Problem 1: Recurrent Neural Operator - one dimensional visco-plasticity (100 points).

Consider the following unit-cell problem that is governed by:

$$
\begin{aligned}
&\epsilon(x,t) = \frac{\partial u(x,t)}{\partial x} && \text{Kinematic relation} \\
&\frac{d\sigma(x)}{dx} = 0 && \text{Equilibrium} \\
&\sigma(x,t) = E(x)\epsilon(x,t) + v(x)\frac{\partial u(x,t)}{\partial t} && \text{Constitutive relation} \\
&u(x,0) = 0,\ \dot{u}(x,0) = 0, && \text{Initial condition} \\
&u(0,t) = 0,\ u(1,t) = \bar{\epsilon}(t) && \text{Boundary condition}
\end{aligned}
$$

where $E(x)$ is Young's modulus, $v(x)$ is viscosity. We consider a 3 phase composites with $E(x)$ and $v(x)$ piecewise constant functions with 3 different values, as depicted in Fig. 1.

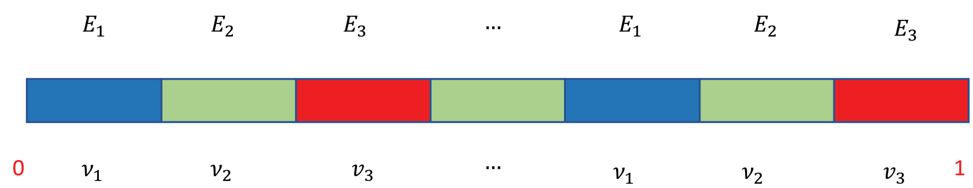

Figure 1: Viscoelastic material with 3 phases

(a) Define the input and output for a macroscopic constitutive model based on the provided unit cell problem.

(b) You are provided with a data set ("viscodata_3mat.mat") containing 400 samples of macroscopic stresses and strains obtained by solving the unit cell problem. Design, build, and train an Recurrent Neural Operator to learn the macroscopic constitutive model of the composite using training data.

(c) Determine the minimum number of internal/hidden variables required in the RNO to fully learn the constitutive relation. Justify your choice.