tropi~ -1      1975 6 28 6 32.4 -78.7 25 1012
## 7 Amy tropi~ -1      1975 6 28 12 33.3 -78 25 1011
## 8 Amy tropi~ -1      1975 6 28 18 34 -77 30 1006
## 9 Amy tropi~ 0       1975 6 29 0 34.4 -75.8 35 1004
## 10 Amy tropi~ 0       1975 6 29 6 34 -74.8 40 1002
## # ... with 10,000 more rows, and 2 more variables: ts_diameter <dbl>,
## # hu_diameter <dbl>
```

Sort the dataframe by year (most recent first) then category of the storm (most severe first). Verify.

```
storms %<>%
  arrange(desc(year), desc(category))
storms
```

```
## # A tibble: 10,010 x 13
##   name status category year month day hour lat long wind pressure
##   <fct> <fct> <ord>   <dbl> <dbl> <int> <dbl> <dbl> <dbl> <int>   <int>
## 1 Joaq~ hurri~ 4       2015 10 1 12 23.1 -73.7 115 942
## 2 Joaq~ hurri~ 4       2015 10 1 18 23 -74.2 115 936
## 3 Joaq~ hurri~ 4       2015 10 2 0 22.9 -74.4 120 931
## 4 Joaq~ hurri~ 4       2015 10 2 6 23 -74.7 120 935
## 5 Joaq~ hurri~ 4       2015 10 2 12 23.4 -74.8 115 937
## 6 Joaq~ hurri~ 4       2015 10 3 0 24.3 -74.3 115 943
## 7 Joaq~ hurri~ 4       2015 10 3 6 24.8 -73.6 120 945
## 8 Joaq~ hurri~ 4       2015 10 3 12 25.4 -72.6 135 934
## 9 Joaq~ hurri~ 4       2015 10 3 18 26.3 -71 130 934
## 10 Joaq~ hurri~ 4       2015 10 4 0 27.4 -69.5 115 941
## # ... with 10,000 more rows, and 2 more variables: ts_diameter <dbl>,
```

```
## # hu_diameter <dbl>
```

Create a new feature wind_speed_per_unit_pressure.

```
storms %<>%  
  mutate(wind_speed_per_unit_pressure = wind / pressure)  
storms
```

```
## # A tibble: 10,010 x 14
```

```
##   name status category year month day hour lat long wind pressure  
##   <fct> <fct> <ord>    <dbl> <dbl> <int> <dbl> <dbl> <dbl> <int>    <int>  
## 1 Joaq~ hurri~ 4      2015 10    1    12 23.1 -73.7 115     942  
## 2 Joaq~ hurri~ 4      2015 10    1    18 23   -74.2 115     936  
## 3 Joaq~ hurri~ 4      2015 10    2     0 22.9 -74.4 120     931  
## 4 Joaq~ hurri~ 4      2015 10    2     6 23   -74.7 120     935  
## 5 Joaq~ hurri~ 4      2015 10    2    12 23.4 -74.8 115     937  
## 6 Joaq~ hurri~ 4      2015 10    3     0 24.3 -74.3 115     943  
## 7 Joaq~ hurri~ 4      2015 10    3     6 24.8 -73.6 120     945  
## 8 Joaq~ hurri~ 4      2015 10    3    12 25.4 -72.6 135     934  
## 9 Joaq~ hurri~ 4      2015 10    3    18 26.3 -71   130     934  
## 10 Joaq~ hurri~ 4      2015 10    4     0 27.4 -69.5 115     941  
## # ... with 10,000 more rows, and 3 more variables: ts_diameter <dbl>,  
## # hu_diameter <dbl>, wind_speed_per_unit_pressure <dbl>
```

Create a new feature: average_diameter which averages the two diameters.

```
storms %<>%  
  mutate(average_diameter = (ts_diameter + hu_diameter) / 2)  
storms
```

```
## # A tibble: 10,010 x 15
```

```
##   name status category year month day hour lat long wind pressure  
##   <fct> <fct> <ord>    <dbl> <dbl> <int> <dbl> <dbl> <dbl> <int>    <int>  
## 1 Joaq~ hurri~ 4      2015 10    1    12 23.1 -73.7 115     942  
## 2 Joaq~ hurri~ 4      2015 10    1    18 23   -74.2 115     936  
## 3 Joaq~ hurri~ 4      2015 10    2     0 22.9 -74.4 120     931  
## 4 Joaq~ hurri~ 4      2015 10    2     6 23   -74.7 120     935  
## 5 Joaq~ hurri~ 4      2015 10    2    12 23.4 -74.8 115     937  
## 6 Joaq~ hurri~ 4      2015 10    3     0 24.3 -74.3 115     943  
## 7 Joaq~ hurri~ 4      2015 10    3     6 24.8 -73.6 120     945  
## 8 Joaq~ hurri~ 4      2015 10    3    12 25.4 -72.6 135     934  
## 9 Joaq~ hurri~ 4      2015 10    3    18 26.3 -71   130     934  
## 10 Joaq~ hurri~ 4      2015 10    4     0 27.4 -69.5 115     941  
## # ... with 10,000 more rows, and 4 more variables: ts_diameter <dbl>,  
## # hu_diameter <dbl>, wind_speed_per_unit_pressure <dbl>,  
## # average_diameter <dbl>
```

Calculate the distance from each storm observation to Miami in a new variable distance_to_miami.

```
MIAMI_COORDS = c(25.7617, -80.1918)  
compute_globe_distance = function(destination, origin){  
  RAD_EARTH = 3958.8  
  dlon = (destination[2] - origin[2]) * 180 / pi  
  dlat = (destination[1] - origin[1]) * 180 / pi  
  a = (sin(dlat/2))^2 + cos(origin[1]) * cos(destination[1]) * (sin(dlon/2))^2  
  c = 2 * atan2( sqrt(a), sqrt(1-a) )  
  RAD_EARTH * c  
}
```

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```

```
storms
```

```
## Source: local data frame [10,010 x 16]
## Groups: <by row>
##
## # A tibble: 10,010 x 16
##   lat long distance_to_mia~ name status category year month day
##   <dbl> <dbl>         <dbl> <fct> <fct> <ord>   <dbl> <dbl> <int>
## 1 23.1 -73.7         5788. Joaq~ hurri~ 4       2015    10    1
## 2 23   -74.2          NaN Joaq~ hurri~ 4       2015    10    1
## 3 22.9 -74.4          NaN Joaq~ hurri~ 4       2015    10    2
## 4 23   -74.7         4340. Joaq~ hurri~ 4       2015    10    2
## 5 23.4 -74.8        10732. Joaq~ hurri~ 4       2015    10    2
## 6 24.3 -74.3          NaN Joaq~ hurri~ 4       2015    10    3
## 7 24.8 -73.6         6425. Joaq~ hurri~ 4       2015    10    3
## 8 25.4 -72.6        11564. Joaq~ hurri~ 4       2015    10    3
## 9 26.3 -71          3385. Joaq~ hurri~ 4       2015    10    3
## 10 27.4 -69.5          NaN Joaq~ hurri~ 4       2015    10    4
## # ... with 10,000 more rows, and 7 more variables: hour <dbl>, wind <int>,
## #   pressure <int>, ts_diameter <dbl>, hu_diameter <dbl>,
## #   wind_speed_per_unit_pressure <dbl>, average_diameter <dbl>
```

At home: convert year, month, day, hour into the variable `timestamp` using the `lubridate` package.

```
storms %<>%
  rowwise() %>%
  mutate(timestamp = ymd_h( paste0( toString(year), "-", toString(month), "-",
    toString(day), " ", toString(hour), sep = ""), locale = "English"))
```

At home: using the `lubridate` package, create new variables `day_of_week` which is a factor with levels “Sunday”, “Monday”, ... “Saturday” and `week_of_year` which is integer 1, 2, ..., 52.

```
storms %<>%
  mutate(day_of_week = wday(timestamp, label = TRUE)) %<>%
  mutate(week_of_the_year = week(timestamp))
```

Create a new data frame `serious_storms` which are category 3 and above hurricanes.

```
serious_storms = storms %>%
  filter(category >= 3)
serious_storms
```

```
## Source: local data frame [779 x 19]
## Groups: <by row>
##
## # A tibble: 779 x 19
##   lat long distance_to_mia~ name status category year month day
##   <dbl> <dbl>          <dbl> <fct> <fct> <ord>   <dbl> <dbl> <int>
## 1 23.1 -73.7          5788. Joaq~ hurri~ 4      2015 10    1
## 2 23    -74.2           NaN Joaq~ hurri~ 4      2015 10    1
## 3 22.9 -74.4           NaN Joaq~ hurri~ 4      2015 10    2
## 4 23    -74.7          4340. Joaq~ hurri~ 4      2015 10    2
## 5 23.4 -74.8         10732. Joaq~ hurri~ 4      2015 10    2
## 6 24.3 -74.3           NaN Joaq~ hurri~ 4      2015 10    3
## 7 24.8 -73.6          6425. Joaq~ hurri~ 4      2015 10    3
## 8 25.4 -72.6         11564. Joaq~ hurri~ 4      2015 10    3
## 9 26.3 -71           3385. Joaq~ hurri~ 4      2015 10    3
## 10 27.4 -69.5           NaN Joaq~ hurri~ 4      2015 10    4
## # ... with 769 more rows, and 10 more variables: hour <dbl>, wind <int>,
## #   pressure <int>, ts_diameter <dbl>, hu_diameter <dbl>,
## #   wind_speed_per_unit_pressure <dbl>, average_diameter <dbl>,
## #   timestamp <dtm>, day_of_week <ord>, week_of_the_year <dbl>
```

In `serious_storms`, merge the variables `lat` and `long` together into `lat_long` with values `lat / long` as a string.

```
serious_storms %<>%
  unite(lat_long, lat, long, sep = " / ")
serious_storms
```

```
## # A tibble: 779 x 18
##   lat_long distance_to_mia~ name status category year month day hour
##   <chr>          <dbl> <fct> <fct> <ord>   <dbl> <dbl> <int> <dbl>
## 1 23.1 / ~          5788. Joaq~ hurri~ 4      2015 10    1    12
## 2 23 / -7~           NaN Joaq~ hurri~ 4      2015 10    1    18
## 3 22.9 / ~           NaN Joaq~ hurri~ 4      2015 10    2     0
## 4 23 / -7~          4340. Joaq~ hurri~ 4      2015 10    2     6
## 5 23.4 / ~         10732. Joaq~ hurri~ 4      2015 10    2    12
## 6 24.3 / ~           NaN Joaq~ hurri~ 4      2015 10    3     0
## 7 24.8 / ~          6425. Joaq~ hurri~ 4      2015 10    3     6
## 8 25.4 / ~         11564. Joaq~ hurri~ 4      2015 10    3    12
## 9 26.3 / ~          3385. Joaq~ hurri~ 4      2015 10    3    18
## 10 27.4 / ~           NaN Joaq~ hurri~ 4      2015 10    4     0
## # ... with 769 more rows, and 9 more variables: wind <int>,
## #   pressure <int>, ts_diameter <dbl>, hu_diameter <dbl>,
## #   wind_speed_per_unit_pressure <dbl>, average_diameter <dbl>,
```

```
## # timestamp <dtm>, day_of_week <ord>, week_of_the_year <dbl>
```

Back to the main dataframe `storms`, create a new feature `decile_windspeed` by binning wind speed into 10 bins.

```
storms %<>%
  mutate(decile_windspeed = factor(ntile(wind, 10)))
storms
```

```
## Source: local data frame [10,010 x 20]
```

```
## Groups: <by row>
```

```
##
```

```
## # A tibble: 10,010 x 20
```

```
##   lat long distance_to_mia~ name status category year month day
##   <dbl> <dbl>          <dbl> <fct> <fct> <ord>   <dbl> <dbl> <int>
## 1 23.1 -73.7          5788. Joaq~ hurri~ 4      2015    10    1
## 2 23   -74.2           NaN Joaq~ hurri~ 4      2015    10    1
## 3 22.9 -74.4           NaN Joaq~ hurri~ 4      2015    10    2
## 4 23   -74.7          4340. Joaq~ hurri~ 4      2015    10    2
## 5 23.4 -74.8         10732. Joaq~ hurri~ 4      2015    10    2
## 6 24.3 -74.3           NaN Joaq~ hurri~ 4      2015    10    3
## 7 24.8 -73.6          6425. Joaq~ hurri~ 4      2015    10    3
## 8 25.4 -72.6         11564. Joaq~ hurri~ 4      2015    10    3
## 9 26.3 -71           3385. Joaq~ hurri~ 4      2015    10    3
## 10 27.4 -69.5           NaN Joaq~ hurri~ 4      2015    10    4
## # ... with 10,000 more rows, and 11 more variables: hour <dbl>,
## #   wind <int>, pressure <int>, ts_diameter <dbl>, hu_diameter <dbl>,
## #   wind_speed_per_unit_pressure <dbl>, average_diameter <dbl>,
## #   timestamp <dtm>, day_of_week <ord>, week_of_the_year <dbl>,
## #   decile_windspeed <fct>
```

Let's summarize some data. Find the strongest storm by wind speed per year.

```
storms %>%
  group_by(year) %>%
  summarize(max_wind_speed = max(wind))
```

```
## Warning: Grouping rowwise data frame strips rowwise nature
```

```
## # A tibble: 41 x 2
```

```
##   year max_wind_speed
##   <dbl>          <int>
## 1 1975             100
## 2 1976             105
## 3 1977             150
## 4 1978              80
## 5 1979             150
## 6 1980              90
## 7 1981             115
## 8 1982             115
## 9 1983             100
## 10 1984            115
## # ... with 31 more rows
```

For each status, find the average category, wind speed, pressure and diameters (do not allow the average to be NA).


```
storms %>%
  group_by(status) %>%
  summarise(avg_category = mean(as.numeric(as.character(category))), avg_wind_speed = mean(wind), avg_p
```

```
## Warning: Grouping rowwise data frame strips rowwise nature
```

```
## # A tibble: 3 x 6
##   status avg_category avg_wind_speed avg_pressure avg_ts_diameter
##   <fct>      <dbl>      <dbl>      <dbl>      <dbl>
## 1 hurri~      1.86        86.0        969.        288.
## 2 tropi~     -1         27.3       1008.         0
## 3 tropi~    0.000229      45.8        999.       160.
## # ... with 1 more variable: avg_hu_diameter <dbl>
```

For each named storm, find its maximum category, wind speed, pressure and diameters (do not allow the max to be NA) and the number of readings (i.e. observations).

```
storms %>%
  group_by(name) %>%
  summarize(max_category = max(category), max_wind_speed = max(wind),
    max_pressure = max(pressure), max_hu_diameter = max(hu_diameter, na.rm = TRUE),
    max_ts_diameter = max(ts_diameter, na.rm = TRUE), readings = n() )
```

```
## Warning: Grouping rowwise data frame strips rowwise nature
```

```
## # A tibble: 198 x 7
##   name max_category max_wind_speed max_pressure max_hu_diameter
##   <fct> <ord>          <int>      <int>      <dbl>
## 1 AL01~ -1           30       1003      -Inf
## 2 AL01~ -1           25       1010      -Inf
## 3 AL02~ -1           30       1009      -Inf
## 4 AL02~ -1           30       1017      -Inf
## 5 AL02~ -1           30       1006      -Inf
## 6 AL02~ -1           30       1010      -Inf
## 7 AL02~ -1           25       1012      -Inf
## 8 AL02~ -1           30       1010      -Inf
## 9 AL02~ 0            45       1008         0
## 10 AL03~ 0            40       1015      -Inf
## # ... with 188 more rows, and 2 more variables: max_ts_diameter <dbl>,
## #   readings <int>
```

For each category, find its average wind speed, pressure and diameters (do not allow the max to be NA).

```
storms %>%
  group_by(category) %>%
  summarize(ave_wind_speed = mean(wind), ave_pressure = mean(pressure), mean(c(hu_diameter, ts_diameter)
```

```
## Warning: Grouping rowwise data frame strips rowwise nature
```

```
## # A tibble: 7 x 4
##   category ave_wind_speed ave_pressure `mean(c(hu_diameter, ts_diameter), ~
##   <ord>      <dbl>      <dbl>      <dbl>
## 1 -1        27.3       1008.         0
## 2 0         45.8       999.       79.8
## 3 1         70.9       982.      168.
## 4 2         89.4       967.      180.
## 5 3        105.       954.      199.
## 6 4        122.       940.      209.
```

```
## 7 5          145.          916.          219.
```

At home: for each named storm, find its duration in hours.

```
storms %>%
  group_by(name) %>%
  mutate (duration = 6*n()) %>%
  arrange(desc(duration))

## Warning: Grouping rowwise data frame strips rowwise nature
## # A tibble: 10,010 x 21
## # Groups:   name [198]
##   lat long distance_to_mia~ name status category year month day
##   <dbl> <dbl>          <dbl> <fct> <fct> <ord>   <dbl> <dbl> <int>
## 1 14.9 -61.4           NaN Emily tropi~ 0      2011 8 2
## 2 15.1 -62.5           NaN Emily tropi~ 0      2011 8 2
## 3 15.4 -63.6        6188. Emily tropi~ 0      2011 8 2
## 4 15.7 -64.8           NaN Emily tropi~ 0      2011 8 2
## 5 16 -66.2           NaN Emily tropi~ 0      2011 8 3
## 6 16.3 -67.7        6570. Emily tropi~ 0      2011 8 3
## 7 16.6 -69.1        9661. Emily tropi~ 0      2011 8 3
## 8 16.8 -70.3        5898. Emily tropi~ 0      2011 8 3
## 9 16.9 -70.7        1345. Emily tropi~ 0      2011 8 4
## 10 16.9 -71.3       4578. Emily tropi~ 0      2011 8 4
## # ... with 10,000 more rows, and 12 more variables: hour <dbl>,
## #   wind <int>, pressure <int>, ts_diameter <dbl>, hu_diameter <dbl>,
## #   wind_speed_per_unit_pressure <dbl>, average_diameter <dbl>,
## #   timestamp <dtm>, day_of_week <ord>, week_of_the_year <dbl>,
## #   decile_windspeed <fct>, duration <dbl>
```

#they are taken every 6 hours so its 6 times the number of readings

For each named storm, find the distance from its starting position to ending position in kilometers.

```
storms %>%
  group_by(name) %>%
  arrange(desc(timestamp)) %>%
  summarize(distance_from_start =
    1.61*compute_globe_distance( c(last(lat), last(long)) , c(first(lat), first(long)) ) )

## Warning: Grouping rowwise data frame strips rowwise nature
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```

```
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## # A tibble: 198 x 2
##   name      distance_from_start
##   <fct>          <dbl>
## 1 AL011993           NaN
## 2 AL012000          9660.
## 3 AL021992         10030.
## 4 AL021994         12555.
## 5 AL021999          7057.
## 6 AL022000         11862.
## 7 AL022001           NaN
## 8 AL022003           NaN
## 9 AL022006          5681.
## 10 AL031987         6271.
## # ... with 188 more rows
```

Now we want to transition to building real design matrices for prediction. We want to predict the following: given the first three readings of a storm, can you predict its maximum wind speed? Identify the y and identify which features you need x_1, \dots, x_p and build that matrix with `dplyr` functions. This is not easy, but it is what it's all about. Feel free to “featurize” (as Dana Chandler spoke about) as creatively as you would like. You aren't going to overfit if you only build a few features relative to the total 198 storms.

```
y = storms %>%
  group_by(name) %>%
  summarize(max_wind_speed = max(wind))
```

```
## Warning: Grouping rowwise data frame strips rowwise nature
```

```
y = y %>%
  arrange(desc(name))
X = storms %>%
  group_by(name) %>%
  arrange(desc(timestamp)) %>%
  filter(timestamp <= nth(timestamp, n()-2)) %>%
  summarize(ave_pressure = mean(pressure), ave_category = mean(as.numeric(as.character(category))),
    distance_from_start = compute_globe_distance( c(last(lat),last(long)) , c(first(lat),first(long)) ),
    ave_ts_diameter = mean(ts_diameter, na.rm = TRUE), ave_hu_diameter = mean(hu_diameter, na.rm = TRUE),
    pressure_by_ts_diameter = ave_pressure * ave_ts_diameter, pressure_by_hu_diameter = ave_pressure * ave_hu_diameter,
    category_by_ts_diameter = ave_category * ave_ts_diameter, category_by_hu_diameter = ave_category * ave_hu_diameter)
```

```
## Warning: Grouping rowwise data frame strips rowwise nature
```

```
## Warning in sqrt(1 - a): NaNs produced
```

```
## Warning in sqrt(1 - a): NaNs produced
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## Warning in sqrt(a): NaNs produced
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## Warning in sqrt(1 - a): NaNs produced
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## Warning in sqrt(1 - a): NaNs produced
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```
## Warning in sqrt(1 - a): NaNs produced
```

[illegible]

```
#Arrange by descending time to get the three earliest observations.
#Take average pressure, average category, how far the storms traveled in 18 hours.
#I included interactions with the diameters because sometimes they were zero.
edge_case = storms %>%
  group_by(name) %>%
  mutate (observations = n()) %>%
  filter(observations < 3) %>%
  summarize(ave_pressure = mean(pressure), ave_category = mean(as.numeric(as.character(category))),
    distance_from_start = compute_globe_distance( c(last(lat),last(long)) , c(first(lat),first(long)) ),
    ave_ts_diameter = mean(ts_diameter, na.rm = TRUE), ave_hu_diameter = mean(hu_diameter, na.rm = TRUE),
    pressure_by_ts_diameter = ave_pressure * ave_ts_diameter, pressure_by_hu_diameter = ave_pressure * ave_hu_diameter,
    category_by_ts_diameter = ave_category * ave_ts_diameter, category_by_hu_diameter = ave_category * ave_hu_diameter)
```

```
X = rbind.data.frame(X, edge_case)
X = X %>%
  arrange(desc(name))
#We check for an edge case, where a storm does not have three observations.
#Then append it to our design matrix then also reorder the names.
y = y %>%
  select(-name)
X = X %>%
```

```

select(-name)
mod = lm(as.matrix(y) ~ as.matrix(X))
summary(mod)$r.squared

```

```
## [1] 0.08750061
```

```
summary(mod)$sigma
```

```
## [1] 35.36272
```

Interactions in linear models

Load the Boston Housing Data from package `MASS` and use `str` and `summary` to remind yourself of the features and their types and then use `?MASS::Boston` to read an English description of the features.

```

data(Boston, package = "MASS")
str(Boston)

```

```

## 'data.frame':    506 obs. of  14 variables:
## $ crim   : num  0.00632 0.02731 0.02729 0.03237 0.06905 ...
## $ zn     : num  18 0 0 0 0 12.5 12.5 12.5 12.5 ...
## $ indus  : num  2.31 7.07 7.07 2.18 2.18 2.18 7.87 7.87 7.87 ...
## $ chas   : int   0 0 0 0 0 0 0 0 0 0 ...
## $ nox    : num  0.538 0.469 0.469 0.458 0.458 0.458 0.524 0.524 0.524 ...
## $ rm     : num  6.58 6.42 7.18 7 7.15 ...
## $ age    : num  65.2 78.9 61.1 45.8 54.2 58.7 66.6 96.1 100 85.9 ...
## $ dis    : num  4.09 4.97 4.97 6.06 6.06 ...
## $ rad    : int   1 2 2 3 3 3 5 5 5 ...
## $ tax    : num  296 242 242 222 222 222 311 311 311 311 ...
## $ ptratio: num  15.3 17.8 17.8 18.7 18.7 18.7 15.2 15.2 15.2 15.2 ...
## $ black  : num  397 397 393 395 397 ...
## $ lstat  : num  4.98 9.14 4.03 2.94 5.33 ...
## $ medv   : num  24 21.6 34.7 33.4 36.2 28.7 22.9 27.1 16.5 18.9 ...

```

```
summary(Boston)
```

```

##      crim              zn              indus              chas
## Min.   : 0.00632   Min.   : 0.00   Min.   : 0.46   Min.   :0.00000
## 1st Qu.: 0.08204   1st Qu.: 0.00   1st Qu.: 5.19   1st Qu.:0.00000
## Median : 0.25651   Median : 0.00   Median : 9.69   Median :0.00000
## Mean   : 3.61352   Mean   : 11.36   Mean   :11.14   Mean   :0.06917
## 3rd Qu.: 3.67708   3rd Qu.: 12.50   3rd Qu.:18.10   3rd Qu.:0.00000
## Max.   :88.97620   Max.   :100.00   Max.   :27.74   Max.   :1.00000
##      nox              rm              age              dis
## Min.   :0.3850   Min.   :3.561   Min.   : 2.90   Min.   : 1.130
## 1st Qu.:0.4490   1st Qu.:5.886   1st Qu.: 45.02   1st Qu.: 2.100
## Median :0.5380   Median :6.208   Median : 77.50   Median : 3.207
## Mean   :0.5547   Mean   :6.285   Mean   : 68.57   Mean   : 3.795
## 3rd Qu.:0.6240   3rd Qu.:6.623   3rd Qu.: 94.08   3rd Qu.: 5.188
## Max.   :0.8710   Max.   :8.780   Max.   :100.00   Max.   :12.127
##      rad              tax              ptratio              black
## Min.   : 1.000   Min.   :187.0   Min.   :12.60   Min.   : 0.32
## 1st Qu.: 4.000   1st Qu.:279.0   1st Qu.:17.40   1st Qu.:375.38
## Median : 5.000   Median :330.0   Median :19.05   Median :391.44
## Mean   : 9.549   Mean   :408.2   Mean   :18.46   Mean   :356.67

```

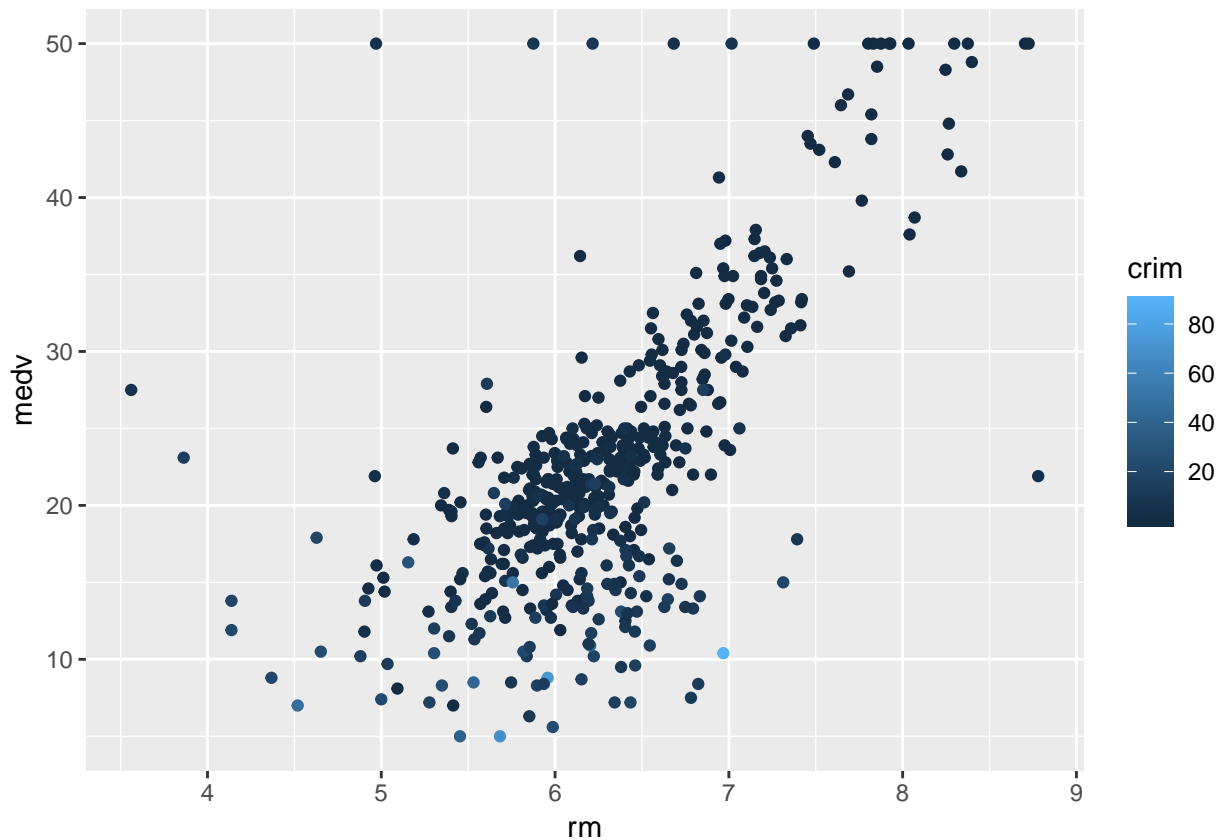
```
## 3rd Qu.:24.000 3rd Qu.:666.0 3rd Qu.:20.20 3rd Qu.:396.23
## Max. :24.000 Max. :711.0 Max. :22.00 Max. :396.90
## lstat medv
## Min. : 1.73 Min. : 5.00
## 1st Qu.: 6.95 1st Qu.:17.02
## Median :11.36 Median :21.20
## Mean :12.65 Mean :22.53
## 3rd Qu.:16.95 3rd Qu.:25.00
## Max. :37.97 Max. :50.00
```

```
?MASS::Boston
```

```
## starting httpd help server ... done
```

Using your knowledge of the modeling problem, try to guess which features are interacting. Confirm using plots in `ggplot` that illustrate three (or more) features.

```
pacman::p_load(ggplot2)
base = ggplot(Boston, aes(x = rm, y = medv))
base + geom_point(aes(col = crim))
```



Once an interaction has been located, confirm the “non-linear linear” model with the interaction term does better than just the vanilla linear model.

```
mod = lm(medv ~ rm * crim, Boston)
coef(mod)
```

```
## (Intercept)      rm      crim  rm:crim
## -37.257338   9.651470  1.462943 -0.287657
```



```
mod_vanilla = lm(medv ~ rm + crim, Boston)
summary(mod_vanilla)$r.squared
```

```
## [1] 0.5419592
```

```
summary(mod_vanilla)$sigma
```

```
## [1] 6.236844
```

```
summary(mod)$r.squared
```

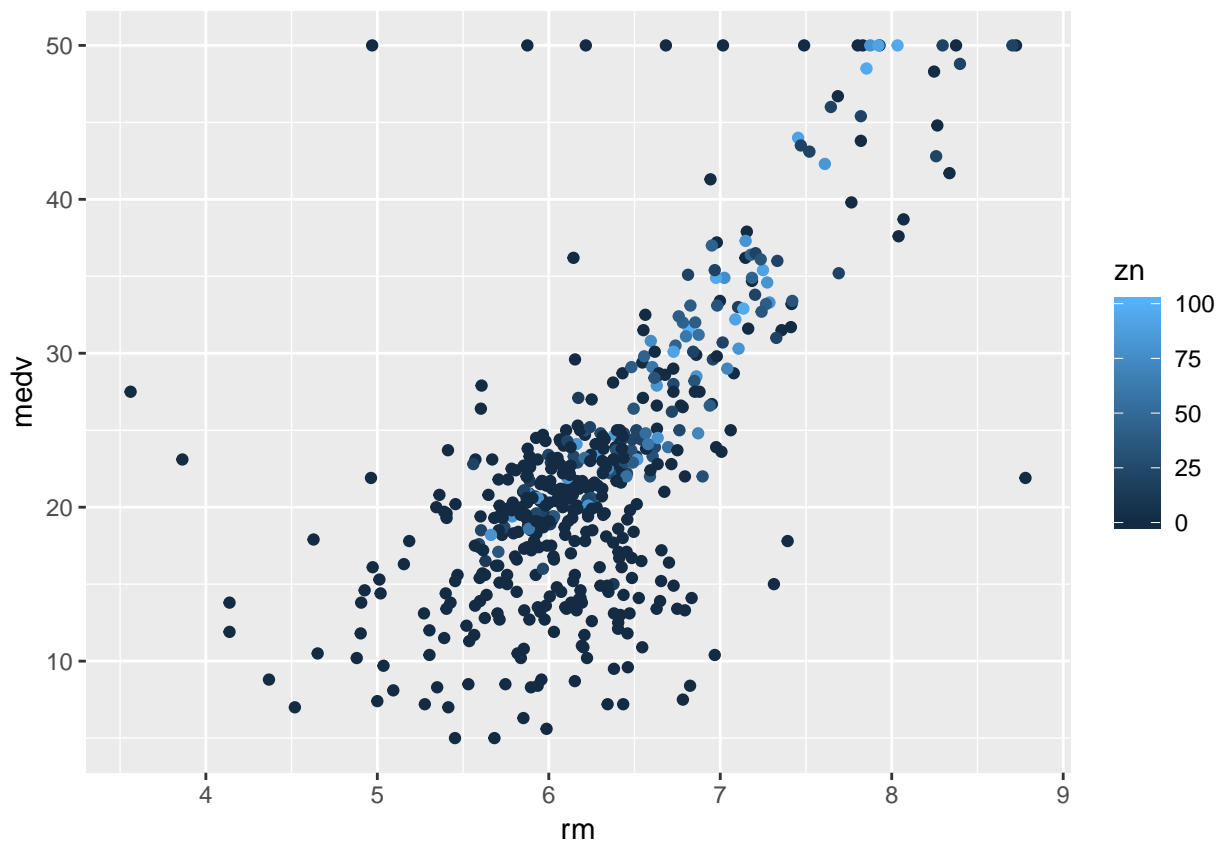
```
## [1] 0.5814763
```

```
summary(mod)$sigma
```

```
## [1] 5.967672
```

Repeat this procedure for another interaction with two different features (not used in the previous interaction you found) and verify.

```
base + geom_point(aes(col = zn))
```



```
mod = lm(medv ~ rm * zn, Boston)
coef(mod)
```

```
## (Intercept)      rm      zn  rm:zn
## -26.9934476  7.7661501 -0.4697937  0.0791624
```

```
mod_vanilla = lm(medv ~ rm + zn, Boston)
summary(mod_vanilla)$r.squared
```

```
## [1] 0.5063381
```

```
summary(mod_vanilla)$sigma
```

```
## [1] 6.474818
```

```
summary(mod)$r.squared
```

```
## [1] 0.5223732
```

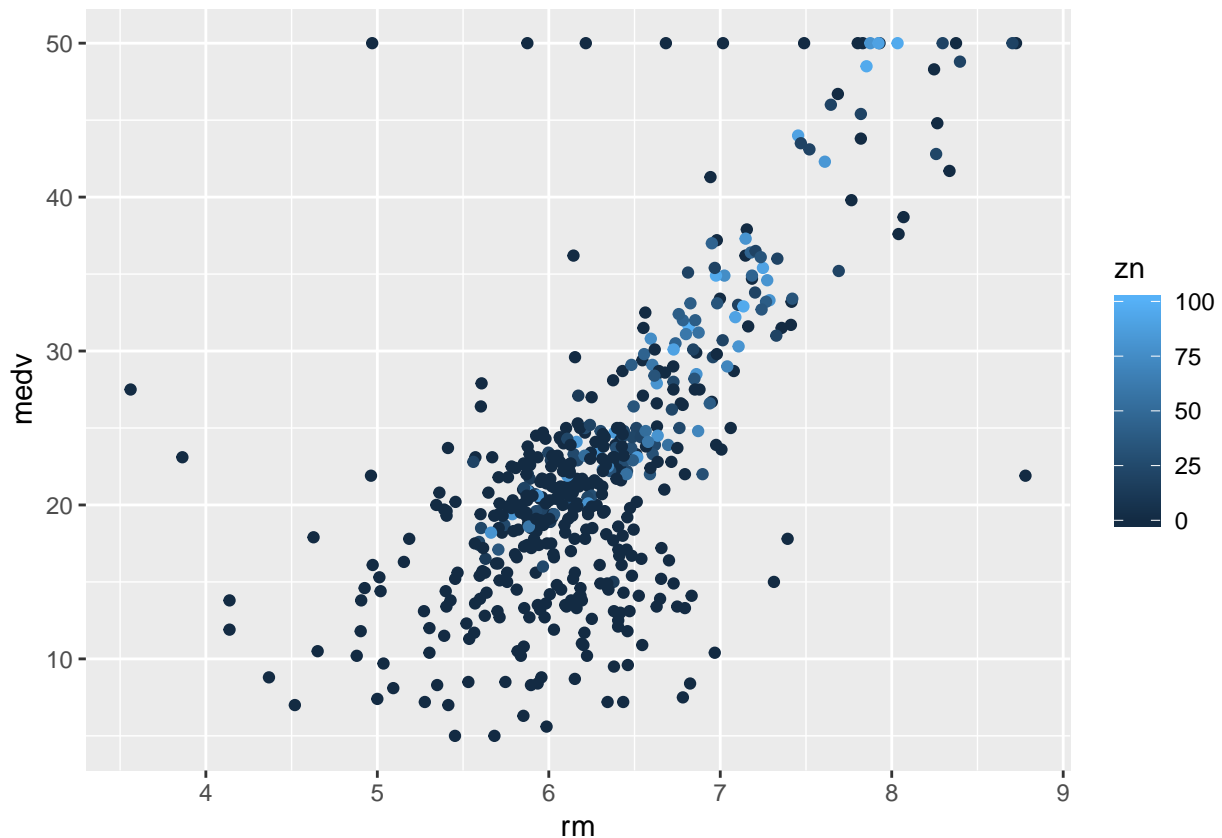
```
summary(mod)$sigma
```

```
## [1] 6.375133
```

Fit a model using all possible first-order interactions. Verify it is “better” than the linear model. Do you think you overfit? Why or why not?

No we didnt overfit n = 509 and we used 91 features

```
base + geom_point(aes(col = zn))
```



```
mod = lm(medv ~ (.)^2 , Boston)
mod_vanilla = lm(medv ~ rm + zn, Boston)
summary(mod_vanilla)$r.squared
```

```
## [1] 0.5063381
```

```
summary(mod_vanilla)$sigma
```

```
## [1] 6.474818
```

```
summary(mod)$r.squared
```

```
## [1] 0.9211876
```

```
summary(mod)$sigma
```

```
## [1] 2.851634
```

CV

Use 5-fold CV to estimate the generalization error of the model with all interactions.

```
pacman::p_load(mlr)
```

```
## Installing package into 'C:/Users/burha/Documents/R/win-library/3.6'
```

```
## (as 'lib' is unspecified)
```

```
## also installing the dependencies 'fastmatch', 'ParamHelpers', 'BBmisc', 'checkmate', 'data.table', 'parallelMap'
```

```
## Warning: unable to access index for repository http://www.stats.ox.ac.uk/pub/RWin/bin/windows/contrib/3.6/
```

```
## cannot open URL 'http://www.stats.ox.ac.uk/pub/RWin/bin/windows/contrib/3.6/PACKAGES'
```

```
## package 'fastmatch' successfully unpacked and MD5 sums checked
```

```
## package 'ParamHelpers' successfully unpacked and MD5 sums checked
```

```
## package 'BBmisc' successfully unpacked and MD5 sums checked
```

```
## package 'checkmate' successfully unpacked and MD5 sums checked
```

```
## package 'data.table' successfully unpacked and MD5 sums checked
```

```
## package 'parallelMap' successfully unpacked and MD5 sums checked
```

```
## package 'XML' successfully unpacked and MD5 sums checked
```

```
## package 'mlr' successfully unpacked and MD5 sums checked
```

```
##
```

```
## The downloaded binary packages are in
```

```
## C:\Users\burha\AppData\Local\Temp\Rtmpmw9d97\downloaded_packages
```

```
##
```

```
## mlr installed
```

```
modeling_task = makeRegrTask(data = Boston, target = "medv") #instantiate the task
```

```
algorithm = makeLearner("regr.lm") #instantiate the OLS learner algorithm on the diamonds dataset and s
```

```
validation = makeResampleDesc("CV", iters = 5) #instantiate the 5-fold CV
```

```
resample(algorithm, modeling_task, validation)
```

```
## Resampling: cross-validation
```

```
## Measures:           mse
```

```
## [Resample] iter 1:    31.6988982
```

```
## [Resample] iter 2:    18.5708417
```

```
## [Resample] iter 3:    27.2670259
```

```
## [Resample] iter 4:    20.0313403
```

```
## [Resample] iter 5:    20.4185830
```

```
##
```

```
## Aggregated Result: mse.test.mean=23.5973378
```

```
##
```

```
## Resample Result
## Task: Boston
## Learner: regr.lm
## Aggr perf: mse.test.mean=23.5973378
## Runtime: 0.0431411
```

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