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Space Reduction of Tentris Hypertrie with Path Compression

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Ort, Datum   Unterschrift

Abstract. The demand for Semantic Web technologies in recent years motivated the construction of a variety of triple stores. Triple stores are a special kind of data management systems designed to store RDF triple data; a standard defined by World Wide Web Consortium (W3C) to represent the data web. Triple stores' performance is a crucial requirement to build scalable and reliable semantic web applications around major industrial fields, including medicine, commerce, and defense. For that, the scientific community dedicates a considerable amount of research to investigating approaches to designing high-quality triple stores. Tentris is a novel triple store developed by the data science research group (DICE) at Paderborn university. It represents an RDF graph as a sparse boolean tensor and maps SPARQL operations to tensor operations. Tentris realizes the tensor concept using a trie-based data structure called Hypertrie that provides efficient translation of tensor operations. In my thesis, I further contribute to the Tentris project by investigating, implementing, and evaluating an approach to reduce Hypertrie memory utilization when storing large RDF data sets.

Contents

Introduction

The amount of data on the world wide web (WWW) increased exponentially in the last few years, and so are the techniques for exploiting them. Semantic web is one of those fields where the work goes toward giving data a contextual meaning so they can be processable by machines. Data on the semantic web are represented by graph of connected resources called Knowledge Graphs. Knowledge graphs are represented by A novel RDF triple based on storing

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Preliminaries and Foundations

This chapter is logically divided into two main parts. The first part presents the tooling and techniques I utilized to implement the space-reduction solution. The first section 2.1 introduces common notations and terminologies used through out the work. Next, the pointer tagging technique is presented in section 2.2). The first part is finalized with a quick overview on template-meta programming in modern C++ (Sec. 2.3).

In the second part, I visit the semantic web topic (Sec. 2.4). Afterwards I delve into Tentris in section 2.5 where I present its concept and implementation. More deeply, I describe how its realized by presenting its main underlying data structure, Hypertrie.

2.1 Notation and Convention

For a function f, the domain and co-domain of f are denoted by $\mathrm{dom}(f)$ and $\mathrm{codom}(f)$, respectively. The set of natural numbers $\mathbb{N}$ includes zero in this work. Further $\mathbb{N}_n$ with $n \in \mathbb{N}$ is equal to $\{0, 1, ..., n-1\}$. Let $\mathbb{B}$ be the set of boolean values; i.e. $\mathbb{B} = \{true, false\}$. We map $true$ to 1 and $false$ to 0. We use angle brackets $< ... >$ to define a tuple t which represents a sequence with fixed order for its elements. The entries $< t_0, t_1, ..., t_{n-1} >$ of a tuple t with length n can be accessed using the square bracket notation (subscript) after the tuple symbol. For example, $t[i] = t_i$ is the tuple t entry at position i. Entries of a tuple t are zero-indexed. The domain of a tuple t, denoted by $\mathrm{dom}(t) = < 0, 1, ..., n-1 >$, is a tuple of t entries' positions.

2.2 Pointer Tagging

Pointer Tagging [13] is a low-level programming technique that uses the spare low bits in a pointer to encode additional information. Using the Pointer tagging technique, pointer value (initially a memory address before tagging) can hold extra information, *a tag*. The tag can hold extra information about the point-to heap object or can be used as meta-data to further describe the usage of the pointer data. Pointer tagging is mainly enabled because of the way heap objects are situated and accessed on modern computer architectures.

2.2.1 Data Structure Alignment

Data alignment (also referred to as data structure padding) is a way in which heap objects are arranged and accessed by the CPU. CPUs in modern computer architecture (say 64-bit archi-

tecture) read data from and write data to memory more efficiently when data is aligned.

On an abstract level, computer memory can be seen as an array of words or bytes, each with its own address. Unlike bytes, the term word has ambiguate meaning. In the context of this work, we are targeting the generic term in the context of CPU architecture. That is, a "processor word" refers to the size of a processor register or memory address register. The term word also refers to the size of CPU instruction, or the size of a pointer depending on the exact CPU architecture [16]. For example, in a 64-bit architecture, the word size (also pointer size) is 64 bits = 8 bytes.

Generally, when a source program is executed, it is loaded into memory and put into a process p for execution. All data objects in the program are mapped at certain point in time (during compilation or execution) to a physical memory address [16]. Let us take, as an example, the following coding snippet written in C++:

```
bool *p1 = new bool(true);      // p1 = 0x011F0
int  *p2 = new int(123);        // p2 = 0x11608
char *p3 = new char('A');       // p3 = 0x117A8
```

The execution of the snippet will metaphorically results in a memory layout similar to the one presented in figure 2.1. According to C++ language specification [17], the size of integer value in memory is 4 bytes, the size of char is 1 byte, and short is 2 bytes. When we execute the previously mentioned statements, however, the compiler (or linker) books 8 bytes of memory to hold the integer value and not 4 bytes as expected. The reason is that, the compiler adds padding to the heap objects in order to align them in memory. The same applies to the character value (Fig. 2.1).

Why data alignment? The CPU can access the memory only in word-sized chunks. So if our data always starts at a word it can be fetched efficiently. If data was to start somewhere in the middle of a word, the CPU will need to wait two or more memory cycles to fetch data from or write data to memory causing an increase in the CPU *stall* period which results in a significant performance overhead [16].

many modern compilers implementations handle data alignment in memory automatically, example includes C, C++, Rust, C# compilers.

2.2.2 Tagged Pointers

Some high-level programming languages, for example C++, offer developers a tool set to work with memory. Using such tool set, developers have access to low level memory abstraction. The main building block that enables memory management is the **pointer data type** and its ecosystem. A variable of type pointer holds a memory address of an object stored in the heap. Due to data alignment (cf. **??**), the memory address of any object in the heap memory is always $\alpha \cdot w$ where $w = 8$ (the word size). This implies all addresses held as a pointer value are multiple of 8. A pointer thus can be 8, 16, 24, 109144, etc. But it can not be 7 or 13.

Consequently, all pointer values share the fact that the first three low significant bits (LSBs) in their binary representation are always 0. As a result, we could exploit those bits to encode additional information without affecting the validity of the pointer (Fig. 2.2).

8 bytes = 1 word

0x0007 0x0006 0x0005 0x0004 0x0003 0x0002 0x0001 0x0000

0x0008 = 0b00000000000001**000**

p1   0x011F0

p2   0x11608

p3   0x117A8

8509   0x011F0 = 0b00001000111110**000**

12   0x11608 = 0b10001011000001**000**

'A'   0x117A8 = 0b100010111111001**000**

Stack

Heap

padding
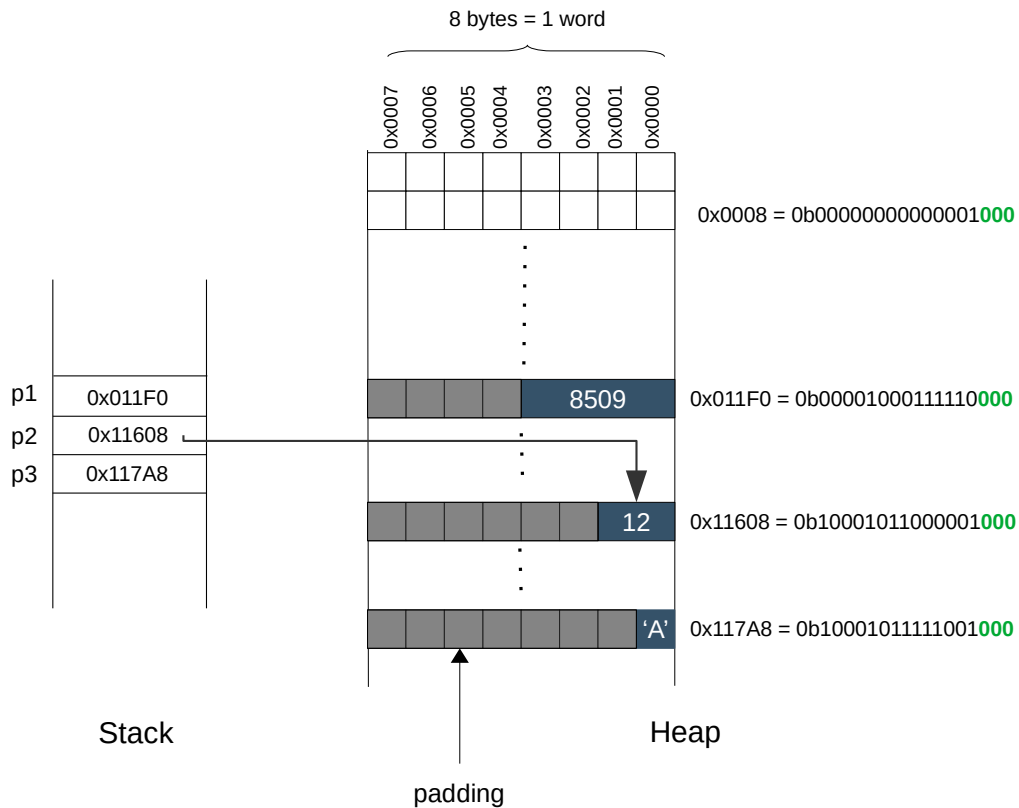
Figure 2.1: An example of memory layout with data objects of various types allocated in the heap. All objects are aligned by 8 bytes so their addresses are always multiple of 8.

address   tag

$64 - N$ bits N bits

Figure 2.2: The structure of a tagged pointer. The number of tagging bits depends on the CPU architecture. Depending on the architecture, a $N = log_2(word\_size)$ of LSBs bits are dedicated for tagging. In 64-bit architecture, $word_size = 8$, thus $N = 3$.

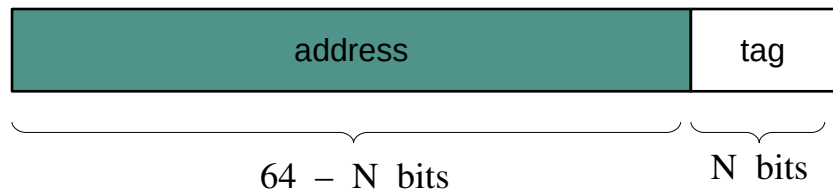As a use case, depending on the tag, pointer tagging allows a dynamic representation of the numeric value held by the pointer. Thus, the actual payload of the pointer could represents a memory address for some time during the process execution but can later express the binary representation of a `char` value for example, depending on the execution context and after a change in the tag value during run-time.

2.2.3 Tagged Pointer Implementation

An approach to implement a tagged pointer is to develop a wrapper object around a pointer type variable. By that, we insure safety by presenting a single point of failure in case of pointer value misuse. The wrapper can be equipped with adequate behaviours that govern the tag/ payload manipulation and retrieval.

Accessing the binary representation of pointer to enable pointer tagging is straightforward in most programming languages and is done by using bit-wise operations.

2.3   Template Meta-programming

In this section I discuss the C++ meta programmin concept.

2.4 Semantic Web

Today World Wide Web (WWW) holds a tremendous amount of different types of data (videos, images, text, geolocation data, books, publications, etc.). Such data comes from different sources (web applications, warehouse systems, GPS devices, smartphones, ATMs, etc.). However, such data are still processed passively by computer system as there is no way to understand its meaning or context. The term **Semantic Web** was coined by Tim Berners-Lee for a *Web of Data* (or *Data Web*) [4] that can be processed by machines. The key technologies of Semantic Web are published by the World Wide Web Consortium (W3C). The Resource Description Framework (RDF), a standard for representing data in the Semantic Web, and SPARQL, a query language for RDF data, are introduced in this section.

2.4.1 Resource Description Framework

The Resource Description Framework (RDF) [1] is part of the W3C standard to define the web of data. Regardless of the nature of the data entity held on the web (blog post, image, publication, newspaper article, list of invoices, etc.), RDF identifies them uniformly as resources. In the standard, each resource is attached to a unique Internationalized Resource Identifier (IRI). IRI is a standard defined by the Internet Engineering Task Force in RFC 3987 [9]. Literals are another sort of resources. A literal comprises a hardcoded value represented as a string; ("Martin," "true", "12.3") are examples of literals. The third resource type is called a blank node. Blank nodes represent anonymous resources and always have local scope where they can be assigned a unique identifier. All definitions in this section are taken from "RDF 1.1 Concepts and Abstract Syntax" [20].

Definition 2.1 (RDF Terms)**.** Let I be the set of *IRIs*, L be the set of *literals* and be B the set of *blank nodes*. Further I, L and B are finite and pair-wise disjoint. Then the set $RT = I \cup L \cup B$ is called the set of RDF terms.

RDF Triple, RDF Graph

On an abstract level, an RDF triple can be seen as a statement that describes the relationship between two resources. An RDF triple can also serve to describe the property of a resource. RDF triple is represented by a sentence composed of three elements in order: *a subject*, *a predicate* and *an object*. The subject is a RDF resource. The subject has a property defined by the predicate and a value for that property set by the object. RDF restricts which RDF terms can be used for subject, predicate and object:

Definition 2.2 (RDF Triple, RDF Graph)**.** An RDF Triple is a triple $(s, p, o) \in (I \cup B) \times I \times RT$ where s is called subject, p is called predicate and o is called object.
A set of of RDF Triples is called RDF graph.

Example 2.1. An example of an RDF graph is given in Table 2.1. The data in the graph presents a list of pioneers along with their occupations and spouses. Table 2.2 defines the abbreviations used in the example.

We use a special language to retrieve data from an RDF graph called SPARQL. SPARQL [12] is a recursive acronym for "SPARQL Protocol And RDF Query Language".

Listing 2.1 shows a simple example of a SPARQL query against the RDF graph in table 2.1.

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subject	predicate	object
ex:PresidentOfUS	rdf:type	madsrdf:Occupation
ex:Professor	rdf:type	madsrdf:Occupation
ex:PoliticalParty	rdf:type	v:Organization
ex:RepublicanParty	rdf:type	ex:PoliticalParty
ex:DemocraticParty	rdf:type	ex:PoliticalParty
ex:AndrewNg	madsrdf:occupation	ex:Professor
ex:BarakObama	madsrdf:occupation	ex:PresidentOfUS
ex:GeraldFord	madsrdf:occupation	ex:PresidentOfUS
ex:BarakObama	ex:party	ex:DemocraticParty
ex:GeraldFord	ex:party	ex:RepublicanParty
ex:BarakObama	foaf:spouse	ex:DemocraticParty
ex:GeraldFord	foaf:spouse	ex:MichelleObama
ex:AndrewNg	foaf:spouse	ex:CarolEReiley

Table 2.1: An RDF graph about a lama and a unicorn, what are their names and what they eat.

abbr.	IRI
rdf:	http://www.w3.org/1999/02/22-rdf-syntax-ns
madsrdf:	http://www.loc.gov/mads/rdf/v1
ex:	https://www.example.com
v:	https://www.w3.org/TR/vcard-rdf/
foaf:	http://xmlns.com/foaf/0.1/

Table 2.2: The list of abbreviations used in table 2.1.

Listing 2.1: A SPARQL query that returns the spouse of each US president whose party is the democratic.

```
SELECT ?spouse
WHERE {
        ?president madsrdf:occupation ex:PresidentOfUS .
        ?president ex:party ex:DemocraticParty .
        ?president foaf:spouse ?spouse }
```

Triple Stores

Triple Stores are a special kind of data management systems designed to store RDF triple data. It can store one or more RDF graphs. Generally, triple stores provide a standard interface to enable performing queries and other semantic operations on the stored RDF triples using a query language such as SPARQL (Sec. 2.4.1). Triple stores usually expose HTTP-based SPARQL endpoints for querying enablement. A triple store client can send a SPARQL query embedded in an HTTP request to the endpoint. The store evaluates the query against the RDF data it holds and return the results to the client in the body of an HTTP response in a format conforms to the system configuration.

2.5 Tentris

There is no standard design guideline for triple stores. Hence different implementations of triple stores co-exist. Each subgroup of these implementations utilizes a category of underlying data structures as well as corresponding algorithms that govern the behavior. In production, triple stores are used to store up to billions of RDF triples. To that extent, quality factors like efficiency and scalability are considered first-class citizens during triple stores' construction. And the selection of internal data structures and the behavior definitions greatly influence the overall system efficiency.

Fuseki, Blazegraph, Virtuoso, and RDF-3X are popular implementations of Triple stores [19, 18, 11, 14]. One of the key design characteristics those triple stores have in common is that they all utilize B+ trees to store the indices. Other categories of triple stores use 3D Boolean tensors to store and process RDF data. Examples of such system include TensorRDF [8] and BitMat [3]. In such systems, each tensor dimension is mapped to a triple data aspect, i.e., subject, predicate, or object. Examples of tensor-based triple stores include systems like TensorRDF and BitMat. In the following, a novel tensor-based RDF triple store is presented.

Tentris is a triple store variant designed by the data science research group at Paderborn university[5]. Tentris is an in-memory storage solution that represents RDF knowledge graphs as sparse order-3 tensors using a novel data structure, called Hypertire. It then uses tensor algebra to carry out SPARQL queries by mapping SPARQL operations to Einstein summation[10]. At the time of this writing, Tentris' SPARQL engine realizes *SPARQL-BGB* (a subset of the SPARQL-algebra). Consequently, the engine can execute SPARQL queries containing the following keywords: `@prefix`, `SELECT`, `WHERE`, and `DISTINCT`[12]. mention that you omit talking about the query aspects: Einstein summation, etc

Since my work represents a further contribution to the project Tentris, I dedicate this section to deliver a preface for the relevant aspects. In subsection 2.5.1, I specify what tensors are. A way of representing RDF graphs in tensors is showed in subsection 2.5.2. Hypertrie data structure is visited in subsection 2.5.3.

2.5.1 Tensor Algebra

In mathematics, the term Tensor holds a representation-independent meaning. According to [15], Tensors are "objects with many indices that transform in a specific way under a change of Basis". This mathematical construct has many applications in physics, artificial intelligence, and other fields. However, for tensors to be applied in a practical context, a more concrete definition should be selected. Among other choices, multi-dimensional arrays are widely used as a representation of tensors. Throughout this work, a finite n-dimensional array for tensor representation is adopted. In the context of Tentris [5], the term Tensor adopts the following concrete definition:

Definition 2.3 (Tensor). A mapping
$$T : \mathbf{K} \to V$$
from a multi-index $\mathbf{K}$ to a codomain V is called tensor. $\mathbf{K}$ is called *key basis* of dimension $n \in \mathbb{N}$ with
$$\mathbf{K} = K_0 \times K_1 \times ... \times K_{n-1}, K_i \subset \mathbb{N}$$
The tuple $\mathbf{k} \in \mathbf{K}$ is called *key*, K_i is called a *key part basis* and $k \in K_i$ is called *key part*.
The dimension of the key basis is also called dimension of the tensor and is denoted by ndim(T)=n.

Further, $\text{nnz}(T) = |\{\mathbf{k} \in \mathbf{K} | T(\mathbf{k}) \neq 0\}|$ denotes the *number of non-zero entries.*

To resolve a value of a tensor, we use the array subscript notation $T[k_0, .., k_{n-1}]$. Moreover, the symbol $V^{\mathbf{K}}$ denotes the set of all mappings $T : \mathbf{K} \to V$. My work exclusively consider tensors T with $\mathbb{B}$ or $\mathbb{N}$ as codomain and only use multi-indexes $K_1 = ... = K_n \subset \mathbb{N}$. An n-dimensional tensor is also called *order-n* tensor.

Example 2.2. In the following, some examples to illustrate the Definition 2.3:

1. A tensor $S \in \mathbb{Z}^{\emptyset}$ is called a *scalar*.
$$S = 1$$
   So $S[\emptyset] = S[]$ is 1.

2. A tensor $X \in \mathbb{Z}^{\mathbb{N}_3}$ is called a *vector*.
$$X = \begin{bmatrix} 4 \\ 7 \\ 15 \end{bmatrix}$$
   where $X[2]$ is 15.

3. Now, we take a three dimensional tensor $Y \in \mathbb{Z}^{\mathbb{N}_2 \times \mathbb{N}_2 \times \mathbb{N}_2}$. It can be visualized by a vector for the first key part which, in turn, has matrices each with dimensions corresponding to second and third key parts. As an example:
$$Y = \begin{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \\ \begin{bmatrix} 7 & 11 \\ 13 & 15 \end{bmatrix} \end{bmatrix}$$
   Such that, $Y[1, 0, 1]$ is 11.

Tensor Operations

This section highlights the core operations on tensors. In particular, I describe what tensor slicing and Einstein summations are. In the following, I skip the formal definitions of the operations, instead I present them informally with examples.

Slices Slicing is a useful operation to retrieve a well-defined portion of a tensor $T \in V^K$ in the form of a lower order tensor. Slicing is done by using a *slice key* $\mathbf{s} \in S = (K_0 \cup \{:\}) \times .... \times (K_{n-1} \cup \{:\})$, and denoted like as we are retrieving a value but with one or more dimensions not bound, e.g. $T[:, x, :]$ (or $T[<:, x, :>]$). When applying the slice key $\mathbf{s}$ to a tensor T, the unbounded dimensions in the slice key (the ones that are marked with :) are kept. A slice key part $s_i \neq:$ removes all entries with other key parts at position i and removes K_i from the resultant tensor domain. e.g., $T[:, 2, :]$ or $T[<:, 2, :>]$.

Example 2.3. Back to the third item in Example 2.2, slicing tensor Y by slice keys $s_1 = <:, 1, :>$ results in an order-2 tensor $Z_1 = Y[:, 1, :]$, and with a slice key $s_2 = < 0, 1, :>$ results in an order-1 tensor $Z_2 = Y[0, 1, :]$:
$$Z_1 = \begin{bmatrix} 1 & 2 \\ 7 & 11 \end{bmatrix}$$
$$Z_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Worth to mention that, slicing operation is associative, meaning that applying a sequence of slicing operations with different grouping to tensor Y brings the same results:
$$Y[1, 0, :] = (Y[1, :, :])[0, :] = Y([:, 0, :])[1, :] = Z_2$$

Einstein Summation Available in many software packages. Allow mutiple operation on different tensors (including vectors, matrices) Einstein Notation Example: Print Three arrays
Calculate the summation result (from the paper)

2.5.2 Storing RDF Graphs in Tensors

Based on [5], let g be an RDF graph with IRIs (I), blank nodes (B) and literals (L) being finite sets, and thus the set of RDF terms RT is also finite (Sec. 2.4.1). We define a bijective function $id_{RT} : RT \to N_n$ where $|RT| = n$. We call the function *index of resources.* id_{RT} maps each RDF term to a unique identifier. Similarly, we define the *resources by indices* function as $res_{RT} = id_{RT}^{-1}$. We omit the use of the subscript RT with the functions' names id_{RT} and res_{RT} when it is clear from the context.

Every triple $< s, p, o >\in g$ is also in $(RT \times RT \times RT)$. For that, we can define an order-3 tensor $T \in \mathbb{B}^{N_n \times N_n \times N_n}$. We store the triple $< s, p, o >$ in T by setting $T[id(s), id(p), id(o)] = 1$ and all other entries in T are set to 0. We call T the tensor representation of the RDF graph g and denoted by T_g. In other words:
$$T[i_s, i_p, i_o] = 1 \iff < res(i_s), res(i_p), res(i_o) >\in g$$

Example 2.4. Figure 2.3 shows a 3D visualization of 3-dimensional tensor that represents the RDF graph g depicted in table 2.3. We notice that the number of distinct RDF terms in g is 7, so we can represent g with a tensor $T_g \in \mathbb{B}^{N_7 \times N_7 \times N_7}$.

Since T_g has the codomain $\mathbb{B}$, we describe T_g as a *boolean tensor*. Subject, predicate, and object are associated with one dimension each. In practice, a dimension range ($|RT| = n$) of an RDF tensor T_g fits much more than the bounded resources' IDs in g for that dimension. In [5], however, it is proven that by choosing equal key part basis for enable efficient dimension matching using Einstein notation. As T_g can store a super set of RDF triple in g, we call T_g a *sparse tensor*

subject	predicate	object
:e1 (1)	foaf:knows (2)	:e2 (3)
:e1 (1)	foaf:knows (2)	:e3 (4)
:e2 (3)	foaf:knows (2)	:e3 (4)
:e2 (3)	foaf:knows (2)	:e4 (5)
:e3 (4)	foaf:knows (2)	:e2 (3)
:e3 (4)	foaf:knows (2)	:e4 (5)
:e2 (3)	rdf:type (6)	dbr:Unicorn (7)
:e4 (5)	rdf:type (6)	dbr:Unicorn (7)

Table 2.3: An RDF graph example. Resources are printed alogn with their corresponding IDs (enclosed in brackets).
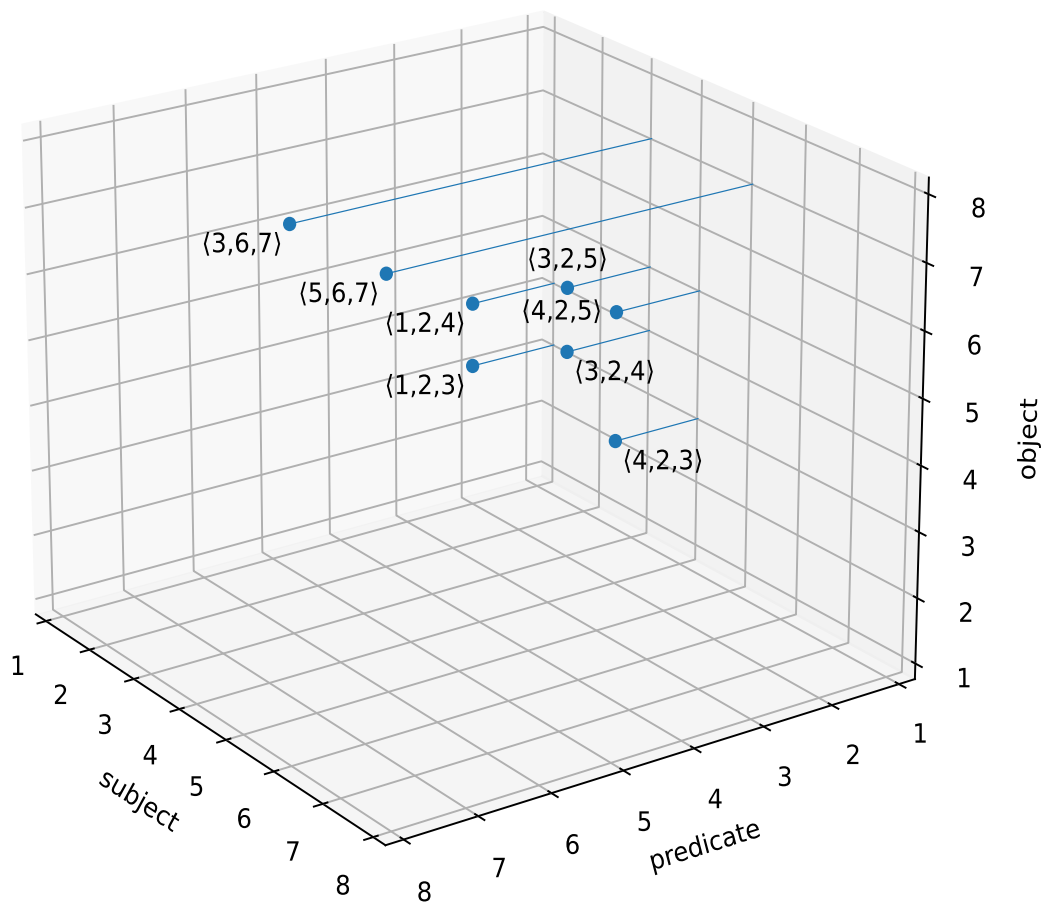
.

Figure 2.3: A 3D plot of an order-3 RDF tensor representing the RDF graph in table 2.3.

2.5.3 Tentris and Hypertrie

Underlying the order-d RDF tensor concept, Tentris realizes the concept using a novel data structure called Hypertrie. Before I discuss hypertrie structure, I first clarify how tensors storing RDF graphs can be represented as tries. Then, I show why this is not engouh for our use case. rephrase the concept.

Trie

A *Trie* [6] is a tree structure used to store *sequences of characters* from an alphabet $\mathbb{A}$. An example of an alphabet is the ASCII codes set. Sequences, in this case, are strings. A node in a Trie contains potentially one outgoing edge for each possible character. Each node in the tree corresponds to a prefix of some sequences of the set, so if the same prefix occurs several times, there is only one node to represent it.

A possible realization of a Trie node is to use a pointer array of the size $|\mathbb{A}|$ [6]. Each pointer points to either another trie node or null. Each array entry corresponds to a character in the alphabet. As an array lookup operation is computationally constant, looking up sequences in the Trie is fast and requires only $O(k)$ time where k is the sequence size. This approach, however, becomes space-inefficient as the size of the alphabet set increases or when it is infinite. An alternative way to store edges is to maintain a hash table *HT* whose size will increase as we add distinct edges. The hash table keys corresponds to *edge labels* (characters) and are mapped to the node pointers (edges). Following that, the structure can still be used efficiently to retrieve sequences as accessing keys in a well-implemented sparse hash table is nearly constant.

A *fixed-depth trie C* is a trie that holds sequences of the same length n, termed as *keys*. In that case, a sequence $l = < l_0, \ldots, l_m > \in \mathbb{A}^m$ of length $m <= n$ forms a *key prefix*. $C[l]$ is defined as the node that is reached from the root node r by walking along the nodes with edge labels equal to the entries of l. Hence, $C[l]$ could be undefined if no appropriate path exists. Moreover,

We can represent a tensor $T_g \in \mathbb{B}^{\mathbb{N}_n \times \mathbb{N}_n \times \mathbb{N}_n}$ that is used to store an RDF graph g with $|RT| = n$ (Sec. 2.5.2) using a fixed-depth trie C_{T_g} of depth $d = \text{ndim}(T_g) = 3$ with the alphabet $\mathbb{N}_n$. We call C_{T_g} a *a trie tensor or trie representation of T_g*, if:
$$\forall \mathbf{k} \in \mathbb{N}_n \times \mathbb{N}_n \times \mathbb{N}_n, v \neq 0 : T_g[k] = v \iff C_{T_g}[k] = v$$

Resolving a tensor trie C_T of depth d by key prefix $l = < k_0, ..., k_m >$ where $m < d$ is equavalent to slicing the corresponding d-order tensor using a slice key $s = < k_0, ..., k_m, e_{m+1}, ..., e_d >$ with $e_i = $ **:**. Then the resultant tensor slice $T[k_0, ..., k_m, e_{m+1}, ..., e_d]$ (d'-order where $d' = d - m$) is represented by an inner trie node or *subtrie* of depth d'.

In the context of this writing, the term *node depth* corresponds to the sequences length that a node holds. For a trie tensor C_T of depth d. The root node is assigned the depth d, an inner node holding prefix of size m, has the depth $d' = d - m$. Leaf nodes share the depth 1. An example of a trie tensor is showed in figure 2.4.
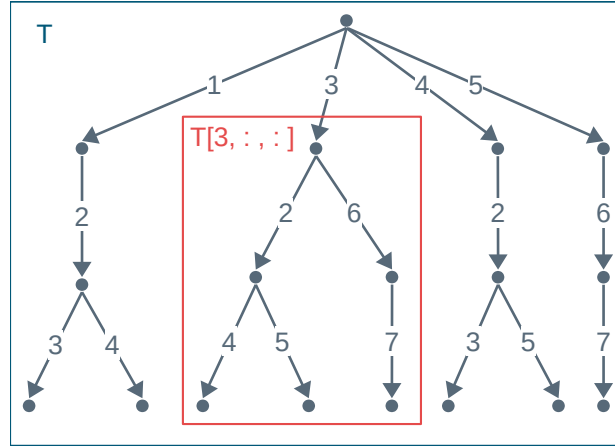
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Figure 2.4: Trie representation of the tensor T_g that depicts the RDF graph g in Table 2.3. A slice $T_g[3, :, :]$ by the first dimension with 3 is shown in the red (inner) box.

Hypertrie

In the previous section we defined a trie as a data strucuture for realizing an order-3 RDF tensor. The structure is memory-efficient as it sparsely encodes a Boolean-valued tensor by storing only the keys that map to 1. As keys with the same key prefixes share the same node, a moderate level of compression is achieved. Slicing is also efficient if the set of edges to children are represented using a well-designed hashtable or search tree[**?**].

A trie, as defined earlier, however, still lake the ability for flexible slicing, that is by any combination of dimensions. The trie tensor as it is defined so far allows slicing positions to only suffix the slice keys, thus using $s = <:, 3, :>$ as a slice key is not possible. Felxible slicing mush alwasy be maintained to enable resolving triple patterns (Sec. ???). Tentris uses *Hypertrie*, a novel trie-based data structure, for holding the keys of an RDF tensor. Hypertrie generalize the normal trie tensor concept by enabling the efficient slicing by any combination of dimensions. Hypertrie is defined formally based on Tentris paper [5] in the following:

Definition 2.4. Let $H(d, A, E)$ with $d \geq 0$ be the set of all hypertries with depth d, alphabet A and values E. If A and E are clear from the context, we use $H(d)$. We set $H(0) = E$ per definition. A hypertrie $h \in H(1)$ has an associated partial function $c_1^{(h)} : A \nrightarrow E$ that specifies outgoing edges by mapping edge labels to children. For $h' \in H(n), n > 1$, partial functions $c_i^{(h')} : A \nrightarrow H(d-1), i \in \mathbb{N}_n$ are defined. Function $c_i^{(h')}$ specifies the edges for resolving the part equivalent to depth i in a trie by mapping edge labels to children. For a hypertrie h, $z(h)$ is the size of the set or mapping it encodes.

Continue like this Informally, a .....

Figure 2.5 shows a hypertrie storing the RDF triples shown in table 2.1. Examples for slicing hypertrie with different slice keys corresponding to various combinations of slicing positions is presented in figure 2.6.

add definition here as needed
Add the concept of node *processed_nodes* $: \mathbb{B}^3 \rightarrow node$
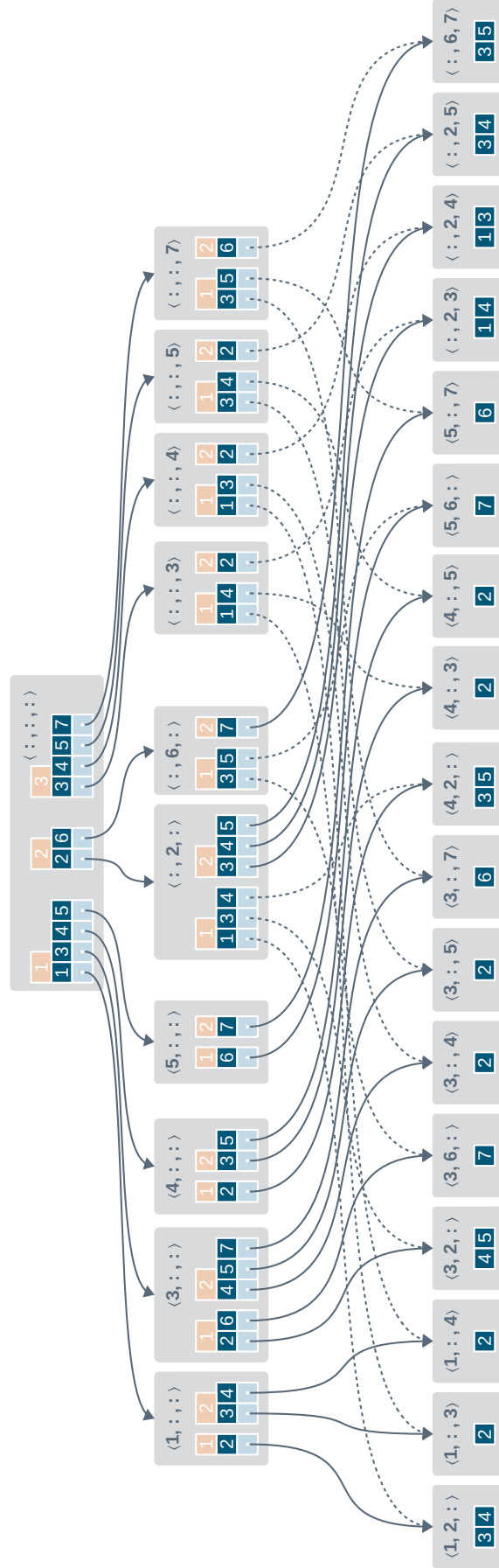
16

Figure 2.5: Trie representation of the tensor T_g that depicts the RDF graph g in Table 2.3. A slice $T_g[3,:,:]$ by the first dimension with 3 is shown in the red (inner) box.
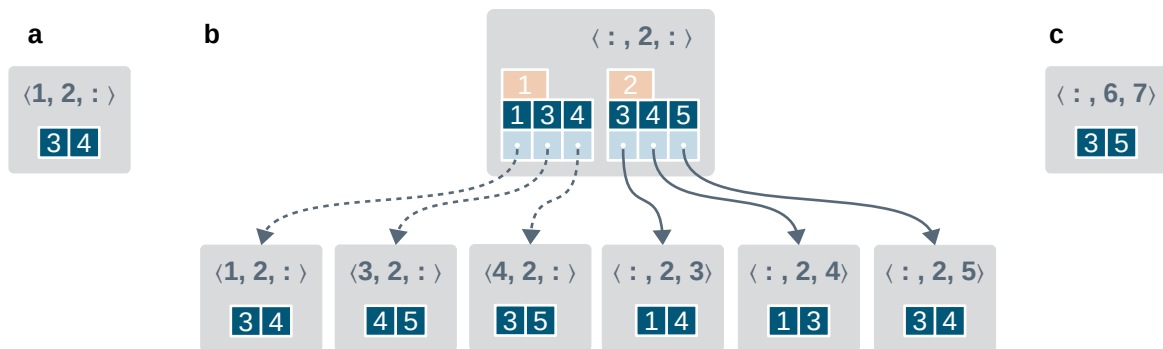
Figure 2.6: Slice.

Related Work 3

asdasdasd

4

Space Reduction Approach

This part of the thesis discusses the approach to substantially mitigates the space inefficiency characteristic of Hypertrie. The technique relies mainly on compressing a Hypertire path with specific characteristics. Worth mentioning that the approach does not neglect the other attempts already realized to minimize Hypertrie memory footprint. In contrast, it can be considered an added feature that further contributes to the space reduction of the overall Hypertrie data structure.

In this chapter, I deliver a motivation to the approach. Afterward, I discuss the new Hypertrie internal nodes' design needed to realize the path compression feature. Finally, algorithms defining the behaviors of the newly designed Hypertrie are also presented.

4.1   Hypertrie Memory Utilization

Despite its operational efficiency, Hypertrie performance comes not without a trade-off. Since Hypertrie is a special kind of a Trie data structure, it inherits some of the fundamental problems of Tries. One of these problems is the excessive space utilization in a worst-case scenario.

The current design and implementation of Hypertrie, however, mitigates the space inefficiency characteristic in two ways. First, for each tensor dimension mapping in each node, the Hypertrie uses custom hash map data structures instead of arrays or linked lists to store the keys. By using a map, Hypertrie's nodes only stores keys that form prefixes to already existed paths. In contrast, arrays utilization in normal Tries considers the whole alphabet set in each node with many array entries store pointers that refer to null.

The adoption of maps in Hypertrie also delivers extra performance as looking up keys in a carefully designed map is nearly constant compared to linked list search where it has a linear complexity $O(n)$. The other solution realized by Hypertrie to reduce the overall space requirement is to store equal nodes (Subhypertrie) only once. In this way, Hypertrie achieves a moderate level of compression in practice.

Despite the previously mentioned attempts to minimize the size of Hypertrie, the excessive memory requirement is still a bottleneck. The case can be witnessed when the set of RDF triples needed to be indexed by Hypertrie increases in size with less overlapping between its elements. As a result, many intermediate nodes store map with a single entry for a particular dimension

where the entry hosts a space for key and a pointer. This becomes a space redundancy issue when the leaf node referenced by the pointer has one key only.

The purpose of the following approach is to try to reach a more space-efficient Hypertrie.

4.2   Space Reduction Solution

The purpose of this section is to give a better intuition on the idea of path compression.

Assuming we want to store the set of RDF triples in listing 4.1, presented in Turtle syntax, in our space-efficient Hypertrie:

Listing 4.1: An example set of RDF triples

```
@prefix rel: <http://www.example.com/schemas/relationship/> .
@prefix ex: <http://www.example.com/schemas/entities/> .
@prefix xsd: <http://www.w3.org/2001/XMLSchema#> .

ex:Germany rel:capital ex:Berlin .
ex:USA rel:capital ex:Washington_DC
ex:USA rel:political_city ex:Washington_DC
ex:Germany rel:population "82.79e6"^^xsd:integer
```

Tentris do not store the actual values of RDF terms (RTs). Instead, it stores their associated identifiers. For generating identifiers, a bijective function $id : RT \rightarrow N$ is used. For the example of RDF data above, a possible mapping for the terms used is given below:

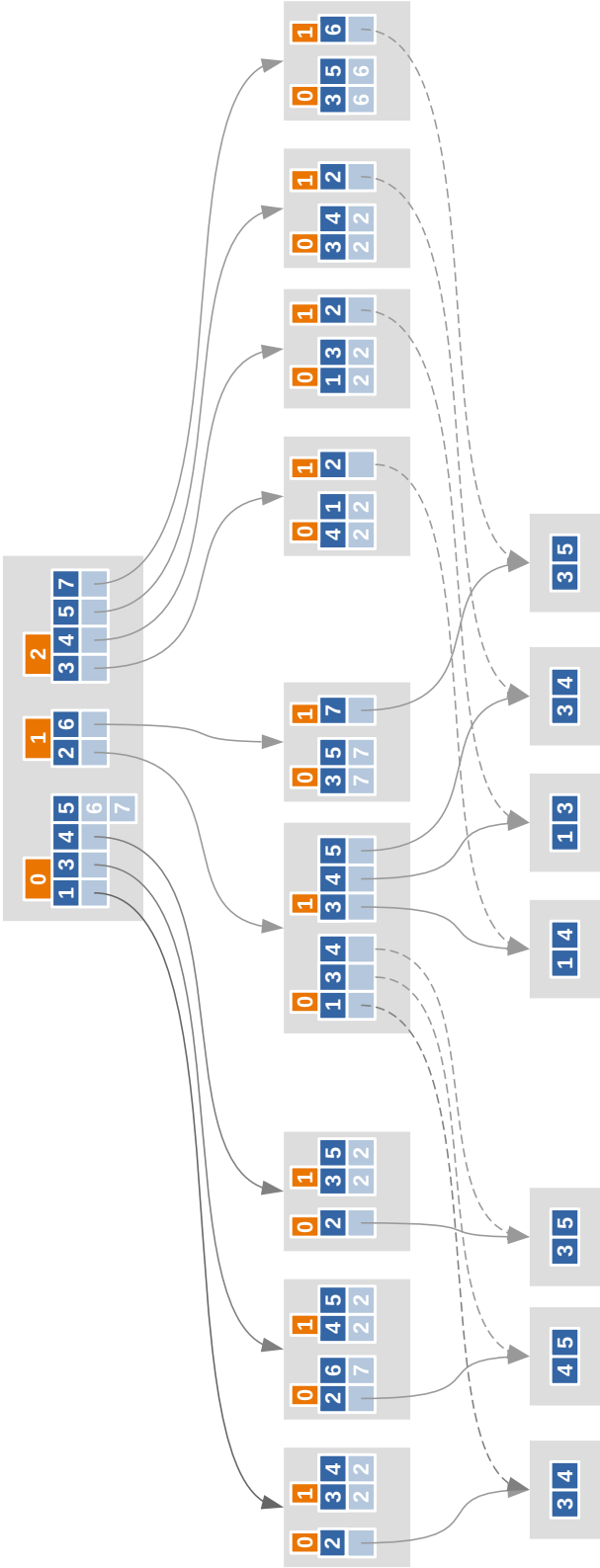| RT	id
ex:Germany	17
rel:capital	4
ex:Berlin	30
ex:USA	20
ex:Washington_DC	40
rel:political_city	5
rel:population	6
82.79e6 (integer)	35

Figure 4.1: Storing RDF in space-efficient Hypertrie.

The Hypertrie will store the triple as shown in Figure **??**, when the path compression technique is applied. It is straightforward to notice that many keys stored in the second level nodes (depth=2) do not need to be branched further to point to other nodes in the third level. As a result, the tree height is cut down, and a substantial amount of memory is saved by storing objects in-place instead of storing them on the heap.

So, the memory for the pointer to the object on the heap is saved. The same method is applied for the root node where keys for a specific dimension are branched by a *lonely path*, i.e. a key path where each element has a single child element.

4.3 Compressed Hypertrie Nodes

In order to achieve path compression in Hypertrie, fundamental design changes need to take place. By that, we can enable the node to store the entire key suffix. Concretely, each group of Hypertrie nodes in certain tree depth will have their own internal node representation.

We create **Adaptive Hypertrie nodes**.

Concretely, each node must hold internal **containers** where key suffixes can be stored.

We reduce the space requirement of Hypertrie by collapsing Hyptertrie nodes to.

From programming point of view, the redesign of Hypertrie nodes' structures is low level. Thanks to C++17 template meta-programming feature, we could separate the compressed nodes realization from the Hypertrie data structure interface. By that, we can still insure a smooth integrity of Hypertrie with other components in Tentris system.

4.3.1 Internal Node Representations

Now I come to the part where the internal structure of space-efficient Hypertrie nodes are discussed. In my approach, it is a requirement to realize the container concept for each node[1] . As a result, each inner node should still be able to expand at certain edges to sub Hypertrie nodes while maintaining a compressed key path in its bounded container for other edges.

The compressed key path container implementation varies depending on the node depth. The idea of having different internal representations comes from the fact that, based on the current structure of nodes on depth two, I found that there is no need to add an additional structure that serves as a container for the key path. Instead, I exploit the space dedicated to pointer value existed as a value in the hash table of store the compressed key path.

Since Hypertrie's internal nodes can be either a root node or depth two nodes, we can distinguish two variants of internal node representations:

Depth 3 Node (root node)

In addition to the set of edges (hash tables) $HT_{3,p}$ for each position $p \in P = \{0, 1, 2\}$, the root node also holds another array of hash tables $CommHT_p$ that maps key parts k_j to static arrays

---

[1]Leaf nodes are not considered.

arr_j. Each array will serve as the container for the key path prefixed by the associated key part k_j at the corresponding position p as depicted in Fig. 4.2. We call the edges k_j stored in $CommHT_p$ **compressed edges**. Clearly, a key part at a particular position p can either represents a compressed or non-compressed edge at a time, so it exists in either HT_p or $CommHT_p$. The remaining key path $k_S =< k_1, k_2 >$ associated with each compressed edge k_j holds the key part chain ordered by their presence in the key.

Worth to mention that the key path associated with each compressed edge still represents a $2D$ sparse tensor S that results from slicing tensor T represented by the root node at position p with key part k_j. The resultant tensor S has a single entry $< k_1, k_2 >$ that evaluates to 1. As a result, it is important to maintain the order of the elements in the compressed path arr_j as each key $arr[i]$ represents the single edge at position i in S whose child is the other array entry.

Depth 2 Node

Internal nodes with $d = 2$ realize the static container concept associated with compressed edges differently than for the root node. Considering the number of internal nodes, it becomes unfeasible, assigning an extra set of hash tables for each node that serve as key part chains containers.

To realize the container concept, we exploit the fact that key suffixes for edges in $d = 2$ $Node_2$ node comprise a single key part. Hence, we could reuse the space already booked to store the pointers to child nodes (leafs) to hold the suffixed key part. For the pointer ptr_j associated with the edge k_j to serve the purpose of either pointing to a child node or holding an integer value, we make it a tagged pointer (cf. 2.2).

Consequently, pointers to children that corresponds to edges k_j in $Node_2$ are denoted by $ptr_j = (value, tag)$. Such pointers carry two pieces of information. (1) A *value* is the actual payload of bits, which can be viewed as either a memory address or a raw integer value depending on the tag value. (2) A *tag* is the value held in the least significant bits (LSB) indicating whether the associated edge k_j is compressed or non-compressed. Figure 4.3 visualizes the structure.
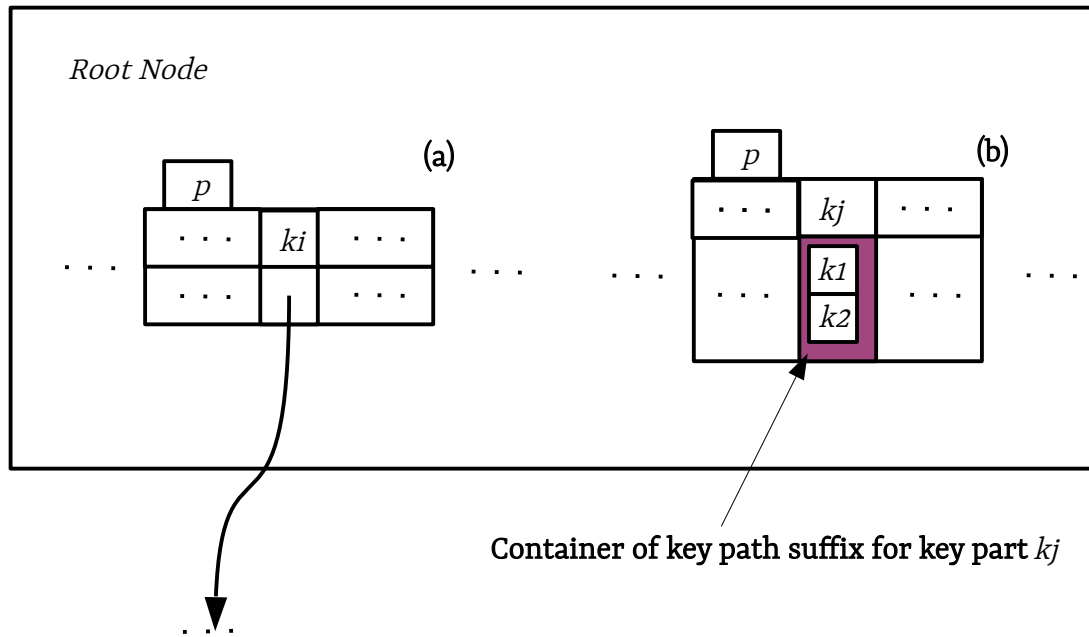
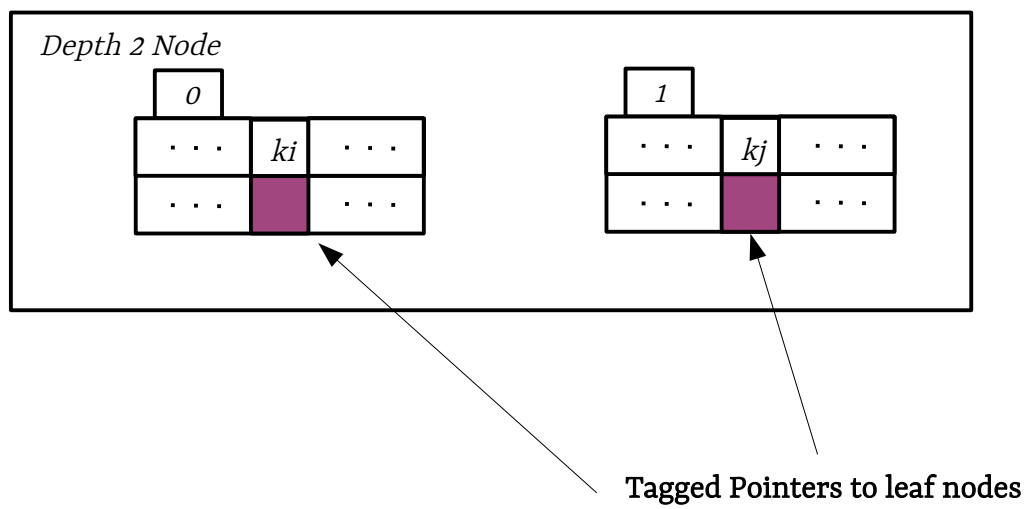about hash capacity

Figure 4.2: Depth 3 Node

Figure 4.3: Depth 2 Node

4.3.2 Node Expansion

Node Expansion text. Node Expansion text.Node Expansion text.Node Expansion text.Node Expansion text.

4.3.3 Virtual Nodes

Due to the fact that some key paths in Hypertrie are collapsed into a single node of depth d, we still need to maintain the concept that collapsed path still represents a tensor of the order $d-1$ with a single entry that evaluates to 1. Maintaining the concept becomes very important when we implement Hypertrie programmatically. In particular, when we want to perform slicing in the position of the collapsed path.

A straight forward solution is to return the container representation of the collapsed path (array, int value) from the slicing call. However, that contradicts with slicing function contract defined in the Hypertrie interface which states that slicing always return a child node. Also, following this approach will add extra code complexity in the context of handling the different types of returned values from slicing. It will also force us to define a different behavior for each returned type from slicing resulting in substantial amount of code redundancy.

A clean solution is to encapsulate the defined key path containers in a special type of Hypertrie nodes. Those nodes realize all the methods defined in the Hypertrie interface. As a result, we treat slicing operations uniformally. I call these wrappers **virtual nodes** to discriminate them from the actual Hypertrie nodes which I call **concrete nodes**.

Back to the previous sub section 4.3.1, I defined two types of key chain static containers. Concretely, a static array of length 2 representing the key suffix of compressed edges in the root node. A tagged pointer that bounds the key suffix (a single key part) of compressed edges in internal depth 2 nodes. Hence, we now have two variants of nodes for each depth (except for the root node). Concrete nodes and virtual nodes.

In the original Hypertrie implementation, slicing method gives enough information about the child node being returned from its execution as shown in listing 4.4. As we conceptually expanded the set of Hypertrie nodes to include virtual nodes, the pointer to a child node of a certain depth returned from a slicing must carry information about the nature of the node being concrete or virtual.

Listing 4.2: Slicing method signituar defined for node where d = *depth* in original Hypertrie

```cpp
#include <array>

template<int depth>
class hypertrie_node {
        ....
        template<typename key_part_type>
        using SliceKey = std::array<std::optional<key_part_type>, depth>;
        ....
        template<int slice_depth>
        hypertrie_node<slice_count>* slice(SliceKey slice_key) {
                ....
        }
        ....
}
```

To accomplish this, we again use tagged pointers. This time, the pointer holds only a memory address of type `void *`. However, the tag here is used to distinguish between the referenced node being concrete or virtual. Accordingly, we can cast the the pointer value to the appropriate type based on the tag value. It becomes the responsibility of the slicing method to create and return that pointer properly. Listing 4.3 shows the structure of the tagged pointer I developed to reference different node types in our space-friendly Hypertrie.

Listing 4.3: Node pointer structure

```cpp
template<typename VirtualNodePtr, typename ConcreteNodePtr, int alignedTo>
class NodePointer {
        private:
                static const intptr_t tagMask = alignedTo − 1;
            static const intptr_t pointerMask = ~tagMask;

        public:
                static constexpr int VIRTUAL = 1;

                static constexpr int CONCRETE = 0;

        protected:
                union {
                        void *asPointer;
                        uintptr_t asBits;
                }

        public:
        inline NodePointer(VirtualNodePtr ptr) {
                // Safer to clear the tag as we set a fresh tag in the setter method
                clearTag();
                asPointer = ptr;
                asBits |= VIRTUAL;
        }

        inline void clearTag() {
                asBits &= pointerMask;
        }


                inline NodePointer(ConcreteNodePtr ptr) {
                clearTag();
                asPointer = node;
                asBits |= CONCRETE;
        }

        /**
        * @param ptr it is already a tagged pointer
        */
        inline NodePointer(void *ptr) {
                asPointer = ptr;
        }

        inline NodePointer() {
                asPointer = nullptr;
        }
}
```

Listing 4.4: Slicing method signituar defined for node where d = *depth* in the space-friendly Hypertrie

```cpp
#include <array>
```

```
template<int depth, bool virtual>
class hypertrie_node {
  ...
  template<typename key_part_type>
  using SliceKey = std::array<std::optional<key_part_type>, depth>;
  ....
  template<int depth>
  using NodePointer =
    TaggedPointer<hypertrie_node<depth, false>, hypertrie_node<depth, true>>;
  ....
  template<int slice_depth>
  NodePointer<slice_depth> slice(SliceKey slice_key) {
        ....
  }
  ....
}
```

4.4   Algorithms

The change in the internal nodes representations in the compressed version of Hypertrie leads to change in how we define its behavior. Hence, the new realization add the logic necessary to consider the compressed paths containers. At the same time, the logic has to have the same contract defined in the original Hypertrie interface. Following that approach will allow us to integrate the new Hypertrie implementation into the Tentris system efficiently.

Next, I list the main algorithms that realize the core behaviors of Hypertrie when the space reduction approach is applied. The great proportion of my work was on expanding the code base of Hypertrie project to adapt the key path compression approach including the implementation of the accompanying algorithms[2].

4.4.1   Key Retrieval

A basic operation in Hypertrie is to check if a given $key \in N^d$ represents a path from a $Node_d$ to a leaf node where d is the node depth. In other words, if key is a non-zero entry in the tensor $T = h^{-1}(Node_d)$. Due to the presence of key path containers in internal nodes, the logic should be expanded to consider the children paths of both the compressed and non-compressed edges in each node.

s the redefini-
of d as I did
earlier

a definition

---

[2]In this section, I list only the main algorithms

Input: *node*3: a depth 3 node, *depth*: current node depth, *key*: array of key parts
$k_i \in K_i$ of size 3

Output: a boolean if a key represents a path in Hypertrie

1 node3_key_retrieval(*node3, depth, key*)

2 begin

3 $p_{min} \leftarrow$ minCardPos(*node3*)

4 $k_i \leftarrow key[p_{min}]$

5 $arr_i \leftarrow HTComm_{p_{min}}[k_i]$

6   **if** $arr_i \,! = NULL$ **then**

7 $l \leftarrow 0$

8 $c \leftarrow 0$

9   **while** $l < depth$ **do**

10   **if** $l \; == \; p_{min}$ **then**

11 $l = l + 1$

12   **else**

13   **if** $arr_i[c] \,! = \; key[l]$ **then**

14   **return** *false*

15   **end**

16 $l = l + 1$

17 $c = c + 1$

18   **end**

19   **end**

20   **return** *true*

21   **end**

22 $child_i \leftarrow HT_{p_{min}}[k_i]$

23   **if** $child_i \,! = \; NULL$ **then**

24 $subkey \leftarrow < 0, 0 >$

25 $l \leftarrow 0$

26 $c \leftarrow 0$

27   **while** $l < depth$ **do**

28   **if** $l \; == \; p_{min}$ **then**

29 $l = l + 1$

30   **else**

31 $subkey[c] \leftarrow key[l]$

32 $l = l + 1$

33 $c = c + 1$

34   **end**

35   **end**

36   **return** node2_key_retrieval(*child_i*, 2, *subkey*)

37   **else**

38   **return** *false*

39   **end**

40 end

Algorithm 1: KEY RETRIEVAL IN THE ROOT NODE

Input: $node2$: a depth 2 node
$depth$: current node depth
key: array of key parts $k_i \in K_i$ of size 2 representing a sub key
Output: a boolean if a key represents a path in Hypertrie
1 node2_key_retrieval($node2$, $depth$, key)
2 begin
3 $p_{min} \leftarrow$ minCardPos($node3$)
4 $k_i \leftarrow key[p_{min}]$
5 $ptr_i \leftarrow HTp_{min}[k_i]$
6   **if** $ptr_i == NULL$ **then return** false
7 $(value_i, tag_i) \leftarrow ptr_i$
8 $next\_pos \leftarrow (p_{min} + 1) \% 2$
9   **if** $tag_i == INT\_TAG$ **then**
10   **return** $value_i == key[next\_pos]$
11   **else**
12 $subkey \leftarrow < key[next\_pos] >$
13 $child_i \leftarrow$ getPointer($value_i$)
14   **return** node1_key_retrieval($child_i, 1, subkey$)
15   **end**
16 end

Algorithm 2: Key Retrieval in depth 2 Nodes

Input: $node1$: a depth 1 node
$depth$: current node depth
key: array of key parts $k_i \in K_i$ of size 2 representing a sub key
Output: a boolean if a key represents a path in Hypertrie
1 node1_key_retrieval($node1$, $depth$, key)
2 begin
3   **return** set of edges in $node1$ holds the key part $key[0]$
4 end

Algorithm 3: Key Retrieval in depth 1 Nodes

4.4.2 Key Insertion

Similar to other tree-based data structures, the key insertion operation is recursive. Due to the fact that nodes at certain depth enjoys a unique internal structure, the algorithm for adding keys will change depending on the target node in the recursion sequence. To achieve key insertion, two auxiliary constructs must be implemented. First, the algorithm should utilize a mechanism that keeps track of the processed key parts so far for each key path in hypertrie. A way to maintain such state information is to use a tuple $processed \in \mathbb{B}^3$, such that:

$$\forall i \in \mathbb{N}_3, processed[i] = \begin{cases} 1, & \text{key part at position } i \text{ is processed} \\ 0, & \text{otherwise} \end{cases}$$

Utilizing the state tuple $processed$ while processing a hypertrie node, we know the *sub key*, the remaining key parts to be added. Hence, the state tuple is updated as we proceed in the construction of the key path during insertion; i.e. processing more key parts.

For that, a couple of utility functions are used to map the main key to sub keys. That is, they map key part indices of the inserted key to dimensions represented by the succeeding child node and vise versa. By that, we know which key part should be inserted at which position. $pos2ids$ maps node positions to key part indices, and $id2pos$ maps key part indices to node positions. Both functions are defined in the following:

$$pos2id(pos, processed) = pos + |\{e|e \in \text{dom}(processed), e \leq pos, processed[e] = 1\}|$$

$$id2pos(id, processed) = id - |\{e|e \in \text{dom}(processed), e \leq id, processed[e] = 1\}|$$

The second requirement to achieve efficient insertion similar to the original hypertrie insertion method is to preserve the concept of pointing to equal child nodes only once. This is done by setting up a dictionary $processed\_nodes : \mathbb{B}^3 \to node$ for each key insertion call. $processed\_nodes$ maps instances of the state tuple $processed$ to inform which hypertrie nodes were already created for which combination of dimensions.

Input: *node3*: a depth 3 node, *depth*: current node depth, *key*: array of key parts
1 insert(*node3, depth, key*)
2 begin
3 $\quad$ $processed \leftarrow\, <0, 0, 0>$
4 $\quad$ $processed\_nodes \leftarrow \{\}$
5 $\quad$ node3_insert(*node3, depth, key, processed, processed_nodes*)
6 end

Algorithm 4: KEY INSERTION MAIN METHOD

Input: $node3$: a depth 3 node, $depth$: current node depth, key: array of key parts, $processed$: state of processed key parts, $processed\_nodes$: dictionary of processed nodes

1 node3_insert($node3, depth, key, processed, processed\_nodes$)

2 begin

3   **foreach** key_pos in $\{e | e \in dom(processed), processed[e] = 0\}$ **do**

4 $key\_part \leftarrow key[key\_pos]$

5 $pos \leftarrow id2pos(key\_pos, processed)$

6 $next\_processed = processed$

7 $next\_processed[key\_pos] \leftarrow 1$

8 $child\_ptr \leftarrow$ get($node3, pos, key\_part$)

9 $(child, tag) \leftarrow child\_ptr$

10   **if** $child$ is $empty$ **then**

11 $arr \leftarrow [0, 0]$

12   **for** i in $[0, 1]$ **do**

13 $arr[i] = key[pos2id(i, next\_processed)]$

14   **end**

15 $HTComm_{pos}[key\_part] = arr$

16   **else if** $tag == NON\_COMPRESSED$ **then**

17   node2_insert($child, 2, key, next\_processed, processed\_nodes$)

18   **else if** $tag == COMPRESSED$ **then**

19 $new\_node \leftarrow$ create a new concrete hypertrie child node of depth 2

20 $HT_{pos}[key\_part] = ptr(new\_node)$

21   node2_insert($new\_node, 2, key, next\_processed, processed\_nodes$)

22 $reconstruced\_key \leftarrow < 0, 0, 0 >$

23 $arr \leftarrow HTComm_{pos}[key\_part]$

24   **for** i in $[0, 1, 2]$ **do**

25   **if** $i == key\_pos$ **then**

26 $reconstruced\_key[i] = key\_part$

27   **else**

28 $reconstruced\_key[i] = arr[id2pos(i, next\_processed)]$

29   **end**

30   **end**

31   erase $HTComm_{pos}[key\_part]$

32   node2_insert($new\_node, 2, reconstruced\_key, next\_processed, processed\_nodes$)

33   **end**

34   **end**

35 end

Algorithm 5: Key insertion for depth 3 node

Input: *node*2: a depth 2 node, *depth*: current node depth, *key*: array of key parts, *processed*: state of processed key parts, *processed_nodes*: dictionary of processed nodes

1 node2_insert(*node2, depth, key, processed, processed_nodes*)
2 begin
3 $processed\_nodes[processed] \leftarrow node2$
4   **foreach** *key_pos in* $\{e | e \in dom(processed), processed[e] = 0\}$ **do**
5 $key\_part \leftarrow key[key\_pos]$
6 $pos \leftarrow id2pos(key\_pos, processed)$
7 $next\_processed \leftarrow$ cloning *processed*
8 $next\_processed[key\_pos] \leftarrow 1$
9 $child\_ptr \leftarrow HTpos[key\_part]$
10 $(value, tag) \leftarrow child\_ptr$
11   **if** *child_ptr is empty* **then**
12 $next\_key\_part = key[pos2id(0, next\_processed)]$
13 $HT_{pos}[key\_part] \leftarrow$ tagged pointer holding *next_key_part* integer
14   **else**
15   **if** $tag = IS\_POINTER$ **then**
16 $child\_node \leftarrow$ node to which address in *value* points
17   node1_insert(*child_node*, 1, *key, next_processed, processed_nodes*)
18   **else if** $tag = IS\_INT$ **then**
19 $created\_node \leftarrow processed\_nodes[next\_processed]$
20   **if** *created_node exists* **then**
21 $value \leftarrow$ hold a pointer to *created_node*
22 $tag \leftarrow IS\_POINITER$
23   **else**
24 $reconstruced\_key \leftarrow <0, 0, 0>$
25   **for** *i in* $[0, 1, 2]$ **do**
26   **if** $processed[i] == 1$ **then** $reconstruced\_key[i] = key[i]$
27   **else if** $i == key\_pos$ **then** $reconstruced\_key[i] = key\_part$
28   **else** $reconstruced\_key[i] = value$
29   **end**
30   **end**
31 $new\_node \leftarrow$ create a new concrete hypertrie child node of depth 1
32 $HT_{pos}[key\_part] \leftarrow$ tagged pointer holding pointer to *new_child*
33   node1_insert(*new_node*, 1, *key, reconstruced_key, next_processed, processed_nodes*)

34   **end**
35   **end**
36   **end**
37 end

Algorithm 6: Key insertion for depth 2 node

Input: $node1$: a depth 1 node, *depth*: current node depth, *key*: array of key parts, *processed*: state of processed key parts, *processed_nodes*: dictionary of processed nodes

1 node1_insert(*node1, depth, key, processed, processed_nodes*)
2 begin
3 $\quad$ $created\_node \leftarrow processed\_nodes[processed]$
4 $\quad$ **if** *created_node does not exist* **then**
5 $\quad\quad$ $\mid$ $processed\_nodes[processed] = node1$
6 $\quad$ **end**
7 $\quad$ $key\_part \leftarrow key[pos2id(0, processed)]$
8 $\quad$ add *key_part* to the set of key parts (edges) held by $node1$
9 end

Algorithm 7: KEY INSERTION FOR DEPTH 1 NODE

Input: $node1$: a depth 1 node, *depth*: current node depth, $key, reconstruced\_key$: array of key parts,
$\quad\quad$ *processed*: state of processed key parts, *processed_nodes*: dictionary of processed nodes

1 node1_insert(*node1, depth, key, processed, reconstruced_key, processed_nodes*)
2 begin
3 $\quad$ $created\_node \leftarrow processed\_nodes[processed]$
4 $\quad$ **if** *created_node does not exist* **then** $processed\_nodes[processed] = node1$
5
6 $\quad$ $key\_part_1 \leftarrow key[pos2id(0, processed)]$
7 $\quad$ $key\_part_2 \leftarrow reconstruced\_key[pos2id(0, processed)]$
8 $\quad$ add key_part and key_part_2 to the set of key parts (edges) held by $node1$
9 end

Algorithm 8: KEY INSERTION FOR DEPTH 1 NODE INCLUDING THE INSERTION OF THE RECONSTRUCTED KEY

4.4.3 Slicing

]

5

Evaluation

How much data compression does the Hypertrie achieve due to applying the space reduction approach presented earlier? Does the new data structure compromise the overall efficiency, more specifically, the query processing speed? On the one side, this chapter describes the space-friendliness of Hypertrie by presenting the compression ratio achieved w.r.t. the original Hypertrie after the bulk loading with different RDF data sets is done for both Hypertrie variants. On the other side, we have a look at how the space enhanced Tentris can still maintain its proven efficiency. For that, I ran a a series of benchmark experiments on real RDF data and queries.

As this work's effort is mainly toward enhancing the Tentris system in the context of space cost, all the experiments consider only the original Tentris' Hypertrie as the reference for comparison opting out other triple store systems. The experimental setup is described in section 5.1. In the succeeding section 5.2, the results are presented.

5.1 Experimental Setup

5.1.1 Data Sets and Queries

I used three different RDF data sets to assess the performance (load time/ size) of the compressed Hypertrie. Concretely, I used Semantic Web Dog Food SWDF (372K triples) as well as the English DBpedia 2015-10 (681 M triples) as real-world data sets. I also chose the 1 billion-triple synthetic data set from WatDiv [2] (see Table 5.1). All the three data sets were used during the size/load speed assessment. However, only SWDF and DBpedia data set were incorporated into the benchmarks.

5.1.2 Test Environment

I used the server machine "Geiser" provisioned and maintained by the data science research group at Paderborn university to run all the experiments. The server machine has two Xeon E5-2683 v4 CPUs and 512 GB of RAM. Geiser runs Ubuntu 18.04.4 LTS 64-bit on Linux Kernel 4.15.0-112-generic with Python 3.6.9 and OpenJDK Java 11.0.8 installed[1]. The benchmark tool and the Tentris triple stores (compressed/ non-compressed) were installed locally on a single

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[1]The server specification may change in the future. However, by the time of the document submission date, the spec mentioned here holds.

Dataset	#T	#S	#P	#O	Type
SWDF	372 k	32 k	185	96 k	real-world
DBpedia	681 M	40 M	63 k	178 M	real-world
WatDiv	1 G	52 M	186	92 M	synthetic

Table 5.1: RDF datasets which were used throughout the evaluation phase. Numbers of distinct triples (T), subjects (S), predicates (P) and objects (O) of each dataset. Additionally, Type classifies the datasets as real-world or synthetic.

server instance to avoid network latency. Data sets held in N-Triples format (`.nt` files) were uploaded to the server and stored on disk.

5.1.3 Bulk Loading

Generally, bulk loading is the activity of loading a large volume of data into a data storage system (database, triple store, etc.) mainly in one call, thus in a relatively small amount of time. There are many use cases in which bulk loading could apply; one example is data import/ export. In this work context, bulk loading is used to load an entire RDF graph into Tentris; thus, it can calculate its memory footprint and the time to load. During the bulk loading experiment, all the three RDF datasets (Sec. 5.1.1) were involved for each variant of the triple stores (compressed, non-compressed). I also expanded the load test to check the space utilization performance of the compressed Hypertrie as the dataset scales. For that, I used generated WatDiv synthetic datasets with increasing scale factors: [10, 100, 1000, 10000]. The bulk loading tests' results were calculated using a set of utility methods inside the Tentris project. Those utilities rely on built-in functions inside C++ language, which interface low-level system services and data to their clients.

Calculating Size

There are many methods to calculate memory utilization of processes or data structures residents. Each method fits a set of use cases and they can vary with accuracy. For the case at hand, I relied on the `proc` virtual file system in the server instance that ran the experiments. Given that, "The `proc` filesystem is a pseudo-filesystem which provides an interface to kernel data structures. It is commonly mounted at `/proc`. Typically, it is mounted automatically by the system, but it can also be mounted manually" [2]. The content of the files inside `proc` has mostly the status of *read-only* as the files are written by the kernel. For each process (with a process id *pid*) currently running in the linux system, there exists a subdirectory `/proc/[pid]`. Underbeneath each subdirectory `/proc/[pid]`, there are a set of files and subdirectories disclosing meta-information about process with *pid*. The file `/proc/[pid]/status` provides status information about the process including various memory consumption information. The information are provided in a human-readable fashion.

After a single bulk load experiment is finished, Tentris access the corresponding status file; i.e. `/proc/slef/status` where the magic symbolic link `self` is automatically evaluated to Tentris' process ID. The field `VmRSS` inside `Status` file provide the current memory utilization of the resident Tentris process including the sizes of resident anonymous memory, resident file mappings and resident shared memory.

[2]https://man7.org/linux/man-pages/man5/proc.5.html

Calculating Load Time

A simple tic-toc approach is adopted to calculate the load speed of RDF graphs into Tentris. More specifically, two start and end time flags are setup immediately before and after the actual loading of keys. The difference between the time checkpoints is then calculated to capture the load speed.

The time checkpoints calculation utilized `steady_clock` instead of `system_clock`. As `system_clock` talks periodically to machine clock to correct itself, it might make minor timing mistakes. As `steady_clock` is independent from the system clock, thus providing more reliable timing insights.

5.1.4 Benchmark Execution

For executing the benchmark, I used the generic SPARQL benchmark execution framework IGUANA v3.0.0-alpha2 [7]. IGUANA is a benchmark suite for executing benchmarks. It takes a benchmark, namely a data set and a possible list of SPARQL queries/updates, as input. Then it simulates a SPARQL user that pushes a series of queries repeatedly in a stress test scenario to a SPARQL endpoint where the next request is sent immediately after returning the last response. IGUANA can execute both synthetic benchmarks and benchmarks based on real data. As part of its execution, the suite returns information on the respective triple store's different behavioral aspects, such as query processing speed for each query and the query result's size. The framework enables different benchmark execution options and fashions (measure performance of triple stores under updates, parallel user requests, etc ...). As Tentris provides an HTTP-based SPARQL interface, I used the HTTP-based benchmark with one user as a benchmark setup. The suite returned the average response time for each query, and the query result's size to consider later for comparison.

5.2 Results
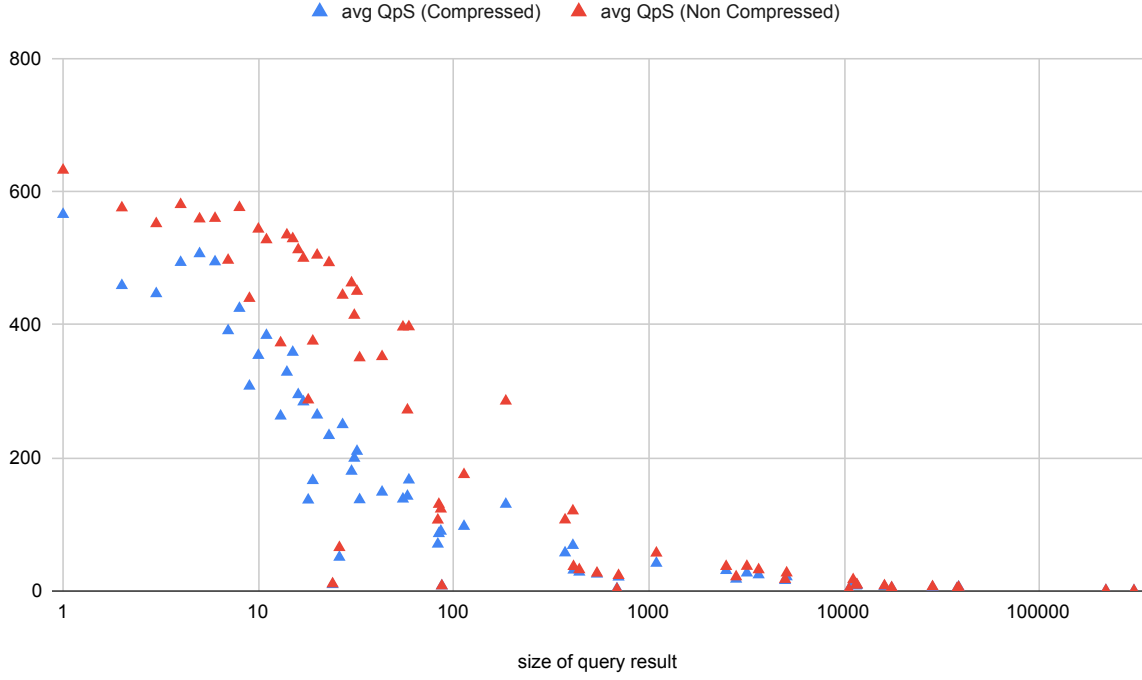
(a) SWDF
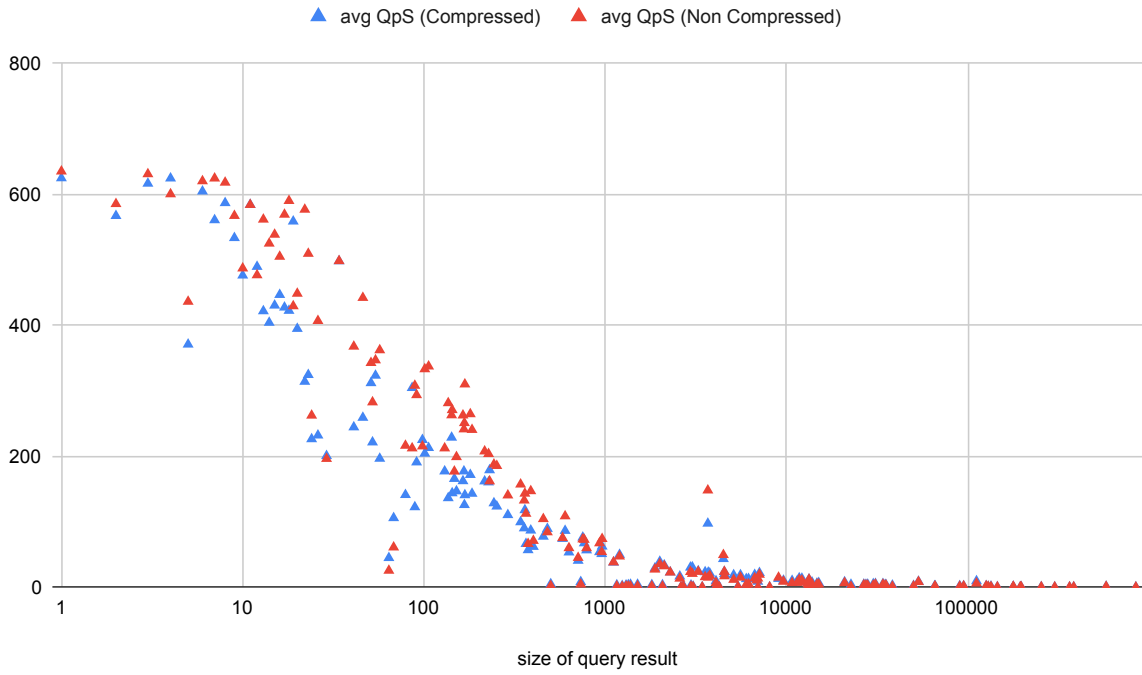
(b) DBpedia

Figure 5.1: Average Queries per Second (avg QpS) by size of the query result considering the two variants of Hypertrie implementations (compressed, non-compressed). Figure 5.1a shows the results of benchmarking against SWDF dataset and associated queries, while figure 5.1b shows the results of benchmarking against DBpedia dataset.

6

Conclusion

Parallelizeation Different types of containers Different types of compression methods

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